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The Place of Meaning in the Teaching of Arithmetic

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Defining Meaning

Increasingly, during the last twenty years, the literature relating to arithmetic instruction has carried the words "meaning," "meaningful," and "meaningfully." For some persons, these terms seem to be no more than words—mere items in the vocabulary of modern elementary education, adopted because, for the moment, they are fashionable. For others, these words serve as symbols of a vague protest against what they call the "traditional" arithmetic, though they have little except pious wishes to offer as a substitute. For still others, the terms are appropriate for use in connection with arithmetic experiences which arise out of felt needs on the part of children. This third usage, unlike the first two, has in its favor a certain definiteness. It implies particular conditions of learning and motivation. Children see the chance to use their arithmetical ideas and skills to further some end, and they use the ideas and skills for this purpose.

We should, however, at this point, distinguish between what I shall designate the *meaning of* a thing and the *meaning of* a thing *for* something else; for the sake of brevity, between *meaning of* and *meaning for.* I know little about the meaning *of* the atomic bomb, because I lack the knowledge of chemistry and physics which are requisite to accurate understanding, but I think I know a good deal about the meaning of the atomic bomb *for* other things—for peace or for the destruction of our culture, for example.

The distinction I am suggesting is no verbal quibble, no bit of theoretical hairsplitting. Failure to recognize the difference between meanings *of* and meanings *for* makes it difficult for those of us who are interested in the improvement of arithmetic instruction to agree on procedures. We use the same words but in different senses. The third usage, namely, that children have meaningful arithmetic experiences when they use arithmetic in connection with real life needs, relates to meanings *for*. On this account some prefer to call such arithmetic experiences "significant" rather than "meaningful."

On the other hand, just as the meaning *of* the atomic bomb is to be found in the related physical sciences, so the meanings *of* arithmetic are to be found in mathematics. They are not to be found in the life-settings in which they are normally imbedded, except by him who already possesses them. They must be sought in the mathematical relationships of the subject itself, in its concepts, generalizations, and principles. In this sense a child has a meaningful arithmetic experience when the situation with which he deals "makes sense" mathematically. He behaves meaningfully with respect to a quantitative situation when he knows what to do arithmetically and when he knows how to do it; and he possesses arithmetical meanings when he understands arithmetic as mathematics. In arithmetic, then, meanings *of* may be defined as mathematical understandings, and it is in this sense that the word will be used throughout this article.

I have spoken of meanings as if they were absolute—as if, one has a meaning, or he has none. In terms of learning, however, meanings are relative, not absolute. There are degrees of meanings; degrees of what may be termed extent, exactness, depth, complexity; and growth in meanings may take place in any of these dimensions. For relatively few aspects of life, for relatively few aspects of the school's curriculum (including arithmetic), do we seek to carry meanings to anything like their fullest development. Moreover, whatever the degree of meaning we want children to have, we cannot engender it all at once. Instead, we stop at different levels with different concepts; we aim now at this level of meaning, later at a higher level, and so on.[1]

[1] William A. Brownell and Verner M. Sims, "The Nature of Understanding," *The Measurement of Understanding*, pp. 27–43. Forty-fifth Yearbook of the National Society for the Study of Education, Part I. Chicago: distributed by the University of Chicago Press, 1946.

Meaningful Arithmetic

"Meaningful" arithmetic, in contrast to "meaningless" arithmetic, refers to instruction which is deliberately planned to teach arithmetical meanings and to make arithmetic sensible to children through its mathematical relationships. Not all possible meanings are taught, nor are all meanings taught in the same degree of completeness. Meaningful arithmetic, then, may be thought of as occupying a place well to the right on a scale of meaningfulness. On the other hand, "meaningless" arithmetic occupies a place well toward the left end of the scale but not at the 0-point; for there can hardly be a wholly meaningless arithmetic. Meaningless arithmetic is only relatively meaningless. Its content is taught with no specific intention of developing meanings, and the meanings which are learned are acquired incidentally and largely through the learner's own efforts.

The meanings of arithmetic can be roughly grouped under a number of categories. I am suggesting four.

1. One group consists of a large list of basic concepts. Here, for example, are the meanings of whole numbers, of common fractions, of decimal fractions, of per cent, and, most persons would say, of ratio and proportion. Here belong, also, the denominate numbers, on which there is only slight disagreement concerning the particular units to be taught. Here, too, are the technical terms of arithmetic—addend, divisor, common denominator, and the like—and, again, there is some difference of opinion as to which terms are essential and which are not.
2. A second group of arithmetical meanings includes understanding of the fundamental operations. Children must know when to add, when to subtract, when to multiply, and when to divide. They must possess this knowledge, and they must also know what happens to the numbers used when a given operation is employed. If the newer textbooks afford trustworthy evidence on the point, the trend toward the teaching of the functions of the basic operations is well established. Few changes in the more recent textbooks, as compared with those of twenty years ago, are more impressive.
3. A third group of meanings is composed of the more important principles, relationships, and generalizations of arithmetic, of which the following are typical: When 0 is added to a number, the value of that number is unchanged. The product of two abstract factors remains the same regardless of which factor is used as multiplier. The numerator and denominator of a fraction may be divided by the same number without changing the value of the fraction.
4. A fourth group of meanings relates to the understanding of our decimal number system and its use in rationalizing our computational procedures and our algorisms. There appears to be a growing tendency to devote more attention to the meanings of large numbers in terms of the place values of their digits. Likewise there is a strong tendency to rationalize the simpler computational operations such as "carrying" in addition and "borrowing" in subtraction; but there is some hesitation about extending rationalizations very far into multiplication and division with whole numbers and fractions.

It is a mistake to suppose meaningful arithmetic is something new, something cut out of the whole cloth, as it were, during the past twenty or twenty-five years. Three years ago I participated in a conference on arithmetic in a southwestern state. I used my time chiefly to show how arithmetic may be made mathematically sensible to children. At the conclusion of the conference, an elderly member, a principal in one of the elementary schools, told me that the superintendent of the system thirty years before had been dismissed largely because he persistently advocated the very procedures which I had described.

If there is anything unique about our present interest in meaningful arithmetic, it is, first, that the interest is more general than ever before and, second, that it embraces, not a segment, but the whole range of arithmetical content. In times past, beginning with Pestalozzi, attempts to make arithmetic meaningful were confined largely to the primary grades. True, some students of the subject and some teachers of secondary-school mathematics were disturbed because the arithmetic of the higher grades (percentage, for instance) seemed senseless to children, but they did little about it. Curiously enough, and inconsistently enough, some of them saw little reason to worry about the equal senselessness of arithmetic as taught in the lower grades. In recent years, these individuals have seen the light, and now they are eager to have *all* arithmetic taught meaningfully, from the kindergarten and Grade I through the intermediate grades to Grade VIII or IX.

Increased Interest in Arithmetical Meanings

The chief reason for our vital interest in arithmetical meanings is to be found, I think, in the demonstrated failure of relatively meaningless programs. The latter programs have not produced the kind of arithmetical competence required for intelligent adjustment to our culture. Evidence of this failure has been accumulating from several sources.

Teachers are familiar with three kinds of evidence of the weakness of the arithmetic taught in the elementary schools of the teens and of the twenties and thirties of this century: (1) anecdotal evidence illustrating the arithmetical incompetence of adults in their practical activities; (2) test evidence and testimony of the armed forces, which have been given wide publicity; and (3) the experience of teachers of mathematics above the elementary grades.

There is still another body of evidence, with similar portent—the findings of research. Long before World War II, investigators within the area of education were revealing shortcomings in arithmetic instruction. Error studies, for example, disclosed faulty procedures which were explicable only as the results of blind groping on the part of children. Test and interview data showed the same uncertainty and confusion. Still other investigations, through a frontal attack on problems of instruction, revealed that meaningful arithmetic actually paid dividends. For one thing, it protected the children from the absurd mistakes commonly made under other programs of instruction.

No explanation for the current interest in meaningful arithmetic would be complete if reasons were sought only within the field of arithmetic itself, however. Arithmetic is but one of the subjects of the elementary-school curriculum. For the past twenty years and more, the elementary-school curriculum has been the subject of continuous and lively discussion. Its content has been reexamined in light of the purposes of elementary education, and the methods of instruction employed in teaching have been critically evaluated. In the midst of this general ferment about the curriculum, it was hardly possible that arithmetic should escape notice, or that our views of arithmetic and of the methodology of instruction should remain unchanged while our views of the elementary-school curriculum as a whole were undergoing drastic reorganization. Our thinking about the curriculum inevitably had its effects on our thinking about arithmetic, and the new insistence on meaningful learning in other subjects, as might have been expected, naturally led to the demand for meaningful learning in arithmetic.

School personnel and, to some extent, the public at large are beginning to awaken to the fallacy of treating arithmetic as a tool subject.[2] To classify arithmetic as a tool subject, or as a skill subject, or as a drill subject is to court disaster. Such characterizations virtually set mechanical skills and isolated facts as the major learning outcomes, prescribe drill as the method of teaching, and encourage memorization through repetitive practice as the chief or sole learning process. In such programs, arithmetical meanings of the kinds mentioned above have little or no place. Without these meanings to hold skills and ideas together in an intelligible, unified system, pupils in our schools for too long a time have "mastered" skills which they do not understand, which they can use only in situations closely paralleling those of learning, and which they must soon forget.

Objections to Teaching Arithmetic Meaningfully

I do not mean that a complete victory has been won for meaningful arithmetic. There is still opposition,[3] though it seems to be declining both in extensiveness and in vigor. Not many persons wish, in 1946–47, to speak out vehemently against meaningful arithmetic. Nevertheless there are lingering doubts, and we had best examine them.

These doubts, stated as questions, take the form of:

1. Are meanings really necessary in the learning of arithmetic?
2. Are not meanings of the kind now called for really too difficult for children to learn?
3. Does it not take an undue amount of time to teach meanings—so much time that other more important aspects of the subject suffer?
4. Suppose that meanings are learned: do they actually function; are they really used; may they not interfere with effective thinking?

Let us start with the first question. "Are meanings essential to the learning of arithmetic?" If this question is interpreted as asking whether we need to understand processes merely in order to compute accurately, the answer is "No." Moreover, this reply is the answer usually given by persons who have little faith in meaningful arithmetic. For them the purpose of arithmetic is to produce habits of quick, correct

[2] Charles H. Judd, "The Fallacy of Treating School Subjects as 'Tool Subjects,'" *Selected Topics in the Teaching of Mathematics*, pp. 1–10. Third Yearbook of the National Council of Teachers of Mathematics. New York: Teachers College, Columbia University, Bureau of Publications, 1928.

[3] See, for example: Irwin Buell, "Let Us Be Sensible about It," *Mathematics Teacher*, XXXVII (November, 1944), 306–8. This statement is typical of arguments against meaningful arithmetic, though it is better expressed than is sometimes the case. It was effectively answered by Harry G. Wheat, "Why Not Be Sensible about Meaning?" *Mathematics Teacher*, XXXVIII (March, 1945), 99–102. A third article in this series is: Walter Bernard, "Why Not Be Sensible about Meaning and Applications?" *Mathematics Teacher*, XXXVIII (October, 1945), 259–63.

computation. In their view, meanings contribute nothing to this purpose and may even defeat it.

The contribution of meanings is obvious, however, if one views arithmetic as a logical system of thinking. Such a system is clearly dependent on meaningful concepts, principles, and expressions of relationships, and little would be gained by laboring this point. Yet, as a matter of fact, meanings contribute to learning regardless of the kind of arithmetic one has in mind, even when one considers its principal purpose to be that of developing skill in computation.

To be of use, computational habits must first of all be retained. As has already been stated, skills which are learned mechanically, with a minimum of meaning, quickly deteriorate. To keep them alive, one must practice them ceaselessly. However, the conditions of life afford little opportunity for continuous practice, and once the unremitting drill of the school is withdrawn, the skills suffer. To be of use, moreover, computational habits must be adaptable to a wide variety of circumstance, and mechanical skills, even when they are retained, cannot meet this test. Therefore, whether the criterion be retention or functional value, meaningless arithmetic defaults on its one claim—the assuring of competence in computation.

The next question pertains to the difficulty of the understandings called for by meaningful arithmetic. By way of illustrating the excessive difficulty of meanings, opponents of meaningful arithmetic seek to reduce the matter to an absurdity by adding the question, "Seriously now, is the fifth-grade child to be expected to rationalize the division of 458,605 by 79, or the multiplication of 3,709 by 638?"

The answer to both questions is, of course, the same: "No." The rationalizations in the two computations cited would be difficult, but not impossible, for the sophisticated adult; but the difficulty would reside, not so much in the mathematical principles involved, as in the language required—both rationalizer and hearer would get lost in the words. No advocate of meaningful arithmetic expects that fifth-grade children shall be able to rationalize examples of this kind, but it is not impossible for fifth-grade children to rationalize the division of 462 by 6 and the multiplication of 8 by 49. If the children can explain the computation in these simpler examples, they can understand that the same principles apply to the more complex processes, and this knowledge gives them confidence in their learning and respect for what they are learning. These attitudes, which are highly prized by exponents of meaningful arithmetic, are apparently of little consequence to their critics. The latter, in raising the question which is intended to reveal the hopelessness of teaching meanings, are guilty of the fallacy of thinking in absolute terms. This fallacy, noted at an earlier place in this article, consists in the belief that meanings, if taught at all, must be carried to the limit of their development. In no program of meaningful arithmetic that I have seen is serious effort expended to extend rationalization very far in the processes of multiplication and division.

The third objection to meaningful arithmetic, expressed as a question, is: "Is not the time necessary for teaching meanings unduly costly? If time is taken for this purpose, will not other aspects of the subject have to be sacrificed?" It does take time to teach meanings. There can be no doubt about that fact, but whether the expenditure is uneconomical is another matter.

With comparatively little research but with considerable experience to support them, advocates of meaningful arithmetic are convinced that it pays to teach understandings. They concede that it takes time to teach place value, for example; but they argue that the total gains fully warrant the time taken. They point out that only through an understanding of place value is it possible really to comprehend the larger numbers. They point out also that understanding of place value helps children to understand many of our computational procedures ("carrying" in addition and "borrowing" in subtraction, for instance). The values of meanings are cumulative. If in order to teach meanings adequately, progress at first seems to be slow, it can be more rapid later on—not only more rapid, but better grounded, with gains in the subject as a whole. In the end, time spent in developing meanings is not lost, but saved.

The last of the four objections to meaningful arithmetic which I am considering has to do with the way in which meanings function in effective quantitative thinking. Are meanings, once they are learned, of any use? Do they actually facilitate thought? Is it not possible that they may even impede the kind of thinking we want in arithmetical situations?

Doubts concerning the functional worth of arithmetical meanings seem to me to have their origin in faulty notions concerning the nature of intelligent thinking. The fallacies are exposed in a criticism offered by those who see little value in arithmetical meanings, to wit:

> You teach children all this tens-and-ones business in "explaining" carrying in addition. Thus, for the example 47 + 36, you have children say, "Seven ones and six ones are thirteen ones. Write the 3 for the ones in the ones' column and carry the one ten of 13. Add the tens' figures: one and four are five; five and three are eight. Write the 8 in the tens' column."
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> What is the sense in having children say all this? Besides, once they learn to say it, won't they use the pattern forever thereafter, thus slowing up their thought-processes needlessly?

There *is* sense in having children employ the full statement in their first experiences with such examples; for, by so doing, they gain insight into the rationale of the process. No one, however, wishes children to continue the long explanation indefinitely, and there is little danger that they will. It is characteristic of the economy of thinking to eliminate and to short-circuit. Needless words tend to be discarded once they have served their purpose.

Exponents of meaningful arithmetic, like their critics, fully expect children to arrive eventually at the abbreviated thought-pattern for the example mentioned: Seven, six, thirteen; write 3; carry one. One, five, eight; write 8. But note that word "eventually." The short form of thinking is not to be attained all at once, but rather by stages, beginning with the first complete statement and proceeding, without loss of understanding, to the final economical pattern.

Furthermore, exponents of meaningful and of meaningless arithmetic, alike, have relatively full understandings of the numbers and of the process represented in the fact 3 + 9 = 12. These understandings do not interfere with arriving at the sum 12 correctly and immediately when the problem 3 + 9 is presented. The response is instantaneous. For such arithmetical items the process of short-circuiting has been carried to its practicable limit. The most careful introspection fails to reveal the operation of meanings, so quickly does the answer come.

Values of Meaningful Arithmetic

So much for the objections most commonly raised to meaningful arithmetic. I have tried to meet these objections. At the same time, I have used these objections as occasions to set forth some of the advantages of meaningful arithmetic. Allow me now to collect these stated advantages and to add to them somewhat in summary.

From the standpoint of the teacher, meaningful arithmetic is interesting to teach. The need to develop understandings is much more stimulating than the task of listening to memorized facts and of administering mechanical drill.

From the standpoint of the pupil meaningful arithmetic—

1. Gives assurance of retention.
2. Equips him with the means to rehabilitate quickly skills that are temporarily weak.
3. Increases the likelihood that arithmetical ideas and skills will be used.
4. Contributes to ease of learning by providing a sound foundation and transferable understandings.
5. Reduces the amount of repetitive practice necessary to complete learning.
6. Safeguards him from answers that are mathematically absurd.
7. Encourages learning by problem-solving in place of unintelligent memorization and practice.
8. Provides him with a versatility of attack which enables him to substitute equally effective procedures for procedures normally used but not available at the time.
9. Makes him relatively independent so that he faces new quantitative situations with confidence.
10. Presents the subject in a way which makes it worthy of respect.

These are ambitious claims—the more so when it must be admitted that not all of them are fully attained, even in the best of arithmetic programs. How much evidence is there to support them?

I wish that I could cite an impressively large body of competent research. I cannot do so. It is probable that of the fifteen hundred to two thousand published reports of investigations, fewer than 5 per cent deal immediately and seriously with meanings. Perhaps another 10 per cent deal indirectly with meanings or have relatively clear implications with respect to the values or the development of meanings.

I do not belittle the worth of the research we have. True, some of the most relevant and promising studies have failed to produce unequivocal findings in favor of meaningful arithmetic. Yet, even these studies have served a purpose, if only to show some of the pitfalls in this kind of research. Research on meaningful learning is extraordinarily difficult. Routine and standardized techniques of control and evaluation have to be considerably modified for the new purpose. We are learning, however, how to plan and manage investigations. Indeed, several of the investigations already reported[4] warrant considerable confidence in meaningful arithmetic.

Even without the assistance of research findings, we can build a fairly strong case for meaningful arithmetic and for the claims made for it. In the first place, as I have stated several times, we have found, through the experience of many teachers, that meaningful

[4] To locate this research, which cannot be mentioned here, the reader is advised to consult the volumes of the *Education Index* using the names: Lester Anderson, W. A. Brownell, Guy T. Buswell, Harry O. Gillet, Charles H. Judd, T. R. McConnell, Herbert F. Spitzer, Ben A. Sueltz, Grace Swenson, C. L. Thiele, and Harry G. Wheat. He would do well to study also the theoretical articles by these same individuals and by B. R. Buckingham and H. Van Engen, to mention some of the principal contributors. He would get considerable stimulation also from E. A. Greening Lamborn, *Reason in Arithmetic*, Oxford, England: Oxford University Press, 1930.

arithmetic "works" and that it yields valuable outcomes.

In the second place, we have negative and deductive evidence. The arithmetic programs of the schools in our times, up to recently, failed to develop arithmetical competence. The element most conspicuously and significantly absent in this instruction was that of meaning. To improve instruction we can choose between two alternatives: (1) We can redouble our efforts with respect to drill along the old lines, or (2) we can change to meaningful arithmetic. The nature of the inadequate results of meaningless arithmetic is such as to warrant greater confidence in the second alternative.

In the third place, we have the unambiguous support of psychological research on meaningful as contrasted with meaningless learning. Without exception; I believe, experimental psychologists have found in favor of meaningful learning, whether the criterion be ease (speed) of learning, retention, or transferability. McGeoch, in his scholarly summary of the results of experimentation on human learning, has the following to say:

> It is probable, on the basis of available data, that there is a very high positive correlation, and perhaps a perfect one when other things are equal, between meaning and rate of learning.[5]
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> When the meaning of a material is not easily available to a learner, he may accelerate his rate of learning by a search for meanings, by the imposition of rhythm and pattern, by new groupings of the items, by noting spatial relations, and by other devices whereby he may make the material more meaningful and thus assimilate it more readily into his already existent patterns of response….
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> The conclusion that there is a high positive correlation between meaningfulness of material and rate of learning holds under a very wide range of conditions.[6]

In the fourth place, the theory of meaningful arithmetic agrees completely with prevailing educational theory in general. Both want children, as children, and later as adults, to live more efficiently, more intelligently, more richly, and more happily in their culture. That culture is highly quantitative and is steadily becoming more so. More and more vital, therefore, is the need for quantitative intelligence; hence, more and more imperative is it that we teach arithmetical meanings.

[5] John A. McGeoch, *The Psychology of Human Learning*, p. 159. New York: Longmans, Green & Co., 1942.
[6] Ibid., p. 167.

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