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Sensitivity study of the CUSUM control chart with an economic model

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Abstract

Economic control chart designs have not been universally implemented in industry for several reasons. For example, the parameters are too numerous and are often difficult to estimate accurately. A possible solution to these problems involves performing a sensitivity analysis of the inputs to determine which parameters are significant and how parameter misspecification impacts the results. Using two-level fractional factorial designs, we identify highly significant parameters in the Lorenzen and Vance economic control chart model under a Cumulative Sum (CUSUM) condition. The response variables examined include the expected cost per time unit and the decision variables sample size, sampling interval, control chart decision interval and reference value. Verification and misspecification analysis support our conclusions with respect to the expected cost per time unit response variable. The sensitivity study highlights the importance of experiment design in understanding the underlying behavior of the model inputs. Results of testing several scenarios indicate that a small subset of model inputs actually drive the cost response, which should make industrial implementation an easier task. The search for significant inputs can be aided by a study of the relative magnitudes of some factors such as the assignable cause rate and the ratio of out-of-control to in-control quality costs.

Keywords: Experiment design; Statistical process control; Cost ratios; Simulation

1. Introduction

Economic considerations are often overlooked as important factors in the design and use of control charts. To monitor and maintain statistical control of a process, control charts are often designed with respect to statistical criteria only. Many times statistically optimal control charts can be more costly than a control chart whose type and design parameters are determined by the economic consequences.

Models that determine control chart parameters based on economic factors are attractive if an organization is interested in minimizing costs related to the control process. These economic models include measures of statistical performance in the total cost equation, so that the optimum cost design incorporates considerations for the level of Type I and Type II error. Extensive research has

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been conducted in the design and development of economic models. Unfortunately, little of this successful research has been adopted by the engineers in industry. One of the concerns most often expressed in attempting to apply these models in real world situations is that there are too many inputs to estimate. One method for reducing the number of terms is to choose an economic model and the appropriate type of control chart and perform a sensitivity analysis on the input variables to determine which are critical.

To help promote the practical use of economic models in industry and help bridge the gap between researchers and practitioners, we have selected a robust economic model and a robust control chart to identify the input parameters significant to a general class of problems. We apply the Lorenzen and Vance (LV) economic model to the CUSUM control chart and perform a sensitivity analysis on the model inputs. Several previously published examples are used to test for robustness of the results. Another example is used to verify the findings of the sensitivity. Also, because the size of the process shift is an important factor, an analysis of a wide range of possible shifts is also conducted. From these analyses we determine the key factors driving cost in the LV CUSUM model, the key factors driving the control chart decision variables, and the extent to which certain input variables may be misspecified without appreciably affecting cost.

2. Literature review

As control charts became more common in industry, cost considerations became an important factor. Cost modeling of quality-control systems was introduced prior to Duncan [1], but he proposed the first fully economic model for single assignable causes, complete with a formal optimization methodology. He developed an economic model for the Shewhart control chart. His paper provided the foundation for much of the subsequent work in this area.

Although the Shewhart chart is very popular and easy to interpret, it is not able to quickly detect small process shifts. The CUSUM chart is being increasingly applied in industry (60 000 charts monitored daily by DuPont alone) because (a) it can quickly detect small process shifts, (b) it is very effective with size one samples, important for the chemical and process industries, (c) the simpler analytic form of the chart is now widely accepted, and (d) the CUSUM can be combined effectively with the Shewhart chart to detect both small and large shifts [2].

The CUSUM control chart was developed by Page [3]. The scheme became popular after Barnard's article [4]. The basic form of the CUSUM for individual continuous variables is

$$S_i = \max(0, cY_i - k + S_{i-1}),$$

where S_i is the CUSUM value, $Y_i = (x_i - T)/s$ is a transformation of x_i, the ith observation, T the target value, s an estimate of the process standard deviation, k the reference value, c multiplier set to $+1(-1)$ to detect increases (decreases) in the process mean.

The reference value prevents early signaling of an out-of-control situation. The initial value of S_i is usually set at zero. The current CUSUM value S_i is compared with the CUSUM control limit, H. The process is deemed out-of-control when $S_i > H$. The one-sided CUSUM has a minimum value of zero and a single control limit H. To apply this procedure for sample averages, x_i is replaced with $\bar{x}_i$ and s with $s/\sqrt{n}$. As in the case of Shewhart charts, if shifts in the process mean in both directions (two-sided case) are of interest, then two separate CUSUM charts are maintained and can be represented as follows:

$$\text{Increases: } SH_i = \max(0, Y_i - k + SH_{i-1}),$$

$$\text{Decreases: } SL_i = \max(0, -Y_i - k + SL_{i-1}).$$

The initial CUSUM control chart was in the form of a "V-mask" which is applied to the plotted cumulative sum values. The V-mask is constructed by specifying Type I and Type II errors and the magnitude of the shift to be detected. From these inputs, the parameters of the V-mask may be determined. The mask is then cut from thick paper or cardboard and placed over the last CUSUM value plotted. If all previously plotted points are inside the open area of the mask (which appears like a notched letter "V"), then no assignable causes of variation are assumed to be present. Otherwise, an out-of-control condition is indicated. A detailed description of the V-mask form of the CUSUM can be found in Montgomery [5].

Research on economic models for CUSUM charts began when Taylor [6] first introduced a CUSUM chart economic model, but his approach required that the sampling interval and sample size be prespecified in order to solve the model. Goel and Wu [7] developed a single assignable cause model, similar to Duncan's, for the CUSUM chart. Their work also provided sensitivity analysis on some of the model parameters. Chiu [8] modified previous approaches to CUSUM economic modeling by working with the analytic form of the CUSUM instead of the V-mask version. The analytic form offers the advantages that it is easier to compute and easier for the operator to understand.

In recent years, vast amounts of successful research have been accomplished in developing economic control chart designs, but very little has actually been implemented in industry. Some of the reasons given by Saniga and Shirland [9] and Chiu and Wetherill [10] are that the mathematical models are complex, and the model input parameters are too numerous and often hard to estimate. Others have noted that the assumptions used in developing the economic models do not apply in real world situations.

The solutions to the economic model implementation problems are steadily surfacing. In part due to the renewed interest in total quality control, statistical quality control computer software is abundant and widely used, making the use of complex models relatively simple. Although the input parameters may in some cases be difficult to estimate, Montgomery [11] noted that the cost response is relatively flat and generally insensitive to errors in parameter estimation. Reducing the number and required precision of input parameters has been studied by Montgomery [12], von Collani [13], Montgomery and Storer [14], and Pignateillo and Tsai [15].

Advances in the applicability of these models to real world situations have also surfaced. Recently proposed economic models, such as that of Lorenzen and Vance [16], are quite robust to the type of control chart used and the assumed assignable cause distribution. Before the Lorenzen and Vance (LV) model was introduced, economic models could only be used for X-bar and fraction defective charts. Because the LV model incorporates null and specified shift average run lengths, most any control chart can be used. Deviations from the traditionally assumed exponential time between occurrences in economic designs has been studied by Hu [17] and Banerjee and Rahim [18 and 19]. Specifically with regard to the LV model, McWilliams [20] found that their design is quite insensitive to the assumed distribution. He performed a sensitivity analysis on model performance for non-exponential assignable cause distributions by applying the LV model under Weibull distributions with varying shape parameters. He found that the LV model was insensitive to the Weibull family assignable cause distribution. This finding provided additional rationale for using the LV model in situations requiring a robust framework.

3. The Lorenzen and Vance economic design

The Lorenzen and Vance model provides the practitioner the most flexibility of any of the widely known single assignable cause models available. By using average run lengths instead of Type I and Type II errors, LV allows the analyst to choose from any type of variable or attribute control chart. The authors also include indicator variables in the model to identify whether production ceases or continues during search and/or repair, so that any possible operational scenario can be appropriately modeled.

The LV model incorporates three types of cost ratios into its formulation: (1) the cost of producing non-conforming items, (2) the cost of false alarms and of search and repair of the true assignable cause, and (3) the cost of sampling. The CUSUM control chart design parameters for the LV model sample size (n), sampling interval (h), decision interval (H) and reference value (k) are chosen to minimize the expected cost per hour function:

$$C = \frac{C_0/\lambda + C_1(-\tau + nE + h(\mathrm{ARL2}) + \delta_1 T_1 + \delta_2 T_2}{\mathrm{ECT}} + \frac{sY/\mathrm{ARL1} + W}{\mathrm{ECT}} + \frac{[(a+bn)/h][1/\lambda - \tau + nE + h(\mathrm{ARL2}) + \delta_1 T_1 + \delta_2 T_2]}{\mathrm{ECT}}, \quad (1)$$

where

$$\mathrm{ECT} = 1/\lambda + (1-\delta_1)sT_0/\mathrm{ARL1} - \tau + nE + h(\mathrm{ARL2}) + T_1 + T_2. \quad (2)$$

ECT represents the expected cycle time, which is the time between successive in-control periods. Table 1 provides a description of each of the model parameters as well as definitions of other terms used in the paper.

A search of possible combinations of the decision variables n, H, k, and h is conducted to find the optimal values n^*, H^*, k^*, and h^* that minimize hourly cost. The optimization procedure includes a grid search on n, H, and k, and a Golden section search on h to minimize expected hourly cost. Grid increments were determined by inspection of the cost deviations between design alternatives in the region of minimum expected cost. Adequate grid granularity was defined as having at least 50 design alternatives within 5% of optimal cost across a variety of scenarios. The resulting grid consisted of sample sizes ranging from 1 to 12. The decision interval values ranged from 0.5 to 6.5 in 0.5 increments. The reference value was varied between 0.125 and 1.000 in increments of 0.125. The CUSUM control chart average run lengths are computed using Brook and Evans' [21] Markov chain approach. Because the matrix inversion routines are computationally time consuming, ARL tables for various combinations of n, k, H, and shift in the process mean (Δ) are developed and a file lookup technique is used for the optimization runs.

Table 1
Explanation of model terms

Symbol	Description
λ	Number of assignable causes per hour
τ	Expected time of occurance of the assignable cause (a function of λ)
s	Expected number of samples taken while in-control (a function of λ)
T_0	Expected search time when false alarm
T_1	Expected time to discover the assignable cause
T_2	Expected time to repair the process
E	Time to sample and chart one item
C_0	Quality cost/hour while producing in-control
C_1	Quality cost/hour while producing out-of-control
W	Cost for search/repair
a	Fixed cost per sample
b	Variable cost per unit
Y	Cost per false alarm
Δ	Mean shift-number of standard deviations slip when out-of-control
n	Sample size
h	Hours between samples or sampling interval
k	CUSUM reference value
H	CUSUM decision interval
δ_1	Flag for whether production continues during searches (1-yes, 0-no)
δ_2	Flag for whether production continues during repairs (1-yes, 0-no)
ARL1	In-control average run length
ARL2	Out-of-control average run length
L	X bar chart – number of standard deviations from control limits to center line
α	Probability of a Type I error or probability of a false alarm
β	Probability of a Type II error or 1.0 minus the power of the test

4. Sensitivity analysis

By examining the cost equation for the LV model it is evident that, although the terms fully describe the economics of the control chart process, there are many parameters to estimate. The purpose of the sensitivity analysis is to determine the key drivers of cost and the four control chart decision variables. We perform experimental design and analysis of the twelve time and cost input parameters. Table 2 provides a description of the inputs and indicates which portion(s) of the model that each input affects. The response variables include the model output, expected cost per time unit, and the optimum cost control chart decision variables consisting of the CUSUM decision interval, CUSUM reference value, sampling interval, and sample size. We study the decision variable responses separately to understand the effects that inputs have on the optimal cost design.

Initial experiments were developed and run against two different previously published examples. These examples were first presented in the LV paper [16] and in the Montgomery text [5, p. 420] and provide a diverse set of realistic scenarios. Because the expected cost per unit time is the response of primary interest, additional experiments are conducted using different previously published scenarios to determine whether the key inputs change. Finally, verification and misspecification analyses are performed using a scenario developed by the authors.

The sensitivity experiments for the examples were designed using two possible process shift ranges. The small shift condition contained low, center and high levels for process shifts of 0.25, 0.75 and 1.25 standard deviations. The large shift condition used process shift levels of 1.25, 1.75 and 2.25 standard deviations.

For each experiment, a resolution IV 2^{12-6} design was selected so the main effects could be estimated by not being confounded with the two factor interactions. A range of $\pm 30\%$ for the baseline variable values was used to calculate the high and low levels. Because the equation used to calculate expected cost was deterministic, a single design replicate and single center point were run resulting in 65 optimization runs. The deterministic nature of the response required us to use a heuristic approach for identifying significant variables. Significant variables were determined by inspection of the normal probability effects plots. Higher order interaction terms were pooled to provide an estimate of error. Significant main effects were identified in an effort to identify the most parsimonious model. The proposed effects were used to develop an analysis of

Table 2
Input variables

Symbol	Description	LV model component		
		Cost of producing bad product	Cost of search and repair (true cause and false alarms)	Cost of sampling
λ	Number of assignable causes per hour	x	x	x
T_0	Expected search time when false alarm	x	x	x
T_1	Expected time to discover the assignable cause	x	x	x
T_2	Expected time to repair the process	x	x	x
E	Time to sample and chart one item	x	x	x
C_0	Quality cost/hour while producing in-control	x		
C_1	Quality cost/hour while producing out-of-control	x		
W	Cost for search/repair		x	
a	Fixed cost per sample			x
b	Variable cost per unit			x
Y	Cost per false alarm		x	
Δ	Mean shift-number of standard deviations slip when out-of-control	x	x	x

variance (ANOVA) model and the effect estimates and standard errors were calculated. Typically, a cutoff point of $p = 0.05$ is used to determine significance. In this situation however, because the cost model is deterministic, there is no noise term in the ANOVA other than the higher order terms. As a result, the standard errors of the effect estimates tend to be very small and most of the main effects and two factor interactions were significant at the 5% level. In the interest of parsimony and dimension reduction, only the major contributors were selected for inclusion into each model.

Example 1. The first scenario used in the sensitivity analysis is an example used by Lorenzen and Vance when they introduced their economic model. They considered the economic implications of the use of a fraction defectives chart (p-chart) in a foundry operation. The purpose of the control chart was to isolate assignable causes for high readings in carbon-silicate content in castings. High levels of carbon-silicate indicated that the castings would have low tensile strength.

We chose to apply the CUSUM control chart using many of the same initial cost and time parameter values. We made small changes to a number of the variables to obtain reasonable symmetric high and low levels for the designed experiment. We also included a non-zero value for the fixed cost per sample term. The high (low) levels for each variable were found by increasing (decreasing) the center point by about 30%. The design points and outputs are listed in Table A.1 in Appendix A. The center point levels are listed below.

$\lambda = 0.03$	$E = 0.333$	$a = \$1.0$
$T_0 = 0.333$	$C_0 = \$115$	$b = \$4.0$
$T_1 = 0.333$	$C_1 = \$950$	$Y = \$975$
$T_2 = 1.5$	$W = \$975$	$\Delta = 0.75$

The runs were made using the algorithm previously described on a 486DX-33 personal computer with each optimization taking about three minutes. The 2^{12-6} fractional design including the center point resulted in 65 runs per scenario. The analysis consisted of determining the significant variables for each of the decision variables (see Table 3).

The results show that four of the 12 inputs significantly drive the cost response. The significant variables include λ, C_0, C_1, and Δ. As the number of assignable causes per hour increases, cost also increases. The two quality cost variables, C_0 and C_1, are also positively correlated with cost. The process shift has a negative correlation, meaning that it costs more to detect smaller shifts. This four variable model accounts for over 90% of the total variability in the cost equation.

The CUSUM reference value (k) is almost entirely dependent on the level of the process shift. This result is consistent with practical guidelines that suggest setting the reference value equal to 1/2 the process shift to be detected [22, 5]. According to Chiu [8] and Lucas [3], this approach gives the smallest out-of-control ARL (ARL2) for a given in-control ARL (ARL1). Because the optimal cost control chart does not exclusively consider statistical performance, the resulting control chart reference values did not always equal $\Delta/2$, but were some function of Δ.

The sensitivity models for the decision variables of H, h, and n captured only about two-thirds of the response variability. Because the LV equation contains many occurrences of these responses both directly and indirectly (via ARL values), we suspected that many terms influenced these optimal cost design parameters. However, it was important to only select the largely significant variables because we were interested in a parsimonious model

Table 3
Example 1 results (shift of 0.25–1.25)

Response	Transform?	R^2	λ	E	C_0	C_1	a	b	Y	Δ
Cost		0.91	+		+	+				−
Reference value		0.88								+
Decision interval		0.67							+	−
Sampling interval	log e	0.64	−	−		−		+		
Sample size	log e	0.67		−						−

describing the underlying relationship between the responses and the influential inputs. The results for the CUSUM decision interval (H) indicated that Δ was the primary influence and Y, the cost of a false alarm, also had an effect. For the sampling interval response, we expected the LV model cost of sampling ratio term to have an impact. Indeed, the major contributors included three terms from that ratio λ, E, and b and a fourth term C_1, the cost while producing out-of-control. The sample size was a function of E and Δ. In our ANOVA model development, we sometimes encountered unequal error variances, requiring a transformation of the response variable. In each case, a logarithmic transformation worked well. We have indicated the models requiring transformations in the tables.

Although the study included five response variables, the most important variable was expected cost per time unit. The results of this example indicate that only four inputs significantly drove the cost response. If this result can be generalized for the LV model using the CUSUM chart, the practitioner's emphasis can be directed toward accurate estimation of this reduced set of variables. The next example will be used to test the generalization.

Example 2. We used a modification of the example shown in [5, p. 420] to test the sensitivity analysis results using somewhat different input values. In this example, control of soft drink bottle thickness is monitored because the manufacturer is interested in detecting whether the wall of the glass is too thin. If this condition occurs, the internal pressure generated during filling will cause the bottle to burst. The Montgomery example used an X-bar chart and applied Duncan's model that assumes the process continues during search and repair of the assignable cause. We used a CUSUM chart and, in the LV model, set the flags (δ_1 and δ_2) to the value of one to simulate the process continuing during search and repair. We also assumed a nominal value for the defect cost during the in-control condition. The process cost and time values used as the center points were developed from the example and the high and low values were obtained using the same range ($\pm$ 30%) as Example 1. The center points are listed below. The design points and response output are listed in Table A.2 in Appendix A.

$\lambda = 0.05$	$E = 0.0833$	$a = \$1.00$
$T_0 = 1.0$	$C_0 = \$5$	$b = \$0.10$
$T_1 = 1.0$	$C_1 = \$100$	$Y = \$50$
$T_2 = 1.0$	$W = \$25$	$\Delta = 0.75$

This example provided an opportunity to test the diversity of the sensitivity results from the first example. It is important to compare the magnitudes of certain cost ratios between examples when searching for significantly different inputs. Some practical cost ratios include the ratio of cost to locate and repair the assignable cause to the quality cost per hour while producing out-of-control (W/C_1). The ratio of quality cost per hour while producing out-of-control to the quality cost per hour while producing in-control (C_1/C_0) is also a practical consideration and may intuitively have an impact on input variable significance. Table 4 shows that several of these ratios are compared for the two examples and the results indicate that the examples are different.

The results of this example are shown in Table 5 listed with the LV example results. Many of the significant variables from the LV example are also significant in the Montgomery example. For the cost response, the only change from the Example 1 results is that C_0 is not significant. For the reference value response, Δ is again the only significant variable, accounting for 99% of the variability. In the decision interval (H) model, two additional variables (a and b) are significant, indicating that sampling costs also affected H. Additional variables are also significant in the sampling interval model. The sample size model consists of three terms E, C_1 and Δ that account for 91% of the variability.

Summarizing the results of both examples, it is clear that, for the cost response, at least three inputs (λ, C_1, and Δ) and possibly a fourth (C_0) are key cost drivers. More scenarios are used later to test

Table 4
Cost ratios

	W/C_1	C_1/C_0	C_1/b	W/Y
Example 1	1.03	8.3	237.5	1.0
Example 2	0.25	20.0	1000.0	0.5

Table 5
Example 1 and 2 results (shift of 0.25–1.25)

Response	Transform?	R^2	λ	E	C_0	C_1	a	b	Y	Δ
Cost										
LV example		0.91	+		+	+				−
Ex. 10-1		0.91	+			+				−
Reference value										
LV example		0.88								+
Ex. 10-1		0.99								+
Decision interval										
LV example		0.67							+	−
Ex. 10-1	log e	0.64					−	−	+	−
Sampling interval										
LV example	log e	0.64	−	−		−		+		
Ex. 10-1		0.58		−		−	+	+	−	+
Sample size										
LV example	log e	0.67		−						−
Ex. 10-1		0.91		−		−				−

this result. In terms of the decision variables, the process shift Δ drives the reference value k. The major inputs determining the decision interval H are Y and Δ. The inputs E, C_1 and b are significant for the sampling interval (h) response in both examples. The combined results of the sampling interval and sample size models indicate that as E and C_1 decrease, the decision variables h and n increase.

4.1. *Small versus large process shifts*

We pointed out earlier that the two examples contained significantly different input values. The only independent variable with identical values between examples is Δ. Because we chose the CUSUM for the sensitivity, small to medium shifts (0.25, 0.75, and 1.25) are used in the designs because the CUSUM is better than the Shewhart in detecting small shifts. We are also interested in determining whether the results of the sensitivity would change if Δ represents medium to large shifts (1.25, 1.75, and 2.25). We selected the Lorenzen and Vance example (Example 1) to perform another 2^{12-6} design with a single center point run. The results are shown in Table 6 together with the small shift case.

For the cost response, the same key drivers are significant regardless of shift size. Both models account for over 90% of the variability. The larger process shift still drives the reference value. The results for the other decision variables are similar to the small shift case, with minor changes in two of the models. As is the case for the examples in the small shift scenario, Δ is significant in each model that was developed under the large shift scenario. These results are not surprising, but they emphasize the importance of correctly specifying the size of the process shift.

4.2. *Cost response study*

Although the two examples initially used in the sensitivity study contain vastly different cost ratios, it may be of further interest to investigate other scenarios. Chiu's development of an economic model for the CUSUM control chart involved the study of 15 case scenarios. These scenarios were formed by perturbing one or two inputs at a time using a basic example (Case 1). We computed key cost ratios for the 15 cases and selected three cases (1, 4 and 7) whose ratios were significantly different from our first two examples.

Table 6
Example 1 using small and large process shifts

Responses	Transform?	R^2	λ	E	C_0	C_1	b	Y	Δ
Cost									
Small shift		0.91	+		+	+			−
Large shift		0.90	+		+	+			−
Reference value									
Small shift		0.88							+
Large shift		0.80							+
Decision interval									
Small shift		0.67						+	−
Large shift		0.60					−		−
Sampling interval									
Small shift	log e	0.64	−	−		−	+		
Large shift	log e	0.76	−			−	+		+
Sample size									
Small shift	log e	0.67		−					−
Large shift		-	Insufficient variability – all but 8 runs had $n = 1$ as optimum sample size						

Table 7
Additional examples versus cost response

	Significant variables						Key parameter values and cost ratios				
	λ	T_2	C_0	C_1	W	Δ	λ	W/C_1	C_1/C_0	C_1/b	W/Y
LV	√		√	√		√	0.03	1.03	8.3	237.5	1.0
Montgomery	√		√	√		√	0.05	0.25	20.0	1000.0	0.5
Chiu Case 1			√				0.01	0.13	3.0	1500.0	2.0
Chiu Case 4			√				0.01	0.01	2.0	20000.0	2.0
Chiu Case 7			√				0.01	1.33	3.0	1500.0	20.0
Chiu Case 1 (mod)	√		√	√		√	0.04	0.13	3.0	1500.0	2.0
Chiu Case 7 (mod)	√	√	√	√	√	√	0.05	1.33	6.0	1500.0	20.0

The same 65-run 2^{12-6} fractional factorial designs are used in each case and the high and low points consisted of $\pm 30\%$ deviations from the initial parameter values. The small shift condition of 0.25, 0.75 and 1.25 was used throughout this study. The design points and response output for Case 1 are provided in Table A.3 in Appendix A. Analysis of the output reveals that these three Chiu cases had only one significant parameter C_0, in each of the ANOVA models, meaning that the in-control quality cost was the only key cost determinant. Study of the input values and associated cost ratios indicates a low assignable cause rate (λ) and a small out-of-control process cost penalty (C_1/C_0 ratio). The other 11 inputs were not significant simply because out-of-control conditions did not occur often enough and when they did, the penalty was not sufficiently severe (see Table 7).

To understand better the influence that the inputs have on the cost equation, we modified two of Chiu's cases (1 and 7). The value of λ for Case 1 was increased from 0.01 to 0.04 so that other terms might become significant. The results indicate that the same four inputs significant in the first example (λ, C_0, C_1, and Δ) are significant here. Case 7 has relatively large values of W and T_2, indicating increased time and cost to repair an assignable cause. In this modification, λ is increased and the value of C_0 is decreased so that the C_1/C_0 ratio increases. The analysis shows that W and T_2 are now significant together with the four other primary cost drivers (λ, C_0, C_1, and Δ).

From these results some general comments can be made. If the assignable cause rate (λ) and cost penalty ratio (C_1/C_0) are not large, then the in-control quality cost (C_0) will be the only cost driver. However, if these elements are significantly large, then other terms can also drive the equation. The other terms typically consist of, but are not limited to the out-of-control quality cost, the assignable cause rate, and the process shift size. It is also evident that if the cost penalty ratio is excessive as in example 2($C_1/C_0 = 20$), then C_0 may no longer be significant. We hesitate to list specific numerical values of the critical input variables or values of input ratios that would be applicable in all cases. At the same time, the results of this sensitivity analysis provide general guidelines to those interested in reducing terms in the LV model and hence make it more convenient for industrial implementation.

4.3. Verification

In order to test our conclusions that the four primary cost drivers (λ, C_0, C_1, and Δ) are significant in models with sufficiently large values of λ and C_1/C_0, we developed a scenario with these conditions and performed a verification study. An analysis was conducted by comparing the range of optimal costs for non-significant variable fluctuation ($\pm$ 30%) versus the optimal cost range for significant variable cost fluctuation. We first ran a baseline case using the following input variable values.

$\lambda = 0.067$	$E = 0.10$	$a = \$0.30$
$T_0 = 0.6$	$C_0 = \$10$	$b = \$0.10$
$T_1 = 0.3$	$C_1 = \$50$	$Y = \$20$
$T_2 = 0.2$	$W = \$10$	$\Delta = 0.75$

The values for λ (0.067) and C_1/C_0 (5.0) are similar in magnitude to the values computed for Example 1 and the modified Case 1 of Chiu (Table 7). The expected relation between each input variable and the response was determined by studying the LV cost function. For instance, it is clear from (1) that as the fixed cost per sample increases, total cost will increase. The relation between cost and other terms such as the expected time to discover the assignable cause (T_1), depend on the value assigned to the process continue or cease flags (δ_1 and δ_2) (see Table 8).

Based on the results of the sensitivity analysis, the four primary cost drivers (λ, C_0, C_1, and Δ) are selected as the group of significant variables. The remaining eight inputs form the group of non-significant variables. The significant variables are first modified by 30% in the direction of increasing cost, while the non-significant variables are held constant. The optimal cost is recorded. Then the significant variables are altered in the direction of decreasing cost, again holding the nonsignificant group constant and the optimal cost is recorded. The difference between the costs is calculated to

Table 8
Results of verification test

	Input variables												Results	
	λ	T_0	T_1	T_2	E	C_0 (\$)	C_1 (\$)	W (\$)	a (\$)	b (\$)	Y (\$)	Δ	Cost (\$)	% change over base
Expected relation with cost	**+**	–	–	–	+	**+**	**+**	+	+	+	+	**–**		
Baseline	**0.067**	0.6	0.3	0.2	0.10	**10**	**50**	10	0.3	0.10	20	**0.75**	16.07	
Modify non-significant variables														
Maximize cost	**0.067**	0.42	0.21	0.14	0.13	**10**	**50**	13	0.39	0.13	26	**0.75**	17.56	
Minimize cost	**0.067**	0.78	0.39	0.26	0.07	**10**	**50**	7	0.21	0.07	14	**0.75**	14.10	21
Modify significant variables														
Maximize cost	**0.087**	0.6	0.3	0.2	0.10	**13**	**65**	10	0.3	0.10	20	**0.25**	23.73	
Minimize cost	**0.047**	0.6	0.3	0.2	0.10	**7**	**35**	10	0.3	0.10	20	**1.25**	10.07	85

Table 9
Results of misspecification test

	Input variables												Results	
	λ	T_0	T_1	T_2	E	C_0 (\$)	C_1 (\$)	W (\$)	a (\$)	b (\$)	Y (\$)	Δ	Cost (\$)	% change over base
Expected relation with cost	+	–	–	–	+	+	+	+	+	+	+	–		
Baseline	**0.07**	0.6	0.3	0.2	0.1	**10**	**50**	10	0.3	0.1	20	**0.75**	16.07	
Modify non-significant variables														
Maximize cost	**0.07**	0.54	0.27	0.18	0.11	**10**	**50**	11	0.33	0.11	22	**0.75**	16.62	
Minimize cost	**0.07**	0.66	0.33	0.22	0.09	**10**	**50**	9	0.27	0.09	18	**0.75**	15.52	7
Modify significant variables														
Maximize cost	**0.07**	0.6	0.3	0.2	0.1	**11**	**55**	10	0.3	0.1	20	**0.68**	18.10	
Minimize cost	**0.06**	0.6	0.3	0.2	0.1	**9**	**45**	10	0.3	0.1	20	**0.83**	14.20	24

determine the impact the significant variables had on cost variability. A similar experiment is conducted by altering the non-significant variables, holding the significant variables constant. The difference in costs for non-significant variable modifications is compared to the cost differences under significant variable modifications. The results (Table 8) indicate that modifying the four significant variables results in an 85% change in optimal cost while altering the eight nonsignificant variables only affect the baseline cost by 21%.

4.4. Misspecification

In order to determine the impact that inaccurate estimation of the input variables has on optimal cost, we developed a scenario using the verification test baseline and altered the variables in groups (significant, then nonsignificant) by $\pm 10\%$. Table 9 shows that modifying the non-significant variables only changes optimal cost by 7%, but misspecifying the significant variables by the same amount affects cost by 24%. Obviously the emphasis on accurate estimation should be placed on the four significant inputs.

5. Conclusion

The sensitivity analysis was designed to provide insight into the significant inputs to the LV model when the CUSUM control chart is employed. We wish to emphasize that in developing scenarios, it is important to ensure that the input values relative to each other are realistic so that inferences drawn from them are valid. By restricting our analysis to highly significant factors only, four main effects: rate of shift, magnitude of shift, and the quality costs (in-control and out-of-control), are candidates for the reduced model. These form the basis for building the reduced model. Whether other variables are added or the basis set is reduced depends on factors such as λ and the ratio C_1/C_0. We have also identified key input variables with respect to the decision variables of the LV model: control limit, reference value, sample size and time between samples. We have verified our conclusions concerning highly significant variables with respect to expected cost per time unit by changing and not changing the highly significant variables and noting the effects on expected cost.

A major obstacle to industrial implementation of the LV model is the large number of terms and difficulties in their estimation. Our results indicate that one could drastically reduce the number of input variables and observe relatively small changes in the cost response relative to the full model. This study provides a basis for the investigation of the use of cost ratios rather than actual cost as a further aid to implementation.

With fewer input values and ultimately with the use of ratios rather than actual numerical inputs, industrial practitioners will find it much easier and convenient to apply the LV economic model. Our results suggest starting points for additional sensitivity studies involving the use of the LV and other models with SPC procedures such as the Shewhart and EWMA. We have demonstrated that statistical design of experiments is a preferred and viable alternative to varying one factor at a time when performing sensitivity studies with deterministic models of any type.

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Appendix A

Table A.1
Example 1: Design points and results

Input variables										Output decision/output variables							Run lengths	
λ	T_0	T_1	T_2	E	C_0	C_1	W	a	b	Y	Δ	Cost	k	H	h	n	ARL1	ARL2
0.04	0.23	0.23	1.0	0.23	80	1250	1275	0.7	2.8	675	0.25	$382.61	0.250	5.5	0.15	3	93.8	22.8
0.02	0.23	0.43	2.0	0.43	150	1250	1275	0.7	2.8	1275	0.25	$376.99	0.250	6.5	0.19	3	162.6	27.9
0.02	0.43	0.43	1.0	0.43	80	650	1275	1.3	2.8	1275	1.25	$152.34	0.625	5.0	0.25	1	1369.8	8.6
0.02	0.23	0.43	2.0	0.23	80	1250	1275	1.3	5.2	1275	0.25	$343.07	0.250	5.5	0.28	3	93.8	22.8
0.02	0.43	0.43	1.0	0.43	150	650	1275	1.3	5.2	675	0.25	$305.84	0.125	5.5	0.46	2	41.8	20.1
0.02	0.43	0.43	2.0	0.43	150	650	675	0.7	2.8	675	1.25	$198.31	0.625	4.5	0.26	1	736.0	7.8
0.02	0.23	0.23	2.0	0.23	150	650	1275	0.7	5.2	675	0.25	$295.72	0.250	4.0	0.56	3	38.5	15.6
0.04	0.23	0.43	1.0	0.23	150	650	1275	1.3	2.8	1275	0.25	$360.74	0.250	6.5	0.29	4	62.6	22.9
0.04	0.43	0.43	1.0	0.43	150	1250	675	0.7	2.8	1275	0.25	$467.58	0.250	6.5	0.15	3	162.6	27.9
0.04	0.23	0.43	2.0	0.43	150	650	675	1.3	5.2	675	0.25	$340.36	0.125	6.5	0.21	1	61.5	34.0
0.02	0.23	0.23	1.0	0.43	80	650	675	0.7	5.2	675	1.25	$143.25	0.625	4.0	0.34	1	393.9	7.0
0.02	0.43	0.43	2.0	0.43	80	650	675	0.7	5.2	1275	0.25	$251.14	0.250	6.0	0.29	2	123.9	33.1
0.02	0.23	0.23	2.0	0.23	80	650	1275	0.7	2.8	1275	1.25	$143.05	0.875	3.5	0.42	2	1094.7	4.7
0.02	0.43	0.43	1.0	0.23	80	650	1275	0.7	2.8	675	0.25	$226.19	0.250	5.5	0.30	3	93.8	22.8
0.04	0.23	0.23	1.0	0.43	80	1250	1275	1.3	2.8	1275	1.25	$233.72	0.625	5.0	0.15	1	1369.8	8.6
0.02	0.23	0.23	2.0	0.43	150	650	1275	1.3	5.2	1275	1.25	$219.24	0.625	4.5	0.37	1	736.0	7.8
0.04	0.43	0.23	2.0	0.43	150	650	1275	0.7	2.8	1275	0.25	$351.92	0.250	6.5	0.23	3	162.6	27.9
0.04	0.23	0.23	1.0	0.23	150	1250	1275	0.7	5.2	1275	1.25	$300.97	0.625	5.0	0.15	1	1369.8	8.6
0.02	0.43	0.23	2.0	0.43	150	1250	675	1.3	5.2	675	0.25	$368.40	0.125	6.5	0.17	1	61.5	34.0
0.04	0.23	0.23	2.0	0.23	80	1250	675	1.3	5.2	675	1.25	$206.73	0.625	4.0	0.18	1	393.9	7.0
0.04	0.43	0.23	2.0	0.43	80	650	1275	0.7	5.2	675	1.25	$190.53	0.625	4.0	0.25	1	393.9	7.0
0.04	0.43	0.43	2.0	0.43	150	1250	1275	1.3	5.2	1275	1.25	$307.47	0.625	4.5	0.18	1	736.0	7.8
0.02	0.23	0.23	1.0	0.43	150	650	675	0.7	2.8	1275	0.25	$290.05	0.250	6.5	0.29	3	162.6	27.9
0.02	0.43	0.43	2.0	0.23	80	650	675	1.3	5.2	675	1.25	$141.43	0.875	2.5	0.72	2	189.0	3.6
0.04	0.23	0.23	2.0	0.23	150	1250	675	1.3	2.8	1275	0.25	$435.67	0.250	6.5	0.15	3	162.6	27.9
0.02	0.43	0.23	1.0	0.43	80	1250	1275	0.7	5.2	1275	0.25	$350.34	0.250	6.5	0.17	2	162.6	36.9
0.04	0.43	0.43	2.0	0.23	80	1250	1275	0.7	2.8	1275	1.25	$221.91	0.625	5.0	0.15	1	1369.8	8.6
0.02	0.43	0.23	1.0	0.43	150	1250	1275	0.7	2.8	675	1.25	$236.22	0.625	4.5	0.17	1	736.0	7.8
0.04	0.43	0.43	2.0	0.43	80	1250	1275	1.3	2.8	675	0.25	$393.49	0.250	5.0	0.15	2	70.4	25.9
0.02	0.43	0.43	1.0	0.23	150	650	1275	0.7	5.2	1275	1.25	$221.28	0.875	3.0	0.67	2	456.8	4.1
0.02	0.23	0.23	1.0	0.23	80	650	675	1.3	5.2	1275	0.25	$251.30	0.250	5.5	0.42	3	93.8	22.8
0.02	0.43	0.43	2.0	0.23	150	650	675	1.3	2.8	1275	0.25	$283.93	0.250	6.5	0.36	4	162.6	22.9
0.02	0.23	0.23	1.0	0.23	150	650	675	1.3	2.8	675	1.25	$199.44	0.875	3.0	0.51	2	456.8	4.1
0.03	0.33	0.33	1.5	0.33	115	950	975	1	4	975	0.75	$247.69	0.375	6.5	0.15	1	529.7	16.9
0.04	0.23	0.23	2.0	0.43	150	1250	675	0.7	2.8	675	1.25	$257.42	0.625	4.5	0.15	1	736.0	7.8
0.02	0.23	0.43	2.0	0.43	80	1250	1275	0.7	5.2	675	1.25	$182.94	0.625	4.0	0.24	1	393.9	7.0

Table A.1 (continued)

Input variables										Output decision/output variables							Run lengths	
λ	T_0	T_1	T_2	E	C_0	C_1	W	a	b	Y	Δ	Cost	k	H	h	n	ARL1	ARL2
0.04	0.23	0.43	1.0	0.43	80	650	1275	0.7	5.2	1275	0.25	$337.30	0.250	6.0	0.23	2	123.9	33.1
0.02	0.43	0.23	2.0	0.43	80	1250	675	1.3	2.8	1275	1.25	$162.07	0.625	5.0	0.17	1	1369.8	8.6
0.02	0.43	0.23	2.0	0.23	80	1250	675	0.7	2.8	675	1.25	$273.88	0.250	6.0	0.17	3	123.9	25.3
0.04	0.43	0.23	2.0	0.23	150	650	1275	1.3	2.8	675	1.25	$239.29	0.875	3.0	0.37	2	456.8	4.1
0.04	0.43	0.23	1.0	0.43	150	650	675	1.3	5.2	1275	1.25	$243.26	0.625	4.5	0.26	1	736.0	7.8
0.02	0.43	0.23	2.0	0.23	150	1250	675	0.7	5.2	1275	1.25	$232.11	0.625	5.0	0.20	1	1369.8	8.6
0.04	0.43	0.23	1.0	0.23	150	650	675	0.7	5.2	675	0.25	$342.91	0.125	5.5	0.34	2	41.8	20.1
0.02	0.43	0.23	1.0	0.23	150	1250	1275	1.3	2.8	1275	0.25	$373.58	0.250	6.5	0.23	4	162.6	22.9
0.04	0.43	0.43	2.0	0.23	150	1250	1275	0.7	5.2	675	0.25	$461.57	0.125	6.0	0.19	2	51.0	22.2
0.04	0.23	0.23	1.0	0.43	150	1250	1275	1.3	5.2	675	0.25	$481.49	0.125	6.0	0.15	1	51.0	30.7
0.04	0.23	0.43	2.0	0.43	80	650	675	1.3	2.8	1275	1.25	$168.39	0.625	5.0	0.18	1	1369.8	8.6
0.02	0.23	0.43	1.0	0.43	150	1250	675	1.3	5.2	1275	1.25	$246.88	0.625	4.5	0.24	1	736.0	7.8
0.02	0.23	0.43	1.0	0.23	80	1250	675	0.7	2.8	1275	1.25	$161.46	0.625	5.5	0.15	1	2542.2	9.4
0.02	0.43	0.23	1.0	0.23	80	1250	1275	1.3	5.2	675	1.25	$180.61	0.625	4.0	0.24	1	393.9	7.0
0.04	0.43	0.43	1.0	0.43	80	1250	675	0.7	5.2	675	1.25	$226.21	0.625	4.0	0.17	1	393.9	7.0
0.04	0.43	0.43	1.0	0.23	150	1250	675	1.3	2.8	675	1.25	$270.55	0.625	4.5	0.15	1	736.0	7.8
0.04	0.23	0.43	2.0	0.23	80	650	675	0.7	2.8	675	0.25	$263.16	0.250	5.5	0.22	3	93.8	22.8
0.04	0.43	0.23	2.0	0.23	80	650	1275	1.3	5.2	1275	0.25	$322.71	0.250	5.0	0.37	3	70.4	20.3
0.02	0.23	0.43	2.0	0.23	150	1250	1275	1.3	2.8	675	1.25	$234.64	0.875	3.0	0.34	2	456.8	4.1
0.02	0.23	0.43	1.0	0.23	150	1250	675	0.7	5.2	675	0.25	$369.95	0.125	6.0	0.25	2	51.0	22.2
0.04	0.23	0.23	2.0	0.43	80	1250	675	0.7	5.2	1275	0.25	$429.27	0.250	6.0	0.15	2	123.9	33.1
0.02	0.23	0.23	2.0	0.43	80	650	1275	1.3	2.8	675	0.25	$227.40	0.250	5.5	0.30	3	93.8	22.8
0.04	0.43	0.43	1.0	0.23	80	1250	675	1.3	5.2	1275	0.25	$440.90	0.250	5.5	0.21	3	93.8	22.8
0.04	0.43	0.23	1.0	0.43	80	650	675	1.3	2.8	675	0.25	$277.63	0.250	5.5	0.18	2	93.8	29.5
0.04	0.43	0.23	1.0	0.23	80	650	675	0.7	2.8	1275	1.25	$163.83	0.625	5.5	0.15	1	2542.2	9.4
0.04	0.23	0.43	1.0	0.23	80	650	1275	1.3	5.2	675	1.25	$199.23	0.875	2.5	0.53	2	189.0	3.6
0.04	0.23	0.43	1.0	0.43	150	650	1275	0.7	2.8	675	1.25	$251.73	0.625	4.5	0.20	1	736.0	7.8
0.02	0.23	0.43	1.0	0.43	80	1250	675	1.3	2.8	675	0.25	$293.86	0.250	6.0	0.15	2	123.9	33.1
0.04	0.23	0.43	2.0	0.23	150	650	675	0.7	5.2	1275	1.25	$232.86	0.625	4.5	0.25	1	736.0	7.8

Table A.2
Example 2: Points and results

Input variables										Output decision/output variables							Run lengths	
λ	T_0	T_1	T_2	E	C_0	C_1	W	a	b	Y	Δ	Cost	k	H	h	n	ARL1	ARL2
0.065	1.3	0.7	1.3	0.058	6.5	70	32.5	1.3	0.07	35	1.25	$21.37	1.000	1.5	1.17	7	46.9	1.2
0.065	0.7	1.3	1.3	0.058	6.5	70	17.5	0.7	0.13	65	1.25	$22.29	1.000	2.0	0.93	7	129.3	1.4
0.035	1.3	0.7	1.3	0.058	3.5	130	17.5	0.7	0.07	35	0.25	$29.17	0.375	3.0	0.36	12	32.9	6.5
0.065	1.3	1.3	1.3	0.058	6.5	130	32.5	0.7	0.13	35	0.25	$48.07	0.375	2.0	0.47	10	13.0	5.0
0.065	1.3	1.3	1.3	0.058	3.5	130	32.5	0.7	0.07	65	1.25	$31.55	1.000	2.0	0.53	6	129.3	1.5
0.065	1.3	0.7	0.7	0.108	6.5	70	17.5	1.3	0.13	65	1.25	$20.54	1.000	2.0	0.83	5	129.3	1.7
0.035	1.3	1.3	0.7	0.058	3.5	70	32.5	0.7	0.07	35	0.25	$20.33	0.375	3.0	0.52	12	32.9	6.5
0.035	1.3	1.3	1.3	0.108	3.5	70	17.5	0.7	0.13	65	0.25	$24.02	0.375	3.5	0.53	10	50.5	8.4
0.065	0.7	0.7	1.3	0.108	6.5	130	17.5	0.7	0.07	35	1.25	$30.77	1.000	2.0	0.39	4	129.3	2.0
0.035	0.7	0.7	0.7	0.058	3.5	70	17.5	1.3	0.13	65	0.25	$22.37	0.375	3.0	0.71	12	32.9	6.5
0.065	1.3	0.7	1.3	0.108	6.5	70	32.5	0.7	0.07	65	0.25	$32.09	0.375	4.0	0.34	10	76.3	9.6
0.065	1.3	1.3	0.7	0.108	3.5	130	17.5	0.7	0.13	35	1.25	$28.82	1.000	1.5	0.53	4	46.9	1.7
0.065	0.7	1.3	1.3	0.058	3.5	70	17.5	0.7	0.07	35	0.25	$27.98	0.375	2.5	0.51	12	21.0	5.5
0.065	0.7	0.7	0.7	0.108	6.5	130	32.5	1.3	0.13	35	0.25	$45.85	0.375	1.5	0.57	7	7.7	4.5
0.035	0.7	1.3	1.3	0.108	6.5	130	32.5	0.7	0.07	65	0.25	$37.03	0.375	4.0	0.29	10	76.3	9.6
0.065	0.7	0.7	0.7	0.058	6.5	130	32.5	0.7	0.13	65	1.25	$27.98	1.000	2.0	0.50	5	129.3	1.7
0.065	1.3	1.3	0.7	0.058	6.5	130	17.5	1.3	0.07	35	1.25	$30.50	1.000	1.5	0.72	6	46.9	1.3
0.035	0.7	1.3	0.7	0.058	3.5	130	17.5	0.7	0.07	65	1.25	$18.58	1.000	2.0	0.72	7	129.3	1.4
0.035	1.3	1.3	1.3	0.108	6.5	70	17.5	0.7	0.07	35	1.25	$16.64	1.000	2.0	0.80	5	129.3	1.7
0.065	1.3	1.3	0.7	0.058	3.5	130	17.5	1.3	0.13	65	0.25	$46.54	0.375	2.5	0.49	12	21.0	5.5
0.035	0.7	1.3	0.7	0.108	6.5	130	17.5	1.3	0.13	65	1.25	$23.58	1.000	2.0	0.75	5	129.3	1.7
0.035	0.7	0.7	1.3	0.108	6.5	70	32.5	1.3	0.13	65	1.25	$17.23	1.000	2.0	1.18	6	129.3	1.5
0.035	0.7	0.7	0.7	0.108	3.5	70	17.5	0.7	0.13	35	1.25	$12.18	1.000	1.5	1.06	5	46.9	1.5
0.035	1.3	1.3	0.7	0.058	6.5	70	32.5	0.7	0.13	65	1.25	$16.15	1.000	2.0	1.13	7	129.3	1.4
0.035	0.7	1.3	1.3	0.108	3.5	130	32.5	0.7	0.13	35	1.25	$22.17	1.000	1.5	0.70	4	46.9	1.7
0.035	1.3	1.3	1.3	0.058	6.5	70	17.5	1.3	0.07	65	0.25	$25.42	0.375	3.5	0.58	12	50.5	7.5
0.035	1.3	1.3	0.7	0.108	6.5	70	32.5	1.3	0.13	35	0.25	$25.17	0.500	1.0	1.53	12	5.6	3.0
0.065	0.7	1.3	0.7	0.058	6.5	70	32.5	1.3	0.07	65	0.25	$32.25	0.375	3.0	0.54	12	32.9	6.5
0.035	0.7	1.3	0.7	0.058	6.5	130	17.5	0.7	0.13	35	0.25	$33.25	0.375	2.0	0.59	11	13.0	4.7
0.065	0.7	1.3	0.7	0.108	3.5	70	32.5	0.7	0.13	65	0.25	$31.38	0.375	3.0	0.48	9	32.9	7.7
0.035	0.7	0.7	1.3	0.108	3.5	70	32.5	1.3	0.07	35	0.25	$22.10	0.375	2.0	0.88	12	13.0	4.5
0.035	0.7	0.7	0.7	0.108	6.5	70	17.5	0.7	0.07	65	0.25	$23.43	0.375	4.0	0.41	11	76.3	9.0
0.065	0.7	0.7	1.3	0.058	6.5	130	17.5	1.3	0.07	65	0.25	$47.03	0.500	2.5	0.36	12	34.1	6.6
0.035	0.7	0.7	0.7	0.058	6.5	70	17.5	1.3	0.07	35	1.25	$14.50	1.000	1.5	1.45	8	46.9	1.2
0.035	1.3	0.7	0.7	0.108	6.5	130	32.5	0.7	0.07	35	1.25	$20.21	1.000	2.0	0.48	4	129.3	2.0
0.035	1.3	0.7	0.7	0.058	3.5	130	32.5	1.3	0.13	35	1.25	$17.72	1.000	1.5	1.03	7	46.9	1.2
0.065	1.3	1.3	1.3	0.108	3.5	130	32.5	1.3	0.07	35	0.25	$47.85	0.375	2.0	0.43	8	13.0	5.6
0.035	1.3	0.7	0.7	0.058	6.5	130	32.5	1.3	0.07	65	0.25	$33.61	0.375	3.5	0.38	12	50.5	7.5

Table A.2 (continued)

Input variables										Output decision/output variables							Run lengths	
λ	T_0	T_1	T_2	E	C_0	C_1	W	a	b	Y	Δ	Cost	k	H	h	n	ARL1	ARL2
0.035	1.3	1.3	0.7	0.108	3.5	70	32.5	1.3	0.07	65	1.25	$14.26	1.000	2.0	1.17	7	129.3	1.4
0.035	0.7	1.3	1.3	0.058	3.5	130	32.5	1.3	0.13	65	0.25	$36.19	0.375	3.0	0.49	12	32.9	6.5
0.065	0.7	1.3	1.3	0.108	3.5	70	17.5	1.3	0.07	65	1.25	$20.97	1.000	2.0	0.87	6	129.3	1.5
0.065	0.7	1.3	0.7	0.058	3.5	70	32.5	1.3	0.13	35	1.25	$19.26	1.000	1.5	1.21	7	46.9	1.2
0.065	0.7	0.7	1.3	0.058	3.5	130	17.5	1.3	0.13	35	1.25	$28.47	1.000	1.5	0.76	6	46.9	1.3
0.065	0.7	0.7	0.7	0.108	3.5	130	32.5	1.3	0.07	65	1.25	$27.47	1.000	2.0	0.48	4	129.3	2.0
0.065	0.7	0.7	1.3	0.108	3.5	130	17.5	0.7	0.13	65	0.25	$46.95	0.375	3.5	0.24	7	50.5	10.6
0.065	1.3	1.3	1.3	0.108	6.5	130	32.5	1.3	0.13	65	1.25	$36.95	1.000	2.0	0.60	5	129.3	1.7
0.065	1.3	0.7	1.3	0.058	3.5	70	32.5	1.3	0.13	65	0.25	$31.25	0.375	2.5	0.72	12	21.0	5.5
0.065	1.3	1.3	0.7	0.108	6.5	130	17.5	0.7	0.07	65	0.25	$47.16	0.375	4.0	0.20	8	76.3	11.2
0.065	0.7	1.3	0.7	0.108	6.5	70	32.5	0.7	0.07	35	1.25	$21.50	1.000	2.0	0.64	5	129.3	1.7
0.065	1.3	0.7	0.7	0.108	3.5	70	17.5	1.3	0.07	35	0.25	$27.65	0.375	2.0	0.66	11	13.0	4.7
0.035	0.7	0.7	1.3	0.058	6.5	70	32.5	0.7	0.13	35	0.25	$23.83	0.375	2.0	0.90	12	13.0	4.5
0.065	1.3	0.7	1.3	0.108	3.5	70	32.5	0.7	0.13	35	1.25	$19.42	1.000	1.5	0.74	4	46.9	1.7
0.065	1.3	0.7	0.7	0.058	3.5	70	17.5	0.7	0.07	65	1.25	$15.90	1.000	2.0	0.77	7	129.3	1.4
0.065	0.7	0.7	0.7	0.058	3.5	130	32.5	0.7	0.07	35	0.25	$38.57	0.375	3.0	0.25	10	32.9	7.3
0.035	0.7	1.3	0.7	0.108	3.5	130	17.5	1.3	0.07	35	0.25	$32.23	0.375	2.0	0.57	10	13.0	5.0
0.035	1.3	1.3	1.3	0.058	3.5	70	17.5	1.3	0.13	35	1.25	$14.27	1.000	1.5	1.48	7	46.9	1.2
0.035	1.3	0.7	1.3	0.108	6.5	130	17.5	1.3	0.13	35	0.25	$35.64	0.375	1.5	0.78	9	7.7	4.1
0.035	1.3	0.7	1.3	0.058	6.5	130	17.5	0.7	0.13	65	1.25	$21.82	1.000	2.0	0.73	6	129.3	1.5
0.035	1.3	0.7	0.7	0.108	3.5	130	32.5	0.7	0.13	65	0.25	$32.84	0.375	3.5	0.31	8	50.5	9.8
0.035	0.7	1.3	1.3	0.058	6.5	130	32.5	1.3	0.07	35	1.25	$24.21	1.000	1.5	1.01	7	46.9	1.2
0.05	1	1	1	0.083	5	100	25	1	0.10	50	0.75	$24.90	1.000	1.5	0.72	8	46.9	2.1
0.065	1.3	0.7	0.7	0.058	6.5	70	17.5	0.7	0.13	35	0.25	$28.76	0.500	1.5	0.78	12	10.5	4.1
0.035	1.3	0.7	1.3	0.108	3.5	130	17.5	1.3	0.07	65	1.25	$20.49	1.000	2.0	0.70	5	129.3	1.7
0.065	0.7	1.3	1.3	0.108	6.5	70	17.5	1.3	0.13	35	0.25	$32.68	0.250	0.5	2.36	9	2.1	1.9
0.035	0.7	0.7	1.3	0.058	3.5	70	32.5	0.7	0.07	65	1.25	$13.06	1.000	2.0	1.05	8	129.3	1.3

Table A.3
Chiu Case 1: Points and results

Input variables										Output decision/output variables							Run lengths	
λ	T_0	T_1	T_2	E	C_0	C_1	W	a	b	Y	Δ	Cost	k	H	h	n	ARL1	ARL2
0.013	0.13	0.07	0.14	0.039	65	105	14	0.650	0.130	13	1.25	$67.06	1.000	1.5	2.57	8	46.9	1.2
0.010	0.10	0.10	0.20	0.030	50	150	20	0.500	0.100	10	0.75	$53.55	1.000	1.0	1.79	12	17.7	1.3
0.013	0.07	0.13	0.14	0.039	65	105	26	0.350	0.070	7	1.25	$66.81	1.000	1.5	1.69	7	46.9	1.2
0.013	0.13	0.13	0.14	0.021	65	195	14	0.650	0.070	7	1.25	$68.34	1.000	1.5	1.10	8	46.9	1.2
0.013	0.07	0.07	0.26	0.039	65	195	14	0.350	0.070	7	1.25	$68.35	1.000	1.5	0.82	6	46.9	1.3
0.007	0.13	0.13	0.14	0.039	35	105	26	0.650	0.070	13	1.25	$36.83	1.000	1.5	2.29	9	46.9	1.1
0.007	0.07	0.13	0.26	0.021	35	195	26	0.650	0.130	13	0.25	$42.65	0.250	0.5	2.46	12	2.1	1.8
0.007	0.13	0.13	0.26	0.021	65	105	14	0.650	0.070	13	0.25	$68.41	0.375	1.0	2.81	12	4.4	2.7
0.013	0.13	0.07	0.26	0.021	35	105	26	0.650	0.130	13	0.25	$41.78	0.250	0.5	2.81	12	2.1	1.8
0.013	0.13	0.07	0.26	0.039	35	105	26	0.350	0.130	7	1.25	$37.71	1.000	1.0	1.45	5	17.7	1.2
0.007	0.07	0.07	0.14	0.021	65	105	14	0.650	0.070	7	1.25	$66.19	1.000	1.5	2.98	9	46.9	1.1
0.013	0.07	0.07	0.14	0.039	35	195	26	0.650	0.070	13	1.25	$39.21	1.000	1.5	0.97	7	46.9	1.2
0.013	0.13	0.13	0.26	0.039	35	195	26	0.650	0.070	7	0.25	$43.99	0.125	0.5	1.51	11	1.8	1.7
0.007	0.07	0.07	0.14	0.039	35	105	14	0.350	0.130	7	1.25	$36.70	1.000	1.0	2.25	6	17.7	1.2
0.007	0.13	0.13	0.14	0.021	65	105	26	0.350	0.130	13	1.25	$66.40	1.000	1.5	3.12	8	46.9	1.2
0.007	0.13	0.07	0.26	0.021	65	195	14	0.350	0.130	13	1.25	$67.52	1.000	1.5	1.53	7	46.9	1.2
0.013	0.13	0.07	0.26	0.021	65	105	26	0.650	0.070	7	1.25	$66.92	1.000	1.5	2.21	9	46.9	1.1
0.013	0.07	0.07	0.26	0.021	35	105	14	0.350	0.070	7	0.25	$40.15	0.250	0.5	2.03	12	2.1	1.8
0.013	0.07	0.07	0.14	0.021	65	195	26	0.350	0.130	13	1.25	$68.64	1.000	1.5	1.15	7	46.9	1.2
0.007	0.13	0.07	0.14	0.039	65	195	26	0.350	0.070	7	1.25	$67.22	1.000	1.5	1.30	7	46.9	1.2
0.007	0.07	0.07	0.14	0.021	35	105	14	0.650	0.130	13	0.25	$39.75	0.250	0.5	3.76	12	2.1	1.8
0.007	0.07	0.07	0.26	0.039	65	105	26	0.650	0.130	13	1.25	$66.54	1.000	1.5	3.26	8	46.9	1.2
0.013	0.13	0.13	0.26	0.039	65	195	26	0.650	0.130	13	1.25	$69.39	1.000	1.5	1.23	7	46.9	1.2
0.007	0.13	0.13	0.26	0.039	35	105	14	0.350	0.130	13	0.25	$39.84	0.375	0.5	3.29	12	2.5	1.9
0.007	0.07	0.13	0.26	0.021	65	195	26	0.650	0.070	7	1.25	$67.47	1.000	1.5	1.60	9	46.9	1.1
0.013	0.07	0.13	0.14	0.039	35	105	26	0.350	0.130	13	0.25	$41.79	0.375	0.5	2.36	12	2.5	1.9
0.007	0.07	0.07	0.26	0.039	35	105	26	0.650	0.070	7	0.25	$38.93	0.125	0.5	3.24	12	1.8	1.6
0.007	0.13	0.07	0.14	0.039	35	195	26	0.350	0.130	13	0.25	$42.56	0.375	0.5	2.14	12	2.5	1.9
0.013	0.07	0.13	0.14	0.021	65	105	26	0.650	0.070	13	0.25	$69.79	0.375	1.0	2.14	12	4.4	2.7
0.013	0.13	0.13	0.14	0.039	35	195	14	0.350	0.130	7	1.25	$39.03	1.000	1.0	0.95	5	17.7	1.2
0.007	0.13	0.07	0.26	0.039	35	195	14	0.650	0.070	13	1.25	$37.88	1.000	1.5	1.38	8	46.9	1.2
0.013	0.13	0.07	0.14	0.021	35	105	14	0.350	0.070	13	1.25	$37.18	1.000	1.5	1.38	8	46.9	1.2
0.013	0.07	0.07	0.26	0.021	35	195	14	0.650	0.130	7	1.25	$39.24	1.000	1.0	1.17	6	17.7	1.2
0.007	0.07	0.13	0.14	0.021	65	195	14	0.350	0.130	7	0.25	$70.38	0.125	0.5	2.46	12	1.8	1.6
0.007	0.13	0.07	0.26	0.021	35	195	14	0.350	0.070	7	0.25	$40.74	0.250	0.5	1.84	12	2.1	1.8
0.007	0.07	0.07	0.26	0.021	65	105	26	0.350	0.130	7	0.25	$67.98	0.125	0.5	3.98	12	1.8	1.6
0.007	0.13	0.13	0.14	0.021	35	105	26	0.350	0.070	7	0.25	$38.71	0.250	0.5	2.81	12	2.1	1.8
0.007	0.07	0.13	0.26	0.039	35	195	26	0.350	0.130	7	1.25	$37.96	1.000	1.0	1.34	5	17.7	1.2

Table A.3 (continued)

Input variables										Output decision/output variables							Run lengths	
λ	T_0	T_1	T_2	E	C_0	C_1	W	a	b	Y	Δ	Cost	k	H	h	n	ARL1	ARL2
0.013	0.07	0.07	0.14	0.021	35	195	26	0.350	0.070	7	0.25	$43.03	0.250	0.5	1.37	12	2.1	1.8
0.007	0.07	0.07	0.14	0.039	65	105	14	0.350	0.070	13	0.25	$68.29	0.375	1.5	1.90	12	7.7	3.5
0.013	0.13	0.07	0.14	0.021	65	105	14	0.350	0.130	7	0.25	$68.94	0.125	0.5	3.29	12	1.8	1.6
0.013	0.13	0.13	0.26	0.021	65	195	26	0.350	0.130	7	0.25	$72.87	0.125	0.5	1.74	11	1.8	1.7
0.013	0.07	0.13	0.26	0.039	35	105	14	0.650	0.070	13	1.25	$37.66	1.000	1.5	1.63	8	46.9	1.2
0.013	0.07	0.07	0.14	0.039	65	195	26	0.650	0.130	7	0.25	$73.04	0.125	0.5	1.60	8	1.8	1.8
0.013	0.07	0.13	0.14	0.021	35	105	26	0.650	0.130	7	1.25	$37.75	1.000	1.0	1.90	7	17.7	1.1
0.013	0.07	0.07	0.26	0.021	65	195	14	0.650	0.070	13	0.25	$73.90	0.375	1.0	1.13	12	4.4	2.7
0.007	0.07	0.13	0.14	0.039	35	195	14	0.650	0.070	7	0.25	$41.06	0.125	0.5	2.09	12	1.8	1.6
0.007	0.13	0.13	0.26	0.039	65	105	14	0.350	0.070	7	1.25	$66.16	1.000	1.5	2.36	8	46.9	1.2
0.007	0.07	0.13	0.14	0.039	65	195	14	0.650	0.130	13	1.25	$67.77	1.000	1.5	1.79	7	46.9	1.2
0.013	0.13	0.13	0.14	0.021	35	195	14	0.650	0.130	13	0.25	$45.37	0.250	0.5	1.86	12	2.1	1.8
0.013	0.07	0.13	0.26	0.039	65	105	14	0.650	0.130	7	0.25	$69.21	0.125	0.5	3.37	12	1.8	1.6
0.013	0.13	0.07	0.26	0.039	65	105	26	0.350	0.070	13	02.5	$69.75	0.375	1.5	1.38	12	7.7	3.5
0.007	0.13	0.07	0.26	0.039	65	195	14	0.650	0.130	7	0.25	$70.73	0.125	0.5	2.42	10	1.8	1.7
0.007	0.07	0.07	0.26	0.021	35	105	26	0.350	0.070	13	1.25	$36.65	1.000	1.5	1.98	8	46.9	1.2
0.013	0.13	0.13	0.26	0.021	35	195	26	0.350	0.070	13	1.25	$39.00	1.000	1.5	0.91	8	46.9	1.2
0.007	0.13	0.07	0.14	0.021	65	195	26	0.650	0.070	13	0.25	$71.36	0.375	1.0	1.53	12	4.4	2.7
0.013	0.07	0.13	0.26	0.021	65	105	14	0.350	0.130	13	1.25	$66.95	1.000	1.5	2.36	8	46.9	1.2
0.007	0.13	0.13	0.26	0.021	35	105	14	0.650	0.130	7	1.25	$36.86	1.000	1.0	2.46	7	17.7	1.1
0.007	0.07	0.13	0.14	0.021	35	195	14	0.350	0.070	13	1.25	$37.44	1.000	1.5	1.26	8	46.9	1.2
0.007	0.13	0.07	0.14	0.021	35	195	26	0.650	0.130	7	1.25	$37.85	1.000	1.0	1.58	6	17.7	1.2
0.013	0.07	0.07	0.26	0.039	35	195	14	0.350	0.130	13	0.25	$45.72	0.375	0.5	1.58	12	2.5	1.9
0.007	0.13	0.13	0.14	0.039	65	105	26	0.650	0.130	7	02.5	$68.09	0.125	0.5	3.98	10	1.8	1.7
0.007	0.07	0.13	0.26	0.039	65	195	26	0.350	0.070	13	0.25	$71.45	0.375	1.5	1.03	12	7.7	3.5
0.013	0.13	0.07	0.14	0.039	35	105	14	0.650	0.070	7	0.25	$40.32	0.125	0.5	2.41	12	1.8	1.6
0.013	0.13	0.13	0.14	0.039	65	195	14	0.350	0.070	13	0.25	$73.82	0.375	1.5	0.75	12	7.7	3.5

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