Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 55-3 (2022) 37–42

Robust Cooperative Attack of Multiple Missiles with External Disturbances and Communication Delay

Ao Li Zhenghong Deng Xiaoxiang Hu*

School of Automation, Northwestern Polytechnical University, Xi'an 710129, China

**(Tel: +86-13696261728; e-mail:xxhu@nwpu.edu.cn).*

Abstract: The problem of cooperative attack of multiple missiles is studied in this paper, and these missiles are supposed to suffer from external disturbances, communication delay. A kinematics model of the multi-missile under the situation of external disturbances and communication delay is firstly developed.And then based on the topology of directed graph, the problem of multi-missile convergence is transformed into the stability analysis of a closed-loop control system. A state feedback robust controller is designed by linear matrix inequality. Finally, a numerical simulation is given to demonstrate the effectiveness of the proposed control strategy.

Keywords: Missile formation; cooperative control; communication delay; linear matric inequality; robust control

1. INTRODUCTION

With the development of Anti-Aircraft Missile Systems (AAMS) and Close-In Weapon System (CIWS) in the world, it is becoming more and more difficult for a single missile to breakthrough defense(Lyu etal, 2019). Missile cooperation provide a new way of combat for modern warfare. Missile cooperation can share mulitiple missles information with each other to get the tactical coordination after launch, and finish the mission as formation group. In this case, the possibility of breaking through defenses and the capability of successfully striking against enemy targets will get a considerable enhancement.

This paper focused on cooperative attack of multiple missiles under cooperative control. This is more difficult than one missile launched from different positions at the same time and aimed at a common enemy target. It is an effective way to improve the probability of completing the task. convergence attack is mainly divided into two ways: one is guidance using cooperative control, and the other is impact time control guidance (ITCG). The cooperative control requires online data connection between missiles, and synchronizes the arrival time by communicating with each other and adjusting their own flight status. cooperative control has been widely studied and can be devided into three aspects: time cooperative guidance (Jeon etal, 2010), directional communication topology (Zhao etal, 2017) and space cooperative guidance (Shaferman etal, 2017). And the ITCG requires the determination of a suitable attacking time before guidance (Hu etal, 2018).

However, missiles cannot carry high powered communication equipment, the quality of connection is difficult to guarantee, so communication delay and interference would occur in both sending and receiving of process. In addition, due to the measurement noise and error, missiles have huge difficult to get accurate information about their status. The problems of communication delay, noise interference and model uncertainty within the missile cooperative control system have not been adequately studied.

Consensus control has been widely studied in the multi-agent system (MAS), and as an important research aspect of the MAS, consensus control has received more and more attention since last decade. Many research works on MAS mainly focused on some special graph, such as undirected graph (Hu etal, 2016), strong connected graph (Yi etal, 2017), balanced graph (Yu etal, 2016), and so on. As for the MAS under external disturbances and network imperfections, (Elahi etal, 2019) presents a distributed control protocol for consensus control MAS.

From the above analysis, it can be found that studies of controller designed for MAS has received much attention and multiple missiles can be treated as MAS. But the design steps of this approach are cumbersome, the solving process of MAS heavily dependent on the state information of other agent, and variable topology will substantially increase the computational effort. All these problems prevent graph theory-based approach from being put to real battlefield use. Therefore, this paper proposes a linear matrix inequality (LMI) method to solve the convergence attack problem. By transforming it into a stability analysis problem A robust controller is then designed and the input of the controller adopts a decentralized structure. To reduce the computational complexity, the input of each controller only depends on itself. Considering the communication delay and interference happened in the battlefield environment, the performance of index $L_2/L_\infty$was chosen to evaluate the constrained output. The method has some practical significance when applied to the convergence attack of multiple missiles.

2. MISSILE MODELING AND PROBLEM DESCRIPTION

A reasonable missile kinematic model is designed and then the control objective is proposed.

10.1016/j.ifacol.2022.05.007

2.1 Missile model with delay and disturbances

Since most of the missiles always fly in a plane, it can be assumed that the attitude of the missile does not change. Then we take external disturbances into account. A missile model is given as follows.

$$\begin{cases} \dfrac{dx}{dt} = V\cos\theta + V_r\cos\theta \\ \dfrac{dy}{dt} = V\sin\theta + V_r\sin\theta \\ \dfrac{d\theta}{dt} = \omega + \omega_r \end{cases} \tag{1}$$

where, V is the velocity of the mass center, V_r is the velocity of disturbance, ω is the angular velocity of the missile in the plumb plane, ω_r is the angular velocity disturbance. θ is the angle between the ground plane called path angle, x, y is the displacement in the horizontal and vertical directions respectively.

Define the state variables and inputs as follows.

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = \begin{bmatrix} x \\ y \\ V\cos(\theta) \\ V\sin(\theta) \end{bmatrix} \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} a \\ \omega \end{bmatrix} \tag{2}$$

where a is the linear acceleration of the missile. According to the new definition, equation (1) and equation (2) can be rewritten as.

$$\dot{\xi} = \begin{bmatrix} \xi_4\cos(\xi_3) \\ \xi_4\sin(\xi_3) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}\eta = f(\xi) + g(\xi)\eta \tag{3}$$

For the convenience of controller design, equation (3) is transformed as follows and defined.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = T(\xi) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_4\cos(\xi_3) \\ \xi_4\sin(\xi_3) \end{bmatrix} \tag{4}$$

$$\eta = \begin{bmatrix} \cos(\xi_3) & \sin(\xi_3) \\ -\dfrac{\sin(\xi_3)}{\xi_4} & \dfrac{\cos(\xi_3)}{\xi_4} \end{bmatrix} u, = M(\xi)u \tag{5}$$

Based on the above substitution of variables, the state mode can be got and.

$$\dot{x} = \frac{\partial T}{\partial \xi}\dot{\xi} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} V_r\cos\xi_3 \\ V_r\sin\xi_3 \\ -\xi_4\omega_r\sin\xi_3 \\ \xi_4\omega_r\cos\xi_3 \end{bmatrix} \tag{6}$$

Then let the disturbance variable $w = \begin{bmatrix} V_r\cos\xi_3 \\ V_r\sin\xi_3 \\ -\xi_4\omega_r\sin\xi_3 \\ \xi_4\omega_r\cos\xi_3 \end{bmatrix}$, we can transform (6) into (7) as follow.

$$\dot{x} = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix} x + \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} u + I_4 \begin{bmatrix} V_r\cos\xi_3 \\ V_r\sin\xi_3 \\ -\xi_4\omega_r\sin\xi_3 \\ \xi_4\omega_r\cos\xi_3 \end{bmatrix} = Ax + Bu + Gw \tag{7}$$

It is necessary to fully consider the delay happened during the process of model building. Therefore the missile motion control model containing input time delay is established as follows.

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Gw(t) \tag{8}$$

where τ is the time delay of the control input.

Through the above analysis, a mathematical model of the missile under the action of external disturbance and input time delay is established, and the model can better describe the motion of the missile.

2.2 Convergence problem description

The missile model with controller input time delay and external disturbances developed above and the convergence problem to be solved are restated for analytical purposes.

In the control problem for convergence attack of multiple missiles, each missile is assumed to have the same control model of the following form.

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t-\tau) + Gw_i(t) \\ y_i(t) = C_1x_i(t) \\ z_i(t) = C_2x_i(t) \end{cases} \tag{9}$$

where $x_i(t) \in R^4, y_i(t) \in R^4, z_i(t) \in R^2, u_i(t) \in R^2, w_i(t) \in R^4$ represent the motion state, the measured output of the control system, the controlled output, the control input and the external disturbance of the ith missile respectively. The constants in (9) are defined as follows.

$$A = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix}, B = \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix}, G = I_4, C_1 = I_4, C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Based on the model for each missile in (9), the Kronecker product is used to derive the model for the entire missile convergence control system as follows.

$$\begin{cases} \dot{x}(t) = A_Dx(t) + B_Du(t-\tau) + G_Dw(t) \\ y(t) = C_{1D}x(t) \\ z(t) = C_{2D}x(t) \end{cases} \tag{10}$$

For a cooperative convergence control system with *N* missiles, the constants in (10) are defined as follows.

$$A_D = I_N \otimes A, B_D = I_N \otimes B, G_D = I_N \otimes G, \\ C_{1D} = I_N \otimes C_1, C_{2D} = I_N \otimes C_{2D} \tag{11}$$

The solution to the convergence problem is to design a state feedback controller for each object with a decentralized controller of the following form.

$$u_i = \sum_{j \in N_i} K_j^{fb} x_j + K_i^{fb} x_i \tag{12}$$

where K_j^{fb} is the feedback gain to be determined. The controller is designed so that the state variables of each object converge to the same vector. For any i, j, we have the limitation as follow.

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0 \tag{13}$$

where $\|*\|$represents the Euclidean norm of the vector.

The LMI approach proposed is now used to design a state feedback controller to implement convergent control of the missile in the following form.

$$u(t) = KL_C x(t) \quad (14)$$

where K is the external controller gain.

The design goal is to have all missiles eventually converge to the origin of the coordinate system we have established.

Under the assumption that the external disturbances to the system are satisfied $w(t) \in L_2[0,\infty)$, consider the use of L_2/L_∞ performance metrics to measure the ability of the control system to attenuate the disturbances. The convergence problem can be briefly stated as follows: consider the system (10), determine the controller gain K in (14) so that multiple missiles converge to the origin of the coordinate system in the condition of time delay and external disturbances. For a given constant μ, the transfer function of the system satisfies the following equation.

$$\frac{\|z(t)\|_\infty}{\|w(t)\|_2} < \mu \quad (15)$$

So, the model of the close-loop system can be expressed as follow.

$$\begin{cases} \dot{x}(t) = A_D x(t) + B_D K L_C x(t-\tau) + G_D w(t) \\ y(t) = C_{1D} x(t) \\ z(t) = C_{2D} x(t) \end{cases} \quad (16)$$

3. MAIN RESULT

Before design the controller of multiple missiles, the following lemmas are given.

Lemma 1: Suppose $a(*) \in R^{n_a}, b(*) \in R^{n_b}, C \in R^{n_a \times n_b}$ all are definedon the interval Ω, then for any matrix $X \in R^{n_a \times n_b}, Y \in R^{n_a \times n_b}, Z \in R^{n_a \times n_b}$, we can get the following equation.

$$-2\int_\Omega a^T(\alpha) C b(\alpha) d\alpha \le \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - C \\ Y^T - C^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha \quad (17)$$

where the matrixX, Y, Zsatisfies$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$.

Lemma 2: Suppose $M, P, Q$is a matrix of suitable dimensions given and satisfies $Q > 0$,then the following inequality is equivalent.

$$P + MQ^{-1}M^T < 0 \Leftrightarrow \begin{bmatrix} P & M \\ M^T & Q \end{bmatrix} < 0 \quad (18)$$

Considering the case of external disturbances and input time delay, the state feedback controller is designed using the form of (14) so that the close-loop system remains stable and each missile finally converges to the target, that is the origin of the coordinate system. Meanwhile the performance indexL_2/L_∞ of the system meets the requirements in (16).

Theorem 1 Consider the system model (16), when there exist matrices with positive definite and symmetric matrices L, R, W and matrices M, N satisfying the following matrix inequalities (19) to (21), the close-loop system is asymptotically stable at a given performance index $\mu > 0$. And for any constant time delay τ satisfies$0 < \tau < \tau_{max}$.

$$\begin{bmatrix} \Phi & B_D V - N & G_D & L A_D^T \tau_{max} \\ * & -W & 0 & V^T T_D^T \tau_{max} \\ * & * & -I & G_D^T \tau_{max} \\ * & * & * & -R\tau_{max} \end{bmatrix} < 0 \quad (19)$$

where $\Phi = \bar{L} A_D^T + A_D \bar{L} + \bar{M}\tau_{max} + \bar{N} + \bar{N}^T + \bar{W}$

$$\begin{bmatrix} M & N \\ N^T & LR^{-1}L \end{bmatrix} > 0 \quad (20)$$

$$\begin{bmatrix} L & LC_{2D}^T \\ C_{2D} & \mu^2 \end{bmatrix} > 0 \quad (21)$$

The state feedback control rate of the multiple missiles convergent system is given by the following equation.

$$u(t) = VL^{-1}x(t) \quad (22)$$

Then the gain of the controller in (22) is $K = VL^{-1}L_C^{-1}$.

Proof: According to the Newton-Leibniz formula, there are:

$$x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(\theta) d\theta \quad (23)$$

Substituting into the first equation in (30), we have:

$$\dot{x}(t) = (A_D + B_D K L_C) x(t) - B_D K L_C \int_{t-\tau}^{t} \dot{x}(\theta) d\theta + G_D w(t) \quad (24)$$

Let $KL_C = \widetilde{K}$, the equation (24) can be abbreviated as:

$$\dot{x}(t) = (A_D + B_D \widetilde{K}) x(t) - B_D \widetilde{K} \int_{t-\tau}^{t} \dot{x}(\theta) d\theta + G_D w(t) \quad (25)$$

Choose the following Lyapunov function.

$$V(x(t)) \triangleq V_1(x(t)) + V_2(x(t)) + V_3(x(t)) \quad (26)$$

Among them:

$$V_1(x(t)) \triangleq x^T(t) P x(t) \quad (27)$$

$$V_2(x(t)) \triangleq \int_{-\tau}^{0} \int_{t+\beta}^{t} \dot{x}^T(\alpha) Z \dot{x}(\alpha) d\alpha d\beta \quad (28)$$

$$V_3(x(t)) \triangleq \int_{t-\tau}^{0} \int_{t+\beta}^{t} x^T(\alpha) Q x(\alpha) d\alpha d\beta \quad (29)$$

where $P = P^T > 0, Z = Z^T > 0, Q = Q^T > 0$ is the matrix to be selected.

Taking the derivative of $V_1(x(t))$ and substituting (25) in it, we can get:

$$\dot{V}_1(x(t)) = x^T(t) \left[(A_D + B_D \widetilde{K})^T P + P(A_D + B_D \widetilde{K}) \right] x(t)$$
$$-2x^T(t) P B_D \widetilde{K} \int_{t-\tau}^{t} \dot{x}(\theta) d\theta + w^T(t) G_D^T P x(t) + x^T(t) P G_D w(t) \quad (30)$$

Defining $a(*) \triangleq x(t), b(*) \triangleq \dot{x}(t), C = PB_D\widetilde{K}$, and using Lemma 1, we get:

$$-2x^T(t)C\int_{t-\tau}^{t}\dot{x}(\alpha)d\alpha \le \int_{t-\tau}^{t}\begin{bmatrix}x(t)\\ \dot{x}(\alpha)\end{bmatrix}^T\begin{bmatrix}X & Y-C\\ Y^T-C^T & Z\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(\alpha)\end{bmatrix}d\alpha$$

$$\le \tau_{max}x^T(t)Xx(t) + 2x^T(t)\left(Y-PB_D\tilde{K}\right)[x(t)-x(t-\tau)] + \int_{t-\tau}^{t}\dot{x}(\alpha)Z\dot{x}(\alpha)d\alpha \quad (31)$$

$$\begin{bmatrix}X & Y\\ Y^T & Z\end{bmatrix}\ge 0 \quad (32)$$

Substituting (31) and (32) into (30), we get:

$$\dot{V}_1(x(t)) \le x^T(t)(A_D^TP + PA_D + \tau_{max} + Y + Y^T)x(t) + 2x^T(t)(Y-PB_D\tilde{K})x(t-\tau) + w^T(t)G_D^TPx(t) + x^T(t)PG_Dwd) + \int_{t-\tau}^{t}\dot{x}(\alpha)Z\dot{x}(\alpha)d\alpha \quad (33)$$

Similarly, taking the derivative of $V_2(x(t)), V_3(x(t))$, we get:

$$\dot{V}_2(x(t)) = \tau\dot{x}^T(t)Zx^T(t) - \int_{t-\tau}^{t}\dot{x}(\alpha)Z\dot{x}(\alpha)d\alpha \le -\tau_{max}(\Gamma)\int_{t-\tau}^{t}\dot{x}(\alpha)Z\dot{x}(\alpha)d\alpha \quad (34)$$

where $\Gamma = A_Dx(t) + G_Dw(t) + B_D\tilde{K}x(t-\tau)$.

$$\dot{V}_3(x(t)) = x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) \quad (35)$$

Combining (33~35), we get the derivative of $V(x(t))$ as:

$$\dot{V}(x(t)) = \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t)) \le x^T(t)\left(A_D^TP + PA_D + \tau_{max}(\Lambda)^TZ(\Lambda)\right) + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) + w^T(t)G_D^TPx(t) + x^T(t)PG_Dw(t) \quad (36)$$

where $\Lambda = A_Dx(t) + G_Dw(t) + B_D\tilde{K}x(t-\tau)$

Under zero initial conditions, we can get $V(x(t))|_{t=0} = 0$ easily, then we consider the following performance index function.

$$J_T \triangleq V(x(t)) - \int_0^t w^T(s)w(s)ds \quad (37)$$

Transforming (37) under zero initial conditions,we get:

$$J_T \triangleq \int_0^t\left[\dot{V}(s) - w^T(s)w(s)\right]ds \quad (38)$$

Let $\eta(t) \triangleq [x(t)x(t-\tau)w(t)]^T$,we can transform (38) into the (39) as follow.

$$J_T = \int_0^t \eta^T(s)\Pi\eta(s)ds \quad (39)$$

where $\Pi = \begin{bmatrix}\Phi_1 & PB_DK - Y + \tau_{max}A_D^TZB_DK & \tau_{max}A_D^TZG_D + PG_D\\ * & -Q + \tau_{max}K^TB_D^TZB_DK & \tau_{max}K^TB_D^TZG_D\\ * & * & -I + \tau_{max}G_D^TZG_D\end{bmatrix}$

and $\Phi_1 = LA_D^T + A_DL + \tau_{max}X + Y + Y^T + Q + \tau_{max}A_D^TZA_D$

If $\Pi < 0$, we have $J_T < 0$ in (37), then we get the following inequality.

$$x^T(t)Px(t) \le V(x(t)) < \int_0^T w^T(s)w(s)ds \quad (40)$$

Using Lemma 3.2, equation (21) is equivalent to (41) as follow.

$$C_{2D}^TC_{2D} < \mu^2P \quad (41)$$

In summary, using (16), (40), and (41), we can draw the conclusions as follow.

$$z^T(t)z(t) = x^T(t)C_{2D}^TC_{2D}x(t) < \mu^2x^T(t)Px(t) < \mu^2\int_0^t w^T(s)w(s)ds \quad (42)$$

Since $\mu^2\int_0^t w^T(s)w(s)ds < \mu^2\int_0^\infty w^T(s)w(s)ds$, for $t > 0$ and $\omega(t) \in L_2[0,\infty)$, we obtain $\|z(t)\|_\infty^2 < \mu^2\|w(t)\|_2^2$, so that the requirements for the control system L_2/L_∞ performance index (15) are met.

Using Lemma 3.2 again, $\Pi < 0$ equivalent to (43) as follow.

$$\Pi = \begin{bmatrix}\Phi_2 & PB_DK - Y & PG_D & \tau_{max}A_D^T\\ * & -Q & 0 & \tau_{max}K^TB_D^TZ\\ * & * & -I & \tau_{max}G_D^TZ\\ * & * & * & -\tau_{max}Z\end{bmatrix} < 0 \quad (43)$$

$\Phi_2 = LA_D^T + A_DL + \tau_{max}X + Y + Y^T + Q$

Next, define$L \triangleq P^{-1}$, and multiplying the (43) first by the left matrix $diag(L \quad L \quad I \quad Z^{-1})^T$, and then by the right matrix$diag(L \quad L \quad I \quad Z^{-1})$, we have (44) as follow.

$$\Pi = \begin{bmatrix}\Phi_3 & B_DKL - LYL & G_D & \tau_{max}LA_D^T\\ * & -LQL & 0 & \tau_{max}LK^TB_D^T\\ * & * & -I & \tau_{max}G_D^T\\ * & * & * & -\tau_{max}Z^{-1}\end{bmatrix} < 0 \quad (44)$$

$\Phi_3 = LA_D^T + A_DL + \tau_{max}LXL + LYL + LY^TL + LQL$

Similarly, multiplying the (32) first by the left matrix $diag(L \quad L)^T$, and then by the right matrix$diag(L \quad L)$, we have (48) as follow.

$$\begin{bmatrix}LXL & LYL\\ LY^TL & LZL\end{bmatrix}\ge 0 \quad (45)$$

Now, let $V \triangleq FL, M \triangleq LXL, N \triangleq LYL, W \triangleq LQL, R \triangleq Z^{-1}$, and substituting them into (44) and (45), we obtain (19) and (20) in Theorem 3.1. The proof of Theorem 3.1 is completed.

However, equation (21) contains $LR^{-1}L$ term , which is not a linear term. Therefore (21) is not a linear matrix inequality. To solve this problem, a new variable S is defined to the following equation.

$$LR^{-1}L \ge S \quad (46)$$

And let $T = S^{-1}, U = L^{-1}, G = R^{-1}$, equation (21) can be transformed into (47).

$$\begin{bmatrix}M & N\\ N^T & S\end{bmatrix}\ge 0, \begin{bmatrix}T & U\\ U & G\end{bmatrix}\ge 0, \begin{bmatrix}S & I\\ I & T\end{bmatrix}\ge 0, \begin{bmatrix}L & I\\ I & U\end{bmatrix}\ge 0, \begin{bmatrix}R & I\\ I & G\end{bmatrix}\ge 0 \quad (47)$$

By solving the linear matrix inequality set formed by the union of (19), (20) and (47).The design of the multiple missilescontroller in the situation of external disturbances and input time delay is finished, and meet the control system L_2/L_∞ performance index.

4. SIMULATION RESULTS

Using the model given in (10), the control method proposed in section 3 is verified by simulation in the Matlab. The flight trajectory of the missiles is given to show the effectiveness of the controller.

4.1 Numerical calculation of controller

Set the controller parameters as:

$$\text{Number of missiles:} N = 3$$

$$\text{Max Input Time Delay:} \tau_{max} = 0.5$$

$$L_2/L_\infty \text{performance index: } \mu = 0.7$$

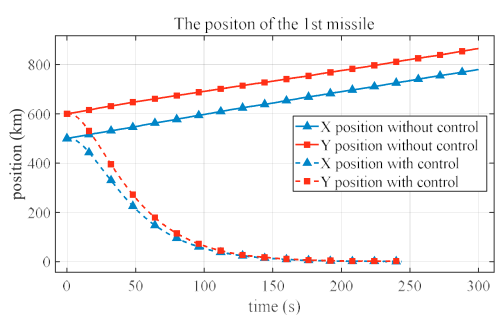
Using the control system model given in equation (30), the set of linear matrix inequalities based on the union of equations (33), (35) and (61) in section 3, and using LMI toolbox in Matlab, the controller gain K is obtained.

4.2 Flight trajectory simulation under small disturbances

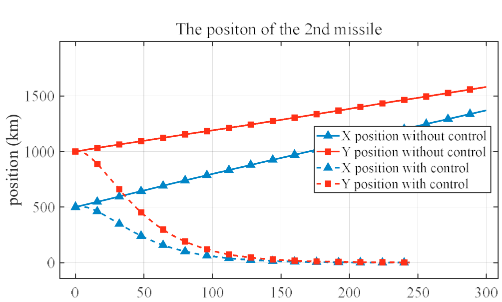
Assume that the initial values of the three missile positions (km) and velocities (km/s) are as follows.

$$x_{10} = [500 \quad 500 \quad 1 \quad 1]^T$$
$$x_{20} = [500 \quad 1000 \quad 3 \quad 2]^T$$
$$x_{30} = [1000 \quad 800 \quad 2 \quad 1]^T$$

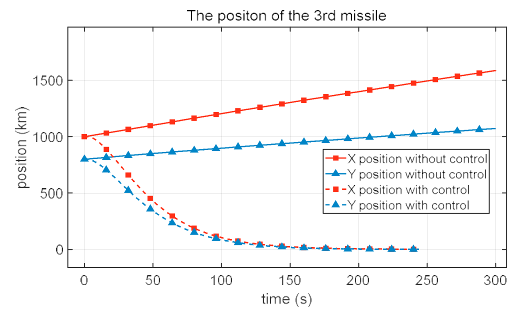
Let the input time delay be 0.2, and we use *rands(12,1)* to simulate the small disturbances during the flight. The time-dependent trajectories of the missiles in the plumb plane under controlled and uncontrolled conditions are as follow.



(a)The position of first missile



(b)The position of second missile



(c)The position of third missile

Fig.1. The position of 3 missiles change with time under small disturbances (with or without control)

From the Fig.1 above we can see that the controller took effect on missiles. But the control effect didn't happen immediately due to the time delay we set on the controller. Missiles flight with the velocities been given on them as initial values, it takes about 2 seconds since simulation began. And then missiles with control converged to zero under small disturbances at about 240 seconds, while missiles without control maintained original speed, deviated from target.

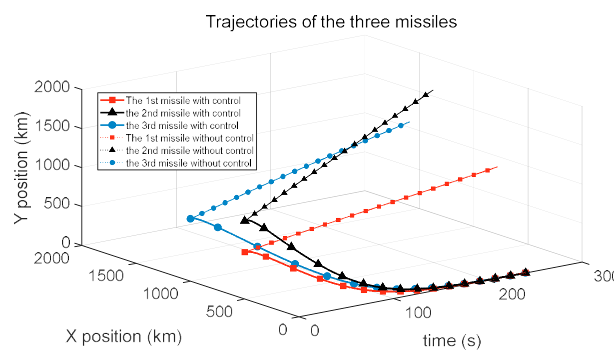
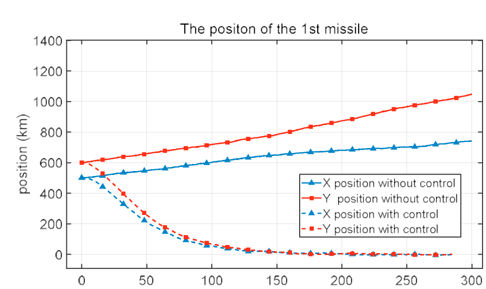


Fig.2. 3D simulation of missiles flight trajectory under small disturbances(with or without control)

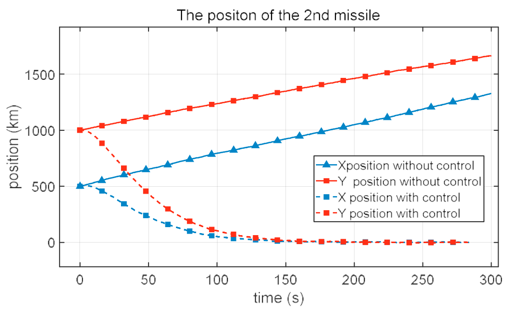
Fig.2 displays an overview of missiles' trajectory in the 3D coordinate system formed by time and plumb plane. We can compare the difference more visually in the flight trajectory of the missile with or without control.

4.3 Flight trajectory simulation under large disturbances

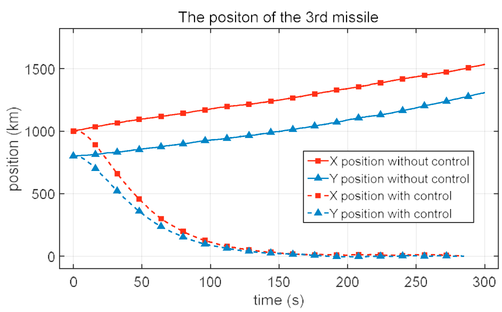
The initial values of missiles are the same with the condition under small disturbances. We use *100*rands(12,1)* to simulate the large disturbances during the flight.



(a)The position of first missile



(b)The position of second missile



(c)The position of third missile

Fig.3. The position of 3 missiles change with time under large disturbances (with or without control)

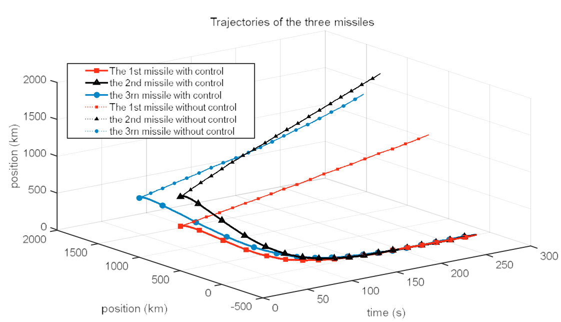


Fig.4. 3D simulation of missiles flight trajectory under large disturbances (with or without control)

In Fig.3, it is apparent that missiles under large disturbances had obvious deviation compared with Fig.1. Although all three missiles converged to the desired position with control under large disturbances, the simulation time became longer. This is more realistic and demonstrates the controller's ability to resist interference.

5. CONCLUSIONS

In this paper, a multiple missiles convergence control method is proposed, and a rubust controller meet the performance index L_2/L_∞ for the purpose of anti-interference is designed. The design of the controller is relatively clear. The solution process is simple, and the gain of the controller can be solved directly by computer, which is convenient to be applied to the flight control system of the missile. The method can be easily combined with other control methods, has potential for use in actual battlefield situation

REFERENCES

Elahi, A., Alfi, A. and Modares, H. (2019). H-infinity consensus control of discrete-time multi-agent systems under network imperfections and external disturbance, *IEEE-CAA JOURNAL OF AUTOMATICA SINICA*, 6(3), pp. 667–675. doi: 10.1109/JAS.2019.1911474.

Hu, WF. Liu, L. and Feng, G. (2016). Consensus of linear multi-agent systems by distributed event-triggered strategy. *IEEE transactions on cybernetics*,*46*(1), pp. 148-57. doi:10.1109/TCYB.2015.2398892

Hu, QL. Han, T. and Xin, M. (2018). New impact time and angle guidance strategy via virtual target approach, *Journal of Guidance, Control, and Dynamics*, 41(8), pp. 1755–1765. doi: 10.2514/1.G003436

Jeon, IS.Lee, JI. and Tahk, MJ. (2010). Homing guidance law for cooperative attack of multiple missiles, *Journal of Guidance, Control, and Dynamics*, 33(1), pp. 275–280. doi: 10.2514/1.40136.

Lyu, T. Guo, YN. and Other, A.N. (2019). Multiple missiles cooperative guidance with simultaneous attack requirement under directed topologies, *Aerospace Science and Technology*, 89, pp. 100 – 110. doi: 10.1016/j.ast.2019.03.037.

Shaferman, V. and Shima, T. (2017). Cooperative differential games guidance laws for imposing a relative intercept angle, *Journal of Guidance, Control, and Dynamics*, 40(10), pp. 2465–2480. doi: 10.2514/1.G002594.

Yi, X., Lu, W. and Chen, T. (2017). Pull-based distributed event-triggered consensus for multiagent systems with directed topologies, *IEEE Transactions on Neural Networks and Learning Systems, Neural Networks and Learning Systems,* 28(1), pp. 71–79.

Yu, ZY. Jiang, HJ. and Other, A.N. (2016). Consensus of second-order multi-agent systems with delayed nonlinear dynamics and aperiodically intermittent communications. *International Journal of Control*, *90*(5), pp. 909-922.doi: 10.1080/00207179.2016.1187305

Zhao, QL. Dong, XW.and Other, A.N. (2017). Distributed cooperative guidance for multiple missiles with fixed and switching communication topologies. *Chinese Journal of Aeronautics*, (4), p. 1570.