Engineering Structures 178 (2019) 37–54

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Punching strength of conventional slab-column specimens

G.I.B. Rankin*, A.E. Long

School of Natural and Built Environment, Queen's University Belfast, N.Ireland, UK

ARTICLE INFO

Keywords:
Reinforced concrete
Flat slabs
Interior slab-column connections
Punching failure
Punching strength

ABSTRACT

This paper presents an improved rational method for predicting the punching strength of conventional reinforced concrete slab-column specimens extending to the nominal line of contraflexure in a flat slab structure. The proposed method of analysis is for square and circular, isotropically reinforced, conventional slab-column specimens, concentrically loaded using square or circular columns. The method is based on an earlier two-phase approach, in which the punching strength was predicted as the lesser of the flexural punching strength and the shear punching strength. The earlier approach had previously been shown to be more reliable than other methods, including the major building code methods, and the proposed method represents a further significant improvement. The improvement in the proposed method is due to the incorporation of slab depth factors for both the flexural and shear modes of punching failure and refinements to the effects of concrete strength, reinforcement percentage and reinforcement yield strength, for the shear mode of punching failure. Comprehensive test data is presented for 217 tests on conventional slab-column specimens reported in the literature in the sixty year period 1956–2016. Analysis of these results by the proposed method resulted in significantly improved correlation over that of the authors' previous two-phase approach. The method is also shown to be significantly more accurate and consistent than the current Eurocode 2 (2004) method, the ACI 318-14 (2014) method and the *fib* Model Code (2010) method for predicting the punching strength of conventional reinforced concrete slab-column specimens.

1. Introduction

Reinforced concrete flat slab structures were introduced in both North America and Europe at the beginning of the twentieth century. Technical and commercial development was primarily instigated by Turner [1] in the USA and Maillart [2] in Switzerland. In the early years, theoretical methods of analysis had not been developed and load capacity was generally proven by full scale load testing, as demonstrated by Lord [3]. The historical development of flat slab construction has been well documented by authors such as Sozen and Seiss [4], Faulkes [5] and Gasparini [6].

Over the years, there have been justifiable concerns amongst structural engineering designers over the potential for punching failure at interior slab-column connections (Fig. 1). Thus, the development of a reliable rational method for predicting the punching strength of reinforced concrete slab-column connections has been the subject of a considerable amount of research.

As a result of flat slab development, the first punching tests on conventional slab-column specimens extending to the nominal line of contraflexure, taken to be at 0.22 L from the column centre (Fig. 2), were carried out by Elstner & Hognestad [7]. Subsequent work by Base [8], Kinnunen and Nylander [9], Moe [10] and many others, through the decades to the present day, has provided a wide range of punching test results on conventional slab-column specimens.

Contemporaneously, with the provision of test results, there have also been many efforts to develop more reliable methods for predicting the punching strength of conventional slab-column specimens. However, to date, there is no widely accepted rational approach to predicting punching strength, as demonstrated by the widely different approaches currently adopted in the major building codes [11,12] and the more recent *fib* Model Code (2010) [13,14] approach, proposed for use in the forthcoming revised Eurocode.

The approach on which the proposed method for predicting the punching strength of conventional slab-column specimens is founded, is that of Long [15] who, by combining a rational flexural approach and an empirically based shear approach, produced a two-phase approach to the prediction of the punching strength of slabs. Long's original two-phase approach was later modified by Rankin and Long [16], to include the effect of slab ductility in the flexural mode of punching failure.

This paper presents the results of two empirical modifications, derived on the basis of statistical analyses of a wide range of test results, to the authors' previous method [16] for predicting the punching strength

* Corresponding author.
E-mail addresses: b.rankin@qub.ac.uk (G.I.B. Rankin), a.long@qub.ac.uk (A.E. Long).

https://doi.org/10.1016/j.engstruct.2018.10.014
Received 29 September 2017; Received in revised form 5 October 2018; Accepted 5 October 2018
0141-0296/

Notation

B	overall side length of conventional square slab-column specimen (mm)
E_s	modulus of elasticity of flexural tensile reinforcement
L	span of slab between columns (mm)
M_u	ultimate moment of resistance (Nmm/mm)
$M_{(bal)}$	balanced moment of resistance (Nmm/mm)
D_{ff}	proposed slab depth factor for flexural mode of punching
D_{fs}	proposed slab depth factor for shear mode of punching
P_t	ultimate punching load test result (kN)
$P_{p(ACI)}$	predicted punching load to ACI 318-14 (2014) method (kN)
$P_{p(EC2)}$	predicted punching load to Eurocode 2 (2004) method (kN)
$P_{p(MC10)}$	predicted punching load to fib Model Code 2010 method (kN)
$P_{p(R\&L)}$	predicted punching load to Rankin & Long's (1987) method (kN)
$P^{**}_{p(R\&L)}$	predicted punching load to Rankin & Long's proposed method (kN)
P_{vf}	predicted flexural punching load to Rankin & Long's (1987) method (kN)
P^{**}_{vf}	predicted flexural punching load to Rankin & Long's proposed method (kN)
P_{vs}	predicted shear punching load to Rankin & Long's (1987) method (kN)
P^{**}_{vs}	predicted shear punching load to Rankin & Long's proposed method (kN)
R^2	coefficient of determination
S	supported span of conventional square slab-column specimen (mm)
$V_{Rd,c}$	Eurocode 2 shear resistance (kN)
b_0	perimeter of critical section for shear (mm)
c	square column side length (mm)
d	effective slab depth (mm)
d_g	fib Model Code 2010 maximum size of aggregate (mm)
$d_{g(est)}$	estimated maximum size of aggregate (mm)
d_v	fib Model Code 2010 shear resisting effective depth (mm)
f_c'	cylinder compressive strength of concrete (MPa)
f_{ck}	characteristic compressive cylinder strength of concrete at 28 days (MPa)
f_y	yield strength of reinforcement (MPa)
k	Eurocode 2 slab depth factor
k_b	ratio of applied load to average internal moment at column periphery
k_{dg}	fib Model Code 2010 shear factor for maximum aggregate size
k_{yl}	ratio of applied load to average moment for overall yield line mechanism
k_φ	factor accounting for openness and roughening of cracks (fib Model Code 2010)
m_s	bending moment acting in design strip (fib Model Code 2010 method)
m_R	bending strength of design strip (fib Model Code 2010 method)
r_f	reduction coefficient to allow for column shape ($r_f = 1.15$ for square columns)
r_s	governing radius of contraflexure of radial bending moments (fib Model Code 2010)
u_1	Eurocode 2 control perimeter for punching shear (mm)
$v_{Rd,c}$	Eurocode 2 shear resistance (MPa)
α_s	ACI coefficient – 40 for interior columns, 30 for edge columns, 20 for corner columns
β	ACI coefficient – ratio of the long side to the short side of a column
γ_c	Eurocode 2 partial factor for concrete
λ	ACI factor to account for concrete density; $\lambda = 1.0$ for normal density concrete
ρ	average reinforcement ratio
ρ_l	Eurocode 2 average reinforcement ratio for longitudinal reinforcement (≤ 0.02)
$\varnothing_B$	overall diameter of conventional circular slab-column specimen (mm)
$\varnothing_c$	circular column diameter (mm)
$\varnothing_S$	supported diameter of conventional circular slab-column specimen (mm)

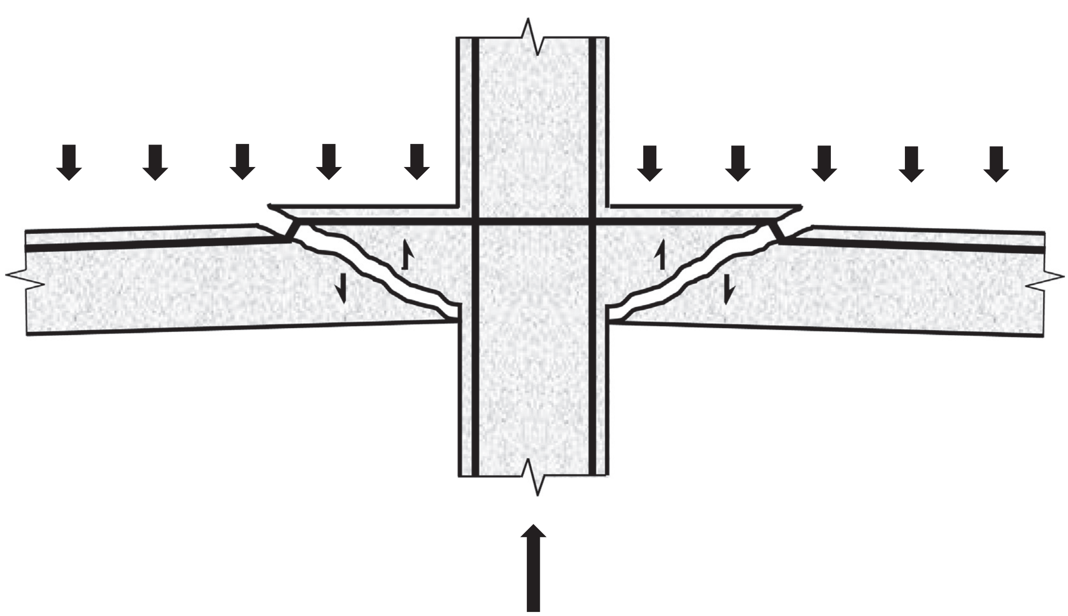

Fig. 1. Punching failure at interior slab-column connection.

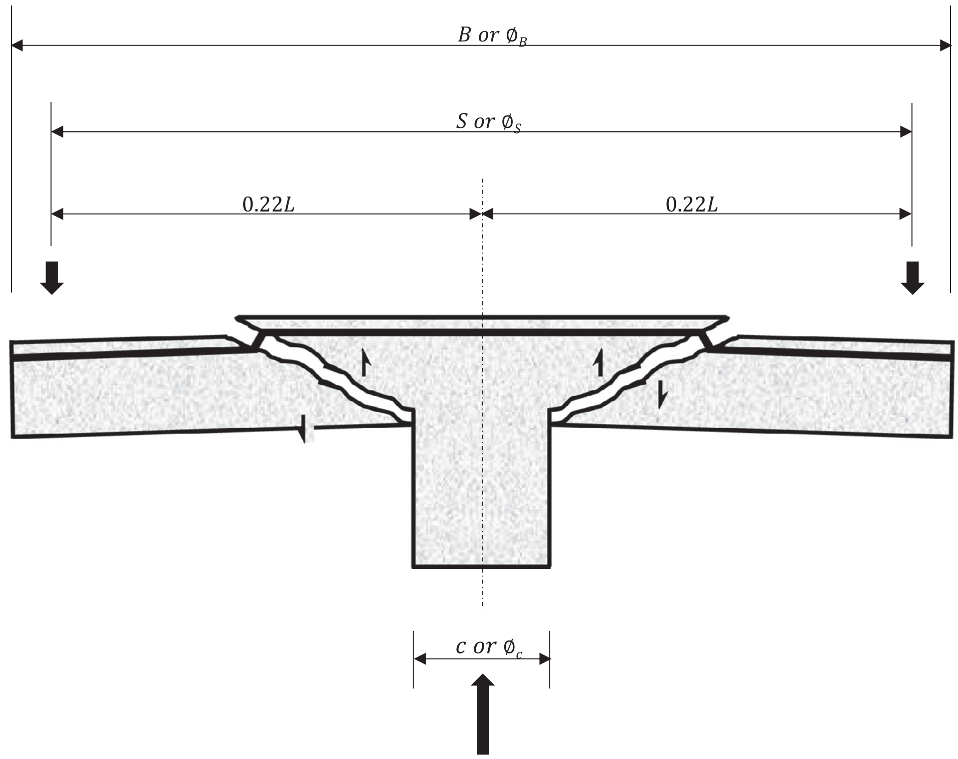

Fig. 2. Punching failure of conventional slab-column specimen.

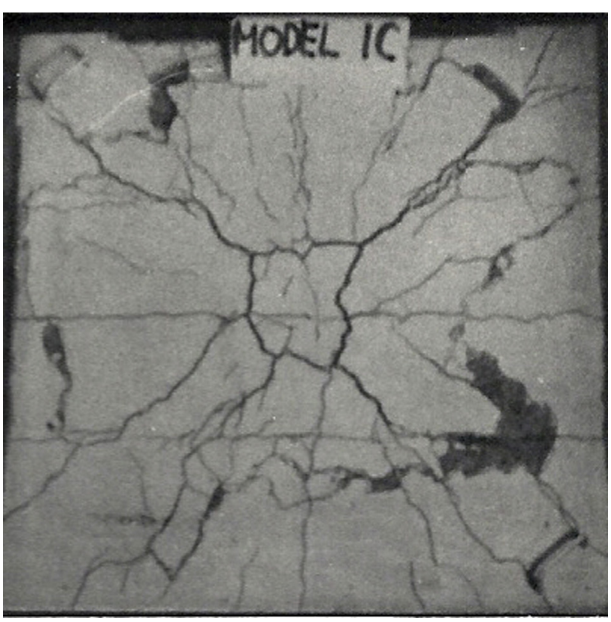

Fig. 3a. Flexural mode of punching failure (Rankin [20]).

of conventional slab-column specimens. In the current paper, the Rankin and Long [16] method is modified by incorporating two empirically derived slab depth factors – one for the flexural mode of punching failure and one for the shear mode of punching failure, and also making refinements to the shear mode punching strength parameters.

The proposed method is shown to give good correlation with a wide range of test results compiled from sixty years of research into punching failure (1956–2016). The coefficient of variation is improved from 0.146, using the previous Rankin & Long method [16], to 0.112 using the proposed method. The method is also shown to be significantly more consistent than both the Eurocode 2, 2004 [11] method, the ACI 318-14, 2014 [12] method and the *fib* Model Code 2010 method [13,14] for predicting the punching strength of conventional slab-column specimens.

To simplify matters, the proposed method of analysis is limited to square and circular, isotropically reinforced, conventional slab-column specimens, concentrically loaded using square or circular columns. However, it is envisaged that the method of analysis could be readily extended to include other configurations such as rectangular slabs, rectangular columns, banded reinforcement, prestressed slabs, and eccentric loading, as proposed previously by Rankin and Long [16].

2. Punching failure

The nature of punching failure has been well documented (eg. Regan and Braestrup [17], the ACI [18] and ACI-*fib* [19], therefore only a brief mention will be made of the main pertinent characteristics.

Punching failure occurs when a concentrated reaction, such as that from a column at a slab-column connection in a flat slab structure (Fig. 1), or a concentrated load, such as that applied in a conventional slab-column punching strength test (Fig. 2), punches a truncated cone of concrete from the reinforced concrete slab. Failure is generally sudden, although most researchers would probably accept that the more reinforced a slab is, the more sudden and explosive punching failure becomes [20]. For this reason, the authors' proposed method takes into account the various modes of punching failure – from lightly reinforced slabs where a significant amount of yielding of the reinforcement occurs prior to punching failure (Fig. 3a), to heavily reinforced slabs where punching failure is more sudden and explosive (Fig. 3b). The latter failure mode may be caused by flexural compression of the concrete or shear failure of the concrete in a localised zone around the concentrated load.

In the Rankin and Long [16] method, slab ductility was taken into account using the ductility parameter ratio $M_u/M_{(bal)}$. Thus, for the hypothetical extreme case of near zero reinforcement - flexural punching failure was predicted by the yield line capacity. For the

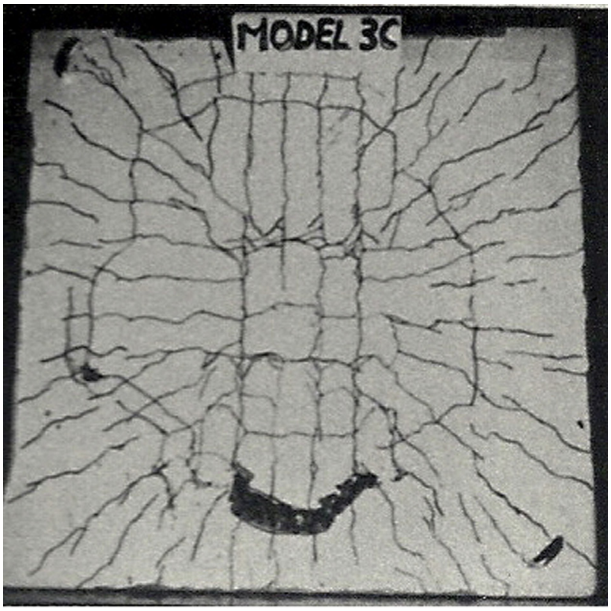

Fig. 3b. Shear mode of punching failure (Rankin [20]).

opposite extreme case of balanced or over-reinforcement, flexural punching failure was predicted when the localised moment at the column periphery reached the balanced moment capacity. Shear punching failure was predicted when the shear capacity at a notional critical perimeter at $d/2$ from the column or loaded area was attained. The punching strength was predicted as the lesser of either the flexural punching strength or the shear punching strength.

3. Summary of current major Code methods [11,12], the *fib* Model Code (2010) method and background to proposed method

3.1. Eurocode 2 (BS EN 1992-1-1, 2004) method [11]

In the Eurocode 2 method [11], the punching shear resistance is calculated at the basic control perimeter u_1 which is taken at a distance of 2d from the column face. The corners of the basic control perimeter are rounded and the control perimeter lengths for both square and circular columns are illustrated in Fig. 4. A check is made that the shear stress at the column face does not exceed the maximum permissible shear stress.

For a square column, the Eurocode 2 [11] critical perimeter for shear is given by:

$$b_0 = 4(c + \pi d) \tag{1}$$

And for a circular column or loaded area, the Eurocode 2 [11] critical perimeter for shear is given by:

$$b_0 = \pi(\varnothing_c + 4d) \tag{2}$$

For non-prestressed slabs, the method given in Eurocode 2 (BS EN 1992:1-1: 2004 + A1: 2014) [11] involves calculation of the shear resistance on the control perimeter as follows:

$$v_{Rd,c} = C_{Rd,c}k(100\rho_l f_{ck})^{\frac{1}{3}} \tag{3}$$

The value of $C_{Rd,c}$ is given in BS EN 1992:1-1: 2004 + A1: 2014 [11] as:

$$C_{Rd,c} = \frac{0.18}{\gamma_c} \tag{4}$$

Note: for laboratory tests, where there is good control on concrete quality, γ_c is normally taken to be 1.0.

From the above expressions, it can be seen that the Eurocode 2 approach [11] allows for the effect of flexural reinforcement ρ_l and also makes an allowance for the slab depth relative to a 200 mm standard depth through the incorporation of the parameter 'k', where:

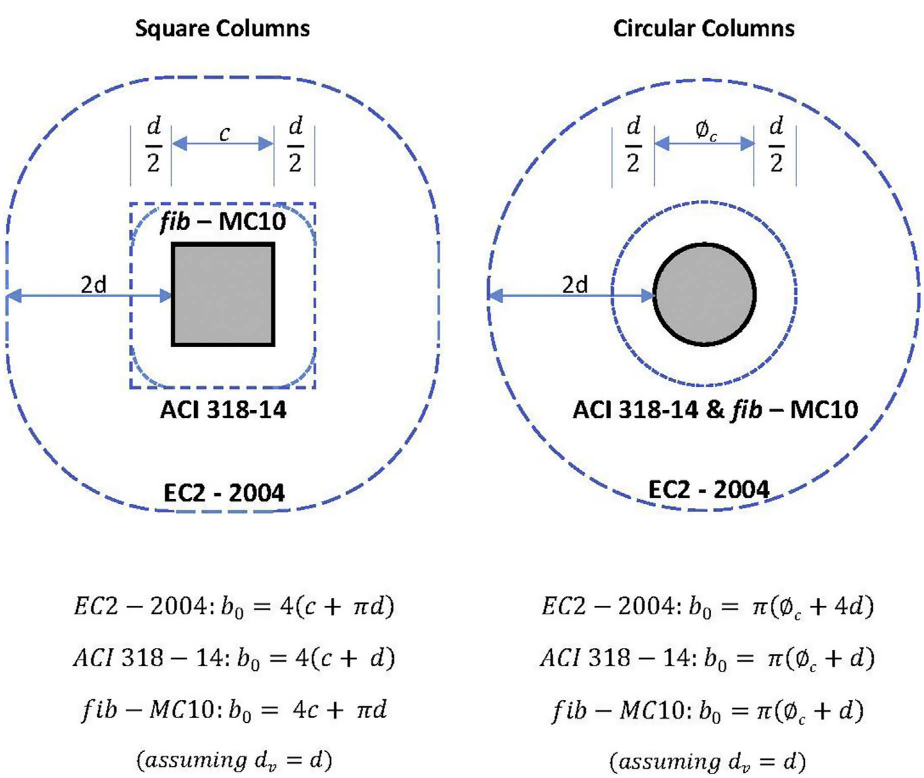

Fig. 4. Basic punching shear control perimeters for building codes [11–14].

$$k = \sqrt{\frac{200}{d}} \leq 2.0 \quad (5)$$

(d in mm)

Thus:

$$V_{Rd,c} = v_{Rd,c} u_1 d \quad (6)$$

Also, the maximum permissible shear stress must not be exceeded at the perimeter of the column, thus:

$$v_{Rd,cmax} = 0.5 v f_{ck} \quad (7)$$

The value of the strength reduction factor 'v' for concrete cracked in shear is given in BS EN 1992:1-1: 2004 + A1: 2014 [11] as:

$$v = 0.6(1 - f_{ck}/250) \quad (8)$$

Therefore the maximum shear load at the perimeter of the column, $V_{Rd,max}$, is given by Eq. (9):

$$V_{Rd,max} = 0.5 u_1 d [0.6(1 - f_{ck}/250)] f_{ck}/\gamma_c \quad (9)$$

3.2. ACI 318-14 method [12]

In the ACI 318-14 method [12], the basic shear perimeter is taken at a distance of 0.5d from the column face. The corners of the basic control perimeter are square and the control perimeter lengths for both square and circular columns are illustrated in Fig. 4. The ACI 318-14 [12] ultimate punching shear resistance for slabs without shear reinforcement is given by the lesser of the following three expressions:

$$V_c = \left(1 + \frac{2}{\beta}\right)\lambda\sqrt{f_c^{'}}\, b_0 d \quad (10)$$

$$V_c = 0.083\left(\frac{\alpha_s d}{b_0} + 2\right)\lambda\sqrt{f_c^{'}}\, b_0 d \quad (11)$$

$$V_c = 0.33\sqrt{f_c^{'}}\, b_0 d \quad (12)$$

Typically Eq. (12) is the lowest and in some sources it is the only expression stated. For a square column, the ACI [12] critical perimeter for shear is given by:

$$b_0 = 4(c + d) \quad (13)$$

And for a circular column, the ACI [12] critical perimeter for shear is given by:

$$b_0 = \pi(\varnothing_c + d) \quad (14)$$

From the above expressions, it can be seen that the ACI [12] approach makes no allowance for the effect of flexural reinforcement and also makes no allowance for the slab depth d relative to any normalised or standard depth.

3.3. *fib* Model Code 2010 [13,14]

The punching provisions of the *fib* Model Code for Concrete Structures 2010 (MC 2010) [13,14] are based on the Critical Shear Crack Theory (CSCT) primarily developed by Muttoni [21]. MC 2010 and its recent revisions, provides the basis for the revisions to the Eurocode, due to be published circa 2018. The corners of the basic control perimeter are rounded and the control perimeter lengths for both square and circular columns are illustrated in Fig. 4.

For a square column, the *fib* Model Code [13,14] critical perimeter for shear is given by:

$$b_0 = 4c + \pi d_v \quad (15)$$

And for a circular column or loaded area, the *fib* Model Code [13,14] critical perimeter for shear is given by:

$$b_0 = \pi(\varnothing_c + d_v) \quad (16)$$

The *fib* Model Code [13,14] provides the following basic design formulations:

$$V_{Rd,c} = k_\varphi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \quad (17)$$

where:

b_0 = *length of control perimeter (set at d_v/2 of edge of supported area)*
d_v = *shear−resisting effective depth of member*

k_φ = *factor accounting for openness and roughening of cracks as given by:*

$$k_\varphi = \frac{1}{1.5 + 0.9 k_{dg}\varphi \cdot d} \leq 0.6 \quad (18)$$

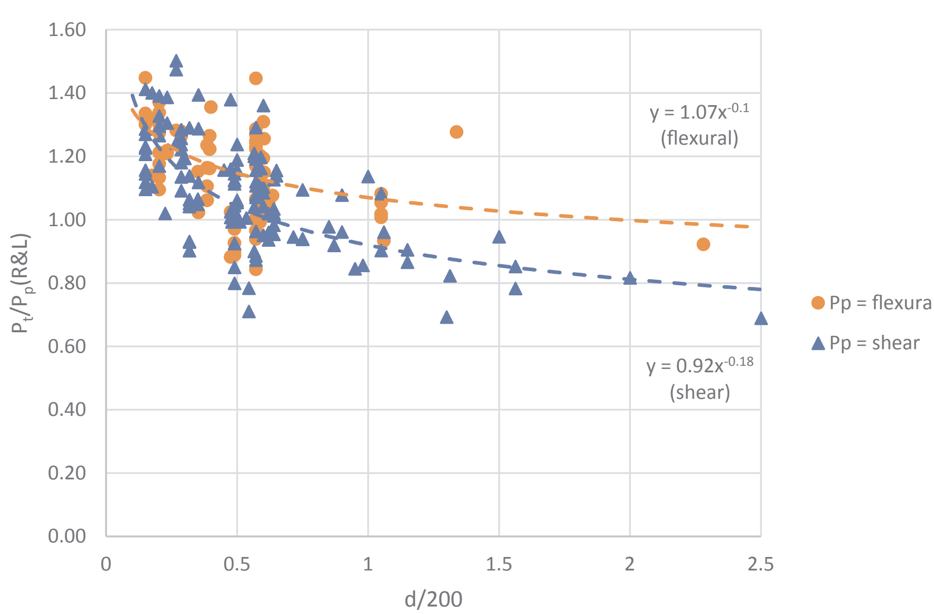

Fig. 5. Correlation of Rankin & Long [16] predictions with relative slab effective depth.

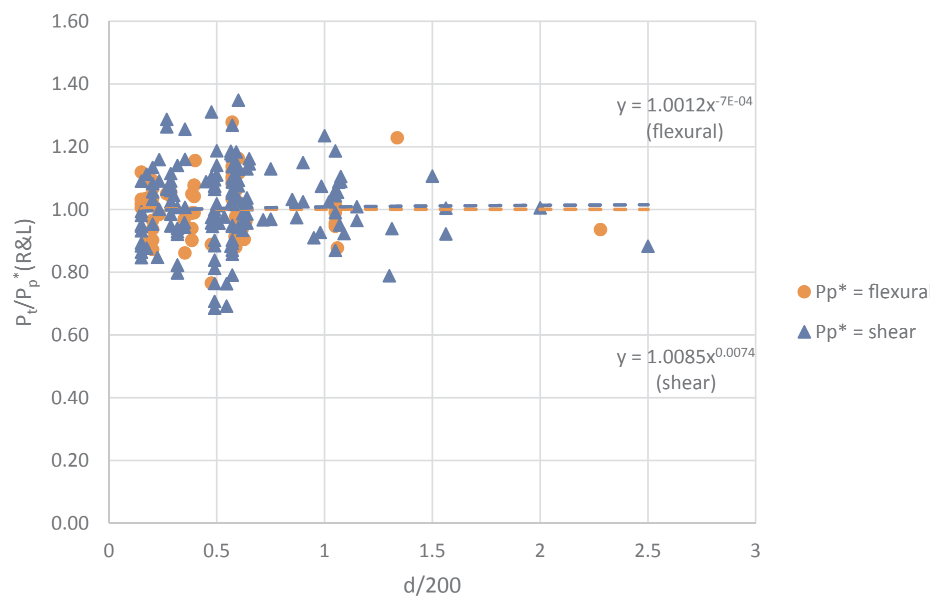

Fig. 6. Correlation of modified Rankin & Long [16] predictions (incorporating slab depth factors only) with relative slab effective depth.

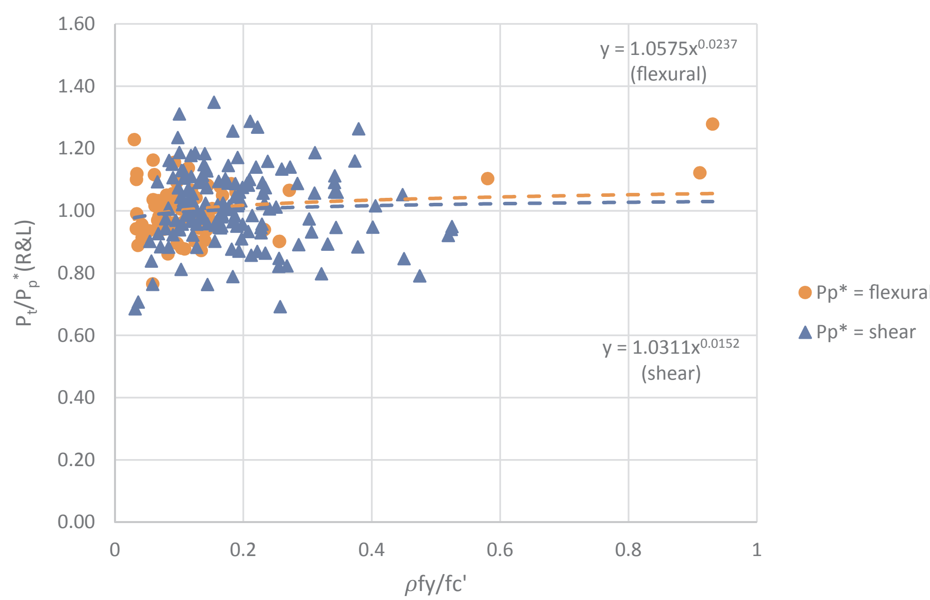

Fig. 7. Correlation of modified Rankin & Long [16] predictions (incorporating slab depth factors only) with reinforcement index.

The factor k_{dg} is defined for shear as:

$$k_{dg} = \frac{32}{(16 + d_g)} \geq 0.75 \tag{19}$$

In Eq. (18), the rotation of the slab 'φ' is the governing parameter. A safe estimate of this value can be obtained by assuming that failure of the slab occurs at full yielding of the flexural reinforcement in the support strip. Thus, for slabs extending to the nominal line of contraflexure, the rotation of the slab can be expressed in terms of the ratio r_s/d where r_s denotes the governing radius of contraflexure of radial bending moments. Thus, in a flat slab structure with regular bays: $r_{sx} = 0.22l_x$ *and* $r_{sy} = 0.22l_y$ where l_x and l_y are the bay spans in the x and y directions respectively. The larger value of r_{sx} *or* r_{sy} governs as r_s. Thus, for a Level of Approximation I (LoA I) approach:

$$\varphi = 1.5\cdot\frac{r_s}{d}\frac{f_y}{E_s} \tag{20}$$

where the normal value for the elastic modulus of steel E_s can be taken as 200,000 Mpa.

An improved Level of Approximation II (LoA II), can be made by taking into account the ratio of the bending moment acting in the design strip and the flexural strength, thus:

$$\varphi = 1.5\cdot\frac{r_s}{d}\frac{f_y}{E_s}\left(\frac{m_s}{m_R}\right)^{1.5} \tag{21}$$

For the slab-column specimens analysed, for the purposes of this analysis and without knowledge of the design loads, it was assumed that the ratio of the bending moment acting in the design strip to the flexural strength was approximately the same as the ratio of the localised elastic bending moment around the column to the yield line bending moment, thus:

$$\left(\frac{m_s}{m_R}\right) \approx \left(\frac{k_b}{k_{yl}}\right) \tag{22}$$

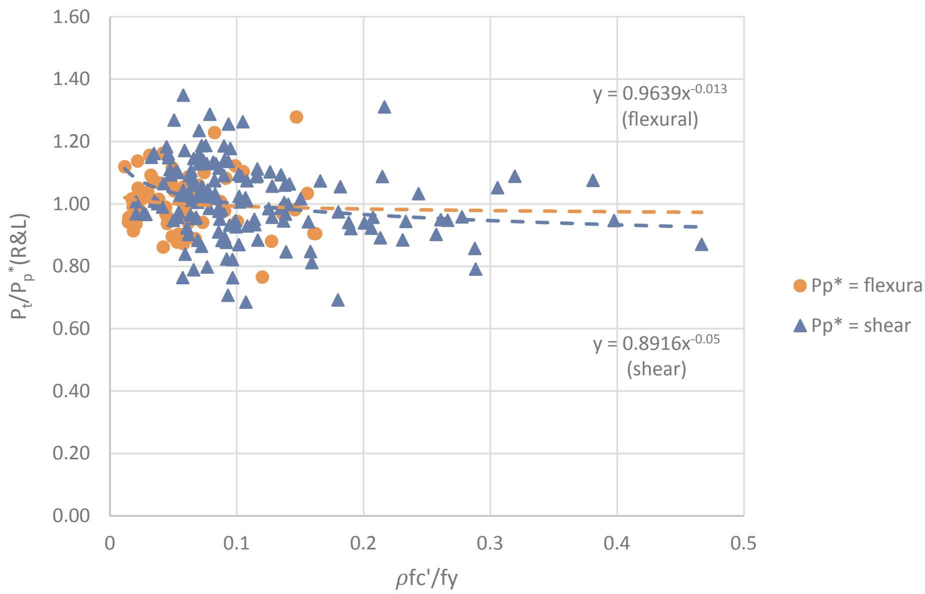

Fig. 8. Correlation of modified Rankin & Long [16] predictions (incorporating slab depth factors only) with material parameters.

3.4. Background to proposed method

The two-phase approach previously proposed by Rankin and Long [16] was shown to give better predictions of punching strength than other methods of the time, such as the method of Regan [22], the BS 8110 (1985) method [23] and the ACI (318-83) [24] method. The method predicted the strengths of two possible modes of punching failure – flexural punching and shear punching - and the predicted strength was taken as the lesser of these two predicted strengths.

The flexural punching strength was given by the rational expression:

$$P_{vf} = \left[k_{yl} - \left(k_{yl} - \frac{k_b}{r_f}\right)\frac{M_u}{M_{(bal)}}\right] M_u \leq \frac{k_b}{r_f} M_{(bal)} \quad (23)$$

where (for conventional square slab and square column specimens):

$$k_{yl} = 8\left(\frac{B}{S-c} - 0.172\right) \quad (24)$$

$$k_b = \frac{25}{\left(\log_e \frac{2.5S}{c}\right)^{1.5}} \quad (25)$$

$$M_u = \rho f_y d^2\left(1 - 0.59\frac{\rho f_y}{f_c'}\right) \quad (26)$$

For conventional circular slabs or columns the moment factor k_{yl} can be calculated using yield line theory and the elastic moment factor k_b can be calculated using Eq. (25), assuming a square column of equivalent perimeter.

It was proposed that $M_{(bal)}$ could be calculated from the simple expression first proposed by Whitney [25] as:

$$M_{(bal)} = 0.333 f_c' d^2 \quad (27)$$

The shear punching strength was predicted from Long's [15] original semi-empirical formula for shear stress on a critical perimeter at $d/2$ from the face of a square column in which it was implicit that the shear strength was given by:

$$v = 0.42\sqrt{f_c'}(100\rho)^{0.25} \quad (28)$$

However, Regan [22] had suggested that, due to the lack of stress concentrations, circular columns give rise to punching strengths approximately 15% greater than square columns of the same perimeter. Thus, Kirkpatrick, Rankin & Long [26] had produced the following expression for the shear punching strength of a slab with a circular column or circular concentrated load:

$$P_{vs} = 1.52\sqrt{f_c'}(\varnothing_c + d)d(100\rho)^{0.25} \quad (29)$$

Allowing for the 15% difference between square and circular columns or concentrated loads, Rankin & Long [16] subsequently gave the following expression for the shear punching strength of a slab with a square column:

$$P_{vs} = 1.66\sqrt{f_c'}(c + d)d(100\rho)^{0.25} \quad (30)$$

4. Proposed modifications to Rankin & Long's (1987) method [16]

4.1. The influence of slab depth

It is widely accepted amongst researchers, and is in fact taken into account in Eurocode 2 [11], that relative slab depth has a significant influence on shear strength (Mitchell, Cook, and Dilger [27] and Birkle and Dilger [28]). It is generally accepted that slab depth has an influence in a relative sense, ie. relative to the strength of a normalised slab effective depth of approximately 200 mm (Mitchell, Cook and Dilger [27]).

The ACI [12] method does not take account of relative slab depth. However, an effective depth of 200 mm is used as a datum for the slab depth factor in Eurocode 2 [11]. Thus, a normalised slab depth of approximately 200 mm is considered an appropriate datum on which to gauge relative strength.

By plotting the ratio of $P_t/P_{p(R\&L)}$ against the relative slab effective depth $d/200$ for the 217 test results analysed, the trend lines for the flexural and shear modes of punching failure were identified as shown in Fig. 5. Thus, statistical analysis gave the following empirical slab depth factors to modify the Rankin & Long [16] method for the flexural and shear punching modes of failure:

$$\text{Flexural mode:} \quad P_t = D_{ff} P_{p(R\&L)} \quad (31)$$

where:

$$D_{ff} = 1.07(d/200)^{-0.10} \quad (32)$$

and:

Table 1
Correlation of various methods for predicting punching strength with 217 test results from various sources (1956–2016).

Test	Type	B or $\varnothing_B$ (mm)	S or $\varnothing_S$ (mm)	c or $\varnothing_c$ (mm)	d (mm)	$d_{g(est)}$ (mm)	ρ (%)	f_y (MPa)	$f_c^{'}$ (MPa)	P_t (kN)	$P_t/P_{p(R\&L)}$	$P_t/P^{**}_{p(R\&L)}$	$P_t/P_{p(EC2)}$	$P_t/P_{p(ACI)}$	$P_t/P_{p(MC10)}$
Elstner & Hognestad (1956) [7]															
A1a	SS	1829	1778	254	117.6	25	1.15	333	14.1	303	1.202	1.066	1.134	1.399	0.917
A1b	SS	1829	1778	254	117.6	25	1.15	333	25.3	366	1.136	1.007	1.127	1.261	0.923[#]
A1c	SS	1829	1778	254	117.6	25	1.15	333	29.1	357	1.065	0.944	1.049	1.147	0.889[#]
A1d	SS	1829	1778	254	117.6	25	1.15	333	36.9	352	0.993	0.880	0.956	1.005	0.861[#]
A1e	SS	1829	1778	254	117.6	25	1.15	333	20.3	357	1.196	1.060	1.183	1.374	0.922[#]
A2a	SS	1829	1778	254	114.3	25	2.47	322	13.7	334	1.248	1.103	1.023	1.624	1.063
A2b	SS	1829	1778	254	114.3	25	2.47	322	19.6	401	1.065	1.030	1.090	1.630	1.067
A2c	SS	1829	1778	254	114.3	25	2.47	322	37.5	468	0.872	0.898	1.025	1.375	0.900
A7b	SS	1829	1778	254	114.3	25	2.47	322	28	513	1.107	1.123	1.239	1.745	1.142
A3a	SS	1829	1778	254	114.3	25	3.7	322	12.8	357	1.446	1.278	0.978	1.796	1.176
A3b	SS	1829	1778	254	114.3	25	3.7	322	22.7	446	1.006	0.989	1.009	1.685	1.103
A3c	SS	1829	1778	254	114.3	25	3.7	322	26.6	535	1.070	1.105	1.148	1.867	1.222
A3d	SS	1829	1778	254	114.3	25	3.7	322	34.6	549	0.963	1.008	1.079	1.680	1.100
A4	SS	1829	1778	356	117.6	25	1.15	333	26.2	401	1.103	0.978	1.049	1.066	0.930[#]
A5	SS	1829	1778	356	114.3	25	2.47	322	27.8	535	0.965	0.920	1.110	1.430	0.922
A6	SS	1829	1778	356	114.3	25	3.7	322	25.1	499	0.843	0.829	0.936	1.404	0.905
A13	SS	1829	1778	356	120.6	25	0.55	294	26.3	236	1.256	1.168[#]	1.168[#]	1.168[#]	1.168[#]
B1	SS	1829	1778	254	114.3	25	0.5	325	14.2	179	1.287	1.137	1.099[#]	1.099[#]	1.099[#]
B2	SS	1829	1778	254	114.3	25	0.5	321	47.7	201	1.245	1.189[#]	1.189[#]	1.189[#]	1.189[#]
B4	SS	1829	1778	254	114.3	25	0.99	304	47.8	334	1.169	1.073[#]	1.073[#]	1.073[#]	1.073[#]
B9	SS	1829	1778	254	114.3	25	2	342	44	506	0.939	0.940	1.127	1.373	0.899
B11	SS	1829	1778	254	114.3	25	3	410	13.5	330	1.269	1.122	0.952	1.616	1.058
B14	SS	1829	1778	254	114.3	25	3	326	50.7	580	0.886	0.934	1.077	1.466	0.960
Base (1959) [8]															
A	SS	610	559	102	57.3	10	1.083	345	26.5	93.9	1.180	1.008	1.318	1.514	0.970
B	SS	610	559	102	57.3	10	1.083	345	28.6	103.9	1.257	1.078	1.422	1.612	1.033
C	SS	610	559	102	57.3	10	1.083	345	26.2	97.9	1.237	1.057	1.379	1.587	1.017
D	SS	610	559	102	57.3	10	1.083	345	27.4	103.9	1.284	1.099	1.442	1.647	1.055
E	SS	610	559	102	57.3	10	0.725	345	29.8	81.9	1.264	1.128[#]	1.264	1.245	1.128[#]
F	SS	610	559	102	57.3	10	0.725	345	27.8	81.9	1.279	1.132[#]	1.293	1.289	1.132[#]
G	SS	610	559	102	57.3	10	1.635	345	29.1	112.9	1.221	1.070	1.339	1.737	1.112
H	SS	610	559	102	57.3	10	1.635	345	26.4	99.9	1.135	0.990	1.224	1.614	1.033
J	SS	610	559	102	57.3	10	3.27	345	28.1	117.9	1.092	0.989	1.123	1.846	1.182
Kinnunen & Nylander (1960) [9]															
1A30(a)24	CC	1829	1710	300	128	30	1.01	456	25.9	430	1.012	0.985	1.232	1.488	0.909
1A30(a)25	CC	1829	1710	300	124	30	1.04	452	24.6	408	1.019	0.985	1.240	1.509	0.922
1A15(a)5	CC	1829	1710	150	117	30	0.8	442	26.3	255	1.107	1.050	1.130	1.535	0.938
1A15(a)6	CC	1829	1710	150	118	30	0.79	455	25.7	275	1.197	1.133	1.214	1.655	1.011
Moe (1961) [10]															
S1-60	SS	1829	1778	254	114.3	38	1.06	400	23.4	390	1.230	1.087	1.325	1.451	0.958[#]
S5-60	SS	1829	1778	203	114.3	38	1.06	400	22.2	343	1.192	1.133	1.294	1.521	1.007
S1-70	SS	1829	1778	254	114.3	38	1.06	484	24.5	393	1.120	1.060	1.315	1.429	0.935
S5-70	SS	1829	1778	203	114.3	38	1.06	484	23.1	379	1.291	1.218	1.411	1.647	1.091
H1	SS	1829	1778	254	114.3	38	1.15	328	26.1	372	1.224	1.082	1.186	1.310	1.003[#]
R2	SS	1829	1778	152	114.3	10	1.15	328	26.6	312	1.156	1.125	1.186	1.506	1.013
M1A	SS	1829	1778	305	114.3	38	1.5	482	20.9	434	1.078	1.030	1.259	1.501	0.974
Taylor & Hayes (1965) [29]															
2S2	SS	889	864	51	63.5	10	1.57	377	26	72.4	1.051	0.927	0.918	1.479	1.026
2S3	SS	889	864	76	63.5	10	1.57	377	24.6	92.9	1.138	1.001	1.091	1.602	1.085
2S4	SS	889	864	102	63.5	10	1.57	377	23.2	87.4	0.929	0.815	0.956	1.308	0.871
2S5	SS	889	864	127	63.5	10	1.57	377	22.1	98.4	0.931	0.815	1.011	1.311	0.863
2S6	SS	889	864	152	63.5	10	1.57	377	18.4	98.4	0.902	0.782	0.998	1.270	0.828
3S2	SS	889	864	51	63.5	10	3.14	377	22.8	79.9	1.042	0.945	0.992	1.744	1.209
3S4	SS	889	864	102	63.5	10	3.14	377	22.6	117.4	1.063	0.964	1.029	1.780	1.185
3S6	SS	889	864	152	63.5	10	1.57	377	21.7	152.8	1.290	1.127	1.467	1.816	1.185
Criswell (1974) [30]															
S2075-1	SS	2134	2032	254	120.6	20	0.75	331	32.5	291	1.149	1.026[#]	1.026[#]	1.026[#]	1.026[#]
S2075-2	SS	2134	2032	254	122.2	20	0.75	331	29.1	273	1.070	0.952	0.943[#]	0.943[#]	0.943[#]
S2150-1	SS	2134	2032	254	124	20	1.5	331	29.7	464	1.053	0.994	1.139	1.376	0.905
S2150-2	SS	2134	2032	254	122.2	20	1.5	331	30.2	441	1.025	0.954	1.102	1.322	0.869
S4075-1	SS	2388	2286	508	127	20	0.75	331	26.7	343	1.076	0.967[#]	0.967[#]	0.967[#]	0.967[#]
S4075-2	SS	2388	2286	508	124	20	0.75	331	32.3	330	1.056	0.967[#]	0.967[#]	0.967[#]	0.967[#]
S4150-1	SS	2388	2286	508	125.5	20	1.5	331	35.5	581	1.014	0.905	0.947	0.929	0.864[#]
S4150-2	SS	2388	2286	508	125.5	20	1.5	331	35.8	582	1.014	0.905	0.946	0.927	0.865[#]
Dragosavic and van den Beukel (1974) [31]															
1	CS	475	425	60	30	8	1.2	425	30.7	32	1.231	0.938	1.443	1.620	1.067
2	CS	475	425	60	30	8	1.2	425	30.7	33	1.270	0.968	1.488	1.671	1.100
3	CS	475	425	60	60	8	1.2	425	27.3	78	1.193	1.024	1.135	1.571	1.075

(*continued on next page*)

Table 1 (*continued*)

Test	Type	B or $\varnothing_B$ (mm)	S or $\varnothing_S$ (mm)	c or $\varnothing_c$ (mm)	d (mm)	$d_{g(est)}$ (mm)	ρ (%)	f_y (MPa)	f_c' (MPa)	P_t (kN)	$P_t/P_{p(R\&L)}$	$P_t/P^{**}_{p(R\&L)}$	$P_t/P_{p(EC2)}$	$P_t/P_{p(ACI)}$	$P_t/P_{p(MC10)}$
4	CS	475	425	40	30	8	1.2	425	30.7	26	1.286	0.980	1.347	1.693	1.139
5	CS	475	425	60	30	8	0.5	425	22	18	1.317	1.166#	1.215	1.166#	1.166#
6	CS	475	425	60	30	8	1.2	425	22.2	31.2	1.412	1.058	1.568	1.858	1.223
Dragosavic and van den Beukel (1974) [31]															
7	CS	475	425	60	30	8	1.73	425	22.2	28	1.156	0.883	1.245	1.667	1.097
13	CS	475	425	60	30	8	0.2	425	24.9	8.9	1.448	1.387#	1.387#	1.387#	1.387#
14	CS	475	425	60	30	8	0.4	425	24.9	15.4	1.335	1.225#	1.225#	1.225#	1.225#
15	CS	475	425	60	30	8	0.6	425	24.9	21.1	1.301	1.143#	1.286	1.186	1.143#
16	CS	475	425	60	30	8	0.9	425	23.6	26	1.226	0.975#	1.409	1.502	0.988
17	CS	475	425	60	30	8	1.3	425	23.6	26	1.118	0.844	1.246	1.502	0.988
18	CS	475	425	60	30	8	1.7	425	23.6	30	1.207	0.923	1.315	1.733	1.140
19	CS	475	425	60	30	8	2.1	425	23.6	30	1.145	0.885	1.226	1.733	1.140
20	CS	475	425	60	30	8	2.5	425	23.6	30	1.096	0.855	1.156	1.733	1.140
Regan (1978) [32]															
SS2	SS	2000	1829	200	77	20	1.2	500	23.4	176	1.061	0.902	1.182	1.292	0.840
SS4	SS	2000	1829	200	77	20	0.92	500	32.3	194	1.163	0.988	1.278	1.212	0.919#
SS6	SS	2000	1829	200	79	20	0.75	480	21.9	165	1.265	1.077	1.273	1.212	0.963#
SS7	SS	2000	1829	200	79	20	0.8	480	30.4	186	1.223	1.041	1.259	1.160	0.993#
Regan (1986) [33]															
1/1	SS	2000	1830	200	77	20	1.2	500	25.76	194	1.106	0.940	1.262	1.358	0.882
1/2	SS	2000	1830	200	77	20	1.2	500	23.44	176	1.061	0.901	1.181	1.291	0.839
1/3	SS	2000	1830	200	77	20	0.92	500	27.44	194	1.235	1.049	1.350	1.315	0.935#
1/4	SS	2000	1830	200	77	20	0.92	500	32.32	194	1.164	0.989	1.278	1.212	0.920#
1/5	SS	2000	1830	200	79	20	0.75	480	28.16	165	1.162	0.989	1.171	1.069	0.941#
1/6	SS	2000	1830	200	79	20	0.75	480	21.92	165	1.265	1.077	1.273	1.211	0.963#
1/7	SS	2000	1830	200	79	20	0.8	480	30.4	186	1.223	1.042	1.259	1.160	0.994#
V/4	SS	1600	1500	102	118	20	0.8	628	36.24	285	1.162	1.098	1.155	1.382	0.954
Narayanan & Darwish (1987) [34]															
S1	SS	780	740	100	45	8	2.01	550	43.28	86.5	1.019	0.861	1.248	1.527	0.999
Rankin & Long (1987) [16]															
1	SS	700	640	100	40.5	6	0.423	530	30.72	36.42	1.278	1.151#	1.169	1.151#	1.151#
2	SS	700	640	100	40.5	6	0.558	530	30.72	49.08	1.371	1.193#	1.436	1.193#	1.193#
3	SS	700	640	100	40.5	6	0.691	530	30.72	56.55	1.338	1.126#	1.541	1.358	1.126#
4	SS	700	640	100	40.5	6	0.821	530	34.8	56.18	1.133	0.945#	1.387	1.268	0.945#
5	SS	700	640	100	40.5	6	0.883	530	34.8	57.27	1.095	0.901#	1.380	1.293	0.901#
6	SS	700	640	100	40.5	6	1.026	530	34.8	65.58	1.169	0.929	1.503	1.480	0.964
7	SS	700	640	100	40.5	6	1.163	530	29.68	70.94	1.327	1.053	1.644	1.734	1.129
8	SS	700	640	100	40.5	6	1.292	530	29.68	71.09	1.296	1.033	1.591	1.737	1.132
9	SS	700	640	100	40.5	6	1.454	530	29.68	78.6	1.391	1.115	1.691	1.921	1.251
10	SS	700	640	100	40.5	6	0.517	530	29.92	43.59	1.301	1.140#	1.320	1.140#	1.140#
11	SS	700	640	100	40.5	6	0.802	530	29.92	55	1.176	0.957#	1.439	1.339	0.957#
12	SS	700	640	100	40.5	6	1.107	530	29.92	67.06	1.265	1.001	1.576	1.632	1.063
13	SS	700	640	100	40.5	6	0.601	530	34	49.39	1.274	1.113#	1.363	1.128	1.113#
14	SS	700	640	100	40.5	6	0.691	530	34	52.45	1.211	1.037#	1.382	1.198	1.037#
15	SS	700	640	100	40.5	6	1.994	530	34	84.84	1.296	1.063	1.570	1.937	1.262
1A	SS	700	640	100	46.5	6	0.422	530	28.8	45.19	1.218	1.089#	1.193	1.089#	1.089#
2A	SS	700	640	100	46.5	6	0.691	530	28.8	66.24	1.209	1.005#	1.483	1.373	1.005#
3A	SS	700	640	100	46.5	6	1.293	530	28.8	89.72	1.386	1.131	1.631	1.859	1.219
4A	SS	700	640	100	46.5	6	1.992	530	30.88	97.43	1.305	1.092	1.498	1.950	1.279
1B	SS	700	640	100	35	6	0.423	530	37.68	28.85	1.319	1.210#	1.210#	1.210#	1.210#
2B	SS	700	640	100	35	6	0.69	530	37.68	37.63	1.140	0.991#	1.200	0.991#	0.991#
3B	SS	700	640	100	35	6	1.292	530	37.68	56.67	1.104	0.867	1.467	1.480	0.958
4B	SS	700	640	100	35	6	1.994	530	30.88	72.52	1.400	1.113	1.736	2.092	1.354
Rankin & Long (1987) [16]															
1C	SS	700	640	100	53.5	6	0.423	530	27.84	62.74	1.282	1.141#	1.335	1.141#	1.141#
2C	SS	700	640	100	53.5	6	0.69	530	32.4	87.86	1.242	1.013	1.510	1.424	1.000#
3C	SS	700	640	100	53.5	6	1.288	530	32.4	124.14	1.502	1.264	1.733	2.012	1.329
4C	SS	700	640	100	53.5	6	1.993	530	27.84	125.94	1.474	1.258	1.599	2.202	1.454
Marzouk & Hussein (1991) [35]															
1	SS	1900	1870	250	120	19	1	460	32.16	475.5	1.194	1.101	1.380	1.431	0.979#
2	SS	1900	1870	250	120	19	1	460	67.16	511.52	1.105	1.005#	1.162	1.065	1.005#
Chana et al (1993) [36]															
1	SS	3000	2400	300	170	20	1	520	43.1	851	0.977	1.015	1.189	1.229	0.814
Ramdane (1996) [37]															
1	CC	1700	1372	150	98	10	0.58	550	88.2	224	0.899	0.855#	1.004	0.947	0.855#
2	CC	1700	1372	150	98	10	0.58	550	56.2	212	0.888	0.819#	1.105	1.122	0.819#
3	CC	1700	1372	150	98	10	0.58	550	26.9	169	1.011	0.905	1.126	1.293	0.790
4	CC	1700	1372	150	98	10	0.58	550	58.7	233	0.971	0.899#	1.197	1.207	0.899#

(*continued on next page*)

Table 1 (*continued*)

Test	Type	B or $\varnothing_B$ (mm)	S or $\varnothing_S$ (mm)	c or $\varnothing_c$ (mm)	d (mm)	$d_{g(est)}$ (mm)	ρ (%)	f_y (MPa)	$f_c^{'}$ (MPa)	P_t (kN)	$P_t/P_{p(R\&L)}$	$P_t/P^{**}_{p(R\&L)}$	$P_t/P_{p(EC2)}$	$P_t/P_{p(ACI)}$	$P_t/P_{p(MC10)}$
5	CC	1700	1372	150	98	12	0.58	550	54.4	190	0.799	0.741	1.001	1.022	0.735[#]
6	CC	1700	1372	150	98	10	0.58	550	101.6	233	0.927	0.886[#]	0.997	0.917	0.886[#]
12	CC	1700	1372	150	98	10	1.28	550	60.4	319	1.045	1.013	1.247	1.629	0.995
13	CC	1700	1372	150	98	10	1.28	550	43.6	297	1.145	1.092	1.294	1.785	1.091
14	CC	1700	1372	150	98	10	1.28	550	60.8	341	1.113	1.079	1.330	1.736	1.061
15	CC	1700	1372	150	98	12	1.28	550	68.4	276	0.849	0.829	1.035	1.324	0.809
16	CC	1700	1372	150	98	10	1.28	550	99.2	362	0.925	0.919	1.199	1.442	0.881
21	CC	1700	1372	150	98	20	1.28	650	41.9	286	1.124	1.062	1.262	1.754	1.071
22	CC	1700	1372	150	98	20	1.28	650	84.2	405	1.123	1.098	1.417	1.752	1.070
Ramdane (1996) [37]															
23	CC	1700	1372	150	100	20	0.87	650	56.4	341	1.237	1.167	1.497	1.752	1.070
24	CC	1700	1372	150	98	6	1.28	650	44.6	270	1.029	0.974	1.167	1.605	0.980
25	CC	1700	1372	150	100	10	1.27	650	32.9	244	1.055	0.987	1.130	1.641	1.003
26	CC	1700	1372	150	100	20	1.27	650	37.6	294	1.189	1.120	1.303	1.850	1.130
27	CC	1700	1372	150	102	20	1.03	650	33.7	227	0.993	0.924	1.081	1.467	0.897
Marzouk et al (1998) [38]															
NS1	SS	1700	1500	150	95	20	1.473	490	42	320	1.160	1.108	1.319	1.607	1.071
HS1	SS	1700	1500	150	95	20	0.491	490	67	178	1.024	0.963[#]	0.963[#]	0.963[#]	0.963[#]
HS2	SS	1700	1500	150	95	20	0.842	490	70	249	0.882	0.797[#]	1.043	0.969	0.797[#]
HS7	SS	1700	1500	150	95	20	1.193	490	74	356	1.025	0.997	1.303	1.347	0.898
HS3	SS	1700	1500	150	95	20	1.473	490	69	356	1.007	0.986	1.243	1.395	0.930
HS4	SS	1700	1500	150	90	20	2.37	490	66	418	1.157	1.146	1.383	1.805	1.199
NS2	SS	1700	1500	150	120	20	0.944	490	30	395	1.360	1.304	1.423	1.686	1.139
HS6	SS	1700	1500	150	120	20	0.944	490	70	489	1.102	1.102	1.328	1.367	0.923
HS8	SS	1700	1500	150	120	20	1.111	490	69	436	0.951	0.958	1.127	1.227	0.829
HS9	SS	1700	1500	150	120	20	1.611	490	74	543	1.042	1.073	1.212	1.476	0.997
HS10	SS	1700	1500	150	120	20	2.333	490	80	645	1.085	1.143	1.239	1.686	1.139
HS11	SS	1700	1500	150	70	20	0.952	490	70	196	1.152	1.026[#]	1.297	1.152	1.026[#]
HS12	SS	1700	1500	150	70	20	1.524	490	75	258	1.049	0.978	1.426	1.466	0.961
HS13	SS	1700	1500	150	70	20	2	490	68	267	1.065	1.002	1.392	1.593	1.045
HS14	SS	1700	1500	150	95	20	1.473	490	72	498	1.379	1.354	1.715	1.910	1.273
Broms (2000) [39]															
9	SS	2600	2000	250	150	25	0.5	510	26.9	408	0.939	0.901	1.101	0.993	0.717[#]
9a	SS	2600	2000	250	150	25	0.5	510	21	360	0.938	0.889	1.055	0.992	0.659
Li (2000) [40]															
P100	SS	925	725	200	100	20	0.98	488	39.4	330	1.061	1.000	1.319	1.328	0.874
P150	SS	1190	990	200	150	20	0.9	465	39.4	583	1.094	1.107	1.224	1.340	0.902
P200	SS	1450	1250	200	200	20	0.83	465	39.4	904	1.136	1.206	1.185	1.364	0.934
P300	SS	1975	1775	200	300	20	0.76	468	39.4	1381	0.946	1.075	0.992	1.111	0.779
P400	SS	1975	1775	300	400	20	0.76	433	39.4	2224	0.816	0.981	0.936	0.959	0.668
P500	SS	1975	1775	300	500	20	0.76	433	39.4	2681	0.689	0.861	0.785	0.809	0.571
Osman et al (2000) [41]															
6	SS	1900	1830	250	120	19	0.5	450	37.8	310.22	1.309	1.204[#]	1.204[#]	1.204[#]	1.204[#]
Salim & Sebastian (2003) [42]															
S1	SS	1200	920	150	113	19	1.06	500	50.4	369.4	1.039	1.016	1.194	1.326	0.893
S2	SS	1200	920	150	113	19	1.06	500	41.6	290.6	0.900	0.872	1.001	1.149	0.773
S3	SS	1200	920	150	113	19	1.06	500	44.8	402.2	1.200	1.167	1.352	1.532	1.031
S4	SS	1200	920	150	113	19	1.06	500	42.4	394.1	1.209	1.172	1.349	1.543	1.039
Oliveira et al (2004) [43]															
L1c	SS	1680	1500	120	107	16	1.09	749	59	318	1.005	0.962	1.129	1.291	0.878
Regan (2004) [44]															
1	SC	2000	1830	50	128	20	0.93	520	38	200	0.954	0.936	1.029	1.374	0.893
2	SC	2000	1830	75	128	20	0.93	520	43.2	260	1.020	1.007	0.893	1.468	0.975
3	SC	2000	1830	100	128	20	0.93	520	46.64	335	1.126	1.116	1.076	1.621	1.094
4	SC	2000	1830	170	128	20	0.93	520	41.68	380	1.034	1.019	1.137	1.488	1.036
5	SC	2000	1830	25	128	20	0.93	520	43.76	190	0.983	0.971	1.745	1.415	0.896
6	SC	2000	1830	50	128	20	0.93	520	30.24	190	1.016	0.985	1.185	1.463	0.951
Regan (2004) [44]															
7	SC	2000	1830	50	124	20	1.71	500	37.36	220	0.960	0.966	1.185	1.609	1.048
Chen & Li (2005) [45]															
SR1C1F0	SS	1000	840	150	70.5	12	0.59	482	16.9	103.9	1.118	0.925	1.280	1.232	0.808
SR1C2F0	SS	1000	840	150	70.5	12	0.59	482	34.4	123.8	1.023	0.901[#]	1.203	1.029	0.901[#]
SR2C1F0	SS	1000	840	150	70.5	12	1.31	482	16.9	146.1	1.287	1.108	1.380	1.732	1.136
SR2C2F0	SS	1000	840	150	70.5	12	1.31	482	34.4	225.7	1.394	1.243	1.682	1.875	1.230
Papanikolaou et al (2005) [46]															
P10-5	SC	750	750	150	80	10	0.54	500	29.1	164	1.355	1.237[#]	1.540	1.594	1.237[#]
P15-5	SC	750	750	150	130	10	0.54	500	32.1	310.4	1.155	1.099	1.218	1.452	1.002

(*continued on next page*)

Table 1 (*continued*)

Test	Type	B or $\varnothing_B$ (mm)	S or $\varnothing_S$ (mm)	c or $\varnothing_c$ (mm)	d (mm)	$d_{g(est)}$ (mm)	ρ (%)	f_y (MPa)	$f_c^{'}$ (MPa)	P_t (kN)	$P_t/P_{p(R\&L)}$	$P_t/P^{**}_{p(R\&L)}$	$P_t/P_{p(EC2)}$	$P_t/P_{p(ACI)}$	$P_t/P_{p(MC10)}$
P15-10	SC	750	750	150	130	10	1.08	500	30.6	355.2	1.138	1.118	1.124	1.702	1.175
P20-5	SC	750	750	150	180	10	0.54	500	30.3	459.1	1.078	1.084	1.021	1.354	0.935
P20-10	SC	750	750	150	180	10	1.08	500	32.1	501.1	0.961	1.003	0.868	1.436	0.992
P25-5	SC	750	750	150	230	10	0.54	500	32.5	587.5	0.905	0.954	0.841	1.137	0.799
P25-10	SC	750	750	150	230	10	1.08	500	29.4	635.7	0.866	0.941	0.754	1.294	0.909
Birkle and Dilger (2008) [28]															
1	CS	2200	2000	250	124	14	1.54	488	36.2	483	0.936	0.934	1.107	1.311	0.863
7	CS	3200	3000	300	190	20	1.3	531	35	825	0.845	0.897	0.942	1.135	0.756
10	CS	4000	3800	350	260	20	1.1	524	31.4	1046	0.692	0.768	0.783	0.892	0.600
Guandalini et al (2009) [47]															
PG-1	SS	3000	3000	260	210	16	1.5	573	27.6	1030.9	1.082	1.160	1.085	1.506	1.018
PG-2b	SS	3000	3000	260	210	16	0.25	552	40.5	439.9	1.054	0.999#	0.999#	0.999#	0.999#
PG-4	SS	3000	3000	260	210	4	0.25	541	32.2	409	1.018	0.956	0.952#	0.952#	0.952#
Guandalini et al (2009) [47]															
PG-5	SS	3000	3000	260	210	4	0.33	555	29.3	562.6	1.082	1.016	0.980#	0.980#	0.980#
PG-10	SS	3000	3000	260	210	16	0.33	577	28.5	539.5	1.007	0.946	0.930	0.906#	0.906#
PG-11	SS	3000	3000	260	210	16	0.75	570	31.5	772.4	0.903	0.941	0.980	1.056	0.714
PG-3	SS	6000	6000	520	456	16	0.33	520	32.4	2163.8	0.922	0.936	0.922	0.848#	0.848#
PG-6	SS	1500	1500	130	96	16	1.5	526	34.7	233.1	0.993	0.939	1.046	1.382	0.929
PG-7	SS	1500	1500	130	100	16	0.75	550	34.7	242.1	1.157	1.062	1.277	1.354	0.912
PG-8	SS	1500	1500	130	117	16	0.28	525	34.7	139.7	1.031	0.964#	0.964#	0.964#	0.964#
PG-9	SS	1500	1500	130	117	16	0.22	525	34.7	115.5	1.064	1.009#	1.009#	1.009#	1.009#
Vollum et al (2010) [48]															
1	SS	3000	2743	270	174	20	1.28	567	24	614	0.919	0.938	0.958	1.229	0.820
Yang et al (2010) [49]															
S1-U	SS	2300	2000	225	109	18	1.18	454	37.2	301	0.784	0.757	0.958	1.027	0.675
S1-B	SS	2300	2000	225	109	18	2.15	445	37.2	317	0.710	0.708	0.826	1.082	0.711
Rizk et al (2011) [50]															
HSS1	SS	2650	2505	400	267.5	19	0.5	460	76	1722	1.277	1.228	1.225#	1.225#	1.225#
HSS3	SS	2650	2505	400	262.5	19	1.42	460	65	2090	0.823	0.966	1.067	1.129	0.754
NSS1	SS	2650	2505	400	312.5	19	1.58	460	40	2234	0.852	1.013	1.002	1.202	0.811
HSS4	SS	2650	2505	400	312.5	19	1.58	460	60	2513	0.783	0.950	0.985	1.104	0.745
Sagaseta et al (2011) [51]															
PT22	SS	3000	2800	260	196	16	0.82	552	67	989	0.856	0.921	1.053	1.024	0.734#
PT31	SS	3000	2800	260	212	16	1.48	540	66.3	1433	0.961	1.080	1.115	1.332	0.901
Ferreira et al (2014) [52]															
S5	SS	2500	2100	300	143	24	1.48	540	50.5	779	0.945	0.977	1.199	1.311	0.861
Einpaul et al (2016) [53]															
PE10	CC	3010	3010	83	210	16	0.77	538	40.4	530	0.952	1.012	0.953	1.307	0.809
PE11	CC	3010	3010	166	215	16	0.75	538	37.5	712	1.003	1.067	0.955	1.369	0.850
PE9	CC	3010	3010	330	218	16	0.74	538	44.1	935	0.836	0.897	1.009	1.137	0.740#
PE12	CC	3010	3010	660	212	16	0.76	538	37.6	1206	0.933	0.877	1.107	1.026	0.871#
PE6	CC	3010	3010	83	215	16	1.46	542	38.4	656	0.989	1.088	1.200	1.594	0.990
PE7	CC	3010	3010	166	213	16	1.47	542	42.5	871	0.989	1.092	0.909	1.596	0.990
PE8	CC	3010	3010	330	214	16	1.47	542	42	1091	0.864	0.954	0.978	1.395	0.866
PE5	CC	3010	3010	660	210	16	1.5	542	36.7	1476	0.793	0.867	1.102	1.286	0.797
PE4	SS	1700	1460	260	197	16	1.59	517	35.1	985	0.991	1.071	1.034	1.399	0.942
PV1	SS	3000	2760	260	210	16	1.5	709	31.1	978	0.967	1.032	0.989	1.346	0.910
PE3	SS	3900	3660	260	204	16	1.54	517	34.2	961	0.939	1.018	0.974	1.315	0.887

SS = Square slab, square column.
CC = Circular slab, circular column.
SC = Square slab, circular column.
CS = Circular slab, square column.
# = yield line predicted failure.

$$\text{Shear mode:} \quad P_t = D_{fs} P_{p(R\&L)} \tag{33}$$

where:

$$D_{fs} = 0.92(d/200)^{-0.18} \tag{34}$$

As can be seen from Fig. 6, incorporation of the slab depth factors given in Eqs. (32) & (34) into the Rankin and Long [16] prediction method resulted in a significant improvement in the correlation with the test results, with the trends of $P_t/P^{*}_{p(R\&L)}$ against the relative slab effective depth factor $d/200$, for both the flexural and shear predicted modes of punching failure, closely approaching the desired straight lines parallel to the x-axis.

4.2. Refinements to shear punching strength material parameters

On further examination of the trend of the $P_t/P^{*}_{p(R\&L)}$ ratio with different material parameters, it was found that there was no significant trend of the predicted results with the familiar arrangement of the reinforcement index $\rho f_y/f_c^{'}$, as shown in Fig. 7. However, as can be seen from Fig. 8, when $P_t/P^{*}_{p(R\&L)}$ was plotted against the material parameters in the form $\rho f_c^{'}/f_y$, there existed the following positively identifiable trend of the predicted results for the ***shear*** mode of punching failure

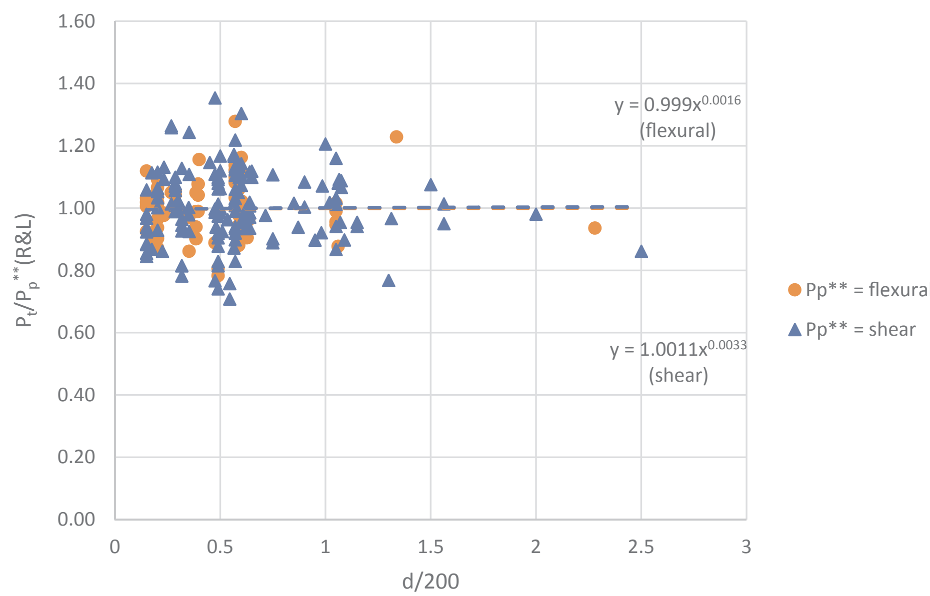

Fig. 9. Correlation of proposed method predictions (incorporating slab depth factors and refined shear strength parameters) with relative slab effective depth.

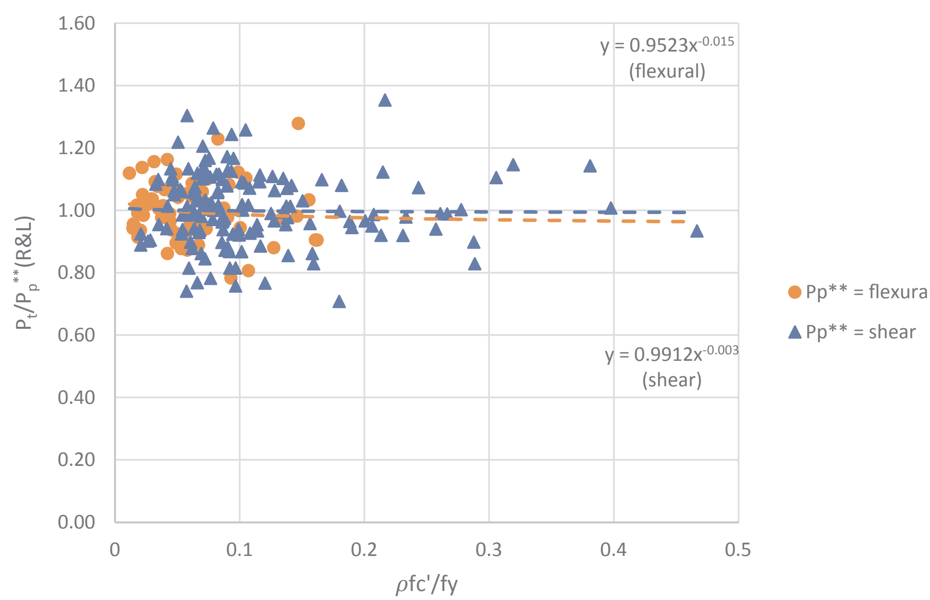

Fig. 10. Correlation of proposed method predictions (incorporating slab depth factors and refined shear strength parameters) with material parameters.

Table 2
Summary of correlation of various methods (including yield line predicted failures#).

Method of prediction	Mean P_t/P_p	Coefficient of variation	R^2 value of linear regression
$P^{**}_{P(R\&L)}$ – Rankin & Long – proposed (2018)	1.018	0.112	0.9822
$P_{p(R\&L)}$ – Rankin & Long (1987) [16]	1.100	0.146	0.9545
$P_{p(MC10)}$ – Model Code 2010 [13,14]	0.988	0.151	0.9348
$P_{p(EC2)}$ – Eurocode 2 [11]	1.196	0.169	0.9649
$P_{p(ACI)}$ – ACI 318-14 [12]	1.384	0.201	0.9251

Yield line predicted failures are indicated in Table 1.

only:

$$P_t/P^{*}_{p(R\&L)} = 0.89\left(\frac{\rho f_c^{'}}{f_y}\right)^{-0.05} \quad (35)$$

This regression analysis indicated that refinements to the effects of concrete strength, reinforcement percentage and reinforcement yield strength, in conjunction with the slab depth factor for the shear mode of punching failure (Eq. (34)), could be made to the Rankin and Long [16] shear punching prediction equation to give further improved correlation with test results.

5. Summary of proposed prediction method

Incorporating the proposed slab depth factors and refinements to the shear punching strength parameters into the previous Rankin & Long method [16], the proposed predicted punching strength is given by the lesser of the flexural (P^{**}_{vf}) and shear (P^{**}_{vs})punching strength

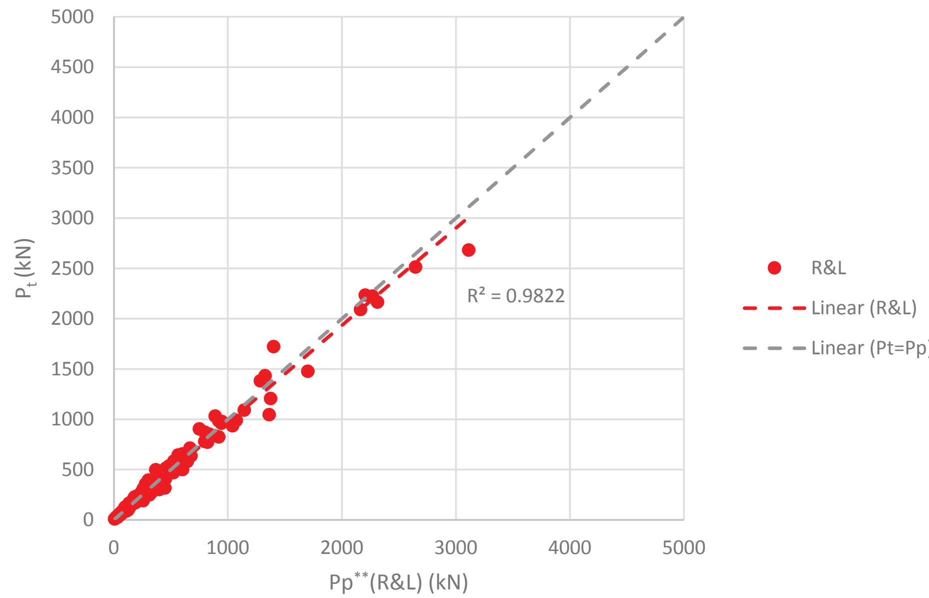

Fig. 11. Overall correlation of 217 test results with proposed method predictions (incorporating slab depth factors and refined shear strength parameters).

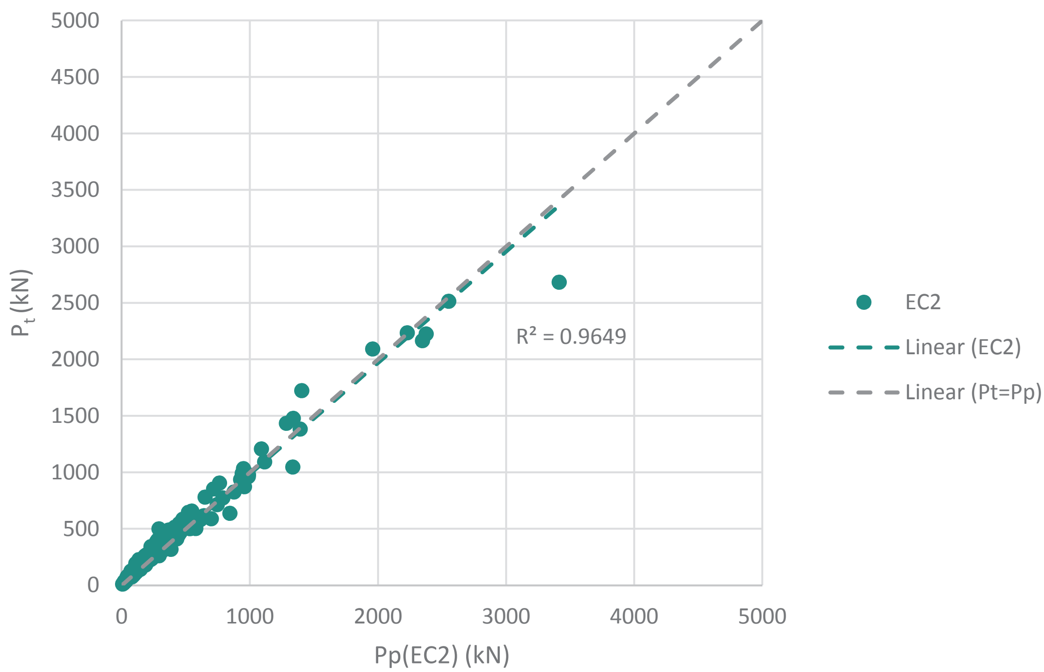

Fig. 12. Overall correlation of 217 test results with Eurocode 2 [11] predictions.

predictions given by the following equations:

(a) Flexural Punching Strength

The flexural punching strength of conventional slab-column specimens with square or circular columns can be predicted from Eq. (36):

$$P_{vf}^{**} = 1.07\left(\frac{200}{d}\right)^{0.1}\left[k_{yl} - \left(k_{yl} - \frac{k_b}{r_f}\right)\frac{M_u}{M_{(bal)}}\right]M_u \le 1.07\left(\frac{200}{d}\right)^{0.1}\frac{k_b}{r_f}M_{(bal)} \tag{36}$$

where:

(i) the moment factor k_{yl} for a conventional square or circular slab-column specimen can be calculated using yield line theory

(ii) the elastic moment factor k_b can be calculated using Eq. (25), in which, for a circular column, a value of 'c' can be used assuming a square column of equivalent perimeter.

(iii) $r_f = 1.15$ *for square columns*

(iv) $r_f = 1.0$ *for circular columns*

(b) Shear Punching Strength

The shear punching strength of conventional slab-column specimens with square or circular columns can be predicted from Eqs. (37) or (38) respectively:

Square columns:

$$P_{vs}^{**} = 1.37f_c^{'0.45}(c + d)d(100\rho)^{0.2}f_y^{0.05}\left(\frac{200}{d}\right)^{0.18} \tag{37}$$

Circular columns:

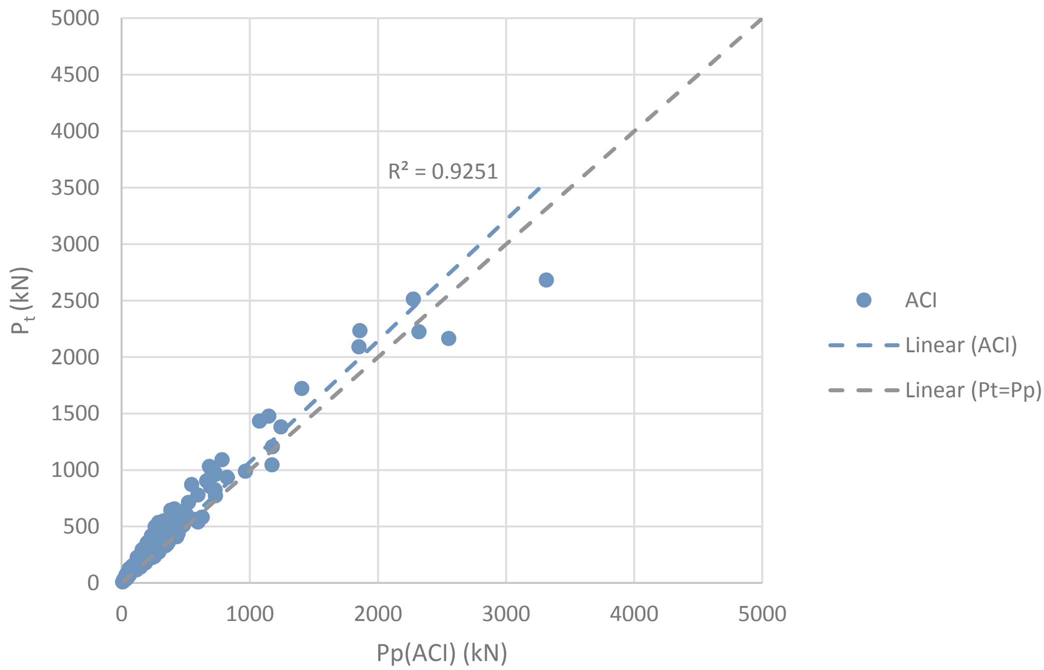

Fig. 13. Overall correlation of 217 test results with ACI 318-14 [12] predictions.

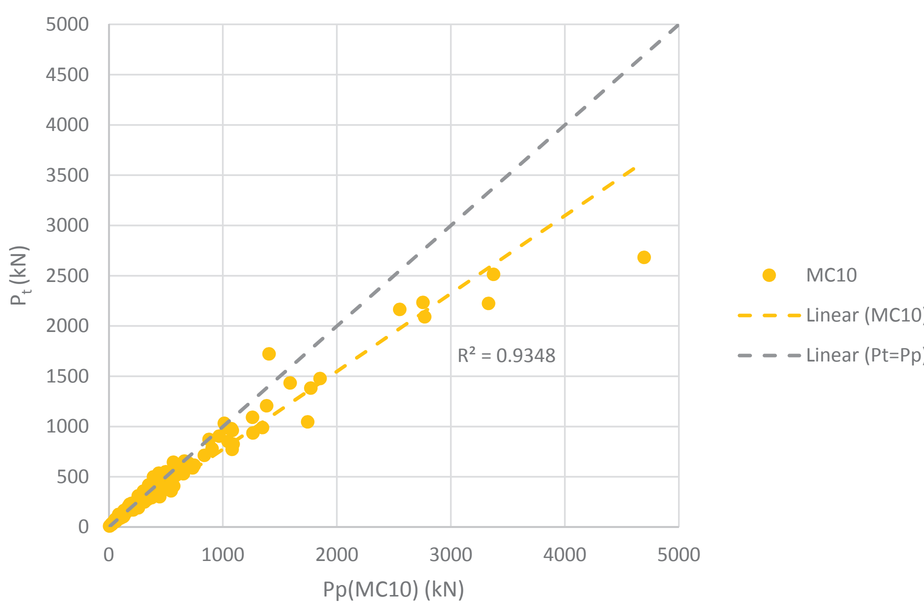

Fig. 14. Overall correlation of 217 test results with *fib* Model Code 2010 [13,14] predictions (Level of Approximation II).

Table 3
Summary of correlation of various methods (†excluding yield line predicted failures#).

Method of prediction	Number of yield line predicted failures#	Mean P_t/P_p^t	Coefficient of variation	R^2 value of linear regression
$P^{**}_{P(R\&L)}$ – Rankin & Long – proposed (2018)	41	1.010	0.108	0.9809
$P_{p(MC10)}$ – Model Code 2010 [13,14]	71	0.984	0.157	0.9356
$P_{p(EC2)}$ – Eurocode 2 [11]	19	1.207	0.169	0.9658
$P_{p(ACI)}$ – ACI 318-14 [12]	28	1.430	0.185	0.9233

Yield line predicted failures are indicated in Table 1.

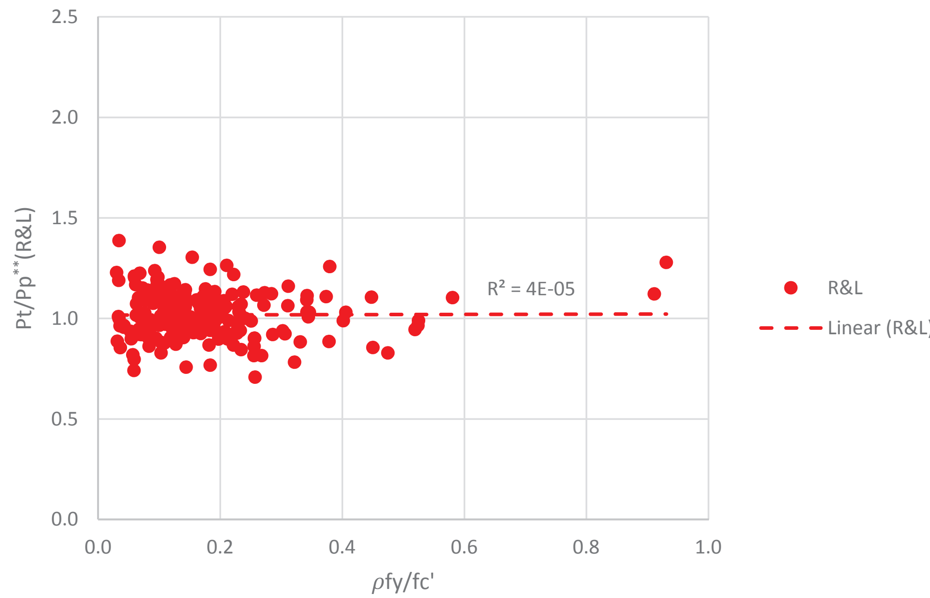

Fig. 15. Correlation of proposed method predictions with reinforcement index.

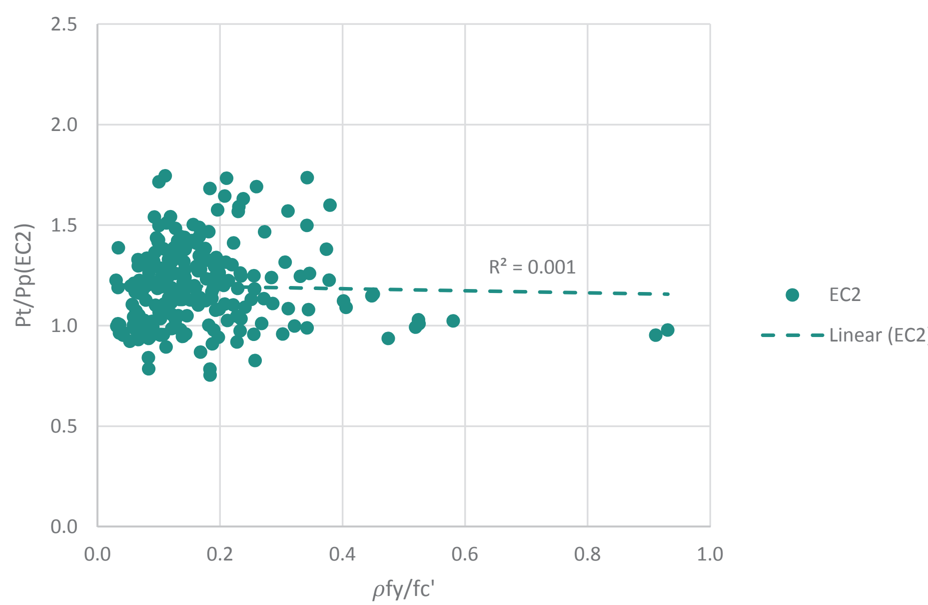

Fig. 16. Correlation of Eurocode 2 [11] predictions with reinforcement index.

$$P_{vs}^{**} = 1.25 f_c^{'0.45} (\varnothing_c + d) d (100\rho)^{0.2} f_y^{0.05} \left(\frac{200}{d}\right)^{0.18} \tag{38}$$

6. Correlation of prediction methods with test results

A wide range of punching test results reported in the literature [7–10,16,28–53] from Elstner & Hognestad's [7] original punching tests in 1956, to Einpaul et al.'s [53] punching tests some 60 years later in 2016, was used in the correlation of predictions using the aforementioned methods. The test results were limited to conventional test models extending to the nominal line of contraflexure, taken to be at 0.22L from the column centre. Thus, the strengths of these slabs were not enhanced by the development of compressive membrane action due to the inherent restraining effect which has been observed in larger slabs extending beyond the nominal line of contraflexure (Rankin and Long [54]). Most published data available to the authors was included in the analysis, with only some tests, where insufficient data was reported, excluded. Results from tests on isotropically reinforced, square and circular, conventional models with either square or circular columns were included in over 200 of the test results compiled by Thompson [55]. Details of each set of test results are given, in chronological order, in Table 1.

The correlation of the proposed method with the relative slab effective depth parameter is shown in Fig. 9 and the correlation with the material parameters is shown in Fig. 10. It can be seen that the trends of $P_t/P_{P(R\&L)}^{**}$ against the relative slab effective depth $d/200$ and the material parameters $(\rho f_c^{'}/f_y)$ for both the flexural and shear modes of punching failure closely approach the desired straight lines parallel to the x-axis.

The correlation coefficients obtained using the proposed method, the previous method by Rankin & Long [16], the two major structural code methods (with safety factors removed), Eurocode 2 [11] and ACI 318-14 [12], and the *fib* Model Code (2010) [13,14], are given in Table 2.

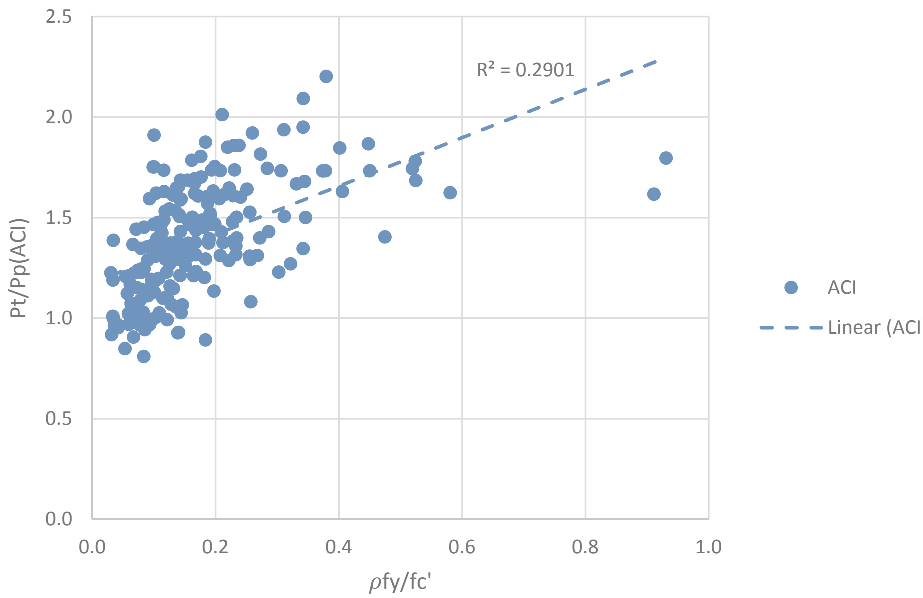

Fig. 17. Correlation of ACI 318-14 [12] predictions with reinforcement index.

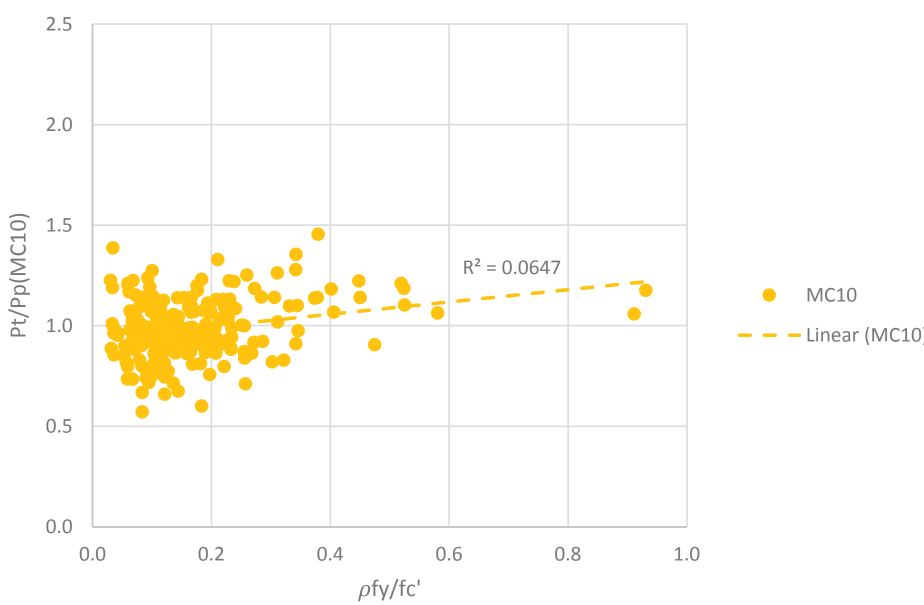

Fig. 18. Correlation of *fib* Model Code 2010 [13,14] predictions with reinforcement index.

This comparison includes all of the 217 results, although some failures were predicted as yield line failures as an upper bound to each method. In Table 2, the methods are ranked in order of consistency, as defined by the lowest coefficient of variation being the most consistent method. The coefficient of variation of 0.112 achieved using the proposed method represents a significant improvement over the authors' previous method which gave a coefficient of variation of 0.146 for the same set of results.

The next best coefficient of variation was 0.151, achieved using the *fib* Model Code (2010) [13,14]. The Eurocode 2 [11] method gave a coefficient of variation of 0.169 and the highest coefficient of variation of 0.201 was given by the ACI 318-14 [12] method.

Regression analysis of the correlations achieved by each method are shown graphically in Figs. 11–14, in which the values of P_t are plotted against the values of P_p for the proposed method, the Eurocode 2 method [11], the ACI method [12] and the *fib* Model Code (2010) method [13,14] respectively.

This analysis revealed a different evaluation of consistency from that given by the coefficient of variation. It can be seen that the proposed method provides a graphically better correlation with the test results than the other methods. It is of note that the Eurocode 2 method also provides a graphically good correlation with the test results. However, the ACI method [12] shows a greater disparity with the test results and the *fib* Model Code method [13,14], although providing the second best coefficient of variation, also shows a greater disparity and, of even more concern, an unconservative trend of predictions with increased load capacities.

From Figs. 11–14, the R^2 values for each data set indicate the proposed method to have the highest R^2 value (0.9822), the Eurocode 2 method [11] to have the next highest R^2 value (0.9649), the *fib* Model code 2010 method [13,14] to have the next highest R^2 value (0.9348) and the ACI method [12] to have the lowest R^2 value (0.9251).

Thus, if the R^2 values were to be used as the correlation indicator instead of the coefficient of variation, the proposed method would provide the best correlation and the Eurocode 2 method [11] would provide better correlation than both the *fib* Model Code 2010 [13,14] and the ACI method [12].

A summary of the correlation obtained by each method, excluding predicted yield line failures, is given in Table 3. Interestingly, use of the *fib* Model Code 2010 [13,14] method results in the greatest number of yield line predicted failures and use of the Eurocode 2 method [11] results in the least number of yield line predicted failures. It can also be seen that, although the key indicator correlation coefficients of mean, coefficient of variation and R^2 values change slightly from the values given in Table 2 (which includes yield line predicted failures), the same general correlation trends still apply.

Use of the proposed method results in significantly improved correlation with a wide variety of test results produced from various sources over the 60 year period (1956–2016).

The mean ratio of $P_t/P^{**}_{p(R\&L)}$ using the proposed method was approximately 1.0 for the 217 tests analysed. This value could easily be made more conservative by incorporating appropriate safety factors in each of the flexural and shear punching mode prediction formulae.

The ratios of P_t/P_p are plotted against the reinforcement index $\rho f_y/f_c^{'}$ for each method in Figs. 15–18. It can be seen that the trend of the ratio with the reinforcement index closely approaches the desired horizontal line for both the proposed method and the Eurocode 2 method [11]. However, the trend given by the ACI method [12] shows generally increasing conservatism as the ratio $\rho f_y/f_c^{'}$ increases. This is also the case, although to a lesser extent, for the *fib* Model Code method [13,14].

Compared to the previous overall correlation obtained by Rankin and Long [16], for a punching test load range of up to approximately 600 kN, the correlation given by the proposed method covers a much larger punching test load range of up to approximately 3000 kN, including square slabs, square columns, circular slabs, circular columns, and all combinations, - thereby representing a significantly improved correlation with a much wider range of punching strengths, geometric variables and material parameters.

7. Conclusions

Slab depth has a significant effect on the punching strength of reinforced concrete slabs.

The effect of slab depth can be empirically related to a normalised slab depth of 200 mm.

The effect of slab depth can by successfully taken into account in predicting the flexural and shear punching strengths of reinforced concrete slabs.

Significantly improved correlation with a wide range of test results from various sources (217 test results) produced over the sixty year period (1956–2016), was achieved by incorporating slab depth factors for both the flexural and shear modes of punching failure and refinements to the effects of concrete strength, reinforcement percentage and reinforcement yield strength, for the shear mode of punching failure, in a modified two-phase approach to predicting the punching strength of conventional slab-column specimens.

The proposed method provides significantly more accurate and consistent correlation with the 217 test results analysed than the methods of Eurocode 2 (2004), ACI (2014) and the *fib* Model Code 2010.

8. Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Turner CAP. Discussion of ‘Reinforced concrete warehouse for North-West Knitting Co., Minneapolis, Minnesota’. Eng News 1905;54(15):383–4.
[2] Maillart R. Konstruktion zur Raumabdeckung (Cover construction). Swiss Patent No. 46928. Zurich, Switzerland: Maillart & Cie; 1909.
[3] Lord AR. A test of a flat slab floor in a reinforced concrete building. Natl Assoc Cement Users (ACI) 1910;7:156–79.
[4] Sozen MA, Seiss CP. Investigation of multiple panel reinforced concrete floor slabs. J Am Concr Instit, Proc 1963;60(8):999–1027.
[5] Faulkes KA. The design of flat slab structures – an historical survey. UNICIV Report No. R-129. University of New South Wales; 1974 April, 42 pp.
[6] Gasparini DA. Contributions of C.A.P. Turner to development of reinforced concrete flat slabs 1905–1909. ASCE J Struct Eng 2002;128(10):1243–52.
[7] Elstner RC, Hognestad E. Shearing strength of reinforced concrete slabs. J Am Concr Instit, Proc 1956;53(1):29–58.
[8] Base GD. Some tests on the punching shear strength of reinforced concrete slabs. Cement and Concrete Association, Slough, Technical Report TRA/321; 1959.
[9] Kinnunen S, Nylander H. Punching of concrete slabs without shear reinforcement. Stockholm, Sweden: Transactions of the Royal Institute of Technology; 1960. p. 112.
[10] Moe J. Shearing strength of reinforced concrete slabs and footings under concentrated loads. Development Department Bulletin D47, Portland Cement Association, Skokie, Illinois; April 1961, 130pp.
[11] Eurocode 2. Design of Concrete Structures—Part 1-1: General Rules and Rules for Buildings, CEN, BS EN 1992-1-1 + A1 (2014), Brussels, Belgium, 225 pp, + UK National Annex + A2 (2014); 2004 24pp.
[12] ACI Committee 318. Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary. Farmington Hills, Michigan: American Concrete Institute; 2014.
[13] Fédération Internationale du Béton (fib). Model Code 2010, final draft, Vol.1, fib Bulletin 65, Lausanne, Switzerland; 2012. 350pp.
[14] Fédération Internationale du Béton (fib). Model Code 2010, final draft, Vol.2, fib Bulletin 66, Lausanne, Switzerland; 2012, 370pp.
[15] Long AE. A two-phase approach to the prediction of the punching strength of slabs. ACI Struct J, Proc 1975;72(2):37–45.
[16] Rankin GIB, Long AE. Predicting the punching strength of conventional slab-column specimens. In: ICE proceedings -structures and buildings, No. 82; 1987, April. p. 327–46.
[17] Regan PE, Braestrup MW. Punching shear in reinforced concrete – a state-of-art report’, Issue 168 of Bulletin d'information/Comité euro-international du béton, Euro-International Committee for Concrete; 1985. 232 pp.
[18] ACI SP-232, Polak, M.A. (Ed). Punching shear in reinforced concrete slabs. Farmington Hills, MI: American Concrete Institute; 2005, 302 pp
[19] ACI-SP-315/fib Bulletin 81. Punching Shear of Structural Concrete Slabs. In: Proceedings of ACI-fib Symposium held at Philadelphia, PA, USA, 25 October 2016, American Concrete Institute, Farmington Hills, MI, and Fédération Internationale du Béton, CH-1015, Lausanne, Switzerland, 2017, 378 pp
[20] Rankin GIB. Punching failure and compressive membrane action in reinforced concrete slabs PhD Thesis Department of Civil Engineering, Queen’s University Belfast; 1982 p. 334.
[21] Muttoni A. Punching shear strength of reinforced concrete slabs without transverse reinforcement. ACI Struct J 2008;105(4):440–50.
[22] Regan PE. Behaviour of reinforced concrete flat slabs. London: Construction Industry Research and Information Association, Report 89; 1981, 90 pp.
[23] BS 8110. Structural use of concrete – Part 1: Code of practice for design and construction. London: British Standards Institution; 1985. p. 162.
[24] ACI Committee 318. Building Code Requirements for Structural Concrete (ACI 318-83) and Commentary. Farmington Hills, Michigan: American Concrete Institute; 1983.
[25] Whitney SC. Design of reinforced concrete members under flexure or combined flexure and direct compression. J Am Concr Instit, Proc 1937;33:483–98.
[26] Kirkpatrick J, Rankin GIB, Long AE. Strength evaluation of M-beam bridge deck slabs. Struct Eng 1984;62B(3):60–8.
[27] Mitchell D, Cook WD, Dilger W. Effects of size, geometry and material properties on punching shear resistance. ACI SP-232 Punching Shear in Reinforced Concrete Slabs. Farmington Hills, MI: American Concrete Institute; 2005. p. 39–56.
[28] Birkle G, Dilger WH. Influence of slab thickness on punching shear strength. ACI Struct J 2008;105(2):180–8.
[29] Taylor R, Hayes B. Some tests on the effect of edge restraint on punching shear in reinforced concrete slabs. Mag Concr Res 1965;17(50):39–44.
[30] Criswell ME. Static and dynamic response of reinforced concrete slab-column connections, Shear in reinforced concrete. ACI, Detroit, SP 42, Vol. 2, Part 4; 1974. p. 721–46.
[31] Dragosavic M, Van Den Beukel A. Punching shear. Heron, Delft, Vol. 20, No. 2; 1974. 48 pp.
[32] Regan PE. Design of reinforced concrete slabs. Construction industry research and information association project record 220, London; 1978.
[33] Regan PE. Symmetric punching of reinforced concrete slabs. Mag Concr Res 1986;36(136):115–28.
[34] Narayanan R, Darwish IYS. Punching shear tests on steel-fibre reinforced microconcrete slabs. Mag Concr Res 1987;39(138):42–50.
[35] Marzouk H, Hussein A. Experimental investigation on the behavior of high strength concrete slabs. ACI Struct J 1991;88(S72):701–13.
[36] Chana PS. A prefabricated shear reinforcment system for flat slabs. Proc - Instit Civ Eng: Struct Build 1993;99(August):345–58.
[37] Ramdane KE. Punching shear of high performance concrete slabs. In: 4th international symposium on utilization of high-strength/high-performance concrete, Paris; 1996. p. 1015–26.
[38] Marzouk H, Emam M, Hilal MS. Effect of high-strength concrete slab on the behavior of slab-column connections. ACI Struct J 1998;95(S21):227–37.

[39] Broms CE. Elimination of flat plate punching failure mode. ACI Struct J 2000;97(S11):94–101.

[40] Li KKL. The influence of size on punching shear strength of concrete slabs MEng Thesis Department of civil engineering and applied mechanicsMontreal, Quebec: McGill University; 2000. p. 78.

[41] Osman M, Marzouk H, Helmy S. Behaviour of high-strength lightweight concrete slabs under punching loads. ACI Struct J 2000;97(S53):492–8.

[42] Salim W, Sebastian WM. Punching shear failure in reinforced concrete slabs with compressive membrane action. ACI Struct J 2003;100(S50):471–9.

[43] Oliveira DRC, Regan PE, Melo GSSA. Punching resistance of RC slabs with rectangular columns. Mag Concr Res 2004;56(3):123–38.

[44] Regan PE. Punching of slabs under highly concentrated loads. Proc - Instit Civ Eng: Struct Build 2004;157(2):165–71.

[45] Chen CC, Li CY. Punching shear strength of reinforced concrete slabs strengthened with glass fiber-reinforced polymer laminates. ACI Struct J 2005;102(S54):535–42.

[46] Papanikolaou KV, Tegos IA, Kappos AJ. Punching shear testing of reinforced concrete slabs and design implications. Mag Concr Res 2005;57(3):167–77.

[47] Guandalini S, Burdet OL, Muttoni A. Punching tests of slabs with low reinforcement ratios. ACI Struct J 2009;106(S10):87–95.

[48] Vollum RL, Abdel-Fattah T, Eder M, Elghazouli AY. Design of ACI-type punching shear reinforcement to Eurocode 2. Mag Concr Res 2010;62(1):3–16.

[49] Yang J-M, Yoon Y-S, Cook WD, Mitchell D. Punching shear behaviour of two-way slabs reinforced with high-strength steel. ACI Struct J 2010;107(S47):468–75.

[50] Rizk E, Marzouk H, Hussein A. Punching shear of thick plates with and without shear reinforcement. ACI Struct J 2011;108(S55):581–91.

[51] Sagaseta J, Muttoni A, Ruiz MF, Tassinari L. Non-axis-symmetrical punching shear around internal columns of RC slabs without transverse reinforcement. Mag Concr Res 2011;63(6):441–57.

[52] Ferreira MP, Melo GS, Regan PE, Vollum RL. Punching of reinforced concrete flat slabs with double-headed shear reinforcement. ACI Struct J 2014;111(2):1–12.

[53] Einpaul J, Bujnak J, Ruiz MF, Muttoni A. Study on influence of column size and slab slenderness on punching strength. ACI Struct J January – February 2016;113:135–45.

[54] Rankin GIB, Long AE. Predicting the enhanced punching strength of interior slab-column connections. In: ICE proceedings -structures and buildings, No. 82; 1987, December. p. 1165–86.

[55] Thompson DRW. Punching failure of interior slab – column connections. Project Dissertation. School of Planning Architecture and Civil Engineering, Queen's University of Belfast; 2014. 149pp.