Renewable Energy 81 (2015) 52–56

Contents lists available at ScienceDirect

Renewable Energy

journal homepage: www.elsevier.com/locate/renene

Some improvements of wind speed Markov chain modeling

Jie Tang [a,*], Alexandre Brouste [b], Kwok Leung Tsui [a]

[a] Department of Systems Engineering and Engineering Management, City University of Hong Kong, Hong Kong, China
[b] Laboratoire Manceau de Mathématiques, Université du Maine, Le Mans, France

ARTICLE INFO

Article history:
Received 27 March 2014
Accepted 2 March 2015
Available online

Keywords:
Wind speed
Forecasting model
Markov chains modeling method

ABSTRACT

In this study, the traditional Markov chain method for wind speed modeling is analyzed and two improvements are introduced. New states categorization step and wind speeds simulation step are presented. They both take advantage of the empirical cumulative distribution function of the wind speed time series. Performances of the new method are tested in terms of modeling and short-term forecasting. The results suggest that this method overperforms the traditional one for modeling.

1. Introduction

Wind characteristics are important for the evaluation of wind resources, performance of wind turbines and their power production. Due to the fact that the Weibull distribution has been unable to represent all the wind regimes encountered in nature, researchers have continued to propose new wind speed probability models (see Ref. [1] for a review).

The Markov chain method is used in numerous studies for modeling various types of wind speed time series [2–7] (see also Refs. [8,9] in the multivariate setting). The procedure for solving such problems using the Markov chain modeling method is relatively clear. Firstly, all of the values of a time series are distributed into several states. Secondly, supposing the series of states is ruled by a homogeneous Markov chain, a transition probability matrix of these states is estimated. Thirdly, this matrix is used to generate a new series of states. Fourth, each state in this new series is converted into a wind speed value with a certain random generator. The final product is a synthetic time series generated from the observed series, with faithfully reproduced statistical parameters.

It is worth emphasizing that certain types of wind speed time series need sometimes more sophisticated methods. Mycielski algorithm [10], neural networks methods [11], semi-Markov models [12,13] can be cited.

This paper studies two improvements of the traditional Markov chain modeling method on the first (states categorization) and fourth (wind speed simulation) steps. They both take advantage of the empirical cumulative distribution function of the wind speed time series. After a brief recall of the traditional method (Section 2), the improvements are presented in Section 3. Then, we evaluate its application on wind speed modeling. Performances of this new method are tested in terms of modeling and short-term forecasting on a real dataset in Section 4. The results suggest that this method overperforms the traditional one for modeling.

2. Traditional first-order Markov chain modeling

In this section, the traditional Markov chain modeling method is introduced. It can be divided into four steps: states categorization, Markov chain transition matrix estimation, Markov chain states simulation and wind speeds simulation.

2.1. Step 1 (states categorization)

The range $[v_{min}, v_{max}]$ of the wind speed training dataset is discretized into several intervals. Considering the operability, a finite number $k \in \mathbb{N}_*$ of disjoint intervals is considered, namely

$$\left\{ \left[V_\ell^i, V_u^i \right], \quad i = 1, \ldots, k \right\}$$

such that $[V_\ell^i, V_u^i] \cap [V_\ell^j, V_u^j] = \varnothing$ for $i \neq j$ and

$$[v_{min}, v_{max}] \subset \bigcup_{i=1}^{k} \left[V_\ell^i, V_u^i \right].$$

* Corresponding author. Tel.: +852 54980165.
E-mail address: tangj4001@gmail.com (J. Tang).

http://dx.doi.org/10.1016/j.renene.2015.03.005
0960-1481/

Generally, the lower limit V_ℓ^1 is taken to be zero. We also denote $E = \{1,\ldots k\}$ the set of the different state indexes.

The categorization is rather arbitrary depending on the purpose. But the methods in the literature can be divided into two families: in the former the intervals are constructed first and the number of intervals follows, in the latter the number of intervals is determined first so as to construct the intervals.

In the first family, we can cite [3] where the intervals boundaries are equal to $\bar{v}_N \pm j s_N$, $j = 0,1,2,\ldots$, until the extremes in the wind speed training dataset are covered. Similarly, the authors of [9] divides the range by finer grid, namely with boundaries $\bar{v}_N \pm j\cdot(0.4 s_N)$, $j = 0,1,2,\ldots$. These methods generally lead to a medium number of states (from 10 to 20). In Refs. [6,5], the authors consider integer values (in m/s) as boundaries and, consequently, intervals of the form $[i-1,i]$, $i = 1,\ldots,k$, $k > v_{max}$. Finer grid for categorization with 0.1 m/s intervals in Ref. [11] and with 0.5 m/s in Ref. [14] are considered. These methods leads to a bigger number of states (from 20 to 30 states in the mentioned studies).

In the second family, the work [8] can be cited where the wind speed time series is split into three states corresponding to 0–3 m/s (weak wind), 3–8 m/s (mean wind) and 8 m/s to the upper limit of wind speeds (strong wind). The improvement presented in Section 3.1 belongs to this family.

Each wind speed values is converted to the index of its belonging interval. The wind speed training dataset $(v_1, v_2,\ldots,v_N)$ is consequently converted into a time series of states $(i_1,i_2,\ldots,i_N)\in E^N$.

2.2. Step 2 (Markov chain transition matrix estimation)

The state time series $(i_1,i_2,\ldots,i_N)\in E^N$ is supposed to be a part of the realization of a homogeneous Markov chain. Then the transition matrix of the underlying Markov chain has to be estimated. First, we give a brief recall on Markov chain theory.

Let $k\in\mathbb{N}_*$ be a positive integer. Let $(S_n)_{n\geq 0}$ be a sequence of random variables valued in the finite set $E = \{1,2,\ldots,k\}$. If $S_n = j$, then the chain is said to be in state j at time n. In an *homogeneous* Markov chain, the transition probabilities satisfy the Markov property

$$P(S_{n+1} = j | S_n = i, S_{n-1} = i_{n-1}, \ldots, S_0 = i_0) = P(S_{n+1} = j | S_n = i) = p_{i,j}$$

for all $n\geq 0$, for all $(i,j)\in E^2$ and for all $(i_{n-1},\ldots,i_0)\in E^n$. We can remark that for an homogeneous Markov chain, these transition probabilities does not depend on the time n. Transition probabilities are determining entirely the Markov chain.

The transition matrix

$$\pi = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,k} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k,1} & p_{k,2} & \cdots & p_{k,k} \end{pmatrix}$$

has to be estimated on the training dataset.

According to [6], the empirical frequencies are the maximum likelihood estimates of the transition probabilities. Therefore, we define the estimators

$$\widehat{p}_{i,j}^{(N)} = \frac{m_{i,j}}{\sum_{j=1}^{k} m_{i,j}}$$

where $m_{i,j}$ is the number of transitions from the state i to the state j on successive time in the training data set (of length N).

2.3. Step 3 (Markov chain states simulation)

Generating a synthetic states time series with transition matrix $\widehat{\pi} = (\widehat{p}_{i,j})_{i=1\ldots k, j=1\ldots k}$ is presented in this step.

Starting from initial states $s_0\in E$ (defined by the user), the successive state is simulated randomly following the discrete probability distribution

$$\left(\widehat{p}_{s_0,1}, \widehat{p}_{s_0,2}, \ldots, \widehat{p}_{s_0,k}\right).$$

Let us note $s_1\in E$ as this generated state. Repeating the same procedure, the $(i+1)$-th state is simulated following the discrete probability distribution

$$\left(\widehat{p}_{s_i,1}, \widehat{p}_{s_i,2}, \ldots, \widehat{p}_{s_i,k}\right).$$

Therefore a time series of states with any length can be generated.

2.4. Step 4 (wind speeds simulation)

The fourth step of the traditional Markov chain method is converting the series of simulated states $(s_1,s_2,\ldots s_N)$ into a time series of wind speeds. It is assumed that the wind speeds in each state follow a certain distribution. Then, wind speed states are converted into wind speeds by

$$\tilde{v}_n = V_\ell^{s_n} + u_n\left(V_u^{s_n} - V_\ell^{s_n}\right), \quad n = 1\ldots N \tag{1}$$

where V_u^i and V_ℓ^i are the wind speed upper and lower boundaries of the i-th state interval (see Step 1). Here u_n is a random value generated from 0 to 1 by a certain distribution generator. In this manner, a time series of wind speeds is generated from the series of wind speed states, with the same length.

In practice, there is no fixed method for choosing the distribution of u_n. Many studies (see Refs. [6,9,14]) considered u_n to be a uniform random variable valued in (0, 1). In Refs. [3,5], the generated wind speeds were assumed to be uniformly distributed only in intermediate intervals whereas they were assumed to follow a shifted exponential distribution in the extreme states.

3. Improved Markov chain modeling

We present in this section two improvements of the traditional Markov chain modeling method. Precisely, we will consider the first and the fourth steps.

3.1. Improvement of the first step

As previously mentioned, the state categorization in the traditional Markov chain modeling method is rather arbitrary and depends on the purpose. The traditional Markov chain method lacks a standardized categorization step. Namely, two problems can spring up in the application:

1. A structure with a huge number of states could be constructed resulting in a large transition matrix that might present additional difficulties in estimation.
2. The number of individuals in certain states could be made much lower than others, generating a great number of small probabilities near 0 in the transition probability matrix.

In order to address this problem, it seems reasonable to determine a relatively small number of states first. Moreover, we emphasize the importance of keeping states distribution nearly uniform.

In the improved method, the number of state $k \in \mathbb{N}_*$ is fixed by the user (or some criterion). We consequently construct the intervals using the empirical quantiles, precisely the boundaries (except the first lower bound equal to 0) are taken to be $\widehat{F}_N^{-1}(j/k)$, $j = 1 \ldots k$ where $\widehat{F}_N(.)$ is the empirical cumulative distribution function

$$\widehat{F}_N(v) = \frac{\sum_{i=1}^{N} \mathbb{1}(v_i \leq v)}{N}$$

where $\mathbb{1}(.)$ is the indicator function.

As we fix k, we can choose a small number. Moreover, with this method, a structure with nearly uniformly distributed states can be constructed without wrong state categorization, which means that this method can solve the aforementioned problems.

The underlying idea of this improvement is the following: suppose that the wind speeds time series is stationary and ergodic. Then, the ergodic theorem shows that the empirical cumulative distribution function $\widehat{F}_N$ is a consistent estimator of the cumulative distribution function F of the invariant measure of this time series.

3.2. Improvement of the fourth step

Regarding the fourth step of the traditional method, either uniform or exponential distribution approximates the real distribution of speeds in each state. But, each real distribution of speeds in every state might have so many alternative approximations that it is very difficult to find the best one, depending merely on observation and experience.

For a large number of states, the uniform distribution is generally valid. But with a small number of states, it could be easy to choose an inappropriate distribution that would induce a big deviation between the generated and original time series.

The idea behind the second improvement is simple. As the number of states is small and there is a relatively large number of individuals in each states, the empirical distribution of the wind speeds in each state is a good approximation of the conditional probability distribution given its state.

4. Applications and tests

4.1. Tests on the Lamma Island dataset

The dataset is a time series that includes 52416 wind speeds measured every 10 min over 364 days since 24 February 2006 on Lamma Island ranging from 0 m/s to 30 m/s approximatively. For comparison purposes, the dataset is divided into two subsets: the first 182 days as the training dataset and the remainder as the testing dataset.

In the following, the training dataset is denoted by $(v_1, v_2, \ldots, v_N)$, $N = 26208$ and the testing dataset is denoted by $w = (w_n)_{n=1 \ldots N}$, N=26208. The former is used to compare the methods for modeling and the latter for forecasting.

4.1.1. Modeling

In this section, we will compare the traditional method with the improved method in terms of modeling.

Step 1 On the training set $(v_1, v_2, \ldots, v_N)$, the number of states generated depends on the method to construct the intervals. For instance

1. using intervals of fixed length equal to 1 m/s (the lower bound of the first is 0), the number of states generated is 30. We will denote this method S1–T1 (Step 1 Traditional Method 1).
2. using intervals with boundaries $v_n \pm j s_n$, the number of states generated is 11. We will denote this method S1–T2 (Step 1 Traditional Method 2). Wind speeds states' boundaries are equals to 0, 1.3, 4.4, 7.5, 10.5, 13.5, 16.6, 19.6, 22.7, 25.7, 28.8, 29.6.
3. using the improved method proposed in Section 3, the number of states generated is 8. This method is denoted S1–I (Step 1 Improved Method). Wind speeds states' boundaries are equals to 0, 1.3, 2.3, 3.1, 4.0, 5.0, 6.0, 7.4 and 29.6 m/s, respectively.

It is worth mentioning that the number of states of the improved S1–I method is similar to that in the classical S1–T2 method (which is much smaller than the number of states obtained with the classical S1–T1 method). Consequently, a particular attention has to be paid to their comparison in Tables 3 and 4.

Steps 2 and 3 The second step is to construct the estimated transition probability matrices. S1–T1 is a 30 × 30 matrix in which there are a huge number of 0s, and nearly all of the non-zero transition probabilities are located at the three nearest states from the diagonal. S1–T2 is 11 × 11 matrix presented in Table 1

For the improved method, the estimated 8×8 transition probability matrix is shown in Table 2.

According to the above table, nearly all of the non-zero transition probabilities are located at the three nearest states from the diagonal.

Step 4. In this numerical study, we consider the uniform distribution generator as the only method used in step 4 as it is done in the traditional method. Therefore, the states are converted using Equation (1). This method is denoted by S4–T (Step 4 Traditional Method) in the following. It can be compared to the improved procedure proposed in Section 3.2 denoted S4–I

4.1.1.1. Comparisons of the methods. Kernel density estimation is adopted with the Gaussian kernel as the kernel function and 0.1 as the bandwidth to obtain the empirical probability density function (epdf) of the training set and the simulated set. Results are presented in Figs. 1 and 2.

It can be seen on Fig. 1 that the observed distribution is a bimodal distribution.[1] The assumption of uniform distribution in traditional Step 4 ignores the high probability of calm or very low wind speeds.

It can also be seen that the categorization method T2 divides the speeds into states with wide intervals, which made the generation with the uniform assumption worse than S1–T1/S4–T1.

With the improved method, on Fig. 2, the observed bimodal distribution is now fitted better by the simulated time series empirical distribution.

In order to compare all this methods with a fixed criterion, we consider the root mean square error (RMSE) defined by

$$RMSE = \sqrt{\frac{\sum_{i=0}^{K} (g_i - o_i)^2}{K + 1}} \quad (2)$$

[1] Cup anemometers, the instruments for extensive use to measure wind force and velocity, typically have relatively high thresholds.

Table 1
Estimated transition matrix for S1–T2 method.

$$\widehat{\pi}_{S1-T2} = \begin{pmatrix} 0.75 & 0.25 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.08 & 0.78 & 0.14 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.18 & 0.72 & 0.10 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.01 & 0.37 & 0.55 & 0.07 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.01 & 0.06 & 0.33 & 0.46 & 0.11 & 0.02 & 0.01 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.01 & 0.07 & 0.30 & 0.38 & 0.16 & 0.07 & 0.01 & 0.01 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.10 & 0.36 & 0.26 & 0.17 & 0.08 & 0.02 & 0.02 \\ 0.00 & 0.00 & 0.00 & 0.02 & 0.06 & 0.10 & 0.38 & 0.25 & 0.13 & 0.06 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.10 & 0.24 & 0.34 & 0.24 & 0.03 & 0.03 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.12 & 0.12 & 0.62 & 0.12 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 \end{pmatrix}$$

Table 2
Estimated transition matrix for S1–I.

$$\widehat{\pi}_{S1-I} = \begin{pmatrix} 0.74 & 0.21 & 0.03 & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.21 & 0.48 & 0.22 & 0.07 & 0.02 & 0.00 & 0.00 & 0.00 \\ 0.04 & 0.24 & 0.39 & 0.24 & 0.07 & 0.02 & 0.01 & 0.00 \\ 0.01 & 0.07 & 0.21 & 0.37 & 0.23 & 0.08 & 0.02 & 0.00 \\ 0.00 & 0.02 & 0.06 & 0.21 & 0.37 & 0.24 & 0.08 & 0.01 \\ 0.00 & 0.00 & 0.02 & 0.07 & 0.26 & 0.36 & 0.24 & 0.06 \\ 0.00 & 0.00 & 0.01 & 0.02 & 0.09 & 0.24 & 0.43 & 0.21 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.01 & 0.05 & 0.21 & 0.72 \end{pmatrix}$$

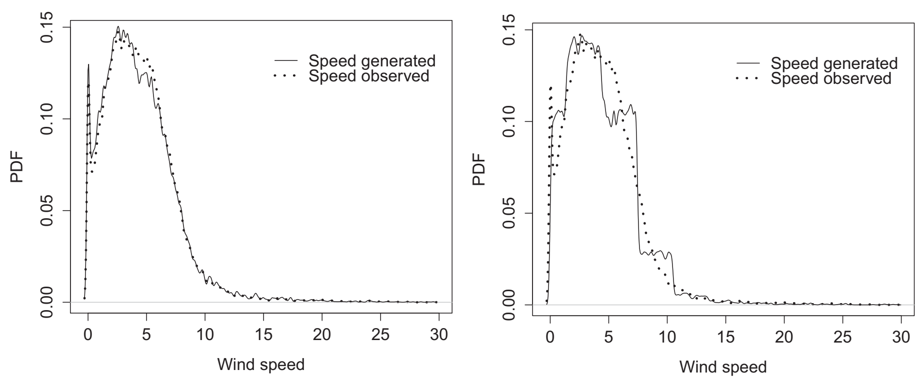

Fig. 1. S1–T1/S4–T method (on the left) and S1–T2/S4–T method (on the right) compared to observed training data.

where g_i and o_i are the epdf of the training sample and the simulated sample respectively, evaluated at the point $i \cdot v_{max}/K$, $i = 0,\ldots K$, of the regular grid of $[0, v_{max}]$ of length $K+1$. RMSE are computed for the different traditional and improved method and summarized in Table 3. This criterion claims that the improved method overperforms the traditional ones in terms of modeling.

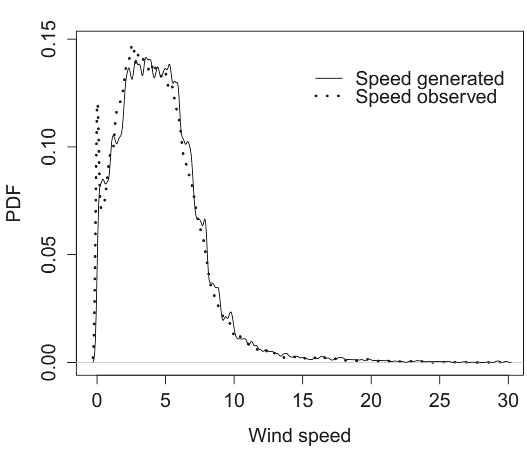

Fig. 2. S1–I/S4–I method compared to the observed training data.

Moreover, it is easy to understand that the computation of a 8×8 matrix is obviously faster than that of a 30×30 matrix.

4.1.1.2. Limitations. In the case where the training set is inhomogeneous with the testing sample, this Step 4 acts as a misspecification of the model and could give negative results. In this case, the uniform distribution is more neutral.

4.1.2. Forecasting

Despite our objective is to model the wind speed, we also compare the traditional methods with the improved method in terms of forecasting.

We consider a one step ahead forecasting procedure. First, the transition matrix $\widehat{\pi}$ is constructed on the training set as in the previous Section 4.1.1.

Let us fix the horizon time to τ (for instance $\tau = 18$ (3h), 36 (6h), 72 (12h) and 144 (1d)). Then we simulate a one step ahead (of horizon time τ) forecast series with the help of the testing sample $(w_1,\ldots,w_N)$. Namely, starting from w_j, $j = 1\ldots N-\tau$, we construct the wind speed (theoretical mean value) forecast $\widehat{v}_{j+\tau}$ of $w_{j+\tau}$ using the estimated transition matrix $\widehat{\pi}$ with

$$\widetilde{v}_{j+\tau} = (0,\ldots,0,1,0,\ldots,0)\widehat{\pi}^{\tau} \cdot (m_1,\ldots,m_k)'$$

Table 3
Modeling comparison (RMSE) between traditional methods and improved method (for $k = 8$ in the improved method).

	S1–T1/S4–T	S1–T2/S4–T	S1–I/S4–I
RMSE	0.007	0.010	0.002

Table 4
RMSE between the testing set and forecasted series (for $k = 8$ in the improved method).

Horizon/Method	S1–T1/S4–T	S1–T2/S4–T	S1–I/S4–I
18	2.36	2.55	2.42
36	2.61	2.65	2.63
72	2.66	2.66	2.66
144	2.66	2.67	2.66

Table 5
RMSE for modeling and forecasting (for $k = 8$ and $\tau = 18$) for the improved method.

RMSE/Turbine	20,500	21,500	22,500	23,500	24,500
RMSE modeling	0.005	0.002	0.003	0.005	0.004
RMSE forecasting	2.32	3.36	3.35	2.81	2.44

where the one is at the place i_j which is the state of the value w_j, m_i is the center of classification interval $[V_\ell^i, V_u^i]$ for i=1,…,k, $\widehat{\pi}$ is the transposition, and $\cdot$ is the scalar product in $\mathbb{R}^k$.

Finally we compute the RSME

$$RSME(\tau) = \sqrt{\frac{\sum_{j=1}^{N-\tau}\left(w_{j+\tau} - \widetilde{v}_{j+\tau}\right)^2}{N}}.$$

We summarized the RSME in Table 4 for different time horizons $\tau = 18,36,72$ and 144.

It can be seen from Table 4 S1–I/S4–I (improved method) overperforms S1–T2/S4–T (traditional 2 method) with similar number of states. Moreover, S1–I/S4–I is comparable to the first traditional method for $\tau \geq 36$.

For big τ, it is comparable to choose the mean value of the training sample as the forecast value.

For $k = 8$ states in the improved method, the forecast for $\tau = 18$ is slightly worse than the traditional method. But, for a bigger number of states, forecasting with this time horizon is comparable. For instance for $k = 16$, RMSE are equal again.

4.2. Performances of the improved version on NREL datasets

The datasets are several time series that includes 52,560 wind speeds measured at the turbine rotor height every 10 min over 365 days in 2005 in US. For instance, the ID 24500 turbine is located 43.48N and 107.29W in Wyoming and wind speeds ranging from 0.09 m/s to 27.75 m/s.

This datasets present no null wind. They are taken from http://wind.nrel.gov.

Every dataset is divided into two subsets: the first 26,280 data as the training dataset and the remainder as the testing dataset. Results presented in Table 5 for modeling (see Section 4.1.1) and forecasting (see Section 4.1.2) show that the improved method works for a large collection of wind.

5. Conclusion

In this study, the application of the traditional Markov chain method for wind speed modeling is analyzed. The first step which is the state categorization might generate a huge and inefficient transition probability matrix.

Moreover, the wind speeds generation from the states simulation in the fourth step might not reproduce some statistical features and stylized facts observed in the data (bimodal distribution, Weibull tails, *etc.*).

To address these problems, two improvements are introduced. The first one is a new state categorization procedure that takes advantage of the empirical probability distribution function of the wind speeds time series. The second improvement relies directly on the empirical distribution of the wind speeds in each state. A new simulation method to generate synthetic wind speeds from the state simulations is given. Application and comparison conducted reveals that the improved method overperforms its traditional counterpart in modeling (with simultaneously comparable results for forecasting).

References

[1] Carta J, Ramirez P, Velazquez S. A review of wind speed probability distributions used in wind energy analysis. Case studies in the Canary Islands. Renew Sustain Energy Rev 2009;13:933–55.
[2] Castino F, Festa R, Ratto C. Stochastic modeling of wind velocities time series. J Wind Eng Ind Aerodynamics 1998;74:141–51.
[3] Sahin AD, Sen Z. First-order Markov chain approach to wind speed modeling. J Wind Eng Ind Aerodynamics 2001;89(3–4):263–9.
[4] Kantz H, Holstein D, Ragwitz M, Vitanov NK. Markov chain model for turbulent wind speed data. Phys A-Stat Mech Appl 2004;342(1–2):315–21.
[5] Nfaoui H, Essiarab H, Sayigh AAM. A stochastic Markov chain model for simulating wind speed time series at Tangiers, Morocco. Renew Energy 2004;29(8):1407–18.
[6] Shamshad A, Bawadi MA, Hussin W, Majid TA, Sanusi SAM. First and second order Markov chain models for synthetic generation of wind speed time series. Energy 2005;30(5):693–708.
[7] Fawcett L, Walshaw D. Markov chain models for extreme wind speeds. Environmetrics 2006;17(8):795–809.
[8] Ettoumi FY, Sauvageot H, Adane AEH. Statistical bivariate modeling of wind using first-order Markov chain and Weibull distribution. Renew Energy 2003;28(11):1787–802.
[9] Yang H, Li Y, Lu L, Qi R. First order multivariate Markov chain model for generating annual weather data for Hong-Kong. Energy Build 2011;43(9):2371–7.
[10] Fidan M, Hocaoğlu FO, Gerek ÖN. Improved synthetic wind speed generation using modified Mycielski approach. Int J Energy Res 2012;36(13):1226–37.
[11] Pourmousavi K, Ardehali M. Very short-term wind speed prediction: a new artificial neural network–Markov chain model. Energy Convers Manag 2011;52(1):738–45.
[12] D'Amico G, Petroni F, Prattico F. Wind speed modeled as an indexed semi-Markov process. Environmetrics 2013;24(6):367–76.
[13] D'Amico G, Petroni F, Prattico F. First and second order semi-Markov chains for wind speed modeling. Phys A-Stat Mech Appl 2013;392(5):1194–201.
[14] Carapellucci R, Giordano L. A new approach for synthetically generating wind speeds: a comparison with the Markov chains method. Energy 2013;49:298–305.