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Sliding-mode Controller for Four-Wheel-Steering Vehicle: Trajectory-Tracking Problem*

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Abstract— **A solution to trajectory-tracking control problem for a four-wheel-steering vehicle (4WS) is proposed using sliding-mode approach. The advantage of this controller over current control procedure is that it is applicable to a large class of vehicles with single or double steering and to a tracking velocity that is not necessarily constant. The sliding-mode approach make the solutions robust with respect to errors and disturbances, as demonstrated by the simulation results.**

Index Terms— **Control of mobile platforms, Sliding-mode control, Trajectory-tracking.**

I. INTRODUCTION

For many years, the control of non-holonomic vehicles has been a very active research field. At least two reasons account for this fact. On one hand, wheeled vehicles constitute a major and ever more ubiquitous transportation system. Previously restricted to research laboratories and factories, automated wheeled vehicles are now envisioned in everyday life (e.g. through car-platooning applications or urban transportation services), not to mention the military domain. On the other hand, the kinematic equations of non-holonomic systems are highly nonlinear, and thus of particular interest for the development of nonlinear control theory and practice. Furthermore, some of the control methods initially developed for non-holonomic systems have proven to be applicable to other physical systems (e.g. underactuated mechanical systems), as well as to more general classes of nonlinear systems.

Research on modelling and control of wheeled vehicles can be divided into two major categories: one that is oriented towards automobiles and terrain vehicles, the other is oriented towards indoor wheeled mobile robots (WMRs). This classification is based on the differences in the design of the vehicles, operational and environmental conditions. Mechanical coupling of the wheels in vehicles often allows the use of a simplified, single-track model of two-wheel-steering and four-wheel-steering cars for the design of motion controllers. In modelling and control of mobile robots with independently steered and/or driven wheels it is required to

Fig. 1.   Seekur - unmanned ground vehicle

explicitly account for each of the wheels of the vehicle. However the controllers in these works are still design under the assumption that the wheels are steered in pairs.

This paper deals with the kinematics model and the feedback control of an autonomous wheeled robot named Seekur (Fig.1) from the Mobile Robots Inc [1].

Outdoor robots face all of the same challenges as indoor robots, such as sensing, data processing, locomotion, navigation, and interaction with the surroundings. Outdoor robots, however, are expected to achieve all of these things in much more complex and unstructured environments such as forests, deserts, and even agricultural fields [2]-[5].

Variable structure control (VSC) has been showing to be a robust approach in different applications and has been successfully applied in control problems as diverse as automatic flight control, control of electric motors, regulation in chemical processes, helicopter stability augmentation, space systems and robotics. One particular type of VCS system is the sliding mode control (SMC) methodology [6]. The theory of SMC has been applied to various control systems, since it has been shown that this nonlinear type of control exhibits some excellent properties, such as robustness against large parameter variation and disturbances [7], [8]. By designing

switch functions of state variables or output variables to form sliding surfaces, SMC can guarantee that when trajectories reach the surfaces, the switch functions keep the trajectories on the surfaces, thus yielding the desired system dynamics. The main advantages of using SMC include fast response, good transient and robustness with respect to system uncertainties and external disturbances.

Kinematics-based control of 4WS vehicle with multiple steering wheels has received considerable attention over the last decade. Controllers that utilize complex dynamic model of the 4WS vehicle are not used in practice due to their computational infeasibility and difficulties related to analytical investigation of their properties.

Recent research and development has shown that four-wheel steering systems can effectively improve the handling behavior of vehicles. These handling improvements provide better maneuverability at low speeds and reduce the delay in path tracking by setting the rear wheel steering angle in the direction opposite to the front wheel steering angle. At high speeds, four-wheel steering systems improve vehicle stability by turning all the wheel steering angles in the same direction.

A. Kinematics model

To discuss a nonholonomic kinematic model of a vehicle,the following assumptions are considered: a) distances between wheels (generally called as *wheelbase*) are strictly fixed; b) the steering axle of each wheel is perpendicular to a surface terrain; c) a vehicle does not consist of any flexible parts.

A kinematic model of vehicle including the lateral slips is shown in Fig. 2. In this model, each wheel has a certain steering angle δ_i and slip angle β_i. The slip angle, which defines how large the wheel generates the lateral slip, is calculated by the longitudinal and lateral linear velocities $v_{x_{wi}}$, $v_{y_{wi}}$ of the wheel as follows:

$$\beta_i = tan^{-1}\left(\frac{v_{y_{wi}}}{v_{x_{wi}}}\right) \tag{1}$$

The subscript i denotes each wheel ID as shown in Fig. 2. (x_{CG}, y_{CG}, ψ) defines the position and an orientation of the center of gravity of the vehicle (CG), while (x_{wi}, y_{wi}) defines the position of the $i-th$ wheel. v and v_i are linear velocities of the vehicle and each wheel, respectively. Also, β denotes the sideslip of the vehicle, which is determined by a similar equation of (1). l_f and l_r means the longitudinal distance from the center of gravity of the vehicle to the front or rear wheels and d defines the wheelbase. Here, based on the assumption as previously pointed, l_f, l_r and d take constant values.

Nonholonomic constraints

In the conventional approach, *Bicycle model* [9] (see Fig. 3, a four-wheel car-like vehicle is approximated as a two-wheel bicycle-like vehicle. However, the bicycle model is hardly
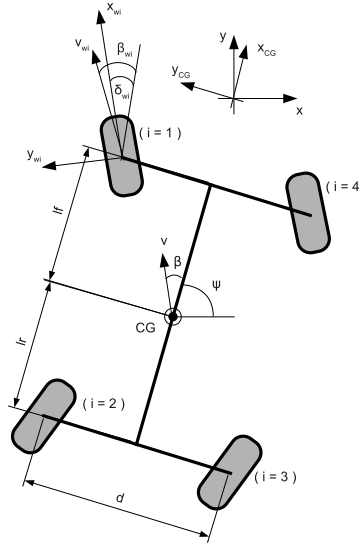
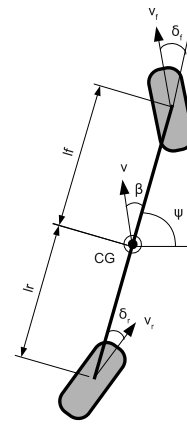Fig. 2.   Kinematics model of 4 wheel vehicle

Fig. 3.   Kinematics model of bicycle

able to deal with the slips of each wheel, strictly. Therefore, taking into account the slips, the nonholonomic constraints are expressed by:

$$\dot{x}_{CG} \cdot sin\left(\beta + \psi\right) + \dot{y}_{CG} \cdot cos\left(\beta + \psi\right) = 0 \tag{2}$$

$$\dot{x}_{wi} \cdot sin\left(\beta_{wi} + \delta_{wi} + \psi\right) + \dot{y}_{wi} \cdot cos\left(\beta_{wi} + \delta_{wi} + \psi\right) = 0 \tag{3}$$

For instance, in terms of a bicycle model ($i = 1, 2$):

$$A_{12} \cdot \dot{q}_0 = 0 \tag{4}$$

where

$$A_{12} = \begin{bmatrix} sin\phi_{w1} & -cos\phi_{w1} & \begin{array}{c} -l_f \cdot cos\left(\phi_{w1} - \psi\right) - \\ -d/2 \cdot sin\left(\phi_{w1} - \psi\right) \end{array} \\ sin\phi_{w2} & -cos\phi_{w2} & \begin{array}{c} l_r \cdot cos\left(\phi_{w2} - \psi\right) + \\ +d/2 \cdot sin\left(\phi_{w2} + \psi\right) \end{array} \\ sin\phi_0 & -cos\phi_0 & 0 \end{bmatrix}$$

$$\dot{q}_0 = \begin{bmatrix} \dot{x}_{CG} \\ \dot{y}_{CG} \\ \dot{\psi} \end{bmatrix}$$

(5)

and $\phi_0 = \beta + \psi$, $\phi_{wi} = \beta_{wi} + \delta_{wi} + \psi$.

Under the basic assumptions of planar motion, rigid body and non-slippage of tire, the four-wheel vehicle can be approximated by a *bicycle model*, as shown in Figure 3. To describe the vehicle motion, a global coordinate $x - y$ is fixed on the horizontal plane on which the vehicle moves. The motion status of the vehicle can be described using the bicycle model as illustrated in Figure 3.

$$\begin{bmatrix} sin\left(\delta_f + \psi\right) & -cos\left(\delta_f + \psi\right) & -l_f \cdot cos\delta_f \\ sin\left(\delta_r + \psi\right) & -cos\left(\delta_r + \psi\right) & l_r \cdot cos\delta_r \\ sin\left(\beta + \psi\right) & -cos\left(\beta + \psi\right) & 0 \end{bmatrix} \cdot$$
$$\cdot \begin{bmatrix} \dot{x}_{CG} \\ \dot{y}_{CG} \\ \dot{\psi} \end{bmatrix} = 0$$

(6)

Using a null-space vector, it is possible to obtain the vector $\dot{q}_0$ satisfying equation (6):

$$\begin{bmatrix} \dot{x}_{CG} \\ \dot{y}_{CG} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} cos\left(\beta + \psi\right) \\ sin\left(\beta + \psi\right) \\ \frac{cos\beta \cdot (tan\delta_f - tan\delta_r)}{l_f + l_r} \end{bmatrix} \cdot v$$

(7)

where $\beta = arctan\frac{l_f \cdot tan\delta_r + l_r \cdot tan\delta_f}{l_f + l_r}$ and v is linear velocity of the vehicle.

In this model, there are three inputs: two steering angles, δ_f and δ_r, and vehicle velocity, v (defined at point CG). The state variables of kinematic motion are the vehicle configuration (x_{CG}, y_{CG}, ψ).

We assume that both the front and rear wheels of this 4WS vehicle can only vary within the following vehicular mechanical range

$$-\delta_{min} \leq \delta_r, \delta_f \leq \delta_{max}$$

(8)

where δ_{max} is the maximum of the steering angles to both sides. The side-slip angle reaches its extreme value only when both front and rear steering angles reach their positive or negative maximum simultaneously.

$$-\beta_{min} \leq \beta \leq \beta_{max}$$

(9)

Two special maneuvers, the so-called *Zero-side-slip Maneuver* and *Parallel Steering Maneuver* [10], take advantage of the special kinematic characteristic of 4WS vehicles and are commonly used. In the following, we will show how
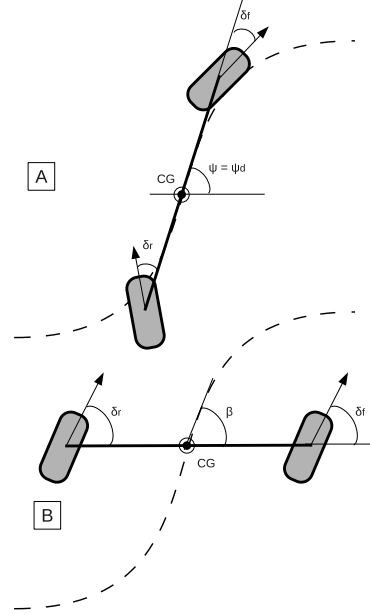

Fig. 4. A. Zero-side-slip maneuver and B. Parallel steering maneuver.

these two maneuvers can be used in our trajectory-tracking problem.

1) Zero-side-slip Maneuver

In this maneuver, the side-slip angle is set to zero from the starting point P_0 to the ending point P_{fin} when the vehicle moves along the path ($\beta(t) = 0$). The orientation of the vehicle $\psi(t)$ is set to match the tangential angle of the desired path $\psi_d(t)$.

$$\psi(t) = \psi_d(t), t : 0 \rightarrow t_{fin}$$

This maneuver is desirable in vehicle motion since the vehicle body is always tangent to the path (see Fig. 4A).

2) Parallel Steering Maneuver

Parallel Steering is defined as that both two wheels are always steered at the same angle in the same direction. In this maneuver, two steering angles is set as follows

$$\delta_f(t) = \delta_r(t) = \beta(t), t : 0 \rightarrow t_{fin}$$

This implies that the vehicle translates without changing its orientation during the motion. Thus we have

$$\psi(t) = \psi_0, t : 0 \rightarrow t_{fin}$$

where ψ_0 is the initial heading angle of the vehicle. This maneuver is very practical in vehicle lanechanging and obstacle-avoidance (see Fig. 4B). The rotation of the vehicle is reduced as well, thus improves the vehicle stability at high speed.

II. CONTROL OF 4WS VEHICLE

The application of SMC strategies in nonlinear systems has received considerable attention in recent years [11], [12], [13], [14]. In trajectory tracking is an objective to control the 4WS vehicle to follow a desired path, with a given orientation relatively to the path tangent, even when disturbances exist. In the case of trajectory-tracking the path is to be followed under time constraints. The path has an associated velocity profile, with each point of the trajectory embedding spatio-temporal information that is to be satisfied by the vehicle along the path. Trajectory tracking is formulated as having the 4WS vehicle following a virtual target 4WS vehicle which is assumed to move exactly along the path with specified velocity profile.

A. Trajectory-tracking errors

Without loss of generality, it can be assumed that the desired trajectory $q_d(t) = [x_d(t), y_d(t), \psi_d(t)]^T$ is generated by a virtual 4WS vehicle (see Fig. 5). When a real vehicle is controlled to move on a desired path it exhibits some tracking error, which, expressed in terms of the vehicle coordinate system, as shown in Fig. 5, is given by

$$\begin{bmatrix} x_e \\ y_e \\ \psi_e \end{bmatrix} = \begin{bmatrix} cos\psi_d & sin\psi_d & 0 \\ -sin\psi_d & cos\psi_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_{CG} - x_d \\ y_{CG} - y_d \\ \psi - \psi_d \end{bmatrix} \tag{10}$$

Consequently one gets the error dynamics for trajectory tracking as

$$\begin{cases} \dot{x}_e = -v_d + v \cdot cos\psi_e + \dot{\psi}_d \cdot y_e \\ \dot{y}_e = v \cdot sin\psi_e - \dot{\psi}_d \cdot x_e \\ \dot{\psi}_e = \dot{\psi} - \dot{\psi}_d \end{cases} \tag{11}$$

In this paper it is assumed that $\delta_r = -\delta_f$ and $|\delta_f| < \pi/2$.
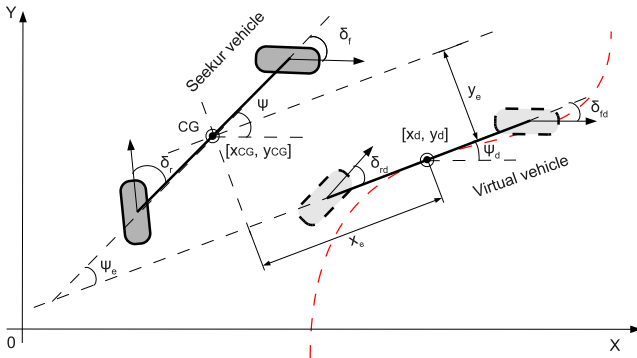

Fig. 5. Lateral, longitudinal and orientation error for trajectory-tracking.

B. Sliding-mode control

A Sliding Mode Controller is a Variable Structure Controller (VSC). Loh and Yeung [16] developed a particular form of speed control relationship, as following:

$$\dot{s}_i = -p_i \cdot exp(\alpha \cdot |s_i|) \cdot sgn(s_i) \tag{12}$$

and the reaching time T_i becomes:

$$T_i = \frac{1}{\alpha \cdot p_i} \cdot [1 - exp(-\alpha \cdot |s_i(0)|)] \tag{13}$$

where s_i is the reaching speed, $s_i(0)$ is the initial value of s_i and p_i is the switching gain, $p_i > 0$, $\alpha > 0$, and $i = 1, 2$.

A new design of sliding surface was proposed in [11] , such that lateral error, y_e, and angular error, ψ_e, are internally coupled with each other in a sliding surface leading to convergence of both variables. For that purpose the following sliding surfaces was proposed:

$$\begin{aligned} s_1 &= \dot{x}_e + k_1 \cdot x_e, \\ s_2 &= \dot{y}_e + k_2 \cdot y_e + k_0 \cdot sgn(y_e) \cdot \psi_e. \end{aligned} \tag{14}$$

here k_0, k_1, k_2 are positive constant parameters, x_e, y_e and ψ_e are the trajectory-tracking errors defined in (10).

If s_1 converges to zero, trivially x_e converges to zero. If s_2 converges to zero, in steady-state it becomes $\dot{y}_e = -k_2 \cdot y_e - k_0 \cdot sgn(y_e) \cdot \psi_e$. For $y_e < 0 \Rightarrow \dot{y}_e > 0$ if only if $k_0 < k_2 \cdot |y_e| / |\psi_e|$. For $y_e > 0 \Rightarrow \dot{y}_e < 0$ if only if $k_0 < k_2 \cdot |y_e| / |\psi_e|$. Finally, it can be known from s_2 that convergence of y_e and $\dot{y}_e$ leads to convergence of ψ_e to zero.

From the time derivative of (14) and using the reaching law defined in (12) yields:

$$\begin{aligned} \dot{s}_1 &= \ddot{x}_e + k_1 \cdot \dot{x}_e = -p_1 \cdot exp(\alpha \cdot |s_1|) \cdot sgn(s_1) \\ \dot{s}_2 &= \ddot{y}_e + k_2 \cdot \dot{y}_e + k_0 \cdot sgn(y_e) \cdot \dot{\psi}_e = \\ &= -p_2 \cdot exp(\alpha \cdot |s_2|) \cdot sgn(s_2) \end{aligned} \tag{15}$$

From (10), (11) and (15), and after some mathematical manipulation, we get the output commands of the sliding-mode trajectory-tracking controller:

$$\begin{aligned} \dot{v}_c &= \frac{1}{cos\psi_e} \cdot \Big(-p_1 \cdot exp(\alpha \cdot |s_1|) \cdot sgn(s_1) - k_1 \cdot \dot{x}_e - \\ &\quad -y_e \cdot \ddot{\psi}_d - \dot{y}_e \cdot \dot{\psi}_d + v \cdot \dot{\psi}_e \cdot sin\psi_e + \dot{v}_d \Big) \\ \dot{\psi}_c &= \frac{1}{v \cdot cos\psi_e + k_0 \cdot sgn(y_e)} \cdot \Big(x_e \cdot \ddot{\psi}_d + \dot{x}_e \cdot \dot{\psi}_d - \\ &\quad -p_2 \cdot exp(\alpha \cdot |s_2|) \cdot sgn(s_2) - k_2 \cdot \dot{y}_e - \dot{v} \cdot sin\psi_e) + \dot{\psi}_d \end{aligned} \tag{16}$$

The $signum$ functions in the sliding surface were replaced by $saturation$ functions, to reduce the chattering phenomenon ([8], [17]). The saturation function is defined as:

$$sat\left(\frac{s}{\tau}\right) = \begin{cases} \frac{s}{\tau} & if \quad \left|\frac{s}{\tau}\right| \le 1 \\ sgn\left(\frac{s}{\tau}\right) & if \quad \left|\frac{s}{\tau}\right| > 1 \end{cases} \tag{17}$$

where constant factor τ defines the thickness of the boundary layer.

III. SIMULATION RESULTS

In this section, some simulation results are presented to validate the proposed control law. To show the effectiveness of the proposed sliding mode control law numerically, experiments were carried out on the trajectory-tracking problem of a 4WS vehicle. The 4WS vehicle is assumed to have the same structure as in Fig. 2.

Fig. 6 is the schematic diagram of the 4WS vehicle control architecture. The control algorithms (including desired motion generation) are written in C++ and run with a sampling time of Ts = 100 ms on a embedded PC, which also provides a user interface with real-time visualization and a simulation environment (MobileSim). MobileSim is software for simulating MobileRobots platforms and their environments, for debugging and experimentation with ARIA (Advanced Robot Interface for Applications). The ARIA software can be used to control the mobile robots like Pionner, PatrolBot, PeopleBot, Seekur etc. ARIA it is an object-oriented Applications Programming Interface (API), written in C++ and intended for the creation of intelligent high-level client-side software

The Trajectory Planner generate the profiles of the velocities (linear v_d and angular $\dot{\psi}_d$), taking account the trajectory example (see in Figs. 7).

The simulation experiments were made for one types of trajectories as shown in Fig. 7. Two experiments were made: A) without initial pose errors ($x_e = 0, y_e = 0, \psi_e = 0$) and B) with initial pose errors (($x_e = 2.0[m], y_e = 1.0[m], \psi_e = 0.0[rad/s]$)).

The trajectory-tracking must perform not only the planning of the curve (spatial dimension) but also the speed profile (temporal dimension). All the experiments had the expected results: the lateral, longitudinal and orientation errors that tends to zero.

Fig. 8 presents the simulations using the Seekur robot without initial errors in case of trajectory shown in Fig. 7. From this figures we can observe that our sliding-mode trajectory-tracking controller is robustness. Figure 9 shows desired, command and Seekur linear velocities (linear and angular) for SM-TT control in case of trajectory tracking without initial pose errors.

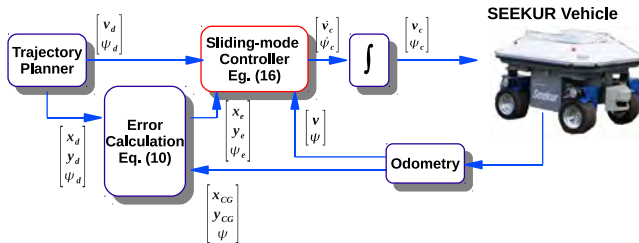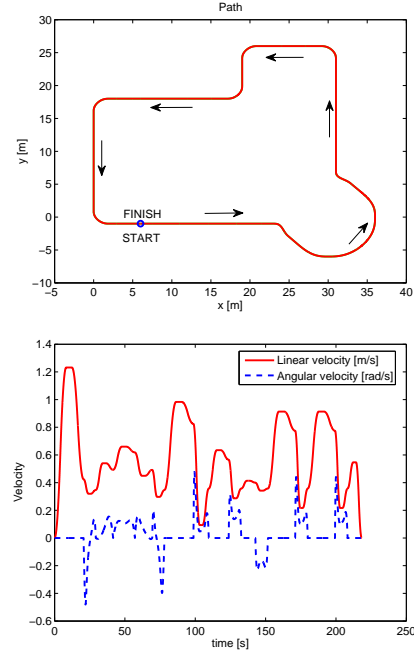As shown in Fig. 10 the Seekur vehicle retrieved quickly

Fig. 7.   Path planning example used in simulations.

Fig. 8.   Simulation results using Aria and MobileSim software.

($\Delta t \approx 15[s]$) and smoothly from its initial state error ($x_e = 2.0, y_e = 1.0, \psi_e = 0.0$), and the tracking errors converge on average to zero with acceptable reduced values along the trajectory.

IV. CONCLUSIONS

In the present paper we have described the nonlinear system architecture of a four-wheel-steering vehicle, focusing on its kinematics and on its control system (sliding-mode controller) used to allow a highly accurate in trajectory-tracking. The proposed solution is based on the sliding-mode approach. The main advantages of using sliding-mode control include fast response, good transient and robustness with respect to system uncertainties and external disturbances.

Simulation results were presented to illustrate the performance of the proposed sliding-mode trajectory-tracking
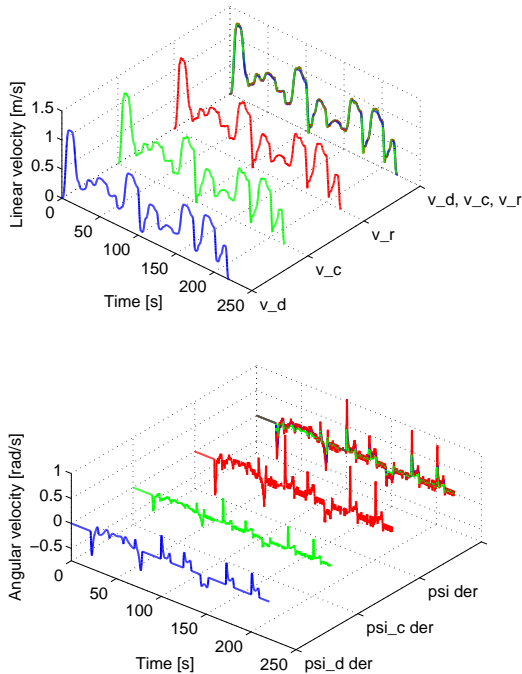

Fig. 6.   Sliding-mode trajectory-tracking control architecture.

Fig. 9. Desired $(v_d, \dot{\psi}_d)$, command $(v_c, \dot{\psi}_c)$ and vehicle $(v, \dot{\psi})$ linear and angular velocities for SM-TT control.

Fig. 10. Simulation results made for SM-TT control starting from an initial pose error $(x_e = 2.0[m], y_e = 1.0[m], \psi_e = 0.0[rad/s])$.

controller. The controller is simply structured and easy to implement. From the simulation results, it is concluded that the proposed strategy achieves the effectiveness of desired performance.

More analytic study on the modeling and control of uncertainties can be pursued as a future research.

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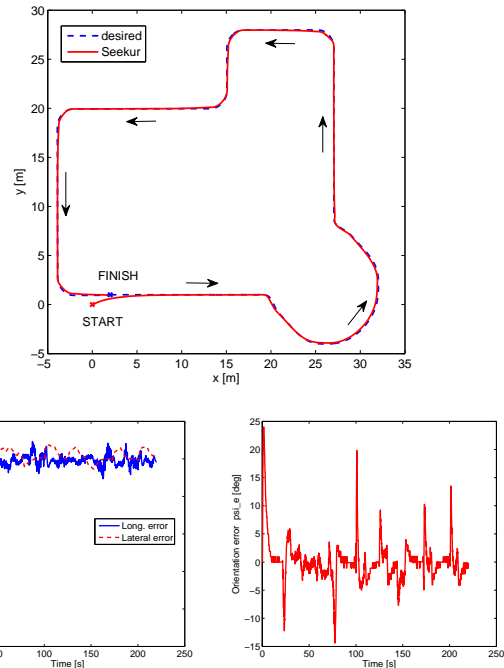[6] V. I. Utkin, *Sliding modes in optimization and control*, Springer-Verlag, New York, 1992.

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