Robustness of weighted networks with the harmonic closeness against cascading failures

Article *in* Physica A: Statistical Mechanics and its Applications · November 2019

DOI: 10.1016/j.physa.2019.123373

ARTICLE IN PRESS
Physica A xxx (xxxx) xxx

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Robustness of weighted networks with the harmonic closeness against cascading failures

Yucheng Hao [a,b], Limin Jia [a,b,c,*], Yanhui Wang [a,b,c]

[a] *State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China*
[b] *School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China*
[c] *Beijing Research Center of Urban Traffic Information Sensing and Service Technology, Beijing Jiaotong University, Beijing 100044, China*

ARTICLE INFO

Article history:
Received 25 January 2019
Received in revised form 26 September 2019
Available online xxxx

Keywords:
Cascading failure
Harmonic closeness
Load
Robustness

ABSTRACT

In order to overcome the limitation of existing weighting methods and mitigate the effect of cascading failures, the harmonic closeness is adopted to define the node weight whose strength is controlled by a weight parameter θ, so that the initial load can be obtained by the node weight. We find that regardless of the average degree $< k >$ in Barabási–Albert (BA networks), Newman–Watts (NW networks), and Erdos–Renyi networks (ER networks), the critical threshold T_c achieves the minimum value under optimal θ. In these artificial networks except for NW and ER networks with big tolerance parameter T, the bigger the value of θ, the smaller the value of the normalized avalanche size CF_N. Through the comparison of different methods, a key finding is that the value of T_c obtained by the method proposed in this paper is significantly smaller than the ones by methods concerning the degree and the betweenness in artificial and real networks. In the range of big T, our method results in the smallest value of CF_N in the networks mentioned above compared with previous methods. These results may be helpful for optimizing the distribution of the initial loads in real-life systems, and extending the research on cascading failures in the light of the harmonic closeness.

1. Introduction

Among various infrastructure networks, due to intentional attacks or random disturbances the failures of one or more nodes are likely to cause successive failures of other nodes, or even the paralysis of the entire network. Owing to the serious impact of the cascading failure, many studies paid a great deal of attention to it [1–4]. Motivated by the pioneering works, a large number of researches focused on the robustness of infrastructure networks against cascading failures, such as the power grid [4–13], traffic network [14–17], water distribution networks [18], supply chain networks [19,20], and energy system [21,22]. Thereinto, Schäfer et al. introduced the modeling of cascading failures in electrical transmission networks and presented a predictive model to identify the key nodes and edges in the power grid before and during the cascading failure process [12]. The performance of the public urban rail transit networks was investigated under different attack strategies concerning the degree, the betweenness, and the clustering coefficient, where the load on a node was dependent on its betweenness [17]. Because cascading failures are closely related to the distribution of the initial loads, how to assign the initial load(namely the weight) is key to improve the robustness.

* Corresponding author.
E-mail address: jialm@vip.sina.com (L. Jia).

https://doi.org/10.1016/j.physa.2019.123373

Please cite this article as: Y. Hao, L. Jia and Y. Wang, Robustness of weighted networks with the harmonic closeness against cascading failures, Physica A (2019) 123373, https://doi.org/10.1016/j.physa.2019.123373.

Based on the initial load on the node determined by its degree, cascading failure models were proposed where a tunable parameter controlled the difference of the node weights. It was found that when the tunable parameter is a specific value, the networks are the most robust by theoretical analysis and numerical simulations [23,24]. In the light of the above cascading model, Fu et al. studied the recovery of the network robustness [25,26]. Similarly, the edge weight was defined by the product of the degrees of end nodes to discuss the robustness of the network under attacks on edges [27,28]. In addition to the degree, according to the definition of the initial load on a node related to its betweenness, cascading failures were simulated in the road traffic network [16] and the supply network [20]. The initial load of the node was defined as the total number of shortest paths [1,2] [29], which is similar to the initial load obtained by the betweenness. Mirzasoleiman et al. utilized the node betweenness centrality to define the edge weight, and found that the network with a specific tuning parameter has the best robustness and their weighting strategy is better than others [30]. Additionally, considering the impact of a centrality measure on cascade failures, Ghanbari et al. carried out simulations on artificial and real networks, and found that the bigger the degree, the smaller the cascade depth while the betweenness and the local rank have a positive correlation with the cascade depth [31]. In the same way, the initial load on an edge equaled its betweenness in this model. Since the initial cascading failure is triggered by the loads of neighboring nodes over their capacities, a method taking into account the information of the single node and its adjacent nodes was put forward [10,11,19,32–34], where the load on a node depended on its degree. Recently, based on the combination of the degree and the betweenness, the method of calculating the initial loads on a node and an edge were proposed, respectively [35,36].

In the existing studies on cascading failures, the initial load is related to its degree or betweenness. Because the failure of a node has effect on more than its adjacent nodes due to the process of cascading failures, it is not accurate enough to obtain the initial load on the node through its degree. Although the betweenness is able to reflect the whole knowledge of a network, the load defined by it cannot significantly improve the robustness, which will be shown in NW and ER networks in this paper. In addition, if the betweenness is equal to zero, the initial load will be probably meaningless for real networks. Owing to those reasons, we adopt the harmonic closeness to define the node weight, the strength of which is governed by a weighting parameter. Note that never before has the research that the distribution of the node weights is obtained by the harmonic closeness. Furthermore, the effect of cascading failures on artificial and real networks is quantified by the critical threshold and the normalized avalanche size. On the basis of this model, we discuss the relationship between the weighting parameter and cascading failures, and find that the harmonic closeness can obviously reduce the cost of avoiding cascading failures and the effect of failed nodes on the robustness compared with the degree and the betweenness. Our model is useful for the design of the capacity in real networks with flow dynamics, such as traffic networks, the power grid, and so on.

2. Cascading failure model with the harmonic closeness

To overcome the drawbacks of the degree and the betweenness, the node weight is computed by the harmonic closeness which can reflect the difficult that the node reaches other nodes in the network. In general, the bigger the harmonic closeness is, the bigger the degree is. The harmonic closeness c_i of node i is defined as follows:

$$h_i = \frac{1}{N-1}\sum_{i\neq j}\frac{1}{d_{ij}} \tag{1}$$

where d_{ij} is the length of the shortest path from node i to node j. N represents the number of the nodes in a network.

Considering Refs. [23,24], we adopt the harmonic closeness to measure the node weight(HW), and assume the weight W_i of node i as

$$W_i = h_i^{\theta} \tag{2}$$

where $\theta(>0)$ is a weight parameter to control the strength of every node weight.

Following the previous cascading model, we assume that the capacity C_i of node i is proportional to its initial load

$$C_i = TL_i(0) \tag{3}$$

where $T(>1)$ is a tolerance parameter. $L_i(0)$ is the initial load on node i and equals its weight. In this study, we construct the cascading model that is characterized by the parameters θ and T. One of our goals is to explore what the value of θ is contributes to reduce the possibility of triggering cascading failures. To this end, the robustness of the network is able to be quantified by the critical threshold T_c. There is the case that when $T \geq T_c$, the nodes except for the attacked node do not fail to work. Once $T < T_c$, cascading failures occur.

In the spread of cascading failures, the bigger weight nodes connected to the broken node take on the more additional load. In the traffic network, the driver in the congested road is prone to arrive at the adjacent road with the high traffic capacity so that they can avoid the possible congestion and reach the destination quickly. It means that this assumption is in accord with the fact. Hence, the additional load $\Delta L_{ij}(t)$ received from the broken node i at step t is proportional to its weight W_j,

$$\Delta L_{ij}(t) = L_i(t)\frac{W_j}{\sum_{k\in\Gamma_i} W_k} \tag{4}$$

where Γ_i is the set of the neighboring nodes of node i. W_k represents the weight of node k that connects with node i. According to the additional load, we update the load of the neighboring node j,

$$L_j(t+1) = L_j(t) + \Delta L_{ij}(t) \tag{5}$$

To measure the effect of cascading failures induced by every failed node on the robustness, we calculate CF_i which denotes the avalanche size and is the number of the broken nodes by attacking node i after the cascading failure process stops. Thus, the value of CF_i is between 0 and $N-1$. Then we quantify the robustness of the network by the normalized avalanche size CF_N, i.e.,

$$CF_N = \frac{\sum_{i=1}^{N} CF_i}{N(N-1)} \tag{6}$$

3. Simulation results and analysis

Because the topological structure of a network is related to the robustness against cascading failures, it is necessary to explore our method in different topological structures. Three classical networks with $N = 1000$ are built, i.e., the Barabási–Albert networks (BA networks) [37], the Newman–Watts networks (NW networks) [38], and the Erdos–Renyi networks (ER networks) [39], respectively. BA networks are constructed as follows: starting from the network of m_0 nodes, a new node attaching $m(m \leq m_0)$ existing nodes is added to the network at each time step, based on the preferential attachment that the probability of connecting to the existing nodes is proportional to the degree of the existing node. NW networks are constructed as follows: from the start of regular rings, each node connects to h adjacent nodes and edges with a certain probability are added between nodes which do not connect. ER networks are constructed as follows: among isolated nodes, edges with a certain probability are randomly added between nodes. Since three typical network models mentioned above have a bit of difference for each build, our data is the average of the simulation on 20 independent networks.

First of all, we focus on the value of T_c influenced by the weight parameter θ under different values of the average degree $\langle k \rangle$. As shown in Fig. 1(a)(b)(c), for three artificial networks the value of T_c decreases first and then increases when the value of θ increases. This is because T_c depends on the hub node and its adjacent nodes. When the value of θ is small, the smaller degree node is the hub node, and its adjacent nodes likely fail to work due to receiving more loads. On the contrary, for the bigger value of θ, the node with the higher load is the hub node. Therefore, when θ exists a reasonable value to minimize T_c, we can obtain the optimal value of the weight parameter, θ^*. In BA, NW, and ER networks with different values of $\langle k \rangle$, $\theta^* \approx 7.8$, $\theta^* \approx 7$, $\theta^* \approx 8$, respectively. Furthermore, it can be seen that there is a negative correlation between T_c and $\langle k \rangle$. From Fig. 1(d), in the case of θ^*, the value of T_c in BA networks is smaller than the ones in NW and ER networks for the given value of $\langle k \rangle$, while the value of T_c in NW networks is similar to the one in ER networks. It means that using our method BA networks more easily prevent cascading failures than NW and ER networks.

In order to explore the relationship between CF_N and θ, CF_N is calculated by our method in three artificial networks for different values of θ when $\langle k \rangle = 6$.

In Fig. 2(a), it is clear that there is a negative correlation between CF_N and θ in BA networks, which implies that the bigger value of θ is helpful to enhance the robustness of the whole network against cascading failures to some extent. The main reason is that for BA networks in the range of bigger θ, the loads of a few nodes and the difference of the initial loads are larger. In the case, owing to the small initial loads of almost of nodes, attacking those nodes cannot trigger cascading failures. That is to say, when the value of θ is smaller, the average of broken nodes induced by attacking each node is smaller and it is necessary to protect the nodes with the higher load. In view of the analysis of CF_N in BA networks, we find that there seems to be a contradiction that in the case of $\langle k \rangle = 6$, the value of T_c is minimum when $\theta = 7.8$ from Fig. 1(a), while the bigger the value of θ is, the smaller the value of CF_N is from Fig. 2(a). A question arises why T_c is different from CF_N in terms of measuring the robustness. For addressing this problem, there is a following analysis in this article.

From Fig. 2(b)(c), it is found that the value of CF_N has a negative correlation with the value of θ when the range of T is between 1.05 and 1.15, which incompletely agrees with the results of BA networks. However, the value of CF_N under $\theta = 10$ is smaller than others in the range of $T > 1.15$. In addition, we can find that the value of CF_N slightly changes for the smaller and bigger value of T in NW and ER networks. When the range of T is between 1.1 and 1.15, the value of CF_N shows a significant reduction. Based on the analysis of CF_N in three networks, there is a similarity that the value of CF_N under $\theta = 6$ is the largest, which means that the network against cascading failures is vulnerable for the smaller value of θ.

From the above discussions of NW and ER networks, measuring the robustness by T_c and CF_N also seems to be contradictory. This is because T_c and CF_N emphasize different aspects of the network robustness with the application of our method. The former, T_c, is determined by the hub node and its neighboring nodes whose failure are easy to cause cascading failures in the network. Namely, it only depends on the most fragile part of the network, and neglects the role of other nodes. Moreover, although the value of T_c reflects the minimal cost that networks can avoid cascading failures, it is hard to know the impact of the broken nodes in the event of cascading failures. The latter, CF_N, pays attention to the average effect of every broken node on the whole network, and measures the robustness of the whole network against

Please cite this article as: Y. Hao, L. Jia and Y. Wang, Robustness of weighted networks with the harmonic closeness against cascading failures, Physica A (2019) 123373, https://doi.org/10.1016/j.physa.2019.123373.

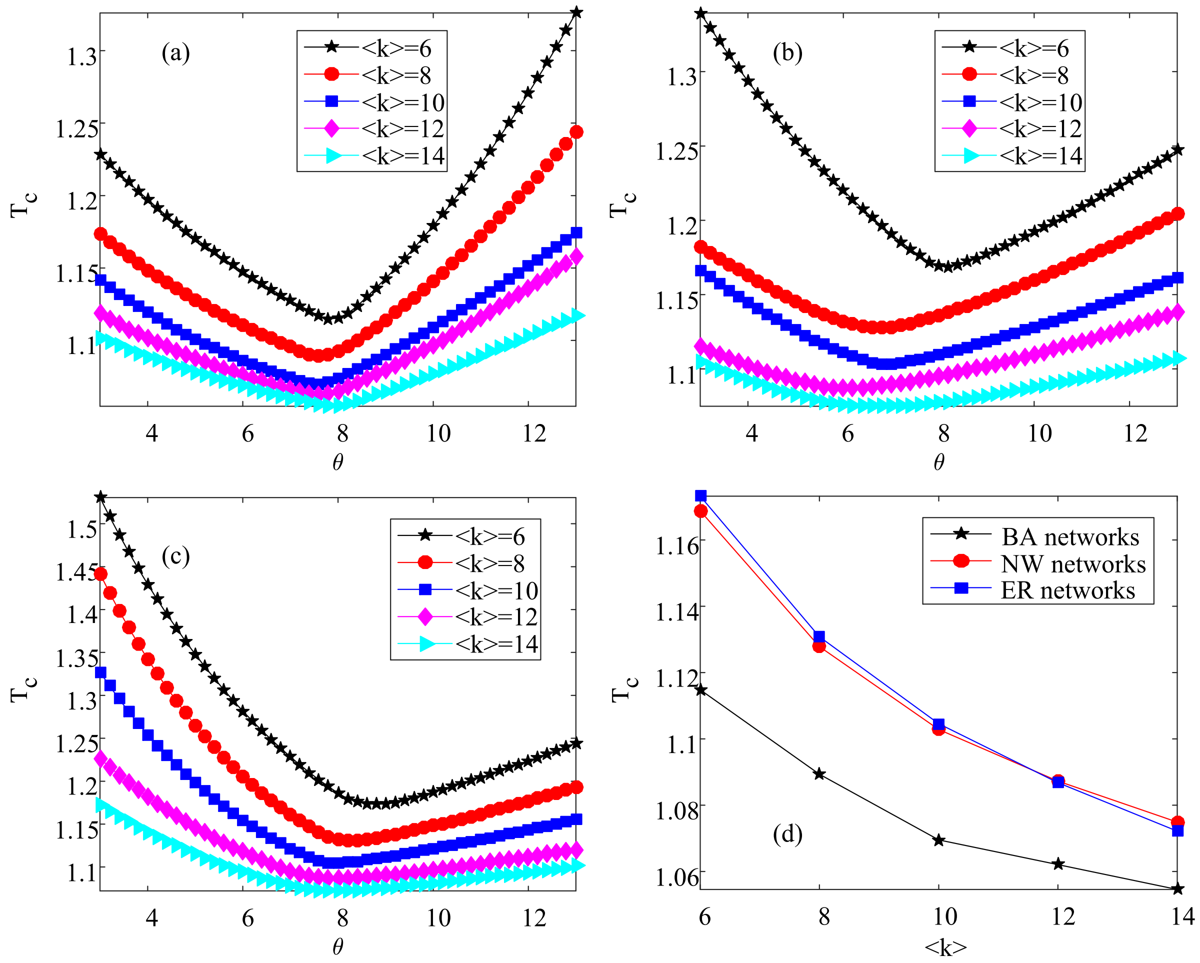

Fig. 1. The critical threshold T_c as a function of the weight parameter θ under different $\langle k\rangle$ for (a) BA networks, (b) NW networks, and (c) ER networks. (d) shows T_c in different artificial networks in the case of θ^*.

cascading failures. For example, from Fig. 2(a) in the case of $\theta = 14$ and $T = 1.09$, the value of CF_N is just small enough, but it is CF_N that does not equal zero. With the increase of T, the value of CF_N under $\theta = 7.8$ first reduces to zero compared with the one under $\theta = 14$, which corresponds with the analysis of Fig. 1(a). Thus, the optimal weight parameters obtained by the indicators mentioned above may not be the same when we evaluate the robustness of the network with the HW.

In the previous cascading models, the load on a node is dependent on its degree [23,24] and betweenness [16], hence for the comparison of our method and existing methods we construct two models with respect to the degree(DW) and the betweenness(BW) to calculate the initial load, as follows:

$$W_i = k_i^{\theta} \tag{7}$$

$$W_i = b_i^{\theta} \tag{8}$$

where k_i and b_i are the degree and the betweenness of node i, respectively. In order to explore whether our method induces the more robust network for different $\langle k\rangle$, the values of θ^* respectively obtained by the HW, the DW and the BW are applied to the comparison of these three methods.

As can be seen in Fig. 3, for different values of $\langle k\rangle$, the value of T_c under the HW is smaller than the ones under the DW and the BW no matter what the kind of the network is. Especially in the range of smaller $\langle k\rangle$, the value of T_c is apparently reduced by the HW. Thus, the HW can save much cost to avoid cascading failures in different artificial networks. In terms of existing methods in BA networks, the value of T_c obtained by the BW is smaller than the one by the DW regardless of $\langle k\rangle$. In NW and ER networks, the BW outperforms the DW in the case of $\langle k\rangle = 6$ while the BW hardly differs from the DW in the range of bigger $\langle k\rangle$. Fig. 4 illustrates that the value of CF_N obtained by the HW is the smallest in BA networks in the case of $T > 1.08$, and the curve of the HW is lower than others in NW and ER networks in the case of $T > 1.15$, which shows that the HW can greatly reduce the impact of the broken nodes on the network when the capacities of nodes are

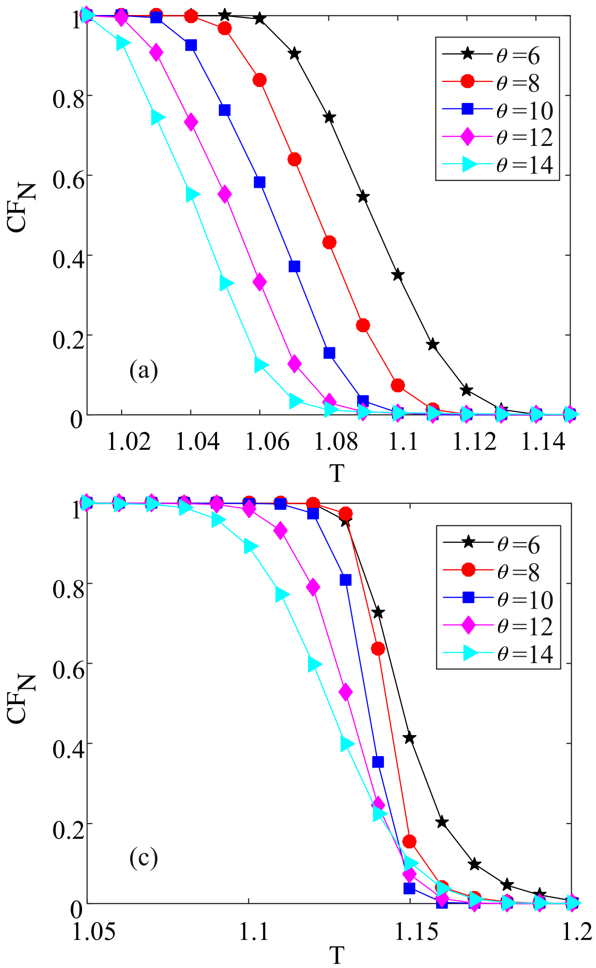

Fig. 2. The normalized avalanche size CF_N as a function of the tolerance parameter T under different θ for (a) BA networks, (b) NW networks, and (c) ER networks.

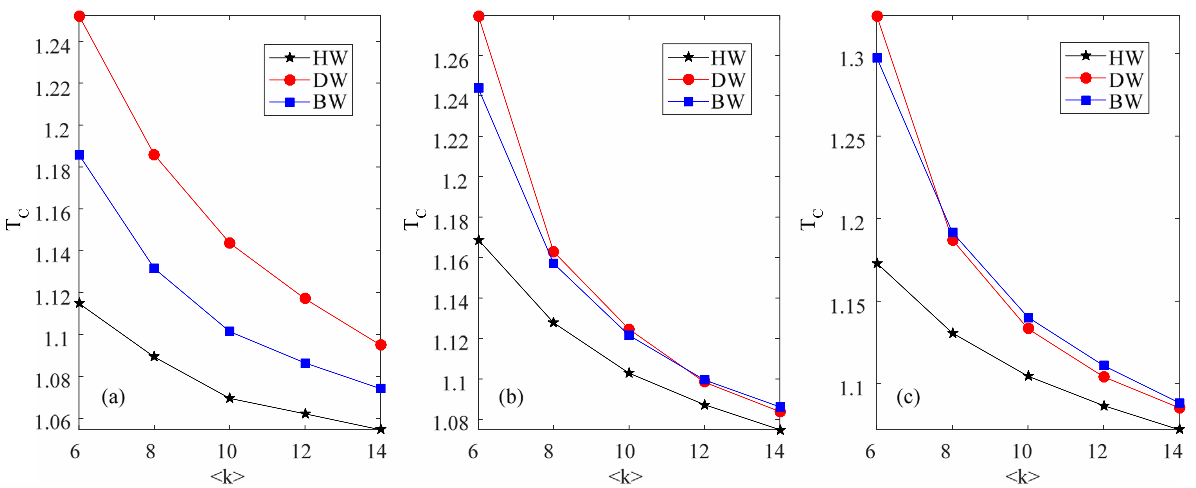

Fig. 3. T_c as a function of the average degree $\langle k\rangle$ with the HW in the case of θ^*, the DW, and the BW for (a) BA networks, (b) NW networks, and (c) ER networks.

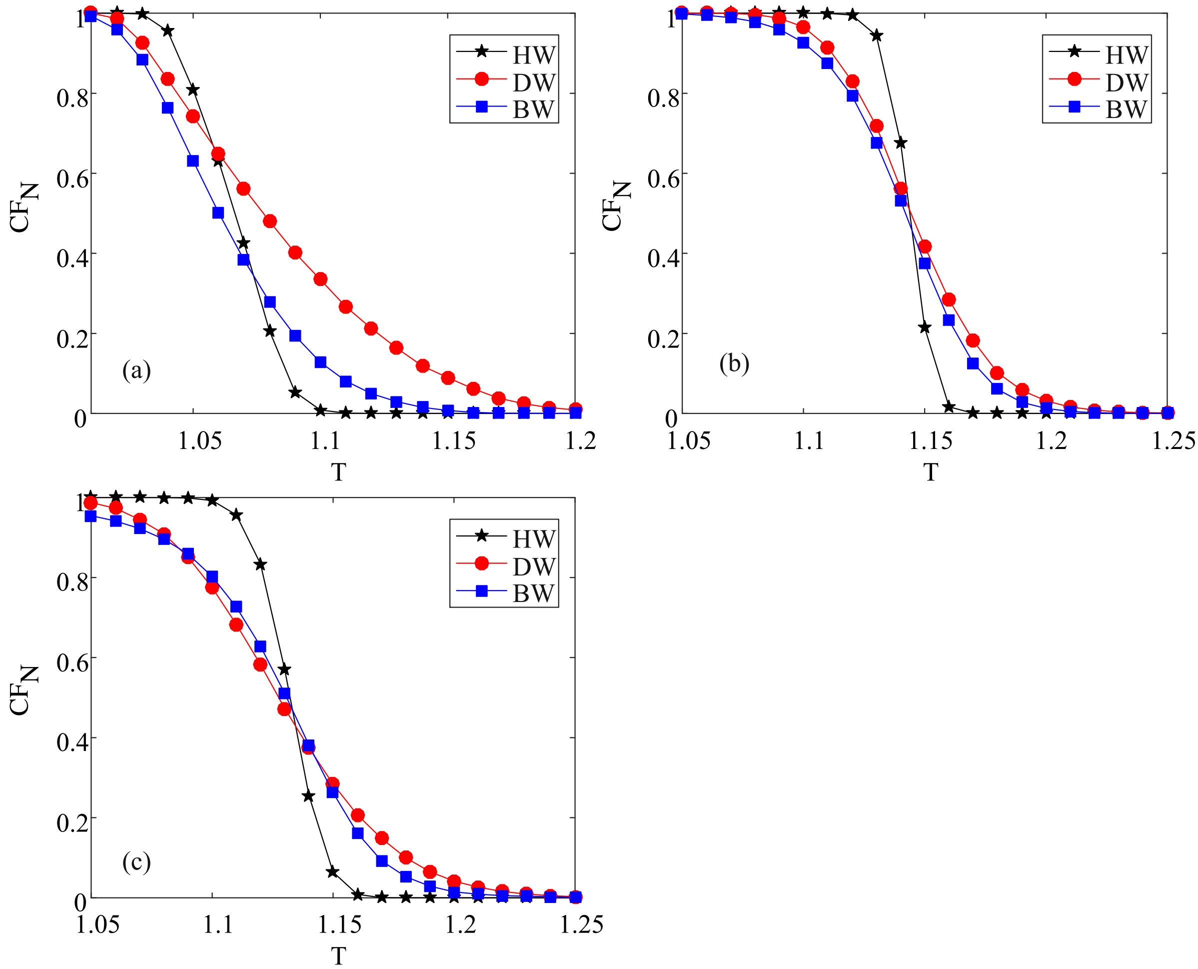

Fig. 4. CF_N as a function of T with the HW in the case of θ^*, the DW, and the BW for (a) BA networks, (b) NW networks, and (c) ER networks.

higher. In addition, for the existing methods, the value of CF_N under the BW is smaller than the one under the DW in BA and NW networks no matter what the value of T is. In ER networks with smaller or bigger T, in the same way, the BW is slightly better than the DW.

4. Case studies

In order to explore the applicability of three weighting methods, we select two classical real-world networks to simulate, i.e., the US power grid [40], and the Internet AS level network [41]. The US power grid consists of 4941 nodes and 6594 edges. The Internet AS level network is composed of 22 963 nodes and 48 436 edges.

From Fig. 5, it is evident that in two real-world networks, the value of T_c obtained by the HW is the smallest, which is completely coincident with the results of artificial networks. In particular for the Internet AS level network, the value of T_c obtained by the HW is just 1.087, while the ones by the DW and the BW are 1.619 and 1.870, respectively. So it is effective to distribute the node weight according to the HW and these two infrastructure networks with the HW are able to prevent cascading failures at low cost. As shown in Fig. 6, the curve of CF_N under the HW is the lowest in the range of $T > 1.26$ in the US power grid, which is similar to the observation of artificial networks. In the Internet AS level network, no matter what the value of T is, the value of CF_N obtained by the HW is the smallest. For the previous methods in these real networks, there is a common ground that the DW has a distinct advantage compared with the BW in the case of using T_c to measure the robustness, which is opposite to the case of using CF_N to measure the robustness. Based on the above simulation results, we can draw a conclusion that the method proposed in this paper yields better performance for real networks in comparison to the previous methods.

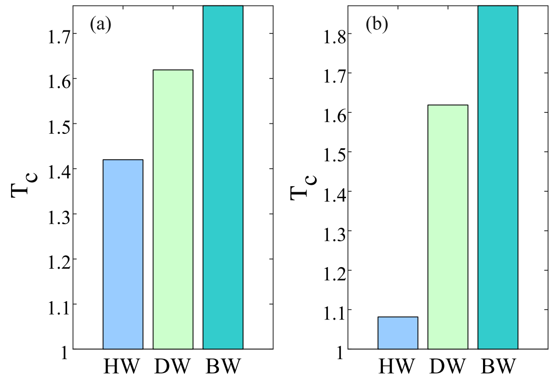

Fig. 5. Comparison of T_c with the HW in the case of θ^*, the DW, and the BW in (a) the US power grid and (b) the Internet AS level network.

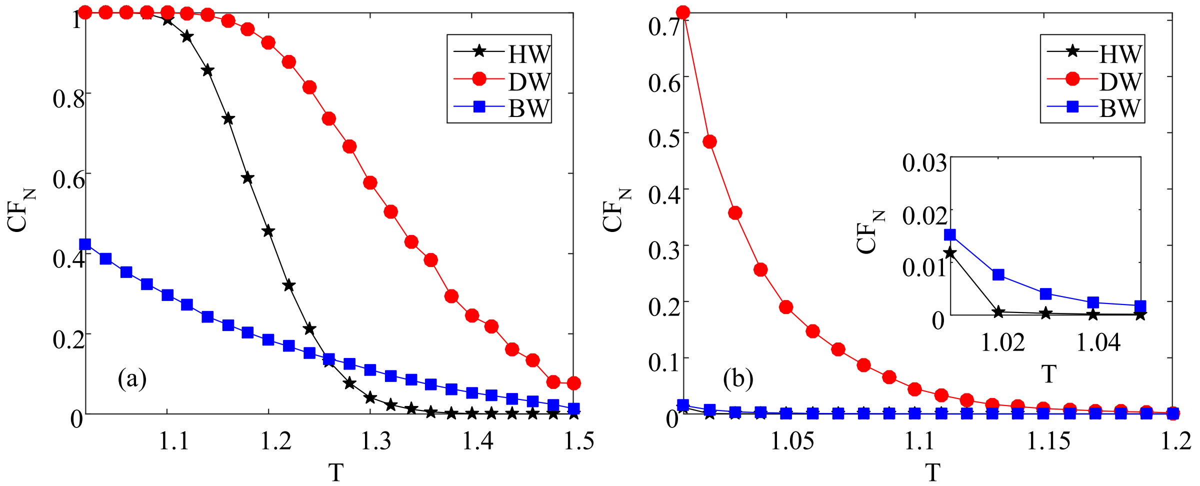

Fig. 6. CF_N as a function of T with the HW in the case of θ^*, the DW, and the BW for (a) the US power grid and (b) the Internet AS level network.

5. Conclusions

In the present cascading models, the weighting methods majorly rely on the degree or the betweenness. From a new perspective, we propose an approach for defining the node weight according to the harmonic closeness with the weight parameter θ, and on which the cascading reaction behaviors of artificial and real-world networks are investigated. We find that when the values of θ are equal to 7.8, 7, and 8, in BA, NW, and ER networks respectively, the values of the critical threshold T_c reach the minimal value. There is a negative correlation between θ and CF_N in BA networks, which agrees with the observation of NW and ER networks with smaller T. According to the comparison of different weighting methods, it is noteworthy that the value of T_c with our method is obviously smaller than the ones with the previous methods in artificial and real networks. Moreover, our method makes CF_N much smaller than existing methods in the range of bigger T. Although our method yields better performance, the harmonic closeness may have larger computational complexity in large-scale networks and the failed node has the more serious impact on the network in the range of small T. In future work, we will study cascading failures induced by attacking edges in weighted networks with the harmonic closeness. In brief, this paper may avail to improve the robustness of the real-world networks, and develop the study of cascading failures in the light of the harmonic closeness.

Acknowledgment

This work was supported by project of State Key Laboratory of Rail Traffic Control and Safety in China under Grant No. RCS2017ZZ002.

References

[1] A.E. Motter, Cascade control and defense in complex networks, Phys. Rev. Lett. 93 (9) (2004) 98701.

[2] A.E. Motter, Y.C. Lai, Cascade-based attacks on complex networks, Phys. Rev. E 66 (2) (2003) 65102.
[3] P. Crucitti, V. Latora, M. Marchiori, Model for cascading failures in complex networks, Phys. Rev. E 69 (2004) 45104.
[4] R. Kinney, P. Crucitti, R. Albert, et al., Modeling cascading failures in the North American power grid, Eur. Phys. J. B 46 (1) (2005) 101–107.
[5] J.M. Reynolds-Barredo, D.E. Newman, B.A. Carreras, et al., The interplay of network structure and dispatch solutions in power grid cascading failures, Chaos 26 (11) (2016) 643–652.
[6] M. Rohden, D. Jung, S. Tamrakar, et al., Cascading failures in ac electricity grids, Phys. Rev. E 94 (3–1) (2016) 32209.
[7] J. Yan, Y. Tang, H. He, et al., Cascading failure analysis with DC power flow model and transient stability analysis, IEEE Trans. Power Syst. 30 (1) (2014) 285–297.
[8] J. Song, E. Cotillasanchez, G. Ghanavati, et al., Dynamic modeling of cascading failure in power systems, IEEE Trans. Power Syst. 31 (3) (2014) 2085–2095.
[9] Q.W. Du, S.L. Xiao, Z. Bo, Analysis of cascading failure in complex power networks under the load local preferential redistribution rule, Physica A 391 (8) (2012) 2771–2777.
[10] J.H. Zhang, Y. Dai, K.S. Zou, et al., Vulnerability analysis of the US power grid based on local load-redistribution, Saf. Sci. 80 (2015) 156–162.
[11] J.W. Wang, L.L. Rong, Cascade-based attack vulnerability on the US power grid, Saf. Sci. 47 (10) (2009) 1332–1336.
[12] B. Schäfer, D. Witthaut, M. Timme, et al., Dynamically induced cascading failures in power grids, Nat. Commun. 9 (1) (2018) 1975.
[13] S.M. Jia, Y.Y. Wang, C. Feng, et al., Cascading failures in power grid under three node attack strategies, Lecture Notes in Comput. Sci. 8589 (2014) 779–786.
[14] J.J. Wu, Z.Y. Gao, H.J. Sun, Effects of the cascading failures on scale-free traffic networks, Physica A 378 (2) (2007) 505–511.
[15] J.J. Wu, H.J. Sun, Z.Y. Gao, Cascading failures on weighted urban traffic equilibrium networks, Physica A 386 (1) (2007) 407–413.
[16] Y. Qian, B. Wang, Y. Xue, et al., A simulation of the cascading failure of a complex network model by considering the characteristics of road traffic conditions, Nonlinear Dynam. 80 (1–2) (2015) 413–420.
[17] R. Ding, N. Ujang, H.B. Hamid, et al., Complex network theory applied to the growth of Kuala Lumpur's public urban rail transit network, PLoS One 10 (10) (2015) e139961.
[18] Q. Shuang, M.Y. Zhang, Y.B. Yuan, Node vulnerability of water distribution networks under cascading failures, Reliab. Eng. Syst. Saf. 124 (124) (2014) 132–141.
[19] Y.C. Wang, F.P. Zhang, Modeling and analysis of under-load-based cascading failures in supply chain networks, Nonlinear Dynam. 92 (2) (2018) 1–15.
[20] Y. Zeng, R.B. Xiao, Modelling of cluster supply network with cascading failure spread and its vulnerability analysis, Int. J. Prod. Res. 52 (23) (2014) 6938–6953.
[21] B. Wu, A. Tang, J. Wu, Modeling cascading failures in interdependent infrastructures under terrorist attacks, Reliab. Eng. Syst. Saf. 147 (2016) 1–8.
[22] S. Wang, L. Hong, M. Ouyang, et al., Vulnerability analysis of interdependent infrastructure systems under edge attack strategies, Saf. Sci. 51 (1) (2013) 328–337.
[23] J.W. Wang, L.L. Rong, L. Zhang, et al., Attack vulnerability of scale-free networks due to cascading failures, Physica A 387 (26) (2008) 6671–6678.
[24] Z.X. Wu, G. Peng, W.X. Wang, et al., Cascading failure spreading on weighted heterogeneous networks, J. Stat. Mech. 2008 (5) (2008) P05013.
[25] C. Fu, Y. Wang, Y. Gao, et al., Complex networks repair strategies: Dynamic models, Physica A 482 (2017) 401–406.
[26] C. Fu, W. Ying, X. Wang, Research on complex networks' repairing characteristics due to cascading failure, Physica A 482 (2017) 317–324.
[27] R. Yang, W.X. Wang, Y.C. Lai, et al., Optimal weighting scheme for suppressing cascades and traffic congestion in complex networks, Phys. Rev. E 79 (2) (2009) 26112.
[28] W.X. Wang, G.R. Chen, Universal robustness characteristic of weighted networks against cascading failure, Phys. Rev. E 77 (2008) 26101.
[29] C.L. Pu, W.J. Pei, A. Michaelson, Robustness analysis of network controllability, Physica A 391 (18) (2012) 4420–4425.
[30] B. Mirzasoleiman, M. Babaei, M. Jalili, et al., Cascaded failures in weighted networks, Phys. Rev. E 84 (2) (2011) 46114.
[31] R. Ghanbari, M. Jalili, X. Yu, Correlation of cascade failures and centrality measures in complex networks, Future Gener. Comput. Syst. 83 (2018) 390–400.
[32] J. Wang, C. Zhang, Y. Huang, et al., Attack robustness of cascading model with node weight, Nonlinear Dynam. 78 (1) (2014) 37–48.
[33] J. Liu, Q.Y. Xiong, X. Shi, et al., Robustness of complex networks with an improved breakdown probability against cascading failures, Physica A 456 (2016) 302–309.
[34] J. Wang, L. Rong, A model for cascading failures in scale-free networks with a breakdown probability, Physica A 388 (7) (2009) 1289–1298.
[35] H.R. Liu, Y.L. Hu, R.R. Yin, et al., Cascading failure model of scale-free topology for avoiding node failure, Neurocomputing 260 (2017) 443.
[36] Z. Ju, J. Ma, J. Xie, et al., Cascading failure model for improving the robustness of scale-free networks, Internat. J. Modern Phys. C 29 (06) (2018) S907828644.
[37] A.L. Barabasi, R. Albert, Emergence of scaling in random networks, Science 286 (5439) (1999) 509–512.
[38] M.E.J. Newman, D.J. Watts, Renormalization group analysis of the small-world network model, Phys. Lett. A 263 (4–6) (1999) 341–346.
[39] P. Erdős, A. Rényi, On the evolution of random graphs, Acad. Sci. 5 (1) (1960) 17–61.
[40] D.J. Watts, S.H. Strogatz, Collective dynamics of "small-world" networks, Nature 393 (6684) (1998) 440.
[41] http://www-personal.umich.edu/~mejn/netdata/.

Please cite this article as: Y. Hao, L. Jia and Y. Wang, Robustness of weighted networks with the harmonic closeness against cascading failures, Physica A (2019) 123373, https://doi.org/10.1016/j.physa.2019.123373.