Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 181 (2007) 331–337
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Decision Support

A new DEA ranking system based on changing the reference set

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Received 21 January 2006; accepted 6 June 2006
Available online 14 August 2006

Abstract

This research proposes a new ranking system for extreme efficient DMUs (Decision Making Units) based upon the omission of these efficient DMUs from reference set of the inefficient DMUs. We state and prove some facts related to our model. A numerical example where the proposed method is compared with traditional ranking approaches is shown.

Keywords: Data envelopment analysis (DEA); Efficiency; Ranking

1. Introduction

One of the main objectives of DEA (Data Envelopment Analysis) is to measure the efficiency of a DMU (Decision Making Unit, e.g. school, public agencies and banks). One of the ways for determining efficiency score of DMUs is to apply the Charnes, Cooper, Rhodes (CCR) model [1] that deals with a ratio of multiple outputs and inputs. One of the interesting research subjects is to discriminate between efficient DMUs. Several authors have proposed methods for ranking the best performers ([2–7] among others). For a review of ranking methods, readers are refereed to Adler et al. [8]. In some cases, the models purposed by [3,6] can be infeasible. In addition to this difficulty, the Andersen and Petersen [3] model may be unstable because of extreme sensitivity to small variations in the data when some DMUs have relatively small values for some of their inputs.

The objective of this work is to propose a new ranking system for extreme efficient DMUs based on the work of Hibiki and Sueyoshi [2]. Our approach does not have the difficulties arising from Andersen and Petersen [3] and Mehrabian et al. [6] models. The main methodological difference of our model in relation to the one of Hibiki and Sueyoshi [2] is that while their approach suggests a measure of efficiency for extreme efficient DMU named DSS (DEA self-sensitivity) which is not dependent of the inefficient DMUs, our methodology proposes a measure that is totally dependent of the inefficient DMUs.

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2006.06.012

This paper is organized as follows. Section 2 briefly introduces the background of DEA. Section 3 introduces our proposal and states and proves some facts related to properties and characteristics of it. A numerical example is given in Section 4 and Section 5 comprehends our conclusions.

2. DEA background

The most basic DEA model is the CCR, which was proposed by Charnes et al. [1] in 1978. The basic idea of the CCR model is the following: the efficiency of an observed DMU, which is the organization to be evaluated, can be measured by the ratio output per input, i.e., how well a DMU can convert its inputs into its outputs. As we usually work in situations where we face multiples inputs and outputs, we are going to form a unique virtual output and a unique virtual input, for the observed DMUp, by the yet unknown weights v_i and u_r. By using linear programming (LP), we can find the weights that maximize the ratio output per input through the model:

$$
\begin{aligned}
\max\ & \sum_{r=1}^{s} u_r y_{rp} \\
\text{s.t.}\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, \quad j=1,\ldots,n, \\
& \sum_{i=1}^{m} v_i x_{ip} = 1, \\
& u_r \geqslant \varepsilon, \quad r=1,\ldots,s, \\
& v_i \geqslant \varepsilon, \quad i=1,\ldots,m,
\end{aligned} \tag{1}
$$

where x_{ij} is the data of the input i on the DMUj, y_{rj} is the data of the output r on the DMUj, v_i is the weight of the input i and u_r is the weight of the output r.

The dual form of Eq. (1) is:

$$
\begin{aligned}
\min\ & \mu = \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip}, \quad i=1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rp}, \quad r=1,\ldots,s, \\
& \lambda_j \geqslant 0, \quad j=1,\ldots,n, \\
& s_i^- \geqslant 0, \quad i=1,\ldots,m, \\
& s_r^+ \geqslant 0, \quad r=1,\ldots,s, \\
& \theta \quad \text{free},
\end{aligned} \tag{2}
$$

where μ is the efficiency measure and ε is a non-Archimedean small and positive number so that the Eq. (1) is feasible and, consequently, objective function of (2) is bounded.

We know that DMUp is CCR-efficient if in Eq. (2) $\theta^* = 1$, $s^{-*} = 0$ and $s^{+*} = 0$, otherwise DMUp is CCR-inefficient. In order to determine the CCR-efficient DMUs, the DEA computer code can use a two-phase LP problem, which may be formalized as follows:

- Phase 1 solves $\theta^* = \min\ \theta$ subject to (2).
- Phase 2 incorporates this value θ^* instead of θ in (2) with a new objective function: $\max\left\{\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right\}$.

For further details in DEA solving procedures readers are referred to [9].

It is important to note that DMUp is extreme efficient if and only if Eq. (2) has unique optimal solution:
$(\lambda_p^* = 1,\ \lambda_j^* = 0,\ j = 1,\ldots,p-1,p+1,\ldots,n,$
$s^+ = 0,\ s^- = 0)$.

The proposal of Hibiki and Sueyoshi [2] known as DEA cross-reference (DCR) is the following:

$$
\begin{aligned}
\min\ & \theta - \left(\sum_{i=1}^{m} \frac{t_i^-}{R_i^-}\right)\Bigg/ m - \left(\sum_{r=1}^{k} \frac{t_i^+}{R_r^+}\right)\Bigg/ s \\
\text{s.t.}\ & -\sum_{j\in J-\{b\}} x_{ij}\lambda_j + \theta x_{ia} - t_i^- = 0, \quad i=1,\ldots,m, \\
& \sum_{j\in J-\{b\}} y_{rj}\lambda_j - t_r^+ = y_{ra}, \quad r=1,\ldots,s, \\
& \sum_{j\in J-\{b\}} \lambda_j = 1, \\
& \lambda_j \geqslant 0, \quad j \in J-\{b\}, \\
& t_r^+ \geqslant 0, \quad r=1,\ldots,s, \\
& t_i^- \geqslant 0, \quad i=1,\ldots,m, \\
& \theta \geqslant 0,
\end{aligned} \tag{3}
$$

where $R_i^- = \max_{1\leqslant j\leqslant n}\{x_{ij}\}$, $i=1,\ldots,m$, $R_r^+ = \max_{1\leqslant j\leqslant n}\{y_{rj}\}$, $r=1,\ldots,s$ and $J=\{1,2,\ldots,n\}$.

In fact, the above model measures the efficiency score of DMUa under the condition that $j \in J-\{b\}$ (i.e., the DMUb is excluded from the reference set of DMUa). Here $\rho_{a,b}^* = \theta^* - \frac{1}{m}\sum_{i=1}^{m}\frac{t_i^{-*}}{R_i^-} - \frac{1}{s}\sum_{r=1}^{s}\frac{t_i^{+*}}{R_r^+}$ at optimality indicates the objective function value.

If in (3) $\{a\} = \{b\}$ then $\rho_{a,b}^*$ equals to the extended DEA measure (EDM) proposed by Andersen and Petersen [3].

The DSS model ranks the BCC extreme efficient's DMUs through:

$$D^*_{a,a} = \rho^*_{a,a} - \eta^*_a, \tag{4}$$

where η^*_a is the BCC efficiency of DMUa.

3. Our proposal

A DMU that is strong efficient by CCR or BBC models will be denoted by *SE* (Strong Efficient).

We noted that by applying the formulation (3), the original efficient frontier will change *iif* DMUb is *SE* and this new efficient frontier (without DMUb) gets closer to the inefficient DMUs (even changing some of these inefficient DMUs to efficient).

The main idea of our proposal is the following: the *SE* DMU that when excluded from the reference set of all the other DMUs allows the efficient frontier to be closest in relation to the inefficient DMUs should be the most efficient *SE* DMU. This idea is exemplified in Fig. 1, where the efficient frontier of the data presented by page 53 of [9] is plotted.

It seems to us that the original idea that motivated Charnes, Cooper and Rhodes in creating the DEA (an envelop that surrounds the data) is preserved in our approach. As in the classical DEA models the DMUs that belongs to this "envelop" are the best performers, it looks like that among them, the one that influences the efficient frontier to get farther in relation to the remaining data should be classified as the best one.

Analyzing the Fig. 1, it is evident that among the *SE* DMUs (DMU5, DMU4 and DMU3), the one that makes the original efficient frontier to get farther from the inefficient DMUs is the DMU5. Consequently, the DMU5 is classified as the most efficient *SE* DMU.

In order to perform our approach, the non-*SE* DMUs should be re-evaluated through:

$$\begin{aligned}
\min \partial_{a,b} &= \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)\\
\text{s.t.}\quad & -\sum_{j\in J-\{b\}} \lambda_j x_{ij} + \theta x_{ia} - s_i^- = 0, \quad i = 1,\ldots,m,\\
& \sum_{j\in J-\{b\}} \lambda_j y_{rj} - s_r^+ = y_{ra}, \quad r = 1,\ldots,s,\\
& \lambda_j \geqslant 0, \quad j \in J-\{b\},\\
& s_r^+ \geqslant 0, \quad r = 1,\ldots,s,\\
& s_i^- \geqslant 0, \quad i = 1,\ldots,m,\\
& \theta \quad \text{free},
\end{aligned} \tag{5}$$

where $J = \{1,2,\ldots,n\}$, $a \in J_n$, $b \in J_e$, J_n is the set of non-*SE* DMUs and J_e is the set of *SE* DMUs.

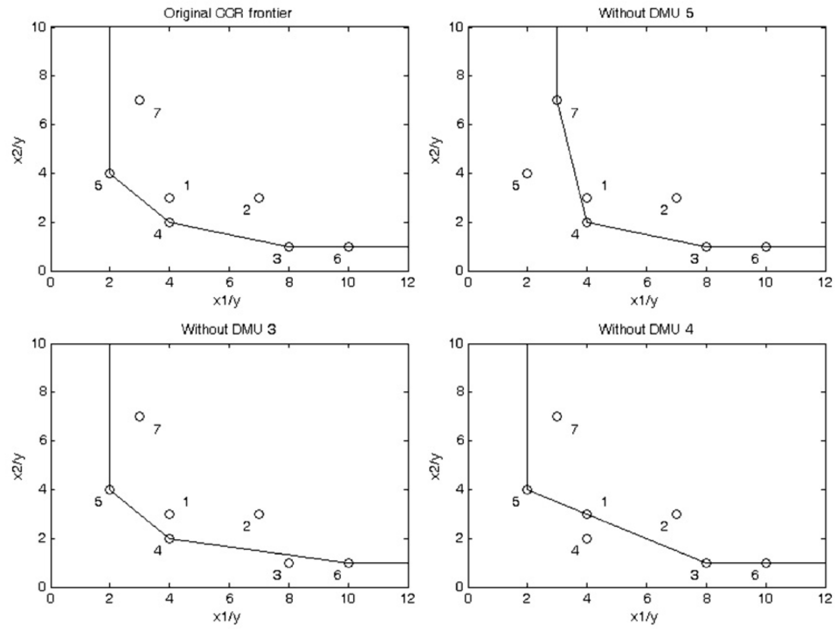

Fig. 1. Main idea of the proposed method.

After calculating the efficiency measure ∂ for all the non-SE DMUs, the efficiency of SE DMUs will be denoted by Ω and will be given by

$$\Omega_b = \frac{\sum_{a \in J_n} \partial_{a,b}}{\tilde{n}}, \tag{6}$$

where b is the evaluated SE DMU and $\tilde{n}$ is the number of non-SE DMUs.

The dual form of Eq. (5) is as follows:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_r y_{ra} \\ \text{s.t.} \quad & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \leqslant 0, \quad j \in J - \{b\}, \\ & \sum_{i=1}^{m} v_i x_{ia} = 1, \\ & u_r \geqslant \varepsilon, \quad r = 1, \ldots, s, \\ & v_i \geqslant \varepsilon, \quad i = 1, \ldots, m. \end{aligned} \tag{7}$$

3.1. Theorems

We state and prove some facts related to properties and characteristics of model (5).

Theorem 1. *For each DMUb*: $\partial^*_{a,b} \geqslant \mu^*_a$.

Proof. The feasible space of (2) is a subset of the feasible space of (5), therefore the optimal objective function value of (5) is greater or equals the optimal function value of (2) and by utilizing the fundamental theorem of duality implies:

$\partial^*_{a,b} \geqslant \mu^*_a$. $\square$

Theorem 2. *If $a \neq b$ then (5) is feasible.*

Proof. The following solution is feasible for (5):

$\lambda_a = 1,\ \lambda_j = 0,\ j \neq a,\ \theta = 0,\ s^+_r = 0,$
$r = 1, \ldots, s,\ s^-_i = x_{ia},\ i = 1, \ldots, m.$ $\square$

Theorem 3. $\partial^*_{a,b} = \mu^*_a$ *if and only if there is an optimal solution for Eq. (2) so that* $\lambda^*_b = 0$.

Proof. Let $(\theta^*, \Lambda^*, S^{+*}, S^{-*})$ be the optimal solution for Eq. (2) in which $\Lambda^* = (\lambda^*_1, \ldots, \lambda^*_{b-1}, \lambda^*_b, \lambda^*_{b+1}, \ldots, \lambda^*_n)$ and $\lambda^*_b = 0$. Since $\lambda^*_b = 0$, Λ^* is a feasible solution for Eq. (5) therefore $\partial^*_{a,b} \leqslant \mu^*_a$ and by Theorem 1 $\partial^*_{a,b} \geqslant \mu^*_a$, consequently, we have that $\partial^*_{a,b} = \mu^*_a$. Conversely, suppose that $\partial^*_{a,b} = \mu^*_a$. We will show that there is an optimal solution for Eq. (2) with $\lambda^*_b = 0$. Let $(\bar{\theta}, \bar{\lambda}_1, \ldots, \bar{\lambda}_{b-1}, \bar{\lambda}_{b+1}, \ldots, \bar{\lambda}_n, t^{-*}_1, \ldots, t^{-*}_m, t^{+*}_1, \ldots, t^{+*}_s)$ be an optimal solution for Eq. (5), obvious that $(\bar{\theta}, \bar{\lambda}_1, \ldots, \bar{\lambda}_{b-1}, \bar{\lambda}_b (= 0), \bar{\lambda}_{b+1}, \ldots, \bar{\lambda}_n, t^{-*}_1, \ldots, t^{-*}_m, t^{+*}_1, \ldots, t^{+*}_s)$ is feasible solution for Eq. (2) and $\partial^*_{a,b} = \bar{\mu}_a$, where $\bar{\mu}_a$ is objective function value of Eq. (2) in the last solution. From hypothesis of theorem we have that $\bar{\mu}_a = \mu^*_a$ therefore the latter solution is optimal for Eq. (2). This completes the proof. $\square$

Theorem 4. *If $a \neq b$ then* $0 < \partial^*_{a,b} \leqslant 1$.

Proof. In Eq. (7) we have $-\sum_{i=1}^{m} v_i x_{ia} + \sum_{r=1}^{s} u_r y_{ra} \leqslant 0$, $\sum_{i=1}^{m} v_i x_{ia} = 1$ and $u_r \geqslant 0$ therefore $0 < \sum_{r=1}^{s} u^*_r y_{ra} \leqslant 1$ at optimality. The fundamental theorem of duality implies that $0 < \partial^*_{a,b} \leqslant 1$. $\square$

Theorem 5. *If $a = b$ and DMUa is CCR-efficient then* $\partial^*_{a,b} \geqslant 1$.

Proof. Since DMUa is CCR-efficient ($\mu^*_a = 1$) then by Theorem 1: $\partial^*_{a,a} \geqslant \mu^*_a = 1$. $\square$

Theorem 6. *If DMUa is CCR-inefficient and DMUb does not belong to the reference set of DMUa then* $\partial^*_{a,b} = \mu^*_a$.

Proof. Since DMUb does not belong to the reference set of DMUa thus $\lambda^*_b = 0$ (for each optimal solutions of (2)). Therefore each optimal solution of (2) is corresponding to feasible solution of (5). By omitting λ^*_b from feasible solutions of (2) results $\partial^*_{a,b} \leqslant \mu^*_a$. Now by Theorem 1: $\partial^*_{a,b} = \mu^*_a$. $\square$

Theorem 7. *If DMUb is inefficient in model (2) then* $\partial^*_{a,b} = \mu^*_a$ *for each DMUa.*

Proof. We consider two cases:

Case 1: DMUb is at the interior PPS (Production Possibility Set). In this case $\lambda^*_b = 0$ in every optimal solutions of (2). Therefore by removing λ_b in all optimal feasible solutions of (2) we have a feasible solution for (5) consequently $\partial^*_{a,b} \leqslant \mu^*_a$. From Theorem 1 the result is obvious.

Case 2: DMUb is on the weak frontier. In this case the constraint corresponding to DMUb is redundant and elimination of DMUb does not change the optimal solution of Eq. (5) then $\partial^*_{a,b} = \mu^*_a$.

Theorem 8. *If DMUa is efficient in (2) and $a \neq b$ then $\partial^*_{a,b} = 1$ for each DMUb.*

Proof. Since DMUa is efficient in (2) then $\mu^*_a = 1$. By Theorem 4 $\partial^*_{a,b} = \mu^*_a = 1$ and the proof is complete. □

Theorem 9. *If $\partial^*_{a,b} < 1$ then DMUa is CCR-inefficient.*

Proof. By Theorem 1 $\mu^*_a < 1$, then DMUa is CCR-inefficient. □

4. Numerical example

In this section we are going to ranking the data listed in Table 1 in order to compare the proposed methodology with other traditional ranking ones. We are also going to evaluate a real word banking data.

4.1. Fictional data

Results are given in Table 2, where in the lines we have the non-*SE* DMUs and in the columns we have the efficiency of original CCR model and the efficiency of (5) without *SE* DMUs (DMUs a, b, c, and d). By removing DMUb from the reference set, the inefficient DMUf becomes efficient, but, by removing DMUa, none of the inefficient DMUs becomes efficient, for example. In other words, the *SE* DMUb has more influence on other DMUs than the *SE* DMUa has. The new efficiency of each *SE* DMU is calculated through (6) and it is listed as the last row of Table 2. By considering average of the magnitude influence of *SE* DMUs on non-*SE* DMUs, this article presents a new ranking system for efficient DMUs based upon the magnitude average in Table 2. Results of ranking using the new method are compared with several other methods in Table 3.

The majority ranking methods analyzed in this section points DMUa as the most efficient DMU. Our approach showed that DMUa has very good predicates (it was classified as the 2nd best one) but the DMU that had the major influence in the definition of the original CCR efficient frontier was the DMUb.

Table 1
DMUs' data (extracted from [8, p. 260])

DMU	Input 1	Input 2	Output 1	Output 2
a	150.000	0.200	14000.000	3500.000
b	400.000	0.700	14000.000	21000.000
c	320.000	1.200	42000.000	10500.000
d	520.000	2.000	28000.000	42000.000
e	350.000	1.200	19000.000	25000.000
f	320.000	0.700	14000.000	15000.000

Table 2
New efficiency evaluation

DMU	CCR	DMUa	DMUb	DMUc	DMUd
e	0.978	0.988	0.994	0.978	1.000
f	0.868	0.894	1.000	0.867	0.875
Ω	–	0.941	0.997	0.923	0.938

4.2. Real word data

We evaluated with our proposal the data of 20 branch banks of Iran. This data was previously analyzed by Amirteimoori and Kordrostami [10] and is listed in Table 4.

The use of our proposal generated the analysis shown in Table 5.

Table 3
DMU scores for several ranking methods

Our results		Other ranking methods [8]									
		CCR		BCC		CEA		CEB		EDM	
b	0.997	a	1.000	a	1.000	a	0.764	a	1.000	a	200.000
a	0.941	b	1.000	b	1.000	b	0.700	d	1.000	b	140.625
d	0.938	c	1.000	c	1.000	d	0.700	e	0.974	c	140.000
c	0.923	d	1.000	d	1.000	e	0.696	b	0.955	d	113.077
e	0.978	e	0.978	e	1.000	c	0.643	c	0.886	e	97.750
f	0.868	f	0.868	f	0.896	f	0.608	f	0.847	f	86.745

CEA is the cross-efficiency-aggressive method, CEB is the cross-efficiency-benevolent method and EDM is the extended DEA measure method [3].

Table 4
DMUs' data (extracted from [10, p. 689])

Branch	Inputs			Outputs			CCR efficiency
	Staff	Computer terminals	Space (m^2)	Deposits	Loans	Charge	
1	0.950	0.700	0.155	0.190	0.521	0.293	1.000
2	0.796	0.600	1.000	0.227	0.627	0.462	0.833
3	0.798	0.750	0.513	0.228	0.970	0.261	0.991
4	0.865	0.550	0.210	0.193	0.632	1.000	1.000
5	0.815	0.850	0.268	0.233	0.722	0.246	0.899
6	0.842	0.650	0.500	0.207	0.603	0.569	0.748
7	0.719	0.600	0.350	0.182	0.900	0.716	1.000
8	0.785	0.750	0.120	0.125	0.234	0.298	0.798
9	0.476	0.600	0.135	0.080	0.364	0.244	0.789
10	0.678	0.550	0.510	0.082	0.184	0.049	0.289
11	0.711	1.000	0.305	0.212	0.318	0.403	0.604
12	0.811	0.650	0.255	0.123	0.923	0.628	1.000
13	0.659	0.850	0.340	0.176	0.645	0.261	0.817
14	0.976	0.800	0.540	0.144	0.514	0.243	0.470
15	0.685	0.950	0.450	1.000	0.262	0.098	1.000
16	0.613	0.900	0.525	0.115	0.402	0.464	0.639
17	1.000	0.600	0.205	0.090	1.000	0.161	1.000
18	0.634	0.650	0.235	0.059	0.349	0.068	0.473
19	0.372	0.700	0.238	0.039	0.190	0.111	0.408
20	0.583	0.550	0.500	0.110	0.615	0.764	1.000

Table 5
New branch banks efficiency evaluation

DMU	CCR	DMU 15	DMU 4	DMU 7	DMU 20	DMU 17	DMU 12	DMU 1
2	0.833	1.000	0.833	0.909	0.833	0.833	0.833	0.833
3	0.991	1.000	0.991	1.000	0.991	0.991	0.991	0.991
5	0.899	1.000	0.899	0.899	0.899	0.929	0.913	0.899
6	0.748	0.950	0.810	0.812	0.748	0.748	0.748	0.748
8	0.798	0.916	1.000	0.798	0.798	0.798	0.798	0.811
9	0.789	0.824	0.816	0.789	0.789	0.808	0.814	0.789
10	0.289	0.444	0.289	0.301	0.289	0.289	0.289	0.289
11	0.604	1.000	0.754	0.612	0.614	0.604	0.604	0.604
13	0.817	0.939	0.817	0.865	0.817	0.817	0.817	0.817
14	0.470	0.560	0.470	0.514	0.470	0.470	0.470	0.470
16	0.639	0.749	0.639	0.648	0.709	0.639	0.639	0.639
18	0.473	0.478	0.473	0.484	0.473	0.473	0.483	0.473
19	0.408	0.408	0.408	0.442	0.408	0.408	0.408	0.408
Ω	–	0.790	0.708	0.698	0.680	0.677	0.677	0.675

As one can see, the use of our methodology full ranked the 7 CCR efficient DMUs, being the branch 15 the most efficient one.

5. Conclusion

This article presented a new ranking system in which all DMUs were evaluated simultaneously. Therefore, this method was able to rank all extreme efficient DMUs.

In Section 2 a brief introduction of DEA where the CCR and all the other models used in this work was presented. Our proposal was introduced in Section 3. Section 4 supplied two numerical examples: one comparing our approach with several other ranking methods and other analyzing a real word banking data.

It seems that our approach is more robust than other methods [3,6] and also more intuitive and coherent with the original idea of Charnes, Cooper and Rhodes.

Initial studies had shown that our approach also can be applied with BCC model. We suggest as future works a deeper analysis in this subject.

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