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Use of mathematical measurement in improving the accuracy (reliability) & meaningfulness of performance measurement in businesses & organizations

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Abstract

A review of the academic and best practice literature indicates that while there is no shortage of publications on performance measurement systems/performance measurement and management system (PMS&PMMS) in the business and organizational perspectives, but there is a dearth of research on how to measure activities/things more precisely and how they contribute to performance using an appropriate measurement theory. Key findings of the weaknesses of PMS&PMMS are: poor definitions of terms, too many terms are used, terms are used interchangeably or tautologically, and, no clear differentiation of measurement and performance measurement (M&PM). These show that the current ways in measuring activities/things and their performance are not reliable and meaningful. To overcome these weaknesses, using mathematical formulation of the hard and soft (social) sciences, a theoretical foundation of the measuring attributes (measures, metrics, indicators) of M&PM is ensconced and empirically tested using a numerical analysis. The key of this theory is accuracy (reliability) and meaningfulness of M&PM.

Keywords: Measure, Metric, Indicator, Measuring attributes, meaningfulness of measurement, Performance measurement, performance measurement and management, mathematical measurement theory

Paper type: Theoretical & research

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1. Introduction

Measurement and performance measurement (M&PM) of activities and things are important for any discipline—be it in the accounting, economic, governmental, management, social science, engineering or medical field. In the accounting, economic governmental, and management field—collectively known as *businesses and organizations* (although in some instances also include non-profit purpose firms, e.g., charities and health-care institutions) M&PM are becoming to use the alternative performance measurement systems (PMS), The growing number of PMS, the contemporary performance measurement and management system (PMMS) [1], and its components—the measuring attributes of an organization, is encouraging in terms of practical significance and academic research. A *measure*, *metric* and an *indicator* in the PMS context are collective, termed by [26] as *measuring attributes* as they possess specific attributes of measurement. These attributes are vital to a measuring system as they are the ones that measure activities/things and provide valuable information on their performance within an organization, and without them, nothing can be measured and managed.

PMS were introduced onto the workplace in the late 1960/early 1970 to replace accounting measures, and the accounting way of measuring performance because many practitioners expressed a general dissatisfaction with the traditional backward-looking accounting-based measurement system (AMS) (see [68], [69], [84], [73], [74], [24], [14], [16], [72], [128], [130], [145], [136], and [10]). The PMS is still largely based on the AMS control concept and is not a formal system but is a collection of many forms of measures, metrics, indicators, methods and systems of measurement and performance measurement. Academic research onto PMS follows, some twenty years later, i.e. from late 1980/early 1990 onwards (see [104], [105][2], [14], [16], [82], [13], [132]). More recently (from mid-2000) research into PMMS appears where a more strategic role in measurement is envisaged (see [32], [91], [132], [118]) however it is still relying on the accounting and management control system.

Thus, in spite of nearly 30 years of research, many essentials of M&PM have not been addressed. [104] conducted a literature review to examine the applications of PMS, measures, and

[1] The abbreviations, PMS and PMMS are used in the singular and plural contexts.
[2] This paper is republished in the same journal in original form in 2005.

metrics within an organization, and managed to identify gaps of the various research reviewed, and proposed a research agenda for the future research for PMS, in particular, its measures and metrics. [16] reviewed the literature on the implementation of PMS, and discovered that there is a dearth of research into the success and failure of the implementation of performance measures, suggesting that there are problems and difficulties in the implementation of performance measurement, which in their view, are just beginning to be recognized. [103] examined the evolution of performance measurement research developments from 1995 to 2005, and one of their key findings was the lack on knowhow to develop measures of unconventional things that are becoming more and more apparent today, for instance, to measure intangible as well as tangible assets for external disclosure as well as internal management?

Theoretical work on M&PM has been limited. [48] found it intriguing that out of 2,240 articles pertaining to business PMS (BPMS) only seventeen offered definitions of BPMS, and that the literature exhibits a diversity on the subject with no coherent explanation of the purpose, function and use of BPMS. In addition, they found that there is a lack of discussions about the measuring attributes such as measures, metrics or indicators by authors that have provided these definitions. [14] reviewed the literature and developed a framework for a PMS consisting of appropriate measures and metrics, and then used this framework for an empirical study in measuring performance. [27] conducted an extensive literature review on PMS in order to identify the required fundamental factors that can be considered necessary in conceptualizing an efficient and effective PMS that is appropriate in the modern organizational setting. [28] undertook a systematic review of the fundamentals of PMS/PMMS, and found that the field is overwhelmingly prescriptive, and terms were used injudiciously and sometimes overlap, are poorly defined, tautological or interchangeable.

These studies have provided us with sufficient insight for us to acknowledge that there is a serious lack of a theoretical foundation of M&PM; in particular the lack of properly defined measures, metrics or indicators and their uses within the confines of an organization. For instance, we would not be able to tell the difference between a measure and a metric, or how would a measure or indicator better used for measuring performance? And how are we going to measure qualitative attribute such as intangibles or customer satisfaction? Moreover the current PMS/PMMS is still overly reliance on the financial (accounting) convention in which the measured activities/things are not differentiated in terms of pure measurement and performance measurement, and worse still, measurement is defined in terms of costs (including real (actual), allocated, apportionment, and forecasted), and values (e.g. sales or replacement or market value).

This is because the field is highly prescriptive with few venturing into the theoretical (scientific, mathematical or/and logical) perspective but instead, measurement practitioners and researchers provide arguments for or against a particular measurement system using a wide array of terms (see [101], [106], [134], [103], [57], [109], [120], [26], [27], [28]), and hence it is necessary to provide a better understanding of the field by moving in the direction of mathematics as the theoretical aspect of measurement is essentially mathematics.

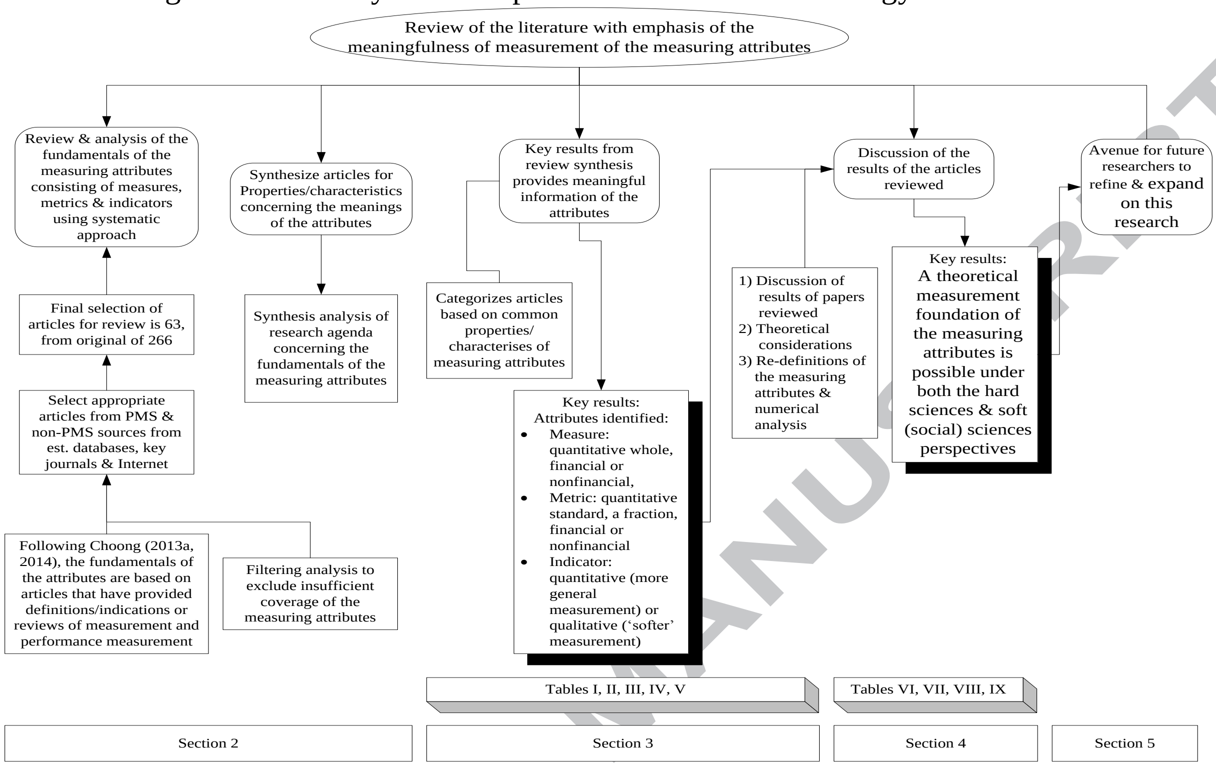
1.1. Purpose of the paper

The aim of this paper is intended to provide a more meaningful explanation of the properties of the measuring attributes for reasons as explained in the previous paragraph. Considering its prescriptive (normative) approach in the discussion of various topics and issues, and its close link with the fields of management, economics and accounting, the study of M&PM in the organizational context can be considered a branch of social science (see [135], [11], [21]). As social science is wide with diverse premises unlike the logical approach as sciences as the former has no clear definition of terms, meanings, and purposes of the roles and functions of a subject (research question, RQ), the research process requires an innovative and novel approach in assisting us in measuring objects in a meaningful way. This paper adopts a rigorous approach that demands the (1) identification of the possible meanings of the measuring attributes: measure, metric and indicator from a review of the literature and sources of best practices in the field of M&PM; (2) a review of existing mathematical measurement theories; and (3) a better understanding of the meaning of the measuring attributes from (1) and (2) would provide us with the tools and theories needed to advance the study of these attributes. The best practices are based on authoritative scientific publications such as the Institute of Electrical and Electronic Engineers (IEEE), and the Software Engineering Institute (SEI). The works of [45], [46]; [135]; and [27] are also important.

1.2. An overview of the paper.

The paper is organized as follows. In §2, a methodology using systematic review and synthesis analysis of the literature and best practices in the field of PMS&PMMS is required for us to understand what the measuring attributes are. §3 is concerned with the results of the review concerning the measuring attributes. Here, it is important to examine the attributes in how well they can fit in the M&PM of organizations, and their relationships with data (variables) in a formal manner. One approach that can help us with this examination is to present the model graphically as a path model. The results of the review of articles are in §4. Finally, §5 concludes this research. The steps (in summary form) for this research are presented in Figure 1.

Figure 1. Summary of the steps of the research methodology for this research



2. Methodology

In pursuit of the aim of this research posed above, the author attempted to conduct an innovative methodology consists of two steps: (1) synthesize the PMS&PMMS and non-PMS&PMMS literature in identifying, classifying, analysing, and interpreting the fundamental characteristics of the measuring attributes; and (2) from the identified fundamental characteristics of the measuring attributes, determination of the appropriate meanings of the attributes of M&PM is established.

A research synthesis requires us to conduct a trustworthy, orderly and ***systematic research***—how investigators ought to find, evaluate and integrate past research ([31]: 1), as well as the author's knowledge of the field developed over the years, in advancing the study of the research topic. Synthesis analysis directs us to review articles by following a list of specific steps to ensure that we can obtain the most relevant information with regard to a specific topic (subject) are obtained in an unbiased manner. Eventually, this ensures the fidelity, completeness and rigorous nature of the review ([54]: 116).

Following [135], [26], [27], [28]), the valid measuring attributes of PMS&PMMS are (1) measure; (2) metric; and (3) indicator, and hence the search keys are *measure*, *metric*, and *indicator* (altogether, three terms) are used to identify articles published between 1990 and 2012 (23 years) in specific business, management and science databases such as ABI/Inform ProQuest, EBSCO, Emerald Full Text, and Science@Direct, as well as the Internet using *Google scholar*. Searches for relevant articles were also made from the ten journals that make up about 97% of all publications pertaining to performance measurement (see [26])[3].

In reviewing the various articles, the RQ (research concept) is about the fundamental characteristics of the measuring attributes, and that the RQ should not be too narrow as it would make the conclusion of the review to be less definitive and less robust ([31]: 7). Following [26], [28]), the fundamentals of M&PM are based on PMS&PMMS articles that have provided definitions/indications or reviews of measurement and performance measurement. The search from five databases, 36 journals, and the Internet pertaining to PMS&PMMS produced 264 articles. A 'filter out process' using inclusion and exclusion criteria is based on [133]; [86] and [28]. In ensuring the selected articles are of upmost relevant and of high quality, exclusion is made for articles that made little/no reference to concepts or rely on weak methodologies or that are too narrowly focused such as industry-specific that may have issues relating to comparability and generalizability of findings/conclusions of their investigations. Thus 195 articles were filtered out and the final number of articles was reduced to 69. The full bibliography of the selected articles appears in Table I and the *Reference section.*

3. Results concerning the measuring attributes.

The results are analyzed in terms of two broad categories: articles search from the organizational (business, public and institutional) PMS&PMMS (they are grouped together since their prescription/description of M&PM are similar) and non-PMS&PMMS (the sciences, and social sciences) to provide a more objective description of terms and explanation of measurement. The article search produced fifty-eight articles from PMS&PMMS and eleven articles from non-PMS&PMMS. These make up a total of 69 selected articles. The selected articles and their coverage of the measuring attributes are presented in Table I:

Table I about here

[3] *Business Process Management Journal, Journal of Operations Management, Management Accounting, Administrative Science Quarterly, Management Accounting Research, International Journal of Operations & Production Management, International Journal of Productivity & Performance Management* (previously, *Work Study*), *Omega, International Journal of Production Research, Journal of Cost Management, Sloan Management Review, Strategic Management Journal, Measuring Business Excellence,* and *Accounting, Organization & Society*.

Table I reveals that about 83% (52+5=57 articles) of all selected authors use the term *performance measure/measure* while about 55% (38 articles) of all selected authors use the terms *performance indicator*/*indicator*, and only 35% (24 articles) of all selected authors use the terms *performance metric/metric*. Only three articles use the term *driver* when discussing PMS&PMMS (not shown in Table). For this paper, the terms *performance measure/measure, performance indicator/indicator, performance metric/metric* and *driver/performance driver/business driver/value driver* are simply referred to as *measure, metric, indicator* or *driver* respectively. The coverage, comprising of definitions/indications and expressions (descriptions, prescriptions and discussions) of the apparent measuring attributes identified, sorted by publication dates chronologically is presented in Table II:

Table II about here

Table II shows that a measure, metric, indicator or driver was described in many different ways. Most authors do not define them, and the few that do are from government agencies (e.g. [140], [141], [138]). Even then, the definitions/indications or expressions of a measure, metric, indicator or driver are not informative and lack clarity of their meanings leading to confusion between terms. Moreover, a measure, metric, indicator and driver are usually used interchangeably in many general discussions about measurement or performance measurement For instance, a metric is used interchangeably with a measure ([88], [104], [105], [100], [16]) (Table II), and an indicator is used interchangeably with a measure ([70], [132], [140]) (Table II).

There is a strong and persistence move by PMS&PMMS authors to discuss the nonfinancial aspect of the measuring attributes. They consider that a (1) measure can be in financial or non financial form ([80], [39], [77], [104], [44], [72], [67], [34], [149], [140], [141]); (2) metric can be in financial or nonfinancial form ([104], [74], [60], [16], [90], [34], [109], [149], [27]); and (3) an indicator can have financial or nonfinancial attribute ([39], [43], [100], [34], [46], [47], [27], [28]).

Since most of the discussions by PMS&PMMS authors about the measuring attributes were broad and not cohesive, a classification of the characteristics of the attributes in terms of common themes was made. The value of classification is associated with its ability to function as a heuristic device, which is useful for the interpretation of substance ([115]), and hence, classification allows us to order the systematic organization of a magnitude of possibilities with regard to the research agenda. Classification is all about simplification ([55]) and is suitable in a practical domain such as measurement where it used in diverse situations with different purposes, e.g., management, production, accounting and others. The synthesis analysis produces the following classification: (1) quantitative or/and qualitative criterion; (2) financial or/and non-financial form; and (3) whole number or/and fraction. The classification of the characteristics of these attributes based on common themes in terms of number of articles is presented in Table III.

Table III about here

While certain aspects of the information within Table III is useful, it offers little insight on how to provide a theory of the measuring attributes because there are too many inter-related characteristics, for example, an equivocal number of PMS&PMMS authors consider that a metric can be quantitative or qualitative in form. To be precise, we just cannot describe an attribute to be either here or there. This suggests that what the PMS&PMMS authors did were mere prescriptions of what measurement (performance measurement) ought to be and the prescriptions of these measuring terms sometimes overlap and are non-specific, tautological and/or interchangeable. Thus, we need non-PMS&PMMS sources to guide us in the appropriate articulation of terms and meanings of the measuring attributes of PMS&PMMS.

A driver cannot be a measuring attribute

First, the analysis of the term *driver* when discussing PMS&PMMS were not explicitly clear as the term could refer to *business drivers* (which is actually an input in cost form) (in the case of [67]) or *performance drivers/indicators.* To [74]), [72] and [67] a *driver* implies (or used interchangeably with) a *measure.* But a *business driver* is not a measurer of performance ([41]) since its meaning can be construed to be 'input' or 'indicators' rather than as an attribute for PMS&PMMS. The term *value driver* refers to any factor that enhances the total value created by an e-business (business) ([2]: 494) suggesting that a driver (of whatever type) cannot be a measuring attribute. Moreover, authoritative non-PMS&PMMS publications such as [63], [64], [119], [111], [135] have indicated that *measure, metric* and *indicator* are the three formal terms commonly used for measurement purposes. [135]), an authority of measurement theory states that a *measure, metric* and an *indicator* are subsets of measurement, and these three terms are distinct, and their differences are due to their complexities in applications. Findings in [26], [27]) affirm their views.

Second, Table III also reveals that not only the discussions of *measures, metrics* and *indicators* lack measurement theory; there were also negligible discussions on how measurement can be used to (1) measure activities; or (2) evaluate performance within an organization. Moreover, very few authors have described how these measuring attributes were constructed with an eye to precision or accuracy nor the relationship between objectivity and subjectivity of the terms.

Thus relying on the PMS&PMMS literature strongly indicates that there is a paradox in the understanding of the meaning and application of the measuring attributes of a PMS&PMMS. Hence a re-examination of their definitions/indications and expressions of the measuring attributes in Table III is needed in order to eliminate those that have provided unclear definitions/indications and expressions of these terms as well as those using these terms interchangeably. The re-evaluated selected articles from PMS&PMMS and non-PMS&PMMS (best practice and scientific institutions) authors are presented in Table IV:

Table IV about here

Table IV reveals that the characteristics of the measuring attributes are now clearer than before the re-evaluation. The re-evaluation provides a more rational basis in providing a more 'definitive' in meaning of a PMS&PMMS to assist practitioners and researchers.

Distinction between measurement and performance measurement

Having articulated that a driver cannot be a measuring attribute the next thing is to articulate the difference between measurement and performance measurement of a PMS. In non-scientific context, the literature does not really differentiate between M&PM. They are used interchangeably, for instance measures is often used to mean measurement. Obviously, these terms are distinct. In practice, we measure of exactness simply for custodial or stewardship purpose, i.e. how much does an executive chair cost, and this cost value is recorded to certify that it is part of a company's asset. That is, the measured cost is scientifically known as *measure* and there is a mathematical theory to attest for this. However to run a business successfully we need to place resources in producing valued-added goods, i.e. we operationalize the measurement process by identifying the measured costs and values to ascertain the effectiveness, efficiency and reliability of activities/things that are placed in the production process—to measure their improvement (deterioration) or performance. Thus a scientific PMS&PMMS provides a two-tier measurement perspective: measurement of exactness and measurement of the improvement (deterioration) of activities/things ([27]: 110)—i.e. there is a M&PM in place.

Normative definition of a *measure*

A *measure* is a quantitative whole, either in monetary (financial) form (e.g. sale values), dimension form (e.g. square meter) or unit form (e.g. production output), and that the choice of the measure depends on circumstances ([135], [26], [27]).

Normative definition of a *metric*

A *metric* is a quantitative standard in fraction form that can be in financial or nonfinancial form because only by assigning a number to the denominator of the fraction then only it can be a metric ([27], [28], [91]). Thus a metric is a comparison of two or more measures ([119]) and can be in the form of a measure deflated by another measure ([119], [121]), or it can be in the form of a measure deflated by a standard unit ([137]). Because of this, the *metric* is needed to be more clearly defined than a *measure.* A *metric* must be specially developed based on the performance objective (goal) relevant to the stakeholders (organization) concerned.

Normative definition of an *indicator*

An *indicator* can be quantitative or qualitative form for measuring things more generally (e.g. in summary form), and is considered to be less precise than a measure or metric, suggesting an indicator is used to infer things more generally than a measure or metric ([135]). Because of this, an indicator can be used to support measurement by measuring the ‘softer’ nonquantitative parameter such as people’s behavior, customer satisfaction and service quality ([27], [28]).

Conceptualization of the measuring attributes using path analysis

As the information contained in Table IV is still hard to comprehend, we can use the *sufficiency* and *justification* methodology of [28] in seeking common grounds in explaining the meaningfulness of the measuring attributes for consideration of a more theoretically-based model of M&PM. The finding will then be conceptualized by a path analysis and presented in graphical form pertaining to the measuring attributes. To provide the path analysis for the measuring attributes, this study employs the Galois Lattice Theory (GLT), and Formal Concept Analysis (FCA). The GLT and the FCA are often jointly used for a variety of theoretical (conceptual) applications; however in this research, the use of GLT is just to provide for the theoretical foundation of a set of classification schemata through the use of relation in connecting to a formal concept structures provided by the FCA (see [147]). To use the Lattice Theory to formally structuring the measuring attributes, the information in Table IV needs to be reclassified and presented in matrix form with the set of objects, O corresponding to the rows of the matrix, the attributes, set L corresponding to the columns of the table, and an indication of which objects have which attributes. Formally, it can be written as $I \subseteq O \times L$, and a concept for a context is defined to be a pair (O_i, L_i) such that $O_i \subseteq O$ *and* $L_i \subseteq L$. Thus we have two pairs of objects, quantitative and nonquantitative, and financial and nonfinancial concepts, which can be represented in Boolean value in cell (O_i, L_i) whenever object O_i, has value L_i where $L_{i,\ldots,n} = a$, b or c. The concept types in terms of the measuring attributes in matrix form are presented in Table V.

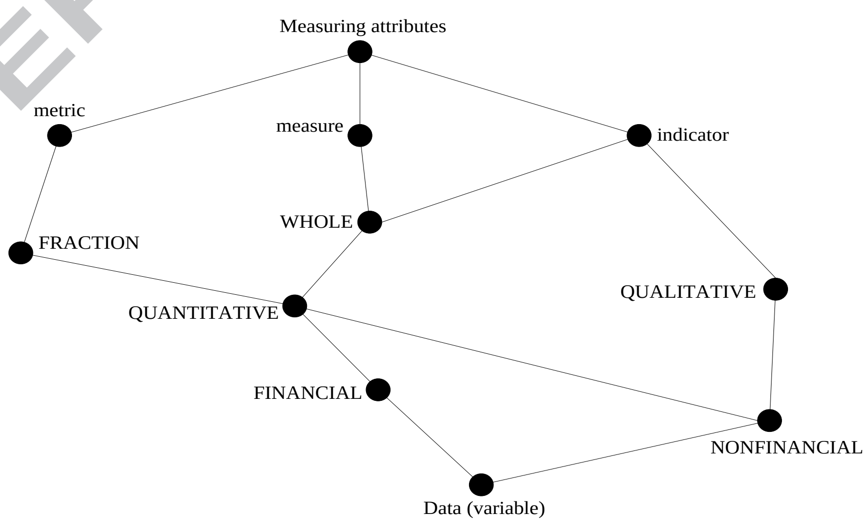
Table V about here

To generate the minimum lattice for classifying the measuring attributes in Table V, the FCA is applied. In FCA, application domains are organized and structured according to Galois connection (Galois Graphs). Given sets X and Y, with the set of objects of a formal concept—its *extent*, and the attribute set of a formal concept—its *intent*, and the two concepts, $N_1 = (X_1, Y_1)$ and $N_2 = (X_2, Y_2)$, $N_1 \leq N_2 \Leftrightarrow X_1 \supset X_2$, which means that the Galois connection have the equivalent relationship $N_1 \leq N_2 \Leftrightarrow Y_1 \subset Y_2$. The Galois connections are related to the concept of order and with its intent and extent purposes play an important role in Lattice theory ([32]: 93). In the measuring context:

O = {*FINANCIAL, NONFINANCIAL, QUANTITATIVE, NONQUANTITATIVE WHOLE, FRACTION*}

A = {*measure, metric, indicator*} and *R* is the specified matrix in Table V shows the resulting Lattice in which the attributes are in lower-case letters, and concept types in upper-case letters.

Figure 2. Lattice of the measuring attributes, constructed using GLT and FCA theories



In Figure 2, the objects are presented, each corresponding to the three attributes: measure, metric, and indicator. Since both a measure and metric are quantitative, they are of the same concept, same as the quantitative concept of indicator. The qualitative concept of indicator is unique and hence a separate node is needed. There are two nodes from the quantitative concept as they can be in financial or nonfinancial form, while there is only one node from the qualitative concept as it must be nonfinancial. Ultimately, all these nodes will conceptualize into the three measuring attributes comprising of a measure, metric or indicator. Although the outcome of the synthesis review is useful, but there is insufficient theoretical foundation of these attributes.

4. Discussion of the results of the articles reviewed

4.1. Discussion of results of papers reviewed

Although formal definitions as well as conception by a path analysis were made concerning the measuring attributes, linking them to mathematical measurement theory is a challenge. This is because M&PM in the business or organizational context is considered a social science which differs from mathematics. In social science definitions and concepts are by and large, not validated or empirically tested because arguments for and against a proposition are usually based on normative or prescriptive arguments. It is not uncommon to encounter situations in social sciences where individual interpretation can lead to ambiguous results, especially when the construct definition and/or nomenclature are inconsistent (see [29]). This issue is corroborated by the fact that the reviewed papers as discussed by various authors differ from the topic of measurement as typically discussed by scientists, mathematicians and psychologists on one hand, and that of measurement academics on the other. Few accounting or PMS/PMMS authors have described about the precision, accuracy, validity or the objectivity or subjectivity on how these measuring attributes were constructed and operationalized because they are more concerned with how to get things measured, i.e. they are generally more than willing to use a ‘good enough’ measure if it can provide useful information quickly ([90]) whereas for the scientists, mathematicians and psychologists, adapting and validating (empirical evidence) measures and metrics to address specific practical applications are more important than getting things done quickly.

Why is that PMS&PMMS authors still telling us in using an archaic form of measurement?—where the accuracy of measurement is at best, second-rate and the meaningfulness of measurement (in most times) is not envisaged. This has led to a second wave of criticisms of PMS&PMMS in which the suitability of performance measurement theories and practices for modern organizations are lacking (see [53],[58],[26],[27],[28],[118]), suggesting that the M&PM in organizations increases fear, reduces trust, promotes ‘command and control’ systems, diminishing employee engagement [118]. Surely this is not due to the lack of effort by PMS&PMMS authors as there is no shortage of publications and many kinds of theories (concepts) were discussed in the literature, and a plethora of performance measurement frameworks emerged since the late 1980 (see [28],[118]). [29] argued that many measurement authors focus only on ‘theoretical’ considerations and resist the temptation to conduct empirical tests; an issue commonly faced in social science as it lacks an empirical theme. We can see this is why? Altogether only 20% (14 out of 69 articles) of the selected articles have provided some kinds of empirical support (mainly case studies and surveys (questionnaires)) and another 12 articles that have discussed sufficiently about how to go about the implementation of metrics and indicators using illustrations or cases provided by past studies. These articles have stressed the importance of empirical work in PMS&PMMS studies but there is insufficient in-dept discussion of the validity and reliability of measurement in practice (i.e. the operationalization of measurement).

[118] offer an insight of the theoretical and empirical dilemma of PMS&PMMS literature. According to [118], the foundations of performance measurement still lie in the organizational and management control theories where the former consists of *Technical-control* (rational, planned, bureaucratic and structural elements of the organization) and *Social-control* (emergent, cultural and behavioral aspects of the organization) mechanisms, and the latter is derived from the management accounting literature. However, the PMS&PMMS literature has not offered the knowledge required for us to design organizational controls in managing processes, and as a result, PMS&PMMS authors fail to establish a clear relationship between performance measurement, management practices, employee engagement and performance. Notwithstanding this tale of woe, some positive developments took place in recent years. There have been attempts to relate measurement to statistical theory ([34],[70],[136],[145]) and using mathematical theory in defining indicators ([45],[46],[47],[149]). In the best practice perspective, [90], [91] have provided an objective definition of a metric and demonstrated how it can be developed in providing a practical insight in the ways M&PM can be applied in the business and organizational perspectives. In addition, [118] proposed a theoretical framework in relating to the interplay between performance measurement (technical-controls), performance management (social-controls), employee engagement and performance

These recent works by PMS&PMMS authors, although paltry in view of the large numbers of PMS&PMMS publications are encouraging. To the PMS&PMMS authors, the reliability of measurement and how measurement is validation is important. This view is consistent to the view of the scientists, mathematicians and psychologists. The PMS&PMMS authors are concerned that a theoretical foundation of M&PM in the social science perspective is also vital. This view is also consistent to the view of the scientists, mathematicians and psychologists. It appears that there is a convergent view of M&PM by PMS&PMMS authors and that of scientists, mathematicians and psychologists, and why a solid measurement theory is still lacking in the PMS&PMMS arena?

To answer this question, this paper uses an analogy. There is no doubt a large number of arrows being fired by PMS&PMMS authors but somehow, they all missed the target. But we cannot say that the works of PMS&PMMS authors are futile; in fact a few are relevant and interesting (which have been described earlier). Consequently, two valuable point of discussions are missing by these authors. First, the discussion of measurement in terms of the philosophy of science is hardly touched by the selected articles. In fact any mathematical measurement theory will have as much mathematics as philosophy, in particular on the epistemic of science with respect to validity, accuracy, exactness, and precision—that forms the foundation of measurement. Second, the root of measurement is hardly touched by PMS&PMMS authors, i.e. mathematics that is concerned with the fundamentals of measurement—i.e. defining measures, metrics and indicators with a view of validating them for accuracy to address specific applications. This sounds intimating as mathematical measurement theory is advanced mathematics and is beyond most academicians and practitioners in the business and organizational perspectives. Intimating or not we will need to go into mathematics for us to have a solid foundation of M&PM.

4.2. Mathematical theoretical considerations

Measurement theory is rooted in mathematics ([2]), and it can be viewed from two perspectives: in (1) hard sciences; and (2) soft sciences such as social sciences and psychology[4]. In both perspectives there is no one single measurement theory. To the scientists, physicists and engineers, measurement involves complex mathematics such as the *Quantum (mechanics) theory*, *Relativity theory*, *Dynamic systems theory*, *Statistical measurement theory*, *Control theory*,

[4] Whether psychology is a (hard) science or social science discipline is still debatable. Although psychologists must remain cognizant of the fact that their prime objects of study are human beings, psychology is a plethora of many other topics and subjects. Measurement wise, psychologists use various scientific and mathematical methods, many of which are statistically and mathematically sophisticated; in particular, safeguards against error–which minimize a host of sources of subtle bias, are *de rigueur* in studies of psychological treatment ([78]). However the theory in measurement that psychologists have initiated is not based on hard sciences, but differently, albeit still based on mathematics, and collectively these alternatives are known as the *modern measurement theory* (MMT). And the MMT has been adopted by almost all of the social science disciplines. Because of this, mathematical measurement theory as discussed in this paper will be categorized between hard sciences and social sciences including psychology.

Circuit theory, etc. To the social scientists and psychologists, the modern measurement theory (MMT) is generally referred to as the *Representational Measurement Theory* (RTM). However the term RTM needs clarification as it is not a single theory per se, but in fact, is a collection of at least three prominent measurement theories: (1) classical theory; (2) representational theory; and (3) operational theory (see [92], [93]; [59]).

According to [65], [66], measurement theory in the hard sciences (MT) belongs to the applied mathematics or theoretical informatics, and is a collection of theories that can be used to provide measurement to objects or events**.** To ([65]: 6), MT is the most fundamental theory—the mathematical representation of "the mechanical world view"—an epistemology to understand and analyze (control) every phenomenon (, e.g., economics, business, psychology, engineering, etc. by an analogy of mechanics. For this to happen, precision and the meaningfulness of measurement are important. To meet these requirements, *Dynamic System Theory* (DST) is first used.

It is well-known that DST starts from the following:

$$DST = \begin{cases} y(t) = g(x(t)), u_1(t), t) \text{ .... measurement equation} \\ \dfrac{dx(t)}{dt} = f(x(t), u_2(t), t),\ x_0 \text{ .... state equation} \end{cases}$$

(1)

Where: u_1, u_2 are external forces (or noises)

Quantum mechanics can be defined as ([143]):

$$\textit{Quantum mechanics} = \underbrace{\textit{measurement}}_{\textit{Born's quantum measurement}} + \underbrace{\textit{rule of time evolution}}_{\textit{Heisenberg's kinetic equation}}$$

(2)

Note that the term measurement appears in (1) and (2), thus measurement theory (MT) is constructed using (2) formulated:

$B(H) \equiv \{T / T$ is a bounded linear operator from a Hilbert space, $H\}$.

Define $\|T\|_{B(H)} = \sup\{\|Tv\|V \ : \ \|v\|V = 1\}$, and $(T_1T_2)(v) = T_1(T_2v)(\forall v \in V\}$ and T^* is defined by the adjoint operator of T. Note that it holds that $\|T^*T\|_{B(H)} = \|T\|^2_{B(H)}$ $(\forall T \in B(H))$. Thus $B(H)$ is a C^*-algebra. C^*-algebra refers to the theory of operator algebras commonly used in classical and quantum mechanics.

A triplet $\left[ƛ, N(\equiv \overline{ƛ}^{W^*}),\ B(H) \right]$ is an operator algebraic structure if $ƛ(\subseteq B(H))$ is the norm closed sub-$*$algebra of $B(H)$, and if $N(\equiv \overline{ƛ}^{W^*}(\subseteq B(H)))$ is the weak*-closure of $ƛ$ in $B(H)$. Thus, $ƛ$ and $\overline{ƛ}^{W^*}$ are C^*-algebras and that $N(\equiv \overline{ƛ}^{W^*})$ is both a C^*-algebra and a W^*-algebra (see [66]).

*W**-algebra is a special type of *C**-algebra, also known as a von *Neumann algebra* [143] is a *-algebra of bounded operators on a Hilbert space that is closed in the weak operator topology and contains the identity operator.

Thus with (2) as a kind of generalization of quantum mechanics, a *Pure Mathematical Theory* (PMT) is proposed (cf. [65], [66]):

$$PMT = \underbrace{\text{measurement}}_{\text{Axiom 1}} + \underbrace{\text{the relation among systems}}_{\text{Axiom 2}}$$

(3)

Therefore we can say (1) + (2) $\subset$ (3).

An element F in $B(H)$ is called self-adjoint if it holds that $F = F^*$. If there exists an element F_0 in $B(H)$ such that $F = F_0^* F_0$, a self-adjoint element F in $B(H)$ is induced as positive (i.e. $F \geq 0$). If $F = F^2$ holds, a positive element F is induced as a projection.

Let $ƛ^*$ be the dual Banach space of $ƛ$, i.e. $ƛ^* \equiv \left\{ p \middle| \begin{array}{l} p : ƛ \to C \text{ is a complex-valued continuous} \\ \text{linear function} \end{array} \right\}$,

and the norm $\|p\|_{ƛ^*}$ is defined by $\sup\{|p(F)| \mid \|F\|_{ƛ} \leq 1\}$ ([66]).

In quantum mechanics, in a quantum state system a **pure state** is a pure quantum state that cannot be written as a mixture of other states which is described mathematically by a vector in Hilbert space. All other states are called **mixed quantum states** of a quantum thermodynamical ensemble that mixes quantum and classical thermodynamical (or statistical) information, which is described mathematically by the density matrix in Hilbert space. The concept of state of a system is particularly useful as we do not need to refer to a concrete set of systems that coexist in space, and therefore, it can help us in describing things/objects or situation in more abstract terms.

First, the *mixed state space,* $\wp^m(ƛ^*)$ is defined:

$$\wp^m(ƛ^*) \equiv \{\rho \in ƛ^* \| \|\rho\|_{A^*} = 1 \text{ and } \rho(F) \geq 0 \text{ for all } F \geq 0\}$$

(4)

From (4), a *mixed state*, $\rho(\in \wp^m(\tilde{\lambda}^*))$ if it satisfies "$\rho = \theta p_1 + (1-0)\rho_2$ for some $\rho_1, \rho_2 \in \wp^m$ $(\tilde{\lambda}^*)$ and $0 < \theta < 1$" implies $\rho = \rho_1 = \rho_2$ is defined as a *pure state*, and the *pure state space*, $\wp^p(\tilde{\lambda}^*)$ is defined as [66]:

$$\wp^p(\tilde{\lambda}^*) \equiv \{\rho^p \in \wp^m(\tilde{\lambda}^*) \big| \rho^p$$

(5)

The normal class state space, $\wp^n(N_*)$:

$$\wp^n(N_*) \equiv \{\rho^n \in N_* \big\| \|\rho^n\|_{N_*} = 1 \text{ and } \rho^n \geq 0 \text{ (i.e. } \rho^n(T^*T) \geq 0 \text{ for all } T \in N)\}$$

(6)

Where $\wp^n(N_*) \equiv \rho^n(\in \wp^n(N_*))$ is called a normal (density) state.

Note that the quantum state system as described above plays an important role in statistical measurement theory.

In (3), the state ρ^p is always assumed to be pure ($\rho^p \in \wp^p(\tilde{\lambda}^*)$). For statistics, the distribution is always of a mixed state ($\rho^m \in \wp^m(\tilde{\lambda}^*)$). Thus, the *Statistical Measurement Theory* (SMT) is represented by:

$$SMT = \underbrace{\text{statistical measurement}}_{\text{Proclaim 1}} + \underbrace{\text{the relation among systems}}_{\text{Axiom 2}} \tag{7}$$

Where:

$$\text{Proclaim 1} = \text{Axiom 1} + \underbrace{\text{Statistical state}}_{\text{the probabilistic interpretation of the mixed state}}$$

(8)

Combining (7) and (8):

$$SMT = \underbrace{\text{PMT}}_{\text{Axiom 1+Axiom 2}} + \underbrace{\text{Statistical state}}_{\text{the probabilistic interpretation of the mixed state}}$$

(9)

Equation (9) indicates that there is no *SMT* without *PMT*, i.e. *SMT* is dependent on *PMT*, thus MT can be written as:

$$MT = \begin{Bmatrix} \text{Pure measurement theory } (PMT) \\ \text{Statistical measurement theory } (SMT) \end{Bmatrix}$$

(10)

Equation (10) and the series of mathematical equations as explained earlier can also be called the *General Dynamical System Theory* (GDST) or *Generalized Quantum Theory* (GQT) which is essentially concerned with **the precision and meaningfulness of measurement**. As seen, the PMT and SMT are derived from axioms, and they are now explained[5].

Axiom 1: *Measurement*

With any *system S*, a *C**-algebra $\hat{\lambda}$ can be associated in which measurement theory of that system can be formulated. A *state* of the system, *S* is represented by a pure state space, ρ^p $(\in \wp^P(\hat{\lambda}^*))$. An *observable* is represented by a *C**-observable *O* ≡ (*X*, $\Im$, *F*) in the *C**-algebra $\hat{\lambda}$. The measurement of an observable *O* for the system *S* with (or, in) the state ρ^p is represented by $M_{\hat{\lambda}}(O, S_{|\rho P|})$ in the *C**-algebra $\hat{\lambda}$. Also, a measured value $x(\in X)$ is derived by the measurement $M_{\hat{\lambda}}(O, S_{|\rho P|})$. The term *observable* usually means a real valued continuous function on a state space Ω [resp. a self-adjoining operator in *B*(*H*) where *B*(*H*) = *T:T* is a bounded linear operator from a Hilbert space *V* into itself].

Therefore, Axiom 1: Consider a measurement $M_{\hat{\lambda}}(O \equiv (X, \Im, F), S_{|\rho P|})$ formulated in a *C**-algebra $\hat{\lambda}$. Assumed that the measured value $x(\in X)$ is obtained by the measurement $M_{\hat{\lambda}}(O, S_{|\rho P|})$, then the probability that the $x(\in X)$ belongs to a set $\Xi(\in \Im)$ is given by $\rho^p(F(\Xi))(\equiv_{\hat{\lambda}^*} \langle \rho^p F(\Xi) \rangle_{\hat{\lambda}})$

Axiom 2: *The relation among systems* is represented by a Markov relation

Let $T \equiv \{0,1,...,N\}, \pi : T \setminus \{0\} \to T)$ be a tree with root 0 and let $S_{|\rho_0^P|} \equiv \left[S_{|\rho_0^P|}, \hat{\lambda}_t \xrightarrow{\Phi_{\pi(t),t}} \hat{\lambda}_{\pi(t)} \left(t \in T \setminus \{0\} \right) \right]$ be a general system with the initial system $S_{|\rho_0^P|}$. Also, let an observable $O_t \equiv (X_t, \Im_t, F_t)$ in *C**-algebra $\hat{\lambda}_t$ be given for each *t*∈*T*. Thus, we have a sequential observable $\left[(O_t)_{t \in T}, \{ \Phi_{t,\pi(t)} : \hat{\lambda}_t \to \hat{\lambda}_{\pi(t)} \}_{t \in T \setminus \{0\}} \right]$. For each *s*(∈*T*), define the observable $\tilde{O}_t \equiv \left(\prod_{t \in T_s}, X_t, \prod_{t \in T_s} \Im_t, \tilde{F}_s \right)$ in $\hat{\lambda}_s$ such that:

$$\tilde{O}_s = \begin{cases} O_s & (\text{if } s \in T \setminus \pi(T)) \\ O_s \overset{qp}{X} \left(\overset{qp}{X}_{t \in \pi^{-1}(\{s\})} \Phi_{\pi(t),t} \tilde{O}_t \right) & (\text{if } s \in \pi(T)) \end{cases}$$

(11)

[5] Because of the extremely complexity of the axioms, only a scaled-down of the Axioms, adapted from [65], [66] are shown here.

If then an observable $\tilde{O}_0$ (i.e. the Heisenberg picture representation of the sequential observable $\left[(O_t)_{t \in T}, \{\Phi_{t,\pi(t)} : \bar{\lambda}_t \to \bar{\lambda}_{\pi(t)}\}_{t \in T \setminus \{0\}} \right]$ in $\bar{\lambda}_0$ exists, then measurement:

$$M_{\bar{\lambda}0}\left(\tilde{O}_0 \equiv \left(\prod_{t \in T} X_t, \prod_{t \in T} \Im_t, \tilde{F}_0 \right), S_{|\rho_0^P|} \right)$$

(12)

Which is called the *Heisenbery picture* representation[6] of the symbol $\aleph\left(O_t\right)_{t \in T}, S_{|\rho_0^P|}$.

Therefore, Axiom 2: *The relation among systems* is represented by a Markov relation $\{\Phi_{t1,t2} : \bar{\lambda}_{t2} \to \bar{\lambda}_{t1}\}_{(t1,t2)} \in T_{\leq}^2$. Let $O_t (\equiv (X_t, \Im_t, F_t))$ be an observable in $\bar{\lambda}_t$ for each $t (\in T)$. If the procedure per equation (11) is possible, a sequential observable $[O_T] \equiv \left[\{O_t\}_{t \in T}, \{\Phi_{t1,t2} : \bar{\lambda}_{t2} \to \bar{\lambda}_{t1}\}_{(t1,t2)} \in T_{\leq}^2 \right]$ can be realized as the observable $\tilde{O}_0 \equiv \left(\Pi_{t \in T} X_t, \Pi_{t \in T} \Im_t, \tilde{F}_0 \right)$ in $\bar{\lambda}_0$.

It is quite important to note that Axiom 2 is stated in terms of $\bar{\lambda}$ and not in terms of $\bar{\lambda}^*$ so that it can be applied to statistical measurement theory as well. Also, we must add the following statement:

Let $S_{|\rho_{t0}^P|} \equiv \left[S_{|\rho_{t0}^P|}, \{\Phi_{t1,t2} : \bar{\lambda}_{t2} \to \bar{\lambda}_{t1}\}_{(t1,t2)} \in T_{\leq}^2 \right]$ be a general system with an initial state $\rho_{t0}^p (\in \wp^P(\bar{\lambda}_{t0}^*))$. Then, a measurement represented by the symbol $\xi(\{\tilde{O}_t\}_{t \in T}, S_{|\rho_{t0}^P|})$ can be realized by $M_{\bar{\lambda}0}(\tilde{O}_0 \equiv \left(\Pi_{t \in T} X_t, \Pi_{t \in T} \Im_t, \tilde{F}_0 \right), S_{|\rho_{t0}^P|})$, if $\tilde{O}_0$ exists.

These two axioms are related—i.e. as long as Axiom 1 holds, and with Axiom 2, a powerful relational structure can be formalized to initiate a ***measurement process*** in order to provide the operationalization of measurement, i.e. ***empiricism*** is facilitated to attain the **desired output**.

Statistical Measurement Theory

The SMT is defined in (9). PMT has been defined in Axiom 1 and Axiom 2. The statistical state is defined as:

$$\text{Statistical state} = \underbrace{\text{mixed state}}_{\text{Mathematically derived}} + \text{probabilistic interpretation}$$

(13)

[6] Is quantum mechanics in which the operators (e.g. observables) incorporate a dependency on time, but the state vectors are time-independent.

The mixed state, $\rho(\in \wp^m(\hat{\lambda}^*))$ has been defined using (4). Therefore based on (7) and (8), we only need to define **Proclaim 1.**

Proclaim 1: ***The probabilistic interpretation of the mixed states.***

Consider a statistical measurement $M_A\left(O \equiv (X, \Im, F), S_{[*]}\left(p^m\right)\right)$ formulated in a C^*-algebra $\hat{\lambda}$. Then the probability that $x\left(\in X\right)$, the measured value obtained by the statistical measurement, $M_A\left(O, S_{[*]}\left(p^m\right)\right)$, belongs to a set $\Xi(\in \Im)$ is given by:

$$\rho^m(F(\Xi))\left(\equiv_{\hat{\lambda}^*} \left\langle \rho^m F(\Xi)\right\rangle_{\hat{\lambda}}\right) \tag{14}$$

Representational Measurement Theory

Measurement can also be theorized from a representational approach that focuses on empirical structures, with an ordering attribute, leading to numerical or geometric representations ([61]; [62]; [19]; [97] and [123], [124]). Although [97] had made extensive use of mathematical axioms in formalizing the fundamentals: i.e. quantitative and qualitative measurements, Narens, Duncan, Krantz, Suppes and Tversk are instrumentals in developing axioms in formulating various virtues of measurement under the ambit of the RMT (see cf. e.g., [79]; [98], [99]; [76]). The works of ([123], [124]) are invaluable with his famous *scales of measurement* (SoM) (Levels of measurement). The SoM comprises of four scales: *nominal, ordinal, interval* and *ratio* (it was enlarged to more scales in a series of developments); each scale has certain properties, and the ratio scale is highest by virtue of it having satisfy the most number of measurable properties. Collectively the works of these authors are catered towards the social sciences and psychology. The ***outcome (output)*** of the RTM is that, the numbers must be meaningful, and **meaningfulness** comes from the ***empirical support*** describing the interrelationships of various ***measurement process*** ([99]). This is interesting as the concept of meaningfulness as explained here is consistent to MT.

4.3. Empiricism, precision, accuracy and meaningfulness of measurement

As discussed in the preceding subsection, on both counts (i.e. in hard and soft sciences), the concepts of precision and meaningfulness of measurement is of paramount importance. That is, the meaningfulness of measurement state that there is a purpose of measurement, and consists of the vital elements: object, foundation of measurement, measurement process and the determination of precise outputs. Since the fundamental of measurement is rooted in mathematics (see [2]) the authors consider that it is imperative that the foundation of measurement is based on axioms.

[126] indicated that there is *no absolute accuracy of measurement* even in a seemingly upmost accurate measurer—the meter bar in Paris. ([126]: 1086) argued that in the physical sciences or hard sciences quantitative expressions of measurement accuracy are typically casts in epistemic terms, namely, in terms of uncertainty. Measurement uncertainty is commonly expressed as a value range ([126]: 1085). Accuracy is related to precision. *Measurement precision* is the closeness of agreement among measured values obtained by repeated measurements of the same (or relevantly similar) ***objects*** using the same measurement system ([126]: 1085). Therefore, measurement meaningfulness requires the operationalization of measurement, i.e. be ***empirically tested*** (evaluated) for accuracy. This concept of measurement precision is consistent to [65] where he stated that there is no absolute (100%) exactness in measurement and what is most important is meaningfulness of measurement where there is a quantitative relation among parameters of things to be measured: the requirements to conform to Axiom 1, Axiom 2 and Proclaim 1.

The key to the philosophy of the meaningfulness in measurement in hard sciences is the requirement that the numbers represent properties of physical things, but measurement in the social sciences does not necessarily have this thing-relatedness ([12]: 8). This is because, in social sciences, measurement of things is sometimes impossible; it has other kinds of phenomena such as states, events, and processes that are measured. Thus, in social sciences, we have numbers (quantity) and numerals (a word, letter, figure or symbol 'transformed' in number terms) to represent things and phenomenon respectively (see [98]). In recent measurement writings, 'things' or 'phenomenon' are now termed as objects (see [126]). Both numbers and numerals are concepts, and one can initiate a set of rules to express a number or numerals to establish a measuring system. For example, in the RTM, measurement is operationalized through the SoM (operationalization process) consisting of any rules will do ([125]: 19). Numbers and numerals will be covered in more details in the next sub-section. However, mathematical scales are only one of several means of representation involved in measurement, and often not the most epistemically problematic or interesting ones ([127]). Also, the theory of measurement scale especially in the construction of homomorphisms is revealed as overly restrictive ([142]: 158-166).

Although the use of the scales of measurement differs between the hard sciences and soft sciences, nevertheless, they are appealing to be used in the organizational context because certain measurement exits in abstract form, for example, intangible assets (intellectual capital), forecast of future sales, and intrinsic value of assets (e.g. shares). The other appealing feature of the scales is that it is less complicated to use especially in social sciences because there are no need to have physical instruments like thermometers or galvanometer ([12]). Instead we have mathematical models function as measuring instruments by transforming sets of observations into a measurement result. In other words, models mediate between theories and the data by transferring observations (or phenomenon) into quantitative facts (or numerals).

Illustration:

In physics, measurement must be precise (as exact as possible) while that of engineering and statistics must be useful. In social sciences, precision of measurement is less of a concern, so we are looking at accuracy (reliability) rather than precision, and that the object measured must be purposeful. That is, in all cases the meaningfulness of measurement is a requirement.

Physics:

To build a spacecraft to travel to the moon and back we need to calculate the lunar-distance (Earth-Moon distance, etc.) (LD) ($\Delta \oplus L$). It measures the average distance from the center of Earth to the center of the Moon, or more technically, it is the mean semi-major axis of the geocentric lunar orbit. The actual distance varies over the course of the orbit of the Moon, from 356,500 km at the perigee to 406,700 km at apogee, resulting in a differential range of 50,200 km [95]. Moreover there are various theories and tools used to measure the LD. In spite of this the precision of this measurement to a few parts in a trillion has useful implications for testing various theories (applications). For example, if the 'actual' LD is 380,000 km and if we err the measurement by 0.001% (a very low margin), the spacecraft will miss the target by 380 km.

So how do we test the application?

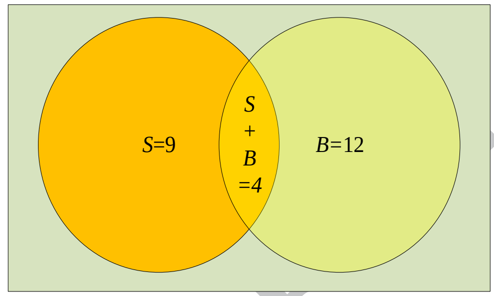
- Is the LD theory true is (Meaningless as we are not testing the distance between the moon and the earth).
- Is the Laser Ranging Theory (LRT) experimentally true or not is (Meaningful as the Spacecraft project team is using the LRT to measure the LD).

Statistics:

KL Ltd is a firm engaging in the manufacturing of generic motor vehicle batteries and the management wants to improve productivity in view of the current competitive market. It employs **100 workers** in its plant. In a survey conducted recently, **9 workers** were found to be slow and did not complete their tasks on schedule while **12 workers** had produced sub-standard work. Out of these **21** (9 + 12) **workers**, **4** had produced work which was both late and sub-standard.

What is the probability that a worker selected will be a slow worker or/and a sub-standard worker?

- Let S = slow work, B = sub-standard work
- $P(S,B)=\frac{21}{100}=0.21$ is $\left(\begin{array}{l}\text{Meaningless as nonsensical}\\ \text{method is used}\end{array}\right)$
- $P(S,B)=\frac{4}{100}=0.4$ is $\left(\begin{array}{l}\text{Meaningless as wrong}\\ \text{method (i.e. mutually}\\ \text{exclusive) is used}\end{array}\right)$
- $P(S,B)=\frac{9}{100}+\frac{12}{100}-\frac{4}{100}=0.17$ is $\left(\begin{array}{l}\text{Meaningful as}\\ \text{correct method}\\ \text{(i.e. mutually}\\ \text{not exlusive) is}\\ \text{used}\end{array}\right)$



Social sciences (e.g. accounting):

A firm sells two products A and B. The inventory level (balance) at 31 December 20X1 are:

- Product A: 10 units @ cost $20 each & 10 units @ cost $22 each.
- Product B: 6 units @ cost $30 each & 6 units @ cost $35 each.

What is the value of inventory at 31 December 20X1?

The results are tabulated below.

Product A						Product B					Total		
Date	Qty	Cost	Total	Av. cost		Qty	Cost	Total	Av. cost		Qty	Av. cost	Total
31-Dec	10	$20.00	$200.00	$21.00		6	$30.00	$180.00	$32.50		20	$25.31	$380.00
31-Dec	10	$22.00	$220.00			6	$35.00	$210.00			12		$430.00
	20		$420.00			12		$390.00			32		$810.00

- The total number of units of 32 is meaningless as unit of different objects cannot be aggregated
- The total average cost of $25.31 is meaningless as different units of objects cannot be averaged
- The total cost of $810.00 is meaningful.
- The total unit of either Product A (20) or Products B (12) is meaningful
- The total cost of either Product A ($420) or Product B ($390) is meaningful
- The total cost of the 1st and 2nd batch of 10 units of Product A ($200 & $220) is meaningful
- The total cost of the 1st and 2nd batch of 6 units of Product B ($180 & $210) is meaningful
- The average cost of Product A ($21) or Product B ($32.50) is meaningful, albeit is considered a good enough (second best) measurement

4.4. Numbers and numerals in terms of mathematical properties

Recent philosophical writings state the outcomes (outputs) of measurement: (1) the specific area in the parameter space where the item is located in a logical space ([142]: 141-181), and in which the final states of an instrument reading after each measurement run is complete. Typical examples are digits appearing on a display, and marks on a multiple-choice questionnaire; and (2) an abstract measurement outcome in the form of knowledge claims about the state of the object of interest. Outcomes are often expressed in the form of a quantity *X* associated with, say, an object *O* that has a value, say *q* with uncertainty, say U, although in general outcomes need not be expressed numerically. While Point (1) is attributable to hard sciences, (2) is suitable for use in soft sciences. In either way, a set of axiomatic rules consisting of the concept of ***quantity***, and associated concepts such as ***order, equality, transitive*** and ***asymmetric relations, magnitudes*** and ***monotonic order*** are required in measurement. The concept of *quantity, order* and *equality* allow us to compare things (objects) in terms of numbers (>, <, =) or numerals ($\succ, \prec, =$). *Transitive* is a relation such that when applied between successive members of a sequence, it must also apply between any two members taken in order. Thus with order and equality, and transitive relation, *asymmetric relation* is established. In mathematics, a *monotonic order* is a function that facilitates increasing or decreasing order. Thus all the properties as explained allow us to compare things (objects). For quantitative properties, we require mathematical operations such as '+', '−', 'x' and '÷', i.e. ***additivity*** and ***multiplication*** to help us accomplish works and make comparison. In mathematics, additivity also includes subtraction, and likewise, multiplication includes division because the operations of the pair property are the transformation of the other.

As there will be many values in an experiment or measurement process, the precision of measurement is determined relative to an established measurement standard. By definition a standard is the target or benchmark in which different results of measured activities are compared with, and hence, standards (e.g. unit definitions) are not chosen arbitrarily. The most common way of evaluating operational accuracy is by calibration—by modelling an instrument in a manner that establishes a relation between its indications and standard quantity values ([126]: 1085-1086). As calibration is not usually used in the social sciences, and if we take the case of the RTM, standard setting is strictly related to the notion of *representation-target* (see [45]: 296). They defined a representation-target "is the operation aimed to make a context, or parts of it, 'tangible' in order to perform evaluations, make comparisons, formulate predictions, take decisions, etc."

The elaborate discussion of measurement in terms of hard sciences (with emphasis of MT) and soft sciences (with emphasis of the RTM) have both positive and negative implications in relation to the measuring attributes: measures, metrics and indicators (Table VI).

Table VI about here

As shown in the table above, a theoretical foundation of measurement is possible for the measuring attributes. And researchers can use either the hard sciences or soft sciences approach since the philosophical and theoretical basis of measurement appeared to have been bridged, and this is achieved through the meaningfulness of measurement.

4.5. Re-definition of the measuring attributes and numerical analysis of the revised properties of the measuring attributes

We need to redefine a few things concerning the measuring attributes in view of the discussion in §4.1 to §4.4. This is to take into account of the theoretical foundation of measurement. Note that all the theoretical foundation is based on axiomatic approach[7].

Re-definition of a *subjective indicator*

Theoretically and mathematically, we cannot have an attribute which is expressed either here or there in terms of a quantitative and qualitative form. Thus we need to differentiate these two properties by having a *subjective indicator* (to be discussed here) and an *objective indicator* (to be discussed later).

A *subjective indicator* conforms to the properties (definitions) of **order** and **equality, transitive and asymmetric** relation, and **monotonic** order. To define a *subjective indicator*, we assume there exists a binary total order relation ($\succ$) that can be applied to objects in the domain (e.g., this product is preferred over that product). For a domain of object *Q, magnitudes* a_i (say to represent product A) and a_j (to represent Product B), and a set of values $X(a's)$, $a \in Q$, then an indicator is defined as:

$$a_i \succ a_j \Leftrightarrow X(a_i) \rangle X(a_j), \forall a_i, a_j \in Q \quad (15)$$

Thus it can be seen that Product A is preferred (more likeable, etc.) than Product B, and not that Product A is better (as this is measured in quantitative term) than Product B. Note that as discussed earlier, we can also use statistical measurers here, e.g., **median** and **mode**. **Therefore, the meaningfulness of measurement is achieved.**

[7] An axiomatic approach differs from axiom as the latter is based on logic with precise logical properties & mathematical constructs while the former is based on reasonable (though logical) expression.

Re-definition of *a measure*

Previously a *measure* has been defined normatively as a quantitative whole. Here, theoretically, epistemically (philosophically) and mathematically, a *measure* is a **quantitative attribute**, and it should conform to the properties (definitions) of **order, equality, transitive and asymmetric relation**, and **monotonic** order. In addition, **additivity and multiplication properties** are required.

To define a *measure*, we assume there exists a magnitude comparison operation ($\varnothing$) that can be applied to objects in the domain (e.g., the price of this product is more than the other product). For a domain of object *Q, magnitudes* a_i (say to represent sale price of product A) and a_j (to represent sale price of Product B), and a set of values *X*(*a's*), $a \in Q$, then a measure is defined as:

$$a_i \varnothing a_j \sim a_h \Leftrightarrow X(a_i)/X(a_j) = X(a_h), \forall a_i, a_j, a_h \in Q \quad (16)$$

Assuming that the sale price per unit of product A is $400 and that of Product B is $200. Thus, it can be seen that the sale price of Product A is twice that of product B. That is, it can be quantitatively compared between two things (objects). **Therefore, the meaningfulness of measurement is achieved.**

Re-definition of a *metric* and an *objective indicator*

Here we re-define a *metric* and an *objective indicator* together since mathematically, they are both quantitative and are **mathematical ratios,** and they should conform to the properties (definitions) of **order, equality, additivity, multiplication, transitive and asymmetric relation**, and **monotonic order**. In addition, they must be based on the same mathematical operations and object domains of a measure. A likely candidate for the numbers used here are *rational numbers.* For a domain of object *Q, magnitudes* a_{i1}, a_{i2} and a_{j1}, a_{j2}, and a set of values *X*(*a's*), $a \in Q$, then a *metric* or an *objective indicator* is defined as:

$$\frac{a_{i1}}{a_{i2}} \varnothing \frac{a_{j1}}{a_{j2}} \sim a_k \Leftrightarrow X\left(\frac{a_{i1}}{a_{i2}}\right) \Bigg/ X\left(\frac{a_{j1}}{a_{j2}}\right) = X(a_k), \forall a_{i1}, a_{i2}, a_{j1}, a_{j2} a_k \in Q \quad (17)$$

The only difference is that a metric is a quantitative standard (standard unit) whereas an objective indicator is not. For example for a metric, *magnitudes* a_{i1}, a_{i2} (say to represent sale price per unit per month of product A) and a_{j1}, a_{j2} (to represent sale price per unit per month of Product B). For an objective indicator, *magnitudes* a_{i1}, a_{i2} (say to represent the sale price per month of product A) and a_{j1}, a_{j2} (to represent the sale price per month of Product B). Note that the 'per unit' is ommited for an objective indicator. This definition is similar to an accounting (financial) or organizational ratio, e.g. returns on equity (ROE).

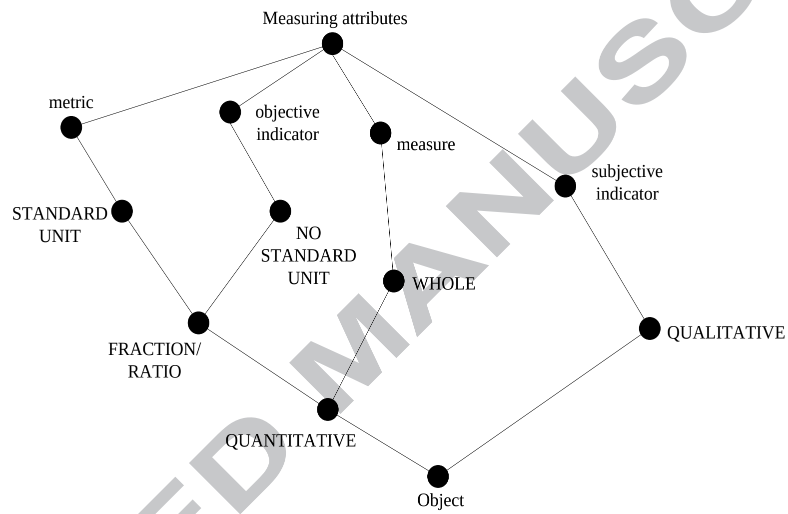
The revised GLT and FCA in the measurement theory in mathematical context:

O = {QUANTITATIVE, QUALITATIVE, *WHOLE*, *FRACTION*, *STANDARD UNIT*, *NO STANDARD UNIT*}

A = {*measure, metric, objective indicator, subjective indicator*} and *R* is the specified matrix in Table VII. as well as measurement theory and measuring attributes.

The revised lattice and Galois connection is depicted in Figure 3.

Figure 3. Revised lattice of the measuring attributes, constructed using GLT and FCA theories



There are some marked differences between Figure 3 and Figure 2 where in the former there is no data but object (to take into consideration of abstract object), and there is no need for the distinction between financial and nonfinancial attributes.

Numerical analysis

To illustrate, a numerical analysis of a company's activities for the month of January 2018 is provided. The company is a retailer & sells two products of Louis IV to VI chairs: Fauteuil rounded back & guided curved detailed chair (Product A) and Bergère tub-shaped back chair (Product B).

Source: Not on the High Street (2016). http://www.notonthehighstreet.com/system/product images/ images/001/137/915/original_antique-gold-louis-chair.jpg

The case

The purchases of Product A & Product B are:

- Product A: 16 units @ cost $100 each on Jan. 1st. & 24 units @ cost $120 on Jan. 4th.
- Product B: 12 units @ cost $200 each on Jan. 2nd. & 8 units @ cost $220 on Jan. 8th.

The sales of Product A & Product B are:

- Product A: 15 units @ $160, 10 units @ $160 and 12 units @ $170 on Jan. 8st., 11th. & 23rd. respectively.
- Product B: 4 and 6 units on Jan. 9th. & 18th. respectively, all @ price $400 each.

One unit of Product A was discovered to be beyond repair when checking for stock re: sale.

The total administration & operating expenses were $1,600, split equally between the two products.

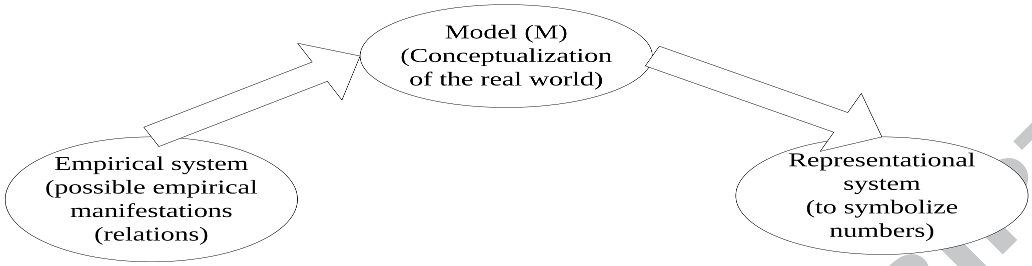
- Current M&PM: the company uses the traditional accounting measurement, average cost for inventory valuation. In addition, the company prepares PMMS information.
- New M&PM: based on the measuring attributes comprising of measures, metrics & indicators and the First-in-First-Out (FIFO) inventory valuation.

Prepare analysis of the two M&PM systems including an income statement.

The solution

For the new M&PM system, a modified of the [46] approach is used. They state that for a good measurement system, three elements must be considered: (1) the model (i.e. the conceptualization of the real world); (2) the representation-target; and (3) the rules to determine the related set of measures, metrics and indicators together with their associated relations. The representation does not hold until these three elements are not delineated. The three elements of this innovative and pragmatic approach are depicted in Figure 4.

Figure 4. The representational approach of an empirical system (adapted from [46]: Fig. 3.2)



For this numerical analysis, we want to identify which product performs better:

(1) The representation-target is "finding the better-performed product";

(2) The model is given by "how we evaluate different products' credentials" (different methods may lead to different classifications);

(3) The measures, metrics & indicators and the associated relations originate from the rules established for obtaining a final score.

The above needs elaboration. For Point (1) 'better-performed' means (1) higher sales quantities and numerical values; and (2) greater acceptance by customers. This means qualitative analysis is required (not possible in the old M&PM and it will be made available by the new M&PM). In Point (3), for empiricism to succeed, the set of rules governing a subject matter needs to be adhered with. This is more so in social sciences, for example, in the accounting convention there are three main approaches: FIFO, Last-in-First-Out (LIFO) and the average method in valuing inventory balances and each gives different numbers, and you can choose any one of them. In this case the most objective method (i.e. one that gives you precision of measurement) must be used, and in so doing you avoid the pitfall of using a 'commonsense approach' as what is usually done in practice. A 'commonsense approach' is not an objective approach in whatever sense as different people have different commonsense, and this often lead to conflicting and nonsensical measurement or at best, a second-rate solution materializes. The results of the various M&PM elements are presented in Table VIII.

Table VIII about here

The discussion of the current and new M&PM are as follows. First, we do not need so many terms, namely cost, value, revenue, etc. to mean the same or different things. For the new M&PM, all we need are the three terms (measuring attributes): measure, metric and indicator where the term 'total' is used to state that an attribute has undergone mathematical operations (e.g. addition or multiplication). Second, the use of average (mean) should be avoided if a direct measurement is possible as the former is not informative (accurate) enough and thus the issue or inventory value is considered to provide only a good enough (second best) measurement. Averages also distort information, viz, the write-off cost under the old M&PM is $112 against the actual cost of $120. Moreover, the average as used in inventory valuation is unstable as it changes each time a new purchase unit measure (cost) changes. But it does not mean the mean is not meaningful at all as it can be meaningful in the right circumstance (see later). Third, as the treatment of write-off (obsolescence) under the accounting convention is not consistent to the meaningfulness of measurement because by including the write-off element under the Cost-of-Goods Sold (COGS) category will result in lowering the gross income (%) (average unit selling price). This is not allowed in the new system as a write-off of an inventory item without a sales value distorts the sale margin. Fourth, for the new M&PM, all measurement conforms to the measuring attributes and they have adhered strictly to mathematical properties (e.g. additivity & multiplication), meaning that the same term is used to represent the same real-world phenomenon, Therefore, for meaningfulness of measurement, a strict matching rule between purchases of differential unit measures (costs) and issues for sales & issues for write-off were instituted using the FIFO concept.

The income statement is presented in Table IX.

Table IX about here

In the current system, the most obvious (commonsense approach) is to focus on the gross income and net income. We can see that Product A performs financially better than Product B (i.e. gross income: $1,672 (28%) vs. $1,920 (48%); net income: $872 (14%) vs. $1,120 (28%)). Since this company is progressive, it prepares supplementary information (PMMS). The scanty supplementary information indicates that Product A sold 3.7 times more than Product B, the purchase units of product A is twice that of product B but only two units remained unsold for Product A and ten for product B, suggesting that either Product A sells better or that its inventory management is better than Product B. This indicates there are some merits of Product A.

In the new system, a whole range of measurement including financial & supplementary measurements adhere strictly to measurement theory. First, both the gross and net incomes of Product A performed financially better than Product B (i.e. gross income: $1,800 (30%) vs. $2,000 (50%); net income: $880 (15%) vs. $1,200 (30%)). Although this tells us the same thing as in the current system; however, the gross and net incomes are higher by about 2% in the new system. This may not be large; however the difference can be higher if there are more transactions, and in today's competitive business environment, a 2% difference in reported 'distorted incomes' is asymmetry information that may cause managers to make erroneous decisions. Second, the range of measures, metrics, and indicators can help us in making more informed decisions. The Mean Sales Measure of Product A is about two-and-the-half times cheaper than Product B, and that the measure of the former is more volatile than Product B (see standard deviation & index). The Mean Purchase Measure of Product A is about 54% cheaper than Product B and has a 10% higher fluctuation than Product B (see index). Product B performs better than Product A in terms of profitability: four times in direct income and one-third in terms of Mean-Sales-Purchase metric.

Third, in the modern business, quantitative measurement will not be all-compelling as qualitative measurement is also important. To illustrate, a simple survey consisting of a questionnaire with pictures of the two chairs and how customers or expected customers would rank (prefer) these chairs in terms of 'appeal', 'comfort', 'style', 'versatile', and 'space'. Assuming the survey received feedbacks from 500 respondents (see Table X).

Table X about here

Since the content in Table X is qualitative measurement, we are more interested in the raw or rank score (numerals) because we cannot perform additive and multiplication operations, and our concern is to see which item (product) is preferred over another. That is, we cannot measure the properties of Product A and Product B directly; what we are doing is to measure headcount in deducing that Product A is preferred to Product B. The qualitative scores indicate that (potential) customers consider that Product A has 'appeal' and more 'space friendly' than Product B while Product B is preferred over Product A in terms of 'comfort', 'style' and 'versatile'. All in all, Product A is preferred over Product B by 53% of the (potential) customers. While this approach may not be the most accurate, at least the measured information for the two products is meaningful.

Last, notice that the current system in general is too financially-based and people are taught to focus on one measure (usually income) which suggests that Product B performed better than Product A, and even when various PMS per unit (i.e. more specific) measures were used, there is only a slight comprehension that Product A might have some merits. Although the financial performance of the products using the new system supports the assertion of the old system, however with a range of measures, metrics and indicators (including qualitative measurement) of the new system pointing at Product A to have greater potentials, suggesting that its performance would surpass that of Product B in the future, and thus, any rational person would prefer Product

A to Product B. That is, the new system consisting of the measuring attributes in which each attribute adheres to a strict set of rules in realizing real world phenomena and conform to the representation target in ensuring that measurement is meaningful. But can the company still rely on the current system? Measurement in social sciences is intriguing as imprecise or even meaningless measurement seem to work unlike in physics where measurement must be precise to succeed. Given a company run by incompetent managers and inept accountants, it goes without saying that the inherent measurement system is a bad one, such that the company earns $1 a day whereas a good system can give the company $100 a day. Then the company begins to lose $100 a day. Ultimately the losses are so enormous that the company goes bankrupt. So, in the long run, a good measurement system in the social sciences count just like in physics.

5. Summary and concluding remarks

A review of the PMS&PMMS and best practice literature indicates that while there is no shortage of publications on M&PM in the business and organizational perspectives, but there is a dearth of research on how to measure activities/things more precisely and how they contribute to performance using an appropriate measurement theory. Measurement is rooted in mathematics ([2]), and broadly, mathematical measurement theories can be viewed from the hard sciences and soft (social) sciences perspectives. However recent philosophical writings and the axiomatization of many aspects of the measurement concepts in soft sciences, much differences have been bridged between these two perspectives. With this development, using mathematical formulation of the hard sciences (MT) and social sciences (RTM), a theoretical measurement foundation of the measuring attributes for M&PM is ensconced and empirically tested using a numerical analysis. The measuring attributes can measure quantitative as well as qualitative objects. The key of this theory is accuracy (reliability) and meaningfulness of things measured.

This research has important implications to both PMS&PMMS academicians and practitioners. First, businesses and organizations are still relying on quantitative financial measures, and the more progressive ones use a few PMS/PMMS measures or indicators. Second, measurement in the social sciences is intriguing as imprecise or even meaningless measurement seem to work. However in the long run only a good M&PM system counts just like in physics as a bad system creates information asymmetry, encourage poor decisions, and increase inefficiency as illustrated in the numerical analysis. For an organization to succeed there is no choice but to embark on a good M&PM. The keys of this M&PM: (1) able to measure quantitative and qualitative objects; and (2) adhere to the definitions of the measuring attributes. The definitions of the attributes and elements of the new M&PM (§4.5) are important. To operationalize measurement, there must be (1) a model (i.e. conceptualization of the real world); (2) the representation-target; and (3) the rules to determine the related set of measures (quantitative whole), metrics (quantitative standard), indicators (quantitative numbers & qualitative numerals) together with their associated relations for obtaining the final score. Note that qualitative measurement is also important. The operationalization process is explained in the numerical analysis.

As this work is interesting, the author invites future researchers to expand on this seminal work in the following areas: (1) as this paper uses complex mathematics, using other (simplier) mathematical formulation may be easier for lay readers to understand the theoretical measurement foundation; (2) refine the epistemology that allows us to analyze the mechanical world view in a more ‘usual life’ manner; (3) explore more real-world empirical settings (e.g. online shopping, manufacturing) to convince readers on how a bad M&PM system can fail a business; and how a good system can bring success to a business; and (4) develop a taxonomy of measures, metrics and indicators for practical use.

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Table I. Measuring attributes identified

Authors	Measure	Metric	Indicator
(a) PMS&PMMS authors (58)			
Rogers, 1990	√		√
Lynch & Cross, 1991	√		
Eccles & Pyburn, 1992	√		√
McGee, 1992	√	√	√
Dumond, 1994	√		
Fry, 1995	√		√
Lebas, 1995	√		
Neely *et al*., 1995	√	√	√
Flapper *et al*., 1996	√		√
Kaplan & Norton, 1996 *	√	√	√
Atkinson *et al*., 1997	√		√
Bititci *et al*., 1997	√		
Harbour, 1997	√	√	√
Nadzam & Nelson, 1997	√		√
Atkinson, 1998	√		√
Neely, 1998	√	√	√
Gates, 1999	√		
Marshall *et al*., 1999	√		√
Otley, 1999	√		
Forza & Salvador, 2000	√		√
Gross *et al*., 2000			√
UK CAG, 2000	√		√
US DOE, 2000	√		√
US GAO, 2000	√		
Kueng *et al*., 2001			√
Maisel, 2001	√		
Toni & Tonchia, 2001	√		√
Kanji, 2002 *	√		√
McAdam & Bailie, 2002	√		
Neely, 2002	√	√	√
Bourne *et al*., 2003	√	√	√
Drongelen & Fisscher, 2003	√	√	√
Ittner *et al*., 2003 *	√		
Matos *et al*., 2003			√
Rouse & Martin, 2003	√		
Tangen, 2003	√		

Melnyk *et al*., 2004		√	
Bourne *et al*., 2005	√		
Denton, 2005	√	√	√
Tangen, 2005	√		
Walsh, 2005	√		
Al Bento & Lourdes, 2006	√		
Henri, 2006	√	√	√
Gunasekaran & Kobu, 2007	√	√	√
Julnes, 2007	√		√
Franceschini *et al.*, 2007, 2008			√
Marchand & Raymond, 2008	√		
Petersen *et al*., 2009		√	
Yen, 2009	√	√	√
Bryceson & Slaughter, 2010	√		
Tung *et al*., 2011	√		
Franco-Santos *et al*., 2012	√		
Taticchi *et al.*, 2012	√		√
US DOE, 2012	√		
US GAO, 2012	√		
Choong, 2013b	√	√	√
Melnyk *et al*., 2014	√	√	
Smith & Bititci, 2017	√	√	
(b) Non-PMS&PMMS authors (11)			
IEEE, 1983	√		
IEEE, 1990		√	√
SEI, 1994	√	√	
Ragland, 1995	√	√	√
Virginia Tech, 1997			√
Perkins, 2003		√	
University of California, 2003		√	
Poll, 2006			√
Trochim, 2006	√	√	√
Mckinsey, 2007		√	
Serrat, 2010	√		√
	52+5	17+7	32+6

Note: "√" indicates that the article devotes substantial discussion (not just passing reference) of these apparent measuring attributes.

CAG = Comptroller & Auditor General, DOE = Department of Energy, GAO = General Accounting Office

- Measures are used interchangeably with drivers (and indicators in Kanji, 2002. Norton & Kaplan, 1996); but their descriptions relate to measures, metrics or indicators. Since the term driver is not an official term for a measuring attribute (Fitz-Gibbon, 1990), this term is not considered one of the measuring attributes.

Table II. Definitions/indications and expressions of the measuring attributes in terms of measure, metric and indicator (Note: articles appearing in each of the attributes are non-mutually exclusive)

Authors	Definitions/indications & expressions
(a) PMS&PMMS authors	
Measure	
Rogers, 1990	Not defined. Measures are used to plan & estimate costs, revenues & growth of plans & programs

Lynch & Cross, 1991	Not defined, a pyramid-shaped "map" is used for understanding & defining the relevant objectives & measures for each level of the business organisation. Measures are financials & nonfinancial form that are used to provide yardsticks of performance such as *R&D*, market, delivery that satisfy/not satisfy strategic goals. Non-financial measures are needed to integrate with financial measures to support a PMS
Eccles & Pyburn, 1992	Not defined. Measures are dominated by financial terms but nonfinancial measures are needed to supplement financial measures due to several limitations of financial measures
McGee, 1992	Not defined. Measures & metrics are components of a PMS. Measures & metrics are used in the measurement process to link to strategic objective, to capture data to measure performance, generate decision-useful information & the dissemination of information to stakeholders. Measures & metrics are treated similarly
Dumond, 1994	Not defined. Performance measures are established to support the achievement of goals and are provided with the intent to motivate, guide and improve an individual's decision making. These measures can be categorized into areas such as workload, quality, operations or price. Additionally, two common categories are those of efficiency and effectiveness. (p. 17)
Fry, 1995	Discussed a need for measures to be used to gauge performance such as productivity (p. 935)
Lebas, 1995	A measure is difficult to define but writer elaborated on "why & what to measure, & how measure leads to better performance". Provided detailed list of measures used by US government
Neely *et al*., 1995	A performance measure is defined as a *metric used to quantify the efficiency and /or effectiveness of an action* (p. 81). A measure is categorised in accordance to activities, namely shipments, inventories. Measures are categorised into dimensions such as quality, time, flexibility & cost.
Flapper *et al*., 1996	A measure is defined in per unit or dimension basis (p. 35)
Kaplan & Norton, 1996	Not defined, measures are used to gauge performance of the four strategic measurement perspectives: financial, customer, internal & language & growth.
Atkinson *et al*., 1997	Not defined. A PMS consists of financial & nonfinancial performance measures (sub-divided into primary & secondary measures) that are used to measure the strategic goals for the organization's stakeholders. For example, the primary & secondary measures for investors are *ROE* & revenue growth respectively.
Bititci *et al*., 1997	Performance measure objectively measures the performance of an individual business process (p. 526)
Harbour, 1997	Not defined. Measures are tailored to different user needs (p. 8)
Nadzam & Nelson, 1997	Not defined. Quantitative & statistical measures are vital in gauging patients' satisfaction, quality of care & in measuring continuous performance
Atkinson, 1998	Not defined. A PMS consists of financial & nonfinancial performance measures (sub-divided into primary & secondary measures) that are used to measure the strategic goals for the organization's stakeholders. For example, the primary & secondary measures for investors are *ROE* & revenue growth respectively.
Neely, 1998	Same as Neely *et al.*, 1995, 2005. Elaboration on how to--and when to-measure business performance is a global management issue.
Marshall *et al*., 1999	Not defined. Identify & determine what are the measures that can be used to evaluate policy & (government & community) goals
Gates, 1999	Not defined. Measures can be financial (e.g. cash flow, *EPS*), nonfinancial (e.g. customer satisfaction, quality), quantitative (e.g. dollar value) or qualitative (e.g. brand)
Otley, 1999	Not defined. Measures are developed from a stated objective (goals) that are needed to control & assess the organization's various business performances. Measures are financial or nonfinancial and may be qualitative (e.g. measures of

	employee morale). Measurement process must strike a balance between output measures (e.g. sales) and input measure (e.g. cost of production)
Forza & Salvador, 2000	Measures are empirically developed. Measures are developed based on scales based on multiple items from surveys provided by practitioners from 5 countries. The measures are then statistically tested for reliability (yield consistent result) & validity (close approximation of the real thing)
UK CAG, 2000	Formally defined. Performance measure is defined as "a measurable indicator of performance which may or may not be targeted." (pp. 61). Performance measures must first be defined to ensure they are appropriate to measure things in accordance to their needs. "Performance measures assist organisations to communicate objectives & priorities, measure what they deliver and report publicly on what they have achieved." (pp. 10) Good performance measures can assist readers/stakeholders of the annual report to understand and access those aspects of performance which are central to the organization/unit's
US DOE, 2000	Formally defined. "Performance measures quantitatively tell us something important about our products, services, and the processes that produce them. They are a tool to help us understand, manage, and improve what our organizations do. Effective performance measures can let us know: (1) How well we are doing; (2) If we are meeting our goals; (3) If our customers are satisfied; (4) If our processes are in statistical control, and (5) If and where improvements are necessary. They provide us with the information necessary to make intelligent decisions about what we do. A performance measure is composed of a number and a unit of measure. The number gives us a magnitude (how much) and the unit gives the number a meaning (what). (p. 3)
US GAO, 2000	Performance measures may address the type or level of program activities conducted (process), the direct products and services delivered by a program (outputs), and/or the results of those products and services (outcomes)
Maisel, 2001	Conducted surveys in US & found financial measures are more important than nonfinancial measures. The most common nonfinancial measures are revenues, growth, gross margin, *EBIT*, net operating income, *EPS*, *ROI*, *EVA* & cash flows & most common nonfinancial measures are customer satisfaction, market share, quality & process-related, innovation/product develop., time, speed, agility, supplier, regulatory & compliance, productivity, employee turnover & demographics.
Toni & Tonchia, 2001	Measures are varied, formal or informal & they can be financial or nonfinancial that measures & are used to measure various activities e.g. responsibility of individuals, inventory & accounting transactions
Kanji, 2002	Not defined, call for integrated & balance performance measures. Balance means the use of nonfinancial measures (e.g. leadership) in support of financial measures. Critical of traditional financial measures
McAdam & Bailie, 2002	Not defined. Performance measures are developed to align with strategic goals to evaluate performance. These measures are based on accounting & nonaccounting frameworks such as TQM, BSC, etc.
Neely, 2002	Same as Neely *et al*., 1995; Neely, 1998
Bourne *et al.,* 2003	Formally defined. A performance measure can be defined as a metric that is used to quantify the efficiency and/or effectiveness of action". "The set of measures is multi-dimensional as it includes both financial and nonfinancial measures, it includes both internal and external measures of performance and it often includes both measures which qualify what has been achieved as well as measures which are used to help predict the future." (p. 3).
Drongelen & Fisscher, 2003	Not defined, performance measures direct people's behavior to ensure that attention will be paid to all of the organizational objectives
Ittner *et al.,* 2003	Not defined. Measures are financial or nonfinancial that help evaluate performance

	of business activities
Rouse & Martin, 2003	Performance measures must relate to organizational goals & strategy, holistic, etc. & be dictated by instrument to measure performance (p. 802)
Tangen, 2003	Not defined. Use of performance measures is an effective way to increase the competitiveness & profitability of a manufacturing firm (p. 347).
Bourne *et al*., 2005	They consider that many of the contextual, process & content factors are common in an organization & thus allowing them to focus on the use of the measures to investigate the use of measurement & impact on performance (p. 3)
Denton, 2005	Financial measures tend to be lagging indicators that only capture the results after-the-fact (p. 281)
Tangen, 2005	The design of an individual performance measure is not about answering the question ‘‘What to measure?’’. That is an issue that should already have been solved when designing the structure of PMS. The design of an individual performance measure is mainly about answering the question ‘‘How to measure?’’. Designing a performance measure must be based on a formula that may suite the particular purpose of the measure. The measures should be easily measured & easily understood, objective (rather than subjective), ratios should be used instead of absolute numbers & they should stimulate improvement (p. 5)
Walsh, 2005	Developed taxonomy of ‘less than accurate’ measures in the form of surrogate measures in measuring performance. He argued that objective measures are too rigid & as such they may not be able to measure many organizational activities such as intangibles
Al Bento & Lourdes, 2006	Not defined. Financial measures are not sufficient in measuring performance & should include nonfinancial performance measures for use in strategic PMS
Henri, 2006	Performance measures are used to provide feedback regarding expectations & to communicate with various stakeholders (monitoring) (p. 80)
Gunasekaran & Kobu, 2007	Adopt definition of Neely *et al*., 1995
Julnes, 2007	Measures or indicators are components of a PMS. They are used to measure and provide information about an organization’s performance in terms of inputs, outputs & outcomes
Marchand & Raymond, 2008	Not specific. Measures referred to those specified in PMS literature
Yen, 2009	The five perspectives and their related measure which is given in parenthesis are: customer perspective (response time), process perspective (completion time), product perspective (processing time), system perspective (throughput) and resource perspective (utilization rate) (p. 867)
Bryceson & Slaughter, 2010	Discussion based on past papers
Tung *et al*., 2011	Adopt definition of Kaplan & Norton
Franco-Santos *et al*., 2012	A Contemporary PM system exists if financial and non-financial performance measures are used to operationalize strategic objectives.(p. 80)
Taticchi *et al*., 2012	Adopts the definition per Kaplan & Norton
US DOE, 2012	Formally defined. “Performance measure(s) encompassing the quantitative basis by which objectives are established and performance is assessed and gauged. Includes performance objectives and criteria (POCs), performance indicators, and any other means that evaluate the success in achieving a specified goal. Also, the quantitative results used to gauge the degree to which an organization has achieved its goals.”
US GAO, 2012	Formally defined. “Performance measures quantitatively tell us something important about our products, services, and the processes that produce them. They are a tool to help us understand, manage, and improve what our organizations do.”
Choong, 2013b	Formally defined. “A *measure* to be in quantitative form that is a whole number, expressed either in monetary (financial) form (e.g. sale values), dimension form (e.g. square meter) or unit form (e.g. production outputs). A measure is most suitable to be used for *measurement*, for example to find the exact purchase or

	sales value. Thus a measure resembles accounting values."
Melnyk *et al*., 2014	Per definition of Neely *et al*., 1995
Metric	
McGee, 1992	Not defined. Measures & metrics are components of a PMS. Measures & metrics are used in the measurement process to link to strategic objective, to capture data to measure performance, generate decision-useful information & the dissemination of information to stakeholders
Neely *et al*., 1995	Metrics are used interchangeably with measures
Kaplan & Norton, 1996	Metrics are used to gauge returns to investments, e.g. shareholders value. Metric can represent performance driver (p. 76)
Harbour, 1997	Specific performance measurement such as cycle or quality yield (p. 9)
Neely, 1998	Same as Neely *et al*., 1995, 2005. Discuss quite detail on roles of different kinds of metrics & explain how to quantify variables into metrics in evaluating business performance
Neely, 2002	Same as Neely *et al.*, 1995; Neely, 1998
Bourne *et al*., 2003	Metric is synonymous with measure
Drongelen & Fisscher, 2003	Not defined. Metrics must be selected for any appropriate kind of performance evaluation
Melnyk *et al.*, 2004	Indication provided. A metric is a verifiable measure, stated in either quantitative or qualitative terms and defined with respect to a reference point. Ideally, metrics are consistent with how the operation delivers value to its customers as stated in meaningful terms (p. 211)
Denton, 2005	Not defined but indicated that performance metric can be used for measuring customer satisfaction (p. 278)
Henri, 2006	Firms should move away from the rigid traditional financial metrics & instead focus more on loose form of control, planning & performance evaluation nonfinancial metrics (p. 87)
Gunasekaran & Kobu, 2007	Adopt definition of Neely *et al.*, 1995
Petersen *et al*., 2009	The goal of all of these metrics has been to gain some quantifiable measure of any number of business goals, from measuring the effectiveness of marketing campaigns to proxies of firm value (p. 97)
Yen, 2009	Each goal should be represented by a set of metrics that measure the effectiveness of business process (p. 867)
Choong, 2013b	Formally defined. "A *metric* is a fraction that is expressed as a standard or unit of measurement that can be in financial or nonfinancial form. In essence, it is a measure deflated by a common (consistent) denominator—that is not subjected to change easily."
Melnyk *et al.*, 2014	A metric is more than a performance measure. In the language used in this study, a metric has three distinct elements: 1. A performance measure that quantifies what is happening. 2. A performance standard, or target, that indicates what is considered good & bad performance so guides the direction of the organisation. 3. Consequences relating to being on, below or above target. While a measure is informative, a metric is critical from a business perspective. All three elements are necessary; removing any one of these elements effectively cripples the metrics and diminishes its effectiveness from a business perspective. For us, metrics are the fundamental building blocks of a PMM system (p. 175).
Indicators	
Rogers, 1990	Outcome-related indicators are used to provide indication of improved performance or activities that achieved/not achieved targets
Eccles & Pyburn, 1992	Financial measures show up in multiple financial reports (systems) but they provide poor indicator of performance. Hence new indicators like customer satisfaction and an organization's status & professionalism) should provide more

	meaningful performance information
McGee, 1992	Measures & metrics operate as leading indicators of performance against strategic goals & initiatives. Measures & metrics are used interchangeably
Fry, 1995	Primary indicators of performance are budget variances (p. 944)
Neely *et al*., 1995	Used to substantiate the meaning of measure in relating to performance
Flapper *et al.*, 1996	Indicators are important for everyone inside an organisation, as they tell what has to be measured and what are the control limits the actual performance should be within. Indicators are expressed in monetary & non-monetary units (p. 27). Indicators include reals (e.g. cost, physical or per unit) & intrinsic values (p. 30)
Kaplan & Norton, 1996	Not defined. Lag indicators, e.g. return on investment; lead indicators, e.g. customers satisfied survey
Harbour, 1997	Indicators are comparative performance metric used to address a specific issue (p. 8)
Nadzam & Nelson, 1997	Not defined. Indicator could mean quantitative & qualitative such as job satisfaction, increased physicians/staff efficiency, quality of improvement
Atkinson *et al*., 1997; Atkinson, 1998	Indicator is mentioned once as key performance indicator (KPI)
Neely, 1998	Formally defined. Performance indicators can be defined to identify financial & nonfinancial performance. Also mentioned that measures & metrics can be used to provide leading and other indicators to evaluate business performance
Marshall *et al*., 1999	Indicators are developed to measure performance (p. 13). Indicators are used to guide and allocate resources in aligning strategies. Some of the common indicators used by public sector bodies are indicators of education, economy & public safety
Gross *et al*., 2000	Indicators are developed for specific cases and are based on established PMS to ensure that the indicators developed are well-defined & must convey information related to the goals and objectives of activities to be measured
Forza & Salvador, 2000	Not defined. Performance indicators are used to highlight performance. Indicators have the following characteristics: cost, quality, fast delivery, on-time delivery & flexibility.
UK CAG, 2000	Indicators are used interchangeably with performance measures. Using the Custom & Excise agency as an example, an indicator comprises of two elements: (1) “sized” element – total value of drugs physically seized; and (2) “additional preventive” – estimate of value of additional drugs that successful investigations have prevented individuals/organizations from importing due to (1).
US DOE, 2000 (revised 2012)	Formally defined. “Performance indicator is a parameter useful for determining the degree to which an organization has achieved its goals. Also, a quantifiable expression used to observe and track the status of a process. Also, the operational information that is indicative of the performance or condition of a facility, group of facilities, or site.”
Kueng *et al*., 2001	Formally defined “.. performance indicators reflect the stakeholders’ interests; each individual performance indicator is part of an integrated system.” (p. 14), nonfinancial indicators such as IT performance, future environmental factors are important leading indicators
Toni & Tonchia, 2001	Based on prior studies, indicators are based on four distinct performance dimensions: (1) costs/productivity; (2) time; (3) flexibility; and (4) quality.
Neely, 2002	Same as Neely, 1998
Kanji, 2002	Performance indicators provide accountability & intend to drive future resource allocation decisions. Indicators can be financial & nonfinancial (e.g. customer satisfaction) but most financial indicators are lag indicators
Bourne *et al*., 2003	Performance indicators are designed from the objective & decision variables (factors which the decision maker can vary in pursuit of the business objectives) (p. 14)
Drongelen & Fisscher, 2003	Indicator is synonymous (used in adjective) with metric

Matos *et al*., 2003	Very specialised case of development of indicators, i.e. in a water service industry. Indicators developed are based on relation of materials, quantities, qualities, environments, etc.
Bourne *et al*., 2005	Not defined. Indicators are described as lead, formal & informal; their usage depends on managers' discretion. Indicators play this role (p. 13)
Denton 2005	Indicators are referred to as lag and lead measures of performance. Leading measures/indicators are outcome are those that precede, anticipate, or impact the future, e.g. customer satisfaction. Lagging measures/indicators are static data based on historical outcomes or a set of events that have occurred "after the fact", for example accounting reports. They are of little use in predicting the future or in identifying the root causes of poor performance (p. 281)
Walsh, 2005	Dumb measures fail to measure & only; taxonomy of appropriate measures must be developed to measure relevant attributes of performance as defined by strategy.
Gunasekaran & Kobu, 2007	Adopt definition of Neely *et al*., 1995
Julnes, 2007	Indicators are synonymous with measures
Franceschini *et al*., 2007, 2008	In general an indicator (I) is an application that – according to the representation-target – homomorphically maps the empirical manifestations into corresponding symbolic manifestations (see Figure 1) (taken from Franceschini *et al.*, 2006). They then went on to provide an indicator based on mathematical theory
Taticchi *et al.*, 2012	Not defined, implied KPI
Choong, 2013b	Indication provided. A good indicator to be (1) relevant to the goal; (2) easily measured and understandable to users; and (3) provide reliable information, either in quantitative or qualitative (characteristic) form—financial or nonfinancial.
Non-PMS&PMMS authors	
Measures	
IEEE, 1983	Formally defined. "A measure is to ascertain or appraise by comparing to a standard."
SEI, 1994	Formally defined. "A standard or unit of measurement; the extent, dimensions, capacity, etc., of anything, especially as determined by a standard; an act or process of measuring; a result of measurement."
Ragland, 1995	Adopt definition of IEEE, 1983 & SEI, 1994
UK CAG, 2000	Formally defined. Performance measure is defined as "a measurable indicator of performance which may or may not be targeted." (p. 61). Performance measures must first be defined to ensure they are appropriate to measure things in accordance to their needs. "Performance measures assist organisations to communicate objectives & priorities, measure what they deliver and report publicly on what they have achieved." (p. 10). Good performance measures can assist readers/stakeholders of the annual report to understand and access those aspects of performance which are central to the organization/unit's success (p. 16). Performance measures can be quantitative (e.g. size or value of drug seized) or qualitative (e.g. detection of fraud on an insurance claim). Measures need to be complex to be informative & simple measures are not indicative of performance
Trochim, 2006	Indication provided. A measure is quantitative & expressed in 'wholeness'
Serrat, 2010	Mentioned performance measures found in literature in which the purposes & characteristics of different performance measures are discussed. For example the purpose of control measure is to regulate inputs
Metrics	
IEEE, 1990	Formally defined. "A metric as a quantitative measure of the degree to which a system, component, or process possesses a given attribute."
SEI, 1994	Formally defined. "A calculated or composite indicator based upon two or more measures. A quantified measure of the degree to which a system, component, or process possesses a given attribute"
Ragland, 1995	Adopt definition of IEEE, 1990 & SEI, 1994

Perkins, 2003	Formally defined. "Metrics are measurements of different aspects of an endeavor that help us determine whether we are progressing toward the goal of that endeavor." (p 9). "These metrics are defined and associated with their appropriate questions and goals. Each metric requires two or more measurements (p. 10)
Univ. California, 2003	Performance metrics should be constructed to encourage performance improvement, effectiveness, efficiency, and appropriate levels of internal controls. They should incorporate "best practices" related to the performance being measured and cost/risk/benefit analysis, where appropriate.
Trochim, 2006	A *metric* as a quantitative expression, and it is based on a standard or unit of measurement, like cost per unit. Because of this a metric is more precise than a measure because it needs to be defined or specially developed based on a performance objective that is relevant to the stakeholders
Mckinsey, 2007	Should move away from traditional accounting metrics to use new metrics. New metrics refer to things (activities) like knowledge, relationships, reputations, and other intangibles created by talented people take sufficient notice of the real engines of wealth today
Indicators	
IEEE, 1990	Formally defined. Indicator as "a device or variable that can be set to a prescribed state based on the results of a process or the occurrence of a specified condition."
Ragland, 1995	Adopt definition of IEEE, 1990
Virginia Tech, 1997	"An indicator is a tool that helps you and your organization knows how far your project is from achieving your goals and whether you are headed in the right direction. Choosing the right indicator is essential for effectively evaluating your progress. The right indicator should (1) be relevant to the project; (2) be easily understandable to everyone interested in your project; (3) be easily measured; and (4) provide reliable information."
Poll, 2005	Very specific discussion on indicators – in libraries. Provided elaborate factors for the design of good indicators for use in libraries based on ISO
Trochim, 2006	Indicators are used to infer things more generally than measures or metrics and hence, indicators are not very useful for measurement

Note: CAG = Comptroller & Auditor General, DOE = Deprt. of Energy, GAO = General Accounting Office

Table III. Classifying measuring attributes discussed by PMS&PMMS authors in accordance with common themes, in terms of number of articles reviewed (Note: *N* taken from Table I, numbers appearing in each of the common theme are non-mutually exclusive)

	N	Quantitative	Qualitative	Financial	Nonfinancial	Whole	Fraction	Std.
Measure	52	49	20	49	45	41	41	6
Metric	17	17	6	17	18	12	15	3
Indicator	32	32	15	24	29	26	26	4
Driver	3	3	3	3	3	1	1	0

Table IV. Matching meaningfulness (represented by number where 1 indicates affirmative and 0 indicates no affirmation) of definitions/indications and expressions of measuring attributes by PMS&PMMS authors with Non-PMS&PMMS authors

	Quantitative	Qualitative	Financial	Nonfinancial	Whole	Fraction	Std.
Measure							
PMS&PMMS authors							
US DOE, 2000	1	0	1	1	1	0	0

US GAO, 2000	1	0	1	1	1	0	0
US DOE, 2012	1	0	1	1	1	0	0
US GAO, 2012	1	0	1	1	1	0	0
Choong, 2013b	1	0	1	1	1	0	0
Total	5	0	5	5	5	0	0
Non-PMS&PMMS authors							
IEEE, 1983	1	0	0	1	0	1	1
SEI, 1994	1	0	0	1	0	1	1
Trochim, 2006	1	0	1	1	1	0	0
Total	3	0	1	3	1	2	2
Metric							
PMS&PMMS authors							
Melnyk *et al.*, 2004	1	1	1	1	1	1	1
Choong, 2013b	1	0	1	1	0	1	1
Total	2	1	2	2	1	2	2
Non-PMS&PMMS authors							
IEEE, 1990	1	0	0	1	0	1	1
SEI, 1994	1	0	0	1	0	1	1
University of California, 2003	1	0	1	1	0	1	1
Trochim, 2006	1	0	1	1	0	1	1
Total	4	0	2	4	0	4	4
Indicator							
PMS&PMMS authors							
Flapper *et al.*, 1996	1	1	1	1	1	1	1
US DOE, 2000	1	0	0	1	1	1	0
Choong, 2013b	1	1	1	1	1	0	0
Total	3	2	2	3	3	2	1
Non-PMS&PMMS authors							
Virginia Tech, 1997	1	1	1	1	1	1	0
Poll, 2006	1	1	1	1	1	0	0
Trochim, 2006	1	1	1	1	1	0	0
Total	3	3	3	3	3	1	0

Table V. Concept types, based on literature & best practices

Concept types	Measure	Metric	Indicator
FINANCIAL	√	√	√
NONFINANCIAL	√	√	√

QUANTITATIVE	√	√	√
QUANLITATIVE			√
WHOLE	√		√
FRACTION		√	

Table VI. Measurement in terms of hard sciences (with emphasis of MT) and social sciences (with emphasis of the RTM)

	Hard sciences (MT)	Social sciences (RTM)	Discussion
Philosophical			
Purpose	Meaningfulness of measurement	Meaningfulness of measurement	Although the words used differs both perspectives realize that there is no exactness (100% accuracy) of measurement & the meaningfulness of measurement is deemed as prerequisite for good measurement.
Object	Specifies that object must be physical & observable (thing)	Often referred to as data. But this can be construed as object & that it can be numbers or numerals	Although the definition of object in hard science is specific it is very restrictive. The RTM is not specific as not all items to be measured is data, e.g. intangibles (abstract). The concept gap of object appears to have been bridged as recent philosophical writers from both perspectives consider 'thing' to be measured as an object & it can consist of number or in abstract (conceptual) term.
Mathematical			
Axioms	The foundation of measurement (MT) is specified in axiomatic terms	The foundation of measurement is not specified in axiomatic terms; however many aspects of the RTM is now aximotized	MT is rooted in concepts (e.g. value set, elements, relations among systems) & mathematical foundation. Conceptually RTM is based on the assignment of numbers (numerals) & only not all aspects of RTM are mathematically articulated, and hence measurement in some instances are subjected to interpretations & misconceptions.
Empiricism			
Measurement process	Measurement process is validated for accuracy through repeated measurement using same measurement system by calibration (use of an instrument) Since there will be many values in an experiment, the precision of measurement is determined relative to an established measurement standard.	One can initiate a set of rules (any rules) to express a number or numerals to establish a measuring system (model). A model is usually operationalized by a scale system (SoM) that mediates between theories & object to produce out-comes. Thus measurement is riddled with problems as it can be operationalized in an almost infinite number of ways.	Recent philosophical writings conceptually divide measurement procedures into 2 levels: (1) a concrete process involving interactions between an object of interest, an instrument & the environment; and (2) a theoretical and/or statistical representation of that process. Although (2) is applicable to social sciences, issues remain concerning to what roles do theoretical & statistical assumptions about a measurement process play in establishing the representational adequacy of outcomes?
Outputs	Are represented by numbers	Are represented by numbers	The merits of measurement in hard sciences are that outputs are represented in

(outcomes)	in terms of accuracy, precision & error (uncertainty)	(observables) & numerals (phenomenon). No indication of accuracy, precision or error.	terms of accuracy, precision & error, while that of the social sciences are that outputs are differentiated in terms of objects based on observables & phenomenon.

Table VII. Revised concept types, based on measurement theories

Concept types	Measure	Metric	Indicator
FINANCIAL	√	√	√
NONFINANCIAL	√	√	√
QUANTITATIVE	√	√	√
QUANLITATIVE			√
WHOLE	√		√
FRACTION		√	

Table VIII. Various M&PM elements of the current & new M&PM

A **Purchases & Inventory balances** Current & New M&PM

Product A Fauteuil rounded back & guided curved detailed chair

	Purchase			Sale/Issue			Balance			
Date	Qty	Cost	Total	Qty	Cost	Total	Qty	Cost	Total	Av. Cost
Jan 1	16	$100.00	$1,600.00				16	$100.00	$1,600.00	$100.00
Jan 4	24	$120.00	$2,880.00				24	$120.00	$2,880.00	$120.00
			$4,480.00				40	$112.00	$4,480.00	$112.00
Total (Product A&B)			$8,640.00				60			$144.00

Product B Bergère tub-shaped back chair

	Purchase			Sale/Issue			Balance			
Date	Qty	Cost	Total	Qty	Cost	Total	Qty	Cost	Total	Av. Cost
Jan 2	12	$200.00	$2,400.00				12	$200.00	$2,400.00	$200.00
Jan 6	8	$220.00	$1,760.00				8	$220.00	$1,760.00	$220.00
			$4,160.00				20	$208.00	$4,160.00	$208.00

Current M&PM:	Explanation (comment)
Product A On Jan 1, 16 units were purchased at the purchase cost of $100 per unit & the total purchase amount of $1,600 is called purchases (total purchases). Any purchases will go straight into inventory balance. On Jan 4, 24 units were purchased at a unit cost of $120, thus giving a total purchase of $2,880. Again there were no issues/sales and the number of units, unit cost and the purchase amount will go into inventory balance. Thus, at Jan 4, the total inventory value (balance) stood at $4,480 with an average unit cost of $112 (i.e. the mean of total inventory value).	Too many terms: *units, purchase cost, cost per unit (unit cost), total purchase amount (purchases or total purchases), unit cost, number of units, total inventory value (balance), average unit cost.* The use of average unit cost means that the average unit cost changes each time the purchase unit cost changes. Although is meaningful but the value is considered to provide only a good enough (second best) measurement. That is, the measured cost is an *objective indicator*.

Product B	Too many terms (see above)
On Jan 2, 12 units were purchased at a unit purchase cost of $200 to arrive at the total purchases of $2,400. These numbers will go straight into inventory balance. On Jan 6, the units purchased was 6 at a unit cost of $220, thus giving a total purchase of $1,760. These numbers will go into inventory balance. Thus, at Jan 6, the total inventory value stood at $4,160 with an average unit cost of $208.	The use of average unit cost although is meaningful but is considered to provide only a good enough (second best) measurement. That is, the measured cost is an *objective indicator*.
Total (*Product A* + *Product B*) Total purchases & inventory values are $8,640 comprising of 60 units or average unit cost of $144 ($8,640/60).	Total purchases & inventory values are meaningful. However the average unit cost is nonsense because there are different average unit cost simply because the number and total value change each time a new purchase with different unit cost is made.
New M&PM:	
Product A On Jan 1, the 16 units is called quantities, $100 is purchase measure & $1,600 is called total purchase measure. This numbers will also appear at inventory balance. On Jan 4, the quantity, measure and total measure for both purchases and inventory are 24, $120 & $2,880 respectively. Thus at Jan 4, the quantities, measure and total inventory measure will need to be separated in terms of 16, $100, $1,600 & 24, $120, $2,880 due to different purchase measure.	Only two terms used: *quantity(ies) & measure(s)* to represent unit or total, which can be used in different contexts, e.g. purchases & inventory. This is neat & less: thus, making things simple. The inventory is valued at FIFO basis & this avoids the averaging of unit cost, thus giving more precise measurement to purchases & inventory as in each purchase, its measures (unit & total) & quantities are matched with the corresponding inventory measures & quantities. The quantities & measures conform to the properties of the measuring attributes & total measure has additivity & multiplication properties; thus the measurement is meaningful.
Product B	Only two terms used (see above)
On Jan 2, 12 quantities @ $200 measure were purchased, giving a total measure of $2,400. All these numbers go straight into inventory balance. On Jan 8, the quantity, measure and total measure for both purchases and inventory are 8, $220 & $1,760 respectively. Thus at Jan 6, the quantities, measure and total inventory measure will need to be separated due to different purchase measure (see Table).	The inventory is valued at FIFO basis & the objectivity of matching of quantities, unit & total measures with purchases is possible and thus measurement is more precise. The quantities & measures conform to the properties of the measuring attributes; thus the measurement is meaningful (see above).
Total (*Product A* + *Product B*) Total purchases & inventory measures are $8,640 comprising of (1) Product A: 16 quantities @ $100 = $1,600 & 24 quantities @ $120 = $2,880; (2) Product B: 12 quantities @ $200 = $2,400 & 8 quantities @ $220 = $1,760.	Quantities, unit & total measures for purchases are meaningfully matched to inventory as the numbers have mathematical measurement properties (see above).

B **Sales** Current & New M&PM

Product A Fauteuil rounded back & guided curved detailed chair

Date	Qty	Price	Total
Jan 8	15	$160.00	$2,400.00
Jan 11	10	$160.00	$1,600.00
Jan 23	12	$170.00	$2,040.00
	37		$6,040.00

Product B Bergère tub-shaped back chair

Date	Qty	Price	Total
Jan 9	4	$400.00	$1,600.00
Jan 18	6	$400.00	$2,400.00
	10		$4,000.00

C **Issues/sales & Inventory balances** Current M&PM

Product A Fauteuil rounded back & guided curved detailed chair

		Sale/Issue			Balance		
Date		Qty	Av. Cost	Total	Qty	Av. Cost	Total
Jan 8	Sales	15	$112.00	$1,680.00	25	$112.00	$2,800.00
Jan 11	Sales	10	$112.00	$1,120.00	15	$112.00	$1,680.00
Jan 11	W/off	1	$112.00	$112.00	14	$112.00	$1,568.00
Jan 23	Sales	12	$112.00	$1,344.00	2	$112.00	$224.00
		38	$112.00	$4,256.00			

Product B Bergère tub-shaped back chair

		Sale/Issue			Balance		
Date		Qty	Av. Cost	Total	Qty	Av. Cost	Total
Jan 9	Sales	4	$208.00	$832.00	16	$208.00	$3,328.00
Jan 18	Sales	6	$208.00	$1,248.00	10	$208.00	$2,080.00
		10	$208.00	$2,080.00			

D **Issues/sales & Inventory balances** New M&PM

Product A Fauteuil rounded back & guided curved detailed chair

	Purchase	Sale/Issue			Balance		
Date		Qty	Cost	Total	Qty	Cost	Total
Jan 8	Sales	15	$100.00	$1,500.00	1	$100.00	$100.00
Jan 8					24	$120.00	$2,880.00
							$2,980.00
Jan 11	Sales	1	$100.00	$100.00			
Jan 11	W/off	1	$120.00	$120.00	23	$120.00	$2,760.00
Jan 11	Sales	9	$120.00	$1,080.00	14	$120.00	$1,680.00
Jan 23	Sales	12	$120.00	$1,440.00	2	$120.00	$240.00
		38	$111.58	$4,240.00			

Product B Bergère tub-shaped back chair

	Purchase	Sale/Issue			Balance		
Date		Qty	Cost	Total	Qty	Cost	Total
Jan 9	Sale	4	$200.00	$800.00	8	$200.00	$1,600.00
					8	$220.00	$1,760.00
							$3,360.00
Jan 18	Sale	6	$200.00	$1,200.00	2	$200.00	$400.00
					8	$220.00	$1,760.00
		10	$200.00	$2,000.00			$2,160.00

Current M&PM:	Explanation (comment)
Product A	Too many terms (see above).
On Jan 8, 15 units were sold, and the issue cost would be based on the average unit cost of $112, and the inventory balance was reduced to 25 units @ $112, thus making a total inventory balance of $2,800.	The use of average unit cost means that this cost will be used for all issues (sales, write-off, obsolescence, etc.) and this makes measurement

On Jan. 11, when checking for stock one item was found to be broken beyond repair and was written off based on the average unit cost, and the sale of 10 units would be based on the average unit cost, and the inventory balance was reduced to 14 units @ $112, thus making a total inventory balance of $1,568. On Jan. 23, the sale of 12 units would be based on the average unit cost of $112.50, and the inventory balance of $224.	meaningless.
Product B On Jan. 9, 4 units were issued for an average unit cost of $208 and the inventory balance would become 16 units @ $208 and a total value of $3,328. On Jan. 18, 6 units were sold @ $208 & the inventory balance was reduced to $2,080.	Too many terms (see above). Since the use of average unit cost here is solely for issue & hence it offers a limited meaningfulness of measurement (i.e. only a good enough (second best)) because average here lacks precision (i.e. measurement is *objective indication*).
New M&PM:	
Product A On Jan 8, 15 units were sold, and the issue cost was based on the FIFO measure of $100, and the inventory balance was reduced to one quantity @ $100. All the inventory belonging to the 24 quantities @ $120 remains unchanged. The total inventory balance was $2,980. On Jan. 11, when checking for stock one item was found to be broken beyond repair and was written off based on the purchase metric of $120 (the cost of that item purchased), and the sale of 10 quantities would have to be spilt into one quantity @ $100 & 9 quantities @ $120. Thus the inventory balance was 14 quantities @ $120, making a total inventory measure of $1,680. On Jan. 23, the sale of 12 quantities @ $120 and the balance of 2 quantities would be based on $120, and the inventory balance stood at $240.	All issue measures were matched to actual items to be sold including the write-off, which was accounted for separately from sales. Also, the quantities & measures conform to the properties of the measuring attributes & thus the measurement is meaningful.
Product B On Jan. 9, 4 quantities were issued for the FIFO measure @ $200 and the inventory was reduced to 8 quantities @ $200. All the inventory belonging to the 8 quantities @ $220 remains unchanged. On Jan. 18, 6 quantities were sold @ $200 and the inventory balance were reduced to 2 quantities @ $200. All the inventory belonging to the 8 quantities @ $220 remains unchanged. Therefore, the total inventory balance stood at $2,160.	The issue measures were matched to actual items to be sold. Also, the quantities & measures conform to the properties of the measuring attributes & thus the measurement is meaningful.

Table IX. Income statement of Product A & Product B

	Current M&PM								**New M&PM**						
	Product A				Product B				Product A				Product B		
	$	$	%		$	$	%		$	$	%		$	$	%
Income statement (abstract)															
For the month ended 31 January 2018															
Sales		6,040				4,000				6,040				4,000	
Less: Cost of Goods sold															
Purchases	4,480				4,160				4,480				4,160		
Less: end inventory	224				2,080				240				2,160		
		4,256				2,080				4,240				2,000	
Inventory w/off		112													
Gross income		1,672	0.28			1,920	0.48			1,800	0.30			2,000	0.50
Less: Expenses															
Admin & operating expenses		800				800				800				800	
Inventory w/off										112					
Net income		872	0.14			1,120	0.28			888	0.15			1,200	0.30

Supplimentary information (PMMS & measures, metrics & indicators)															
Sale units/quantities	*Measure*	37				10				37				10	
Write-off units/quantities	*Measure*	1								1					
Purchase units/quantities	*Measure*	40				20				40				20	
Inventory units/quantities	*Measure*	2				10				2				10	
Mean Sales Measure (price)	*Obj. Indicator*									163				400	
Std. Dev. Sale Measure (price)	*Obj. Indicator*									5.77				0.00	
Sales Measure (price) Index	*Metric*									1.06				1.00	
Mean Purchase Measure (cost)	*Obj. Indicator*									112				208	
Std. Dev Purchase measure (cost)	*Obj. Indicator*									14.14				14.14	
Purchase Measure (cost) index	*Metric*									1.20				1.10	
Direct income per sales quantity	*Metric*									49				200	
Mean Sales-to-Purchase	*Metric*									1.48				1.90	

Table X. respondents of a simple survey consisting of a questionnaire with pictures of the two chairs and how customers or expected customers have ranked (prefer) these chairs in terms of ‘appeal’, ‘comfort’, ‘style’, ‘versatile’, and ‘space’

Product	**A**	**B**	***N***
Appeal (attractiveness)	300	200	500
Comfort	170	330	500
Style	240	260	500
Versatile (be used for resting, welcoming, dining, etc.)	220	280	500
Space friendly (can be placed in small homes)	400	100	500
N score	1,330	1,170	2,500
N%	53	47	

D **Issues/sales & Inventory balances** New M&PM

Product A Fauteuil rounded back & guided curved detailed chair

Product B Bergère tub-shaped back chair

Date	Purchase	Sale/Issue Qty	Cost	Total	Balance Qty	Cost	Total	Date	Purchase	Sale/Issue Qty	Cost	Total	Balance Qty	Cost	Total
Jan 8	Sales	15	$100.00	$1,500.00	1	$100.00	$100.00	Jan 9	Sale	4	$200.00	$800.00	8	$200.00	$1,600.00

Jan 8					24	$120.00	$2,880.00						8	$220.00	$1,760.00
							$2,980.00								$3,360.00
Jan 11	Sales	1	$100.00	$100.00				Jan 18	Sale	6	$200.00	$1,200.00	2	$200.00	$400.00
Jan 11	W/off	1	$120.00	$120.00	23	$120.00	$2,760.00						8	$220.00	$1,760.00
Jan 11	Sales	9	$120.00	$1,080.00	14	$120.00	$1,680.00			10	$200.00	$2,000.00			$2,160.00
Jan 23	Sales	12	$120.00	$1,440.00	2	$120.00	$240.00								
		38	$111.58	$4,240.00											

	Current M&PM						New M&PM					
	Product A			Product B			Product A			Product B		
	$	$	%	$	$	%	$	$	%	$	$	%
Income statement (abstract)												
For the month ended 31 January 2018												
Sales		6,040			4,000			6,040			4,000	
Less: Cost of Goods sold												
Purchases	4,480			4,160			4,480			4,160		
Less: end inventory	224			2,080			240			2,160		
		4,256			2,080			4,240			2,000	
Inventory w/off		112										
Gross income		1,672	0.28		1,920	0.48		1,800	0.30		2,000	0.50
Less: Expenses												
Admin & operating expenses		800			800			800			800	
Inventory w/off								112				
Net income		872	0.14		1,120	0.28		888	0.15		1,200	0.30
Supplimentary information (PMMS & measures, metrics & indicators)												
Sale units/quantities	*Measure*	37			10			37			10	
Write-off units/quantities	*Measure*	1						1				
Purchase units/quantities	*Measure*	40			20			40			20	
Inventory units/quantities	*Measure*	2			10			2			10	
Mean Sales Measure (price)	*Obj. Indicator*							163			400	
Std. Dev. Sale Measure (price)	*Obj. Indicator*							5.77			0.00	
Sales Measure (price) Index	*Metric*							1.06			1.00	
Mean Purchase Measure (cost)	*Obj. Indicator*							112			208	
Std. Dev Purchase measure	*Obj. Indicator*							14.14			14.14	

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(cost)			
Purchase Measure (cost) index	*Metric*	1.20	1.10
Direct income per sales quantity	*Metric*	49	200
Mean Sales-to-Purchase	*Metric*	1.48	1.90

Soft indicators

Product	**A**	**B**	***N***
Appeal (attractiveness)	300	200	500
Comfort	170	330	500
Style	240	260	500
Versatile (be used for resting, welcoming, dining, etc.)	220	280	500
Space friendly (can be placed in small homes)	400	100	500
N score	1,330	1,170	2,500
N%	53	47	
Score	1,330	2,340	
Mean rank	1835	2340	
SD	88.2	88.2	
Skew	0.87784	-0.87784	