Mean-Variance Analysis of Option Contracts in a Two-Echelon Supply Chain

Wenyan Zhuo , Lusheng Shao , Honglin Yang

PII: S0377-2217(18)30433-8
DOI: 10.1016/j.ejor.2018.05.033
Reference: EOR 15146

To appear in: *European Journal of Operational Research*

Received date: 2 May 2017
Revised date: 20 March 2018
Accepted date: 16 May 2018

Please cite this article as: Wenyan Zhuo , Lusheng Shao , Honglin Yang , Mean-Variance Analysis of Option Contracts in a Two-Echelon Supply Chain, *European Journal of Operational Research* (2018), doi: 10.1016/j.ejor.2018.05.033

Highlights

1. Studying option contracts with risk considerations in a two-echelon supply chain.
2. Constructing the mean-variance models of option contracts.
3. Investigating the channel coordination of option contracts with risk constraints.
4. Designing a new minimum option quantity commitment for the supplier.

Mean-Variance Analysis of Option Contracts in a Two-Echelon Supply Chain

Wenyan Zhuo[1], Lusheng Shao[2], Honglin Yang[1*]

[1] School of Business Administration, Hunan University, Changsha, Hunan Province 410082, PR China
[2] Faculty of Business and Economics, University of Melbourne, Melbourne, Victoria 3010, Australia

Abstract: This paper studies the implications of risk considerations for option contracts in a two-echelon supply chain. Under the mean-variance framework, we first investigate the conditions for coordinating the supply chain by using option contracts. We find that supply chain coordination is not always achieved, contrasting with the result that properly designed option contracts can always coordinate a supply chain in the absence of risk considerations. Second, we analyze the Stackelberg game for a decentralized supply chain in two cases, depending on whether the retailer's risk aversion threshold is known to the supplier. We show that when the threshold is public information, there exists a unique equilibrium in which the supplier with a higher risk tolerance prefers to reduce the exercise price, and thus, the retailer's order quantity increases. When the retailer's risk aversion threshold is private information, the retailer has an incentive to pretend to be less risk averse. To curb this incentive distortion, we design a new minimum option quantity commitment for the supplier. We complement our theoretical results with numerical simulations.

Keywords: supply chain management; option contract; risk constraint; mean-variance model; supply chain coordination

---

[*]Corresponding author. Tel.: +86 731 88823895; Fax: +86 731 88823670.
E-mail address: ottoyang@126.com (H.Yang).

1. Introduction

Risks are pervasive in firms' operational decisions, and thus, risk management plays a vital role in the success of firm operations. Indeed, inappropriate risk management may lead to significant financial losses. For example, rapidly weakening demand coupled with locked-in supply agreements incurred a $2.5 billion inventory write-off for Cisco Systems, Inc. in the second quarter of 2001 (Norrman and Jansson, 2004). In the third quarter of 2001, Nike lost $100 million in sales revenue due to an inventory shortage (Norrman and Jansson, 2004). Therefore, incorporating risk factors into supply chain decisions has drawn heightened attention from practitioners. For instance, Hewlett-Packard established a procurement risk management system to evaluate and control supply chain risks. Through this system, Hewlett-Packard saved at least $100 million in sourcing costs in 2008 (Nagali et al., 2008). Although firms have begun to realize the importance of supply chain risk management, determining how to make a tradeoff between profit and risk remains a significant challenge.

Previous studies have mainly focused on two types of supply chain risk: disruption risks, such as those associated with wars, earthquakes, diseases and terrorist attacks (Qi et al., 2004; Sodhi et al., 2012; Ray and Jenamani; 2016), and operational risks, such as those emanating from supply reliability and demand uncertainty (Wei and Choi, 2010; Liu and Nagurney, 2011; Xue et al., 2016, Zeng and Yen, 2017, Fan et al., 2017). Researchers have proposed various methods for managing a supply chain under risk constraints. The most widely used method is the mean-variance framework, originating from Markowitz's portfolio theory. Markowitz originally proposed the mean-variance framework to analyze risk diversification of financial assets and to help investors design an optimal portfolio (Markowitz, 1959).

The mean-variance framework is widely explored within the realm of operational decisions to address various supply chain risks, particularly those arising from uncertain market demand (Tomlin, 2006; Choi et al., 2008a; Wu et al., 2009; Choi and Chiu, 2012; Liu et al., 2016; Chiu and Choi, 2016). Specifically, Choi et al. (2008a) carry out a mean-variance analysis for the newsvendor problem in which decision makers are risk-averse, risk-neutral, or risk-taking. They analytically investigate the effective frontiers for each case. With the same objective function, Choi and Chiu (2012) study the mean-downside-risk and mean-variance newsvendor models for both the exogenous and endogenous retail price cases. They find that the retailer orders the same stocking amounts in the mean-downside-risk and mean-variance models. Some studies have also analyzed how to achieve supply chain coordination within the mean-variance framework (Gan et al., 2004; Gan et al., 2005; Choi et al., 2008b; Choi et al., 2008c; Wei and Choi, 2010; Chiu et al., 2011). For instance, Choi et al. (2008c) study the coordination of a buyback contract under the mean-variance framework. They find that channel coordination is not always achievable under risk constraints, in contrast to results indicating that the conventional buyback contract can always coordinate a supply chain (Pasternack, 1985; Tsay, 2001; Lee and Rhee, 2007). Chiu et al. (2011) investigate the channel coordination problem for a target sales rebate contract in which supply chain parties make decisions based on a mean-variance analysis. Wei and Choi (2010) explore the coordination of both a wholesale price and a profit-sharing contract under the mean-variance model. They analytically characterize the necessary and sufficient conditions under which supply chain coordination is achieved. The present paper contributes to this line of research by providing a mean-variance analysis of option contracts, with a focus on supply chain coordination and equilibrium analysis for a two-echelon supply chain.

Contract arrangements provide a useful risk management mechanism in supply chains under demand uncertainty. For example, option contracts allow a buyer to determine how much to purchase according to the realized market demand, while providing a supplier with upfront payments. Option contracts have seen widespread application in a number of industries, including IT, telecommunications, semiconductors, and electricity (Wu and Kleindorfer, 2005; Anderson et al., 2017). For example, many giant companies, such as IBM, Sun Microsystems and Hewlett-Packard, have taken a portfolio procurement strategy with options (Tsay and Lovejoy, 1999). In addition, option contracts have been used in the agriculture industry (Zhao et al., 2013; Wang and Chen, 2017). For example, to facilitate vegetable sales in Ishikawa, Japan, some local farmers established an agriculture agency as their representative to negotiate with vegetable buyers such as stores and restaurants. The agency has offered the option contracts of vegetables to its buyers since 2008. The buyers first place an option quantity before the growing season and pay 4% of the total value as deposits. The farmers then grow the vegetable based on the buyers' orders. During the selling season, the buyers purchase an amount of the vegetable up to the option quantity from the farmers at a pre-determined exercise price to satisfy the realized market demand. Figure 1 illustrates this transaction process.

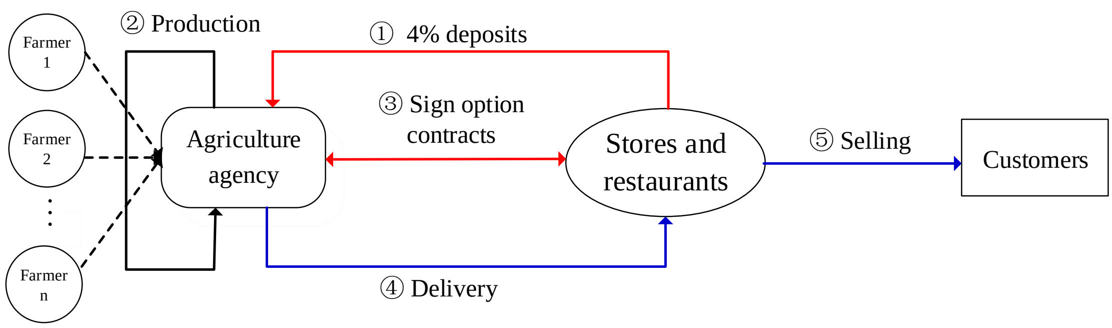

Fig. 1. Transaction process

An increasing number of studies have focused on the application of option contracts to procurement risk management (Wu and Kleindorfer, 2005; Wang et al., 2006; Wang et al., 2012; Zhao et al., 2013; Liu et al., 2014; Nosoohi and Nookabadi, 2016; Paul et al., 2016; Sawik, 2016; Anderson et al., 2017; Zhao et al., 2018). Zhao et al. (2013) examine the feedback effects of the bidirectional option on the retailer's initial order strategy. Nosoohi and Nookabadi (2016) investigate the manufacturer's decisions in two stages under call, put and bidirectional options. Wang et al. (2006) analyze a call option contract and show that this contract improves the buyer's performance. Wang et al. (2012) show that in a two-stage model, the buyer has a higher expected profit in the first stage, whereas the supplier may have worse performance in the second stage compared with the case without option contracts. Those studies compare the optimal decisions relating to option contracts with the conventional newsvendor model within a risky environment. However, little research has examined how risk (as an exogenous factor) affects the supply chain decisions involved in option contracts. Introducing risk constraints into a mean-variance model, we examine supply chain coordination and optimal decisions for option contracts. We find that channel coordination depends on the retailer's risk attitude and may not always be achieved, in contrast to results indicating that properly designed option contracts can always coordinate a supply chain (Gomez_Padilla and Mishina, 2009; Wang and Liu, 2007; Zhao et al., 2010; Wang et al., 2015).

In this paper, we study the mean-variance model under option contracts in a two-echelon supply chain. We investigate the tradeoff between profit and risk faced by supply chain parties. Our proposed mean-variance model of option contracts is in sharp contrast to that of a buyback contract presented by Choi et al. (2008 c) and that of a profit sharing contract presented by Wei and Choi (2010). Table 1 summarizes the difference of the three contracts with risk constraints.

Table 1. The comparisons of three contracts with risk constraints

Contracts	Control parameter	Degree of control	Coordination
Buyback contract (Choi et al. 2008c)	Returns price b	Weak	Not always
Profit sharing contract (Wei and Choi 2010)	Wholesale price w and profit sharing ratio λ	Strong	Always
Option contract (our paper)	Option price o and exercise price e	Strong	Not always

In terms of model setup, the supplier in Choi et al. (2008c) only decides the returns price b while the wholesale price is fixed. The manufacturer in Wei and Choi (2010) decides the wholesale price w and the profit sharing ratio λ. In our option contract, the supplier decides the option price o and exercise price e. These three contracts therefore provide the supplier with different degrees of control in achieving supply chain coordination. As a result, the coordination outcomes are different as well. In Choi et al. (2008c), a higher buyback price offers the retailer higher expected profit and lower risk but imposes on the supplier lower expected profit and higher risk. The results seem inconsistent with classical investment theory, in which a high risk is often accompanied by a great expected profit. Wei and Choi (2010) show the necessary and sufficient conditions that coordinate the supply chain, and they characterize the equilibrium for the Stackelberg game in a decentralized supply chain when the proportion of the profit and risk sharing is predetermined. Our proposed model, however, shows that when the supplier (retailer) bears a lower risk, she (he) obtains a lower expected profit. In addition, in our mean-variance models, the proportion of risk sharing between the supplier and retailer is not predetermined; instead, it is determined by the option price and exercise price. By changing the prices, the allocation of expected profit and risk sharing will be altered. Our results are in line with the observation that with a higher risk tolerance, the supplier always prefers to reduce the exercise price, and the retailer increases the option quantity to enjoy more profit.

We summarize the key results of our paper as follows. We first study the channel coordination of option contracts with risk constraints. We find that supply chain coordination depends on the retailer's risk tolerance and, more importantly, may not always be achieved. We next study a decentralized supply chain in which the supplier and retailer are maximizing their own profits subject to risk constraints. We consider two cases: (1) the retailer's risk aversion threshold is public information; and (2) the threshold is private information of the retailer. In the former case, we find that changing the option price and exercise price reallocates expected profit and risk sharing between the retailer and the supplier. We also show that a unique equilibrium exists in the Stackelberg game, and the equilibrium outcomes depend largely on the supplier's and retailer's risk tolerance levels. In the latter case, we first show that the retailer benefits from pretending to be less risk averse. To avoid untruthful information reporting on the part of the retailer, we propose a minimum option quantity commitment for the supplier.

The contribution of our paper is twofold. First, our paper contributes to the option contract literature by investigating the implication of risk considerations for option contracts. In doing so, we develop insights into how risk considerations shape the coordination results and equilibrium outcomes. Second, this paper contributes to the literature on contract design with risks by studying supply chain coordination and equilibrium analysis in an option contract setting. In particular, we study both symmetric and asymmetric information settings in which the retailer's risk attitude may be known or unknown to the supplier.

The rest of this paper is organized as follows. Section 2 describes the notation and assumptions. Section 3 studies supply chain coordination with option contracts in the mean-variance model. Section 4 conducts the equilibrium analysis in the decentralized case under both symmetric information and asymmetric information cases. Section 5 concludes and provides some management insights.

2. Model description

We consider a two-echelon supply chain consisting of a supplier and a retailer in which the newsvendor-like retailer orders products from the supplier to satisfy an uncertain demand X. The probability density function (PDF) of X is $f(x)$, the cumulative distribution function (CDF) is $F(x)$, and the complementary CDF is $\bar{F}(x)$. The mean and standard variance of X are u and σ, respectively. For convenience, we refer to the supplier as "she" and the retailer as "he" in the following analysis.

We are interested in a situation in which option contracts are used to capture the contractual relationship between the supplier and the retailer. As discussed earlier, option contracts have been used in many industries, such as telecommunications, IT and agriculture. The option contract is characterized by an option price o and an exercise price e. By convention, e is the price that the retailer pays to the supplier per unit of the product purchased, and o is the price paid in advance for the reserved options. In practice, the option price o can be thought of as an upfront payment made by the retailer for reserving one unit of the product before the sales season.

The supplier decides the option price o and exercise price e, and the retailer decides how much to reserve from the supplier, which we term the option quantity q. Based on the realized demand x, the retailer purchases $min\{x,q\}$ units from the supplier at the exercise price e. The expected sales quantity is given by $S(q){:=}E(min\{X,q\})$, where the expectation is taken over the random demand X. It is straightforward to show that $S(q)=q-\int_0^q F(x)dx$. To avoid triviality, we assume $o<c<o+e<p$ (Zhao et al., 2010; Feng et al., 2014). In fact, assuming $o<c$ avoids the unreasonable case in which the supplier's production is risk-free. Moreover, assuming $c<o+e$ and $o+e<p$ avoids the trivial cases in which the supplier is unwilling to produce and the retailer is unwilling to order. We also consider zero salvage values for the unsold products. Such a setting is appropriate for seasonal products with a long lead time and a short selling horizon. The above setting has been used in many supply chain management papers, such as Kouvelis and Zhao (2012), Kouvelis and Zhao (2016), Feng et al. (2015) and Chen (2015). The sequence of events is illustrated in Fig. 2.

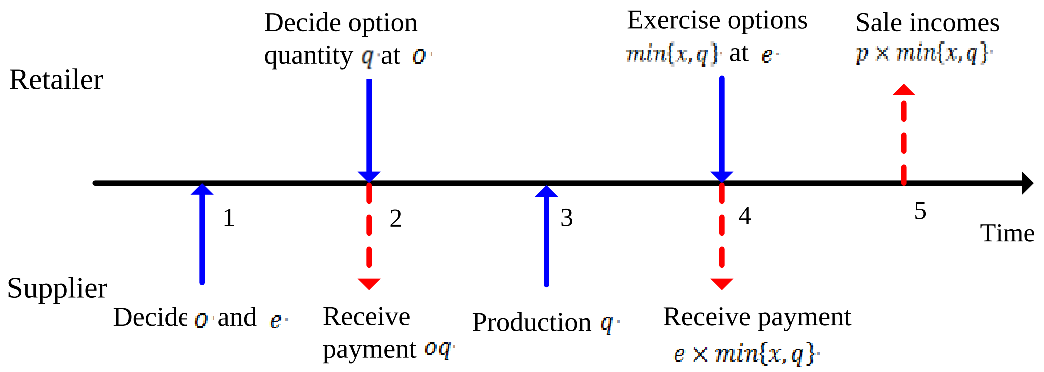

Fig. 2. Sequence of events

Incorporating risk preferences of supply chain members, we construct the mean-variance model to analyze the decisions involved in an option contract. Each supply chain decision maker aims to maximize their expected profits given their risk constraints. For clear interpretation, we list the main notation in Table 2.

Table 2. Summary of notations.

Notation	Definition
p	Selling price
o	Per-unit option price
c	Procurement cost
e	Per-unit exercise price
x	Market demand level

$f(x)$	Probability density function of market demand
$F(x)$	Cumulative distribution function of market demand
u	Mean of market demand
σ	Standard variance of market demand
q	Commodity option quantity
$S(q)$	Expected sales
$q_{SC,EP}$	Optimal order quantity that maximizes the expected profit of the supply chain
$q_{R,EP}$	Optimal order quantity that maximizes the expected profit of the retailer
EP_i	Expected profit of agent i, where $i = SC, S, R$
P_i	Profit of agent i, where $i = SC, S, R$
SP_i	Standard deviation of the expected profit of agent i, where $i = SC, S, R$
K_i	Risk aversion threshold of agent i, where $i = SC, S, R$
$q_{SC,SP}$	Supply chain's maximum order quantity that satisfies the risk constraints
$q_{S,SP}$	Supplier's maximum order quantity that satisfies the risk constraints
$q_{R,SP}$	Retailer's maximum order quantity that satisfies the risk constraints
$q_{SC,MV}$	Optimal order quantity that maximizes the mean–variance optimization problem for the supply chain
$q_{R,MV}$	Optimal option quantity that maximizes the mean–variance optimization problem for the retailer

For convenience, we use the following notations throughout the paper: EP is short for expected profit, SP is short for standard deviation of profit, and MV is short for mean-variance. Furthermore, the subscripts SC, S and R denote supply chain, supplier and retailer, respectively.

3. Supply chain coordination

In this section, we examine whether or not supply chain coordination can be achieved and, if achievable, under what conditions. Following the approach for supply chain coordination under the mean-variance model (Choi et al., 2008c), the supplier is supposed to play the role of supply chain coordinator. The supplier's objective is to set an optimal option price and an optimal exercise price such that the retailer's order quantity equals the supply chain's optimal quantity.

3.1 Supply chain optimal solution

To begin, we first look at the supply chain optimal solution. Suppose the production quantity is q. With the realized demand x, the sales quantity is given by $min\{x, q\}$, and the supply chain's profit is given by $P_{SC} = pmin\{x, q\} - cq = (p-c)q - p(q-x)^{+}$. Therefore, the supply chain's expected profit is given by

$$EP_{SC} = pS(q) - cq = (p-c)q - p\int_0^q F(x)dx. \tag{1}$$

The standard deviation of the supply chain profit is given by $SP_{SC} = p\sqrt{\xi(q)}$, where

$$\xi(q) = Var((q-X)^{+}) = 2\int_0^q (q-x)F(x)dx - (\int_0^q F(x)dx)^2. \tag{2}$$

We now formulate the mean-variance model of the supply chain as follows:

$$\max_{q} \; EP_{SC}\,(q)$$

$$s.t. \;\; SP_{SC}(q) \le K_{SC}. \tag{P1}$$

The objective of (P1) is to maximize the supply chain's expected profit subject to a constraint on the standard deviation of the supply chain's profit, where $K_{SC} \ge 0$ is the supply chain's risk-aversion threshold. A small K_{SC} means a low risk tolerance of the supply chain. Compared with the case in which the risk is not considered, the supply chain uses a more conservative strategy for the mean-variance model, as illustrated in the following proposition.

Proposition 1. *(i)* $SP_{SC}(q)$ *is increasing in* q*; (ii) The supply chain optimal solution is* $q_{SC,MV} = min\{q_{SC,EP}, q_{SC,SP}\}$*, and* $q_{SC,MV}$ *is non-decreasing in* K_{SC}*, where*

$q_{SC,EP} = F^{-1}(\frac{p-c}{p})$ *and* $q_{SC,SP} = argmax_q \{SP_{SC}(q) \leq K_{SC}\}$.

It is worth noting that $SP_{SC}(q)$ is increasing in q, indicating that as the option quantity increases, so too does the standard deviation of the supply chain profit. The intuition is as follows. The leftover inventory $q - S(q)$ can be viewed as the risk associated with the demand. Knowing $q - S(q) = \int_0^q F(x)dx$, we observe that the leftover inventory increases in q, which suggests that the risk indicated by SP_{SC} also increases in q. Proposition 1(i) is consistent with the findings in Choi et al. (2008a,b,c). In the centralized case, the supply chain risk always increases with order quantity but is independent of the adopted contract.

Proposition 1(ii) gives the analytical solution of the supply chain optimal order quantity $q_{SC,MV}$ under the mean-variance model. First, if the risk constraint is absent, then the supply chain optimal solution is given by $q_{SC,EP} = F^{-1}\left(\frac{p-c}{p}\right)$. When considering the risk constraint, it is intuitive that the optimal order quantity $q_{SC,MV}$ will not exceed $q_{SC,EP}$. We find that the supply chain optimal quantity is the minimum of $q_{SC,EP}$ and $q_{SC,SP}$, the latter of which is the largest quantity satisfying the risk constraint. Specifically, when $SP_{SC}\left(q_{SC,EP}\right) > K_{SC}$, the risk constraint becomes active and the optimal solution will be $q_{SC,SP}$, which is increasing in K_{SC}. When $SP_{SC}\left(q_{SC,EP}\right) \leq K_{SC}$, the impact of risk aversion on the supply chain performance is ignored and the optimal solution will be $q_{SC,EP}$, which is independent of K_{SC}. Proposition 1(ii) shows that the order quantity in the centralized case increases with the supply chain risk tolerance when the risk is active. This result is similar to Choi et al. (2008c) in that with a buyback contract, the order quantity increases with the risk in the interval $K_{SC} \in [0, SP_{SC}\left(q_{SC,EP}\right)]$.

In the mean-variance model, we can see that a tradeoff is made between the expected value and the standard deviation of the supply chain profit. Such a tradeoff can be illustrated by using Fig. 3, which depicts how the supply chain's expected profit and the standard deviation vary with the option quantity. In the figure, we consider three cases of risk-aversion thresholds K_{SCj}, where $j = 1,2,3$. Correspondingly, $q_{SC,SPj}$ is the supply chain's maximum order quantity, given K_{SCj}. Fig. 3 (also the proof of Proposition 1) reveals that the supply chain's expected profit is a concave function of the option quantity and that the standard deviation increases with the option quantity. As can be seen from the figure, there are three scenarios for the supply chain optimal quantity. If $K_{SC} = K_{SC1} < SP_{SC}(q_{SC,EP})$, then $q_{SC,MV1} = q_{SC,SP1}$, and the SP_{SC} constraint is active. If $K_{SC} = K_{SC2} = SP_{SC}(q_{SC,EP})$, then $q_{SC,MV2} = q_{SC,SP2} = q_{SC,EP}$, and the risk constraint is inactive. If $K_{SC} = K_{SC3} > SP_{SC}(q_{SC,EP})$, then $q_{SC,MV3} = q_{SC,EP}$, implying that the risk constraint does not play a role either.

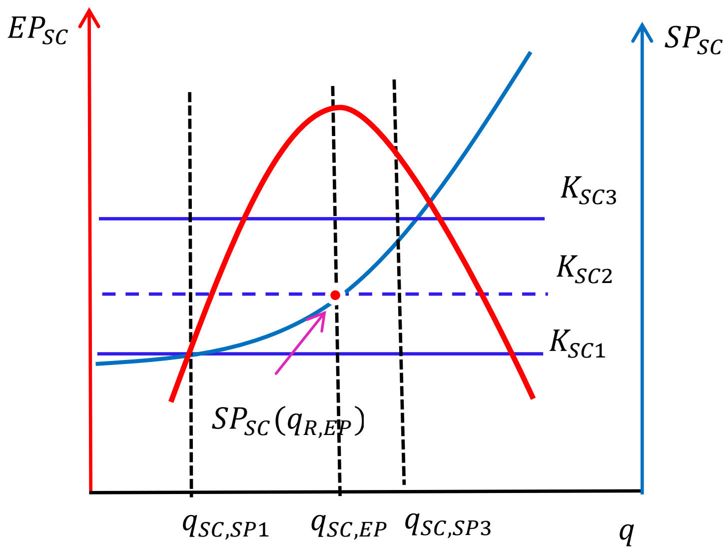

Fig. 3. $EP_{SC}(q)$ and $SP_{SC}(q)$

3.2 Retailer's ordering decision

As mentioned earlier, supply chain coordination refers to a situation where the retailer's order matches the supply chain optimal quantity. In this subsection, we look at the retailer's ordering decision for any given option contract (o, e). With the contract (o, e), the retailer first pays the upfront fee oq for the q units of options before the sales season. During the sales season, according to the realized demand, the retailer exercises the option by paying the exercise price. Let x be the realized demand. Then, the retailer orders $min\{x, q\}$ units from the supplier at the exercise price e. The retailer's profit is given by $P_R = (p-e)min\{x,q\} - oq = (p-e-o)q - (p-e)(q-x)^+$. Therefore, the retailer's expected profit and the standard deviation are respectively

$$EP_R = (p-e-o)q - (p-e)\int_0^q F(x)dx \quad (3)$$

$$SP_R = (p-e)\sqrt{\xi(q)}, \quad (4)$$

where $\xi(q)$ is given in Equation (2). It is noted that the standard deviation of the retailer's profit depends on the exercise price e but not the option price o.

In the mean-variance model, the objective of the retailer is to maximize EP_R subject to a constraint on the standard deviation of the retailer's profit. Therefore, the model of the retailer is formulated as follows:

$$\max_q \; EP_R\,(q)$$

$$s.t.\; SP_R(q) \le K_R, \quad \text{(P2)}$$

where $K_R \ge 0$ is the retailer's risk-aversion threshold. We define $q_{R,SP}(e) = argmax_q\{SP_R(q) \le K_R\}$, which gives the retailer's maximum quantity that satisfies the condition $SP_R(q) \le K_R$.

Proposition 2. *For the retailer's ordering problem, we obtain the following: (i) The retailer's optimal order quantity is given by* $q_{R,MV}(o,e) = min\{q_{R,EP}(o,e), q_{R,SP}(e)\}$, *where* $q_{R,EP}(o,e) = F^{-1}(\frac{p-e-o}{p-e})$; *(ii) There exists a threshold* $e_1 = p - \frac{o}{\bar{F}\left(q_{R,SP}(e_1)\right)}$, *where* $q_{R,MV}(o,e)$ *is increasing in* e *for* $e \in (c-o, e_1)$ *and decreasing in* e *for* $e \in [e_1, p-o)$.

Proposition 2(i) characterizes the retailer's optimal order quantity for any given option contract (o,e). In the absence of the risk constraint, the retailer's order quantity is given by $q_{R,EP}(o,e)$. In the presence of the risk constraint, however, the retailer's order quantity may not maximize his expected profit. When K_R is a small value, the retailer has a low risk tolerance and, hence, orders a relatively conservative quantity $q_{R,MV}(o,e) = q_{R,SP}(e)$. When K_R is a large value (i.e., high risk tolerance), then the retailer orders the same amount as in the absence of risk constraint, that is, $q_{R,MV}(o,e) = q_{R,EP}(o,e)$.

As noted earlier, the standard deviation of the retailer's profit depends on the exercise price e but not the option price o. Proposition 2(ii) examines how the retailer's optimal order quantity changes with the exercise price. Specifically, $q_{R,MV}(o,e)$ first increases and then decreases in e. This is somewhat counterintuitive because common sense suggests that a higher price may lead to a smaller order quantity. Indeed, when the risk constraint is absent, the retailer's order quantity $q_{R,EP}(o,e)$ decreases in the exercise price e. However, in our mean-variance model, the retailer's order quantity is the minimum between $q_{R,EP}(o,e)$ and $q_{R,SP}(e)$, and it can be readily shown that $q_{R,SP}(e)$ increases in e. As a result, when e is small (i.e., $e \in (c-o, e_1)$), the risk constraint is active, and the retailer's order quantity is equal to $q_{R,SP}(e)$. Thus, a further increase in e will lead to a larger order quantity. This explains why the retailer's optimal order quantity first increases in the exercise price. On the other hand, when e is large (i.e., $e \in [e_1, p-o)$), the retailer's order quantity is equal to $q_{R,EP}(o,e)$. Therefore, for this case, the retailer's order quantity decreases in the exercise price.

3.3 Coordination mechanism

In this subsection, we identify the conditions of the option price o and exercise price e under which the retailer will order the supply chain optimal quantity $q_{SC,MV}$. Proposition 3 summarizes the results.

Proposition 3. *When $SP_R(q_{R,EP}((o,e)_M)) \le K_R$, then the supply chain is coordinated under any option contract (o,e) in the set $M = \{(o,e): o = (p-e)\bar{F}(q_{SC,MV})\}$; When $SP_R(q_{R,EP}((o,e)_N)) \ge K_R$, then the supply chain is coordinated under any option contract (o,e) in the set $N = \{(o,e): e = p - \frac{K_R}{\sqrt{\xi(q_{SC,MV})}}\}$; When $SP_R(q_{R,EP}((o,e)_N)) < K_R < SP_R(q_{R,EP}((o,e)_M))$, then the supply chain cannot be coordinated, where $(o,e)_M$ denotes $(o,e) \in M$ and $(o,e)_N$ denotes $(o,e) \in N$.*

Proposition 3 shows that if $SP_R(q_{R,EP}(\ (o,e)_N)) < K_R < SP_R(q_{R,EP}((o,e)_M))$, then there are no option contracts that can coordinate the supply chain; otherwise, we are able to characterize the conditions under which the supply chain is coordinated by adjusting the prices, as shown in Proposition 3. It is noted that the conditions involve a one-to-one relationship of the option and exercise prices. Specifically, when the option price is fixed: if $SP_R(q_{R,EP}((o,e)_M)) \leq K_R$, setting $e = p - \frac{o}{\bar{F}(q_{SC,MV})}$ can achieve the supply chain coordination; if $SP_R(q_{R,EP}((o,e)_N)) \geq K_R$, setting $e = p - \frac{K_R}{\sqrt{\xi(\ q_{SC,MV})}}$ can achieve the supply chain coordination. When the exercise price is fixed: if $SP_R(q_{R,EP}((o,e)_M)) \leq K_R$, setting $o = (p-e)\bar{F}\big(q_{SC,MV}\big)$ can achieve the supply chain coordination; and if $SP_R(q_{R,EP}((o,e)_N)) \geq K_R$, the supply chain coordination is achieved only when $e = p - \frac{K_R}{\sqrt{\xi(\ q_{SC,MV})}}$. This is because in this situation, the retailer's risk constraint is active. Furthermore, the retailer's option quantity $q_{R,SP}(e)$ only depends on e and is independent of o.

Choi et al. (2008c) show that when the risk constraints are considered, supply chain coordination is not always achievable by setting a returns price only. In this setting, maximizing the supply chain's expected profit may hurt the supplier. A larger returns price brings the retailer greater expected profit and lower risk. However, it leads to smaller expected profit and higher risk for the supplier. Hence, offering a returns price that maximizes the supply chain's expected profit may not be a wise decision for the supplier. With an option contract, the supplier can balance the tradeoff between the expected profit and risk by setting the option price and exercise price. Hence, compared to the buyback contract in Choi et al. (2008c), the option contract seems more powerful in terms of supply chain coordination.

Fig. 4 illustrates whether it is possible to coordinate the supply chain under the mean-variance model for a given option price o, where $e_M,\ e_{M'} \in M,\ e_N,\ e_{N'} \in N$. For $q_{SC,MV1} \leq q_{R,MV}(e_1)$, the proof of Proposition 4 shows that $q_{R,SP}(e_N) = q_{SC,MV1}$ and $q_{R,EP}(e_M) = q_{SC,MV1}$, as indicated by points A and B in Fig. 4. If $SP_R(q_{R,EP}(e_M)) \leq K_R$, then $q_{R,MV}(e_M) = q_{R,EP}(e_M) = q_{SC,MV1}$. Therefore, setting

$e = e_M$ coordinates the supply chain. If $SP_R(q_{R,EP}(e_N)) \geq K_R$, then $q_{R,MV}(e_N) = q_{R,SP}(e_N) = q_{SC,MV1}$. Setting $e = e_N$ coordinates the supply chain. For $q_{SC,MV2} > q_{R,MV}(e_1)$, the proof of Proposition 3 shows that $q_{R,EP}(e_{M'}) = q_{SC,MV2}$ and $q_{R,SP}(e_{N'}) = q_{SC,MV2}$, as illustrated by points C and D in Fig. 4. If $SP_R(q_{R,EP}(e_{N'})) < K_R < SP_R(q_{R,EP}(e_{M'}))$, then $q_{R,SP}(e_{M'}) < q_{R,EP}(e_{M'})$ and $q_{R,EP}(e_{N'}) < q_{R,SP}(e_{N'})$. Clearly, $q_{R,MV}(e_{M'}) = q_{R,SP}(e_{M'}) < q_{SC,MV2}$ and $q_{R,MV}(e_{N'}) = q_{R,EP}(e_{N'}) < q_{SC,MV2}$. The retailer will not order at the supply chain's optimal order quantity level. Therefore, coordination of the supply chain cannot be achieved.

The above findings are significantly distinct from the results in Zhao et al. (2010). Their findings show that option contracts without risk constraints can always coordinate the supply chain and achieve Pareto improvement. This implies that the risk constraint has an important impact on the coordination of a supply chain. Therefore, ignorance of risk considerations may lead to misalignment between the retailer's order and the supply chain optimal order.

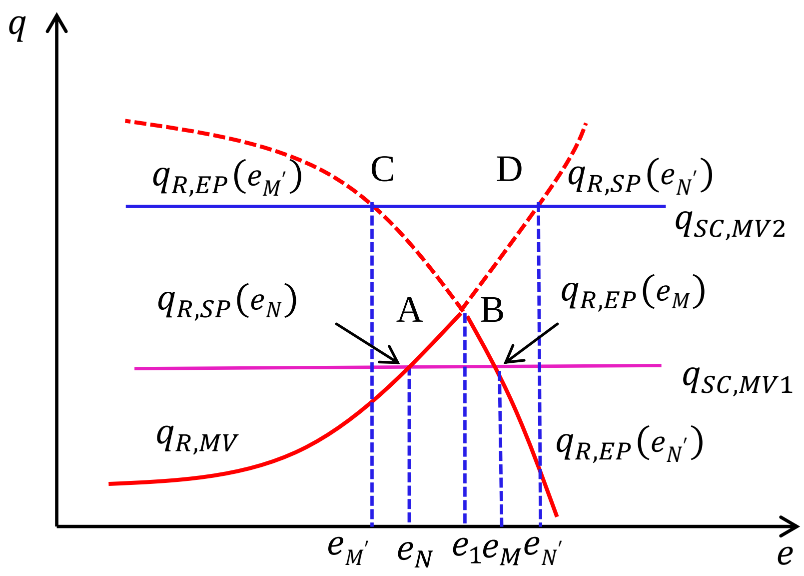

Fig. 4. $q_{R,EP}(e)$, $q_{R,SP}(e)$ and $q_{R,MV}(e)$

To gain further insight into the supply chain coordination conditions, we use Table 3 to demonstrate how the supplier sets her exercise price to coordinate the supply chain under different risk aversion thresholds. We maintain the following assumptions over the simulations: (1) the random demand is normally distributed with u =100 and σ =50; and (2) p =100 and c =15 (Choi et al., 2008c). We also set o =10, with a focus on examining how the exercise price changes with the risk aversion thresholds. Table 3 illustrates that e is decreasing in K_R and K_{SC}, while $q_{SC,MV}$ is increasing in K_R and K_{SC}. As K_{SC} and K_R increase, the risk tolerance of both parties becomes greater, and hence, they are more willing to take the profit risk. The supplier therefore decreases e and then the retailer orders more, which brings both parties more profit. We also note that under the option contract, the expected profit of the retailer with risk constraints and his standard deviation are substantially reduced compared to the risk-neutral case. In contrast, the expected profit and the standard deviation of the supplier's profit with risk constraints are significantly increased. This is because the retailer reduces the order quantity to reduce the risk. Therefore, the mean-variance model is quite different from the risk-neutral case, and it provides important insights into how to achieve the coordination of the supply chain with risk considerations.

Table 3. Exercise prices that coordinates the supply chain in the mean-variance model

Parameters		Optimal decision		Expected profit			Standard deviation		
K_{SC}	K_R	e	$q_{SC,MV}$	EP_{SC}	EP_R	EP_S	SP_{SC}	SP_R	SP_S
∞	∞	33.3	157.0	7727.8	5151.9	2575.9	4180.3	2786.9	1393.4
3200	1200	72.7	120.2	7367.0	1303.3	6063.7	3200.0	874.2	2325.8
3000	1000	75.7	114.2	7224.0	1031.6	6192.2	3000.0	729.6	2270.4
2500	800	80.9	100.0	6773.8	582.1	6191.7	2500.0	478.0	2022.0
2000	1000	84.2	86.2	6185.4	319.6	5865.8	2000.0	315.3	1684.7
1500	1000	86.4	71.9	5420.8	164.9	5255.9	1500.0	204.0	1296.0
1000	500	88.0	56.2	4422.6	69.9	4352.7	1000.0	120.0	880.0

4. Decentralized supply chain

In this section, we study the strategic interaction between the supplier and the retailer in a decentralized supply chain. For this setting, each player's objective is to maximize their own expected profit subject to their respective risk constraints. In the centralized case, the information about the retailer's risk aversion threshold is common knowledge for the players. However, in the decentralized case, the supplier may not know this information. Therefore, we consider two information cases in the decentralized case. We first study the information symmetry case in which the retailer's risk aversion threshold is public information. Then, we explore the information asymmetry case in which the retailer's risk aversion threshold is private. The timeline of the game is the following: The supplier determines o and e, and given these prices, the retailer decides the option quantity q and pays oq to the supplier. After that, the demand is realized, and the retailer purchases $min\{x, q\}$ units from the supplier at the exercise price e. As can be seen, this is a Stackelberg game where the supplier is the leader and the retailer is the follower. Following the standard backward induction approach, we start by analyzing the retailer's ordering decision and then look at the supplier's pricing decision.

4.1. Symmetric information: the retailer's risk aversion threshold is public

The retailer's ordering decision is the same as that for the supply chain coordination analysis (see Subsection 3.2). From Proposition 2, we know that the retailer's optimal option quantity is given by $q_{R,MV}(o, e) = min\{q_{R,EP}(o, e),$ $q_{R,SP}(e)\}$.

Under the mean-variance model, the objective of the supplier is to maximize EP_S subject to a constraint on SP_S. Given any option quantity q and the realized demand x, the supplier's profit is given by $P_S = (o - c)q + emin\{x, q\} = (e + o - c)q - e(q - x)^+$. Therefore, the supplier's expected profit and the standard deviation of the supplier's profit are respectively given by

$$EP_S = (o + e - c)q - e\int_0^q F(x)dx \tag{5}$$

$$SP_S = e\sqrt{\xi(q)}, \tag{6}$$

where $\xi(q)$ is given by Equation (2). We formulate the supplier's pricing problem as follows:

$$\max_{o,e} \; EP_S(q_{R,MV}(o, e))$$
$$s.t. \; SP_S(q_{R,MV}(o, e)) \leq K_S \tag{P3}$$

where $K_S \geq 0$ is the supplier's risk aversion threshold and $q_{R,MV}(o,e)$ is the retailer's optimal order quantity. Denote by $q_{S,SP}(e)$ the largest quantity q such that $SP_S(q,e) \leq K_S$, that is, we have $q_{S,SP}(e) = arg\,max_q\{SP_S(q,e) \leq K_S\}$. Regarding the constraint, if $max_e\{SP_S(q_{R,MV}(o,e))\} \leq K_S$, the supplier's risk constraint is inactive. If $max_e\{SP_S(q_{R,MV}(o,e))\} > K_S$, the supplier's risk constraint is active.

Proposition 4. *(i)* $EP_R = \alpha EP_{SC}$ *and* $EP_S = (1-\alpha)EP_{SC}$, *where* $\alpha = \frac{(p-e)S(q)-oq}{pS(q)-cq}$ *and* α *is decreasing with* o *and* e *; (ii)* $SP_R = \beta SP_{SC}$ *and* $SP_S = (1-\beta)SP_{SC}$, *where* $\beta = (1-\frac{e}{P})$ *and* β *is decreasing in* e.

Proposition 4(i) indicates that the supplier's expected profit and the standard deviation of her profit (EP_S and SP_S) are proportional to those of the supply chain (EP_{SC} and SP_{SC}). Similarly, the retailer's expected profit and the standard deviation of his profit are proportional to those of the supply chain. We note that the retailer's profit-sharing proportion α is decreasing in both o and e and that the retailer's risk-sharing proportion β is decreasing in e. Hence, the retailer's EP is decreasing in o and e, and SP is decreasing in e. Correspondingly, the supplier's EP is increasing in o and e, and SP is increasing in e. The results are consistent with the classical investment theory that a higher profit is linked to a larger risk. Therefore, the adjustment of o and e can effectively allocate EP_S and SP_S between the supplier and retailer. However, the above result differs from that of Choi et al. (2008c) in that they find that a higher returns price brings the retailer greater expected profit and lower risk but leads to smaller expected profit and higher risk for the supplier. This result seems inconsistent with the classical investment theory, in which a high risk is often accompanied by a great expected profit. In our models, when the player bears a higher risk, it will obtain a greater expected profit. In particular, the supplier will balance the tradeoff between the expected profit and risk by adjusting the option price and exercise price.

With option contracts, we next examine the channel player's optimal decisions under the risk-neutral situation. To obtain analytical results, we consider demand distributions with the increasing generalized failure rate (IGFR). Define $h(x) = \frac{xf(x)}{\bar{F}(x)}$ as the general failure rate. When $\frac{\partial h(x)}{\partial x} \geq 0$, the demand distribution satisfies IGFR (Wei and Choi, 2010). As is well known, IGFR is a mild condition that many commonly used distributions can satisfy, including Normal (as well as truncated normal at zero), Uniform, Exponential, Power, Gamma with shape parameter ≥ 1, Beta with parameters ≥ 1, and Weibull distribution with shape parameter ≥ 1. distributions (Lariviere and Porteus, 2001; Wei and Choi, 2010; Kouvelis and Zhao, 2012).

Lemma 1. *Given* o, $EP_R(q_0, e_0) = \max_{q} EP_R(q, e)$ *and* $EP_S(q_0, e_0) = \max_{e} EP_S(q, e)$, *where* $p\bar{F}(q_0) - c - \frac{of(q_0)}{\bar{F}(q_0)^2}\int_0^{q_0} \bar{F}(x)\, dx = 0$ *and* $e_0 = p - \frac{o}{\bar{F}(q_0)}$.

The risk-neutral case is equivalent to the mean-variance model, where K_R and K_S go to infinity. This is because when K_R and K_S go to infinity, the risk constraints will not affect either party's optimal decisions. Lemma 1 characterizes the supplier's optimal exercise price e_0 and the retailer's optimal option quantity q_0. Consider the extreme case where the option price is zero ($o = 0$). We observe that $q_0 = \bar{F}\left(\frac{c}{p}\right)$, which gives the supply chain optimal quantity, and the exercise price is given by $e_0 = p$.

When K_R and K_S are not significantly large, the retailer's order quantity may satisfy $q_{R,MV}(o, e) \leq q_0$. In this case, the supplier will set e according to $q_{R,MV}(o, e)$. Proposition 5 gives the optimal exercise price e^* for different risk thresholds.

Proposition 5. *Given* o, *(i) if* $K_S \geq max\{SP_S\left(q_{R,MV}(o, e_0)\right), SP_S(q_{R,MV}(o, e_1))\}$, *then* $e^* = max\{e_0, e_1\}$; *(ii) if* $K_S < max\{SP_S\left(q_{R,MV}(o, e_0)\right), SP_S(q_{R,MV}(o, e_1))\}$, *then* $e^* = argmax\{EP_S\left(q_{R,MV}(o,e_l)\right), EP_S\left(q_{R,MV}(o,e_u)\right)\}$, *where* e_t *satisfies* $q_{R,MV}(o,e_t) = q_{S,SP}(e_t)$, $t = l, u$.

Because $K_S \geq max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$, the supplier's risk constraint is inactive. The equilibrium exercise price e^* is dependent on K_R. Hence, $e^* = max\{e_1,\ e_0\}$. Because $K_S < max\{SP_S(q_{R,MV}(o,e_0)), SP_S(q_{R,MV}(o,e_1)\}$, the supplier's risk constraint is active. The equilibrium exercise price e^* is dependent on K_S and K_R. Hence, $e^* = argmax\{EP_S\left(q_{R,MV}(o,e_l)\right), EP_S\left(q_{R,MV}(o,e_u)\right)\}$ and $SP_S\left(q_{R,MV}(o,e^*)\right) = K_S$. In the decentralized case, Choi et al. (2008c) find that if the optimal returns price for the supplier is unique, then there will exist a unique equilibrium. In line with this finding, we obtain a unique equilibrium for the Stackelberg game with option contracts (Proposition 5).

Based on the settings in Table 3, we examine how the supplier's EP and SP change with e in Fig. 5. Given K_R, the solid lines are the supplier's expected profits, and the dashed lines are the supplier's standard deviations. We can also calculate the equilibrium prices using the results in Lemma 1 and Proposition 5. For the risk-neutral case, $e^* = e_0$ =78.5. For the risk constraints case, given K_{R1} =1,000, we have $e_1 = 70 < e_0 = 78.5$. When $K_S \geq 2338.7$, then $e^* = e_0 = 78.5$. When $K_S = 1000 < 2338.7$, then e_l =50 and e_u =87.6. The efficient region of e^* is $e \in (5, 50] \cup [87.6, 90)$. We have $e^* = e_u$ =87.6. Given $K_{R2} = 400$, we have e_1 =82.9> e_0 =78.5. When $K_S \geq 2000$, then $e^* = e_1$ =82.9. When $K_S = 1000 < 2000$, then e_l =71.5 and e_u =87.6. The efficient region of e^* is $e \in (5, 71.5] \cup [87.6, 90)$. We have $e^* = e_u$ =87.6.

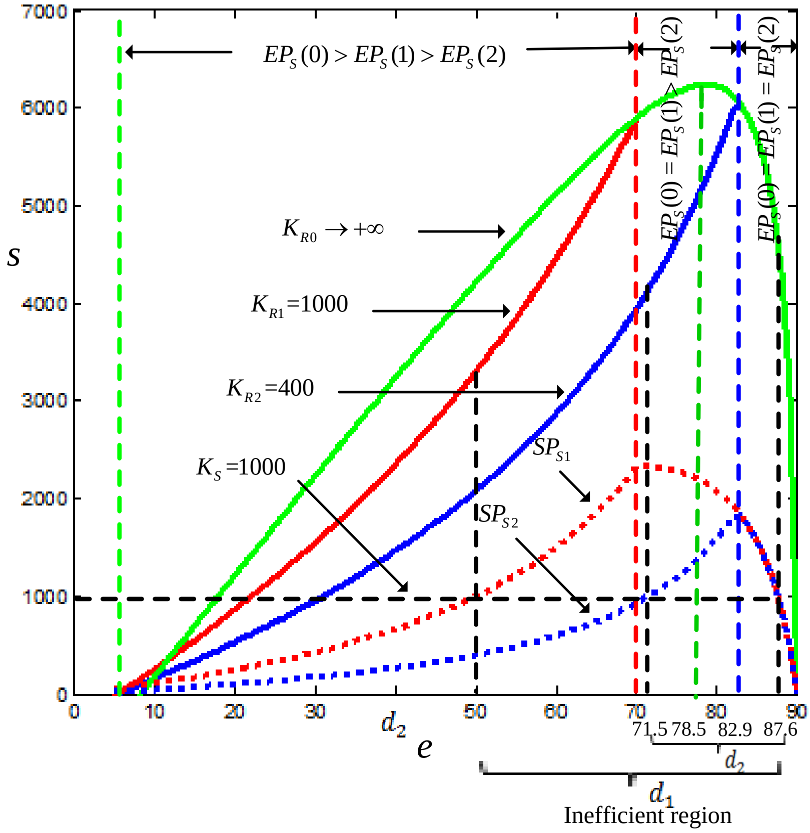

Fig. 5. $EP_S(e)$ and $SP_S(e)$

To show the impact of risk aversion on the equilibrium decisions of the Stackelberg game in the decentralized supply chain, given different values of K_S and K_R, Table 4 shows different values of e^*, $q_{R,MV}(o,e)$ and the corresponding EP and SP. By comparing Table 3 with Table 4, we find that both players in the centralized case have greater expected profit (EP) than in the decentralized case. In the meantime, we note that both players with greater expected profit in the centralized case also bear larger risk (SP) than in the decentralized case. This finding is consistent with the classical investment theory in that a high expected profit is often accompanied by a large risk. We observed that as K_S increases, e^* decreases, whereas the EPs and SPs of the retailer, supplier and supply chain all increase. The decreasing e^* leads to increasing $q_{R,MV}(o,e)$, which further leads to increasing EPs and SPs. As K_R increases, the e^*, EPs and SPs exhibit the same changes as those indicated above. We also note that as K_R (K_S) increases, the retailer's (the supplier's) profit sharing proportion α and risk sharing proportion β increase (see Proposition

4). This suggests that the party facing a higher risk always requires more profit to compensate for potential losses. Our findings are distinct from the results in Choi et al. (2008c), in which, with a buyback contract, a larger return price brings the supplier a lower profit sharing proportion and a higher risk sharing proportion.

Table 4. The optimal e^* and $q_{R,MV}$ for different K_S and K_R in the decentralized case.

K_S	K_R	e^*	$q_{R,MV}$	EP_{SC}	EP_R	EP_s	SP_{SC}	SP_R	SP_S	EP_s/EP_{SC}
500	100	89.00	39.7	3228.9	23.9	3204.9	561.9	61.9	500.0	99.2%
	200	89.00	39.7	3228.9	23.9	3204.9	561.9	61.9	500.0	99.2%
	500	89.00	39.7	3228.9	23.9	3204.9	561.9	61.9	500.0	99.2%
	1000	89.00	39.7	3228.9	23.9	3204.9	561.9	61.9	500.0	99.2%
	1500	89.00	39.7	3228.9	23.9	3204.9	561.9	61.9	500.0	99.2%
750	100	88.36	50.9	4054.8	51.4	4003.4	848.7	98.7	750.0	98.7%
	200	88.36	50.9	4054.8	51.4	4003.4	848.7	98.7	750.0	98.7%
	500	88.36	50.9	4054.8	51.4	4003.4	848.7	98.7	750.0	98.7%
	1000	88.36	50.9	4054.8	51.4	4003.4	848.7	98.7	750.0	98.7%
	1500	88.36	50.9	4054.8	51.4	4003.4	848.7	98.7	750.0	98.7%
1000	100	88.35	51.0	4072.8	52.2	4020.6	850.0	100.0	750.0	98.7%
	200	87.60	60.9	4736.8	92.3	4644.5	1141.7	141.7	1000.0	98.0%
	500	87.60	60.9	4736.8	92.3	4644.5	1141.7	141.7	1000.0	98.0%
	1000	87.60	60.9	4736.8	92.3	4644.5	1141.7	141.7	1000.0	98.0%
	1500	87.60	60.9	4736.8	92.3	4644.5	1141.7	141.7	1000.0	98.0%
1500	100	88.35	51.0	4072.8	52.2	4020.6	850.0	100.0	750.0	98.7%
	200	86.40	71.0	5370.2	164.9	5205.3	1470.6	200.0	1270.6	96.9%
	500	85.30	79.5	5844.9	237.0	5607.9	1762.2	262.2	1500.0	95.9%
	1000	85.30	79.5	5844.9	237.0	5607.9	1762.2	262.2	1500.0	95.9%
	1500	85.30	79.5	5844.9	237.0	5607.9	1762.2	262.2	1500.0	95.9%
2000	100	88.35	51.0	4072.8	52.2	4020.6	850.0	100.0	750.0	98.7%
	200	86.40	71.0	5370.2	164.9	5205.3	1470.6	200.0	1270.6	96.9%
	500	81.20	98.9	6736.3	558.5	6177.8	2464.1	464.1	2000.0	91.7%
	1000	81.20	98.9	6736.3	558.5	6177.8	2464.1	464.1	2000.0	91.7%
	1500	81.20	98.9	6736.3	558.5	6177.8	2464.1	464.1	2000.0	91.7%

To further depict the impacts of risk on e and the EPs, we examine how e and the EPs change with the standard variance σ. Fig. 6, 7 and 8 illustrate that in both cases, as σ increases, the supplier increases e and that the EPs of the retailer and supply chain both decrease. When facing the high demand risk indicated by a large σ, the retailer reduces the option quantity to lower the potential risk. The supplier correspondingly increases e to compensate the expected profit. Both actions reduce the EPs of the retailer and supply chain.

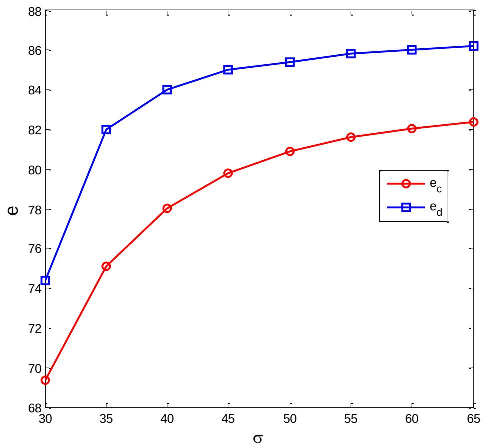

Fig. 6. The changes in e with σ in both cases

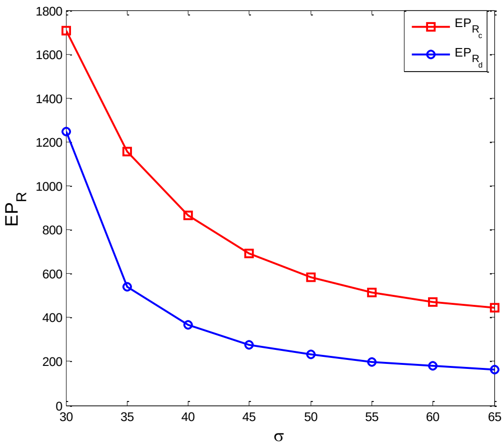

Fig. 7. The changes in EP_R with σ in both cases

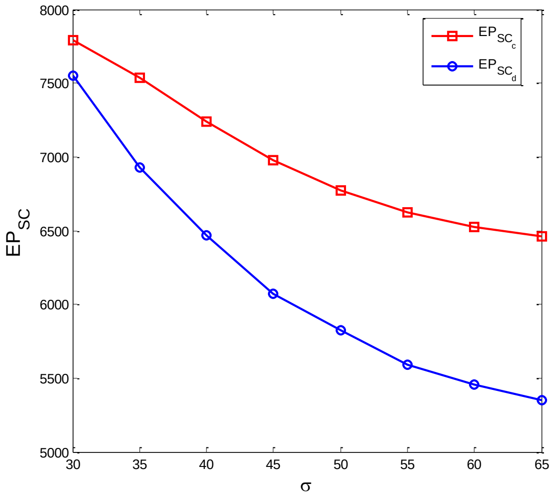

Fig. 8. The changes in EP_{SC} with σ in both cases

4.2 Asymmetric information: the retailer's risk aversion threshold is private

In the previous section, we focused on the information symmetry case in which the retailer's risk attitude is known. However, in practice, the supplier may not know the retailer's risk aversion threshold before offering her prices. It is an interesting question to examine how information asymmetry affects all the decisions in the decentralized case. The sequence of events is as follows: First, the retailer discloses his risk aversion threshold to the supplier, which may not be truthful. Second, the supplier offers an option price and an exercise price to the retailer. Third, the retailer decides the option quantity according to his true risk aversion threshold.

4.2.1 Retailer's problem

Suppose that K_R' is the retailer's risk aversion threshold disclosed to the supplier. Correspondingly, the supplier adjusts the exercise prices e_1 and e_t to $e_1(K_R')$ and $e_t(K_R')$ based on Propositions 2 and 5. Lemma 2 presents the retailer's information disclosure decision.

Lemma 2. *If* $K_R' > K_R$, *then* $EP_R(K_R') \geq EP_R(K_R)$.

Lemma 2 shows that the retailer has an incentive to pretend to be less risk averse. By disclosing a smaller risk aversion threshold, the retailer makes the supplier believe that he will likely order more. This incentivizes the supplier to reduce the exercise price. The proof of Lemma 2 indicates that the retailer's expected profit is decreasing in the exercise price. Therefore, pretending to be less risk averse will lead to a greater profit for the retailer.

4.2.2 Supplier's problem

Lemma 2 shows that the retailer has an incentive to set $K_R' > K_R$. This untruthful disclosure benefits the retailer. What action can the supplier take as a Stackelberg leader to prevent this from happening? Motivated by Wei and Choi (2010), we construct a minimum option quantity contract in which both the exercise price and minimum option quantity $\mathrm{q_{min}}$ are specified. If the retailer chooses a certain contract, then he will order no less than $\mathrm{q_{min}}$ at a corresponding exercise price. Proposition 6 shows how the minimum option quantity contracts are designed.

Proposition 6. *(i) When* $K_S \geq max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$ *, if* $e_0 < e_1(K_R')$*, then* $q_{min} = q_{R,SP}(e_1(K_R'), K_R')$ *and* $e^*(K_R') = e_1(K_R')$ *; if* $e_1(K_R') < e_0$ *, then* $q_{min} = q_{R,EP}(o, e_0)$ *and* $e^*(K_R') = e_0$ *; (ii) When* $K_S < max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$ *, if* $e^*(K_R) = e_l(K_R)$ *, then* $q_{min} = q_{S,SP}(e_l(K_R'), K_R')$ *and* $e^*(K_R') = e_l(K_R')$*; if* $e^*(K_R) = e_u$*, then* $q_{min} = q_{R,EP}(o, e_u)$ *and* $e^*(K_R') = e_u$.

We explain the result in Proposition 6 as follows. First, in the case of $K_S \geq max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$, the supplier is more risk tolerant and the risk constraint is inactive. If the risk constraint with K_R' is active for the retailer, then the supplier sets the minimum option quantity as $q_{min} = q_{R,SP}(e_1(K_R'), K_R')$. If the risk constraint with K_R' is inactive for the retailer, the supplier sets the minimum option quantity as $q_{min} = q_{R,EP}(o, e_0)$.

Second, in the case of $K_S < max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$, the supplier is less risk tolerant and the risk constraint is active. If K_R' is active for the retailer, then the supplier sets the minimum option quantity as $q_{min} = q_{R,MV}(o, e_l(K_R')) = q_{S,SP}(e_l(K_R'))$. If K_R' is inactive for the retailer, then the supplier sets the minimum option quantity as $q_{min} = q_{R,EP}(o, e_u)$.

For K_R to be active, since $SP_S(q_{R,MV}\left(o, e^*(K_R')\right) \geq q_{min}) \geq K_R$, the retailer bears the larger risk. The retailer will have to disclose his true risk aversion information. For K_R to be inactive, the retailer's untruthful disclosure has no impact on the supplier's decision. Thus, the supplier sets $q_{min} = q_{R,EP}\left(o, e^*(K_R')\right)$ only based on her risk constraint. Overall, the contract proposed in Proposition 6 efficiently prevents the retailer from reporting untruthfully.

In Wei and Choi (2010), the retailer's largest order quantity that satisfies his risk constraint is independent of the wholesale price. The retailer can therefore control his risk. However, in our model, the retailer's largest option quantity that satisfies his risk constraint depends on the exercise price, which is determined by the supplier based on the retailer's disclosed risk aversion information and the supplier's risk constraint. Therefore, the information that the retailer intentionally discloses can influence the actual option quantity.

To gain further insights into the supplier's action, we use Table 5 to illustrate how the supplier sets minimum option quantity contracts. The scenario of $K_R = K_R'$ represents the case where the retailer provides the true information of the risk aversion threshold. Compared to case (1a), case (1b) presents the case where the retailer pretends to be less risk averse ($K_R' = 1500 > K_R$ =1000). Then, the supplier sets a smaller exercise price $e^*(K_R')$ =84.50 than $e^*(K_R)$ =86.50 based on the true information. This is consistent with $e_0 < e_1(K_R') < e_1(K_R)$ in Proposition 6. The supplier's estimated option quantity is 84.35. However, based on $e^*(K_R')$ =84.50 and the true K_R =1000, the retailer's actual option quantity is 65.58. This benefits the retailer but hurts the supplier. To prevent this from happening, the supplier sets the minimum option quantity $q_{min} = q_{R,SP}(e_1(K_R'), K_R')$ =84.35. Compared to case (2a), case (2b) is consistent with $e_1(K_R') < e_0 < e_1(K_R)$ in Proposition 6. Then, the supplier sets the minimum option quantity $q_{min} = q_{R,EP}(o, e_0)$ =107.2. Compared to case (3a), case (3b) is consistent with case $e_1(K_R') < e_1(K_R) < e_0$. The supplier's optimal exercise price e_0 is independent of the retailer's risk aversion threshold. The faked information K_R' therefore has no impact on the supplier's decision. The supplier's estimated option quantity equals the retailer's actual option quantity, i.e., $q_{R,MV}(o, e^*(K_R)) = q_{R,MV}(o, e^*(K_R')) = q_{R,EP}(o, e_0)$ =107.2. Similarly, in cases (4) and (5), the faked information K_R' has no impact on the supplier's decision.

Table 5. The optimal e^* and $q_{R,MV}$ for different K_S and K_R when the retailer's risk-aversion threshold is private.

	K_S	K_R	K_R'	$e_a(K_R')(K_R)$	$e^*(K_R')(K_R)$	$q_{R,MV}(K_R')(K_R)$	q_{min}	EP_s	EP_R	SP_S	SP_R
(1a)	2500	200	200	86.50(86.50)	86.50(86.50)	71.2(71.2)	71.2	5218.6	158.4	1275.1	200.0
(1b)	2500	200	300	84.50(86.50)	84.50(86.50)	84.35(65.58)	84.35	4759.9	277.5	1090.2	200.0
(2a)	2500	500	500	80.50(80.50)	80.50(80.50)	101.2(101.2)	101.2	6207.3	613.8	2049.7	500.0
(2b)	2500	500	1000	70.00(80.50)	78.50(80.50)	107.2(95.2)	107.2	5815.9	771.3	1827.3	500.0
(3a)	2500	1000	1000	70.00(70.00)	78.50(78.50)	107.2(107.2)	107.2	6239.0	783.8	2165.5	593.4
(3b)	2500	1000	1500	60.00(70.00)	78.50(78.50)	107.2(107.2)	107.2	6239.0	783.8	2165.5	593.4
(4a)	1000	200	200	87.00(87.00)	88.14(88.14.)	60.58(60.58)	60.58	4652.2	61.6	1000.0	134.6
(4b)	1000	200	300	87.00(87.00)	88.14(88.14.)	60.58(60.58)	60.58	4652.2	61.6	1000.0	134.6
(5a)	1000	500	500	87.00(87.00)	88.14(88.14.)	60.58(60.58)	60.58	4652.2	61.6	1000.0	134.6
(5b)	1000	500	500	87.00(87.00)	88.14(88.14.)	60.58(60.58)	60.58	4652.2	61.6	1000.0	134.6

Note: The symbols "a" and "b" represent the retailer's true and faked information disclosure, respectively.

5. Conclusion and management insights

In this paper, we analyze supply chain coordination and option contract design under the mean-variance model. In this model, each party aims to maximize their expected profits subject to constraints on the risk. Our main results are fourfold.

First, the supply chain is not always coordinated under option contracts with risk constraints. By leveraging the exercise price, the supplier achieves the channel coordination only when the retailer's risk aversion threshold falls within certain intervals. Such a result is distinct from existing research on option contracts without risk constraints, in which the coordination of a supply chain can always be achieved. The option contract with risk constraints may help the supplier balance the tradeoff between the expected profit and risk. In particular, setting the option and exercise prices that maximize the supply chain's expected profit may benefit the supplier.

Second, the adjustment of the option price and exercise price reallocates the proportion of the expected profit and risk sharing between the supplier and retailer. As the option price increases, the supplier's expected profit increases, whereas the retailer's expected profit decreases. As the exercise price increases, the supplier's expected profit and risk increase, whereas the retailer's expected profit and risk decrease. This suggests that a supplier with a higher risk tolerance always prefers to reduce the exercise price and that a retailer with a higher risk tolerance prefers to increase option quantity. This finding is consistent with classical investment theories, in which a higher profit always accompanies a higher risk.

Third, there exists a unique equilibrium for the Stackelberg game in a decentralized supply chain. Both the equilibrium exercise price and the option quantity depend on the risk aversion thresholds and the solutions for the risk-neutral case. When the supplier and retailer both have high risk tolerance, which corresponds to large risk thresholds, they will adopt relatively positive operational strategies.

Finally, in the scenario where the retailer's risk aversion threshold is private, we find that the retailer has an incentive to pretend to be less risk averse. We also find that by constructing a minimum option quantity contract, the supplier is able to prevent the retailer from intentionally disclosing faked information. When the retailer's risk constraint is active, a minimum option quantity commitment proposed by the supplier places a higher risk on the retailer than his true risk tolerance. The retailer is therefore motivated to disclose his true information. When the retailer's risk constraint is inactive, the retailer's faked information has no impact on the supplier's decision, and thus, the supplier sets a minimum option quantity based solely on her risk constraint.

Our work points to two important managerial insights. First, all decisions involved in option contracts are dependent on the risk tolerance. A supplier with a higher risk tolerance always prefers to reduce the exercise price to induce the retailer to order more. A retailer with a higher risk tolerance is willing to increase option quantity to gain more expected profit. Such actions lead to a higher supply chain risk. Second, the adjustment of the option price and exercise price causes the reallocation of the expected profit and risk sharing between the supplier and retailer. By setting prices, the supplier is able to determine who can enjoy more profit or take less risk in the supply chain under an option contract.

Finally, we discuss several potential extensions arising from this research. First, the current paper considers a deterministic selling price. It is a potentially meaningful research direction to consider the selling price as an endogenous decision variable (Choi and Chiu, 2012). Second, both the supplier and the retailer in our model are endowed with unlimited capital. However, in reality, supply chain parties may be financially constrained. Thus, another possible extension is to consider the players' financial constraints in the presence of bankruptcy risk. Third, we model risk within the commonly used mean-variance framework. It would be interesting to explore other modeling frameworks. A notable example is the CVaR framework, which focuses on the decision maker's expected profit from the lower quantile (Lotfi and Zenios, 2018). By considering different risk modeling frameworks, one may draw comparisons with our mean-variance framework. We leave these extensions as future research.

Acknowledgments

This research is supported by the National Natural Science Foundation of China under Grant Nos. 71571065, 71790593 and 71521061; the Program for New Century Excellent Talents in University under Grant No. NCET-13-0193; and the Ministry of Education in China of Humanities and Social Science Project under Grant No. 14YJA630077.

Appendixes

Appendix A (Proof of Proposition 1)

(i) Taking the first-order condition of SP_{SC} with respect to q yields $\frac{\partial SP_{SC}}{\partial q} = \frac{P}{\sqrt{\xi(q)}}\bar{F}(q)\int_0^q F(x)dx$. Obviously, $\frac{\partial SP_{SC}}{\partial q} > 0$. Furthermore, we have that SP_{SC} is increasing in q.

(ii) Taking the first-order condition of EP_{SC} with respect to q yields $\frac{\partial EP_{SC}}{\partial q} = (p-c) - pF(q)$. Since $\frac{\partial^2 EP_{SC}}{\partial q^2} = -pf(q) < 0$, EP_{SC} is concave. Let $\frac{\partial EP_{SC}}{\partial q} = 0$. We have $q_{SC,EP} = F^{-1}(\frac{p-c}{p})$. Since $\frac{\partial \xi(q)}{\partial q} = 2\big(1-F(q)\big)\int_0^q F(x)dx > 0$, $\xi(q)$ is increasing in q. Let $q_{SC,SP} = arg\,max_q\{SP_{SC}(q) \le K_{SC}\}$. When $SP_{SC}(q_{SC,EP}) > K_{SC}$, then $q_{SC,SP} < q_{SC,EP}$. Since EP_{SC} is increasing in q in the interval $(0, q_{SC,EP})$, the optimal option quantity that satisfies (P1) $q_{SC,MV} = q_{SC,SP} < q_{SC,EP}$. When $SP_{SC}(q_{SC,EP}) \le K_{SC}$, then $q_{SC,EP} \le q_{SC,SP}$. The optimal option quantity that satisfies (P1) $q_{SC,MV} = q_{SC,EP} \le q_{SC,SP}$. Therefore, $q_{SC,MV} = min\{q_{SC,EP}, q_{SC,SP}\}$. Since $q_{SC,SP} = argmax_q\{SP_{SC}(q) \le K_{SC}\}$ is increasing in K_{SC} and $q_{SC,EP}$ is independent of K_{SC}, we have that $q_{SC,MV}$ is non-decreasing in K_{SC}.

Appendix B (Proof of Proposition 2)

Taking the first-order condition of EP_R with respect to q yields $\frac{\partial EP_R}{\partial q} = (p-e-o) - (p-e)F(q)$. Since $\frac{\partial^2 EP_R}{\partial q^2} = -(p-e)f(q) < 0$, EP_R is concave.

Let $\frac{\partial EP_R}{\partial q} = 0$. We have $q_{R,EP}(o,e) = F^{-1}(\frac{p-e-o}{p-e})$. Taking the first-order condition of $q_{R,EP}(o,e)$ with respect to e yields $\frac{\partial q_{R,EP}(o,e)}{\partial e} = -\frac{o}{(p-e)^2 f\left(q_{R,EP}(o,e)\right)} < 0$. Therefore, $q_{R,EP}(o,e)$ is decreasing in e.

Since $q_{R,SP}(e) = \underset{q}{argmax}\{ SP_R(q) \le K_R\}$ and $\xi(q)$ is increasing in q, we have $(p-e)\sqrt{\xi(q_{R,SP}(e))} = K_R$ given any risk-aversion threshold K_R. Furthermore, $\frac{\partial q_{R,SP}(e)}{\partial e} = \frac{\xi(q_{R,SP}(e))}{(p-e)\bar{F}\left(q_{R,SP}(e)\right)\int_0^{q_{R,SP}(e)} F(x)dx} > 0$. Therefore, $q_{R,SP}(e)$ is increasing in e.

Let $e_1 = p - \frac{o}{\bar{F}(q_{R,MV}(o,e))}$ be a unique root of $q_{R,EP}(o,e) = q_{R,SP}(e)$. When $e < e_1$, then $q_{R,SP}(e) < q_{R,EP}(o,e)$ and $SP_R(q_{R,EP}(o,e)) > K_R$. The optimal option quantity that satisfies (P1) $q_{R,MV}(o,e) = q_{R,SP}(e) < q_{R,EP}(o,e)$. Hence, $q_{R,MV}(o,e)$ is increasing in e. When $e \ge e_1$, then $q_{R,SP}(e) \ge q_{R,EP}(o,e)$ and $SP_R(q_{R,EP}(o,e)) \le K_R$. The optimal option quantity that satisfies (P1) $q_{R,MV}(o,e) = q_{R,EP}(o,e) \le q_{R,SP}(e)$. Furthermore, $q_{R,MV}(o,e)$ is decreasing in e. Therefore, we have $q_{R,MV}(o,e) = min\{q_{R,EP}(o,e), q_{R,SP}(e)\}$.

Appendix C (Proof of Proposition 3)

Taking o and e as variables and solving the equation $q_{R,EP}(o,e) = q_{SC,MV}$, we obtain the following option contract set, denoted as M, where $M = \{(o,e): o = (p-e)\bar{F}\left(q_{SC,MV}\right)\}$. If $SP_R(q_{R,EP}((o,e)_M)) \le K_R$, then $q_{R,MV}((o,e)_M) = q_{R,EP}((o,e)_M) = q_{SC,MV}$. The supply chain is coordinated under any option contract (o,e) in the set M. Let $q_{R,SP}(e) = q_{SC,MV}$. We have $e = p - \frac{K_R}{\sqrt{\xi(q_{SC,MV})}}$. Taking o and e as variables and solving the equation $q_{R,SP}(e) = q_{SC,MV}$, we obtain the following option contract set, denoted as N, where $N = \{(o,e): e = p - \frac{K_R}{\sqrt{\xi(q_{SC,MV})}}\}$. If $SP_R(q_{R,EP}((o,e)_N)) \ge K_R$, then $q_{R,MV}((o,e)_N) = q_{R,SP}((o,e)_N) = q_{SC,MV}$. The supply chain is coordinated under any option contract (o,e) in the set N. If $SP_R(q_{R,EP}((o,e)_N)) < K_R < SP_R(q_{R,EP}((o,e)_M))$, then $q_{R,EP}((o,e)_M) = q_{SC,MV}$ and $q_{R,SP}((o,e)_N) = q_{SC,MV}$. Since $q_{R,SP}((o,e)_M) < q_{R,EP}((o,e)_M)$ and

$q_{R,EP}((o,e)_N) < q_{R,SP}((o,e)_N)$, we have $q_{R,MV}((o,e)_M) = q_{R,SP}((o,e)_M) < q_{SC,MV}$ and $q_{R,MV}((o,e)_N) = q_{R,EP}((o,e)_N) < q_{SC,MV}$. Therefore, the supply chain cannot be coordinated.

Appendix D (Proof of Proposition 4)

From the expressions of the EPs and SPs of the supplier and retailer, Proposition 4 can be easily obtained. The details are therefore omitted.

Appendix E (Proof of Lemma 1)

From Proposition 2(i), the retailer's optimal order quantity is $q = F^{-1}\left(\frac{p-e-o}{p-e}\right)$. For any given o, $e = p - \frac{o}{\bar{F}(q)}$. Thus, $EP_S = (o+e-c)q - e\int_0^q F(x)dx = \left(p - c - \frac{oF(q)}{\bar{F}(q)}\right)q - \left(p - \frac{o}{\bar{F}(q)}\right)\int_0^q F(x)dx$. The supplier's problem of choosing e is therefore equivalent to choosing q. Taking the first-order condition of EP_S with respect to q yields $\frac{\partial EP_S}{\partial q} = p\bar{F}(q) - c - \frac{of(q)}{\bar{F}(q)^2}\int_0^q \bar{F}(x)dx = p\bar{F}(q) - c - \frac{oh(q)}{q\bar{F}(q)}\int_0^q \bar{F}(x)dx$. Taking the second-order derivative of EP_S with respect to q yields $\frac{\partial^2 EP_S}{\partial q^2} = -pf(q) - \frac{o}{q^2\bar{F}(q)}\{q\frac{\partial h(q)}{\partial q}\int_0^q \bar{F}(x)dx + h(q)(q\bar{F}(q) + (h(q) - 1)\int_0^q \bar{F}(x)dx)\}$. Let $g(q) = q\bar{F}(q) + (h(q)-1)\int_0^q \bar{F}(x)dx$. Then, taking the first-derivative of $g(q)$ with respect to q yields $\frac{\partial g(q)}{\partial q} = \bar{F}(q) - qf(q) + \frac{\partial h(q)}{\partial q}\int_0^q \bar{F}(x)dx + (h(q)-1)\bar{F}(q) = \frac{\partial h(q)}{\partial q}\int_0^q \bar{F}(x)dx$. Since $h(q)$ is increasing in q, $\frac{\partial h(q)}{\partial q} > 0$. Furthermore, $\frac{\partial g(q)}{\partial q} > 0$. Thus, $g(q) > g(0) = 0$. Therefore, $\frac{\partial^2 EP_S}{\partial q^2} < 0$ and EP_S is concave. Based on $\frac{\partial EP_S}{\partial q} = 0$, we have $p\bar{F}(q_0) - c - \frac{of(q_0)}{\bar{F}(q_0)^2}\int_0^{q_0} \bar{F}(x)\,dx = 0$.

Appendix F (Proof of Proposition 5)

We first prove that the supplier's expected profit is concave in e when both the supplier and the retailer are risk-neutral. Then, we prove Proposition 5.

Taking the first-order condition of EP_S with respect to e yields $\frac{\partial EP_S}{\partial e} = \left(o - c + e\bar{F}\left(q_{R,EP}(o,e)\right)\right)\frac{\partial q_{R,EP}(o,e)}{\partial e} + \int_0^{q_{R,EP}(o,e)} \bar{F}(x)dx = \left(p\bar{F}\left(q_{R,EP}(o,e)\right) - c\right)$ $\frac{\partial q_{R,EP}(o,e)}{\partial e} + \int_0^{q_{R,EP}(o,e)} \bar{F}(x)dx$. Since $\bar{F}\left(q_{R,EP}(o,e)\right) = \frac{o}{p-e}$, we have $\frac{\partial q_{R,EP}(o,e)}{\partial e} = -\frac{\bar{F}\left(q_{R,EP}(o,e)\right)^2}{of\left(q_{R,EP}(o,e)\right)}$. Substituting $\frac{\partial q_{R,EP}(o,e)}{\partial e}$ into $\frac{\partial EP_S}{\partial e}$ yields $\frac{\partial EP_S}{\partial e} = -\frac{\bar{F}\left(q_{R,EP}(o,e)\right)^2}{of\left(q_{R,EP}(o,e)\right)}(p\bar{F}\left(\bar{F}\left(q_{R,EP}(o,e)\right)\right) - c - \frac{of\left(q_{R,EP}(o,e)\right)}{\bar{F}\left(q_{R,EP}(o,e)\right)^2}\int_0^q \bar{F}(x)dx)$. From the proof of Lemma 1, $EP_S\left(q_{R,EP}(o,e)\right)$ is increasing in the interval $(0, q_0]$ and decreasing in the interval $[q_0, +\infty)$. Hence, if $q_{R,EP}(o,e) < q_0$, then $p\bar{F}\left(q_{R,EP}(o,e)\right) - c - \frac{of\left(q_{R,EP}(o,e)\right)}{\bar{F}\left(q_{R,EP}(o,e)\right)^2}\int_0^{q_{R,EP}(o,e)} \bar{F}(x)dx) > 0$. If $q_{R,EP}(o,e) \geq q_0$, then $p\bar{F}\left(q_{R,EP}(o,e)\right) - c - \frac{of\left(q_{R,EP}(o,e)\right)}{\bar{F}\left(q_{R,EP}(o,e)\right)^2}\int_0^{q_{R,EP}(o,e)} \bar{F}(x)\,dx < 0$. Since $\frac{\partial q_{R,EP}(o,e)}{\partial e} < 0$, $\frac{\partial EP_S}{\partial e} \geq 0$ if $e \leq e_0$ and $\frac{\partial EP_S}{\partial e} < 0$ if $e > e_0$. Therefore, the supplier's expected profit without risk constraints is concave in e.

(i) If $K_S \geq max_e\{SP_S(q_{R,MV}(o,e))\}$, the supplier's risk constraint is inactive. If $e \leq e_1$, then $q_{R,MV}(o,e) = q_{R,SP}(e) \leq q_{R,EP}(o,e)$ based on Proposition 2(i). Hence, $\frac{\partial EP_S}{\partial e} = \int_0^{q_{R,SP}(e)} \bar{F}(x)dx + \left(o - c + e\bar{F}\left(q_{R,SP}(e)\right)\right)\frac{\partial q_{R,SP}(e)}{\partial e}$. Since $q_{R,SP}(e) = \underset{q}{argmax}\{SP_R(q,e) \leq K_R\}$ and $SP_R(q,e)$ is increasing in $q_{R,SP}(e)$, we have $(p-e)\bar{F}\left(q_{R,SP}(e)\right)\int_0^{q_{R,SP}(e)} F(x)dx\frac{q_{R,SP}(e)}{\partial e} = \xi\left(q_{R,SP}(e)\right) = 2q_{R,SP}(e)$ $\int_0^{q_{R,SP}(e)} F(x)dx - 2\int_0^{q_{R,SP}(e)} xF(x)dx - \left(\int_0^{q_{R,SP}(e)} F(x)dx\right)^2 = q_{R,SP}(e)$ $\int_0^{q_{R,SP}(e)} F(x)dx + \int_0^{\frac{q_{R,SP}(e)}{2}}\left(q_{R,SP}(e) - 2x\right)F(x)dx + \int_{\frac{q_{R,SP}(e)}{2}}^{q_{R,SP}(e)}\left(q_{R,SP}(e) - 2x\right)F(x)\,dx - \left(\int_0^{q_{R,SP}(e)} F(x)dx\right)^2 < q_{R,SP}(e)\int_0^{q_{R,SP}(e)} F(x)dx + F\left(\frac{q_{R,SP}(e)}{2}\right)$ $(\int_0^{\frac{q_{R,SP}(e)}{2}}\left(q_{R,SP}(e) - 2x\right)dx + \int_{\frac{q_{R,SP}(e)}{2}}^{q_{R,SP}(e)}(q_{R,SP}(e) - 2x)dx) -$ $\left(\int_0^{q_{R,SP}(e)} F(x)dx\right)^2 = q_{R,SP}(e)\int_0^{q_{R,SP}(e)} F(x)dx - \left(\int_0^{q_{R,SP}(e)} F(x)dx\right)^2$.

Furthermore, $(p-e)\bar{F}\left(q_{q_{R,SP}(e)}\right)\frac{\partial q_{R,SP}(e)}{\partial e} \leq \int_0^{q_{R,SP}(e)} \bar{F}(x)dx$. Hence, we have

$\frac{\partial EP_S}{\partial e} \geq \frac{\partial q_{R,SP}(e)}{\partial e}(p\bar{F}\left(q_{R,SP}(e)\right) + o - c)$. Let e_s satisfy the equation $q_{R,SP}(e_s) = F^{-1}(\frac{p+o-c}{p})$. If $e_1 < e_s$, then $q_{R,SP}(e) \leq q_{R,SP}(e_1) < q_{R,SP}(e_s)$. Furthermore, $\bar{F}(q_{R,SP}(e)) + o - c > p\bar{F}\left(q_{R,SP}(e_s)\right) + o - c = 0$. Therefore, $\frac{\partial EP_S}{\partial e} > 0$. If $e_1 \geq e_s$, then $\bar{F}(q_0) > \frac{c}{p} > \frac{c-o}{p} = \bar{F}(q_{R,SP}(e_S))$ based on Lemma 1. Furthermore, $q_0 < q_{R,SP}(e_S) \leq q_{R,SP}(e_1) = q_{R,EP}(o, e_1)$. Therefore, $e \leq e_1 < e_0$. Since $q_{R,SP}(e)$ is increasing in e, EP_S is increasing in e. Furthermore, EP_S is increases with e in the interval $(c - o, e_1)$. Therefore, we have $e^* = e_1$. If $e > e_1$, then $q_{R,MV}(o,e) = q_{R,EP}(o,e)$ based on Proposition 2(ii). In this case, the retailer's risk constraint is inactive. Therefore, if $e_0 \geq \mathrm{e}_1$, then $e^* = e_0$. If $e_0 < \mathrm{e}_1$, then EP_S is decreasing in e in the interval $(e_1, p - o)$, and we have $e^* = e_1$. Furthermore, $e^* = max\{e_1,\ e_0\}$.

If $K_S < max_e\{SP_S(q_{R,MV}(o,e))\}$, then there exists $q_{S,SP}(e)$ to satisfy $q_{S,SP}(e) < q_{R,MV}(o,e)$. Since $q_{S,SP}(e) = arg\,max_q\{SP_S(q,e) \leq K_S\}$ and $\xi(q)$ is increasing in q, we have $e\sqrt{\xi(q_{S,SP}(e))} = K_S$ given any risk-aversion threshold K_S. Furthermore, $\frac{\partial q_{S,SP}(e)}{\partial e} = \frac{-\xi(q_{S,SP}(e))}{e\bar{F}\left(q_{R,SP}(e)\right)\int_0^{q_{S,SP}(e)} F(x)dx} < 0$. Therefore, $q_{S,SP}(e)$ is decreasing in e. Furthermore, the equation $q_{R,MV}(o,e) = q_{S,SP}(e)$ has two solutions. Define by e_t the corresponding exercise price such that $q_{R,MV}(o,e_t) = q_{S,SP}(e_t)$, where $t = l, u$.

If $max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\} \leq K_S < \max_e\{SP_S\left(q_{R,MV}(o,e)\right)\}$, then $max\{e_0,\ e_1\} \leq e_l$ or $max\{e_0,\ e_1\} > e_u$. The supplier maximizes the expected profit at point $e = e_0$ or $e = e_1$. Therefore, if $e_0 \geq e_1$, then $e^* = e_0$. If $e_0 < e_1$, then EP_S is increasing in e in the interval $(c - o, e_1]$ and is decreasing in e in the interval $(e_1, p - o)$. Hence, $e^* = e_1$. Furthermore, we have $e^* = max\{e_1,\ e_0\}$.

(ii) If $K_S < max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$, then $e_l \leq max\{e_0,\ e_1\} \leq e_u$. When $e_l < e < e_u$, then $q_{S,SP}(e) < q_{R,MV}(o,e)$. Therefore, the supplier chooses $e \in (c - o, e_l]U[e_u, p - o)$. From the proof of (i), $EP_S\left(q_{R,MV}(o,e)\right)$ is increasing in e in the interval $(c, e_l]$ and decreasing in e in the interval $[e_u, p-o)$. Therefore, we have $e^* = argmax\{EP_S\left(q_{R,MV}(o, e_l)\right), EP_S\left(q_{R,MV}(o, e_u)\right)\}$.

Appendix G (Proof of Lemma 2)

If $K_R' > K_R$, then $q_{R,SP}(e_1, K_R) < q_{R,SP}(e_1, K_R')$. Furthermore, $e_1(K_R') < e_1(K_R)$.

(i) If $K_S \geq max\{SP_S\left(q_{R,MV}(o, e_0)\right), SP_S(q_{R,MV}(o, e_1))\}$, then $e^* = max\{e_0, e_1\}$ and the supplier's risk constraint is inactive. There exist the following three subcases.

Subcase 1: if $e_0 < e_1(K_R') < e_1(K_R)$, then the estimated exercise price is $e^*(K_R') = e_1(K_R')$, while the optimal exercise price based on the true information is $e^*(K_R) = e_1(K_R)$. Thus, $e^*(K_R') < e^*(K_R)$.

Subcase 2: if $e_1(K_R') < e_0 < e_1(K_R)$, then the estimated exercise price is $e^*(K_R') = e_0$, while the optimal exercise price based on the true information is $e^*(K_R) = e_1(K_R)$. Thus, $e^*(K_R') < e^*(K_R)$.

Subcase 3: if $e_1(K_R') < e_1(K_R) < e_0$, then the estimated exercise price is equal to the optimal exercise price based on the true information, i.e., $e^*(K_R') = e^*(K_R) = e_0$.

(ii) If $K_S < max\{SP_S\left(q_{R,MV}(o, e_0)\right), SP_S(q_{R,MV}(o, e_1))\}$, then $e^* = argmax\{EP_S\left(q_{R,MV}(o, e_l)\right), EP_S\left(q_{R,MV}(o, e_u)\right)\}$. Since $K_R' > K_R$, $q_{R,SP}(e, K_R) < q_{R,SP}(e, K_R')$. Furthermore, $q_{R,MV}(e, K_R) < q_{R,MV}(e, K_R')$. From Proposition 5(i), we know that $q_{S,SP}(e)$ is decreasing in e. Since $q_{R,MV}(e, K_R) < q_{R,MV}(e, K_R')$, $e_t(K_R') \leq e_t(K_R)$, where $t = l, u$. From Proposition 2, $q_{R,SP}(e)$ is increasing in e while $q_{R,EP}(o, e)$ is decreasing in e. Therefore, the equation $q_{R,SP}(e) = q_{S,SP}(e)$ has at most one solution, $e = e_l$. The equation $q_{R,EP}(o, e) = q_{S,SP}(e)$ has at least one solution, $e = e_u$. There exist the following two subcases.

Subcase 1: if $e^* = e_l$, then the estimated exercise price is $e^*(K_R') = e_l(K_R')$, while the optimal exercise price based on the true information is $e^*(K_R) = e_l(K_R)$. Therefore, $e^*(K_R') < e^*(K_R)$.

Subcase 2: if $e^* = e_u$, then the estimated exercise price is equal to the optimal exercise price based on the true information, i.e., $e^*(K_R') = e^*(K_R) = e_u$.

To summarize the proofs of (i) and (ii), we have $e^*(K_R') \leq e^*(K_R)$.

Taking the first derivative of EP_R with respect to e yields $\frac{\partial EP_R}{\partial e} = \left((p - e)\bar{F}\left(q_{R,MV}(o,e)\right) - o\right)\frac{\partial q_{R,MV}(o,e)}{\partial e} - \int_0^{q_{R,MV}(o,e)} \bar{F}(x)dx$. When $q_{R,MV}(o,e) = q_{R,SP}(e)$, then $(p-e)\bar{F}\left(q_{R,SP}(e)\right)\frac{\partial q_{R,SP}(e)}{\partial e} \leq \int_0^{q_{R,SP}(e)} \bar{F}(x)dx$ based on the proof of Proposition 5(i). Furthermore, $\frac{\partial EP_R}{\partial e} < 0$. When $q_{R,MV}(o,e) = q_{R,EP}(o,e)$, then $\bar{F}\left(q_{R,EP}(o,e)\right) = \frac{o}{p-e}$. Furthermore, $\frac{\partial EP_R}{\partial e} = -\int_0^{q_{R,MV}(o,e)} \bar{F}(x)dx < 0$. Therefore, EP_R is decreasing in e. Since $e^*(K_R') \leq e^*(K_R)$, $EP_R(K_R') \geq EP_R(K_R)$. The retailer has an incentive to pretend to be less risk averse.

Appendix H (Proof of Proposition 6)

(i) When $K_S \geq max\{SP_S\left(q_{R,MV}(o,e_0)\right), SP_S(q_{R,MV}(o,e_1))\}$, then $e^* = max\{e_0,\ e_1\}$ and the supplier's risk constraint is inactive. If $K_R' > K_R$, then $q_{R,SP}(e_1,K_R) < q_{R,SP}(e_1,K_R')$. Furthermore, $e_1(K_R') < e_1(K_R)$. There exist the following three subcases.

Subcase 1: if $e_0 < e_1(K_R') < e_1(K_R)$, then $e^*(K_R') = e_1(K_R') < e^*(K_R) = e_1(K_R)$. Therefore, the supplier's estimated option quantity is $q_{R,MV}(o, e^*(K_R')) = q_{R,SP}(e_1(K_R'), K_R')$. Based on $e^*(K_R')$ and the true K_R, the retailer orders the option quantity of $q_{R,SP}(e_1(K_R'), K_R)$. Since $K_R' > K_R$, $q_{R,SP}(e_1(K_R'), K_R) < q_{R,SP}(e_1(K_R'), K_R')$. Hence, $q_{min} = q_{R,SP}(e_1(K_R'), K_R')$.

Subcase 2: if $e_1(K_R') < e_0 < e_1(K_R)$, then $e^*(K_R') = e_0 < e^*(K_R) = e_1(K_R)$. Therefore, the supplier's estimated option quantity is $q_{R,MV}(o, e^*(K_R')) = q_{R,EP}(o, e_0)$. Based on $e^*(K_R')$ and the true K_R, the retailer orders the actual option quantity of $q_{R,SP}(e_0, K_R)$. From the proof of Proposition 2, $q_{R,SP}(e)$ is increasing in e, while $q_{R,EP}(o,e)$ is decreasing in e. Furthermore, $q_{R,SP}(e_0, K_R) <$

$q_{R,SP}(e_1(K_R), K_R) = q_{R,EP}\left(o, e_1(K_R)\right) < q_{R,EP}(o, e_0)$. Hence, $q_{min} = q_{R,MV}(o, e^*(K_R')) = q_{R,EP}(o, e_0)$.

Subcase 3: if $e_1(K_R') < e_1(K_R) < e_0$, then $e^*(K_R') = e(K_R) = e_0$. Therefore, the supplier's estimated option quantity equals the retailer's actual option quantity, i.e., $q_{R,MV}(o, e^*(K_R)) = q_{R,MV}(o, e^*(K_R')) = q_{R,EP}(o, e_0)$. Furthermore, the supplier's optimal exercise price e_0 is independent of the retailer's risk aversion threshold. The faked information K_R' has no impact on the supplier's decision. Hence, $q_{min} = q_{R,EP}(o, e_0)$.

(ii) When $K_S < max\{SP_S\left(q_{R,MV}(o, e_0)\right), SP_S(q_{R,MV}(o, e_1))\}$, then $e^* = argmax\{EP_S\left(q_{R,MV}(o, e_l)\right), EP_S\left(q_{R,MV}(o, e_u)\right)\}$. If $K_R' > K_R$, then $e_l(K_R') < e_l(K_R)$ and $e_u(K_R') = e_u(K_R) = e_u$ based on the proof of Lemma 2(ii). There exist the following two subcases.

Subcase 1: if $e^* = e_l$, then $e^*(K_R') = e_l(K_R') < e^*(K_R) = e_l(K_R)$. Therefore, the supplier's estimated option quantity is $q_{R,MV}(o, e^*(K_R')) = q_{R,SP}(e_l(K_R'), K_R')$. Based on $e^*(K_R')$ and the true K_R, the retailer orders the actual option quantity of $q_{R,SP}(e_l(K_R'), K_R)$. Obviously, $q_{R,SP}(e_l(K_R'), K_R) < q_{R,SP}(e_l(K_R'), K_R')$. Hence, $q_{min} = q_{R,SP}(e_l(K_R'), K_R')$.

Subcase 2: if $e^* = e_u$, then $e^*(K_R') = e^*(K_R) = e_u$. Therefore, the supplier's estimated option quantity equals the retailer's actual option quantity, i.e., $q_{R,MV}(o, e^*(K_R')) = q_{R,MV}(o, e^*(K_R)) = q_{R,EP}(o, e_u)$. Therefore, the supplier's optimal exercise price e_u is independent of the retailer's risk aversion threshold. The faked information K_R' has no impact on the supplier's decision. Hence, $q_{min} = q_{R,EP}(o, e_u)$.

References

Anderson, E.J., Chen, B., Shao, L.S., (2017). Supplier competition with option contracts for discrete blocks of capacity. Operations Research, 65 (4), 952-956.

Chen, X. F. (2015) A model of trade credit in a capital-constrained distribution channel. *International Journal of Production Economics*, 159, 347-357.

Chiu, C. H., Choi, T. M., Li, X. (2011) Supply chain coordination with risk sensitive agent under target sales rebate contracts. *Automatica*, 47(8), 1617–1625.

Chiu, C. H., Choi, T. M. (2016) Supply chain risk analysis with mean-variance models: a technical review. *Annals of Operations Research*, 240,489-507.

Choi, T. M., Chiu, C. H. (2012) Mean-downside-risk and mean-variance newsvendor models: implications for sustainable fashion retailing. *International Journal of Production Economics*, 135(2), 552–560.

Choi, T. M., Li, D., Yan, H. (2008a) Mean–variance analysis for the newsvendor problem. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, 38(5), 1169–1180.

Choi, T. M., Li, D., Yan, H., Chiu, C. H. (2008b) Channel coordination in supply chains with agents having mean-variance objectives. *Omega*, 36(4), 565–576.

Choi, T. M., Li, D., Yan, H. (2008c) Mean–variance analysis of a single supplier and retailer supply chain under a returns policy. *European Journal of Operational Research*, 184(1), 356-376.

Fan, H., Li, G., Sun, H., Cheng, T. C. E. (2017) An information processing perspective on supply chain risk management: antecedents, mechanism, and consequences. *International Journal of Production Economics*, 185, 63-75.

Feng, Y., Mu, Y., Hu, B., Kumar, A. (2014) Commodity options purchasing and credit financing under capital constraint. *International Journal of Production Economics*, 153(7), 230-237.

Gan, X., Sethi, S. P., Yan, H. (2004). Coordination of supply chains with risk- averse agents. *Production and Operations Management*, 13(2), 135-149.

Gan, X., Sethi, S. P., Yan, H. (2005). Channel coordination with a risk-neutral supplier and a downside-risk-averse retailer. *Production and Operations Management*, 14(1), 80-89.

Gomez_Padilla, A., Mishina, T. (2009) Supply contract with options. *International Journal of Production Economics*, 122(1), 312-318.

Kouvelis, P., Zhao, W. H. (2012) Financing the Newsvendor: Supplier vs. Bank, and the structure of Optimal Trade Credit Contracts. *Operations Research*, 60(3), 566-580.

Kouvelis, P., Zhao, W. H. (2016) Supply chain contract design under financial constraints and bankruptcy costs. *Management Science*, 62(8), 2341-2357.

Lariviere, M. A., Porteus, E. L. (2001) Selling to the newsvendor: an analysis of price-only contracts. *Manufacturing and Service Operations Management*, 3(4), 293-305.

Lee, C. H., Rhee, B. D. (2007) Channel coordination using product returns for a supply chain with stochastic salvage capacity. *European Journal of Operational Research*, 177(1), 214-238.

Liu, M., Cao, E., Salifou, C. K. (2016) Pricing strategies of a dual-channel supply chain with risk aversion. *Transportation Research Part E: Logistics and Transportation Review*, 90, 108-120.

Liu, Z., Chen, L., Li, L. (2014) Risk hedging in a supply chain: Option vs. price discount. *International Journal of Production Economics*, 151(7), 112-120.

Liu, Z., Nagurney, A. (2011) Supply chain outsourcing under exchange rate risk and competition. *Omega*, 39(5), 539–549.

Lotfi, S., Zenios, S. A. (2018) Robust VaR and CVaR optimization under joint ambiguity in distributions, means, and covariances. *European Journal of Operational Research*, in press.

Markowitz, H. M. (1959) Portfolio Selection: Efficient Diversification of Investment. *John Wiley & Sons, New York.*

Nagali, V., Hwang J., Sanghera D., Gaskins, M., Pridgen, M., Thurston, T., Mackenroth, P., Branvold, D., Scholler, P., Shoemaker, P. (2008) Procurement Risk Management (PRM) at Hewlett-Packard Company. *Interfaces*, 38(1), 51-60.

Norrman, A., Jansson, U. (2004) Ericsson's proactive supply chain risk management approach after a serious sub-supplier accident. *International Journal of Physical Distribution & Logistics Management*, 34(5), 434-456.

Nosoohi, I., Nookabadi, A. S. (2016) Outsource planning through option contracts with demand and cost uncertainty. *European Journal of Operational Research*, 250(1), 131-142.

Pasternack, B. A. (1985) Optimal pricing and return policies for perishable commodities. *Marketing Science*, 4(2), 166-176.

Paul, S.K., Sarker, R., Essam, D. (2016) A quantitative model for disruption mitigation in a supply chain. *European Journal of Operational Research*, 257(3), 881-895.

Qi, X., Bard, J. F., Yu, G. (2004) Supply chain coordination with demand disruptions. *Omega*, 32(4), 301-312.

Ray, P., Jenamani, M. (2016) Mean-variance analysis of sourcing decision under disruption risk. *European Journal of Operational Research*, 250(2), 679-689.

Sawik, T. (2016) Integrated supply, production and distribution scheduling under disruption risks. *Omega*, 62, 131-144.

Sodhi, M. S., Son, B. G., Tang, C. S. (2012) Researchers' perspectives on supply chain risk management. *Production and Operations Management*, 21(1), 1–13.

Tomlin, B. (2006) On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science*, 52(5), 639–657.

Tsay, A. A., Lovejoy, W. S. (1999) Quantity flexibility contracts and supply chain performance. *Manufacturing and Service Operations Management*, 1(2), 89-111.

Tsay, A. A. (2001) Managing retail channel overstock: Markdown money and return policy. *Journal of Retailing*, 77(4), 457-492.

Wang, C., Chen, X. (2017) Option pricing and coordination in the fresh produce supply chain with portfolio contracts. *Annals of Operations Research*, 248(1-2), 471-491.

Wang, Q., Chu, B., Wang, J., Kumakiri, Y. (2012) Risk analysis of supply contract with call options for buyers. *International Journal of Production Economics*, 139(1), 97-105.

Wang, Q., Tang, O., Tsao, D. (2006) A flexible contract strategy in a supply chain with an inflexible production mode. *International Journal of Production Research*, 1(3), 228-248.

Wang, X., Li, F., Liang, L., Huang, Z., Ashley, A. (2015) Pre-purchasing with option contract and coordination in a relief supply chain. *International Journal of Production Economics*, 167, 170-176.

Wang, X., Liu, L. (2007). Coordination in a retailer-led supply chain through option contract. *International Journal of Production Economics*, 110(1-2), 115-127.

Wu, D.J., Kleindorfer, P. R. (2005) Competitive options, supply contracting, and electronic markets. *Management Science*, 51 (3), 452–466.

Wu, J., Li, J., Wang, S., Cheng, T. C. E. (2009) Mean-variance analysis of the newsvendor model with stockout cost. *Omega*, 37(3),724-730.

Wei, Y., Choi, T. M. (2010) Mean–variance analysis of supply chains under wholesale pricing and profit sharing schemes. *European Journal of Operational Research*, 204 (2), 255-262.

Xue, W., Choi, T.M., Ma, L. (2016) Diversification strategy with random yield suppliers for a mean–variance risk-sensitive manufacturer. *Transportation Research Part E: Logistics and Transportation Review*, 90, 90-107.

Zhao, Y., Ma, L., Xie, G., Cheng, T. C. E. (2013) Coordination of supply chains with bidirectional option contracts. *European Journal of Operational Research*, 229(2), 375-381.

Zhao, Y., Wang, S., Cheng, T.C.E., Yang, X., Huang, Z. (2010) Coordination of supply chains by option contracts: a cooperative game theory approach. *European Journal of Operational Research*, 207(2), 668-675.

Zhao, Y., Choi, T. M., Cheng, T. C. E., Wang, S.Y. (2018) Supply option contracts with spot market and demand information updating. *European Journal of Operational Research*, 266(3), 1062-1071.

Zeng, B., Yen, P. C. (2017) Rethinking the role of partnerships in global supply chains: a risk-based perspective. *International Journal of Production Economics*, 185, 52-62.