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Syed Jawad Hussain Shahzad **(Corresponding author)**
Montpellier Business School, 2300 Avenue des Moulins, 34080 Montpellier, France
E-mail address: j.syed@montpellier-bs.com

Jose Arreola Hernandez
ESC Rennes School of Business, Rennes, Brittany, France
E-mail address: jose.arreolah.finance@gmail.com

Stelios Bekiros
Department of Economics, Villa La Fonte, Via delle Fontanelle, Florence, Italy
E-mail address: stelios.bekiros@eui.eu

Muhammad Shahbaz
Montpellier Business School, 2300 Avenue des Moulins, 34080 Montpellier, France
E-mail address: shahbazmohd@live.com

Ghulam Mujtaba Kayani
COMSATS Institute of Information Technology, Islamabad Pakistan
E-mail address: ghulam.mujtaba@comsats.edu.pk

Abstract

We model the downside and upside spillover effects, systemic and tail dependence risks of the DJ World Islamic (DJWI) and DJ World Islamic Financial (DJWIF) indices, and of Islamic equity indices from Japan, USA and the UK. We draw our empirical results and conclusions by implementing a robust modeling framework consisting of Value-at-Risk (VaR), conditional VaR (CoVaR), Delta conditional VaR (ΔCoVaR), canonical vine conditional VaR (c-vine CoVaR), and time-varying and static bivariate and vine copula models. Full sample estimations indicate larger downside spillover effects and systemic risk for the DJ Islamic Financials World and USA Islamic indices, while Islamic indices from Japan and the DJ World financials have greater exposure to upside spillover risk effects. During the financial crisis the USA and UK Islamic indices display higher downside systemic risk; and the strongest negative tail asymmetric dependence occurs between the DJ Islamic Financials World, and the Islamic indices from Japan and the DJ World financials. Implications of the results are discussed.

JEL Classifications: C51; C52; C58; G11; G17
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1. Introduction

Never before has been more important to understand the spillover effects across financial stock markets given the increasing globalization phenomenon, the continuous integration of economies and financial markets, and the increasing important role that stock markets play in determining the performance of world economies. It was during the recent global financial crisis of 2008-2009 when it became evident that portfolio losses are largely influenced by the inability of individual and institutional investors to deal on a timely manner with negative tail co-movements, spillover effects and the systemic risk stemming from the instability of too-big-to-fail financial institutions. Since then, an increasing trend of financial modeling has attempted to better understand and trace the underlying linkages connecting financial markets across countries. Financial regulators in particular, given the size of international Islamic equity markets which has reached trillions in asset value, have began to consider the potential risk effects extreme negative tail movements and co-movements in some countries' Islamic equity markets could exert on Islamic (and conventional) equity markets from other parts of the world. All this in the light of the objectives targeted by the Basel III standards, which have highlighted the importance of maintaining the stability of the financial system and of strengthening its resilience (BCBS, 2011). It is under these circumstances where the problem of accurately and adequately estimating the extent to which some aggregate and representative global financial equity markets influence the performance of other financial equity markets scattered across regions of the world becomes relevant and is worth investigating.

The problem has been examined from various perspectives, including Granger-causality, as in Hernandez et al. (2015), Hiemstra and Jones (1994), Cheung and Mak (1992), or cointegration, following the works by Abduh et al. (2011), Yusof and Bahlous (2013), linear and nonlinear regression e.g., in Ling and Naranjo (1999), Connor and Korajczyk (1995), and vine copula modeling, as in Arreola-Hernandez et al. (2016), Bekiros et al. (2015) and Arreola-Hernandez (2014). Compared with those studies our has the comparative advantage of implementing a robust modeling framework consisting of Value-at-Risk (*VaR*), conditional Value-at-Risk (*CoVaR*), Delta conditional Value-at-Risk (Δ*CoVaR*), canonical vine conditional Value-at-Risk (*c*-vine *CoVaR*) and time-varying and static bivariate and vine copula methods, in an attempt to analyze downside and upside spillover effects, systemic risk and tail dependence risk. We show the usefulness of our empirical approach by modeling Islamic equity indices from Japan, USA and the UK, and the DJ World Islamic Financial and DJ World Islamic indices. The main objective of our research work is to identify the Islamic equity indices, scattered across regions of the world, with greater exposure to downside and upside dependence structure-based spillover, systemic and tail dependence risks caused by extreme movements in the DJ World Islamic Market, a representative of all trading country-based Islamic markets. It is also of interest to discern, through hypothesis testing, whether spillover and contagion effects increase under adverse market scenarios.

We contribute to the relevant literature by conducting under normal and adverse market scenarios (i.e., during the global financial crisis of 2008-2009) a thorough and exhaustive analysis of downside and upside spillover effects, systemic and tail dependence risks in domestic and global Islamic equity markets. The incorporation of vine copulas in our analysis enables us to draw accurate estimates of the risk contribution of global indices to systemic risk, given the precise and adequate measurement we draw of the changing dependence structure corresponding to the global and domestic Islamic equity indices under consideration. The use of both, time-varying and static bivariate and vine copulas makes it possible to conduct a robust analysis of tail dependence risk as a function of the time factor and with respect to its effects on the bivariate and multivariate dependencies. To our knowledge no other research study has implemented the modeling framework we consider on global and domestic Islamic equity markets.

In terms of the modeling approach we pursue our research is broadly linked to studies undertaken by Reboredo and Ugolini (2015a) who by means of vine copula and *CoVaR* models investigate the systemic impact of financial distress of Spanish listed banks on other listed banks that are part of the European financial system. Their results indicate a significant increase of systemic risk in the aftermath of the recent global financial crisis and, to a lesser extent, around the time of the European sovereign debt crisis. Specific findings of their research indicate that the Spanish bank Banco Bilbao Viscaya Argentaria (BBVA) played a predominant role as it both transmitted and received systemic risk to and from the remaining listed banks. Also, while the Santander Bank played a minor role, the smallest banks Sabadell and Bankinter did not play any pivotal role, not even between themselves. The study by Reboredo and Ugolini (2015b), in the context of the European sovereign debt crisis, finds that spillover effects and systemic risk increased slightly in those European countries that did not have sovereign debt problems, after the onset of the Greek debt crisis. On the contrary, countries that had sovereign debt problems during the Greek crisis period experienced smaller spillover effects. Moreover, the Greek economy during the sovereign debt crisis had the largest spillover effect on the Portuguese economy. Girardi and Ergün's (2013) analysis is relevant to our research work in that it modifies the model for systemic risk examination proposed by Adrian and Brunnermeier (2011, 2016). The specific type of model specification they implement is better as to incorporating risk contribution, spillover effects and systemic risk between groups of financial risk factors, and for events characterized by severe distress. Yun and Moon (2014) by fitting the Marginal Expected Shortfall (MES) and the *CoVaR* methods examine the risk contribution and systemic risk between Korean financial institutions. Their incorporation of dynamic conditional correlation estimates into the MES and *CoVaR* methods is indicated to adequately account for banks' dependence structure changes across time and varied market conditions. They also show the risk contribution between banks is dependent on bank's size, leverage ratio and Value-at-Risk exposure.

Reboredo et al. (2016) investigate the downside and upside spillovers between exchange rates and stock prices from emerging economies by first modeling the dependence relationships between the financial variables, and by using the obtained estimates of dependence to adequately and accurately measure the Value-at-Risk exposure. They find evidence of positive correlation between stock prices and currency values for emerging economies, and identify variations in downside and upside asymmetric spillover effects between emerging market currencies, the US dollar and Euro. Regarding the implementation of time-varying bivariate and vine copulas, our study connects to that conducted by Al Janabi et al. (2017), wherein a dynamic conditional correlation *t*-copula is employed along with a liquidity Value-at-Risk model to compare optimal portfolios of international stock and commodity indices. In the same vein Arreola-Hernandez et al. (2016) model the dependence structure of equity portfolios under a systematic copula counting technique proposed for the analysis and interpretation of multivariate dependence risk.

Our empirical results indicate that greater downside spillover effects and systemic risk exists in the DJ Islamic Financials World and USA Islamic indices when considering the full sample period, while equities from the USA and UK Islamic indices are more exposed to downside spillovers during the global financial crisis period. The DJ Islamic Financials World and the Japan Islamic indices have greater exposure to upside systemic risk for the full sample period, while the USA and UK Islamic indices present greater spillovers and risk exposure during the crisis period. The downside and upside *VaR* values are greater for the group of equities belonging to the DJ World Islamic Financial and UK indices. The strongest positive tail asymmetric dependence occurs between the UK and the USA Islamic equity indices and the DJ World Islamic index. The strongest negative tail asymmetric dependence is observed between the DJ Islamic Financials World and Japan Islamic indices and the DJ World Islamic index. The findings may appeal to portfolio and risk managers who hold long and short investment positions and whose concern is risk in the downside and upside.

This paper is organized as follows: the following section presents the modeling framework we apply to undertake our analysis of spillover effects, systemic and tail dependence risks. Section 3 explains the data set under investigation. In Section 4 we display and interpret our empirical results, whilst section 5 concludes the analysis.

2. METHODOLOGY

2.1. Tail modeling and marginal density

As part of the vine copula model specification, we fit an ARMA-FIGARCH process with long-range volatility memory and for the marginal distribution of the standardized residuals; we use the skewed *t* distribution of Hansen (1994) to adequately capture the distributional characteristics of the marginals[1]. The advantages of this specific type of evolution process derive from its FIGARCH component, which provides the required flexibility to adequately model the conditional variance and better explain the dependence of the marginals' volatility through time (Baillie et al., 1996). We specify the marginal densities of the stock market returns (r_t) using an ARMA (*m*, *n*) model of the following type:

$$r_t = \emptyset_0 + \sum_{j=1}^{m} \emptyset_j \, r_{t-j} + \varepsilon_t - \sum_{i=1}^{n} \theta_i \, \varepsilon_{t-i} \quad (1)$$

where *m* and *n* are non-negative integers, and $\emptyset_j$ and θ_i the autoregressive (AR) and moving average (MA) parameters. The parameter σ_t^2 in the expression $\varepsilon_t = \sigma_t z_t$ is the conditional variance that has dynamics determined by an FIGARCH (*p*, *d*, *q*) model. The parameter *d* represents the fractional differencing parameter of the model, where $0<d<1$. This parameter is important as it offers information about the speed and patterns at which effects (or shocks)

[1] We considered four competing GARCH models, namely the ARMA-GARCH, the ARMA-FIGARCH, the ARMA-FIEGARCH and the ARMA-FIAPARCH models, with each of them having a skewed student-t distribution. In selecting the best marginal model for each time series, we have considered multiple and diverse combinations of the lag parameters m, n, p, and q for values of the lags ranging from zero to a maximum of 2. Based on the AIC, we have selected the most adequate model, which in this case is the ARMA-FIGARCH model.

to the volatility process of the model are exerted and propagated. Moreover, the conditioning of *d* to be in the above-specified range is necessary as shock propagation to the mean happens within those values in a slow manner and according to hyperbolic rate. Furthermore, for *d>0* the process displays long memory features, considering its long-term characteristics. For *d=1* the shocks to the conditional variance of the IGARCH component in the FIGARCH continue indefinitely, making it impossible to obtain a finite estimate of forecasted volatility. On the other hand, when *d>1* the conditional variance cannot be defined. In order to adequately account for the dependence of the marginals' volatility, according to Baillie et al. (1996), we use the following model specification:

$$\phi(L)(1-L)^{\xi}\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{2}$$

$$\phi(L) \equiv \phi_1 L + \phi_2 L^2 + \cdots + \phi_q L^q \tag{3}$$

$$\beta(L) \equiv \beta_1 L + \beta_2 L^2 + \cdots + \beta_p L^p \tag{4}$$

$$v_t \equiv \varepsilon_t^2 - h_t \tag{5}$$

The parameter $\{v_t\}$ has a zero mean, is serially uncorrelated and accounts for the changes in the conditional variance, as being determined by new volatility shocks. The parameter h of the FIGARCH model specification, in relation to the v changes or innovations in the conditional variance in Eq. (5), stands for the conditional variance, while the parameter ε_t^2 stands for the square shocks. The parameter L stands for a lag, and if $0 \leq \xi \leq 1$ the model becomes linear. If $\xi = 1$ and $\mathrm{d} = 1$ the process has a unit root, thus reflecting a permanent shock effect. In order to be able to account for the skewness in the negative tail of the return distribution we incorporate in the FIGARCH marginal model specification a skewed Student-t parameterization. In this way, we are in fact implementing a FIGARCH with Student-t innovations to adequately model negatively skewed behavior in the marginal distributions. The density of the skewed-*t*, following Hansen (1994), can be expressed as follows:

$$f(z_t, v, \eta) = \begin{cases} bc\left(1 + \frac{1}{v-2}\left(\frac{bz_t+a}{1-\eta}\right)^2\right)^{-(v+1)/2} & z_t < -{}^{a}/_{b} \\ bc\left(1 + \frac{1}{v-2}\left(\frac{bz_t+a}{1+\eta}\right)^2\right)^{-(v+1)/2} & z_t \geq -{}^{a}/_{b} \end{cases} \quad (6)$$

The parameters v and η represent the degrees of freedom and $(2 < v \leq \infty)$.When $(-1 <$ η*<1* the Student-*t* models the tails symmetrically. The *a*, *b* and *c* are constants represented by the following equalities:

$$a = 4\,\eta c\left(\frac{v-2}{v-1}\right) \quad (7)$$

$$b^2 = 1\ + 3\eta - a^2 \quad (8)$$

$$c = \Gamma\left(\frac{v+1}{2}\right) / \sqrt{\pi(v-2)}\,\Gamma\left(\frac{v}{2}\right) \quad (9)$$

As the degrees of freedom we are considering are finite, the skew-*t* converges to the symmetric Student-t distribution (i.e. $\eta = 0$ and v is finite). However, if we have $\eta = 0$ and $v \rightarrow \infty$, the skew-t converges to the standard Gaussian distribution.

2.2. Canonical vine copula

The canonical vines (*c*-vines) represent a subset of the regular vines and are acknowledged for being statistically adequate to account for the dependence structure of multivariate series with one variable exhibiting the strongest correlations (i.e. exerting the greatest influence) among the rest of the variables in a data set (Arreola-Hernandez et al., 2016; Arreola-Hernandez, 2014; Czado, 2010). A regular vine is called *canonical* or *c-vine* if its "trees" are formed by nodes and edges (where the node with maximal degree in T_1 of a canonical vine is the root) and each tree T_i has a unique node of degree $n - i$. The c-vines are subject to the proximity condition which states that for $i = 2, \dots, n-1$, if $\{a, b\} \epsilon E_i$, then $\#a\Delta b = 2$, (Δ denotes a union without the intersection). This means that, if a and b are nodes of a tree T_i connected by an edge, where $a = \{a_1, a_2\}$ and b= $\{b_1, b_2\}$, then exactly one of the a_i equals one of the b_i. As such, canonical vines have a star-like shape and for every tree T_i, $\in$ $\{1, \dots, n-1\}$, a root node is selected based on the criterion of having the strongest correlation with the rest of the variables in the tree. We select the c-vine copula model instead of others types of vine copulas (e.g., d-vines, r-vines) as it provides the flexibility and accuracy required to measure the nonlinear dependence between the pairs of financial variables considered (see Bekiros et al., 2015; Arreola-Hernandez, 2014; Brechmann and Czado, 2013; Nikoloulopoulos et al., 2012). Thus, we expect the selected c-vine copula model to improve the accuracy of the implemented CoVaR and Delta CoVaR models. The model we use in our work to separate multivariate densities $f(\boldsymbol{x}) = f(x_1, \ldots, x_n)$ and to infer pair c-vine copula structures, was introduced by Aas et al. (2009):

$$f(\boldsymbol{x}) = \prod_{k=1}^{n} f_k(x_k) \cdot \prod_{i=1}^{n-1} \prod_{j=1}^{n-i} c_{i,i+j|1:(i-1)} \left(\mathrm{F}(x_i|x_1, \ldots x_{i-1}), \mathrm{F}\left(x_{i+j}\middle|x_1, \ldots, x_{i-1}\right)\middle|\boldsymbol{\theta}_{i,i+j|\,1:(i-1)}\right) \quad (10)$$

2.3. Time-varying and static bivariate copula models

We employ static and time-varying parameter bivariate copula models to examine the upper and lower tail dependence between the pairs of country-based and global Islamic equity indices. The bivariate copulas employed have been built on the proposition of the theorem of Sklar (1952), which asserts that a joint distribution $F_{XY}(X,Y)$ of two continuous random variables X and Y can be expressed in terms of a copula function C(u, v) and the marginal distribution functions $F_x(x)$, $Fy(y)$, so that:

$$F_{XY}(X,Y) = C(u,v) \quad (11)$$

where $u = F_x(x)$ and $v = F_Y(y)$. Eq. (11) shows that a bivariate copula can be used to account for a specific type of dependence relationship which two uniform marginal distributions hold, as inferred by their joint distribution.[2] In implementing the bivariate copulas, a parametric distribution function (e.g. Student-*t*, Gaussian) is used to capture and shape the marginal empirical distribution. The drawn parametric distributions from the marginals could either be embedded or not embedded inside the bivariate copula set-up. The following joint probability

[2] See Joe (1997) and Nelsen (2006) for more details. For an overview of copula applications to finance see Cherubini et al. (2004).

density of X and Y, resulting from the copula density $c(u,v) = \frac{\partial^2 C(u,v)}{\partial_u \partial_v}$, shows that a bivariate copula is capable of modeling separately the marginals and the dependence structure from a pair of variables' joint distribution:

$$f_{XY}(X,Y) = C(u,v) f_Y(y) f_X(x) \tag{12}$$

In Eq. (12), $f_Y(y)$ and $f_X(x)$ represent the marginal density functions of Y and X respectively. On a more practical level, a bivariate copula provides information about extreme upper and lower co-movements or what is commonly known as *tail dependence*. The smaller the tail dependence parameters the stronger the correlation observed in the negative tail of the joint distributions, and vice versa. An analytical measure of upper and lower tail dependence can be expressed as follows:

$$\lambda_U = \lim_{u\to 1} \Pr\,[X \geq F_X^{-1}(u) \mid Y \geq F_Y^{-1}(u) = \lim_{u\to 1} \frac{1-2u+C(u,v)}{1-u} \tag{13}$$

$$\lambda_L = \lim_{u\to 0} \Pr\,[X \leq F_X^{-1}(u) \mid Y \leq F_Y^{-1}(u) = \lim_{u\to 0} \frac{C(u,v)}{u} \tag{14}$$

where $\lambda_U, \lambda_L \in [0,1]$. The lower (upper) tail dependence implies that $\lambda_L > 0$ ($\lambda_U > 0$), showing that a non-zero probability of observing an extremely small (or large) value for one series together with an extremely small (or large) value for another series. In our copula application, we implement various types of static and time-varying parameter bivariate copula families including elliptical and Archimedean, and rotated versions of them. The rotated bivariate copulas are employed to better capture joint distributional characteristics, which the standard non-rotated bivariate copulas cannot account for.[3] Among the bivariate copulas we apply are the Gaussian, Clayton, 180-degrees rotated Clayton, Plackett, Frank, Gumbel, 180-degrees rotated, Gumbel, Student-t and Joe-Clayton. The Clayton and 180-degrees rotated Gumbel bivariate copulas are suitable models to measure stronger dependence in the negative tail of the joint distributions. The 180-rotated Clayton, Gumbel

[3] The time-varying parameter bivariate copulas enable capturing the changes in the dependence structure through time.

and Joe-Clayton copulas are adequate models to measure stronger dependence in the positive tail of the joint distributions. The Student-t copula is designed to symmetrically account for dependence in both tails of the joint distributions.[4] The Frank and Gaussian bivariate copulas serve for nonlinear and linear dependence modeling in the center of the joint distribution, respectively. The Plackett is useful to identify potential independence between the pairs of variables under consideration. Hence, we strategically employ a robust bivariate copula-modeling framework that targets the modeling of dependence at various locations of the joint distributions. Table 1 presents the analytical design of each of the bivariate copulas we fit in our research study.

[PLEASE INSERT TABLE 1 HERE]

The change in the parameters of the time-varying copulas is defined by an evolution equation. As to the Gaussian and Student-t bivariate copulas, the evolution of the linear dependence parameter ρ_t follows an ARMA(1,q)-type process (Patton, 2006):

$$\rho_t = \Lambda\left(\Psi_0 + \Psi_1\rho_{t-1} + \Psi_2 \frac{1}{q}\sum_{j=1}^{q} \Phi^{-1}(u_{t-j}).\Phi^{-1}(v_{t-j})\right) \qquad (15)$$

where $\Lambda(x) = (1 - e^{-x})(1 + e^{-x})^{-1}$ is the modified logistic transformation which ensures that ρ_t is in the range [-1, 1]. This implies the dependence parameter is determined by a constant parameter Ψ_0, the explanatory factor of the historical correlation as scaled by Ψ_1, and by the average product of the last q observations of the transformed variables, Ψ_2. The dynamic time-varying parameters of the Student-*t* bivariate copula are also explained by Eq. (15) if one substitutes $\Phi^{-1}(x)$ by $t_v^{-1}(x)$. The evolution dynamics of the time-varying parameters of the asymmetric Gumbel and rotated Gumbel bivariate copulas follows an ARMA(1,q) process with the following specification:

$$\delta_t = \omega + \beta\delta_{t-1} + \alpha \frac{1}{q}\sum_{j=1}^{q}\left|u_{t-j} - v_{t-j}\right| \qquad (16)$$

[4] The bivariate copulas that measure the tail dependence symmetrically are the Gaussian, Student-t, Frank and the Plackett. The copulas for asymmetric dependence modeling are the Gumbel (for upper tail dependence), the rotated Gumbel (for lower tail dependence) and Clayton (for lower tail dependence).

The evolution of the SJC tail dependence parameters is formulated according to:

$$\lambda_t^U = \Delta\left(\omega_U + \beta_U \rho_{t-1} + \alpha_U \frac{1}{q}\sum_{j=1}^{q}\left|u_{t-j} - v_{t-j}\right|\right) \qquad (17)$$

$$\lambda_t^L = \Delta\left(\omega_L + \beta_L \rho_{t-1} + \alpha_L \frac{1}{q}\sum_{j=1}^{q}\left|u_{t-j} - v_{t-j}\right|\right) \qquad (18)$$

where $\Delta(x) = (1 + e^{-x})^{-1}$ is a logistic transformation ensuring that λ_t^U and λ_t^L are in the range of [0, 1].

2.4. VaRs, CoVaRs and Delta CoVaRs risk measures

We fit Value-at-Risk (*VaR*), conditional Value-at-Risk (*CoVaR*), Delta conditional Value-at-Risk (Δ*CoVaR*), canonical vine conditional Value-at-Risk (*c*-vine *CoVaR*), while at the same time we perform hypothesis testing to measure, understand and validate potential losses in Islamic equity portfolio investments, stemming from market volatility and systemic dependence deriving from spillovers the World Islamic Market Index may have on country-based Islamic indices. The *VaR* measures the probable losses a portfolio of assets, which may incur for a specified time investment horizon (e.g., 1-day, 10-day) and a chosen confidence level (e.g., 95%, 99%). The *VaR* can be fitted to forecast investment losses in the downside (for long investment positions) or upside (for short investment positions). The downside risk for a portfolio position at time *t* in the future for a $1-\alpha$ confidence level is equal to:

$$Pr\left(r_t \leq VaR_{\alpha,t}\right) = \alpha \qquad (19)$$

$$VaR_{\alpha,t} = \mu_t + t_{v,\eta}^{-1}(\alpha)\sigma_t, \qquad (20)$$

where the parameters μ_t and σ_t are the conditional mean and standard deviation of the Islamic equity indices' time series modeled, respectively. Each of these parameters evolves according to an ARMA-FIGARCH process as presented in Section 2.1. In Eqs. (19) and (20), the parameter $t_{v,\eta}^{-1}(\alpha)$ stands for the α quantile of the skewed Student-*t* distribution. The upside risk of a portfolio position at time *t* and for a $1-\ \alpha$ confidence level is equal to:

$$Pr\left(r_t \geq VaR_{1-\alpha,t}\right) = \alpha \qquad (21)$$

$$VaR_{1-\alpha,t} = \mu_t + t_{\upsilon,\eta}^{-1}(1-\alpha)\sigma_t \tag{22}$$

Next, we implement *CoVaR* modeling following Adrian and Brunnermeier (2011) and Girardi and Ergün (2013). The *CoVaR* for the asset *i* is defined as the *VaR* for *i* conditional on asset *j* exhibiting extreme movement. Asset *j* represents a variable capable of triggering systemic risk (e.g., a too-big-to-fail financial institution or a representative index of a financial market, as in our work) through spillover effects on other variables with which it holds some degree of dependence. Analytically, we let r_t^s be the returns for Islamic equities and r_t^o be the returns of the DJ World Islamic Market Index (i.e., the DJ World Islamic Market Index is the variable capable of triggering systemic risk on the country-based Islamic equity indices through spillover effects).[5] The downside risk of an investment in Islamic equity securities for a $1-\beta$ confidence level, given an extreme downward movement in the prices of the DJ World Islamic Market is the β-quantile of the conditional distribution of r_t^s:

$$r\left(r_t^s \leq CoVaR_{\beta,t}^s \middle| r_t^o \leq VaR_{\alpha,t}^o\right) = \beta \tag{23}$$

From the above equation the parameter $VaR_{\alpha,t}^o$ stands for the α-quantile of the DJ World Islamic Market Index price distribution and quantifies the maximum loss of a long investment position in equity indices, for a specific time horizon and confidence level $1-\alpha$. The probabilistic expression $\Pr\left(r_t^o \leq VaR_{\alpha,t}^o\right) = \alpha$ accounts for the maximum loss a portfolio position in Islamic equity indices may experience. The upside *CoVaR* for an extreme upward movement in the prices of the DJ World Islamic Market Index can be expressed as follows:

$$Pr\left(r_t^s \geq CoVaR_{\beta,t}^s \middle| r_t^o \geq VaR_{1-\alpha,t}^o\right) = \beta \tag{24}$$

[5] The DJ World Islamic market index measures the global universe of investable equities and it represents the most comprehensive and widely employed index of Islamic stocks in the world. This index contains data for over 12,000 companies from 77 countries, although most of the stocks in the DJ World Islamic market universe are located in non-Muslim developed countries.

In this case, the parameter $VaR^{o}_{1-\alpha,t}$ quantifies the maximum loss of a short investment position in Islamic equity indices, for a specific investment horizon and confidence level. The specification of the *CoVaR* in the form of copula functions is given as:

$$C\left(F_{r_t^s}\left(CoVaR^{s}_{\beta,t}\right), F_{r_t^o}\left(VaR^{o}_{\alpha,t}\right)\right) = \alpha\beta \tag{25}$$

$$1 - F_{r_t^s}\left(CoVaR^{s}_{\beta,t}\right) - F_{r_t^o}\left(VaR^{o}_{1-\alpha,t}\right) + C\left(F_{r_t^s}\left(CoVaR^{s}_{\beta,t}\right), F_{r_t^o}\left(VaR^{o}_{1-\alpha,t}\right)\right) = \alpha\beta \tag{26}$$

where $F_{r_t^s}$ and $F_{r_t^o}$ represent the distributions of the marginals or the distribution of the returns corresponding to both, the country-based Islamic equity indices and the DJ World Islamic equity index, respectively.

In estimating the *CoVaR* for the set of domestic and global Islamic equity indices we follow the process introduced by Reboredo and Ugolini (2015a):

Step 1: We could solve for either Eq. (25) or Eq. (26) in order to draw a value for $F_{r_t^s}\left(CoVaR^{s}_{\beta,t}\right)$, respectively considering the *VaR* and *CoVaR* significance levels, as well as the $\beta_,$.

Step 2: We employ the distribution functions of the DJ World Islamic Market and the Islamic equity indices considering the marginal specification model of Eqs. (1) – (9) and we estimate the *CoVaR* for the equities as $F^{-1}_{r_t^s}\left(F_{r_t^s}\left(CoVaR^{s}_{\beta,t}\right)\right)$.

Furthermore, the systematic risk triggered by extreme movements in the prices of the DJ World Islamic Market Index and reflected on the country-based Islamic equity indices, is defined following Brunnermeier (2011) and Girardi and Ergün (2013) as the risk contribution of a stock market s on single or groups of asset classes. It can be estimated as the delta $CoVaR$ ($\Delta\, CoVaR$), representing the difference between the VaR of the stock market as a whole (DJ World Islamic Market Index), conditional on the distressed state of market s (global financial crisis period in our study) i.e., $\left(R^{s}_{t} \leq VaR^{s}_{\alpha,t}\right)$, and the VaR of the stock market as a whole conditional on the benchmark state of market s. The latter is formulated as the median of the return distribution of market s or the VaR for $\alpha = 0.5$. The systemic risk contribution of market s (DJ World Islamic Market) is eventually defined as:

$$\Delta CoVaR_t^{d/s} = \frac{\left(CoVaR_{\beta,t}^{d/s} - CoVaR_{\beta,t}^{d/s,\alpha=0,5}\right)}{CoVaR_{\beta,t}^{d/s,\alpha=0,5}} \quad (27)$$

The main advantage of the $\Delta CoVaR$ lies in accounting for the marginal risk contribution of market s vis-à-vis the overall risk.

In order to find out if the estimated systemic risk is significant at the considered confidence level, we employ the KS bootstrapping test developed by Abadie (2002). We test the hypothesis of no systemic impact between the returns of the DJ World Islamic Market Index and the country-based Islamic equity indices as:

$$H_0: CoVaR_{\beta,t}^{s} = VaR_{\beta,t}^{s} \quad (28)$$

The KS test measures the difference between two cumulative quantile functions by employing empirical distribution functions, as opposed to using parametric distribution functions. The test is defined as follows:

$$KS_{mn} = \left(\frac{mn}{m+n}\right)^{\frac{1}{l}} \sup_x |F_m(x) - G_n(x)| \quad (29)$$

where $F_m(x)$ and $G_n(x)$ are the cumulative *CoVaR* and *VaR* distribution functions, respectively, and *n* and *m* are the sizes of the two samples.

3. Data analysis

The data set comprises daily equity closing prices of country-based Islamic equity indices (Japan, USA, UK), the global DJ World Islamic Market Index and the DJ Islamic Financials World Index.[6] This global DJ World Islamic Market Index includes data of more than 12,000

[6] The Dow Jones Islamic market (DJIM) incorporates Islamic equities listed in USA, UK, Japan and it is used as a proxy for the global Islamic stock market. As noted in Naifar et al. (2016), the DJIM was launched in February 1999 and constitutes the first global Islamic index created for investors seeking equities in compliance with *Sharia*. Data of the index prior to that year it was launched is obtained according to an index methodology employed by Dow Jones today. The Dow Jones Islamic Market Financials Index is a subset of the DJIM World Index and measures the performance of financial stocks traded globally that pass the screens for compliance

companies from 77 countries and most of the stocks in the index are from the non-Muslim developed countries. According to the Dow Jones website, the U.S. at the end of October 2016 has the highest country allocation (60.53%) in the index, followed by Japan (7.63%), Switzerland (4.67%) and the UK (3.69%). The U.S., the UK and Japan stock indices are selected because they are currently the three largest stock markets in the world and cover different geographical areas. All the stock market indices are expressed in U.S. dollar terms to have a homogenous dataset and to avoid issues of exchange rate risk[7]. We choose to include the Islamic equity financial sector because the timeline of the price and return series we model, covers the global financial crisis of 2008-2009, thus we could expect this sector to experience significant systemic risk-derived effects. The data set ranges from January 1, 1996 to December 31, 2015, covering a total of 19 years and 5220 daily observations.

Our motivation for selecting country-based and global Islamic equity indices is that the number of studies focusing on the analysis of spillover effects, tail dependences and Value-at-Risk between country-based and global Islamic equity indices is still very small. Moreover, the findings resulting from our study would constitute a benchmark for relative comparisons of spillover effects, systemic risk and dependence risk between Western-type and Islamic-type equity markets. We select daily frequency price series because higher frequency data poses a comparative advantage relative to weekly, monthly or quarterly data as reported in Chortareas et al. (2011) and Liu (2009). As in most studies that undertake bivariate and vine copula modeling we estimate and use “copula data” lying in the range [0,1]. For the estimation of *VaR* and *CoVaR* we use logarithmic returns. All Islamic equity price series have been downloaded from Thomson Reuters Datastream International.

with the Islamic investment guidelines. It represents the Financial Industry as defined by Industry Classification Benchmark (ICB) and is a float market cap weighted index.

[7] In order to avoid asynchronicity issues caused by different time zones, we have matched equity prices from day *t* for the global Islamic stock market, the DJ Islamic Financials World Index, the U.S., and UK stock markets with equity prices from day *t+1* for the Japanese stock market.

The descriptive statistics displayed in Table 2 indicate that the USA Islamic equity index and the Dow Jones World Islamic Equity Index have the largest historical means. A look at the equity historical standard deviation shows that the USA Islamic equity index and the Dow Jones World Islamic Equity Index also present the lowest volatility scores, making them the best investment choices from the entire set of country-based and global Islamic equity markets modeled. A distributional feature perhaps not appealing to potential investors is that both have a negative skewness, i.e., indicating a tendency to yield short-term trends of negative returns once one or two negative returns have been realized. The correlation matrix of the country-based and global Islamic equity indices reveals the strongest correlations between the USA Islamic equity index and the DJ Islamic Financials World Index, as well as between the UK and USA Islamic equity indices. Also the USA and UK Islamic equity indices are the most strongly correlated with the Dow Jones World Islamic Index. Figure 1 displays the price series plots of the country and global Islamic indices under consideration. It can be seen that UK, USA and world Islamic indices appear to be the most volatile, followed by the equities of the Japan and DJ Islamic Financials World Index. All price series show the market downturns corresponding to the burst of the Internet Bubble occurring between 2001 and 2003, and the Global Financial Crisis of 2008-2009. However, over the long rung there is an escalating trend in Islamic equity security prices. Figure 2 depicts the c-vine copula structure for the investigated markets. Lastly, the estimates of the ARMA-FIGARCH process with long-range volatility memory and normal standardized residuals for the marginals of the vine copula modeling are reported in Table 3.

[PLEASE INSERT TABLES 2, 3 AND FIGURES 1, 2 HERE]

4. EMPIRICAL RESULTS

Our analysis focuses on the examination of lower and upper tail dependence, Value-at-Risk, spillover effects and systemic risk between the pairs of country-based and global Islamic equity indices. With respect to lower and upper tail dependence we find that, according to each of the bivariate copulas considered, the strongest tail dependence occurs between the Dow Jones World Islamic Index and the UK and USA Islamic equity indices. A comparison of the tail copula parameters derived by the Clayton and Gumbel 180-degrees rotated, and the Clayton 180-degrees rotated ones, shows that the UK and the USA Islamic equity indices illustrate stronger asymmetric positive tail dependence with the DJ World Islamic index, while the equities from the DJ Islamic Financials World and Japan Islamic indices present stronger asymmetric negative tail dependence with the DJ World Islamic index. On the other hand, a comparison of the tail dependence parameters, drawn from the Frank and Gaussian bivariate copulas indicates that the dependence between the DJ World Islamic Index and the USA and UK Islamic equity indices is nonlinear in the center of the joint distributions or when the financial stock markets are calm. These results are confirmed by the fit of the time-varying bivariate copulas as shown by the tail dependence parameters displayed in Panel B of Table 5. Figures 3 and 4 illustrate the temporal characteristics of c-vine and bivariate copula estimated parameters.

[PLEASE INSERT TABLE 4 AND 5 HERE]

[PLEASE INSERT FIGURE 3 AND 4 HERE]

Based on Table 6 and 7, the *VaR* results in relation to downside risk when considering the full sample period, indicate that long investment positions in equities of the DJ Islamic Financials World and the Japan Islamic indices have greater *VaR* exposure stemming from market volatility. During the global financial crisis long investment positions in the DJ Islamic Financials World and the UK Islamic indices show higher risk exposure. With respect to upside risk, the *VaR* results for the full sample indicate that short investment positions in the DJ Islamic Financials World and the Japan Islamic indices have higher *VaR* scores. Also, during the global financial crisis, short investment positions in the DJ Islamic Financials World and the UK Islamic indices show greater risk exposure too.

[PLEASE INSERT TABLES 6 AND 7 HERE]

The *CoVaR* empirical findings for downside risk in the full sample period indicate that higher systemic risk exists in the DJ Islamic Financials World and USA Islamic indices, given the spillover effects stemming from downturns or decreases in the prices of the DJ World Islamic Index. During the global financial crisis the equities from the USA and UK Islamic indices are more exposed to systemic risk, as demonstrated by the spillover effects from downturns in the prices of the DJ World Islamic Index. As to upside risk, when we consider the full period the systemic *CoVaR* estimates are larger for the DJ Islamic Financials World and the Japan Islamic indices, due to spillovers derived by the increases in the prices of the DJ World Islamic Index. During the global financial crisis the *CoVaR* estimates are larger for the USA and UK, as a result of the up-trending prices of the DJ World Islamic Market Index. The Delta *CoVaR* identifies the greatest systemic risk in the USA Islamic index and the DJ Islamic Financials World Index for both financial periods under consideration. Figure 5 displays the dynamics of the VaR and CoVaR estimated parameters, whilst Figure 6 depicts the time series of the VaR and CoVaR estimates per se.

[PLEASE INSERT FIGURES 5 AND 6 HERE]

Next, the vine *CoVaR* estimates for the full sample reveal that on the downside the DJ Islamic Financials World and the UK Islamic equity indices present the strongest dependence. Under adverse market circumstances namely during the Global Financial Crisis, the UK and USA Islamic equity indices show the strongest dependence. From the upside vine copula *CoVaR* estimates at the 99% confidence level, we observe the strongest dependence between the USA and UK Islamic equity indices. Tables 8 and 9 highlight the statistical significance of the estimated risk metrics via the utilization of the KS bootstrapping tests at various confidence levels (Abadie, 2002). Overall, the same applies during the Global Financial Crisis. The downside and upside Delta *CoVaR* results for both period scenarios confirm the assumption that the USA and UK Islamic equity indices demonstrate the strongest dependence.

[PLEASE INSERT TABLES 8 AND 9 HERE]

5. CONCLUSIONS

As the size and value of Islamic equity markets increases in trillions the importance of measuring and understanding country-based Islamic market marginal and joint dependence behavior cannot be underscored. This is particularly so because Islamic equity markets are subsets of conventional equity markets in most countries, thus posing a degree of systemic risk to country equity markets through spillover risk effects. The present study, in order to draw and provide new and useful insights about the characteristics of interdependence, systemic risk and spill over risk effects in domestic and global Islamic equity models the downside and upside spillover effects, the systemic risk and the tail dependence risk of Islamic equity market indices from Japan, USA and the UK, and for the DJ Islamic Financials World and USA Islamic indices. The empirical results are obtained by implementing Value-at-Risk (*VaR*), conditional VaR (*CoVaR*), Delta conditional VaR (Δ*CoVaR*), canonical vine conditional VaR (*c*-vine *CoVaR*), and time-varying and static bivariate and vine copulas models.

We find evidence of larger downside spillover effects and systemic risk for the DJ Islamic Financials World and USA Islamic indices when considering the entire sample period, while Islamic equities from the USA and UK have greater exposure to downside systemic risk during the global financial crisis. Larger upside spillover effects are identified in the DJ Islamic Financials World and the Japan Islamic indices for the full sample period, whilst the USA and UK Islamic indices are more exposed to systemic risk during the crisis. The downside and upside *VaR* values are larger for the group of equities belonging to the DJ World Islamic Financial and UK indices. The strongest positive tail asymmetric dependence occurs between the UK and the USA Islamic equity indices and the DJ World Islamic index.

Further, the strongest negative tail asymmetric dependence is observed between the DJ Islamic Financials World and Japan Islamic indices and the DJ World Islamic index. Our findings could be important to portfolio managers, policy makers and investors who hold long and short investment positions and whose concern is risk in the downside and upside for systemic risk management purposes.

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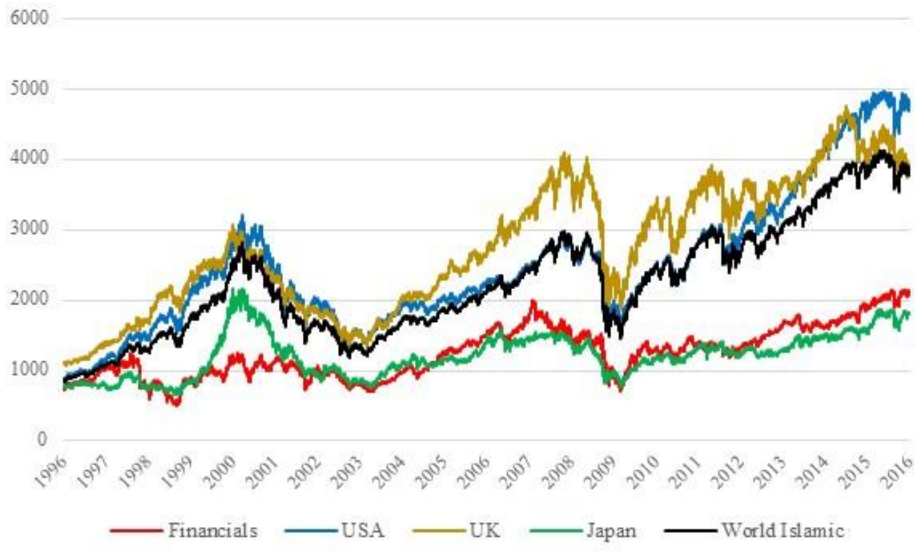
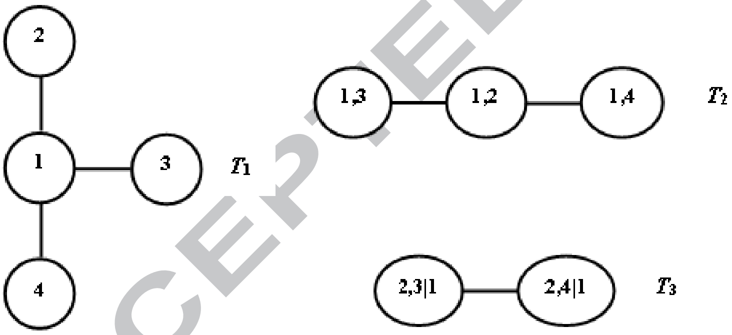
FIGURE 1: TIME SERIES OF ISLAMIC STOCK INDICES



Notes: The plot displays the price series corresponding to the Islamic stock indices. It can be seen that the UK Islamic index appears to be more volatile, relative to the USA, Japan, Islamic world and Financials.

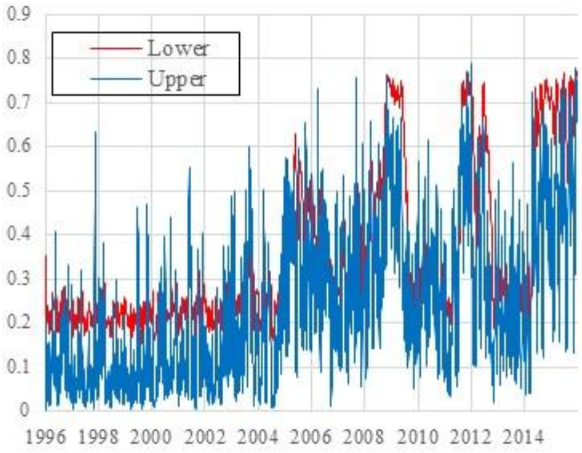
FIGURE 2: C -VINE COPULA STRUCTURE



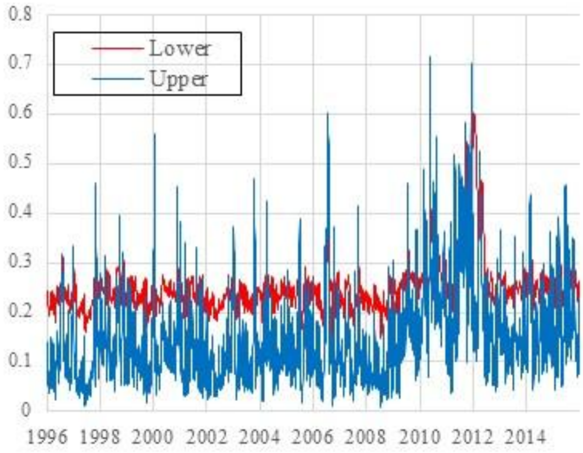
Notes: 1.– Financial, 2.– UK, 3.– USA, 4.– Japan

FIGURE 3: PLOTS OF C-VINE COPULA PARAMETERS

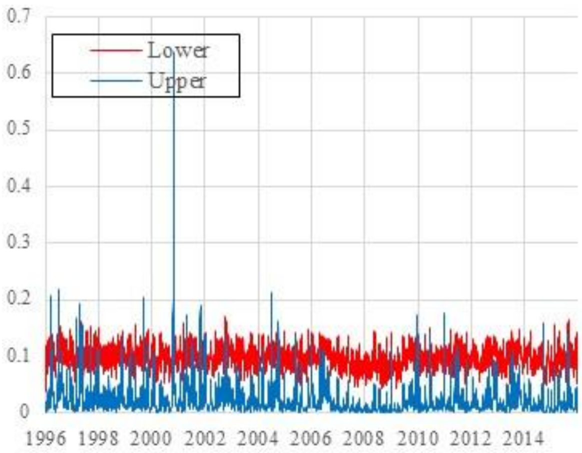
a) T1: (1,2) – TVP SJC



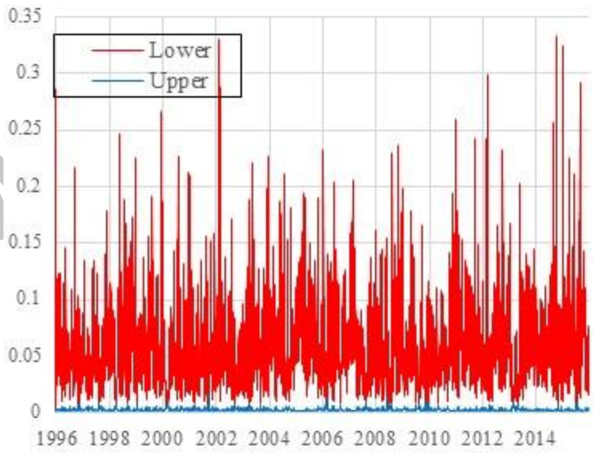
b) T1: (1,3) – TVP SJC



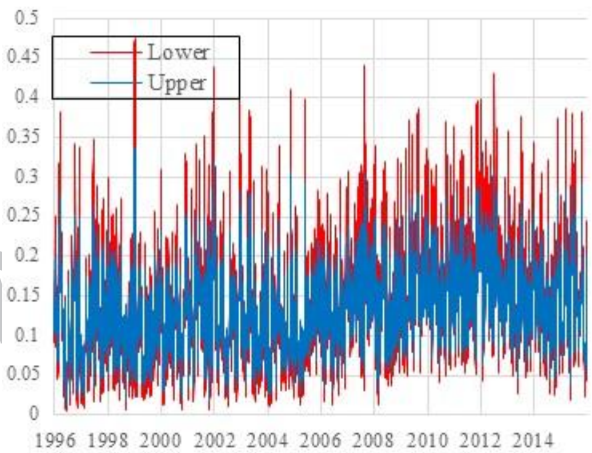
c) T1: (1,4) – TVP SJC



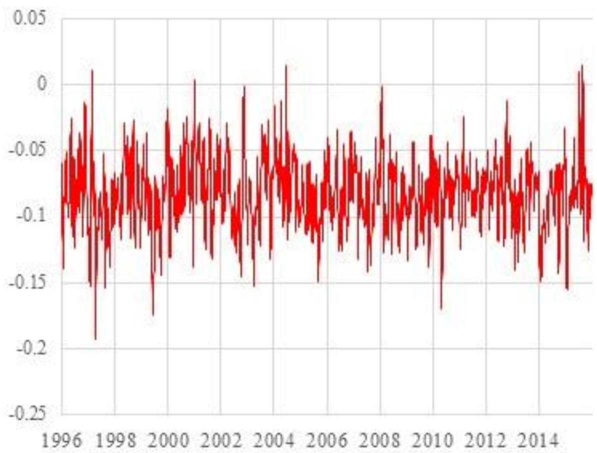
d) T2: (2,3|1) – TVP SJC



e) T2: (2,4|1) – TVP SJC



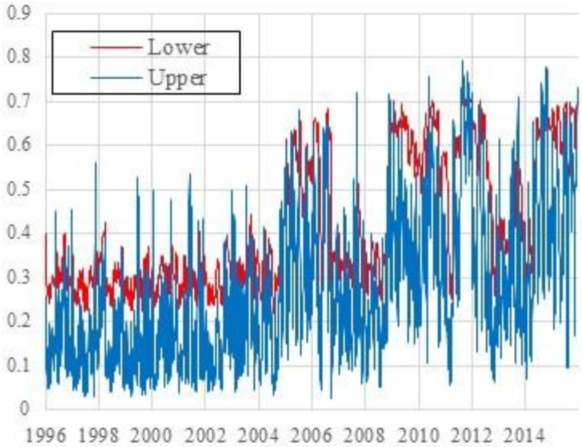
f) T3: (3,4|1,2) – TVP Gaussian



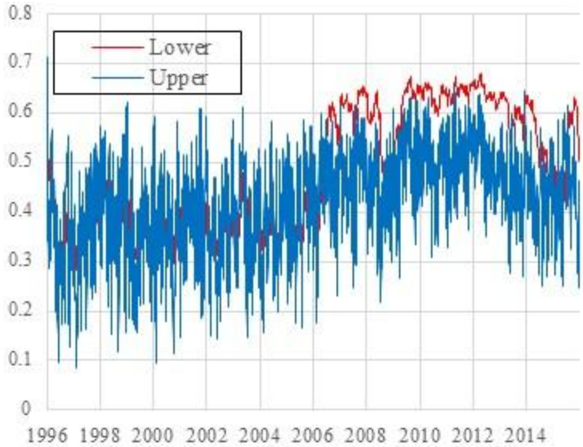
Notes: 1.– Financial, 2.– UK, 3.– USA, 4.– Japan

FIGURE 4: BIVARIATE COPULA PARAMETERS

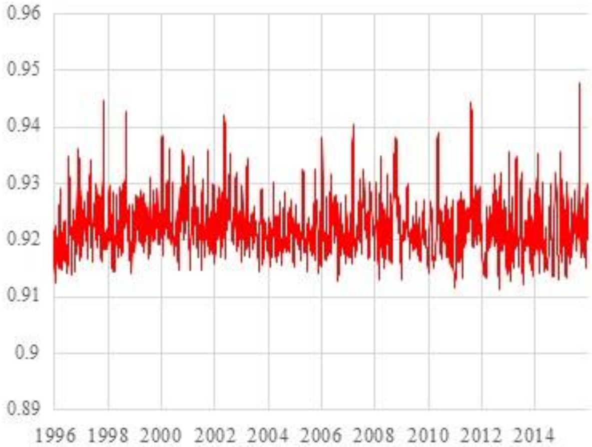
a) Financials – TVP SJC



b) UK – TVP SJC



c) USA – TVP SJC



d) Japan – TVP SJC

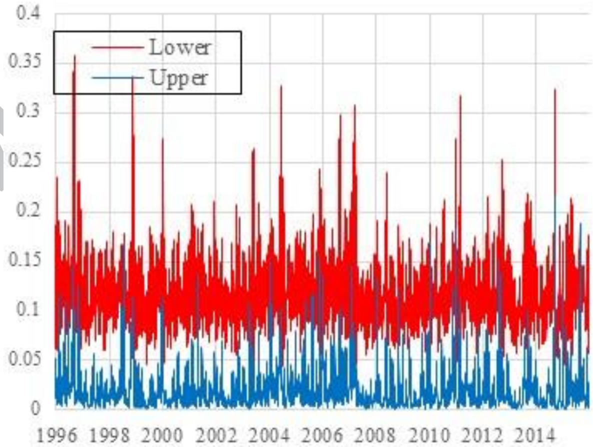
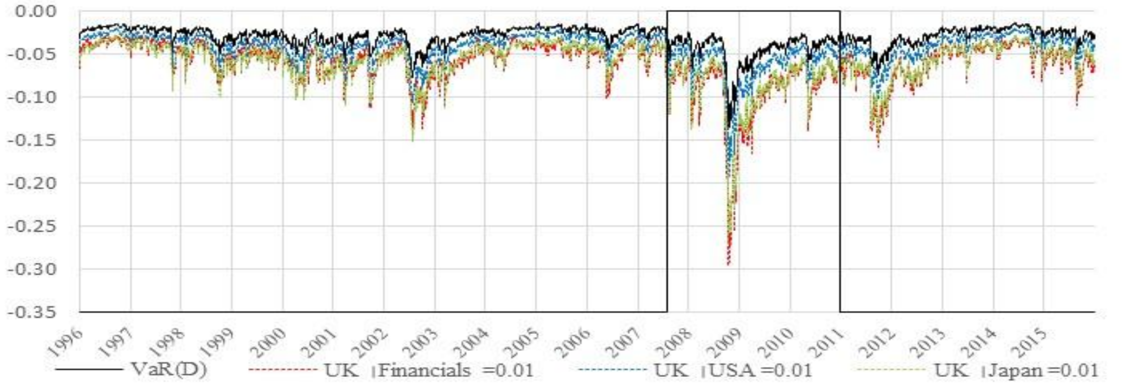


FIGURE 5: PLOTS OF VAR AND COVAR ESTIMATED PARAMETERS

Panel A: Downside VaR and CoVaR

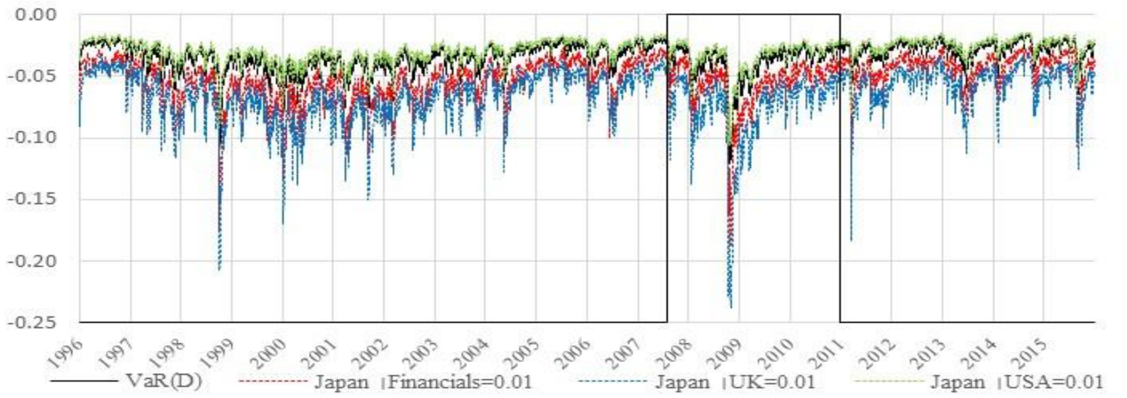
a) Financials

b) UK



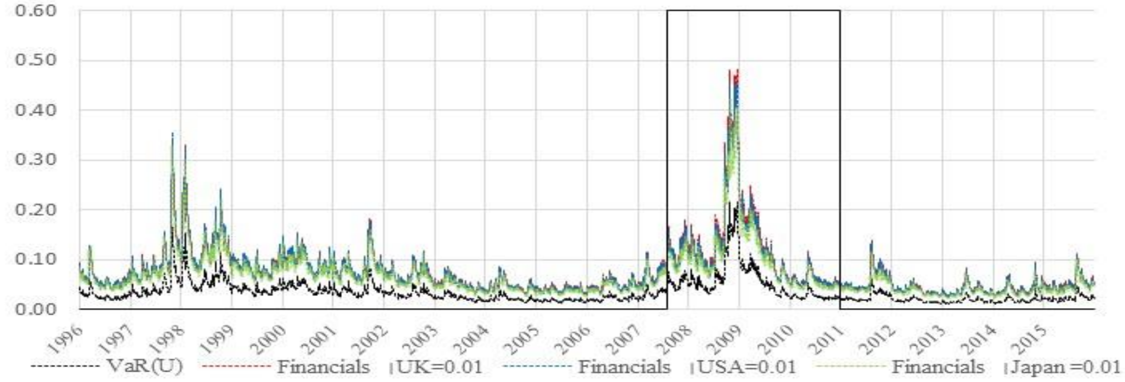
c) USA

d) Japan

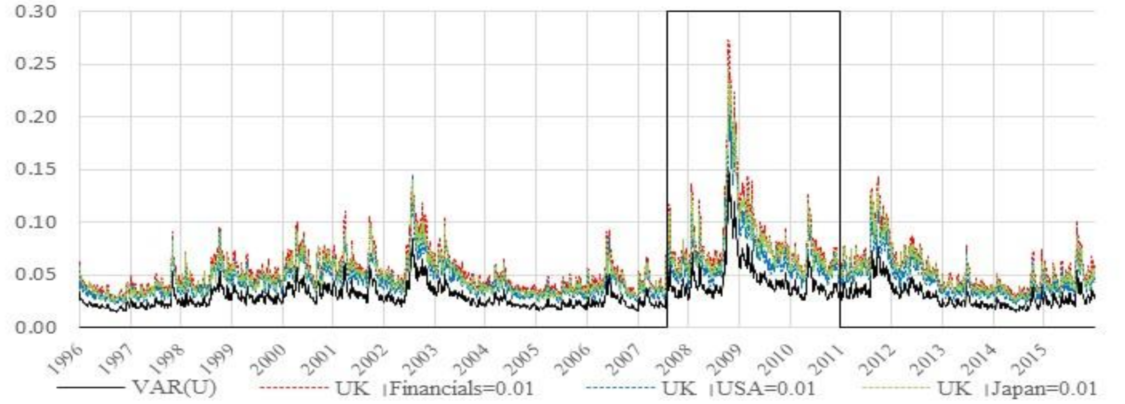


Panel B: Upside VaR and CoVaR

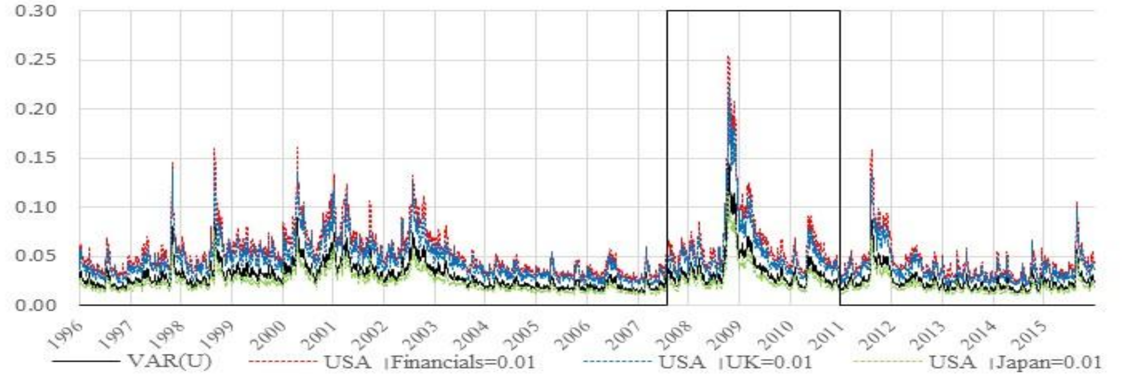
a) Financials



b) UK



c) USA



d) Japan

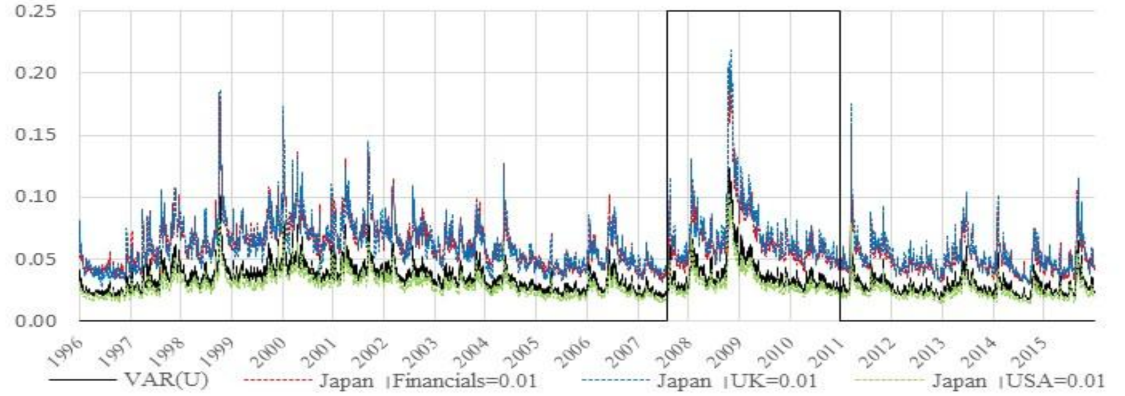
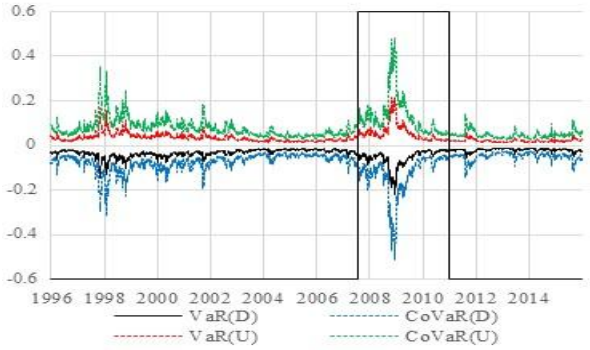


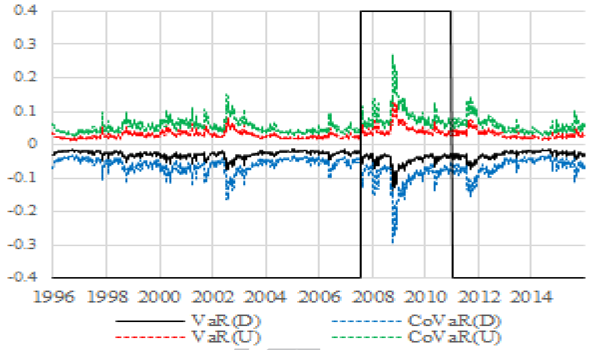
FIGURE 6: TIME SERIES OF VAR AND COVAR MEASURES

Panel A: VaR of equity indices and CoVaR from the DJ World Islamic index

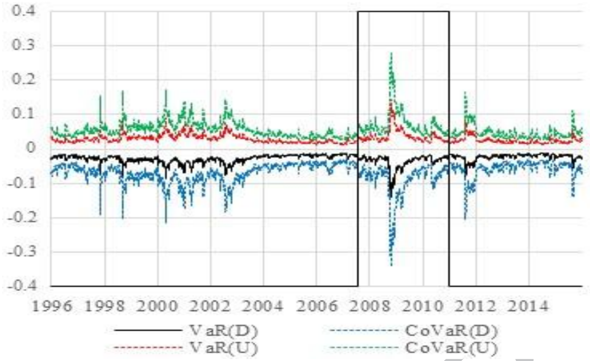
a) Financials



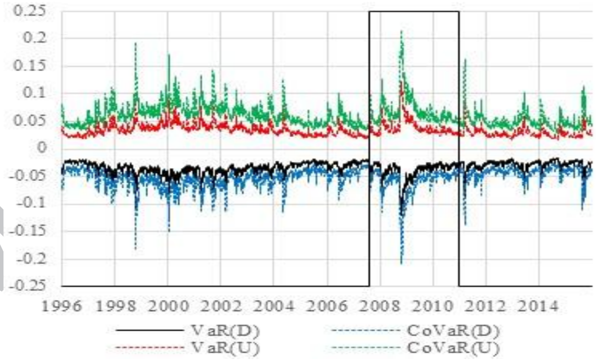
b) UK



c) USA

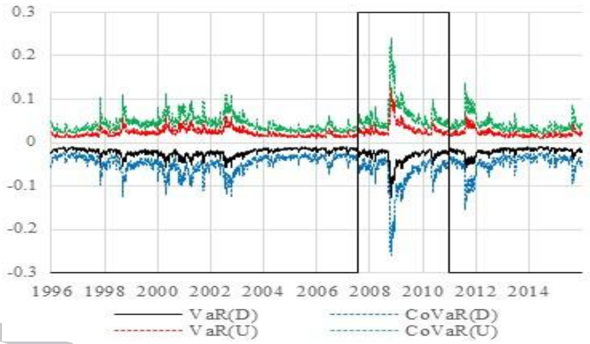


d) Japan

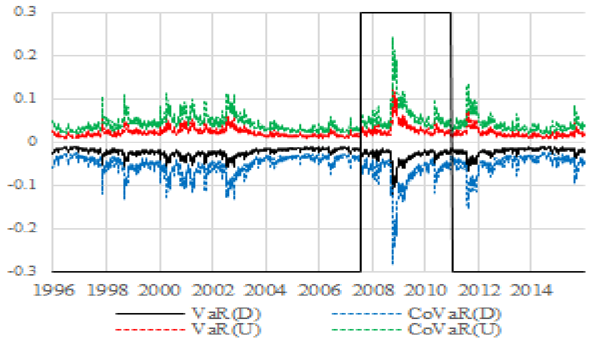


Panel B: VaR of the DJ World Islamic market and CoVaR from the four equity indices

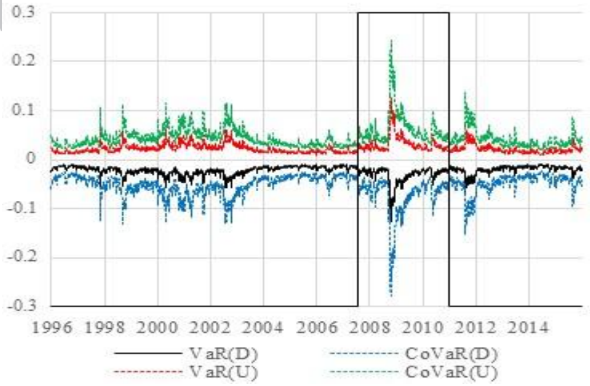
a) Financials



b) UK



c) USA



d) Japan

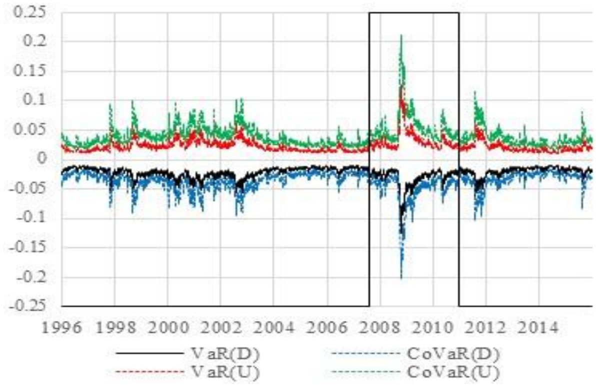


TABLE 1: BIVARIATE COPULA FUNCTIONS

Copula Name	Formula	Parameter	Tail dependence
Normal (N)	$C_N(u,v,\rho) = \Phi(\Phi^{-1}(u),\Phi^{-1}(v))$	$\rho \in [-1\ 1]$	Zero tail dependence: $\lambda_L = \lambda_U = 0$
Student-t (t)	$C_{ST}(u,v,\rho,\upsilon) = T(t_\upsilon^{-1}(u),t_\upsilon^{-1}(v))$	$\rho \in [-1\ 1]$	Symmetric tail dependence: $\lambda_U = \lambda_L = 2t_{\upsilon+1}\left(-\sqrt{\upsilon+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right) > 0$
Clayton (CL)	$C_{CL}(u,v;\delta) = \max\left\{\left(u^{-\delta}+v^{-\delta}-1\right)^{-1/\delta},0\right\}$	$\alpha \in [-1,\infty)\backslash\{0\}$	Asymmetric tail dependence: $\lambda_L = 2^{-1/\delta}, \lambda_U = 0$
Gumbel (Gu)	$C_G(u,v;\delta) = \exp\left(-\left((-\log u)^\delta + (-\log v)^\delta\right)^{1/\delta}\right)$	$\delta \in [1,\infty)$	Asymmetric tail dependence $\lambda_L = 0, \lambda_U = 2-2^{1/\delta}$
Rotated Gumbel	$C_{RG}(u,v;\delta) = u+v-1+C_G(1-u,1-v;\delta)$		upper tail independence and lower tail dependence
Frank (F)	$C_F(u,v;\delta) = \delta\log\left(\left[\left(1-e^{-\delta}\right)-\left(1-e^{-\delta u}\right)\left(1-e^{-\delta v}\right)\right]\big/\left(1-e^{-\delta}\right)\right)$	0<δ<∞	Zero tail dependence: $\lambda_L = \lambda_U = 0$
Placket	$C_P(u,v;\theta) = \frac{1}{2(\theta-1)}(1+(\theta-1)(u+v)) - \sqrt{\left(1+(\theta-1)(u+v)\right)^2 - 4\theta(\theta-1)uv}$	θ	Zero tail dependence: $\lambda_L = \lambda_U = 0$
SJC	$C_{SJC}(u,v;\lambda_U,\lambda_L) = 0.5(C_{JC}(u,v;\lambda_U,\lambda_L) + C_{JC}(1-u,1-v;\lambda_U,\lambda_L) + u+v-1)$	$\lambda_L \in (0,1)$ $\lambda_U \in (0,1)$	$\lambda_U = \lambda_L$
Joe Clayton	$C_{JC}(u,v;\lambda_U,\lambda_L) = 1-\left(1-\left\{\left[1-(1-u)^k\right]^{-\gamma}+\left[1-(1-v)^k\right]^{-\gamma}-1\right\}^{-1/\gamma}\right)^{1/k}$	$\lambda_L \in (0,1)$ $\lambda_U \in (0,1)$	$\lambda_t^U = \Delta\left(\omega_U + \beta_U\rho_{t-1} + \alpha_U\frac{1}{q}\sum_{j=1}^{q}\lvert u_{t-j}-v_{t-j}\rvert\right)$ $\lambda_t^L = \Delta\left(\omega_L + \beta_L\rho_{t-1} + \alpha_L\frac{1}{q}\sum_{j=1}^{q}\lvert u_{t-j}-v_{t-j}\rvert\right)$

Notes: λ_L and λ_U denote the lower and upper tail dependence, respectively. For the Normal copula, $\Phi^{-1}(u)$ and $\Phi^{-1}(v)$ are the standard normal quantile functions and Φ is the bivariate standard normal cumulative distribution function with correlation ρ. For the Student-t copula, $t_\upsilon^{-1}(u)$ and $t_\upsilon^{-1}(v)$ are the quantile functions of the univariate Student-t distribution with υ as the degree-of-freedom parameter and T is the bivariate Student-t cumulative distribution function with υ as the degree-of-freedom parameter and ρ as the correlation. For the SJC copula, $\kappa = 1/\log_2(2-\lambda_U)$, $\gamma = -1/\log_2(\lambda_L)$

TABLE 2: DESCRIPTIVE STATISTICS AND CORRELATIONS

	Financials	USA	UK	Japan	DJWI
Mean	0.00020	0.00032	0.00024	0.00016	0.00028
Std. Dev.	0.01596	0.01250	0.01362	0.01427	0.01027
Sharpe Ratio	0.01290	0.02574	0.01761	0.01155	0.02782
Range	0.34259	0.21434	0.21245	0.20666	0.17959
Skewness	0.22064	-0.13302	-0.10180	-0.05325	-0.35462
Kurtosis	17.7764	9.64358	9.51970	6.70383	9.89863
Jarque-Bera	47522.6***	9613.39***	9252.43***	2985.63***	10458.4***
ADF	-73.402***	-54.786***	-35.588***	-55.3729***	-50.741***
PP	-73.472***	-76.885***	-75.352***	-77.0722***	-62.566***
KPSS	0.0323	0.1505	0.13586	0.0680	0.1186
Q(20)	66.999***	60.308***	99.081***	51.735***	145.68***
Q^2(20)	6144.9***	5320.9***	5544.9***	2600.1***	6927.0***
ARCH(20)	1842.5***	1573.4***	1528.2***	1053.5***	1434.0***
Observations	5219	5219	5219	5219	5219
Correlation Matrix					
Financials	1.0000	0.4693***	0.3507***	0.1676***	
		(38.382)	(27.048)	(12.279)	
USA		1.0000	0.4426***	0.0390***	
			(35.652)	(2.8162)	
UK			1.0000	0.2121***	
				(15.677)	
Japan				1.0000	
Correlation with DJ World Islamic (DJWI) market					
Overall	0.5255***	0.9130***	0.7080***	0.2649***	
	(44.618)	(161.66)	(72.408)	(19.839)	
GFC	0.6609***	0.9043***	0.7971***	0.2439***	
	(26.606)	(64.011)	(39.883)	(7.6000)	

Notes: ADF, PP and KPSS are the empirical statistics of the Augmented Dickey-Fuller (1979), and the Phillips-Perron (1988) unit root tests, and the Kwiatkowski et al. (1992) stationarity test, respectively. Q(k) and Q^2(k) refer to the empirical statistics of the Ljung-Box test for autocorrelation of the returns and squared returns series with lag k, respectively. The ARCH-LM(k) test of Engle (1982) checks the presence of the ARCH effect. The asterisk (***) denotes the rejection of the null hypotheses of normality, no autocorrelation, unit root, non-stationarity, and conditional homoscedasticity and significance of correlation at the 1% significance level. Values in parenthesis are t-statistics.

TABLE 3: MAXIMUM LIKELIHOOD ESTIMATES OF ARMA-FIGARCH WITH SKEWED-*T*

	Financials	USA	UK	Japan	DJWI
Panel A: *mean equation*					
ϕ_0	0.0005***	0.0007***	0.0006***	0.0003**	0.0006***
	0.0001	0.0001	0.0001	0.0002	0.0001
ψ_1	0.3161***	0.7123***	0.8009***	0.6007***	
	0.1069	0.0620	0.0466	0.0798	
ψ_2	-0.2023**	-0.8014***	-0.8791***	-0.7051***	0.1727**
	0.0994	0.0462	0.0297	0.0766	0.0675
Panel B: *variance equation*					
ω	0.0157***	0.0285***	0.0423***	0.1041**	0.0151**
	0.0049	0.0087	0.0143	0.0422	0.0065
d-Figarch	0.7377***	0.5586***	0.4224***	0.3510***	0.5132***
	0.0877	0.0866	0.0465	0.0455	0.0656
α_1	0.1881***	0.0786**	0.1620***	0.0381*	0.0708*
	0.0448	0.0367	0.0542	0.1231	0.0438
β_1	0.8061***	0.6082***	0.5181***	0.3318***	0.5554***
	0.0560	0.0874	0.0743	0.1522	0.0835
Asymmetry	-0.0247*	-0.1306***	-0.0811***	-0.0452**	-0.1001***
	0.0178	0.0193	0.0201	0.0196	0.0188
Tail	6.9990***	8.0366***	9.9633***	9.1307***	8.4681***
	0.6369	0.9934	1.3297	1.0545	1.0350
Panel C: *Diagnostic tests*					
LogLik	15982.9	16500.1	15938.8	15375.5	17571.0
ARCH(20)	[0.3826]	[0.3571]	[0.4321]	[0.3735]	[0.5982]
Q(20)	[0.2267]	[0.2471]	[0.1869]	[0.7534]	[0.3521]
$Q^2(20)$	[0.3409]	[0.3204]	[0.4458]	[0.3960]	[0.5639]
K-S	[0.3066]	[0.1344]	[0.4046]	[0.8721]	[0.6538]
LiMcLeod	[0.2276]	[0.2474]	[0.1879]	[0.7528]	[0.3525]
Hosking	[0.2271]	[0.2475]	[0.1872]	[0.7537]	[0.3525]

Notes: We report the maximum likelihood (ML) estimates and the z statistics (in parentheses) for the parameters of the marginal distribution model defined in Eqs. (1)-(9). The lags *p*, *q*, *r* and *m* are selected using the loglikelihood (logLik) for different combinations of values ranging from 0 to 2. Q(k) and $Q^2(k)$ are the Ljung-Box statistics for serial correlation in the model residuals and squared residuals, respectively, computed with k lags. ARCH(k) is the Engle LM test for the ARCH effect in the residuals up to the *k*th order. K-S denotes the Kolmogorov-Smirnov test (for which the p-values are reported), representing the adequacy of the Student-t distribution model. Hosking (1980) and McLeod and Li (1983) are the autocorrelation tests until lag 20. The p-values [in the square brackets] below 0.05 indicate the rejection of the null hypothesis. The asterisks (***), (**) and (*) represent significance at the 1%, 5% and 10% levels, respectively.

TABLE 4: ESTIMATES FOR C-VINE PAIR COPULAS

Panel A: Static copulas

	T_1			T_2		T_3
	(1,2)	(1,3)	(1,4)	(2,3 ǀ 1)	(2,4 ǀ 1)	(3,4 ǀ 1,2)
Gaussian						
ρ	0.465*	0.371*	0.197*	0.152*	0.302*	-0.083*
	(0.011)	(0.012)	(0.013)	(0.014)	(0.013)	(0.014)
AIC	-1270.156	-770.794	-207.416	-122.740	-498.882	-36.278
Clayton's						
α	0.727*	0.502*	0.244*	0.181*	0.357*	0.001*
	(0.025)	(0.022)	(0.019)	(0.018)	(0.021)	(0.014)
AIC	-1216.489	-710.352	-214.074	-125.084	-385.113	0.039
Rotated Clayton						
α	0.610*	0.427*	0.189*	0.124*	0.340*	0.001*
	(0.024)	(0.021)	(0.019)	(0.017)	(0.020)	(0.015)
AIC	-896.073	-516.642	-129.355	-62.562	-394.970	0.099
Plackett						
δ	4.836*	3.183*	1.805*	1.559*	2.599*	0.781*
	(0.184)	(0.125)	(0.075)	(0.064)	(0.105)	(0.032)
AIC	-1372.639	-759.908	-194.194	-113.388	-505.677	-35.920
Frank						
δ	3.247*	2.364*	1.162*	0.890*	1.902*	0.001*
	(0.092)	(0.088)	(0.085)	(0.084)	(0.087)	(0.083)
AIC	-1272.497	-731.315	-187.559	-112.444	-482.745	0.021
Gumbel						
δ	1.416*	1.280*	1.120*	1.100*	1.212*	1.100*
	(0.015)	(0.013)	(0.011)	(0.011)	(0.012)	(0.012)
AIC	-1194.914	-673.563	-172.040	-75.271	-464.085	283.921
Rotated Gumbel						
δ	1.448*	1.301*	1.136*	1.100*	1.221*	1.100*
	(0.016)	(0.014)	(0.011)	(0.010)	(0.012)	(0.012)
AIC	-1402.835	-794.088	-239.142	-125.407	-469.101	216.440
Student's t						
ρ	0.482*	0.375*	0.199*	0.153*	0.305*	-0.083*
	(0.011)	(0.012)	(0.014)	(0.014)	(0.013)	(0.014)
υ	5.180*	9.377*	11.560*	36.195*	10.732*	10.000*
	(0.442)	(1.379)	(2.085)	(18.065)	(1.743)	(6.943)
AIC	-1473.215	-830.773	-243.579	-126.766	-548.195	-36.160
SJC						
λ_U	0.213*	0.127*	0.020*	0.003*	0.122*	0.001*
	(0.020)	(0.019)	(0.012)	(0.004)	(0.018)	(3.981)
λ_L	0.354*	0.242*	0.094*	0.056*	0.129*	0.001*
	(0.015)	(0.017)	(0.017)	(0.015)	(0.019)	(3.336)
AIC	-1411.336	-820.099	-245.860	-133.736	-517.604	48.506

Panel B: Time-varying copulas

	T_1			T_2		T_3
	(1,2)	(1,3)	(1,4)	(2,3 ǀ 1)	(2,4 ǀ 1)	(3,4 ǀ 1,2)
TVP-Gaussian						
Ψ_0	0.041*	0.009	0.358	0.335	0.869	-0.037
	(0.013)	(0.006	(0.312	(0.241	(0.223	(0.040
Ψ_1	0.316*	0.087	0.279	0.105	0.270	0.049
	(0.035)	(0.015	(0.219	(0.104	(0.108	(0.049
Ψ_2	1.871*	2.005	-0.024	-0.290	-1.094	1.510
	(0.053)	(0.022	(1.750	(1.640	(0.763	(0.527
AIC	-1645.612	-838.748	-222.295	-124.175	-506.493	**-40.536**
TVP-Clayton						
ω	0.951*	1.504	0.355	0.865	1.304	0.000
	(1.605)	(0.079	(0.054	(0.148	(0.136	(0.454
α	0.303*	-0.594	0.767	-0.876	-0.436	-2.401
	(0.912)	(0.038	(0.092	(0.216	(0.131	(1.675
β	-1.355*	-1.908	-0.166	-0.927	-2.016	0.000
	(1.605)	(0.079	(0.054	(0.148	(0.136	(0.454
AIC	-1598.041	-743.626	-222.967	-129.172	-414.376	0.039
TVP-Rotated Clayton						
ω_U	0.836*	0.613	0.893	0.575	1.094	0.000
	(1.665)	(0.041	(0.194	(0.328	(0.150	(0.162
α_U	0.346*	0.472	-0.440	-0.821	-0.334	-2.264
	(6.384)	(0.115	(0.389	(1.838	(0.214	(1.060
β_U	-1.238*	-0.644	-1.271	-0.393	-1.414	0.000
	(1.665)	(0.041	(0.194	(0.328	(0.150	(0.162
AIC	-1273.305	-591.144	-141.755	-63.285	-419.386	0.100
TVP-Gumbel						
ω	0.514*	0.000	1.353	1.649	1.358	0.000
	(0.048)	(0.731	(0.547	(0.868	(0.349	(3.078
α	0.345*	0.543	-0.629	-1.127	-0.443	0.000
	(0.013)	(2.461	(0.436	(0.783	(0.246	(3.024
β	-1.642*	-0.661	-1.016	-0.523	-1.294	0.000
	(0.048)	(0.731	(0.547	(0.868	(0.349	(3.078
AIC	-1656.399	-749.519	-185.370	-83.147	-496.272	-0.038
TVP-rotated Gumbel						
ω_L	0.502*	-0.033	-0.713	1.787	1.558	0.000
	(0.052)	(0.131	(0.158	(0.424	(0.257	(4.134
α_L	0.347*	0.554	0.992	-1.181	-0.562	0.000
	(0.014)	(2.607	(0.116	(0.377	(0.176	(4.047
β_L	-1.527*	-0.539	-0.154	-0.593	-1.458	0.000
	(0.052)	(0.131	(0.158	(0.424	(0.257	(4.134
AIC	-1849.386	-847.345	-250.157	-128.647	-498.909	-0.123
TVP-SJC						
ω_U	0.866*	-0.492	-0.322	-8.718	0.771	-19.899
	(0.543)	(1.141	(0.941	(14.423	(1.000	(23.320
β_U	-11.328*	-7.243	-14.015	8.034	-8.622	-1.205
	(1.950)	(2.497	(4.995	(5.914	(3.315	(9.546
α_U	1.052*	2.606	3.471	-2.985	-3.336	-0.003
	(0.625)	(0.679	(1.188	(2.190	(2.658	(1.032
ω_L	-1.926*	-2.010	-0.069	1.705	1.990	-19.762
	(0.019)	(0.992	(0.824	(0.716	(0.847	(3.889
β_L	-0.814*	-0.651	-4.510	-13.324	-13.112	-4.403
	(0.107)	(2.661	(3.140	(2.827	(3.427	(1.498
α_L	4.136*	4.254	-8.666	-12.267	-3.624	-0.013

	(0.016)	(1.506	(2.587	(2.621	(1.317	(1.100
AIC	**-1914.310**	**-888.734**	**-259.323**	**-141.080**	**-553.346**	97.934
TVP-Student-t						
Ψ_0	0.036*	0.010	0.423	0.262	0.666	-0.152
	(0.011)	(0.005	(0.183	(0.110	(0.645	(0.144
Ψ_1	0.205*	0.058	0.224	0.163	0.105	0.102
	(0.021)	(0.014	(0.100	(0.084	(0.098	(0.104
Ψ_2	1.939*	2.020	-0.343	0.111	-0.243	0.088
	(0.037)	(0.027	(0.992	(0.764	(2.212	(1.759
υ	7.669*	10.452	11.995	25.000	10.961	25.000
	(0.852)	(1.671	(2.227	(7.653	(1.795	(5.457
AIC	-1756.324	-882.103	-255.995	-132.397	-551.451	-32.329

Notes: The table reports the ML estimates for the different dynamic bivariate copulas. The standard error values (given in parenthesis) and the AIC values adjusted for the small-sample bias are provided for these different models. For the TVP Gaussian and Student-t copulas, q in Eq. (7) is set to 10. The asterisk (*) indicates significance at the 5% level. The bold values indicate the best copula.

TABLE 5: ESTIMATES FOR BIVARIATE COPULAS

Panel A: Static copulas

	Financials	UK	USA	Japan
Gaussian				
ρ	0.518*	0.920*	0.648*	0.219*
	(0.010)	(0.000)	(0.008)	(0.013)
AIC	-1627.890	-9890.60	-2840.081	-257.528
Clayton's				
α	0.845*	3.760*	1.144*	0.269*
	(0.026)	(0.060)	(0.029)	(0.019)
AIC	-1517.641	-8070.30	-2338.872	-257.351
Rotated Clayton				
α	0.720*	3.520*	1.071*	0.195*
	(0.025)	(0.060)	(0.028)	(0.019)
AIC	-1153.239	-7639.45	-2131.953	-133.508
Plackett				
δ	5.907*	5.340*	8.833*	1.911*
	(0.220)	(1.590)	(0.308)	(0.078)
AIC	-1766.298	-9275.27	-2812.136	-240.983
Frank				
δ	3.766*	13.450*	5.052*	1.303*
	(0.094)	(0.180)	(0.101)	(0.085)
AIC	-1648.660	-8891.08	-2693.057	-237.484
Gumbel				
δ	1.498*	3.610*	1.741*	1.125*
	(0.016)	(0.040)	(0.019)	(0.011)
AIC	-1530.988	-9300.99	-2660.165	-173.083
Rotated Gumbel				
δ	1.532*	3.700*	1.763*	1.147*
	(0.017)	(0.040)	(0.020)	(0.011)
AIC	-1753.212	-9565.82	-2788.679	-251.638
Student's t				
ρ	0.537*	0.900*	0.654*	0.221*
	(0.010)	(0.120)	(0.008)	(0.014)
υ	5.260*	5.770*	7.339*	3.569*
	(0.449)	(2.400)	(0.832)	(1.543)
AIC	-1833.324	-9843.18	-2968.502	-262.667
SJC				
λ_U	0.259*	0.790*	0.419*	0.013*
	(0.019)	(0.720)	(0.015)	(0.010)
λ_L	0.400*	0.810*	0.475*	0.112*
	(0.014)	(2.030)	(0.013)	(0.017)
AIC	-1755.475	-9516.53	-2856.949	-264.217

Panel B: Time-varying copulas

	Financials	UK	USA	Japan
TVP-Gaussian				
Ψ_0	0.020*	-1.470*	-0.127*	0.000*
	(0.012)	(4.820)	(0.085)	(0.004)
Ψ_1	0.259*	0.090*	0.131*	0.022*
	(0.024)	(0.050)	(0.034)	(0.009)
Ψ_2	1.981*	5.000*	2.465*	2.018*
	(0.038)	(5.270)	(0.162)	(0.028)
AIC	-1846.713	-9898.441	-2893.076	-271.423
TVP-Clayton				
ω	1.000*	1.000*	1.430*	1.093*
	(0.040)	(1.000)	(1.468)	(0.082)
α	0.278*	-1.000*	-0.999*	-0.911*
	(0.011)	(1.000)	(0.176)	(0.033)
β	-1.501*	0.000	0.042*	-1.088*
	(0.040)	(1.000)	(1.468)	(0.082)
AIC	-1804.421	-122.833	-1459.061	-268.196
TVP-Rotated Clayton				
ω_U	0.845*	1.000*	1.490*	0.480*
	(0.302)	(1.000)	(0.495)	(0.122)
α_U	0.327*	-1.000*	-1.350*	0.528*
	(0.000)	(1.000)	(0.674)	(0.248)
β_U	-1.063*	0.000	-0.146*	-0.494*
	(0.302)	(1.000)	(0.495)	(0.122)
AIC	-1445.460	-186.810	-1465.596	-143.597
TVP-Gumbel				
ω	0.531*	2.445*	0.313*	-0.245*
	(0.064)	(0.323)	(0.032)	(0.479)
α	0.327*	-0.116*	0.390*	0.627*
	(0.020)	(0.072)	(0.010)	(0.376)
β	-1.500*	-4.716*	-0.722*	-0.375*
	(0.064)	(0.323)	(0.032)	(0.479)
AIC	-1873.308	-9380.162	-2833.185	-182.675
TVP-rotated Gumbel				
ω_L	0.508*	1.952*	0.292*	2.048*
	(0.046)	(0.707)	(0.032)	(0.050)
α_L	0.334*	0.013*	0.395*	-1.230*
	(0.013)	(0.154)	(0.010)	(0.048)
β_L	-1.376*	-3.573*	-0.643*	-0.845*
	(0.046)	(0.706)	(0.032)	(0.050)
AIC	-2086.023	-9642.847	-2940.436	-262.800
TVP-SJC				
ω_U	-0.391*	1.294*	1.737*	-0.145*
	(0.456)	(2.123)	(0.611)	(2.103)
β_U	-6.867*	-0.022*	-8.455*	-14.971*
	(1.748)	(1.126)	(1.665)	(8.324)
α_U	2.526*	-0.012*	-1.269*	0.622*
	(0.518)	(3.012)	(0.880)	(1.331)
ω_L	-1.902*	1.577*	-1.979*	0.518*
	(0.035)	(2.088)	(0.016)	(0.421)
β_L	-0.588*	0.064*	-0.242*	-5.266*
	(0.119)	(1.149)	(0.056)	(1.505)
α_L	4.016*	-0.184*	4.043*	-9.106*
	(0.036)	(3.285)	(0.017)	(1.165)

AIC	**-2159.330**	-9550.504	**-3072.378**	-276.549
TVP-Student-t				
Ψ_0	0.026*	-4.806*	-0.097*	-0.001*
	(0.013)	(2.432)	(0.073)	(0.004)
Ψ_1	0.166*	0.058*	0.100*	0.019*
	(0.021)	(0.032)	(0.024)	(0.008)
Ψ_2	2.017*	8.648*	2.427*	2.025*
	(0.037)	(2.654)	(0.139)	(0.026)
υ	6.693*	11.283*	7.777*	24.013*
	(0.642)	(1.793)	(0.890)	(6.705)
AIC	-1995.418	**-9957.547**	-3021.220	-275.065

Notes: The table reports the ML estimates for the different dynamic bivariate copulas. The standard error values (given in parenthesis) and the AIC values adjusted for the small-sample bias are provided for these different models. For the TVP Gaussian and Student-t copulas, q in Eq. (3) is set to 10. The asterisk (*) indicates significance at the 5% level. The bold values indicate the best copula.

TABLE 6: DESCRIPTIVE STATISTICS FOR VAR AND VINE COVAR MEASUREMENT FOR ALL ISLAMIC EQUITY INDICES

Variable	Vine Structure	Stats	VaR(D) 1%		Vine CoVaR(D) 1%		VaR(U) 1%		Vine CoVaR(U) 1%		Vine ΔCoVaR(D)		Vine ΔCoVaR(U)	
			Full	GFC	Full	GFC	Full	GFC	Full	GFC	Full	GFC	Full	GFC
Financials		Mean	-0.034	-0.054			0.035	0.056						
		Std. Dev.	(0.023)	(0.037)			(0.023)	(0.037)						
Financials ׀ UK	(1,2)	Mean			-0.074	-0.128			0.077	0.126	0.910	0.991	0.960	0.982
		Std. Dev.			(0.053)	(0.090)			(0.052)	(0.086)	(0.152)	(0.064)	(0.033)	(0.029)
Financials ׀ USA	(1,3)	Mean			-0.071	-0.115			0.076	0.122	0.873	0.883	0.946	0.953
		Std. Dev.			(0.048)	(0.076)			(0.051)	(0.081)	(0.113)	(0.121)	(0.027)	(0.026)
Financials ׀ Japan	(1,4)	Mean			-0.055	-0.084			0.068	0.107	0.524	0.460	0.805	0.794
		Std. Dev.			(0.036)	(0.055)			(0.045)	(0.071)	(0.143)	(0.128)	(0.041)	(0.043)
UK		Mean	-0.031	-0.045			0.032	0.047						
		Std. Dev.	(0.014)	(0.019)			(0.014)	(0.020)						
UK ׀ Financials	(1,2)	Mean			-0.062	-0.095			0.058	0.086	0.895	0.788	0.869	0.732
		Std. Dev.			(0.029)	(0.042)			(0.026)	(0.038)	(0.795)	(0.650)	(0.729)	(0.609)
UK ׀ USA	(2,3,1)	Mean			-0.043	-0.063			0.049	0.071	0.297	0.299	0.538	0.534
		Std. Dev.			(0.019)	(0.027)			(0.021)	(0.032)	(0.027)	(0.031)	(0.086)	(0.092)
UK ׀ Japan	(2,4,1)	Mean			-0.060	-0.090			0.054	0.080	0.725	0.741	0.641	0.676
		Std. Dev.			(0.027)	(0.038)			(0.024)	(0.034)	(0.058)	(0.042)	(0.083)	(0.065)
USA		Mean	-0.028	-0.036			0.030	0.038						
		Std. Dev.	(0.014)	(0.020)			(0.014)	(0.021)						
USA ׀ Financials	(1,3)	Mean			-0.055	-0.071			0.053	0.067	0.647	0.313	0.657	0.315
		Std. Dev.			(0.027)	(0.038)			(0.025)	(0.037)	(0.571)	(0.489)	(0.541)	(0.440)
USA ׀ UK	(2,3,1)	Mean			-0.042	-0.054			0.046	0.058	0.246	0.039	0.494	0.246
		Std. Dev.			(0.020)	(0.030)			(0.022)	(0.033)	(0.335)	(0.183)	(0.394)	(0.237)
USA ׀ Japan	(3,4,1,2)	Mean			-0.026	-0.033			0.024	0.030	-0.091	-0.094	-0.086	-0.088
		Std. Dev.			(0.013)	(0.018)			(0.011)	(0.016)	(0.030)	(0.027)	(0.028)	(0.025)
Japan		Mean	-0.034	-0.039			0.034	0.039						
		Std. Dev.	(0.012)	(0.016)			(0.012)	(0.015)						
Japan ׀ Financials	(1,4)	Mean			-0.053	-0.058			0.060	0.068	0.745	0.221	0.963	0.423
		Std. Dev.			(0.018)	(0.022)			(0.020)	(0.026)	(0.672)	(0.501)	(0.708)	(0.536)
Japan ׀ UK	(2,4,1)	Mean			-0.065	-0.076			0.061	0.072	1.045	0.545	0.940	0.498
		Std. Dev.			(0.022)	(0.030)			(0.021)	(0.028)	(0.629)	(0.294)	(0.586)	(0.283)
Japan ׀ USA	(3,4,1,2)	Mean			-0.030	-0.034			0.029	0.033	-0.242	-0.174	-0.206	-0.143
		Std. Dev.			(0.010)	(0.014)			(0.010)	(0.013)	(0.226)	(0.205)	(0.226)	(0.208)

Notes: This table presents the average and the standard deviation (in parenthesis) of the VaR, CoVaR and ΔCoVaR metrics.

TABLE 7: DESCRIPTIVE STATISTICS FOR VAR AND VINE COVAR FOR WORLD AND ISLAMIC EQUITY INDICES

		VaR(D) 1%		CoVaR(D) 1%		VaR(U) 1%		CoVaR(U) 1%		ΔCoVaR(D)		ΔCoVaR(U)	
		Full	GFC	Full	GFC	Full	GFC	Full	GFC	Full	GFC	Full	GFC
Panel A: VaR and CoVaR of four selected markets													
Financial	Mean	-0.034	-0.054			0.035	0.056						
	Std. Dev.	(0.023)	(0.037)			(0.023)	(0.037)						
Financials ∣ World	Mean			-0.076	-0.127			0.078	0.126	0.953	0.991	0.976	0.990
	Std. Dev.			(0.053)	(0.088)			(0.052)	(0.085)	(0.101)	(0.065)	(0.026)	(0.024)
UK	Mean	-0.031	-0.045			0.032	0.047						
	Std. Dev.	(0.014)	(0.019)			(0.014)	(0.020)						
UK ∣ World	Mean			-0.066	-0.097			0.059	0.087	0.829	0.829	0.751	0.760
	Std. Dev.			(0.029)	(0.042)			(0.026)	(0.037)	(0.025)	(0.022)	(0.021)	(0.022)
USA	Mean	-0.028	-0.036			0.030	0.038						
	Std. Dev.	(0.014)	(0.020)			(0.014)	(0.021)						
USA ∣ World	Mean			-0.070	-0.089			0.057	0.072	0.983	0.978	0.841	0.845
	Std. Dev.			(0.033)	(0.049)			(0.027)	(0.040)	(0.028)	(0.026)	(0.027)	(0.025)
Japan	Mean	-0.034	-0.039			0.034	0.039						
	Std. Dev.	(0.012)	(0.016)			(0.012)	(0.015)						
Japan ∣ World	Mean			-0.052	-0.059			0.061	0.070	0.442	0.422	0.701	0.698
	Std. Dev.			(0.018)	(0.024)			(0.020)	(0.027)	(0.109)	(0.110)	(0.044)	(0.040)
Panel A: VaR and CoVaR of DJ World Islamic market													
World	Mean	-0.023	-0.031			0.024	0.033						
	Std. Dev.	(0.011)	(0.017)			(0.012)	(0.018)						
World ∣ Financials	Mean			-0.051	-0.072			0.046	0.063	0.456	0.262	0.428	0.226
	Std. Dev.			(0.025)	(0.039)			(0.022)	(0.034)	(0.537)	(0.458)	(0.500)	(0.428)
World ∣ UK	Mean			-0.053	-0.073			0.046	0.063	0.437	0.306	0.372	0.256
	Std. Dev.			(0.026)	(0.039)			(0.022)	(0.034)	(0.277)	(0.186)	(0.254)	(0.175)
World ∣ USA	Mean			-0.054	-0.074			0.046	0.063	0.583	0.698	5.532	6.160
	Std. Dev.			(0.026)	(0.040)			(0.022)	(0.034)	(0.204)	(0.173)	(1.218)	(1.204)
World ∣ Japan	Mean			-0.037	-0.051			0.041	0.056	0.002	0.164	0.197	0.397
	Std. Dev.			(0.018)	(0.027)			(0.020)	(0.030)	(0.305)	(0.293)	(0.353)	(0.334)

Notes: This table presents the average and the standard deviation (in parenthesis) of the VaR, CoVaR and ΔCoVaR.

TABLE 8: HYPOTHESIS TESTING OF VAR, COVAR AND DELTA COVAR ASYMMETRIES

	Vine structure	$H_0: CoVaR(D) = VaR\ (D)$ $H_1: CoVaR(D) < VaR\ (D)$		$H_0: CoVaR(U) = VaR\ (U)$ $H_1: CoVaR(U) > VaR\ (U)$		$H_0: \frac{CoVaR}{VaR}(D) = \frac{CoVaR}{VaR}(U)$ $H_1: \frac{CoVaR}{VaR}(D) \neq \frac{CoVaR}{VaR}(U)$		$H_0: \Delta CoVaR(D) = \Delta CoVaR\ (U)$ $H_1: \Delta CoVaR(D) \neq \Delta CoVaR\ (U)$	
		Full	GFC	Full	GFC	Full	GFC	Full	GFC
Financials									
Financials ׀ UK	(1,2)	0.591	0.515	0.598	0.504	0.418	0.580	0.335	0.315
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Financials ׀ USA	(1,3)	0.555	0.505	0.585	0.502	0.352	0.334	0.509	0.460
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Financials ׀ Japan	(1,4)	0.390	0.340	0.508	0.454	0.799	0.889	0.869	0.931
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
UK									
UK ׀ Financials	(1,2)	0.671	0.794	0.625	0.735	0.736	0.963	0.034	0.064
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.004]	[0.044]
UK ׀ USA	(2,3 ׀ 1)	0.357	0.504	0.446	0.608	0.733	0.697	0.953	0.932
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
UK ׀ Japan	(2,4 ׀ 1)	0.669	0.785	0.527	0.707	0.856	0.938	0.446	0.450
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
USA									
USA ׀ Financials	(1,3)	0.633	0.688	0.570	0.629	0.757	0.776	0.032	0.041
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.009]	[0.419]
USA ׀ UK	(2,3 ׀ 1)	0.391	0.488	0.427	0.511	0.394	0.360	0.299	0.361
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
USA ׀ Japan	(3,4 ׀ 1,2)	0.096	0.146	0.228	0.289	0.850	0.906	0.085	0.118
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Japan									
Japan ׀ Financials	(1,4)	0.536	0.608	0.672	0.738	0.727	0.840	0.144	0.188
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Japan ׀ UK	(2,4 ׀ 1)	0.756	0.800	0.683	0.759	0.543	0.665	0.070	0.075
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.012]
Japan ׀ USA	(3,4 ׀ 1,2)	0.174	0.251	0.234	0.342	0.394	0.428	0.077	0.063
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.050]

Notes: This table summarizes the results of the Kolmogorov–Smirnov (KS) test. The values in brackets [ ] are the p-values of the K-S test.

TABLE 9: HYPOTHESIS TESTING OF VAR, COVAR AND DELTA COVAR ASYMMETRIES

	$H_0: CoVaR(D) = VaR\ (D)$ $H_1: CoVaR(D) < VaR\ (D)$		$H_0: CoVaR(U) = VaR\ (U)$ $H_1: CoVaR(U) > VaR\ (U)$		$H_0: \frac{CoVaR}{VaR}(D) = \frac{CoVaR}{VaR}(U)$ $H_1: \frac{CoVaR}{VaR}(D) \neq \frac{CoVaR}{VaR}(U)$		$H_0: \Delta CoVaR(D) = \Delta CoVaR\ (U)$ $H_1: \Delta CoVaR(D) \neq \Delta CoVaR\ (U)$	
	Full	GFC	Full	GFC	Full	GFC	Full	GFC
Financial								
Financial ׀ World	0.608	0.541	0.607	0.520	1.000	1.000	0.308	0.289
	[0.000]	[0.000]	[0.004]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
UK								
UK ׀ World	0.755	0.815	0.643	0.742	1.000	1.000	0.922	0.922
	[0.000]	[0.000]	[0.009]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
USA								
USA ׀ World	0.783	0.798	0.623	0.682	1.000	1.000	0.999	1.000
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Japan								
Japan ׀ World	0.550	0.625	0.689	0.754	0.781	0.818	0.893	0.909
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
World								
World ׀ Financials	0.737	0.760	0.641	0.685	0.866	0.946	0.036	0.056
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.003]	[0.118]
World ׀ UK	0.766	0.781	0.646	0.690	0.994	1.000	0.102	0.137
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
World ׀ USA	0.772	0.783	0.650	0.691	0.999	1.000	1.000	1.000
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
World ׀ Japan	0.507	0.580	0.554	0.617	0.406	0.503	0.258	0.281
	[0.003]	[0.118]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

Notes: This table summarizes the results of the Kolmogorov–Smirnov (KS) test. The values in brackets [ ] are the p-values of the K-S test

A SYSTEMIC RISK ANALYSIS OF GLOBAL ISLAMIC EQUITY MARKETS USING VINE COPULAS AND DELTA COVAR MODELING

HIGHLIGHTS

- We analyze downside and upside spillover effects, systemic & tail dependence risks
- We utilize VaR, CoVaR, ΔCoVaR, c-vine CoVaR, static & TV vine copula approaches
- The investigation involves equity indices from Japan, USA, UK, DJWIF & DJWI
- Our findings could be important to portfolio managers, policy makers and investors
- Market agents are interested in accurately estimating downside & upside systemic risk