Petroleum 2 (2016) 307–312

Contents lists available at ScienceDirect

Petroleum

journal homepage: www.keaipublishing.com/en/journals/petlm

Original article

The optimal model of oilfield development investment based on Data Envelopment Analysis

Yihua Zhong, Jiao Zhao*

School of Science, Southwest Petroleum University, Chengdu 610500, China

ARTICLE INFO

Article history:
Received 25 November 2015
Received in revised form
21 April 2016
Accepted 23 April 2016

Keywords:
Data envelopment analysis
Oilfield development
Investment optimization
Reserves and productions

ABSTRACT

The investment problem of oilfield development is to trade off the investment exploration investment and development investment. With low return on investment got by using the existing method to solve this problem, we construct an optimal model to improve it based on Data Envelopment Analysis (DEA) method and the relations about investment and proven reserves, investment and output as well as production cost. Data Envelopment Analysis (DEA) method is used to present a method to determine the optimal scale of productivity construction investment in unit production. The relation between total cumulated proven reserves and cumulative exploration investment is denoted as an exponential model. The relation among productions and remaining recoverable reserves as well as production cost may be described as an exponential operational cost function. Based on above two relation models and investment effectiveness coefficients of every block, we establish an optimal model whose objective function is net present value (NPV) profit maximum, whose constrain conditions include investment, reserve/production ratio, production and some equality constraints under the mode of sustainable development. It can be solved by genetic algorithms. The result of case study shows that this optimal investment of oilfield development has multi-stage investment structure under given conditions; the model can provide scientific basic theory for oil companies to make a long-term strategic program and investment plan in oil exploration and development, may decrease the subjective blindness in the investment and bring about a reasonable and orderly exploration and development of oil resources.

1. Introduction

Thorough investment analysis is crucial to the decision makers in the oilfield development due to the large costs associated with production fields, processing facilities, compressor stations, pipelines and other infrastructures. Many scholars have studied the optimal problem of oilfield development and investment. For example, Kjetil Trovik Midthun [1] presented an optimization model for analysis of system development for natural gas fields, processing and transport infrastructure. Zhang Daoyong [2] proposed two relation models which are Gompertz model and Exponential model about the drill footage and proved reserves. Chermak J M and Patrick R H [3] generalized the existing economic theory about exhaustible resource production and made a type test for extended theory of Halvorsen and Smith [4] using successfully a sample of natural gas resources. Livernois J [5] made empirical evidence analysis about cost functions for conventional oil extraction and obtained three meaningful conclusions. Under technical and geological uncertainties, Almeida L F et al. [6] studied the optimization system of valve control in intelligent wells based on decision support system, constructed optimal models maximizing the net present value (NPV), presented an evolutionary algorithm to solve them, and formed proactive well control strategies as well as optimal operation schemes for oilfield.

Although people have obtained some experience for investing optimally to develop oilfield, its return on investment

* Corresponding author.
E-mail addresses: 991441036@qq.com, 470365787@qq.com (J. Zhao).
Peer review under responsibility of Southwest Petroleum University.

Production and Hosting by Elsevier on behalf of KeAi

http://dx.doi.org/10.1016/j.petlm.2016.04.004

is often very low, especially that of Chinese oil companies is much lower. Its reason is not only the changes of oilfield exploitation environment but also some unreasonable investment to reduce the oilfield development benefit. In this paper, we shall use Data Envelopment Analysis (DEA) method to study the problem of return on investment by considering synthetically production and reserves as well as investment. And we shall establish investment yield structure relation model and a multiple objective optimal model about maximum profit and return on investment as well as above investment-yield-structural relationship to improve return on investment of oilfield development.

2. The investment problem of oilfield development and Data Envelopment Analysis

2.1. The investment problem of oilfield development

Exploration and development of oil are the two parts depending on each other of the oil industry, because oil exploration may provide backup reserves for oil development, oil development is to make the oil be exploited from the underground. Thus the investment problem of oilfield development is to trade off the investment between oil exploration and oil development. Because the investment amount of oil exploration and oil development is limited for an oilfield enterprise, if the investment in exploration of oil is too much, then the investment in oil development is surely too less, and funds will be backlogged in the form of reserves. In the same way, if the investment in development of oil is too much, then the investment in oil exploration is surely too less, and oilfield may be exploited overmuch. Whether excessive development for oilfield or the lack of investment in oil exploration may all decrease the explored oil reserves, which will make oilfield enterprise can not conducive to the long-term stable development of oilfield. Therefore, we have to make sure a reasonable scale of investment for oil and gas exploration and development to coordinate the proportion of investment in exploration and development of oil and gas. Only in this way, can we give full play to the overall function of the system and achieve the best economic effect. Then the oilfield enterprises can realize a sustainable and continuous development.

2.2. Data Envelopment Analysis

Data envelopment analysis (DEA) is a nonparametric method in operations research and economics for the estimation of production frontiers. It is used to empirically measure productive efficiency of decision making units (DMUs). When the production process presents a structure of multiple inputs and outputs, it is essentially a linear programming methodology to measure the efficiency of DMUs by as follows model.

Let the number of DMUs as n, in which there are υ input variables x_{ij}, $i=1,2,...,\upsilon, j=1,2,\cdots,n$ and s output variables $y_{rj}(r=1,2,...,s)$. If the maximum ratio of the linear combination of output variables and the linear combination of input variables in j_0-th DMU is set as the goal, and that the ratio of the linear combination of output variables and input variables for all DMUs is less than or equal to 1 is set as constraint conditions, then we get a DEA model of comprehensive relative efficiency evaluation for j_0-th DMU. By the Charnes Cooper conversion, its specific form is as Equation (1-1) [6].

$$\min \quad \theta-\varepsilon\left(\sum_{i=1}^{\upsilon} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$
$$s.t.\begin{cases}\sum_{j=1}^{n}\lambda_j x_{ij}+s_i^-=\theta x_{ij_0}, & i=1,2,...,\upsilon\\ \sum_{j=1}^{n}\lambda_j y_{rj}-s_r^+=y_{rj_0}, & r=1,2,...,s\\ \lambda_j\ge 0, s_i^-\ge 0, s_r^+\ge 0, j=1,2,...,n\end{cases} \tag{1-1}$$

Assume that we solve model (1-1) and get its optimal solution $\theta^*,\lambda_j^*,s_i^{*+},s_r^{*-},\quad j=1,2,\cdots,n\ ;\ i=1,2,\cdots,\nu\ ;\ r=1,2,\cdots,s.$ If $\theta^*=1$, then the j_0-th DMU is weak effective of DEA; if $\theta^*=1,\ s^{*+}=s^{*-}=0$, then the j_0-th DMU is effective of DEA; if $\theta^*<1$, then the j_0-th DMU is not effective of DEA.

3. The method and models about oilfield development investment

3.1. The method to determine the optimal scale of productivity construction investment based on DEA

It will first be solved to determine the investment scale of development engineering construction in unit production in considering oilfield development investment plan, which is always studied by using Empirical analysis method and averaging method. That is to calculate the average investment scale of construction investment for unit productivity in study area according to available data. When use these two methods, the premise is that all the happened investment is reasonable and effective, it does not exclude occasional factor and does not make analysis on the input/output result for previous investment. Therefore, the investment scale for unit productivity obtained by above methods is not always the optimal investment scale. With this weak point, we use the DEA method to determine optimal investment scale of development engineering construction by considering of oilfield practices in this paper.

The optimal scale of productivity construction investment in unit production is determined by using DEA method as follows:

(1) Collect history data of oil development in one year before beginning programming year, and classify the data of different types of developed reservoirs with method of clustering analysis.

In order to determine accurately the optimal scale of productivity construction investment in unit production, we first need to classify the data of different types of developed reservoirs to construct of decision making units (DMUs) of evaluating productive efficiency in one year before beginning programming year.

Let the number of reservoir samples as l, each sample $X_h(h=1,2,\ldots,l)$ represents a category reservoir. In which there are μ input indexes $X_{h\tau}$, $h=1,2,...,l;\tau=1,2,\cdots,\mu$ (such as reserves, drilling investment, surface engineering construction investment, recovery factor, etc.). Then sample matrix is written as:

$$X=\begin{bmatrix}X_1\\ \vdots\\ X_h\\ \vdots\\ X_l\end{bmatrix}=\begin{bmatrix}X_{11} & \cdots & X_{1\tau} & \cdots & X_{1\mu}\\ \vdots & & \vdots & & \vdots\\ X_{h1} & \cdots & X_{h\tau} & \cdots & X_{h\mu}\\ \vdots & & \vdots & & \vdots\\ X_{l1} & \cdots & X_{l\tau} & \cdots & X_{l\mu}\end{bmatrix} \tag{2-1}$$

First, calculate the distance matrix D_0 between two samples according to Equation (2-3):

$$D_0 = \begin{bmatrix} d_{11} & \cdots & d_{1h} & \cdots & d_{1l} \\ \vdots & & \vdots & & \vdots \\ d_{h1} & \cdots & d_{hh} & \cdots & d_{hl} \\ \vdots & & \vdots & & \vdots \\ d_{l1} & \cdots & d_{lh} & \cdots & d_{ll} \end{bmatrix} \tag{2-2}$$

$$d_{hk}^2 = \sum_{\tau=1}^{\mu} (X_{h\tau} - X_{\kappa\tau})^2 \quad h,k = 1,2,\cdots,l \tag{2-3}$$

Then, find the smallest distance $d_{i'j'}^2$, and accordingly merge $X_{i'}$ and $X_{j'}$ as a new category, written as X_{l+1}:

$$X_{n+1} = \begin{bmatrix} \frac{x_{i'1}}{x_{j'1}} & \frac{x_{i'2}}{x_{j'2}} & \cdots & \frac{x_{i'\mu}}{x_{j'\mu}} \end{bmatrix} \tag{2-4}$$

Re-calculate category distance between new category X_{l+1} and other categories according to Equation (2-4), obtain first order category distance matrix D_1, and merge two nearest categories into one new category, written as X_{l+2}. Repeat above calculation till the reasonable number of categories (ϖ) is acceptable, that is, the reservoir samples are finally classified into ϖ categories, which are treated as ϖ DMUs of DEA model (1-1).

(2) Analyze respectively on the investment effectiveness for every category of reservoirs.

For one group of data on each category of reservoir, by using model (1-1), let investment data (input variable) as $g_{ij}, i = 1,2,$ $j = 1,2,\cdots,\varpi$, output data (output variable) as production q_{rj}, $r = 1$, calculate investment effectiveness judgment factor θ^* for each group of data according to DEA model (2-5) in the following way, and determine whether investment locates in effective set.

$$\min\ \theta^* = \theta - \varepsilon\left(\sum_{i=1}^{2} s_i^- + s_r^+\right)$$
$$s.t.\begin{cases} \sum_{j=1}^{\varpi} \lambda_j g_{ij} + s_i^- = \theta g_{ij} & i = 1,2 \\ \sum_{j=1}^{\varpi} \lambda_j q_{rj} - s_r^+ = q_{rj} & r = 1, j = 1,2,\cdots,\varpi \\ \lambda_j \ge 0,\ s_i^- \ge 0,\ s_r^+ \ge 0 \end{cases} \tag{2-5}$$

For the $\varpi - th$ category of reservoir, if there are r_ω groups of input–output data whose investment effectiveness factor $\theta^* = 1$, then the sum of drilling investment in production wells is expressed as $\sum_{l=1}^{r_\omega} g'_{1l}$; the sum of surface engineering construction investment is expressed as $\sum_{l=1}^{r_\omega} g'_{2l}$; the sum of oil production is expressed as $\sum_{l=1}^{r_\omega} q'_{1l}$.

Thus, the effective drilling investment α for unit productivity in each category of reservoirs is calculated as:

$$\alpha = \frac{\sum_{l=1}^{r_\omega} g'_{1l}}{\sum_{l=1}^{r_\omega} q'_{1l}} \tag{2-6}$$

The effective surface engineering construction investment β for unit productivity for each category of reservoir is calculated as:

$$\beta = \frac{\sum_{l=1}^{r_\omega} g'_{2l}}{\sum_{l=1}^{r_\omega} q'_{1l}} \tag{2-7}$$

3.2. The model of investment and output on oilfield development

3.2.1. A model of exploration investment and proven reserves growth

The growth of total proven reserves of oil companies usually experiences the process of low-high-slow, which has been used to establish many prediction models using extrapolation method by many scholars [7–10]. Wang YuTao et al. [7] analyze changing rule of the proved reserves and exploration benefit in xinjiang oilfield, come at the exploration investment growth is an important guarantee of reserves continue to grow steadily. Other mathematic models for predicting production or recoverable reserves include Logistic model [8], Hubbert model [9] and Gompertz model [10].

These models have certain rationality for the prediction of oil company reserves growth. However, as the models illustrate, the rules that accumulated proven reserves grow with time are just a superficial phenomenon. Intrinsically, exploration results are achieved based on the exploration workload in certain period, whereas, the completion of exploration workload is on the premise of certain exploration investment. So it is more reasonable to establish a relation model between proven reserves and exploration workload, which is usually considered the exponent curve type. Of course, its relation model can also be expressed between proven reserves N and exploration investment f as Equation (2-8) [11].

$$N = b - a\exp(-kf) \tag{2-8}$$

This model indicates the growth rules of proven reserves to be predicted for geological unit with simple geology condition and less types of reservoirs, that is, larger scale oil reservoirs are discovered in primary period of investment, proven reserves rise faster in primary period of exploration; however, as the major reservoirs and main layers are discovered successively, exploration difficulty gradually increases and reserves rise slowly in their middle and later periods. Equation (2-8) shows the relation of cumulate proven reserves and exploration investment, whose coefficients $a > 0, b > 0, k > 0$ may be computed by fitting historical data of cumulate proven reserves and cumulate exploration investment in the different exploration period using Least Square Method.

3.2.2. The relationship model of output and investment

Oilfield output depends on the developed blocks and the exploration investment after production. So the relationship model of output and exploration investment is as follows [11].

$$q_t = \sum_{k=1}^{t} \lambda_{t-k+1} w_k + w_t \quad t = 1,2,\ldots,T \tag{2-9}$$

$$w_t = \frac{1}{\alpha + \beta} I_t^D \quad t = 1,2,\ldots,T \tag{2-10}$$

3.2.3. The model of production cost

According to Chinese accounting system, development investment of an oil company is divided into two parts: one part is the investment of new productivity construction, namely oilfield development engineering construction investment, which includes drilling investment and surface engineering construction investment; the other part is the investment maintaining production capacity, namely oilfield operating cost.

Many studies have been done on production cost of un-renewable resources [3–5,12], and production cost is thought to be a regression model about wells, water content and oil price etc. [4], or a function of accumulated output and remaining recoverable reserves [3,5]. Through the studies on various regression models, it is found that the logarithmic form of oil production cost model (2-11) has the best fitting degree.

$$C_t = \exp(a_1 + b_1 \ln N_{pt} + b_2 \ln R_t + b_3 \ln \tau_t) \tag{2-11}$$

4. The optimization model of investment on petroleum exploration and development

4.1. Decision-making variables

The goal for investment optimization of exploration and development is to find a rational ratio of exploration investment and development investment in a long period of time, so that an oil company can get the maximum profit. Two decision-making variables are set as I_t^E,I_t^D, where I_t^E means exploration investment in the t-th year and I_t^D means development investment in the t-th year.

4.2. Objective function

For the Oil Company with m oil bearing blocks, the objective function of n years developing these m oil bearing basins is net present value (NPV) maximum, which can be described as Equation (3-1).

$$\max \quad NPV = \sum_{t=1}^{n}\left[\sum_{j=1}^{m}\left(P_t q_{t,j} - I_j^E - I_j^D - C_{t,j}\right)(1+r_t)^{-t}\right] \tag{3-1}$$

4.3. Restrictive conditions

(1) Boundary condition

The sum of I_t^E and I_t^D should not exceed I_t, where I_t means total investment limits in the t-th year. That is,

$$I_t^E + I_t^D \le I_t \tag{3-2}$$

(2) Reserve/production ratio restriction

In order to determine the reserves, we need to consider the quality of its reserves, the decline rate and moisture content in the oilfield exploration and development stage. According to the actual oilfield to determine the reserves which can stable and increase yield, this is expressed as [13].

$$\phi_t \le \sum_{j=1}^{m} N_{t,j} \Big/ \sum_{t=1}^{n} q_{t,j} \tag{3-3}$$

(3) Production restriction

In order to ensure continuity of oilfield development activities, avoid excessive and destructive development every year, we should have exploration and development activities ensuring the continuity of a maximum yield limit, assumed to max(q_t) [13], the annual production should meet the following constraints:

$$q_t \le \max(q_t) \tag{3-4}$$

(4) Development investment constraint

Exploration investment is the preparation of development, so before drilling investment, exploration investment no less than a regulation coefficient ρ of development investment

$$\sum_{k=1}^{t} I_k^E \le \rho \sum_{k=1}^{t} I_k^D \tag{3-5}$$

4.4. Constructing optimal model

$$\max \quad NPV = \sum_{t=1}^{n}\left[\sum_{j=1}^{m}\left(P_t q_{t,j} - I_j^E - I_j^D - C_{t,j}\right)(1+r_t)^{-t}\right]$$

$$s.t.\begin{cases} I_t^E + I_t^D \le I_t \\ \phi_t \le \sum_{j=1}^{m} N_{t,j} \Big/ \sum_{t=1}^{n} q_{t,j} \\ q_t \le \max(q_t) \\ (1/\rho)\sum_{k=1}^{t} I_k^E \le \sum_{k=1}^{t} I_k^D \end{cases} \tag{3-6}$$

This model is a non-linear programming model related with complicated variables. This optimization model can't be solved with traditional methods. But it can be resolved with genetic algorithms. In genetic algorithms objective function is demanded to be calculable, rather than continuous or differentiable. And genetic algorithms is an optimal search in the whole scope, especially suitable for the complicated non-linear problems, therefore it is adopted to resolve the above problem.

5. Case study

The case calculation is based on the data of several operating blocks of an oilfield in china. Suppose the oil company needs to draw up an investment plan for exploration and development investment in the next ten years.

The data of drilling investment, surface engineering construction investment and oil output data of 10 reservoir blocks are shown in Table 1 in 2005. Because the type of these reservoir blocks is simple, they are directly taken as 10 DMUs of DEA model (1-1). Investment effectiveness coefficients (θ) of every block can be got by using the method presented in Section 2.1. As shown in Table 1, oil development engineering investments in the blocks of No.1, 5, 8 and 10 are effective according to DEA method. Thus the relative effective coefficients of productivity investments are calculated with Equations (2-6) and (2-7) to get $\alpha = 110$ Yuan/t; $\beta = 60$ Yuan/t.

The data of exploration investment and new incremental proven reserve in the past 10 years are listed in Table 2. We used Least Square Method to get the relation model (4-1) of

Table 1
The data of productivity investment and output for each block in 2005.

Serial no. of block	Drilling investment (10^4 yuan)	Surface engineering construction investment (10^4 yuan)	Output (10^4 t)	θ
1	649	778	15.3	1.0000
2	812	772	11.9	0.9455
3	715	1172	17.3	0.9310
4	752	88	12.2	0.9250
5	760	1130	15.4	1.0000
6	649	663	6.9	0.9996
7	648	921	18.6	0.9872
8	715	546	9.5	1.0000
9	718	1468	171	0.9265
10	678	342	8.3	1.0000

Table 2
Data of exploration investment and proven reserves.

Serial no. of investment year	Exploration investment (10^4 yuan)	Increased proven reserves (10^4t)	Relative error ratio (%)
2006	4396	40.45	20.24
2007	4098	59.62	42.51
2008	1000	20.76	20.40
2009	1001	21.21	4.49
2010	1771	10.61	0.72
2011	19,869	72.19	1.60
2012	10,430	91.35	11.06
2013	12,208	131.2	19.83
2014	19,159	115.94	6.72
2015	18,120	140.45	7.14

Table 3
Oil operating cost, output and remaining recoverable reserves.

Serial no. of investment year	Output (10^4 t)	Remaining recoverable reserves (10^4 t)	Actual cost (Yuan/t)	Predicted cost (Yuan/t)	Relative error ratio (%)
2006	84	2674	340.42	354.58	4.16
2007	84	2617	352.81	359.81	1.98
2008	87	2557	350.68	360.54	2.66
2009	99	2490	348.17	360.01	3.55
2010	114	2413	362.17	389.35	7.50
2011	128	2327	421.57	422.14	0.14
2012	127	2242	509.20	506.16	−0.60
2013	126	2158	534.92	559.39	4.57
2014	128	2069	577.23	601.52	4.21
2015	131	1982	590.07	626.45	6.16

accumulated exploration investment and proven reserves growth, which is by MATLAB R2013b software to fit regression Equation (2-8) of the exponential proved reserves model and its corresponding parameters a, b, k according to the real historical data of 10 years. At the same time, we also used obtaining model (4-1) to predict the exploration investment of in past 10 years.

$$N = 1.056 - 120.4\exp\left(-1.993\times 10^{-5}f\right) \tag{4-1}$$

The data of oilfield operating cost and oil output as well as remaining recoverable reserves in the past ten years are listed in Table 3. The oilfield operating cost function model (4-2) is fitted according to Equation (2-11) with the relative data in Table 3 by MATLAB R2013b software.

$$\ln c_t = 242.1625 - 16.0159\ln N_{Pt} - 7.4598\ln R_t + 10.8764\ln \tau_t \tag{4-2}$$

Based on the optimization model of (3-6) and the data on each year total investment and comprehensive production annually declining rate in the Table 4 as well as above prediction results, we constructed the optimal model to make a planning of exploration and development investment from 2006 to 2015. Moreover, we solved it by using genetic algorithm, and got annual exploration investment and annual development investment as well as the net present value, which were shown in Table 4. In order to check the model, the real net present value was also shown in Table 4.

The rational scale of exploration investment on this oilfield is 4.583 billion Yuan, development investment is 11.777 billion Yuan, the average of exploration and development investment ratio is 7.18%, the total predictive net present value of exploration and development investment is 15.4492 billion Yuan, and the real net present value of exploration and development investment is 14.3191 billion Yuan. Comparing the theoretical net present value to the real net present value of oilfield, we found that these results fit with the actual and show that the oilfield exploration and development investment efficiency is better, which may laid a solid foundation for Long-term and stable development of oilfield enterprises.

6. Conclusions

With respect to the investment structure in exploration and development of oil companies, the following ideas are drawn: the relation between proven reserves and exploration investment can be described by exponent model; the relation model between oil productivity and oil development engineering

Table 4
The data of investment.

Serial no. of investment year	Total investment (10^4 yuan)	Comprehensive production annually declining rate (%)	Construction investment in unit productivity (Yuan.t^{-1})	Annual exploration investment (10^4 yuan)	Annual development investment (10^4 yuan)	Net present value (10^4 yuan)	The real net present value (10^4 yuan)
2006	85,000	0	110	25,300	59,700	112,547	107,289
2007	80,000	0	120	24,100	55,900	136,472	130,932
2008	98,000	2	132	28,700	69,300	160,287	151,900
2009	118,000	1	133	41,500	76,500	181,910	171,520
2010	120,000	3	159	42,000	78,000	227,715	208,923
2011	115,000	6	160	25,600	89,400	200,523	189,477
2012	153,000	10	158	32,200	120,800	170,347	153,645
2013	238,000	12	172	67,100	170,900	126,074	119,863
2014	278,000	14	183	72,300	205,700	119,904	99,223
2015	351,000	15	212	99,500	251,500	109,141	99,134
Total/average	1,636,000	7.18	1539	458,300	1,177,700	1,544,920	1,431,906

construction investment can be got with DEA method; the optimal solution of long-term total NPV model can be got with genetic algorithms and simple programming. Above methods can provide foundation for petroleum companies to make long-term exploration and development strategic planning. Moreover, they may also provide references for reducing blindness and subjectivity during exploration and development investment.

Nomenclature

θ judgment coefficient of investment effectiveness, dimensionless, $0 \le \theta \le 1$;
θ^* optimal solution of investment effectiveness, dimensionless, $0 \le \theta^* \le 1$;
ε infinitesimal of non-Archimedes
$x_{ij}(i = 1, 2, ..., \upsilon,\ \ j = 1, 2, ..., n)$ investment amount of j-th DMU for i-th input variable;
$y_{rj}(r = 1, 2, ..., s)$ output quantity of the j-th DMU for the r-th output variable;
s_i^-, s_r^+ slack variable;
λ_j decision-making variable of DEA model;
$X_h(h = 1, 2, ..., l)$ the $h - th$ reservoir sample;
$X_{h\tau},\ \ h = 1, 2, ..., l; \tau = 1, 2, \cdots, \mu$ the $\tau - th$ input index of the $h - th$ reservoir sample;
$g1j$ drilling investment of development wells in $j-th$ category reservoir, 10^4 Yuan;
$g2j$ surface engineering construction investment of development wells in $j-th$ category reservoir, 10^4 Yuan;
q_{1j} oil output of development wells in $j-th$ category reservoir, 10^4t;
l serial number of samples of effective investment reservoir ($l = 1, 2, ..., r_\omega$);
g'_{1l} drilling investment of development wells in $l - th$ effective investment reservoir, 10^4 Yuan;
g'_{2l} surface engineering construction investment of development wells in $l - th$ effective investment reservoir, 10^4 Yuan;
q'_{1l} oil output of development wells in $l - th$ effective investment reservoir, 10^4t;
α effective drilling investment for unit productivity in each category of reservoir, Yuan/t;
β effective surface engineering construction investment for unit productivity in each category of reservoirs, Yuan/t;
N accumulated proven reserves, 10^4t;
f accumulated exploration investment, 10^6Yuan;
a,b,k coefficients to be determined, $a > 0, b > 0, k > 0$;
$q_{t,j}$ oil output of the j-th oil bearing blocks in the t-th year, 10^4t;
λ_{t-k+1} decline factor of output in the $t - k + 1$-th years, %;
w_t new capacity in the t-th year, 10^4t;
I_t^D development investment in the t-th year, 10^4 Yuan;
C_t oil & gas production operating cost in the t-th year, 10^4 Yuan;
a_1,b_1,b_2,b_3 fitting coefficient;
N_{pt} production accumulated to the t-th year, 10^4t;
R_t remaining recoverable reserves at the t-th year, 10^4t;
τ_t oil & gas production time accumulated to the t-th year, a;
NPV objective function, represent profits, 10^4 Yuan;
P_i oil price in the t-th year, Yuan/t;
I_t^E exploration investment in the t-th year, 10^4 Yuan;
I_t total investment limits in exploration and development in the t-th year, 10^4 Yuan;
ϕ_t reserve-production ratio in the t-th year;
$q_{t,j}$ output of the j-th oil bearing block in the t-th year, 10^4t;
max (q_t) maximum annual output limits in the t-th year, 10 t;
ρ regulation coefficient, $0 \le \rho \le 1$;

References

[1] K.T. Midthun, M. Fodstad, L. Hellemo, Optimization model to analyse optimal development of natural gas fields and infrastructure, Energy Procedia 64 (2015) 111–119.
[2] Zhang Daoyong, Statistic model based on drill footage proved reserves and its application, J. Daqing Pet. Inst. 30 (2) (2006) 20–22.
[3] J.M. Chermak, R.H. Patrick, A micro econometric test of the theory of exhaustible resources, J. Environ. Econ. Manag. 42 (1) (2001) 82–103.
[4] R. Halvorsen, T.R. Smith, A test of the theory of exhaustible resources, Q. J. Bus. Econ. 106 (1) (1992) 123–140.
[5] J. Livernois, Empirical evidence on the characteristics of extractive technologies: the case of oil, J. Environ. Econ. Manag. 14 (1) (1991) 72–86.
[6] Ye Jin Gen, Zhang Zai Xun, The data envelopment analysis method application in Shengli oilfield investment benefit evaluation, J. Univ. Pet. 25 (2) (2001) 119–124 (in Chinese).
[7] Wang YuTao, Jiang ShaoBin, Li Na, et al., 10 years in xinjiang oilfield petroleum exploration investment benefit and influence factors analysis, Oil Forum 1 (2012) 12–16 (in Chinese).
[8] Liu Guoquan, Jia Li, Liu Juanxia, et al., Prediction method of proven oil reserve growth trend and its application, China Pet. Explor. 19 (4) (2014) 70–74.
[9] Zhu Jie, Che Chang Bo, Liu Cheng lin, et al., Contrastive analysis of reserves increasing prediction models, Xi'an Shiyou Univ. Nat. Sci. Ed. 23 (5) (2008) 21–23.
[10] Li Feng, Zhang XiaoHui, Qu DeBin, et al., Based on the principal component regression model of water drive oilfield operation cost prediction, Oil Gas Technol. 34 (9) (2012) 136–143.
[11] Zheng YuHua, Luo DongKun, Investment optimization in oil and gas plays, Pet. Explor. Dev. 36 (4) (2009) 535–540 (in Chinese).
[12] B. Helmi-Oskoui, R. Narayanan, T. Glover, et al., Optimal extraction of petroleum resources: an empirical approach, Resour. Energy 14 (3) (1987) 267–285.
[13] Bing Yi Li, Yan Bing Lin, Xiao Han Zhao, et al., Based on the dynamic model of oil field exploration and development project investment optimization, Math. Pract. Theory 45 (18) (2015) 20–27 (in Chinese).