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QUEUING THEORY MODEL USED TO SOLVE THE WAITING LINE OF A BANK -A STUDY ON ISLAMI BANK BANGLADESH LIMITED, CHAWKBAZAR BRANCH, CHITTAGONG

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ABSTRACT

Waiting lines and service systems are important parts of the business world. In this article we describe several common queuing situations and present mathematical models for analyzing waiting lines following certain assumptions. Those assumptions are that: (1) arrivals come from an infinite or very large population, (2) arrivals are Poisson distributed, (3) arrivals are treated on a FIFO basis and do not balk or renege, (4) service times follow the negative exponential distribution or are constant, and (5) the average service rate is faster than the average arrival rate.

The model illustrated in this Bank for customers on a level with service is the multiple-channel queuing model with Poisson Arrival and Exponential Service Times (M/M/S). After a series of operating characteristics are computed, total expected costs are studied, total costs is the sum of the cost of providing service plus the cost of waiting time. Finally we find the total minimum expected cost.

Keywords: Service; FIFO; M/M/s; Poisson distribution; Queue; Service cost; Utilization factor; Waiting cost; Waiting time, Optimization

INTRODUCTION

Queuing theory had its beginning in the research work of a Danish engineer named A.K. Erlang. In 1909 Erland experimented with fluctuating demand in telephone traffic. Eight years letter he published a report addressing the delays in automatic dialing equipment. At the end of World War II, Erlang's early work was extended to more general problems and to business applications of waiting lines.

The study of waiting lines, called queuing theory, is one of the oldest and most widely used quantitative analysis techniques. Waiting lines are an everyday occurrence, affective people shopping for groceries buying gasoline, making a bank deposit, or waiting on the telephone for the first available airline reservationists to answer. Queues, another term for waiting lines, may also take the form of machines waiting to be repaired, trucks in line to be unloaded, or airplanes lined up on a runway waiting for permission to take off. The three basic components of a queuing process are arrivals, the actual waiting line and service facilities.

CHARACTERISTICS OF A QUEUING SYSTEM:

We take a look at the three parts of queuing system: (1) the arrival or inputs to the system (sometimes referred to as the calling population), (2) the queue# or the waiting line itself, and (3) the service facility. These three components have certain characteristics that must be examined before mathematical queuing models can be developed.

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Arrival Characteristics

The input source that generates arrivals or customers for the service system has three major characteristics. It is important to consider the size of the calling population, the pattern of arrivals at the queuing system, and the behavior of the arrivals.

Size of the Calling Population

Population sizes are considered to be either unlimited (essential infinite) or limited (finite). When the number of or arrivals on hand at any customers given moment is just a small portion of potential arrivals, the calling population is considered unlimited. For practical purpose, in our examples the limited customers arriving at the bank for deposit cash. Most queuing models assume such an infinite calling population. When this is not the case, modeling becomes much more complex. An example of a finite population is a shop with only eight machines that might deposit cash break down and require service.

* The word queue is pronounced like the letter Q, that is, “kew”

Pattern of arrivals at the system

Customers either arrive at a service facility according to some known schedule customers or else they arrive randomly. Arrivals are considered random when they are independent of one another and their occurrence cannot be predicted exactly. Frequently in queuing problems, the number of arrivals per unit of time can be estimated by a probability distribution known as the Poisson distribution., For any given arrival rate, such as two passengers per hour, or four airplanes per minute, a discrete, Poisson distribution can be established by using the formula :

$$P(n;t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \text{ for n} = 0,1,2,3,4,\ldots\ldots$$

Where

P (n;t) = probability of n arrivals

λ = average arrival rate

e = 2.18

n = number of arrivals per unit of time

Behavior of the Arrival

Most queuing models assume that an arriving passenger is a patient traveler. Patient customer is people or machines that wait in the queue until they are served and do not switch between lines. Unfortunately, life and quantitative analysis are complicated by the fact that people have been known to balk or renege. Balking refers to customers who refuse to join the waiting lines because it is to suit their needs or interests. Reneging customers are those who enter the queue but then become impatient and leave the need for queuing theory and waiting line analysis. How many times have you seen a shopper with a basket full of groceries, including perishables such as milk, frozen food, or meats, simply abandon the shopping cart before checking out because the line was too long? This expensive occurrence for the store makes managers acutely aware of the importance of service level decisions.

Waiting Line characteristics

Queue

The waiting line itself is the second component of a queuing system. The length of a line can be either limited or unlimited. A queue is limited when it cannot, by law of physical restrictions, increase to an infinite length. Analytic queuing models are treated in this article under an assumption of unlimited queue length. A queue is unlimited when its size is unrestricted, as in the case of the tollbooth serving arriving automobiles.

Queue discipline

A second waiting line characteristic deals with queue discipline. The refers to the rule by which passengers in the line are to receive service., Most systems use a queue discipline known as the first in, first out rule (FIFO). This is obviously not appropriate in all service system, especially those dealing with emergencies.

In most large companies, when computer-produced pay checks are due out on a specific date, the payroll program has highest priority over other runs.

Service Facility Characteristics

The third part of any queuing system is the service facility. It is important to examine two basic properties: (1) the configuration of the service system and (2) the pattern of service times.

Basic Queuing System Configurations

Service systems are usually classified in terms of their number of channels, or number of servers, and number of phases, or number of service stops, that must be made.

The term FIFS (first in, first served) is often used in place of FIFO. Another discipline, LIFS (last in, first served), is common when material is stacked or piled and the items on top are used first.

A single-channel system, with one server, is typified by the drive in bank that has only one open teller. If, on the other hand, the bank had several tellers on duty and each customer waited in one common line for the first available teller, we would have a multi-channel system at work. Many banks today are multi-channel service systems, as are most large barbershops and many airline ticket counters.

A single-phase system is one in which the customer receives service from only one station and then exits the system. Multiphase implies two or more stops before leaving the system.

Service Time Distribution

Service patterns are like arrival patterns in that they can be either constant or random. If service time is constant, it takes the same amount of time to take Care of each customer. More often, service times are randomly distributed in many cases it can be assumed that random service times are described by the negative exponential probability distribution. This is a mathematically convenient assumption if arrival rates are Poisson distributed.

The exponential distribution is important to the process of building mathematical queuing models because many of the models' theoretical underpinning is based on the assumption of Poisson arrivals and exponential services. Before they are applied, however, the quantitative analyst can and should observe, collect, and pilot service time data to determine if they fit the exponential distribution.

Mathematical Models

Single-Channel Queuing Model with Poisson Arrivals and Exponential service times (M/M/1):

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We present an analytical approach to determine important measures of performance in a typical service system. After these numerical measures have been computed, it will be possible to add in cost data and begin to make decisions that balance desirable service levels with waiting line service costs.

Assumptions of the Model

The single-channel, single-phase model considered here is one of the most widely used and simplest queuing models. It involves assuming that seven conditions exits:

1. Arrivals are served on a FIFO basis.
2. Every arrival waits to be served regardless of the length of the line; that is, there is no balking or reneging.
3. Arrivals are independent of preceding arrivals, but the average number of arrivals (the arrival rate) does not change over time.
4. Arrivals are described by a Poisson probability distribution and come from an infinite or very large population.
5. Service time also varies from one passenger to the next and is independent of one another, but their average rate is known.
6. Both the number of items in queue at anytime and the waiting line experienced by a particular item are random variables.
7. Service times occur according to the negative exponential probability distribution.
8. The average service rate is greater than the average arrival rate.
9. The waiting space available for customers in the queue is infinite.

When these seven conditions are met, we can develop a series of equations that define the queue's operative characteristics. [Figure]

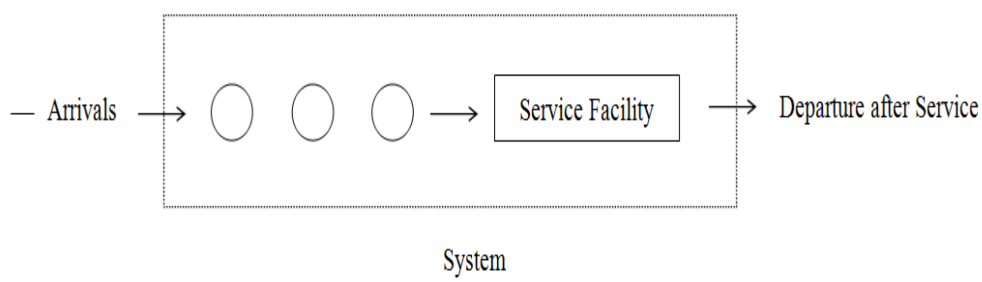

Figure 1. Single-Channel Waiting Line

Queuing Equations

λ = mean number of arrivals per time period (for example, per hour)

μ = mean number of people or items served per time period.

When determining the arrival rate (λ) and the services rate (μ), the same time period must be used. For example, if the λ is the average number of arrivals per hour, then μ must indicate the average number that could be served per hour.

The Queuing Equations Follow

1. The average number of customers or units in the system, Ls, that is, the number in line plus the number being served :

$$Ls = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

2. The average time a customer spends in the system, Ws, that is, the time spent in line plus the time spent being served :

$$Ws = \frac{1}{\mu-\lambda}$$

3. The average number of customers in the queue, Lq:

$$Lq = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

4. The average time a customer spends waiting in the queue, Wq:

$$Wq = \frac{\lambda}{\mu(\mu-\lambda)}$$

5. The utilization factor for the system ρ, that is, the probability that the service facility is being used :

$$\rho = \frac{\lambda}{\mu}$$

6. The present idle time, Po, that is, the probability that no one is in the system:

$$\text{Po} = 1 - \frac{\lambda}{\mu}$$

7. The probability that the number of customers in the system is greater than k, P n>k:

$$P\, n > k = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

8. Length of the non empty queue ; $L_q' = \dfrac{\mu}{\mu-\lambda}$

9. Probability that waiting time is more than t is the system = $e^{-(\mu-x)}t - (\mu-\lambda)t$

10. Probability that waiting time is more than t is the queue = $e^{-(\mu-\lambda t)} \times \dfrac{\lambda}{\mu}$

Multiple-Channel Queuing Model with Poisson Arrivals and Exponential service Times (M/M/S)

The next logical step is to look at a multiple channel queuing system, in which two or more servers or channels are available to handle arriving passengers. Let us still assume that travelers waiting service from one single line and then proceed to the first available server. Each of these channels has an independent and identical exponential service time distribution with mean $1/\mu$. The arrival process is Poisson with rate λ. Arrivals will join a single queue and enter the first available service channel.

The multiple-channel system presented here again assumes that arrivals follow a Poisson probability distribution and that service times are distributed exponentially. Service is first

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come, first served, and all servers are assumed to perform at the same rate. Other assumptions listed earlier for the single-channel model apply as well.

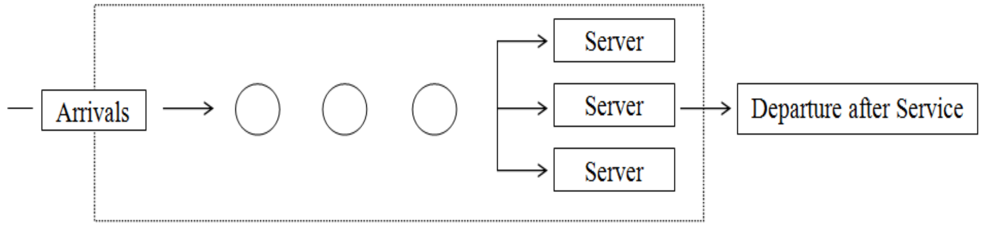

Figure 2. Multiple-Channel Waiting Line

Equations for the Multi-channel queuing Model

If we let

S = number of channels open,

λ = average arrival rate, and

μ = average service rate at each channel.

The following formulas may be used in the waiting line analysis:

1. Utilisation rate : $\rho = \dfrac{\lambda}{s\mu}$
2. The probability that there are zero customers or units in the system:

$$\text{Po}= \frac{1}{\sum_{n=0}^{S-1} \frac{(\rho)^n}{n!} + \frac{(\rho)^S}{S!\left(1-\frac{\rho}{S}\right)}}$$

3. The probability that there are n number of customers in the system:

$$P_n \begin{cases} \frac{(\rho)^n}{n!} f_0,\ if \quad n < S \\ \frac{\rho^n}{S!S^{n-S}} f_0,\ if \quad n \geq S \end{cases}$$

4. Probability that a customer has to wait :

$$P(n \geq S) = \frac{\mu(\rho)}{(S-1)!\,(S\mu - \lambda)} \rho_0$$

5. The average number of customers or units in the system :

$$L_s = \left[\frac{(\rho)^S}{\mu S!S\left(1-\frac{\rho}{S}\right)^2} P_0 + \frac{1}{\mu} \right] \lambda$$

6. The average time a unit spends in the waiting line or being serviced (namely, in the system) :

$$Ws = \frac{(\rho)^S}{\mu S!S\left(1-\frac{\rho}{S}\right)^2} P_0 + \frac{1}{\mu} \text{ Or } W_S = L_S / \lambda$$

7. The average number of customers or units in line waiting for service :

$$L_q = \frac{(\rho)^{S+1}}{S!S\left(1-\frac{\rho}{S}\right)^2} P_0 \text{ Or } L_q = L_s - \rho$$

8. The average time a customers or unit spends in the queue waiting for service:

$$W_q = \frac{(\rho)^S}{\mu S!S\left(1-\frac{\rho}{S}\right)^2} P_0 \text{ Or } W_q = W_s - \frac{1}{\mu}$$

These equations are obviously more complex than the ones used in the single-channel model, yet they are used in exactly the same fashion and provide the same type of information as did the simpler model.

Some General Operation Characteristic Relationship

Certain relationships exist among specific operating characteristics for any queuing system in a steady state. A steady state condition exists when a queuing system is in its normal stabilized operating condition, usually after an initial or transient state that may occur. John D.C little is credited with the first of these relationships, and hence they are called little's Flow Equations.

$$W_q = L_q / \lambda \ (or \ L_q = \lambda W_q)$$
$$W_s = L_s / \lambda \ (or \ L_s = \lambda W_s)$$

A third condition that must always be met is:

Average time in system = average time in queue + average time receiving service

$$W_s = W_q + \frac{1}{\mu}$$

The advantage of these formulas is that once one of these four characteristics is known, the other characteristics can easily be found. This is important because for certain queuing models, one of these may be much easier to determine than the other. There are applicable to all of the queuing system except the finite population's model.

WAITING LINE COST

There are two basic types of costs associated with waiting-line problems. First, there are the fairly 'tangible' costs involve in operating each service facility like the costs for equipment, materials, labor, etc. these cost of course , rise as the number of service facilities put into operation increase. On the other hand, there are the relatively 'intangible' costs associated with causing customers to have to wait in line for some period of time prior to being waiting upon- physical discomfort, adverse emotional reactions, reduced or lost sales and so on. Of course, as the number of service facilities in operation increases, the time the customer has to wait in line, on the average, decreases, and hence so too do these costs.

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As shown in the adjoin figure, the total of these two basic types of costs goes to a minimum at some specific number of facilities. This then is the optimum number of service facilities which should be scheduled by the operations manager- optimum because it minimize the total cost of both operation service facilities and waiting to be served at them.

MANAGERIAL APPLICATIONS OF QUEUING THEORY

Queuing theory is a valuable tool for business decision-making. It can be applied to a wide variety of situations for scheduling. Some of these are given below-

1. Mechanical transport fleet
2. Scare defense equipment
3. Issue and return of tools from tool cribs in plants
4. Aircrafts at landing and take-off from busy airports
5. Jobs in production control
6. Parts and components in assembly lines
7. Routing sales persons
8. Inventory analysis and control
9. Replacement of capital assets
10. Minimization of congestion due to traffic delays at booths.

Queuing theory has generally been applied by factories, transport companies, telephone exchanges, computer centers, retail stores, cinema houses, restaurants, banks, insurance companies, traffic control authorities, hospitals, etc.

ADVANTAGES AND LIMITATIONS OF QUEUING THEORY

It Offers the Following Benefits

I. Queuing theory provides models that are capable of determining arrival pattern of customers or most appropriate number of service stations.
II. Queuing models are helpful in creating balance between the two opportunity costs for optimization of waiting costs and service costs.
III. Queuing theory provides better understanding of waiting lines so as to develop adequate service with tolerable waiting.

Major Limitations of Queuing Theory Are

1. Most of the queuing models are very complex and cannot be easily understood. The element of uncertainty is there in almost all queuing situations. Uncertainty arises due to:
   i. We may not know the form of theoretical probability distribution which applies.
   ii. We might not know the parameters of the process even when the particular distribution is known.
   iii. We would simply be known only the probability distribution of out-comes and not the distribution of actual outcomes even when (i) and (ii) are known.
2. In addition to the above complications, queue discipline may also impose certain limitations. If the assumption of 'First come first served' is not a true one (and this happens in many cases) queuing analysis becomes more complex.

3. In many cases, the observed distributions of service times and time between arrivals cannot be fitted in the mathematical distributions of usually assumed in queuing models. For example, the Poisson distribution which is generally supposed to apply may not fit many business situations.
4. In multi-channel queuing, the departure from one queue often forms the arrival of another. This makes the analysis more difficult.

Application of the queuing system configurations and discussion of waiting cost of customers: A study on Islamic Bank Bangladesh Limited, Chawkbazar Branch, Chittagong.

The Estimated Characteristic of the System:

1. Average number of customers arriving at the system, $\lambda = 133.61$
2. The mean number of customers served per hour, $\mu = 17.083$
3. The utilization factor for the system, $\rho = \lambda/\mu = 7.821$
4. The traffic intensity for the system, $\psi = \rho / S = 7.821/10 = 0.7821 < 1$
5. Probability that there are zero customers in the system Po = 0.00034
6. The average number of customers in line waiting for service (check in) $L_q = 1.3298$
7. The average number of customers in the system $L_s = 9.1508$
8. The average time a customer spends in the queue waiting for service $W_q = 0.01$
9. The average time a customers spends in the waiting line or being serviced (namely, in the system) : $W_s = 0.0685$
10. The number of unoccupied booth $\bar{\rho} = 10(1-0.7821) = 2.179$ (nearly two counters are inactive)
11. The probability that customer wait before to be served P(>o) ≈ 36.82%
    i. The probability that the system is empty (Po) increases;
    ii. The average length of the queue (Lq) and in the system (Ls) decline;
    iii. The average waiting time in the queue (Wq) and the average time in the system (Ws) decrease also;

Expected Total Cost

The Objective Function

Min {E (TC) = E (SC) + E (WC)}

Where

TC: Total Expected Cost;

SC: Cost of Providing Service;

WC: Cost of Waiting Time.

E (TC) = Co + S.Cs + Ca.Ls

Where

Co: The fixed cost of operation system per unit of time;

Cs: The marginal cost of a registration agent per unit of time (or Total hourly service cost);

Ca: The cost of waiting based on time in the queue and in the system.

Note: For this problem, an analytical solution does not exist, and it be necessary to solve the problem by groping especially that to be question of convex function in other words whether her curve is in form of U, therefore it just takes to estimate E(TC) for the values of S growing up until the cost cease to decrease.

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Cost of Waiting Time (WC)

I. One unit of waiting time of a traveller was estimated on the basic wage of the latter.
II. The mean waiting time cost per passenger is: Ca = Σ WiCi.

Where

Ci: Hour-wage of a passenger belong to the socio-professional category i.
Wi: Weight of the category i is extracted from the total of the sample.

Empirical Result

Size of the sample: N = 100 customers after a questionnaire accomplished by Islami Bank. As mentioned inside of this sample, it was found among these categories: Our effective sample is of size N = 900 travelers.

This analysis is summarized in Table 1.

Table 1

Socio-Professional Category	*Number Of Customers*	*Theoretical Frequency (%)*
[200 ; 400[	339	38%
[400 ; 600[	249	28%`
[600 ; 800[	163	18%
[800 ; et +[	149	16%
Total	900	100%

Remark: We have cancel the socio-professional category [0; 200 [because the lack into gain for this class is low.

Table 2. The Optimal Configurations

No. of booths (S)	*Total Hourly service Cost E(SC)=S.CS*	*The Average Number of Passengers In The System Ls=Lq+p*	*Total Hourly Service Cost E(WC)=Ca Ls*	*Total Expected Cost E(TC)=E(SC)+E(WC)*
4	53.782	60.85	337.235	391.017
5	53.162	60.16	336.152	389.314
6	54.172	59.27	334.185	388.357
7	55.166	49.13	332.176	387.342
8	62.184	48.39	329.923	392.107
9	69.957	11.79	80.3842	150.3412
10	*77.73*	*9.1508*	*62.3916*	*140.1216*
11	85.503	8.3495	56.926	142.429
12	93.276	8.0502	54.886	148.162
13	101.049	7.9198	53.997	155.046
14	108.822	7.863	53.609	162.431
15	116.595	7.838	53.439	170.034

S* = 10

CONCLUSION

Queuing models have found widespread use in the analysis of service facilities, production and many other situations where congestion or competition for scarce resources may occur. This paper has introduced the basic concepts of queuing models, and shown how linear programming, and in some cases a mathematical analysis, can be used to estimate the performance measures of system. The key operating characteristics for a system are shown to be (1) utilization rate, (2) percent idle time, (3) average time spent waiting in the system and in the queue, (4) average number of customers in the system and in the queue, and (5) probabilities of various numbers of customers in the system. The article presents especially the total minimum expectation cost of the bank. Unlike the discrete and continuous probability distribution used in the analysis of queuing models.

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