ISA Transactions 52 (2013) 335–341

Contents lists available at SciVerse ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research Article

Chattering free adaptive multivariable sliding mode controller for systems with matched and mismatched uncertainty

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ARTICLE INFO

Article history:
Received 28 September 2012
Received in revised form
4 December 2012
Accepted 19 December 2012
Available online 26 January 2013
This paper was recommended for publication by Dr. Jeff Pieper

Keywords:
Sliding mode control
Mismatched uncertainty
Adaptive tuning
Vertical take-off and landing (VTOL) aircraft

ABSTRACT

In this paper, a chattering free adaptive sliding mode controller (SMC) is proposed for stabilizing a class of multi-input multi-output (MIMO) systems affected by both matched and mismatched types of uncertainties. The proposed controller uses a proportional plus integral sliding surface whose gain is adaptively tuned to prevent overestimation. A vertical take-off and landing (VTOL) aircraft system is simulated to demonstrate the effectiveness of the proposed control scheme.

1. Introduction

Physical systems suffer from performance degradation and instability due to uncertainties existing in nature which can be broadly classified into matched and mismatched types. Uncertainties acting on the system through the input channel are called matched uncertainties, whereas perturbations in the system parameters are termed as mismatched uncertainties. Classical control techniques like adaptive control [1], optimal control [2], sliding mode control [3] and intelligent control methods like fuzzy logic control [4] have been extensively used in control systems perturbed by matched uncertainty. Among these methods, sliding mode control has received wide acceptance owing to its robustness and simplicity. However, designing sliding mode controllers for systems perturbed by the mismatched type of uncertainty still remains a challenge to the research community. The difficulty lies in the fact that the dynamics of the uncertain system are affected even after reaching the sliding mode.

Active research is continuing in the control community for developing sliding mode controllers for multi-variable systems affected by mismatched type of uncertainty [5–8]. One significant research finding is that the stability of the system is guaranteed if the system trajectory is driven to a bounded region [9–11]. Hence to ensure asymptotic stability, restriction of keeping an upper bound on uncertainties is imposed in most of the research works. By designing a sliding mode controller for certain states of the system which are provided as inputs to a reduced order system can take care of the mismatched uncertainties. However, limitation of this method is that uncertainties should lie in the range space of certain matrix of the nominal system [3]. A fuzzy logic-based sliding mode controller proposed in [11] was successful in achieving quadratic stability for systems with mismatched uncertainty. Even this method could handle mismatched uncertainty of a certain form only provided its bound was known a priori [12–14]. By introducing two sets of switching surfaces for the subsystems and hence reducing the rank of the uncertainty, asymptotic stability was achieved in [15]. Dynamic output feedback sliding mode controllers were attempted in [16] and nonlinear integral type sliding surface was used to deal with mismatched uncertainties in [17]. All these works required prior knowledge about the upper bound of the mismatched uncertainty which is in general difficult to obtain. Hence, a strategy to obtain the upper bound of the system uncertainty or a method that does not require this knowledge is needed. The adaptive sliding mode controller proposed in [18–20] provided a solution to this problem. However, this adaptive method yielded gains which were overestimated in many cases giving rise to large control efforts and high chattering [21,22].

Although the sliding mode controller guarantees robustness, chattering is its main drawback. Chattering is the high frequency bang-bang type of control action which leads to premature wear and tear or even breakdown of the system being applied to. Chattering is caused due to the fast dynamics which are usually neglected in the ideal model utilizing digital controllers with a finite sampling rate. This disadvantage of chattering could be reduced by techniques such as nonlinear gains, dynamic

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http://dx.doi.org/10.1016/j.isatra.2012.12.007

extensions or by using more recent strategies such as higher order sliding mode control. In [23,24] an algorithm has been proposed based on the block control and quasi-continuous higher order sliding mode techniques for nonlinear systems subjected to mismatched uncertainty. The core idea that drives the higher order sliding mode control is that it keeps the sliding surface as well as its higher order derivatives to zero. The higher order sliding mode controller ensures good tracking performance, robustness and finite time stabilization of the controlled system. Past few decades witnessed tremendous improvement in the second-order sliding mode (SOSM) controller. Twisting and super twisting [25], suboptimal [26,27], drift algorithm [28–30] are the existing SOSM control algorithms. Nonlinear sliding surface is mostly used to design a second-order sliding mode controller for uncertain systems. Wen and Cheng [19] proposed an adaptive variable structure controller for a class of dynamic systems with matched and mismatched perturbations. The controller proposed by Wen and Cheng [19] achieved asymptotic stability without having prior knowledge about the upper bounds of perturbations. However, this control scheme suffered the drawback of severe chattering in the control input. Similar kind of problem was cited in [20] too.

The major contributions of this paper are the following:

- An adaptive integral sliding mode controller (SMC) is proposed for stabilization of a class of MIMO systems affected by both matched and mismatched uncertainties.
- An adaptive tuning law is designed and by using that law the mismatched perturbations are rejected during the sliding mode while ensuring asymptotical stability of the overall system.
- The adaptive tuning law ensures that there is no gain overestimation with respect to the unknown uncertainties.
- The proposed controller eliminates chattering in the control input and hence is suitable for practical applications.

The design procedure can be divided into two steps. The first step is to build the sliding surface using an adaptive technique that eliminates the need of prior knowledge about the upper bounds of system perturbations except for those at the input. In the next step, a derivative control law is developed which contains the discontinuous sign function. The actual control is obtained by integrating the derivative control and thereby the control becomes continuous, smooth and chattering free.

The outline of this paper is as follows. Section 2 describes the system and the problem is formulated. The design procedure for the proposed adaptive integral sliding mode controller (SMC) is explained in Section 3. Effectiveness of the proposed controller is demonstrated in Section 4 by performing simulation studies. Conclusions are drawn in Section 5.

2. System description and problem formulation

Let us consider the following dynamic system:

$$\dot{x}(t) = Ax + B[u + \xi(t,x)] + p(t,x) \tag{1}$$

where $x \in R^n$ is the state vector and $u \in R^m$ is the control input. Moreover, A and B are known matrices with proper dimension and B has full rank. Furthermore, $\xi(t,x)$ and $p(t,x)$ represent the unknown matched and mismatched uncertainties, respectively. The objective of the proposed control scheme is to design an adaptive chattering free sliding mode scheme for a class of MIMO systems with matched and mismatched perturbations. The design of the sliding mode controller involves two key steps, viz. (i) designing the sliding surface and (ii) designing the control input which obeys the reaching law property that the sliding manifold approaches zero in finite time.

The sliding surface σ is designed as

$$\sigma = Sx \tag{2}$$

where $S \in R^{m*n}$ is a constant matrix designed by selecting the eigenvalues suitably (all negative) to make the system stable [31]. By using the coordinate transformation $\left[\begin{smallmatrix} z \\ \sigma \end{smallmatrix}\right] = Mx$, where the transformation matrix $M = \left[\begin{smallmatrix} W_g \\ B_g \end{smallmatrix}\right]$, Eq. (1) can be transformed to

$$\begin{aligned} \dot{z} &= W_gAWz + W_gAB\sigma + W_gp(t,x) \\ \dot{\sigma} &= B_gAWz + B_gAB\sigma + u + \xi(t,x) + Sp(t,x) \end{aligned} \tag{3}$$

here $S = B_g$ and W_g, B_g satisfy $B_gB = I_m$, $B_gW = 0$, $W_gB = 0$, and $W_gW = I_{n-m}$. The matrix W is chosen in such a way that $J = W_gAW$ has the desired eigenvalues [32], where J is a symmetric matrix. It can be verified that

$$M^{-1} = [W \;\; B] \tag{4}$$

and it can be observed that $x = Wz + B\sigma$.

When the system is in the sliding mode, it satisfies the conditions $\sigma = 0$ and $\dot{\sigma} = 0$. Then, the perturbation term in Eq. (3) becomes $W_gp(t,x) = W_gp(t,Wz) = p_r(t,z)$. Now the reduced order equation becomes

$$\dot{z} = Jz + p_r(t,z) \tag{5}$$

If the mismatched perturbation $p_r(t,z)$ satisfies $\|p_r(t,z)\| \le \phi_r\|z\|$, where $\phi_r < -\lambda_{\max}(J)$, $\lambda_{max}(J)$ being the maximum eigenvalue of the J matrix, then by choosing the Lyapunov function $V = (1/2)\|z\|^2$, it can be proved that [19,20]

$$\dot{V} = z^TJz + z^Tp_r(t,z) \le \lambda_{\max}(J)\|z\|^2 + \phi_r\|z\|^2 = [\lambda_{\max}(J) + \phi_r]V < 0 \tag{6}$$

The above condition means that the system will be asymptotically stable once the sliding mode is reached. However, it is obvious from the above discussion that the sliding surface design requires the bounds of the uncertainties to be known a priori [33] which is extremely difficult practically. Hence, the need arises for designing the sliding surface in such a way that prior knowledge about the bounds of the uncertainties is not required.

2.1. The adaptive sliding surface design

Let us consider the sliding surface

$$\sigma = S(t)x \tag{7}$$

The sliding coefficient matrix $S(t) \in R^{m*n}$ can be designed as [19]

$$S(t) = B^+ + N(t)W_g \tag{8}$$

where $B^+ = (B^TB)^{-1}B^T \in R^{m*n}$ is the Moore–Penrose pseudo-inverse [34] of B and $N(t) \in R^{m*n}$ is designed using an adaptive technique to be explained later. Let us consider the transformation

$$\begin{bmatrix} z \\ \sigma \end{bmatrix} = \begin{bmatrix} W_g \\ S(t) \end{bmatrix} x = M(t)x \tag{9}$$

Now defining $W(t) = W_g^+ - BN(t) \in R^{n*(n-m)}$ and $W_g^+ = W_g^T(W_gW_g^T)^{-1} \in R^{n*(n-m)}$, it can be verified that

$$M(t)^{-1} = [W(t) \;\; B] \tag{10}$$

From (9) and (10), it can be observed that

$$x = W(t)z + B\sigma \tag{11}$$

So, Eq. (1) gets transformed to

$$\dot{z} = W_gAW(t)z + W_gAB\sigma + W_gp(t,x) \tag{12}$$

$$\dot{\sigma} = S(t)AW(t)z + S(t)AB\sigma + u + \dot{N}(t)z + \xi(t,x) + S(t)p(t,x) \tag{13}$$

When the system is in the sliding mode, it satisfies the conditions $\sigma = 0$ and $\dot{\sigma} = 0$. Then, the perturbation term in Eq. (12) becomes $W_gp(t,x) = W_gp(t,W(t)z) = \overline{p}(t,z)$ and Eq. (12) transforms into a reduced order equation as

$$\dot{z}=\overline{A}z+\overline{B}\nu(t)+\overline{p}(t,z) \tag{14}$$

where $\overline{A}=W_gAW_g^+\in R^{(n-m)*(n-m)}$, $\overline{B}=W_gAB\in R^{(n-m)*m}$ and $\nu(t)=-N(t)z\in R^m$.

Theorem 1. *Let us consider the perturbed dynamic equation (14) under the assumption that $n\le 2$ m. Suppose that $\overline{B}$ has full rank and the mismatched perturbations in the domain of interest satisfy $\|\overline{p}(t,z)\|\le\phi_2\|z\|$ [35–37], where ϕ_2 is an unknown positive constant. If the feedback gain $N(t)$ of the controller is designed as [20]*

$$N(t)=K_2+[\hat{\phi}_2(t)+\rho]\overline{B}^+ \tag{15}$$

where ρ is a positive constant, $K_2=\overline{B}^+\overline{A}$, $\overline{B}^+=\overline{B}^T(\overline{B}\overline{B}^T)^{-1}\in R^{m*(n-m)}$ and $\hat{\phi}_2(t)$ is an adaptive gain given by

$$\hat{\phi}_2(t)=\int_{t_0}^{t}\theta\|z\|^2\,d\tau+\hat{\phi}_2(t_0) \tag{16}$$

with $\theta>0$ being a positive constant and $\hat{\phi}_2(t_0)=0$ being the initial condition, $\hat{\phi}_2(t)$ is bounded and the trajectories z (14) and state x will be asymptotically stable in the sliding mode.

Proof. Let us consider the Lyapunov function $V_2(z,\tilde{\phi}_2)=\frac{1}{2}[\|z\|^2+\theta^{-1}\tilde{\phi}_2(t)^2]$. Here $\tilde{\phi}_2(t)$ is the estimation error of the adaptive gain given by $\tilde{\phi}_2(t)=\hat{\phi}_2(t)-\phi_2(t)$, where $\hat{\phi}_2(t)$ is the estimated adaptive gain and $\phi_2(t)$ is the actual adaptive gain [19]. Then

$$\begin{aligned}\dot{V}_2(z,\tilde{\phi}_2)&=z^T\overline{A}z+z^T\overline{B}\nu+z^T\overline{p}+\theta^{-1}\tilde{\phi}_2\dot{\hat{\phi}}_2\\&\le z^T\overline{A}z+z^T\overline{B}\nu+\|z\|\|\overline{p}\|+\theta^{-1}\tilde{\phi}_2\dot{\hat{\phi}}_2\\&\le z^T\overline{A}z+z^T\overline{B}\nu+\phi_2\|z\|^2+(\hat{\phi}_2-\phi_2)\|z\|^2\\&\le z^T\overline{A}z+z^T\overline{B}\nu+\hat{\phi}_2\|z\|^2\le-\rho\|z\|^2\le 0\end{aligned} \tag{17}$$

It is obvious from the above discussion that $z\in L_2\cap L_\infty$ and $\tilde{\phi}_2(t)\in L_\infty$. Hence from Eqs. (14) and (15) and the fact that $\|\overline{p}(t,z)\|\le\phi_2\|z\|$, it can be shown that $\dot{z}\in L_\infty$ as well as $\ddot{V}_2\in L_\infty$. From Barbalat's lemma [3], it is found that $z\to 0$ as $t\to\infty$. The bound of the adaptation law is $0\le\phi_2(t)\le(\|z(t_0)\|^2+\phi_2^2)/\rho$. Moreover, it can be seen from Eq. (15) that $N(t)$ is bounded since $\tilde{\phi}_2(t)\in L_\infty$ and hence the state $x(t)=W(t)z=[W_g^+-BN(t)]z$ becomes asymptotically stable as the system reaches the sliding mode. □

Remark 1. However in complex higher order systems it is often observed that the sliding surface does not converge to zero in the steady state but undergoes oscillations within a small bound. Thus, $\dot{\hat{\phi}}_2$ does not become exactly zero in finite time and thereby increases the adaptive parameter $\hat{\phi}_2$ boundlessly. A simple way of overcoming this disadvantage is to modify the adaptive tuning law (16) by using the dead zone technique [3].

3. Design of adaptive integral sliding mode controller

In contrast with conventional sliding mode control, the system motion under integral sliding mode has dimension equal to that of the state space. In integral sliding mode control, the system trajectory always starts from the sliding surface [38–40]. Accordingly, the reaching phase is eliminated and robustness in the whole state space is promised.

The sliding surface σ' is chosen as [18,34]

$$\sigma'=S(t)x-B^+(A+BK)\int_{t_0}^{t}x\,d\tau \tag{18}$$

where $K\in R^{m*n}$ is the design matrix and $B^+=(B^TB)^{-1}B^T\in R^{m*n}$ is the Moore–Penrose pseudo-inverse of B [34]. The sliding coefficient matrix $S(t)$ is designed as explained in Section 2.1. The design matrix $K\in R^{m*n}$ is chosen satisfying the inequality condition

$$Re[\lambda_{max}(A+BK)<0] \tag{19}$$

Taking the derivative of σ' and using Eqs. (7), (11) and (13) yields

$$\dot{\sigma}'=S(t)AW(t)z+S(t)AB\sigma+u-B^+(A+BK)(W(t)z+B\sigma)+d(t,x) \tag{20}$$

where

$$\begin{aligned}d(t,x)&=\dot{N}(t)z+\xi(t,x)+S(t)p(t,x)\\&=\theta\|W_gx\|^2\overline{B}^+W_gx+\xi(t,x)+S(t)p(t,x)\end{aligned} \tag{21}$$

The second time derivative of σ' can be expressed as

$$\begin{aligned}\ddot{\sigma}'&=\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma+S(t)AB\dot{\sigma}\\&\quad-B^+(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})+\dot{u}+\dot{d}(t,x)\end{aligned} \tag{22}$$

In the above equation, $\dot{d}(t,x)$ is considered as unknown disturbance or perturbation.

Remark 2. The above design procedure assumes that the disturbance or perturbation is continuous, bounded and its derivatives exist.

Assumption 1. The disturbance $\dot{d}(t,x)$ in (22) is assumed to be bounded and satisfy the following condition:

$$\|\dot{d}(t,x)\|\le\sum_{i=0}^{r}\beta_i\|x\|^i \tag{23}$$

here r is a positive integer determined by the designer in accordance with the knowledge about the order of the perturbations. For example, if the perturbations contain a term x_1^3, then one may choose r=3. However, if x_1^4 exists in the perturbation, then the inequality might not be satisfied for certain domain of x if one still chooses r=3 [20]. The upper bound of the uncertainty is termed as β_i which cannot be determined easily in real environment.

The main idea behind the second-order sliding mode is to act on the second-order derivative of the sliding variable σ' rather than the first derivative as in conventional sliding mode. The second-order sliding mode is determined from the basic equality condition $\sigma'=\dot{\sigma}'=0$ reaches in finite time, whereas the proposed controller reaches the condition asymptotically.

Let us define the sliding manifold $l(t)$ such that

$$\begin{aligned}l(t)&=\dot{\sigma}'+k\sigma'\\\dot{l}(t)&=\ddot{\sigma}'+k\dot{\sigma}'\end{aligned} \tag{24}$$

where $k\in\text{diag}(k_1,k_2,\ldots k_{n-m})$ is a diagonal matrix. Using the above equations (22)–(24) yields

$$\begin{aligned}\dot{l}(t)&=\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma+S(t)AB\dot{\sigma}\\&\quad-B^+(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})+\dot{u}+\dot{d}(t,x)+k\dot{\sigma}'\end{aligned} \tag{25}$$

Assuming that the sliding surface (7) has a relative degree one implies that the derivative of the sliding manifold $l(t)$ can be expressed as

$$\dot{l}(t)=\Phi(t,z,u)+\psi(t,z)\dot{u} \tag{26}$$

where $\Phi(t,z,u)=\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma+S(t)AB\dot{\sigma}-B^+(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})+\dot{d}(t,x)+k\dot{\sigma}'$ collects all the uncertain terms not involving $\dot{u}$ and $\psi(t,z)=1$. If it is possible to bring σ' and $\dot{\sigma}'$ to zero by using a discontinuous control signal $\dot{u}$, then the actual control u obtained by integrating $\dot{u}$ becomes continuous. The undesired high frequency oscillations which are always present in the control input of the conventional first-order sliding mode controllers are thus eliminated in the proposed controller.

From Eq. (25), the equivalent control $\dot{u}_{eq}$ for controlling the nominal system can be designed as

$$\dot{u}_{eq}=-[\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma$$

$$+S(t)AB\dot{\sigma}-B^{+}(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})+k\dot{\sigma}'] \quad (27)$$

In practice, the bounds of the system uncertainty are often unknown in advance and hence it is difficult to find the error term $\dot{d}(t,x)$ in Eq. (22). So an adaptive tuning law is proposed to estimate $\dot{d}(t,x)$. Now the proposed adaptive controller for tackling the system uncertainty is designed as [41]

$$\begin{aligned}\dot{u}_{adp} &= -\sum_{i=0}^{r}\hat{\beta}_i\|x\|^i\ \mathrm{sign}(l(t)) \quad \text{if } l(t)\neq 0\\ &=0 \quad \text{otherwise}\end{aligned} \quad (28)$$

where $\hat{\beta}_i$ is the adaptive parameter which is tuned using the following adaptive rule:

$$\begin{aligned}\dot{\hat{\beta}}_i &= \theta_i(-\rho_i\hat{\beta}_i+\|l(t)\|\|x\|^i) \quad \text{if } l(t)\neq 0\\ &=0 \quad \text{otherwise}\end{aligned} \quad (29)$$

here ρ_i and θ_i are positive constants, $\hat{\beta}_i(0)=0$ is the initial condition for $0\le i\le r$.

The switching control law $\dot{u}_s$ can be designed as

$$\begin{aligned}\dot{u}_s &= -\tau l(t)-\eta\ \mathrm{sign}(l(t)) \quad \text{if } l(t)\neq 0\\ &=0 \quad \text{otherwise}\end{aligned} \quad (30)$$

where τ and η are positive constants.

Now the control law $\dot{u}$ can be obtained as

$$\dot{u}=\dot{u}_{eq}+\dot{u}_{adp}+\dot{u}_s \quad (31)$$

where $\dot{u}_{eq}$ is the equivalent control part, $\dot{u}_{adp}$ is the adaptive control part and $\dot{u}_s$ is the switching control.

Theorem 2. *Let us consider the system (1) with the adaptive sliding surface given by (8) and (15). The trajectory of the closed loop system (1) can be driven onto the sliding manifold $l(t)$ in finite time by using the controller given by*

$$\begin{aligned}\dot{u} &= -[\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma\\ &\quad +S(t)AB\dot{\sigma}-B^{+}(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})+k\dot{\sigma}']\\ &\quad -\sum_{i=0}^{r}\hat{\beta}_i\|x\|^i\ \mathrm{sign}(l(t))-\tau l(t)-\eta\ \mathrm{sign}(l(t)) \quad \text{if } l(t)\neq 0\\ &=0 \quad \textit{otherwise}\end{aligned} \quad (32)$$

Proof. Let us define a Lyapunov function V_0 as follows [22,41]:

$$V_0=\frac{1}{2}l(t)^T l(t)+\frac{1}{2}\sum_{i=0}^{r}\theta_i^{-1}\tilde{\beta}_i(t)^2 \quad (33)$$

where $\tilde{\beta}_i(t)=\hat{\beta}_i(t)-\beta_i(t)$ are the estimation errors of the adaptive gains. The time derivative of V_0 is obtained as

$$\begin{aligned}\dot{V}_0 &= l(t)^T\dot{l}(t)+\sum_{i=0}^{r}\theta_i^{-1}\tilde{\beta}_i\dot{\hat{\beta}}_i\\ &= l(t)^T[\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma\\ &\quad +S(t)AB\dot{\sigma}-B^{+}(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})\\ &\quad +k\dot{\sigma}'+\dot{u}+\dot{d}(t,x)]+\sum_{i=0}^{r}\theta_i^{-1}\tilde{\beta}_i\\ &= l(t)^T[\dot{S}(t)AW(t)z+S(t)A\dot{W}(t)z+S(t)AW(t)\dot{z}+\dot{S}(t)AB\sigma\\ &\quad +S(t)AB\dot{\sigma}-B^{+}(A+BK)(\dot{W}(t)z+W(t)\dot{z}+B\dot{\sigma})+k\dot{\sigma}'+\dot{u}\\ &\quad +\dot{d}(t,x)]+\sum_{i=0}^{r}\theta_i^{-1}(\hat{\beta}_i(t)-\beta_i)\theta_i(-\rho_i\hat{\beta}_i+\|l(t)\|\|x\|^i)\end{aligned}$$

Using the relations from (27)–(32) yields

$$\begin{aligned}\dot{V}_0 &\le \Bigg[-\sum_{i=0}^{r}\hat{\beta}_i\|x\|^i\|l(t)\|-\sum_{i=0}^{r}\beta_i\|x\|^i\|l(t)\|-\tau\|l(t)\|-\eta\|l(t)\|\\ &\quad +\sum_{i=0}^{r}\hat{\beta}_i\|x\|^i\|l(t)\|+\sum_{i=0}^{r}\beta_i\|x\|^i\|l(t)\|\Bigg]-\rho_i(\hat{\beta}_i^2-\hat{\beta}_i\beta_i)\\ &\le -\eta l(t)\ \mathrm{sign}(l(t))-\tau l(t)^2-\rho_i\left(\hat{\beta}_i-\frac{1}{2}\beta_i\right)^2+\frac{1}{4}\rho_i\beta_i^2\\ &\le -\eta\|l(t)\|-\tau l(t)^2+\frac{1}{4}\rho_i\beta_i^2\end{aligned}$$

It is clear that $\dot{V}_0<0$ if $l(t)>\sqrt{\delta_1/4\tau_{min}}$ or $\|l(t)\|>\delta_1/4\eta_{min}$, where η,τ are positive design parameters and $\delta_1=\rho_i\beta_i^2$. The decrease of V_0 eventually drives the trajectories of the closed loop system into $l(t)<\sqrt{\delta_1/4\tau_{min}}$ and $\|l(t)\|<\delta_1/4\eta_{min}$ [41]. Therefore, the trajectories of the closed loop system are bounded ultimately as

$$\lim_{t\to\infty} l(t)\in\left(l(t)<\sqrt{\frac{\delta_1}{4\tau_{min}}}\right)\bigcap\left(\|l(t)\|<\frac{\delta_1}{4\eta_{min}}\right) \quad (34)$$

which is a small set containing the origin of the closed loop system. In order to guarantee bounded motion around the sliding surface, the positive parameters η and τ are chosen to be large enough such that $\dot{V}_0<0$ when V_0 is out of the bounded region which contains an equilibrium point [22]. It can be observed that $\dot{V}_0<0$ is achievable which implies that the sliding manifold $l(t)$ will approach zero in finite time. Therefore, the control law given by (32) guarantees that the sliding mode will be reached in finite time and sustained thereafter [41]. □

Remark 3. Once sliding mode is established, the proposed gain adaptation law (29) allows the gain $\hat{\beta}_i$ to decrease. Thus, it is observed that the proposed gain adaptation law, while maintaining the sliding mode, keeps the gain $\hat{\beta}_i$ at the smallest possible value to ensure a given tracking accuracy in presence of uncertainties.

Thus, the adaptive gain tuning law is modified as

$$\begin{aligned}\dot{\hat{\beta}}_i &= -\theta_i\rho_i\hat{\beta}_i+\theta_i\|l(t)-\wp\|\|x\|^i \quad \text{if } l(t)\neq 0\\ &=0 \quad \text{otherwise}\end{aligned} \quad (35)$$

where $0\le i\le r$, $\wp$ is a small positive number and r is a desired positive integer.

Remark 4. The parameters τ and η in the controller (32) are very crucial as these are some of the parameters responsible for determining the convergence rate of the sliding surface. It is clear that a large value of τ and η will force the system states to converge to the origin at a high speed. Since high τ and η will require a very high control input which is not desirable in reality, these parameters cannot be selected too large. Hence, a compromise has to be made between the response speed and the control input.

Remark 5. The parameters ρ_0 and ρ_1 in (35) determine the convergence rate of the estimated bounds $\hat{\overline{B}}_0$ and $\hat{\overline{B}}_1$. The design parameters ρ_0,ρ_1 are used to determine the band of the bounded region and we can choose ρ_0 and ρ_1 to be small enough in order to guarantee the motion along the sliding surface. However, a compromise is made between the band of the bounded region and the convergence speed of the estimated bounds $\hat{\overline{B}}_0$ and $\hat{\overline{B}}_1$. Too small ρ_0 and ρ_1 will lead to a very low convergence rate of the estimated bounds $\hat{\overline{B}}_0$ and $\hat{\overline{B}}_1$. Thus, the parameters ρ_0,ρ_1 cannot be selected too small. Practically, any positive value of ρ_0 and ρ_1 can be used to estimate the uncertainty but a higher value only is used for faster estimation of uncertainty.

4. Effectiveness

Let us compare the proposed controller with the adaptive sliding mode controller (SMC) designed by Wen and Cheng [19] and given below:

$$u = u_f + u_{adp} + u_s \tag{36}$$

where

$$u_f = -S(t)AW(t)z - S(t)AB\sigma$$

$$u_{adp} = -\sum_{i=0}^{r} \hat{\beta}_i \|x\|^i \frac{\sigma}{\|\sigma\|} \quad \text{if } \sigma \neq 0$$
$$= 0 \quad \text{otherwise}$$

$$u_s = -\eta \frac{\sigma}{\|\sigma\|} \quad \text{if } \sigma \neq 0$$
$$= 0 \quad \text{otherwise} \tag{37}$$

The adaptive parameter $\hat{\beta}_i$ is tuned using the following adaptive rule:

$$\dot{\hat{\beta}}_i = \theta_i \|x\|^i \quad \text{if } \sigma \neq 0$$
$$= 0 \quad \text{otherwise} \tag{38}$$

where θ_i are positive constants, $\hat{\beta}_i(0) = 0$ is the initial condition for $0 \le i \le r$.

Implementation of the adaptive SMC (36) and (37) is limited by the obvious drawback of the gain $\hat{\beta}_i$ being susceptible to overestimation and thereby increasing the chattering in the system. Hence, this approach is not directly applicable but requires modifications regarding the sign function which needs replacement by a saturation function. However, the width of the boundary layer in the saturation function affects the accuracy and robustness of the SMC. Furthermore, no methodology for tuning the boundary layer width is provided in [19].

The proposed chattering free adaptive integral sliding mode control method offers the major advantage that no a priori knowledge about the bounds of $\hat{\beta}_i$ is required as it adaptively estimates the bounds of $\hat{\beta}_i$ and also ensures that the adaptive gain does not get overestimated. Another major benefit of the proposed controller is that the discontinuous sign function is contained in the derivative control input and the actual control is obtained after integrating the discontinuous derivative control. As such, the actual control is continuous, smooth and chatterless which makes the proposed controller superior to the conventional adaptive SMC proposed in [19].

4.1. Application

The vertical take-off and landing (VTOL) aircraft is a highly nonlinear complex system whose aerodynamic parameters vary considerably during the flight. Let us consider the typical load and flight conditions for a VTOL aircraft at the nominal airspeed of 135 knots [19]. The linearized dynamics of this VTOL aircraft in the vertical plane can be described as Eq. (1) with

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix} \xi(t,x) = \begin{bmatrix} -5\sin(0.5t) + x_3 \\ 2\cos(0.2x_1)x_2 + 3 \end{bmatrix}$$

$$p(t,x) = \begin{bmatrix} \sin(0.1x_2)(0.1x_3 + 0.7x_4) \\ -0.3x_4\cos(0.3x_4 t) \\ -2x_1 - 0.3x_2 \\ (0.2x_1 + 0.4x_4)\sin(0.4x_3) \end{bmatrix} \tag{39}$$

where x_1 is the horizontal velocity (knots), x_2 is the vertical velocity (knots), x_3 is the pitch rate (degrees per second) and x_4 is the pitch angle (degrees). Furthermore, u_1 is the collective pitch control which alters the pitch angle (angle of attack with respect to air) of the main rotor blades collectively to provide the vertical movement and u_2 is the longitudinal cyclic pitch control which tilts the main rotor disc by varying the pitch of the main rotor blades individually to provide the horizontal movement. Moreover, u_1 and u_2 have cross-coupling effects on the horizontal and vertical velocities, respectively. The matched and mismatched perturbations are represented by $\xi(t,x)$ and $p(t,x)$, respectively [19].

4.2. Simulation results

The proposed adaptive integral sliding mode controller is applied to the above VTOL aircraft (39) affected by both matched and mismatched uncertainties. The simulation is carried out in MATLAB Simulink platform by using the ODE 4 solver with a fixed step size of 0.001 s.

4.3. Stabilization of the VTOL aircraft

The reference output is chosen as $x_d = [0\ 0\ 0\ 0]^T$ since all the states are to be driven to zero to stabilize the aircraft. For comparison purpose, the associated design parameters of the proposed adaptive integral sliding mode controller are chosen following [19]. As such W_g is selected as

$$W_g = \begin{bmatrix} 0.0666 & 0.0076 & 0.0102 & 0.72 \\ 0.8879 & 0.1010 & 0.136 & -0.028 \end{bmatrix} \tag{40}$$

Hence, $W_g B = 0$ [32]. Next, the pseudo-control input, the adaptive controller and the adaptive gains are designed in accordance with Eqs. (15), (16), (29) and (35), where $\theta = 0.1474$, $\theta_0 = 0.1271$, $\theta_1 = 0.1251$, $\rho_0 = \rho_1 = 1.188$, $\eta = 2$, $\tau = 10$, $k = \text{diag}(1.25, 1.25)$, $\rho = 0.1519$ and K is so chosen that the eigenvalues are placed at -0.5, -0.7, -15, -25 [34]. The initial state is assumed as $x(0) = [2\ -2\ 1\ 1]^T$.

The adaptive tuning laws used for stabilization are given by

$$\dot{\hat{\phi}}_2 = \theta \|z\|^2$$
$$\dot{\hat{\beta}}_1 = -0.151\hat{\beta}_1 + \theta_1 \|l(t) - \wp\| \|x\|$$
$$\dot{\hat{\beta}}_0 = -0.151\hat{\beta}_0 + \theta_0 \|l(t) - \wp\|$$

The initial values of $\hat{\phi}_2$, $\hat{\beta}_1$ and $\hat{\beta}_0$ are chosen as 0, 1 and 1, respectively. The small positive constant $\wp$ is selected as $\wp = 0.01$.

The adaptive first-order sliding mode controller proposed by Wen and Cheng [19] (36)–(38) is next applied to the VTOL aircraft system (39). The design parameters are chosen as $\theta = 0.3$, $\theta_0 = 0.21$, $\theta_1 = 0.27$, $\eta = 4$ and $\rho = 1$. The initial state is assumed as $x(0) = [2\ -2\ 1\ 1]^T$. From simulation results obtained in Figs. 1 and 2, it can be observed that although the system states converge to the equilibrium, the control inputs are not smooth and contain excessive chattering.

Simulation results obtained by using the proposed adaptive integral sliding mode controller to the VTOL aircraft system are shown in Figs. 3–6. It is observed from Fig. 3 that all the states converge to the origin quickly. Moreover, comparison of Fig. 3 with Fig. 1 reveals that the states converge in lesser time in the case of the proposed controller as compared to Wen and Cheng's controller [19]. From Figs. 2 and 4, it is evident that chattering

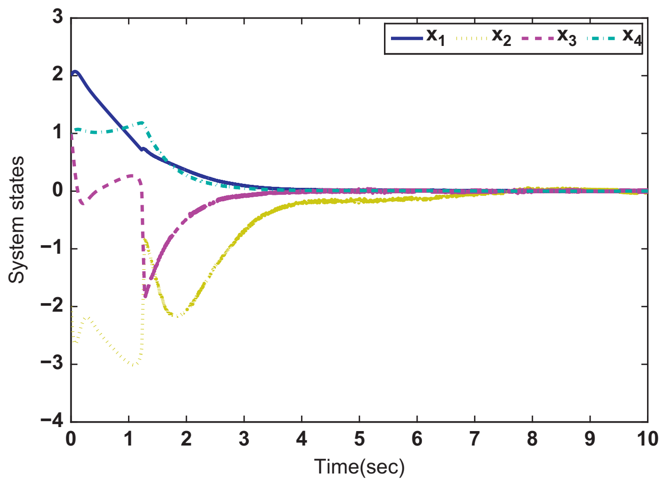

Fig. 1. State responses with the method proposed by Wen and Cheng [19]: x_1 (solid line), x_2 (dash–dot line), x_3 (dash line), and x_4 (dot line).

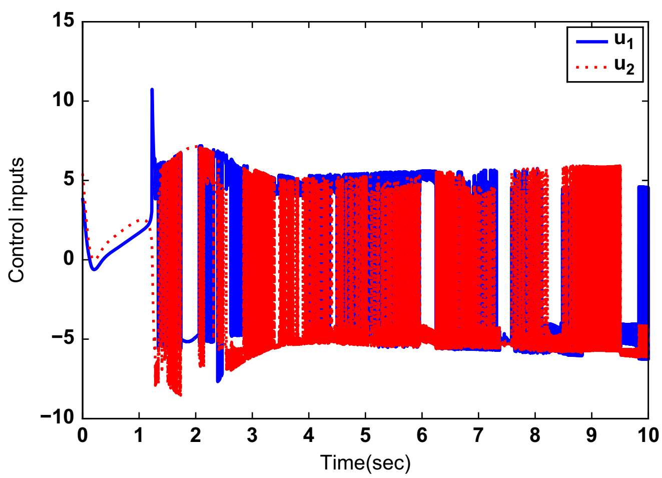

Fig. 2. Control inputs with the method proposed by Wen and Cheng [19]: u_1 (solid line) and u_2 (dot line).

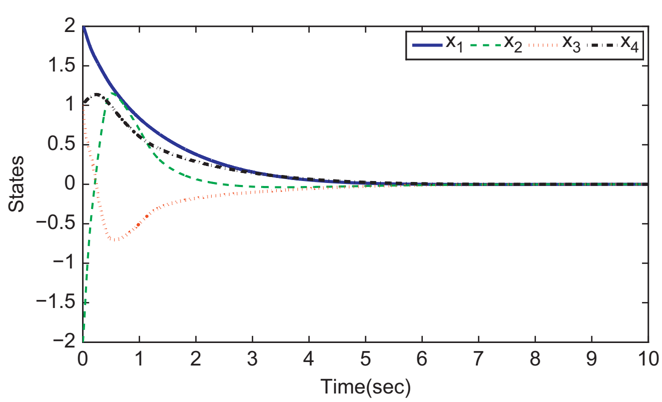

Fig. 3. State responses using the proposed chattering free adaptive integral sliding mode controller.

present in the control inputs obtained by using the proposed adaptive integral sliding mode controller is significantly lesser as compared to that in Wen and Cheng's method [19]. The bounded convergence of the adaptive gains $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\phi}_2$ is confirmed in Fig. 5. From Fig. 6, it can be observed that the proportional plus integral sliding surface σ' and the sliding manifold $l(t)$ are smooth and both approach zero quickly.

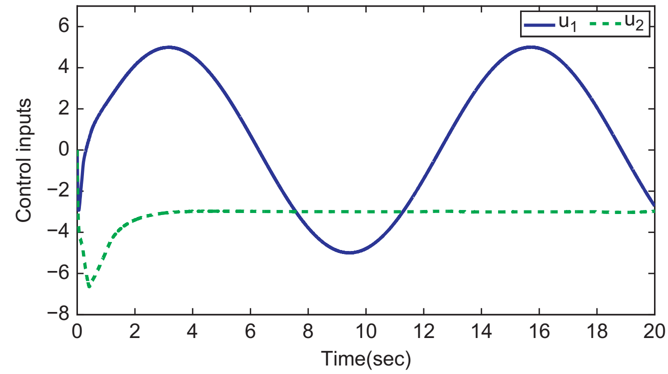

Fig. 4. Control inputs using the proposed chattering free adaptive integral sliding mode controller.

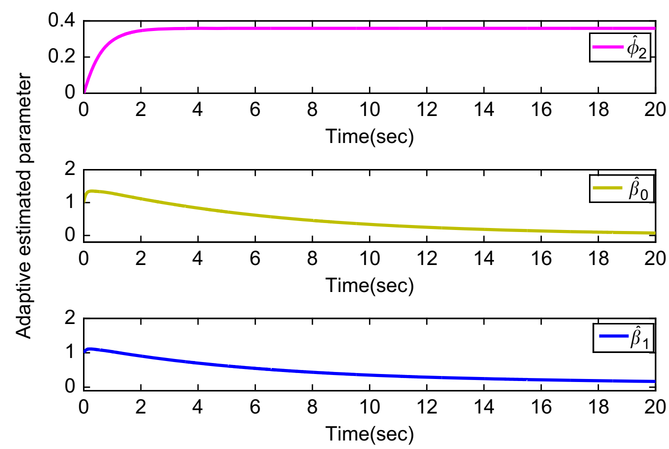

Fig. 5. Estimated parameters using the proposed chattering free adaptive integral sliding mode controller.

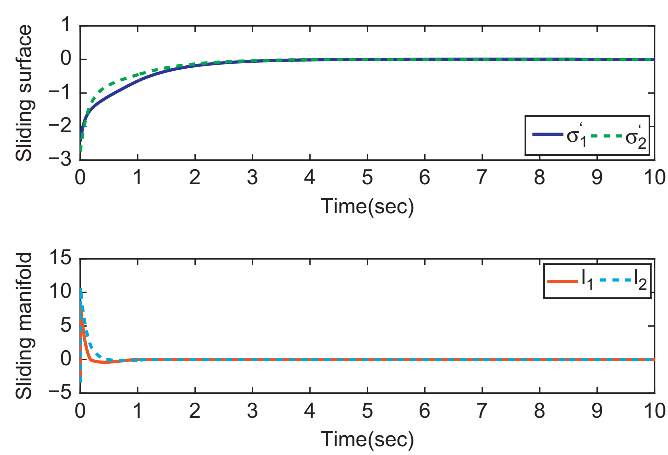

Fig. 6. Sliding surface and sliding manifold using the proposed chattering free adaptive integral sliding mode controller.

5. Conclusion

This paper proposes an adaptive integral sliding mode controller for multi-input multi-output (MIMO) systems affected by unknown uncertainties of any kind, matched or mismatched. The proposed controller can effectively overcome perturbations to achieve quick asymptotical stability. Moreover, the adaptive gain tuning mechanism ensures that the gain is not overestimated with respect to the actual unknown value of the uncertainty. The proposed controller is applied for stabilization of the vertical take-off and landing (VTOL) aircraft system which is a highly complex nonlinear and uncertain MIMO system. From simulation results, the proposed controller is found to be superior in mitigating chattering in the control input than the existing first-order adaptive sliding mode controllers. Faster convergence of the system states is a remarkable benefit of the proposed adaptive integral sliding mode controller. For application to uncertain systems affected by severe matched and mismatched uncertainties, the proposed adaptive integral sliding mode controller promises to be a suitable strategy. For future study, the case of $n>2$ m is worth considering.

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