Cross-efficiency evaluation in data envelopment analysis based on prospect theory

Hui-hui Liu , Yao-yao Song , Guo-liang Yang

PII: S0377-2217(18)30668-4
DOI: https://doi.org/10.1016/j.ejor.2018.07.046
Reference: EOR 15284

To appear in: *European Journal of Operational Research*

Received date: 27 January 2018
Revised date: 12 June 2018
Accepted date: 29 July 2018

Please cite this article as: Hui-hui Liu , Yao-yao Song , Guo-liang Yang , Cross-efficiency evaluation in data envelopment analysis based on prospect theory, *European Journal of Operational Research* (2018), doi: https://doi.org/10.1016/j.ejor.2018.07.046

Highlights

- Common cross-efficiency evaluation models fail to consider the risk attitude.
- This paper investigates the cross-efficiency evaluation based on prospect theory.
- We define a prospect value of decision-making unit to describe the risk attitude.
- A new prospect cross-efficiency model is developed based on the prospect value.
- The case study shows that the risk preference in?uences the evaluation result.

Cross-efficiency evaluation in data envelopment analysis based on prospect theory

Hui-hui Liu[a,b], Yao-yao Song[a,b], and Guo-liang Yang[a,b,*]

[a]Institutes of Science and Development, Chinese Academy of Sciences, Beijing 100190, China

[b]University of Chinese Academy of Sciences, Beijing 100049, China

June 11, 2018

Emails: Hui-hui Liu (liuhuihui@casipm.ac.cn)
Yao-yao Song (songyaoyao15@mails.ucas.ac.cn)
Guo-liang Yang (glyang@casipm.ac.cn)

Abstract: Cross-efficiency evaluation in data envelopment analysis (DEA) is a useful tool in evaluating the performance of decision-making units (DMUs). It is generally assumed that decision makers (DMs) are completely rational in common cross-efficiency evaluation models, which fail to consider the DM's risk attitude that plays an important role in the evaluation process. To fill this gap, we investigate the cross-efficiency evaluation in DEA based on prospect theory. First, we introduce a prospect value of the DMU to capture the non-rational psychological aspects of a DM under risk. Second, based on the prospect value, we propose a new cross-efficiency model termed the prospect cross-efficiency (PCE) model. Particularly, some existing cross-efficiency evaluation models can be deemed as the special cases of the PCE model with suitable adjustments of the parameters. Furthermore, this paper provides an empirical example to evaluate cross-efficiency with several selected universities directly managed by the Ministry of Education of China to illustrate the effectiveness of the PCE model in ranking DMUs.

Keywords: Data envelopment analysis, cross-efficiency, prospect theory, risk attitude

[*] Corresponding author. Address: Institutes of Science and Development, Chinese Academy of Sciences, P.O. Box 8712, Beijing 100190, PR China. Tel.: +86 10 59358816; fax: +86 10 59358608. E-mail address: glyang@casipm.ac.cn (G.-l. Yang).

1. Introduction

Cross-efficiency evaluation, developed by Sexton, Silkman, and Hogan (1986), has been widely accepted as a discriminative assessment tool for data envelopment analysis (DEA). It is generally used for distinguishing efficient decision-making units (DMUs) from one another (Despotis, 2002). Each DMU in cross-efficiency evaluation has a self-evaluated efficiency derived by its own set of optimal weights and $n-1$ peer-evaluated efficiencies obtained by the optimal weights of other DMUs. Consequently, a final efficiency for ranking DMUs is aggregated based on n efficiencies. The major characteristics of cross-efficiency evaluation are the following: (1) ranking the DMUs in a unique order (Doyle & Green, 1995), (2) eliminating unrealistic weight schemes without predetermining any weight restrictions (Anderson, Hollingsworth, & Inman, 2002), and (3) effectively differentiating between good and poor performers among the DMUs (Boussofiane, Dyson, & Thanassoulis, 1991). Due to these advantages, cross-efficiency evaluation has been used in a variety of applications, including project ranking (Green, Doyle, & Cook, 1996), the measurement of the labour assignment in a cellular manufacturing system (Ertay & Ruan, 2005), sports rankings (Wu, Liang, & Chen, 2009), corporate philanthropic selection (Partovi, 2011), the supplier selection problem in public procurement (Falagario, Sciancalepore, Costantino, & Pietroforte, 2012), and portfolio selection (Lim, Oh, & Zhu, 2014; Mashayekhi & Omrani, 2016).

Despite the many advantages and wide applications of cross-efficiency, its usefulness is possibly reduced by the non-uniqueness of the optimal weights (Doyle & Green, 1994). Specifically, the possible existence of multiple optimal weights in the evaluation leads to different sets of cross-efficiency scores for each DMU. This situation may reduce the discriminative capability of the cross-efficiency evaluation. To alleviate this problem, Sexton et al. (1986) and Doyle and Green (1994) presented well-known aggressive and benevolent formulations as secondary goals to select a unique solution from multiple optimal weights. The core idea of the aggressive formulation is to obtain a solution by minimizing the cross-efficiencies of the other DMUs while retaining the self-efficiency of the evaluated DMU at a predetermined optimal level. In contrast, the benevolent formulation maintains the self-efficiency while maximizing the cross-efficiencies of the other DMUs. Subsequently, numerous secondary goal models for cross-efficiency evaluations have been proposed on the basis of this idea. For example, Liang, Wu, Cook, and Zhu (2008) extended the aggressive and benevolent models of Doyle and Green (1994) by introducing various secondary objective models based on deviation from the target efficiency of each DMU, and each of these models could be applied in different practical circumstances. Wang and Chin (2010b) further proposed alternative secondary goal models by replacing the target efficiency from ideal point 1, which was used by Liang et al. (2008), to CCR efficiency.

Similar ideas also appeared in Lim (2012), in which a minimax or a maximin type secondary objective is incorporated into the aggressive and benevolent formulations of cross-efficiency.

However, as noted in Wang and Chin (2010a), the aggressive and benevolent formulations do not guarantee that a consistent conclusion would be derived. To avoid the dilemma of choosing between two different formulations, Wang and Chin (2010a) and Ramón, Ruiz, and Sirvent (2010) proposed the secondary goal models that primarily focus on the individual viewpoint of each DMU without considering their effect on other DMUs. The cross-efficiencies become neutral and logical this way. In addition, Wang, Chin, and Luo (2011) provided four neutral models for cross-efficiency evaluation from the perspective of multiple criteria decision analysis. Refer to Oukil and Amin (2015), Ruiz and Sirvent (2012), Wu, Chu, Sun, Zhu, and Liang (2016), Wu, Sun, and Liang (2012) regarding computing the cross-efficiencies using other secondary goal models without being neither aggressive nor benevolent.

As a matter of fact, cross-efficiency evaluation is used as a decision-making technique to rank the DMUs. A consensus exists that the psychology of a decision maker (DM) plays an important role in the decision-making process (Berman, Sanajian, & Wang, 2017; Borgonovo, Cappelli, Maccheroni, & Marinacci, 2018; Lejarraga & Müller-Trede, 2017; Smith & Ulu, 2017). Nevertheless, the aforementioned cross-efficiency evaluation models assume that DMs are completely rational and generally fall into the expected utility theory framework. The expected utility theory, however, has several unexplained phenomena, such as the Allais paradox (Allais, 1953) and the Ellsberg paradox (Ellsberg, 1961). Noting the limitations of the expected utility theory, Kahneman and Tversky (1979) proposed prospect theory, which can capture the non-rational psychological aspects of DMs under risk. Prospect theory has three principal conclusions: (1) the DMs exhibit the risk-avoiding tendency for gains and the risk-seeking tendency for losses, (2) the DMs usually perceive gains or losses according to a reference point, and (3) the DMs are more sensitive to loss than gain. Since prospect theory is considerably consistent with the actual behaviours of humans, the decision-making method based on prospect theory has recently become a research hotspot (Bleichrodt, Schmidt, & Zank, 2009; Borgonovo et al., 2018; Krohling & de Souza, 2012; Lahdelma & Salminen, 2009; Wang, Wang, & Martínez, 2017; Qin, Liu, & Pedrycz, 2017; Scholz, Dorner, Schryen, & Benlian, 2017; Vipin & Amit, 2017). For example, Wang et al. (2017) considered the psychological factors of experts and developed an emergency group decision-making method based on prospect theory. Borgonovo et al. (2018) provided a translation method that allowed the use of prospect-theory-like models into a decision-making analysis. Qin et al. (2017) proposed an extended TODIM (an acronym in Portuguese for Interactive and Multicriteria Decision Making) method based on prospect theory to solve multiple criteria group decision-making problems. Scholz et al. (2017) applied prospect theory to develop an attribute weight elicitation method, which incorporated the behaviour of DMs.

Vipin and Amit (2017) studied decision bias in a newsvendor problem based on prospect theory by considering loss aversion and stochastic reference point.

To the best of our knowledge, although prospect theory has been extensively applied in the decision-making method, little research has been done on the cross-efficiency evaluation method based on prospect theory. Therefore, we attempt to partially fill this gap by modelling the psychological factors of DMs in the cross-efficiency evaluation process. We introduce the prospect values of each DMU and develop a prospect cross-efficiency (PCE) model that characterizes the psychological factors in the cross-efficiency evaluation process. We analyse the performance of the Science and Technology (S&T) activities of universities supervised by the Ministry of Education (MoE) in China and further compare the PCE model with classical cross-efficiency evaluation models, such as the models of Doyle and Green (1994) and Wang et al. (2011).

The rest of this paper is organized as follows. Section 2 reviews the cross-efficiency evaluation process and its main formulations. Section 3 provides a brief introduction to prospect theory. Section 4 proposes a new cross-efficiency evaluation model based on prospect theory. Section 5 presents an illustrative example to demonstrate the potential applications of the new model. Conclusions are provided in Section 6.

2. Cross-efficiency evaluation

As an extension of the DEA model, the cross-efficiency evaluation method is implemented via two stages, including self-evaluation and peer-evaluation. It assesses the overall performance of each DMU by considering not only its own weights but also the weights of all DMUs. In this section, we briefly review cross-efficiency evaluation and its main formulations.

2.1. Self-evaluation

Assume $D=\{DMU_1, DMU_2, \cdots, DMU_n\}$ is the set of n DMUs to be evaluated, and each DMU produces s outputs by consuming m inputs. For convenience, let $N=\{1,2,\cdots,n\}$ for $k \in N$, $M=\{1,2,\cdots,m\}$ for $i \in M$, and $S=\{1,2,\cdots,s\}$ for $r \in S$. Variables y_{rk} and x_{ik} are the output and input values, respectively, of DMU_k (see Table 1), whose relative efficiency E_{kk} is denoted by the ratio of outputs to inputs:

$$E_{kk}=\sum_{r=1}^{s} u_{rk} y_{rk} \,/\, \sum_{i=1}^{m} v_{ik} x_{ik}, \tag{1}$$

where u_{rk} and v_{ik} are the non-negative weights assigned to s outputs and m inputs, respectively. Especially under self-evaluation, the efficiency of DMU_k relative to the other

DMUs is measured by the Charnes, Cooper and Rhodes (CCR) model (Charnes, Cooper, & Rhodes, 1978):

$$
\begin{aligned}
\max E_{kk} = & \sum_{r=1}^{s} u_{rk} y_{rk} / \sum_{i=1}^{m} v_{ik} x_{ik} \\
s.t. \quad & \sum_{r=1}^{s} u_{rk} y_{rj} / \sum_{i=1}^{m} v_{ik} x_{ij} \le 1, \ \ j \in N, \\
& u_{rk}, v_{ik} \ge 0, r \in S, \ i \in M.
\end{aligned} \tag{2}
$$

Note that Model (2) is a fractional linear programming model. It can be equivalently transformed into the following linear programming model by the Charnes-Cooper transformation (Charnes & Cooper, 1962):

$$
\begin{aligned}
\max E_{kk} = & \sum_{r=1}^{s} u_{rk} y_{rk} \\
s.t. \quad & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \ \ j \in N, \\
& \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
& u_{rk}, v_{ik} \ge 0, \ \ r \in S, \ i \in M.
\end{aligned} \tag{3}
$$

Let u_{rk}^* and v_{ik}^* be the output and input optimal weights, respectively, of the above model. Then, $E_{kk}^* = \sum_{r=1}^{s} u_{rk}^* y_{rk}$ is referred to as the CCR-efficiency of DMU_k, which represents the optimal relative efficiency of DMU_k by self-evaluation. If $E_{kk}^* = 1$ and all the optimal weights u_{rk}^* and v_{ik}^* are positive, then DMU_k is CCR-efficient. Otherwise, it is CCR-inefficient.

Table 1. The output and input values of the DMUs.

DMUs	DMU_1	DMU_2	$\cdots$	DMU_n
Output values	y_{11}	y_{12}	$\cdots$	y_{1n}
	y_{21}	y_{22}	$\cdots$	y_{2n}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	y_{s1}	y_{s2}	$\cdots$	y_{sn}
Input values	x_{11}	x_{12}	$\cdots$	x_{1n}
	x_{21}	x_{22}	$\cdots$	x_{2n}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	x_{m1}	x_{m2}	$\cdots$	x_{mn}

2.2. Peer-evaluation

Under the self-evaluation Model (3), each DMU is evaluated with its most favourable weights; this manner may lead to the status in which many DMUs are evaluated as CCR-efficient, and the CCR-efficient DMUs fail to be further distinguished. To tackle this issue, Sexton et al. (1986) suggested the cross-efficiency evaluation, which can appraise the overall performance of each

DMU by using the weights of all DMUs. More specifically, if u_{rk}^* and v_{ik}^* are the respective optimal weights of outputs and inputs of Model (3) for a given $DMU_k (k \in N)$, then the cross-efficiency score of DMU_d is defined as follows:

$$E_{dk} = \sum_{r=1}^{s} u_{rk}^* y_{rd} / \sum_{i=1}^{m} v_{ik}^* x_{id}, \ d \in N, \ d \neq k, \tag{4}$$

which reflects the peer-evaluation of DMU_k to DMU_d.

Model (3) should be solved n times each time for a target DMU_k for the acquisition of the cross-efficiency scores of all DMUs. Consequently, the n sets of input and output weights become available for the n DMUs, and each DMU obtains $n-1$ cross-efficiency scores and the optimal CCR-efficiency score. All of these scores can be shown as a $n \times n$ cross-efficiency matrix where the diagonal elements present the CCR-efficiency scores E_{kk}^* (see Table 2).

Table 2. Cross-efficiency matrix of the DMUs.

DMU	Target DMU				Average cross-efficiency
	DMU_1	DMU_2	$\cdots$	DMU_n	
DMU_1	E_{11}	E_{12}	$\cdots$	E_{1n}	$\sum_{k=1}^{n} E_{1k}/n$
DMU_2	E_{21}	E_{22}	$\cdots$	E_{2n}	$\sum_{k=1}^{n} E_{2k}/n$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
DMU_n	E_{n1}	E_{n2}	$\cdots$	E_{nn}	$\sum_{k=1}^{n} E_{nk}/n$

To obtain the overall performance of each DMU, the most extensively used approach is to average the cross-efficiency scores in each row of cross-efficiency matrix (see the last column in Table 2). In this manner, the cross-efficiency score of DMU_d is defined as follows:

$$\bar{E}_d = \sum_{k=1}^{n} E_{dk}/n, \ d \in N. \tag{5}$$

The cross-efficiency score $\bar{E}_d$ provides a peer-evaluation of DMU_d, and correspondingly these n DMUs can be fully compared or ranked.

Note that multiple optimal solutions (input and output weights) may exist in Model (3), which may lead to the non-unique cross-efficiency scores for DMUs. Sexton et al. (1986) suggested introducing a secondary goal to derive unique optimal input and output weights. The most commonly used secondary goals developed by Doyle and Green (1994) are presented below:

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_{rk} \Big(\sum_{j=1,\, j \neq k}^{n} y_{rj} \Big) \\
s.t. \quad & \sum_{r=1}^{s} u_{rk} y_{rk} - E_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \\
& \sum_{i=1}^{m} v_{ik} \Big(\sum_{j=1,\, j \neq k}^{n} x_{ij} \Big) = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \;\; j \in N,\; j \neq k, \\
& u_{rk}, v_{ik} \geq 0, \;\; r \in S, \;\; i \in M,
\end{aligned} \tag{6}
$$

and

$$
\begin{aligned}
\min \quad & \sum_{r=1}^{s} u_{rk} \Big(\sum_{j=1,\, j \neq k}^{n} y_{rj} \Big) \\
s.t. \quad & \sum_{r=1}^{s} u_{rk} y_{rk} - E_{kk}^{*} \sum_{i=1}^{m} v_{ik} x_{ik} = 0, \\
& \sum_{i=1}^{m} v_{ik} \Big(\sum_{j=1,\, j \neq k}^{n} x_{ij} \Big) = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \leq 0, \;\; j \in N,\; j \neq k, \\
& u_{rk}, v_{ik} \geq 0, \;\; r \in S, \;\; i \in M.
\end{aligned} \tag{7}
$$

Model (6) is known as the benevolent formulation of the cross-efficiency evaluation that maximizes the cross efficiencies of other DMUs to some extent, whereas Model (7) is known as the aggressive formulation of the cross-efficiency evaluation that minimizes the cross efficiencies of other DMUs.

Models (6) and (7) optimize the input and output weights in two different ways. As a result, they are not guaranteed to lead to the same efficiency ranking or conclusion for DMUs (Wang & Chin, 2011; Yang, Ang, Xia, & Yang, 2012). To solve this problem, Wang and Chin (2010a) proposed a neutral DEA model for the cross-efficiency evaluation that does not require the DM to select between the aggressive and benevolent formulations. The cross-efficiencies computed in this manner become neutral and logical. Subsequently, Wang et al. (2011) proposed several improved neutral DEA models for cross-efficiency evaluation from the perspective of multiple criteria decision analysis.

Nevertheless, the aforementioned models for cross-efficiency evaluation generally assume that DMs are completely rational and fail to consider the risk preference of DMs. Noting that prospect theory captures the risk attitude of DMs (which is considered consistent with the actual decision-making behaviours of humans) (Abdellaoui & Kemel, 2014; Borgonovo et al., 2018; Long & Nasiry, 2015), we attempt to solve this problem by developing a new cross-efficiency evaluation model based on prospect theory. For discussion purposes, we briefly introduce prospect theory in the next section.

3. Prospect theory

Prospect theory was initially proposed by Kahneman and Tversky (1979) as a descriptive theory for the decision behaviour of an individual under risk. Since its inception, prospect theory has been regarded as one of the most influential behavioural decision theories (Wang et al., 2017). Prospect theory involves the following important principles (Kahneman & Tversky, 1979).

(i) Reference dependence. The DM generally perceives outcomes as gains or losses relative to a reference point. Thus, the prospect value curve of a DM is divided into two parts by the reference point, the gain and the loss domains.

(ii) Loss aversion. The DM is more sensitive to losses than to absolute commensurate gains (Abdellaoui, Bleichrodt, & Paraschiv, 2007). In this way, the prospect value curve is steeper in the loss than in the gain domain.

(iii) Diminishing sensitivity. The DM shows a risk-averse tendency for gains and a risk-seeking tendency for losses. Correspondingly, the prospect value curve is concave in the gain domain and convex in the loss domain. That is, the marginal value of both gains and losses decreases with their size.

The meaning of above three principles can be described by an asymmetric S-shaped value curve, as illustrated in Figure 1. The function of this value curve (the prospect value function) is described as follows:

$$v(\Delta z)=\begin{cases}(\Delta z)^{\alpha}, & (\Delta z\geq 0),\\ -\theta(-\Delta z)^{\beta}, & (\Delta z<0),\end{cases} \tag{8}$$

where $\Delta z = z - z_0$ is used to measure the value z deviation from the reference point z_0. If the outcome is larger than the reference point ($\Delta z \geq 0$), then the outcome is viewed as a gain. Otherwise, the outcome is deemed as a loss ($\Delta z < 0$). The parameters α and β represent the bump degree of the value function in the gain and loss regions, respectively, where $0<\alpha<1$ and $0<\beta<1$. Variable θ is the loss-aversion coefficient, and $\theta>1$ which shows that the region value function is considerably steeper for the losses than for the gains.

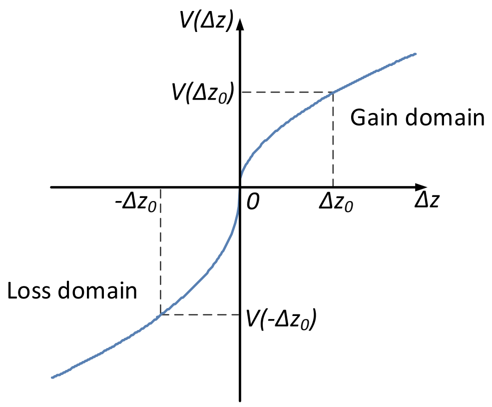

Figure 1. Prospect value curve (here $\alpha=0.55$, $\beta=0.45$, and $\theta=2.25$).

As mentioned in Section 2, the existing cross-efficiency evaluation methods assume that DMs are completely rational and generally fall into the expected utility theory framework. Noting that prospect theory is considerably consistent with the actual decision-making behaviours of humans, the following section proposes a new cross-efficiency evaluation model based on prospect theory.

4. Prospect cross-efficiency model for cross-efficiency evaluation

Prospect theory reveals that DMs usually reflect sensitivity depending on the states of the outcomes with respect to the status quo (i.e., reference point). That is, they reflect whether the outcomes are better or worse than the status quo. The reference point can be specified by the following methods: (1) the zero point, (2) the mean value, (3) the medium value, (4) the worst value, and (5) the best value. In this paper, we use the worst and best values together to derive the cross-efficiency evaluation matrix based on prospect theory, which is further explained in the next section.

For the DMs, the worst DMU generally expends the most inputs to produce the least outputs, and the best DMU consumes the least inputs to yield the most outputs. According to prospect theory, relative gains can be regarded as the value of a DMU above the worst DMU, and the DMU in such case is deemed as a gain. A relative loss can be deemed as the value of a DMU below the best DMU, and the DMU in such case is viewed as a loss.

For the sake of discussion, let $N=\{1,2,\cdots,n\}$ for $k\in N$, $M=\{1,2,\cdots,m\}$ for $i\in M$, and $S=\{1,2,\cdots,s\}$ for $r\in S$. Assume that there are n DMUs to be evaluated, and the output and input values of DMU_k ($k\in N$) are y_{rk} ($r\in S$) and x_{ik} ($i\in M$), respectively. Based on the above analysis, the prospect values of DMU_k are defined as follows.

Definition 1. If the reference point of the DM is the worst DMU, then the prospect gain values with respect to *i*th input and *r*th output of DMU_k are defined as:

$$V_{I_{ik}}^{+}=(x_i^{-}-x_{ik})^{\alpha},\ V_{O_{rk}}^{+}=(y_{rk}-y_r^{-})^{\alpha}, \tag{9}$$

where $x_i^{-}=\max_k\{x_{ik}\}$ and $y_r^{-}=\min_k\{y_{rk}\}$ are *i*th input and *r*th output of the worst DMU, respectively.

Definition 2. If the reference point of the DM is the best DMU, then the prospect loss values with respect to *i*th input and *r*th output of DMU_k are defined as

$$V_{I_{ik}}^{-}=-\theta(x_{ik}-x_i^{+})^{\beta},\ V_{O_{rk}}^{-}=-\theta(y_r^{+}-y_{rk})^{\beta}, \tag{10}$$

where $x_i^{+}=\min_k\{x_{ik}\}$ and $y_r^{+}=\max_k\{y_{rk}\}$ are *i*th input and *r*th output of the best DMU, respectively.

From the perspective of the DM, he/she always selects a unique set of input and output weights to make the gains of DMU_k as much as possible, as follows:

$$\max \sum_{r=1}^{s} u_{rk} V_{O_{rk}}^{+} + \sum_{i=1}^{m} v_{ik} V_{I_{ik}}^{+}. \tag{11}$$

Therefore, the gain model for cross-efficiency evaluation can be constructed as follows:

$$\begin{aligned} \max\ & \sum_{r=1}^{s} u_{rk} (y_{rk} - y_r^{-})^{\alpha} + \sum_{i=1}^{m} v_{ik} (x_i^{-} - x_{ik})^{\alpha} \\ s.t.\ & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ \ j \in N, \\ & u_{rk}, v_{ik} \ge 0,\ \ r \in S,\ \ i \in M. \end{aligned} \tag{12}$$

In addition, the DM always chooses a unique set of input and output weights to make the losses of DMU_k as small as possible, as follows:

$$\min \sum_{r=1}^{s} u_{rk} V_{O_{rk}}^{-} + \sum_{i=1}^{m} v_{ik} V_{I_{ik}}^{-}. \tag{13}$$

Correspondingly, the loss model for the cross-efficiency evaluation can be established as follows:

$$\begin{aligned} \min\ & \sum_{r=1}^{s} u_{rk} \theta (y_r^{+} - y_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik} \theta (x_{ik} - x_i^{+})^{\beta} \\ s.t.\ & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ \ j \in N, \\ & u_{rk}, v_{ik} \ge 0,\ \ r \in S,\ \ i \in M. \end{aligned} \tag{14}$$

Combining the gain model (12) and the loss model (14), a new model for cross-efficiency evaluation can be constructed as follows:

$$\begin{aligned} \max\ & \lambda \Big(\sum_{r=1}^{s} u_{rk} (y_{rk} - y_r^{-})^{\alpha} + \sum_{i=1}^{m} v_{ik} (x_i^{-} - x_{ik})^{\alpha}\Big) - (1-\lambda)\Big(\sum_{r=1}^{s} u_{rk} \theta (y_r^{+} - y_{rk})^{\beta} + \sum_{i=1}^{m} v_{ik} \theta (x_{ik} - x_i^{+})^{\beta}\Big) \\ s.t.\ & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^{*}, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ \ j \in N, \\ & u_{rk}, v_{ik} \ge 0,\ \ r \in S,\ \ i \in M, \end{aligned} \tag{15}$$

which is termed the prospect cross-efficiency (PCE) model, and λ stands for the relative importance degree towards gains satisfying $0 \le \lambda \le 1$.

In this optimization model, different λ values can be used as indicators of the varied attitudes of the DM. For example, if $0 \le \lambda < 0.5$, then the DM will focus more on the losses than the gains. If $\lambda = 0.5$, then the DM will argue that the factors of gains and losses are equally important. If $0.5 < \lambda \le 1$, then the DM will focus considerable attention to the gain preference. During the decision process, the DM can opt for a suitable λ according to his/her preference.

Particularly, if we consider the possible values of parameters λ, α, β, and θ in the PCE model, we can obtain two special cases shown as follows.

Case 1. If λ=0, $\beta \to 1$, and $\theta \to 1$ in Model (15), then we derive:

$$
\begin{aligned}
\min\ E_k^- &= \sum_{r=1}^{s} u_{rk}(x_{ij} - x_j^+) + \sum_{i=1}^{m} v_{ik}(y_j^+ - y_{ij}) \\
s.t.\quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^*, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ \ j \in N, \\
& u_{rk}, v_{ik} \ge 0,\ \ r \in S,\ \ i \in M,
\end{aligned}
\tag{16}
$$

which is simply Model I of Wang et al. (2011).

Case 2. If λ=1 and $\alpha \to 1$ in Model (15), then we derive

$$
\begin{aligned}
\max\ E_k^+ &= \sum_{r=1}^{s} u_{rk}(y_{ij} - y_j^-) + \sum_{i=1}^{m} v_{ik}(x_j^- - x_{ij}) \\
s.t.\quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\
& \sum_{r=1}^{s} u_{rk} y_{rk} = E_{kk}^*, \\
& \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ \ j \in N, \\
& u_{rk}, v_{ik} \ge 0,\ \ r \in S,\ \ i \in M,
\end{aligned}
\tag{17}
$$

which is exactly Model II of Wang et al. (2011).

In addition, the parameter α presents the concave degree of the value function within the gain region. A large value of α implies a large steepness of the utility curve; the DM in such a case tends to be risk-seeking. Therefore, with the value of α tending to 0, the risk attitude of the DM becomes considerably risk-averse in the evaluation process. Correspondingly, the evaluation result of the PCE model is inclined to a relatively conservative solution. Conversely, the parameter β shows the convex degree of the value function within the loss region. A large value of β implies the large steepness of the utility curve for the relative losses. In such a case, the DM becomes sensitive towards losses and considerably conservative. Therefore, with the value of β tending to 0, the DM is prone to risk-seeking in the evaluation process. Correspondingly, the evaluation result of the PCE model is inclined to a relatively adventurous solution.

Therefore, the PCE model is the extension of the models proposed by Wang et al. (2011). In addition, the PCE model can capture the risk attitudes of the DMs and maintain certain fairness by setting the secondary goal for cross-efficiency evaluation without using the information of the other DMUs. In this manner, the optimal input and output weights for calculating the cross-efficiency score comply with the facts, and the DMs do not have to face a difficult choice between the aggressive and benevolent models.

5. An illustrative example

This section presents an illustrative example to analyse the validity of the PCE model developed in Section 4. First, we utilize the PCE model to evaluate cross-efficiency within several selected universities directly managed by the Ministry of Education of China (MoE) of China. Second, we perform a sensitivity analysis to illustrate the influence of the psychological factors of DMs on the evaluation results. Third, we further compare the proposed model with several classic models, including the aggressive and the benevolent models in Doyle and Green (1994), and Models I and II in Wang et al. (2011).

5.1. A case of Chinese universities

The MoE of China has operated the Educational Revitalization Action Plan for the 21st Century since 1998. It is committed to constructing world-class and high-level universities in China, which is termed as the 985 Project. The universities related to the 985 Project, which are the main institutions of scientific research activities, represent the highest level of university research in China. It is essential to evaluate these universities with respect to the efficiency of scientific research activities. We select 10 universities from the 985 Project as the DMUs to be evaluated, including Beijing University, Tsinghua University, China Agricultural University, Tianjin University, Fudan University, Tongji University, Shanghai Jiaotong University, East China Normal University, Sun Yat-sen University, and South China University of Technology. All 10 are top-level universities in first-tier cities of China (i.e., Beijing, Shanghai, Guangdong, and Tianjin), where the level of GDP development is among the highest.

To analyse the performance of the Science and Technology (S&T) activities of the 10 universities selected above, we use the indicators and corresponding data of these universities from the S&T statistics compilation in 2013, which was published by the MoE in China (see Table 3 and Table 4 for details).

After the data gathering, we use the proposed cross-efficiency evaluation method to appraise the efficiency of scientific research activities among the 10 universities, which is shown as follows.

Step I. Self-efficiency evaluation

According to the data in Table 4, we calculate the self-efficiency of the 10 DMUs using the CCR model (see Model 3), and the result is shown in the last column of Table 4. From Table 4, we can find that the self-efficiency scores of many universities are equal to 1 (e.g., DMU_2, DMU_5, $DMU_{7,}$ and DMU_9), resulting in the need for further discrimination. Therefore, as the next step, we compute the cross-efficiency score of each DMU using the PCE model to obtain the ordering of the 10 DMUs.

Step II. Cross-efficiency evaluation

Without loss of generality, we assume that the DM will argue that the gain and loss factors are equally important (i.e., $\lambda = 0.5$). The other parameters α, β, and θ in the PCE model are 0.89, 0.92, and 2.25, respectively (c.f., Kahneman and Tversky, 1979). Based on the self-efficiency scores derived from Step I and the PCE model (see Model 15), the weights for the inputs and outputs are obtained in Table 5. Using the weights in Table 5, we can then generate the prospect cross-efficiency matrix in Table 6. Consequently, we average the cross-efficiency scores in each row of the matrix and obtain the overall performance of the 10 universities (see the penultimate column of Table 6).

Table 3. The evaluation indicators of 10 universities.

Indicators	Type	Units	Notations	Explanations
Technology transfer revenue	Output	RMB in thousand	*Tec.*	The income from the process of technology transfer in a university within the statistical year.
Publication papers	Output	Number	*Pub.*	The number of international papers indexed by the Web of Science published by Thompson Reuters (that is, SCI/SSCI) within the statistical year.
Research and development fund	Input	RMB in thousand	*R&D Fun.*	The fund for the activities of fundamental research, experimental development research and application research within the statistical year.
Research and development staff	Input	Full-time equivalent	*R&D Sta.*	The person engaged in research, management and supporting activities of research and development, including persons in the project teams, persons engaged in the management of S&T activities of enterprises and supporting staff providing direct service to the research projects within the statistical year. This indicator reflects the size of personnel engaged in research and development activities with the independent intellectual property.

Table 4. Inputs and outputs of 10 universities in 2013.

Universities	Outputs		Inputs		Self-efficiency by CCR model
	Tec.	*Pub.*	*R&D Fun.*	*R&D Sta.*	
DMU_1	32441	2174	2159718	1868	0.350
DMU_2	741905	6570	4351036	2762	1.000
DMU_3	4316	1782	983639	642	0.635
DMU_4	35819	2540	2053945	1234	0.510
DMU_5	1960	5078	1754690	1141	1.000
DMU_6	2460	2992	2342515	836	0.807
DMU_7	159634	7189	2876203	1882	1.000
DMU_8	1855	1465	429710	594	0.885
DMU_9	790	4515	1016343	1829	1.000
DMU_{10}	75736	2267	1388514	975	0.690

Table 5. The weights obtained by the PCE model.

Universities	The weights for the outputs		The weights for the inputs	
	Tec.	*Pub.*	*R&D Fun.*	*R&D Sta.*
DMU_1	1.065E-06	1.451E-04	2.942E-07	1.952E-04
DMU_2	1.348E-06	4.239E-18	2.298E-07	9.710E-20
DMU_3	3.786E-06	3.625E-04	1.211E-06	4.905E-04
DMU_4	1.363E-06	1.816E-04	5.952E-22	8.104E-04
DMU_5	7.390E-20	1.969E-04	2.518E-19	8.764E-04
DMU_6	2.035E-06	2.681E-04	2.019E-20	1.197E-03
DMU_7	8.794E-07	1.196E-04	2.424E-07	1.608E-04
DMU_8	4.208E-06	5.988E-04	1.214E-06	8.054E-04
DMU_9	1.854E-15	2.215E-04	9.839E-07	3.255E-12
DMU_{10}	1.978E-06	2.383E-04	5.027E-07	3.097E-04

Table 6. The prospect cross-efficiency of 10 universities in 2013.

DMU	Target DMU										Average cross-efficiency	Ranking
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}		
DMU_1	0.350	0.088	0.258	0.290	0.262	0.290	0.350	0.349	0.227	0.350	0.281	10
DMU_2	0.958	1.000	0.783	0.985	0.534	0.989	0.959	0.940	0.340	0.997	0.849	1
DMU_3	0.635	0.026	0.635	0.633	0.624	0.633	0.635	0.634	0.408	0.625	0.549	7
DMU_4	0.481	0.102	0.341	0.510	0.462	0.510	0.481	0.479	0.278	0.478	0.412	9
DMU_5	1.000	0.007	0.688	1.000	1.000	1.000	1.000	1.000	0.651	0.983	0.833	3
DMU_6	0.512	0.006	0.337	0.807	0.804	0.807	0.512	0.512	0.288	0.500	0.509	8
DMU_7	1.000	0.325	0.728	0.999	0.858	1.000	1.000	0.994	0.563	1.000	0.847	2
DMU_8	0.885	0.025	0.663	0.558	0.554	0.558	0.885	0.885	0.767	0.882	0.666	5
DMU_9	1.000	0.005	0.770	0.554	0.555	0.554	1.000	1.000	1.000	1.000	0.744	4
DMU_{10}	0.684	0.320	0.513	0.652	0.522	0.653	0.684	0.678	0.368	0.690	0.576	6

According to the average cross-efficiency in Table 6, we derive the ranking order of the 10 universities (see the last column of Table 6), and the most efficient university is the university denoted as DMU_2. Table 7 presents the efficiency scores of the CCR model and the PCE model. From Table 7, we can find that the PCE model can discriminate between the performances of

DMU_2, DMU_5, DMU_7, and DMU_9, whereas the efficiency scores of these DMUs in the CCR model are all equal to 1.

Table 7. The efficiency scores of the CCR model and the PCE model.

Models	Efficiency scores									
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
CCR model	0.350	1.000	0.635	0.510	1.000	0.807	1.000	0.885	1.000	0.690
PCE model	0.281	0.849	0.549	0.412	0.833	0.509	0.847	0.666	0.744	0.576

In addition, the cross-efficiency scores of the PCE model are less than the self-efficiency scores of the CCR model (see Figure 2). This finding can be explained through the definition of cross-efficiency evaluation, which allows the overall efficiencies of the DMU to be appraised through self-evaluation and peer-evaluation. Self-evaluation is assessed using the most favourable weights of the DMU to achieve its maximum efficiency, whereas peer-evaluation is appraised by the weights determined by other DMUs. As a result, the efficiency score of the peer-evaluation of the DMU cannot be higher than the efficiency score of its self-evaluation. Therefore, the cross-efficiency score of the DMU, which is the average of the self-efficiency score and peer-efficiency scores, cannot be higher than its self-efficiency score (see Doyle and Green, 1994).

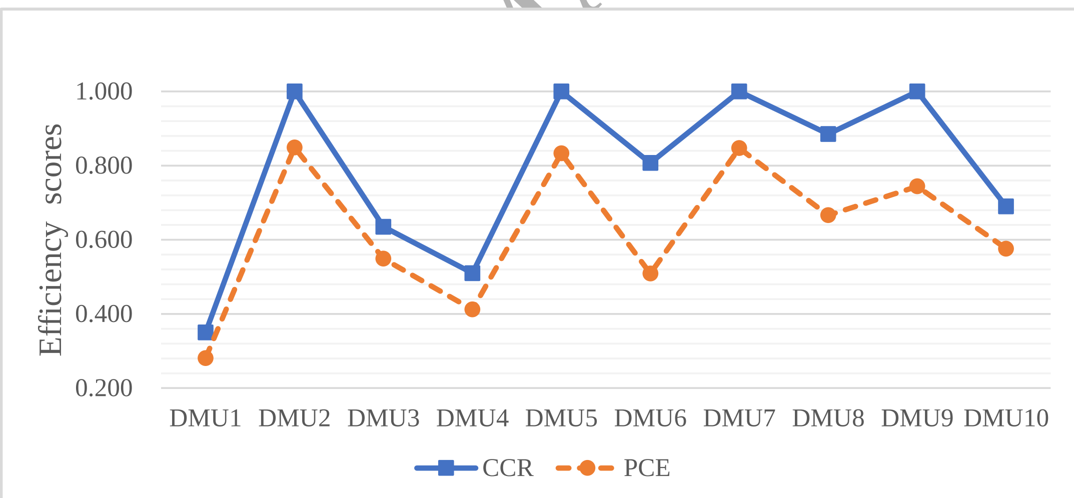

Figure 2. Comparison between the CCR model and the PCE model.

5.2. Sensitivity analysis

In this subsection, we perform a sensitivity analysis to analyse how the different risk attitudes of the DM (that is, parameters α, β, and θ) affect the evaluation results. In addition, we show the variations of evaluation results under different psychological preferences of the DM (i.e., parameter λ).

5.2.1. Different risk attitudes of the DM

Considering the efficiency evaluation case in Section 5.1, we calculate the evaluation results among different values of parameters α, β, and θ. Without loss of generality, we suppose that the original values of parameters α, β, and θ are 0.5, 0.3, and 3, respectively. In this way, Figures 3 and 4 show how different values of parameters α, β, and θ in the PCE model affect the efficiency evaluation result of the 10 universities, which involve changing the single parameter α (resp. β or θ) (see Figure 3), two parameters α and β (resp. α and θ, β and θ) (see Figure 4), respectively.

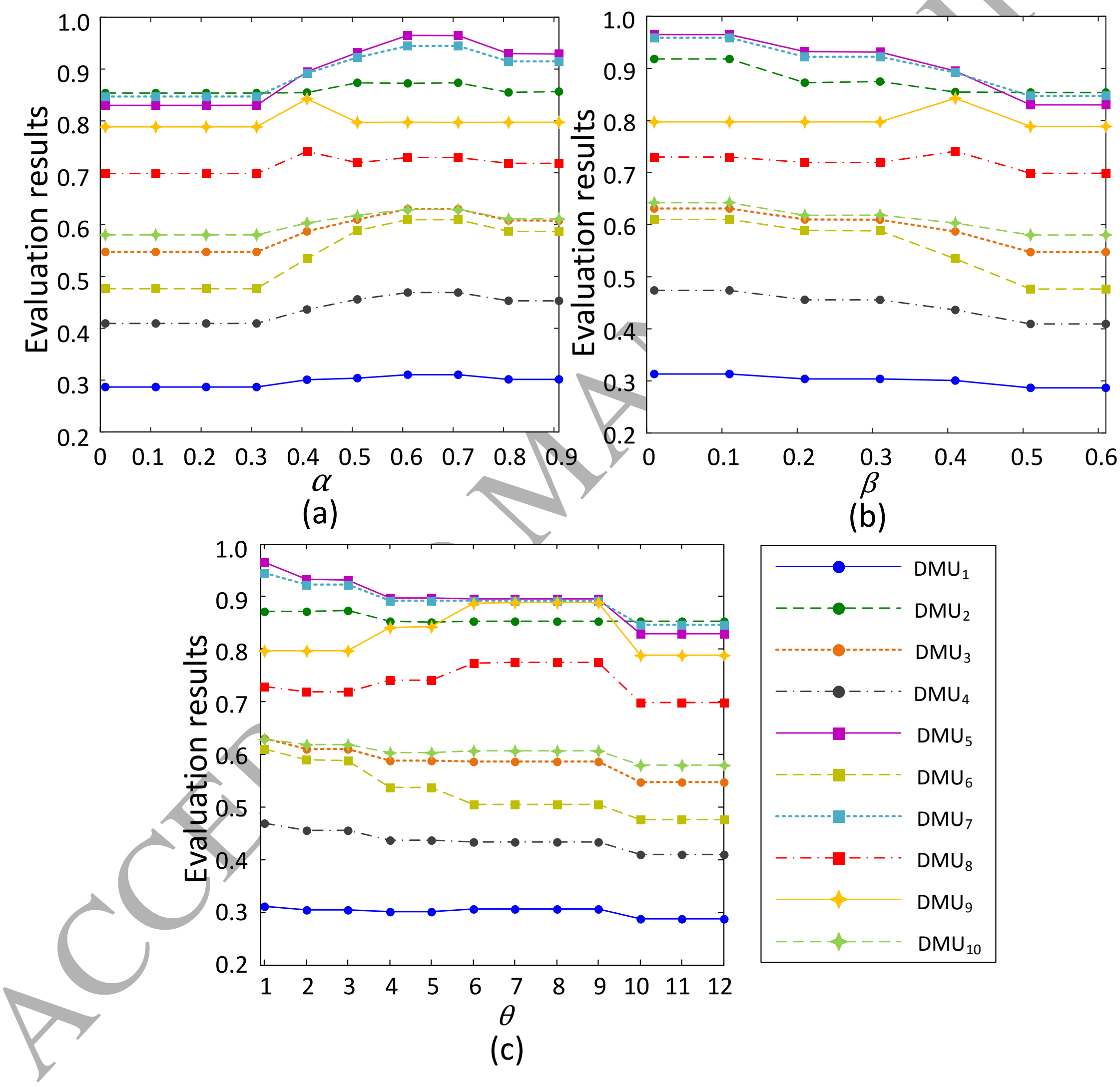

Figure 3. Sensitivity analysis of the evaluation results about single parameter: (a) α, (b) β, and (c) θ.

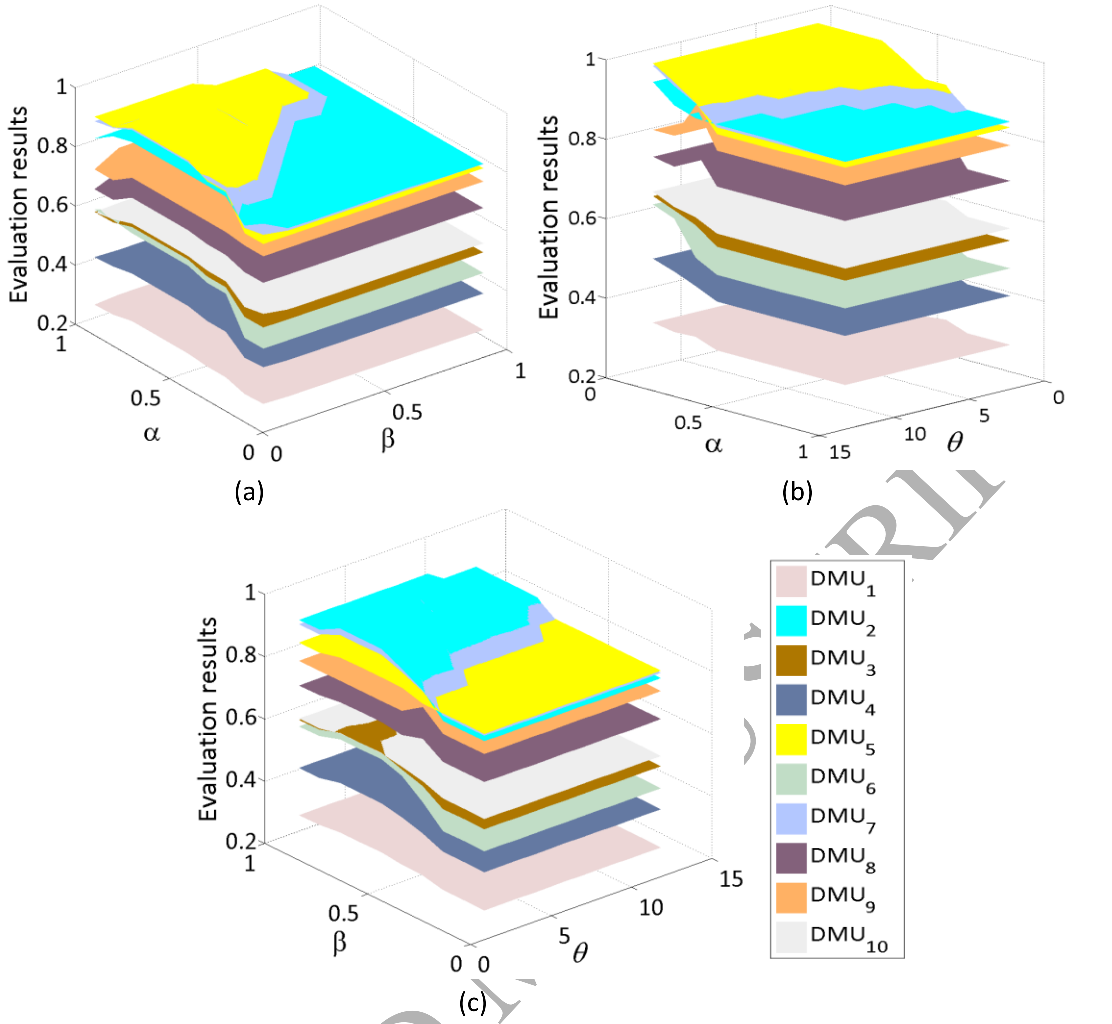

Figure 4. Sensitivity analysis of the evaluation results about two parameters: (a) α and β , (b) α and θ , and (c) β and θ .

From Figure 3, we can find that changing the single parameter affects the efficiency evaluation result. In particular, the most efficient DMU changes as α increases or as β (or θ) decreases to a certain point. For instance, the most efficient DMU switches from DMU_2 to DMU_7 as the parameter α increases from 0.1 to approximately 0.32, and the most efficient DMU changes from DMU_7 to DMU_5 with α continuously increasing to approximately 0.43. Similarly, we can find the corresponding phenomenon about the parameters β and θ . The most efficient DMU shifts from DMU_5 to DMU_7 as the parameter β (resp. θ) increases from 0 (resp. 1.1) to approximately 0.41 (resp. 9.1), and the most efficient DMU changes from DMU_7 to DMU_2 with β (resp. θ) continuously increasing to approximately 0.49 (resp. 9.8). In addition, we notice that changing the two parameters in Figure 4 also affects the evaluation results of DMUs. For example, Figure 4(a) shows that if $0<\alpha\le 0.46$ and $\beta\ge 0.27$, the most efficient DMU is DMU_5, whereas it becomes DMU_2 if $\alpha\ge 0.67$ and $0<\beta\le 0.19$.

The above changes in Figures 3 and 4 can be explained by the implication of the parameters α, β, and θ. The parameter α is the concave degree of the value function within the gain region. A large value of α implies a large steepness of the utility curve. Thus, the DM tends to be risk-seeking. In contrast, the parameter β presents the convex degree of the value function within the loss region, and the parameter θ shows that the utility curve of the value function is steeper for the loss region than for the gain region. Large values of β and θ imply the large steepness of the utility curve for the relative losses; hence, the DM becomes sensitive towards losses and is considerably inclined to be risk-averse. Indeed, the high risk-aversion of the DM means that he/she is considerably conservative (Guiso & Paiella, 2008). Therefore, with the value of α decreasing and β (or θ) increasing, the risk preference of the DM regarding the efficiency evaluation becomes considerably prudent.

In fact, from Table 4, we can find that DMU_2 is superior to DMU_5 with respect to *R&D Fun.* and *Tec.* That is, *R&D Fun.* and *Tec.* of DMU_2 are 4,351,036 and 741,905 *thousand RMB*, respectively, whereas the corresponding items of DMU_5 are 1,754,690 and 1,960 *thousand RMB*, respectively. These imply that the inputs of DMU_2 are approximately 2.5 times more than those of DMU_5, and the outputs of DMU_2 are approximately 378 times more than those of DMU_5. Therefore, DMU_2 consumes the least inputs to yield the most outputs compared with DMU_5. Note that *R&D Fun.* and *Tec.* denote the research and development fund and the technology transfer revenue, respectively, which generally draw more attention from the DM than research and development staff (*R&D Stu.*) and publication papers (*Pub.*) when the DM becomes increasingly conservative. Therefore, the DM will prefer DMU_2 over DMU_5 with α decreasing and β (or θ) increasing. Particularly, from Figures 3(a) and 3(b), the most efficient university is DMU_2 when $\alpha \to 0$ (i.e., the DM becomes prudent); whereas it becomes DMU_5 when $\beta \to 0$ (i.e., the DM tends to be risk-seeking). On the basis of the aforementioned analysis about DMU_2 and DMU_5, this conclusion is consistent with the implication of the PCE model when $\alpha \to 0$ or $\beta \to 0$ (see Section 4).

5.2.2. Different psychological preferences of the DM

We use the example in Section 5.1 to calculate the cross-efficiency scores among different values of the parameter λ. The results are listed in Table 8, which shows that the evaluation results of the 10 universities change with the variations of the parameter λ. For example, the most efficient DMU is DMU_2 when the values of λ are 0, 0.2, 0.4, and 0.6, whereas it becomes DMU_5 when the values of λ are 0.8 and 1.

Table 8. The evaluation results of the PCE model under different values of λ.

DMUs	λ=0		λ=0.2		λ=0.4		λ=0.6		λ=0.8		λ=1	
	Results	Rankings	Results	Rankings	Results	Rankings	Results	Rankings	Results	Rankings	Results	Rankings
DMU_1	0.287	10	0.287	10	0.281	10	0.281	10	0.295	10	0.296	10
DMU_2	0.848	1	0.848	1	0.849	1	0.849	1	0.852	3	0.857	3
DMU_3	0.549	7	0.549	7	0.549	7	0.549	7	0.588	7	0.608	7
DMU_4	0.410	9	0.410	9	0.412	9	0.412	9	0.440	9	0.456	9
DMU_5	0.833	3	0.833	3	0.833	3	0.833	3	0.897	1	0.931	1
DMU_6	0.479	8	0.479	8	0.509	8	0.509	8	0.567	8	0.617	6
DMU_7	0.847	2	0.847	2	0.847	2	0.847	2	0.892	2	0.915	2
DMU_8	0.699	5	0.699	5	0.666	5	0.666	5	0.708	5	0.686	5
DMU_9	0.788	4	0.788	4	0.744	4	0.743	4	0.798	4	0.753	4
DMU_{10}	0.579	6	0.579	6	0.576	6	0.576	6	0.599	6	0.608	8

The change tendency in Table 8 can be explained by the implication of the parameter λ. As presented in Model (15), the PCE model is established based on the gain model (12) and the loss model (14). In the gain model, the DM perceives all outcomes as gains because the reference point of the DM is the worst DMU, and in such a case, the DM is completely optimistic towards the outcomes. In the loss model, the DM perceives all outcomes as losses because the reference point of the DM is the best DMU, and in such a case, the DM is totally pessimistic towards the outcomes. Note that the relative importance degree of the gain model is characterised by the parameter λ in the PCE model. Therefore, the DM becomes considerably pessimistic towards the outcomes with the decrease in value of λ. In addition, Table 4 shows that DMU_2 is superior to DMU_5 with respect to *R&D Fun.* and *Tec.*, which generally draw more attention from the DM than *R&D Stu.* and *Pub.* when the DM becomes increasingly prudent. As a result, the most efficient DMU changes from DMU_5 to DMU_2 with the decrease in value of λ.

Now we explain why we use the worst and best values together as the reference points in the PCE model as follows.

(1) According to Table 8, the most efficient university is DMU_5 in the case of using only the worst value (i.e., λ=1 in the PCE model), whereas it is DMU_2 in the case of using only the best value (i.e., λ=0 in the PCE model). This finding indicates that the evaluation results of using only the worst value and using only the best value are significantly different. In such a case, the DM has to face a difficult choice between the results of using either the worst or best value.

(2) As we mentioned above, the DM can be regarded as completely optimistic when he/she perceives all outcomes as gains (i.e., choosing the worst value as the reference point), whereas the DM can be perceived as totally pessimistic when he/she perceives all outcomes as losses (i.e., choosing the best value as the reference point). However, some psychological studies indicate that optimism and pessimism belong to two opposing extremes, and the DM might have

optimistic and pessimistic tendencies simultaneously (Houlding & Coolen, 2012; Verbunt & Rogge, 2018). Therefore, in comparison with using only the worst or best value, the use of the worst and best values together is more consistent with reality.

On the basis of the above analysis, the use of the worst and best values together in the PCE model can steer clear of making a difficult choice between the results of using only the worst or best value. In addition, this manner considers the preference of the DM towards the outcomes in optimistic and pessimistic views. Particularly, the optimism degree of the DM is characterised by the parameter λ in the PCE model. This way, the DM can select different values of the parameter λ according to his/her psychological preference and obtain the result that is closest to his/her actual preference.

5.3. Comparison of different models

To analyse the validity of the PCE model, this subsection further compares the PCE model with several models, including the aggressive and benevolent models in Doyle and Green (1994), and Models I and II in Wang et al. (2011).

Considering the efficiency evaluation case in Section 5.1, we use the aggressive and the benevolent models of Doyle and Green (1994) to evaluate the performance of the 10 universities (see Tables 9-12); furthermore, we present the results of Models I and II of Wang et al. (2011) that are exactly the cases I and II of the PCE model. Moreover, we present the results of the PCE model with different values of parameter for comparison. Here, we consider the parameter α to illustrate the characteristics of the PCE model because similar conclusions can be derived when considering other parameters. The evaluation results are shown in Table 13.

Table 9. The weights obtained by the benevolent model.

Universities	Outputs		Inputs	
	Tec.	*Pub.*	*R&D Fun.*	*R&D Sta.*
DMU_1	1.496E-07	1.955E-05	4.015E-08	2.603E-05
DMU_2	1.822E-07	2.173E-05	4.600E-08	2.816E-05
DMU_3	1.335E-07	1.821E-05	3.692E-08	2.451E-05
DMU_4	1.358E-07	1.790E-05	5.846E-10	7.901E-05
DMU_5	1.396E-07	1.899E-05	3.850E-08	2.554E-05
DMU_6	1.315E-07	1.733E-05	2.233E-21	7.736E-05
DMU_7	1.667E-07	1.989E-05	4.210E-08	2.577E-05
DMU_8	1.324E-07	1.780E-05	3.622E-08	2.388E-05
DMU_9	1.544E-07	1.842E-05	3.899E-08	2.387E-05
DMU_{10}	1.522E-07	1.834E-05	3.869E-08	2.384E-05

Table 10. The benevolent cross-efficiency of 10 universities in 2013.

DMU	Target DMU										Average cross-efficiency
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}	
DMU_1	0.350	0.350	0.350	0.291	0.350	0.290	0.350	0.350	0.350	0.350	0.338
DMU_2	0.971	1.000	0.958	0.989	0.959	0.989	1.000	0.963	1.000	0.997	0.983
DMU_3	0.632	0.624	0.635	0.633	0.635	0.633	0.624	0.634	0.624	0.625	0.630
DMU_4	0.480	0.478	0.481	0.510	0.481	0.510	0.478	0.481	0.478	0.478	0.485
DMU_5	0.994	0.981	1.000	1.000	1.000	1.000	0.981	0.998	0.981	0.983	0.992
DMU_6	0.508	0.499	0.512	0.799	0.512	0.807	0.499	0.511	0.499	0.500	0.565
DMU_7	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
DMU_8	0.884	0.882	0.885	0.561	0.885	0.558	0.882	0.885	0.882	0.882	0.819
DMU_9	1.000	1.000	1.000	0.558	1.000	0.554	1.000	1.000	1.000	1.000	0.911
DMU_{10}	0.686	0.690	0.684	0.653	0.684	0.653	0.690	0.685	0.690	0.690	0.681

Table 11. The weights obtained by the aggressive model.

Universities	Outputs		Inputs	
	Tec.	*Pub.*	*R&D Fun.*	*R&D Sta.*
DMU_1	1.442E-07	1.966E-05	3.986E-08	2.644E-05
DMU_2	3.384E-07	3.991E-19	9.180E-20	9.090E-05
DMU_3	1.971E-07	1.887E-05	4.965E-08	2.497E-05
DMU_4	1.343E-07	1.788E-05	1.693E-16	7.981E-05
DMU_5	3.309E-17	1.780E-05	2.544E-16	7.923E-05
DMU_6	6.575E-11	8.672E-09	1.095E-24	3.869E-08
DMU_7	1.431E-07	1.886E-05	8.915E-23	8.417E-05
DMU_8	1.253E-07	1.783E-05	3.615E-08	2.399E-05
DMU_9	6.249E-19	1.227E-05	5.453E-08	6.864E-16
DMU_{10}	1.533E-07	1.830E-05	3.824E-08	2.447E-05

Table 12. The aggressive cross-efficiency matrix of 10 universities in 2013.

DMU	Target DMU										Average cross-efficiency
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}	
DMU_1	0.350	0.065	0.308	0.290	0.262	0.290	0.290	0.349	0.227	0.349	0.278
DMU_2	0.958	1.000	0.948	0.985	0.534	0.990	0.990	0.940	0.340	1.000	0.868
DMU_3	0.635	0.025	0.635	0.633	0.624	0.634	0.633	0.634	0.408	0.624	0.548
DMU_4	0.481	0.108	0.414	0.510	0.462	0.511	0.510	0.479	0.278	0.478	0.423
DMU_5	1.000	0.006	0.832	1.000	1.000	1.000	1.000	1.000	0.651	0.981	0.847
DMU_6	0.512	0.011	0.415	0.807	0.804	0.807	0.807	0.512	0.288	0.501	0.546
DMU_7	1.000	0.316	0.880	0.999	0.858	1.000	1.000	0.994	0.563	1.000	0.861
DMU_8	0.885	0.012	0.774	0.558	0.554	0.558	0.558	0.885	0.767	0.875	0.643
DMU_9	1.000	0.002	0.888	0.554	0.555	0.554	0.554	1.000	1.000	0.990	0.710
DMU_{10}	0.684	0.289	0.619	0.652	0.522	0.653	0.653	0.678	0.368	0.690	0.581

Table 13. The evaluation results of different models.

Models	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}
Benevolent model (Doyle & Green, 1994)	0.338	0.983	0.630	0.485	0.992	0.565	1.000	0.819	0.911	0.681
Aggressive model (Doyle & Green, 1994)	0.278	0.868	0.548	0.423	0.847	0.546	0.861	0.643	0.710	0.581
Model I (Wang et al. 2011)	0.273	0.852	0.548	0.416	0.833	0.538	0.846	0.632	0.699	0.570
Model II (Wang et al. 2011)	0.285	0.857	0.547	0.410	0.830	0.477	0.847	0.697	0.788	0.578
PCE model (α =0.7)	0.310	0.874	0.630	0.469	0.965	0.610	0.945	0.729	0.797	0.629
PCE model (α =0.2)	0.287	0.854	0.547	0.409	0.830	0.477	0.847	0.699	0.789	0.580

Figure 5 presents the ranking orders of the five models, and some of them are markedly different. For example, DMU_7 and DMU_5 rank first when the benevolent model (Doyle & Green, 1994) and the PCE model (when α=0.7) are used, respectively. However, DMU_2 is assessed as the most efficient university according to the aggressive model (Doyle & Green, 1994), Models I and II (Wang et al., 2011), and the PCE model (when α=0.2).

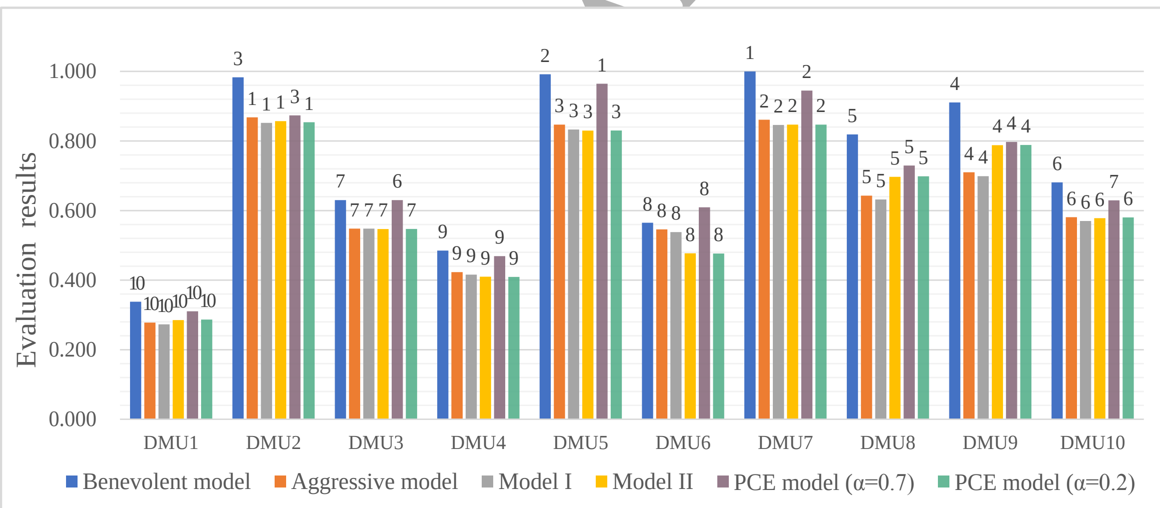

Figure 5. The ranking orders of the models.

Based on the ranking results in Figure 5, the main characteristics of the PCE model can be concluded as follows.

(1) The PCE model can successfully produce a full ranking of the DMUs, in which the DM does not have to face a difficult choice between the aggressive and benevolent models in Doyle and Green (1994). For example, the most efficient university is DMU_2 using the aggressive model and DMU_7 using the benevolent model. Determining which result is correct is difficult because the aggressive and benevolent models evaluate the DMUs from different perspectives and may

thus inevitably produce different results for the same group of DMUs. In such a case, the DM has to judge which between DMU_2 and DMU_7 is better. This situation may reduce the effectiveness of cross-efficiency evaluation to some degree. However, the PCE model can provide a unique result, which considers all possible cases for the DM, instead of only under an extreme case (i.e., aggressive or benevolent). Therefore, the PCE model is more reasonable and closer to the actual preference of the DM.

(2) The PCE model can degenerate to Models I and II in Wang et al. (2011) by considering the possible values of parameters λ, α, β, and θ; thus, the models of Wang et al. (2011) can be viewed as the particular case of the PCE model (see Models (15), (16) and (17)). For example, according to Figure 5, the most efficient university is DMU_2 when Models I or II in Wang et al. (2011) is used, and this result is similar to one case of the PCE model (i.e., when α=0.2). However, DMU_5 is assessed as the most efficient university in another case of the PCE model (i.e., when α=0.7), and this result is different with the cases of Models I and II in Wang et al. (2011). This result is caused by the fact that the PCE model allows the DM to select different values of parameter according to his/her behaviour (see Model (15)). In such manner, the PCE model can consider a wide range of scenarios in the evaluation process.

(3) The PCE model can capture the risk attitudes of the DM in the cross-efficiency evaluation process, whereas the aggressive and benevolent models (Doyle & Green, 1994) and Models I and II (Wang et al., 2011) neglect this point. For example, according to Figure 5, the most efficient university is DMU_2 by using the PCE model when α=0.2 (i.e., the DM tends to be risk-averse), whereas it becomes DMU_5 by using the PCE model when α=0.7 (i.e., the DM is inclined to risk-seeking; see Section 5.2.1 for further detail). Therefore, the evaluation results change with the variations of the parameter α; this finding confirms the significance of capturing the risk factors of the DM in the evaluation process. However, the models of Doyle and Green (1994) and Wang et al. (2011) do not consider the risk attitudes of the DM, that is, they deem that the DM has the same risk attitude with respect to different DMUs. As a result, the evaluation results of these models overlook the risk factors of the DM, which may be inconsistent with reality.

6. Conclusions and discussions

Cross-efficiency in DEA has been widely accepted as a useful tool for evaluating the performance of DMUs and has been used in a variety of theoretical studies and practical applications. However, common cross-efficiency evaluation models assume that DMs are completely rational and generally fail to consider the risk attitude of a DM that plays an important role in the evaluation process. Noting that prospect theory can capture the non-rational psychological aspects of DMs under risk, this paper investigates the cross-efficiency evaluation based on prospect theory and proposes a novel cross-efficiency model termed the PCE model in which the prospect values of each DMU are introduced. The PCE model can capture the risk attitude of the DMs and maintains a certain degree of fairness. That is, the secondary goal for cross-efficiency evaluation is set without the use of information from other DMUs. Furthermore, this paper provides a numerical example to illustrate the potential applications of the PCE model and its effectiveness in ranking DMUs compared with several classical cross-efficiency models. The case study shows that the risk preference of DMs characterized by the parameters α, β, and θ in the PCE model influences the results of performance evaluation.

To the best of our knowledge, this is the first time that prospect theory is used to evaluate efficiency in DEA. The proposed approach can be effectively applied to different evaluation problems, such as investment selection and financial management. Nevertheless, we do not consider the regret of DMs when using the PCE model for evaluation problems. Despite our interest on introducing the regret theory in the cross-efficiency model, we leave that point for future research since it may result in sophisticated calculations and our model cannot be applied to an extended framework.

Acknowledgments

This work was supported by the National Science Foundation of China (No. 71671181). The authors would like to thank the editors of European Journal of Operational Research and three anonymous reviewers for their insightful comments and suggestions.

References

Abdellaoui, M., Bleichrodt, H., & Paraschiv, C. (2007). Loss aversion under prospect theory: a parameter-free measurement. Management Science, 53(10), 1659-1674.

Abdellaoui, M., & Kemel, E. (2014). Eliciting prospect theory when consequences are measured in time units: "Time is not money". Management Science, 60(7), 1844-1859.

Allais, M. (1953). Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'ecole americaine. Econometrica, 21(4), 503-546.

Anderson, T. R., Hollingsworth, K. B., & Inman, L. B. (2002). The fixed weighting nature of a cross-evaluation model. Journal of Productivity Analysis, 18(1), 249-255.

Berman, O., Sanajian, N., & Wang, J. M. (2017). Location choice and risk attitude of a decision maker. Omega, 66, 170-181.

Bleichrodt, H., Schmidt, U., & Zank, H. (2009). Additive utility in prospect theory. Management Science, 55(5), 863-873.

Borgonovo, E., Cappelli, V., Maccheroni, F., & Marinacci, M. (2018). Risk analysis and decision theory: a bridge. European Journal of Operational Research, 264(1), 280-293.

Boussofiane, A., Dyson, R. G., & Thanassoulis, E. (1991). Applied data envelopment analysis. European Journal of Operational Research, 52(1), 1-15.

Charnes, A., & Cooper, W. W. (1962). Programming with linear fractional functional. Naval Research Logistics Quarterly, 9, 181-185.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European Journal of Operational Research, 2(3), 429-444.

Despotis, D. K. (2002). Improving the discriminating power of DEA: focus on globally efficient units. Journal of the Operational Research Society, 53(3), 314-323.

Doyle, J., & Green, R. (1994). Efficiency and cross-efficiency in DEA: derivations, meanings and uses. Journal of the Operations Research Society, 45(5), 567-578.

Doyle, J., & Green, R. (1995). Cross-evaluation in DEA: improving discrimination among DMUs. INFOR: Information Systems and Operational Research, 33(3), 205-222. doi: 10.1080/03155986.1995. 11732281.

Ellsberg, D. (1961). Risk, ambiguity and the savage axioms. The Quarterly Journal of Economics, 75(4), 643-669.

Ertay, T., & Ruan, D. (2005). Data envelopment analysis based decision model for optimal operator allocation in CMS. European Journal of Operational Research, 164(3), 800-810.

Falagario, M., Sciancalepore, F., Costantino, N., & Pietroforte, R. (2012). Using a DEA-cross efficiency approach in public procurement tenders. European Journal of Operational Research, 218(2), 523-529.

Green, R. H., Doyle, J. R., & Cook, W. D. (1996). Preference voting and project ranking using DEA and cross-evaluation. European Journal of Operational Research, 90, 461-472.

Guiso, L., & Paiella, M. (2008). Risk aversion, wealth, and background risk. Journal of the European Economic Association, 6(6), 1109-1150.

Houlding, B., & Coolen, F. P. A. (2012). Nonparametric predictive utility inference. European Journal of Operational Research, 221(1), 222-230.

Kahneman, D., & Tversky, A. (1979). Prospect theory: an analysis of decision under risk. Econometrica, 47(2), 263-292.

Krohling, R. A., & de Souza, T. T. M. (2012). Combining prospect theory and fuzzy numbers to multi-criteria decision making. Expert Systems with Applications, 39(13), 11487-11493.

Lahdelma, R., & Salminen, P. (2009). Prospect theory and stochastic multicriteria acceptability analysis (SMAA). Omega, 37(5), 961-971.

Lejarraga, T., & Müller-Trede, J. (2017). When experience meets description: how dyads integrate experiential and descriptive information in risky decisions. Management Science, 63(6), 1953-1971.

Liang, L., Wu, J., Cook, W. D., & Zhu, J. (2008). Alternative secondary goals in DEA cross-efficiency evaluation. International Journal of Production Economics, 113(2), 1025-1030.

Lim, S. (2012). Minimax and maximin formulations of cross-efficiency in DEA. Computers & Industrial Engineering, 62(3), 726-731.

Lim, S., Oh, K. W., & Zhu, J. (2014). Use of DEA cross-efficiency evaluation in portfolio selection: an application to Korean stock market. European Journal of Operational Research, 236(1), 361-368.

Long, X. Y., & Nasiry, J. (2015). Prospect theory explains newsvendor behavior: the role of reference points. Management Science, 61(12), 3009-3012.

Mashayekhi, Z., & Omrani, H. (2016). An integrated multi-objective Markowitz-DEA cross-efficiency model with fuzzy returns for portfolio selection problem. Applied Soft Computing, 38, 1-9.

Oukil, A., & Amin, G. R. (2015). Maximum appreciative cross-efficiency in DEA: a new ranking method. Computers & Industrial Engineering, 81, 14-21.

Partovi, F. Y. (2011). Corporate philanthropic selection using data envelopment analysis. Omega(39), 522-527.

Qin, J. D., Liu, X. W., & Pedrycz, W. (2017). An extended TODIM multi-criteria group decision making method for green supplier selection in interval type-2 fuzzy environment. European Journal of Operational Research, 258(2), 626-638.

Ramón, N., Ruiz, J. L., & Sirvent, I. (2010). On the choice of weights profiles in cross-efficiency evaluations. European Journal of Operational Research, 207, 1564-1572.

Ruiz, J. L., & Sirvent, I. (2012). On the DEA total weight flexibility and the aggregation in cross-efficiency evaluations. European Journal of Operational Research, 223, 732-738.

Scholz, M., Dorner, V., Schryen, G., & Benlian, A. (2017). A configuration-based recommender system for supporting e-commerce decisions. European Journal of Operational Research, 259(1), 205-215.

Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: critique and extensions. In: Silkman, R.H. (Ed.), Measuring Efficiency: An Assessment of Data Envelopment Analysis. Jossey-Bass, San Francisco, CA, 32, 73-105. doi: 10.1002/ev.1441.

Smith, J. E., & Ulu, C. (2017). Risk aversion, information acquisition, and technology adoption. Operations Research, 65(4), 1011-1028.

Verbunt, P., & Rogge, N. (2018). Geometric composite indicators with compromise Benefit-of-the-Doubt weights. European Journal of Operational Research, 264(1), 388-401.

Vipin, B., & Amit, R. K. (2017). Describing decision bias in the newsvendor problem: a prospect theory model. Omega. doi: 10.1016/j.omega.2017.12.008.

Wang, L., Wang, Y. M., & Martínez, L. (2017). A group decision method based on prospect theory for emergency situations. Information Sciences, 418-419, 119-135.

Wang, Y. M., & Chin, K. S. (2010a). A neutral DEA model for cross-efficiency evaluation and its extension. Expert Systems with Applications, 37, 3666-3675.

Wang, Y. M., & Chin, K. S. (2010b). Some alternative models for DEA cross-efficiency evaluation. International Journal of Production Economics, 128(1), 332-338.

Wang, Y. M., & Chin, K. S. (2011). The use of OWA operator weights for cross-efficiency aggregation. Omega, 39(5), 493-503.

Wang, Y. M., Chin, K. S., & Luo, Y. (2011). Cross-efficiency evaluation based on ideal and anti-ideal decision making units. Expert Systems with Applications, 38(8), 10312-10319.

Wu, J., Chu, J. F., Sun, J. S., Zhu, Q. Y., & Liang, L. (2016). Extended secondary goal models for weights selection in DEA cross-efficiency evaluation. Computers & Industrial Engineering, 93, 143-151.

Wu, J., Liang, L., & Chen, Y. (2009). DEA game cross-efficiency approach to Olympic rankings. Omega, 37(4), 909-918.

Wu, J., Sun, J. S., & Liang, L. (2012). Cross efficiency evaluation method based on weight-balanced data envelopment analysis model. Computers & Industrial Engineering, 63, 513-519.

Yang, F., Ang, S., Xia, Q., & Yang, C. (2012). Ranking DMUs by using interval DEA cross efficiency matrix with acceptability analysis. European Journal of Operational Research, 223(2), 483-488.