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N-sigma stability of stochastic systems with sliding mode control

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Abstract

In this paper, sliding mode control for discrete time systems with stochastic noise in their input channel has been discussed. The idea of process control using control charts has influenced this new approach towards dealing with systems with stochastic noise. The new approach approximates the stochastic noise as a bounded uncertainty, similar to having bounds in the control charts for stochastic process control data. For discrete time systems, this results in a bounded stability in probability of the quasi sliding mode, which is referred to as the N-sigma bounded stability. The probability associated with the stability notions is not fixed and the control engineer may desire lower or higher degrees of stability in terms of this probability. Thus one has design flexibility while implementing the theory in practice, where one might have to adjust the desired degree of stability due to hardware limitations.

1. Introduction

Stochastic systems have been finding quite a lot of interest in the control community over the years. Researchers have attempted to develop the theory and control for stabilization of such systems in both continuous time and discrete time [1,2,4–8]. Several approaches have been taken by researchers, which can be broadly separated into their dealing of the system dynamics using ordinary difference [5–8] in case of discrete time systems and stochastic differential [1,2,4] in case of continuous time systems. All of them have been able to achieve either a notion of stability with certain probability [7,8] or have been able to assign the mean and covariance to them [6].

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Some have proposed mean square stability or stochastic stability of the system [1,2,5]. Such and other stability ideas had been discussed in [13] in details.

Sliding mode control has generated a lot of interest starting from the works in [16–19]. They have opened up considerable research opportunities in this area of control, whose most prominent feature is rejection of disturbance in the sliding mode. However, most of the works have considered only matched or unmatched bounded uncertainty in system dynamics even in advanced development of the subject [21,23,24]. But in practice, a system is always corrupted by stochastic noise, especially if the communication channel is long. Only few works on sliding mode control consider stochastic noise in the system dynamics for analysis [3,7,9,8,10–12]. The approach in this paper to deal with stochastic noise is different to those in these preceding works. It is mathematically less involved and can be readily understood from basic concepts of probability, statistics and process control. The development of the stability notion and henceforth application of the theory has been done to help readers with a basic understanding of sliding mode and statistical analysis of signals to easily grasp the idea.

A new concept of stability is proposed in this paper for discrete systems, which is derived from existing ideas of stability [13] of stochastic and deterministic systems. It serves a more practical idea of stability in a probabilistic sense and is used to develop stability notions of the sliding mode. The main work in this paper is to show that a sliding mode control designed to take care of bounded disturbances in a system will bring the same system to these notions of stability in probabilistic sense when the disturbance is not bounded but stochastic in nature.

For discrete systems, this idea of stability requires the sliding motion to be confined in a certain band with a certain probability after it enters this band in finite steps. This is different to the works in [3,7,9,8,10,11] as the probability associated with stability arises due to our approximation of stochastic noise as bounded disturbance and thereafter propagates into the stability notion. To the authors' knowledge, such approach to approximate stochastic noise as bounded disturbance had not been considered in any preceding control and stability studies for stochastic systems (Figs. 1–5).

The structure of the paper is as follows. In Section 2, we discuss the already known concepts like stochastic signal statistics, with a more specific study of white Gaussian noise, whereby several probability bands are discussed. Also we discuss bounded stability for discrete systems as already existing in the literature. In Section 3, we modify the above mentioned stability idea to

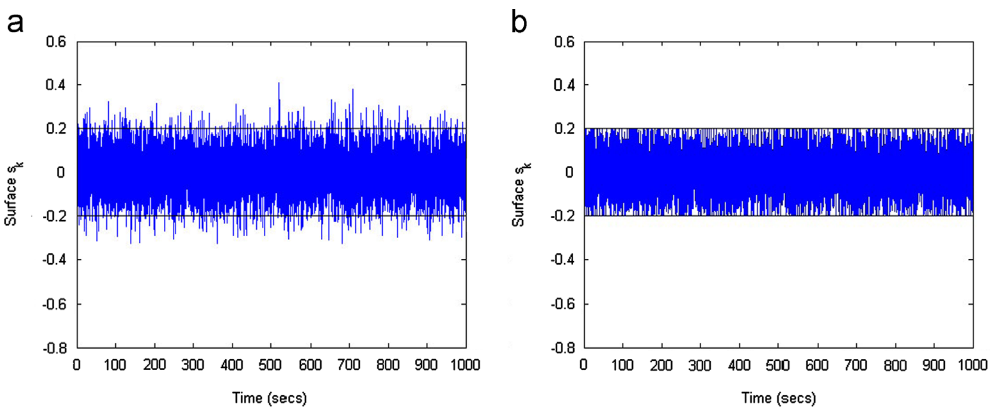

Fig. 1. Sliding surface dynamics for actual and approximated system along with the ultimate band, for $N=2$. (a) Sliding surface for actual system. (b) Sliding surface for approximated system.

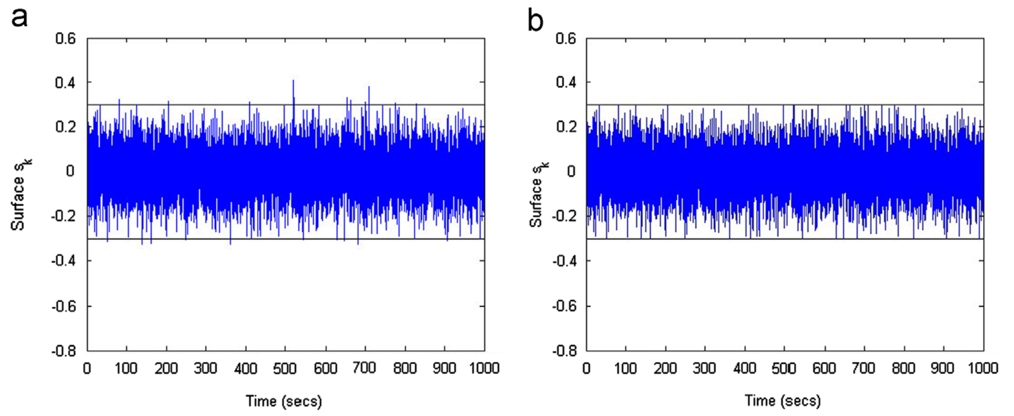

Fig. 2. Sliding surface dynamics for actual and approximated system along with the ultimate band, for $N=3$. (a) Sliding surface for actual system. (b) Sliding surface for approximated system.

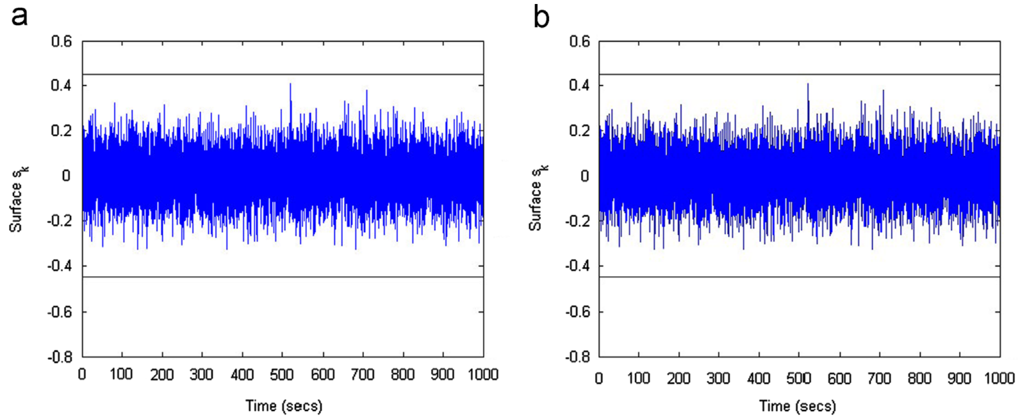

Fig. 3. Sliding surface dynamics for actual and approximated system along with the ultimate band, for $N=4.5$. (a) Sliding surface for actual system. (b) Sliding surface for approximated system.

incorporate a notion of probability in it. Henceforth, we put forward the idea of N-sigma stability that we achieve for discrete time stochastic systems, when the stochastic noise is approximated by a bounded uncertainty. In Section 4, we give simulation examples to illustrate our results for discrete systems. Finally, we draw conclusions in Section 5.

2. Review of available literature

2.1. Stochastic signal statistics

Any stochastic signal $X(t)$ takes random values $x(t)$ at time t. In most cases investigated in control systems, this stochastic signal $X(t)$ is a continuous random variable since it is the stochastic noise entering through the input channel and can take any value within a given range,

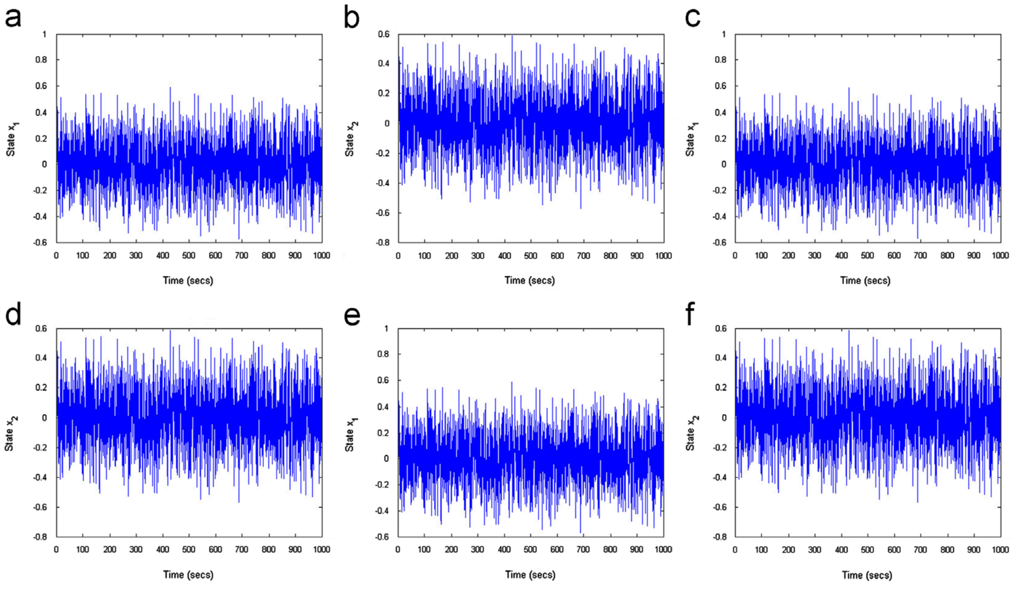

Fig. 4. States x_1 and x_2 of actual system, for different values of N. (a) State x_1 for $N=2$. (b) State x_2 for $N=2$. (c) State x_1 for $N=3$. (d) State x_2 for $N=3$. (e) State x_1 for $N=4.5$. (f) State x_2 for $N=4.5$.

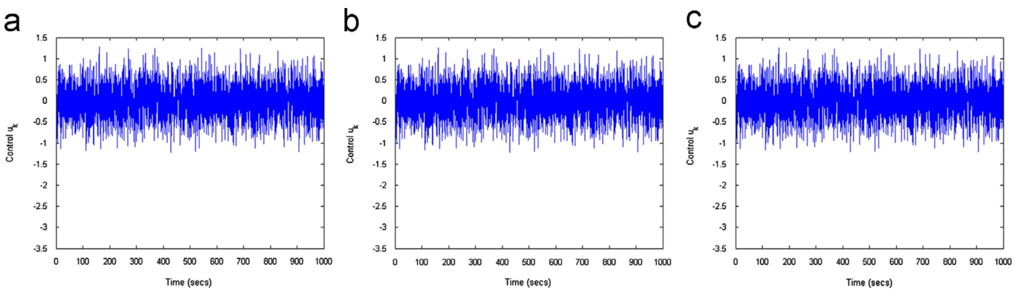

Fig. 5. Control input for actual system, for different values of N. (a) Control input for $N=2$. (b) Control input for $N=3$. (c) Control input for $N=4.5$.

determined by the noise profile or noise distribution. For such continuous random variables, we get a probability density function (pdf) $f(x)$, which gives the probability of the random variable $X(t)$ to assume a value between a and b ($a<b$) as $P[a \le X(t) \le b] = \int_a^b f(x)\,dx$, which is the area under the pdf between a and b.

Given the pdf, the expected value or mean and the variance of the stochastic signal can be easily calculated from well-known formulas as

(1) $\mu = E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$
(2) $E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$
(3) $\sigma^2 = Var[X] = E[X^2] - E^2[X]$

The mean μ gives the most likely value $x(t)$ to be taken up by $X(t)$ at any given time t. The variance indicates the spread of the random variable around the mean. Square root of variance is the standard deviation and denoted by σ.

We may want to know the probability of $X(t)$ taking values within the bounds a and b which are equidistant from the mean μ. If the distance be d, then let the probability be $p(d)$ which is $p(d) = \int_{\mu-d}^{\mu+d} f(x)\,dx$.

We can always write $d = N\sigma$, where N is non-negative. So the probability $p(d)$ will now change to $p(N)$ relative to the bounds $\pm N\sigma$ about the mean μ.

It is obviously understood that $p(N)$ is increasing and $p(0) = 0$. Thus we become more sure that $X(t)$ takes values within the bounds $\pm N\sigma$ about μ as we increase the bounds, i.e., as we increase N.

Most of the engineering system models approximate the noise entering in input channel as a zero-mean white Gaussian noise, which follows a pdf $f_g(x) = (1\sigma\sqrt{2\pi})/e^{-x^2/2\sigma^2}$ at any time instant t and the value taken at any instant is uncorrelated to the values at other instants.

The values for the probabilities $p(N) = p_g(N)$ for different values of N for such a pdf are widely known and a few are given below:

- $p(1) = 0.6826$
- $p(2) = 0.9544$
- $p(3) = 0.9974$
- $p(4) = 0.9999$

with precision up to four decimal places. If we extend our bounds to $N=4.5$, then the probability of noise signal being outside the bounds $\pm N\sigma$ is as less as 3.4 parts per million. The famous six sigma methodology of process quality control [15] assumes $N=6$ in designing their quality control bounds for process control, assuming Gaussian distribution of the data, wherein the probability of defects escaping the bounds comes down to 0.002 parts per million. With $N=6$ taken for their processes, they also have a tolerance incorporated in their design for any future deviation of the mean. The tolerance is 1.5σ, which if the process deviates from the mean in the course of time, will still guarantee $N=4.5$ level of quality control.

2.2. Ultimately bounded stability with probability one

In [13], there had been much discussion on stability concepts for stochastic systems. For a discrete process

$$y_{k+1} = g_k(y_k) \tag{1}$$

the stability concept that interest us is the *ultimately bounded stability with probability one and bound m*, which is defined below [13]. We shall modify it in the following section to suit our needs of stability of discrete time systems in a probabilistic sense.

Definition 1. A process as in Eq. (1) is ultimately bounded with probability one and with bound m if and only if, for each y_0 in a neighborhood E of the origin, there is a finite-valued random instant $K(y_0)$ such that

$$P\left\{ \sup_{\infty > k \geq K(y_0)} \|y_k\| \leq m \right\} = 1 \tag{2}$$

3. N-sigma stability

In this section, we formulate the new concepts of stability in probabilistic sense which we will try to attain for discrete stochastic systems using a sliding mode controller. We shall eventually arrive at the concept of N-sigma stability which owes its name to the approximation approach adopted in this work to deal with stochastic noises acting in the input channel of the systems as bounded matched uncertainties.

Sliding mode control for discrete time systems was developed by discretization of the reaching laws in continuous systems [18]. It was seen that one cannot reach the sliding mode $s(x) = 0$ because of the finite sampling time, but can only achieve to remain within a quasi-sliding band [18] about $s(x) = 0$. In the works [19,21], new approaches toward discrete sliding mode control had been discussed which would bring the surface $s(x)$ to zero, i.e., sliding mode would be reached, when there is no disturbance present in the system. But with the presence of disturbance, sliding mode can never be reached and one arrives at the concept of quasi-sliding mode and quasi-sliding mode band as proposed in [18].

Hence we need a notion of finite time bounded stability in probability for the designed stable sliding surface $s(x)$. We modify Definition 1 by relaxing it in terms of probability and making it stronger in the sense of generalizing to any sample y_k inside the bound m.

Definition 2. The process as in Eq. (1) is ultimately bounded with probability ρ and with bound m if and only if, for each y_0 in a neighborhood E of the origin, there is a finite-valued random instant $K(y_0)$ such that

$$P\{\|y_k\| \le m\} = \rho \ \forall \ \infty > k \ge K(y_0) \tag{3}$$

This means once the random variable y_k enters the bound m, which it does at some finite sampling instant $K(y_0)$, the chances of it coming out of the bound m is $(1-\rho)$. We will use this concept of stability for our surface variable $s(x)$ and will show how our approach brings it inside a desired ultimate band and thereafter keeps it inside the band with probability as high as we design.

Note that no probability is attached with the finite sampling instant $K(y_0)$. This is because we always expect a finite $K(y_0)$ such that y_k enters the bound m. This $K(y_0)$ may not be same for repetitions of the experiment to bring y_k to finite time bounded stability using the same controller u, but we are always sure to have a finite $K(y_0)$. This will change from one experiment to another, but always influenced by the initial condition y_0. Our only concern is Eq. (2), which may not be achievable with absolute certainty (probability 1), and hence we incorporate a probability ρ to the occurrence of this event as in Eq. (3).

Let us have a discrete time system

$$x_{k+1} = a_k(x_k) + b_k(x_k)u + w_k \tag{4}$$

where u is the sliding mode controller and w_k is a stochastic noise acting in the input channel.

The main idea of this paper is to approximate the stochastic noise present in the input channel as a bounded uncertainty. If we do that in Eq. (4), we get the deterministic system

$$x_{k+1} = a_k(x_k) + b_k(x_k)u + d_k \tag{5}$$

where d_k is a bounded matched uncertainty. Sliding mode controllers for such systems can be readily designed which will bring the sliding motion inside an ultimate band in finite time and henceforth keep it inside that band.

Theorem 1. *Let w_k be a white Gaussian noise with zero mean and variance σ^2 acting in the input channel of the discrete time system (4) and u be a sliding mode controller designed for the system (Eq. 5) with a matched uncertainty d_k with bounds $\pm N\sigma$. Then the dynamics of the sliding variable $s_w(x_k)$ for system (4) will match the dynamics of the sliding variable $s_d(x_k)$ for system (5) with a probability $p_g(N)$.*

Proof. Let us denote the events of finite time reaching and quasi sliding of $s_w(x_k)$ as R_w and S_w, respectively, and that of $s_d(x_k)$ as R_d and S_d, respectively. Now, we can always design a sliding mode control u to make the events R_d and S_d occur. Such a control u takes care of the disturbance d_k with bounds $\pm N\sigma$. The disturbance d_k can be of any nature but with bounds $\pm N\sigma$.

The same control u is now given to the system (4) which has input noise w_k. For any instant k, if w_k takes value inside the bounds $\pm N\sigma$, u will be able to take care of w_k and the events R_w and S_w will occur. So, we can say that the events R_w and S_w are both equivalent to the event $(w_k \in \pm N\sigma)$. Therefore,

$P(R_w) = (w_k \in \pm N\sigma) = p_g(N)$

$\Rightarrow R_w$ will follow R_d with a probability $p_g(N)$

Similarly, $P(S_w) = P(w_k \in \pm N\sigma) = p_g(N)$

$\Rightarrow S_w$ will follow S_d with a probability $p_g(N)$

Overall, the dynamics of the sliding variable $s_w(x_k)$ for system (4) will match the dynamics of the sliding variable $s_d(x_k)$ for system (5) with a probability $p_g(N)$. □

Since the stability of the system (4) using this approximation approach is directly related to the bounds $\pm N\sigma$ and the associated probability $p_g(N)$, we may call this as N-sigma bounded stability of the quasi sliding mode. The stability notion is exactly similar as that proposed in Definition 2 at the beginning of this subsection, only given a different name which is peculiar to the approximation approach taken in this work.

Remark 1. From the discussion in Section 2.1, it is obvious that higher the N value chosen, higher is the probability that our approximated system dynamics will match the exact system dynamics using the same sliding mode controller. Hence, with higher N, N-sigma bounded stability for discrete time systems can be achieved with a stronger probability as per Theorem 1.

Remark 2. It may be noted here that the optimum value of N that may be chosen can be taken from the famous industrial quality control strategy of six sigma as $N=6$, which will not only guarantee almost sure matching of the approximated system dynamics with the actual system dynamics in the short term, but also in a long term as it takes care of a shifting of the noise mean from zero to an amount up to 1.5σ and still guaranteeing almost same level of performance [15]. In our simulation, however, we choose only up to $N=4.5$ considering no mean shifting effect.

4. Simulation example

In this section, we take a second order discrete LTI system

$$x_{k+1} = \Phi x_k + \Gamma u_k + \Gamma w_k \tag{6}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}$, and w_k is a zero-mean white Gaussian noise with variance σ^2 acting in the input channel.

Using our approximation theory, the approximated deterministic system will be

$$x_{k+1} = \Phi x_k + \Gamma u_k + \Gamma d_k \tag{7}$$

where d_k is taken as a zero-mean uncertainty with bounds $\pm N\sigma$, where we can choose N as per our requirement of the degree of stability in terms of probability.

We use the controller as designed in [20] for a linear sliding surface

$$s = Gx_k \tag{8}$$

where $s \in \mathbb{R}$ and designed to be stable by proper choice of G [22].

The controller is [20]

$$u_k = -(G\Gamma)^{-1}[GAx_k - Gx_k + \mu\tau Gx_k + \epsilon\tau sgn(Gx_k)] \tag{9}$$

where

$$\mu\tau = \frac{2\tilde{d}_1}{\delta} - 1 \tag{10}$$

and

$$\epsilon\tau = -\mu\tau\delta \tag{11}$$

with τ as the sampling period and the ultimate band value δ chosen such that $\tilde{d}_1 < \delta < 2\tilde{d}_1$ and $\tilde{d}_1 = G\Gamma N\sigma$ [20].

We use the same controller (9) for the stochastic system (6) and perform simulations for three different choices of N, viz., $N=2$, $N=3$ and $N=4.5$. The sliding variable is plotted for the approximated system (7) as well for easy comparison between actual and approximated surface dynamics. It is seen that among the three values chosen, $N=4.5$ yields the best approximation, and $N=2$ gives the worst approximation. This is as predicted by the theory in Section 3.

We consider an example system as below, which is inherently unstable, with a white Gaussian noise at its input channel with variance 0.01, i.e., $\sigma = 0.1$.

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ 3 & -0.5 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(k) \tag{12}$$

A stable sliding surface for this system is

$$s(k) = Gx(k) = [-0.8 \;\; 1]x(k) \tag{13}$$

We now replace the noise $w(k)$ with bounded uncertainty in terms of N. We consider three N values $N=2$, $N=3$ and $N=4.5$ in the simulation results. We shall see that for $N=2$ and $N=3$, the sliding motion comes out of the ultimate band at a few instants, but with $N=4.5$, it always remains within the ultimate band for the chosen sample range of time instants k.

4.1. With $N=2$

For $N=2$, the sigma bounds are ± 0.2. We choose the ultimate band slightly more than $\tilde{d}_1 = 0.2$ as per the theory. With ultimate band chosen as $\delta = 0.201$, the controller settings are calculated as $\mu = 9.9005$ and $\epsilon = -1.99$ for a sampling time of $\tau = 0.1$ s, from relations (10) and (11). The initial state $[1 \;\; 0]^T$ is taken in the example.

The simulation plots for sliding variable for actual and approximated systems are shown together, which clearly shows the mismatch between the actual and approximated dynamics in this case.

It is observed from the simulation plots that 419 samples out of 10,000 are out of our ultimate band. Thus sliding variable values are inside the ultimate band with 0.9581 probability, which is almost same as predicted by the theory, but might not be desirable from a control perspective. The maximum and minimum values of the sliding variable are 0.4109 and −0.3286 which are also a bit away from the ultimate band.

The simulation plots of the individual states are also presented which clearly show that they too are bounded in nature. The control input plot is also shown.

4.2. With $N=3$

For $N=3$, the sigma bounds are ± 0.3. We choose the ultimate band slightly more than $\tilde{d}_1 = 0.3$ as per the theory. With ultimate band chosen as $\delta = 0.301$, the controller settings are calculated as $\mu = 9.9336$ and $\epsilon = -2.99$ for a sampling time of $\tau = 0.1$ s, from relations (10) and (11). The initial state $[1\ 0]^T$ is taken in the example.

The simulation plots for sliding variable for actual and approximated systems are shown together, which clearly shows the mismatch between the actual and approximated dynamics in this case. This mismatch is less than that for $N=2$, but still identifiable.

It is observed from the simulation plots that only 21 samples out of 10,000 are out of our ultimate band. Thus sliding variable values are inside the ultimate band with 0.9979 probability, which is almost same as predicted by the theory. The maximum and minimum values of the sliding variable are 0.4109 and −0.3287 which are not too astray from the ultimate band as well.

The simulation plots of the individual states are also presented which clearly show that they too are bounded in nature. The control input plot is also shown.

4.3. With $N=4.5$

For $N=4.5$, the sigma bounds are ± 0.45. We choose the ultimate band slightly more than $\tilde{d}_1 = 0.45$ as per the theory. With ultimate band chosen as $\delta = 0.451$, the controller settings are calculated as $\mu = 9.9557$ and $\epsilon = -4.49$ for a sampling time of $\tau = 0.1$ s, from relations (10) and (11). The initial state $[1\ 0]^T$ is taken in the example.

The simulation plots for sliding variable for actual and approximated systems are shown together, which clearly shows the strong resemblance between the actual and approximated dynamics in this case.

It is observed from the simulation plots that no samples out of 10,000 came out of our ultimate band. However, there is always a probability that a sample may come out (which is 3 or 4 per million). If we had increased our sampling instants to a million, then we would observe some samples coming out of the ultimate band. From the plot of 10,000 samples it is observed that the maximum and minimum values of the sliding variable are 0.4109 and −0.3288 which are well within our ultimate band.

The simulation plots of the individual states are also presented which clearly show that they too are bounded in nature. The control input plot is also shown.

We could go on increasing N which would yield even lesser probability of the sliding variable to come outside the ultimate band. However, that would result in an increased ultimate band. Hence there is a trade-off in the choice of N for a desired ultimate band size as well as a high probability of the sliding variable to be contained inside the ultimate band.

5. Conclusions

In this work, a new and simpler approach towards dealing with systems subjected to stochastic noise in input channel is proposed, wherein the stochastic signal is approximated as a bounded uncertainty of appropriate bounds as per stability requirement. Practical notion of stability in probability is proposed and something called N-sigma stability is arrived by the approximation approach. It is shown through simulations that the sliding motion becomes N-sigma bounded stable for discrete systems as desired and predicted by the approximation theory. An optimum probability level for the stability is also suggested, referring the widely accepted and celebrated methodology of six sigma used in process quality control. The work as in this paper can be readily applied by design engineers to discrete stochastic systems taking the bounds of uncertainty as required for their desired degree of stability in terms of probability.

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