Journal of Curriculum Studies

ISSN: 0022-0272 (Print) 1366-5839 (Online) Journal homepage: http://www.tandfonline.com/loi/tcus20

# The role that mathematics plays in college- and career-readiness: evidence from PISA

Leland S. Cogan, William H. Schmidt & Siwen Guo

**To cite this article:** Leland S. Cogan, William H. Schmidt & Siwen Guo (2018): The role that mathematics plays in college- and career-readiness: evidence from PISA, Journal of Curriculum Studies, DOI: 10.1080/00220272.2018.1533998

**To link to this article:** https://doi.org/10.1080/00220272.2018.1533998

Published online: 15 Oct 2018.

JOURNAL OF CURRICULUM STUDIES
https://doi.org/10.1080/00220272.2018.1533998

Routledge
Taylor & Francis Group

Check for updates

# The role that mathematics plays in college- and career-readiness: evidence from PISA

Leland S. Cogan, William H. Schmidt and Siwen Guo

Michigan State University, East Lansing, MI, USA

**ABSTRACT**

Many studies have found a strong relationship between the mathematics students study in school and their performance on an academic or school mathematics assessment but not on an assessment of mathematics literacy (ML). With many countries, like the USA, placing emphasis on finishing secondary education being mathematically literate and prepared for college or career, this raises the question about the relationship between the mathematics studied in school and any ML students may have. The Programme for International Student Assessment (PISA) ML assessment is embedded in real-world contexts that provide an important window on how ready students are to tackle the situations and problems that await them whether they intend to pursue further education beyond high school or intend to go directly into the labour force. In this paper, we draw upon the PISA 2012 data to investigate the extent to which the cumulative exposure to rigorous mathematics content, such as that embedded in college- and career-ready standards, is associated with ML as assessed in PISA. Results reveal that both exposure to rigorous school mathematics and experiencing the instruction of this mathematics through real-world applications are significantly related to all the real-world contextualized PISA ML scores.

**KEYWORDS**

Mathematics literacy; mathematics content; opportunity to learn; career ready; college ready

In the 2010 reauthorization of the *Elementary and Secondary Education Act (ESEA)*, the U.S. Department of Education adopted the idea of college- and career-ready standards (U.S. Department of Education, 2010). The perspective that elementary and secondary schooling needs to prepare students not only for college but for careers that may not require 4 years of college, or for that matter any additional formal education, has been around for a while (Kliebard & Franklin, 2003; Lappan & Wanko, 2003; Smith, 1999). However, one impact of the *ESEA* reauthorization is to have increased the attention given to mathematics and how K-12 schooling contributes to students' preparedness. Articulating the rigorous content needed to prepare students in an increasingly technological society to be both college- and career-ready led in the USA to the recent development and adoption of the Common Core State Standards in Mathematics (CCSSM) and similar standards by the vast majority of US states (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The implicit concept of mathematics literacy (ML) in these documents is the sense that students need to complete their secondary education mathematically prepared and ready to engage in any career path they choose whether this entails further formal education or not.

Historically the idea that ML is a goal of education has always been embraced. Debate, however, has swirled around what mathematics is needed to be taught and to whom: is vocational

**CONTACT** William H. Schmidt bschmidt@msu.edu Counseling, Educational Psychology, and Special EducationMichigan State University, 620 Farm Lane, Rm 238, East Lansing, MI 48824, USA

mathematics sufficient for all students; do college bound students need more formal mathematics; and should mathematics instruction always be grounded in the real world first or even exclusively (Blum & Niss, 1991; Donoghue, 2003; Kliebard & Franklin, 2003). Given the rather rigorous content outlined in the US' CCSSM, the stance would appear to be that all students need a good measure of rigorous, formal mathematics in order to be literate, prepared for whatever career path students choose upon completion of their secondary education whether they choose to enter immediately the work force; to enter a technical, trade or vocational career path, or to continue their formal education at a college or university.

The Organization for Economic Cooperation and Development's (OECD's) Programme for International Student Assessment (PISA) assesses 15-year-old students' literacy in mathematics, reading and science every 3 years, with a particular focus on one of these areas each cycle. The PISAs are not developed in conjunction with any system's standards but are intended to reflect the extent to which students are able to apply what they know in common, real-world situations encountered by 'constructive, engaged and reflective citizen(s)' (p. 25, OECD, 2013a; Stacey & Turner, 2015). More specifically, the framework for PISA 2012, the most recent PISA to focus on mathematics, defined ML as 'an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts' which include personal, occupational, societal and scientific situations (OECD, 2013a, p. 25; Stacey, 2015).

The mathematics education standards of many systems include rigorous content for all students yet the relationship between schooling—the academic-oriented learning opportunities provided in schools—and any measurement of their ML such as in PISA is not clear. The inclusion in PISA 2012 of several approaches to measure students' opportunities to learn (OTL) mathematics enables an examination of the relationship between schooling and ML (Cogan & Schmidt, 2015). Although this does not address the question of the relationship between rigorous standards and ML, PISA 2012 does provide an opportunity to address the question: to what extent is the cumulative exposure to rigorous mathematics content—such as that called for in rigorous college- and career- ready standards—associated with ML as assessed in PISA?

## Background

The common sense notion that the exposure students experience related to a topic is related to what they learn about that topic has played an important role in education research for over a century. This psychological perspective on OTL is an idea that was evident in the writings of psychologists Edward Thorndike and William James as far back as around the turn of the last century (Cogan & Schmidt, in press; James, 1983; Thorndike, 1913).

OTL describes a focal aspect of schooling—the coverage of content in the classroom that is directly related to what a student learns. Learning theories have a long and rich heritage in the psychological and education literatures but Carroll's (1963) model was among the first to specifically address classroom learning. It defines classroom learning as an interaction between the instruction provided and what is needed by the student in order to learn, e.g. aptitude, ability and perseverance. Bloom and his colleagues made use of Carroll's theory in the OTL measures developed and used in the First International Mathematics Study which became, in turn, the example for relating OTL to learning for much education research over the past 50 years especially in the international comparative literature (Cogan & Schmidt, 2015). Typically teachers are the ones providing the information on OTL; however, obtaining such information from students has the benefit of reflecting the student's perception which is likely affected by the student factors included in Carroll's model.

Nonetheless, the focus of most of the research that has investigated the relationship of OTL to what students know has been conducted with measures of students' learning of the formal/academic mathematics taught in school. The emphasis of this paper on literacy and the *application* of mathematics knowledge to real-world, everyday situations raises the question as to what specifically occurs in schools that might support the development of ML (de Lange, 2003)? This uncertainty about what type of OTL in schooling may be related to the development of literacy is related to at least two issues. One has to do with the nature of learning and the other has to do with the nature of instruction.

Research on learning has found that children develop competencies in number, space, shape, pattern and measurement even before they begin formal schooling. How these everyday competencies found in preschool children are related to and further developed by the academic experiences in school continues to be a focus of research (Gelman & Greeno, 1989; Ginsburg, Cannon, Eisenband, & Pappas, 2008). *How Students Learn* focuses on three 'fundamental and well-established principles' that were highlighted in the earlier National Research Council (NRC) publication, *How People Learn* (National Research Council, 2005, p. 1). Applying one of these principles to the development of 'mathematics proficiency' requires students to have 'a foundation of factual knowledge (procedural fluency), tied to a conceptual framework (conceptual understanding), and organized in a way to facilitate retrieval and problem solving (strategic competence)' (Fuson, Kalchman, & Bransford, 2005, p. 218). Although all three of these competencies should be outcomes of schooling, ML would appear to be most closely associated with the last of these although supported by the other two.

However, the issues of learning and teaching (instruction) are inextricably linked. As soon as one specifies something to be learned, a competency to be developed, a learning objective to be achieved, the question 'how shall this be accomplished' immediately follows. This becomes an instructional or pedagogical issue: what is to be done in the classroom to move students in the desired direction, to develop the desired competencies? Responses to this question have historically contrasted a focus on 'pure' versus 'applied' approaches to mathematics teaching (Blum & Niss, 1991; de Lange, 1996). Instruction focused on 'pure' mathematics centres on the study of the academic aspect of mathematics: its definitions, relations and structure. Upon mastery of these, students may then be asked to apply their knowledge to real-world situations that require this type of mathematics knowledge for a solution.

In contrast, the applied approach focuses on connecting mathematics to the real world at all times. The meaning and value of mathematics' rules and definitions must be illustrated through practical, real-world situations suggesting the relevance of a different and a more applied type of OTL in understanding performance on a literacy test. This emphasis on teaching mathematics through applications has been linked with recognizing mathematics as a useful tool in the social sciences and society more generally (de Lange, 2003). If de Lange is correct in his assessment, this suggests the potential relevance of such OTL to ML.

The PISA definition of ML was chosen to reflect an emphasis on 'mathematical knowledge put to functional use' in contrast to school mathematics (OECD, 1999, p. 41). Consequently, the issue addressed in this paper is how ML may be related to schooling, or students' OTL, as these have been defined and measured in PISA 2012. More specifically, PISA first defined ML as,

> an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. (OECD, 1999, p. 41)

That definition has been fairly consistent across studies; however, the framework for the 2012 study provided the first major change further elaborating mathematics knowledge and what it means to apply this in the everyday world.

For PISA 2012 mathematics, literacy is defined as,

> ... an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to

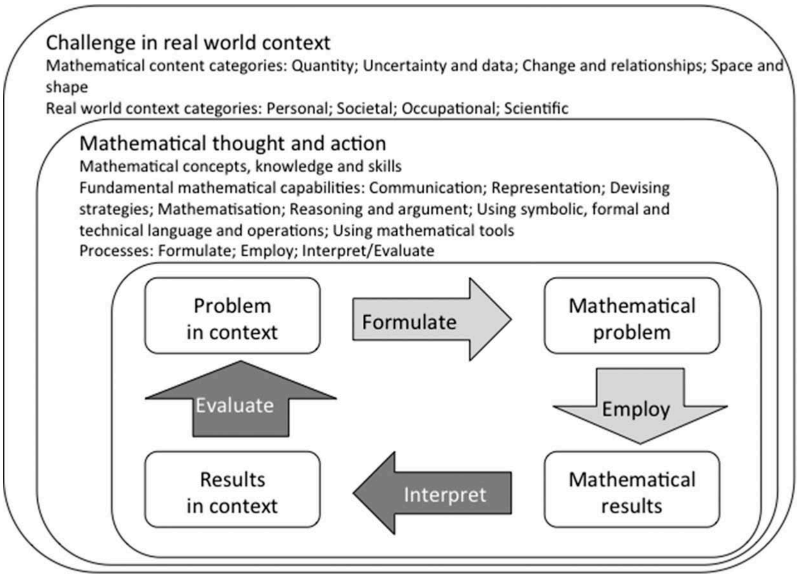


**Figure 1.** A model of mathematical literacy in practice. From OECD, 2013b.

> make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013a, p. 25)

This definition implies a model detailing the processes of what it means to be able to apply one's knowledge of mathematics to the real world—see Figure 1 (OECD, 2013a, p. 26). The definition of literacy also groups mathematics into four broad content areas that are measured and reported as PISA literacy sub-scales: quantity; uncertainty and data; change and relationships; and space and shape.

The definition also emphasizes the importance of applying mathematics knowledge 'in a variety of contexts' specifically identifying the four included in PISA: personal, occupational, societal and scientific which includes the application of mathematics to mathematics (Stacey, 2015). The effect of this most recent revision would appear to lower the wall between some of the historical dichotomies separating areas of study. In addition, these refinements provide possible points of connection with the instruction that typically occurs in schooling.

The iterative model of ML in Figure 1 bears a close resemblance to the 'Conceptual Mathematization' model of de Lange (1996; OECD, 1999). According to the processes specified in Figure 1, ML requires 15 year olds to be able to use their mathematics knowledge first to recognize the mathematical nature of a challenge situation (problem) encountered in the real word and then to formulate it in mathematics terms. This transformation is a critical component of the PISA definition. Once the student has transformed the challenge and formulated it as a mathematics problem, it must then be solved using the school mathematics definitions, algorithms and procedures typically taught in schools. The final component requires the student to take the results of the mathematics computation, put this back into the original real-world situation and provide an interpretation ensuring that it is appropriate for the situation in which the challenge (problem) was originally encountered.

The real-world orientation of PISA ML is evident in the example items included in the PISA frameworks and report (OECD, 2013a, 2013b). These items include reading and interpreting data from statistical displays such as those found in newspapers; determining the amount of wood needed to build a fence or the amount of tile needed to cover an irregular floor; calculating speed; using map scales to determine distances; interpreting medical data and studies; determining appropriate quantities for dosages of medicine or for recipes; using shape to organize spaces such as furniture in a room; and making decisions in a work context using probability to name a few.

As noted above, a substantial amount of literature, both domestic and international, has examined the relationship of OTL, defined in terms of specific content coverage and student performance. It almost exclusively, however, has focused on student assessments that reflect the K-12 classroom instructional content, i.e. the school curriculum (Schmidt & Maier, 2009). Research based on the The Third International Mathematics and Science Study (TIMSS), for example, provides strong evidence that exposure to rigorous and demanding mathematics is associated with student performance on a test designed to reflect an internationally defined school curriculum at grades 4 and 8 (Schmidt et al., 2001). The outstanding research question, and the focus of this paper, is whether a student's ability to apply these concepts as measured in the PISA of ML is similarly related to exposure to rigorous school mathematics content.

The PISA ML assessment has been used in many studies to explore its relationship with, for example, socio-economic status (SES) and gender (Thien, 2016), exit exams (Jürges, Schneider, Senkbeil, & Carstensen, 2012), SES and school resources (Nonoyama-Tarumi & Willms, 2010) and the effects of systemic reform on private–public school gaps (Cheema, 2015). For the first time PISA 2012 included a set of questions in the student background questionnaire that addressed their mathematics learning opportunities in their schools. Consequently, until PISA 2012 it has not been possible to explore the relationship between students schooling, their mathematics OTL and the type of ML assessed in PISA. Consequently, we make use of the PISA 2012 data to see if ML may be related to students' in-school mathematics learning opportunities as well as their application-oriented instructional experiences.

More specifically, we explore four related research questions:

(1) What is the interplay between the three types of OTL measured in PISA 2012 and grade level?
(2) What is the relationship of the amount of exposure to school mathematics content to performance on the PISA literacy test in the USA, OECD countries and other PISA 2012 participating countries/economies?
(3) What is the relationship between students' OTL—their exposure to real-world applications during instruction (including traditional 'word problems') and to more rigorous school mathematics topics—to the various definitions of literacy, e.g. those based on the four mathematics content areas (quantity, uncertainty and data, change and relationship, space and shape); the three processes (formulate, employ and interpret); and the two computer-based versions of literacy (problem solving and the computer-based ML assessment?
(4) What can be learned further about the USA from the participation of three US states PISA 2012?

## Methodology

### *Programme for International Student Assessment*

The PISA, conducted every 3 years since 2000 by the OECD, measures 15-year-old students' literacy in reading, mathematics and science. Each administration assesses all three literacies with one, on a rotating basis, receiving greater emphasis. Mathematics was most recently the focus in PISA 2012. All 34 OECD countries participate in PISA together with a varying number of other countries and economies. Over 65 countries/economies participated in PISA 2012 including three US states, Connecticut, Florida and Massachusetts.

For each participating entity, PISA randomly samples all 15-year-old students in those schools that have been randomly selected from an entity-specific sampling frame. The target population is defined as 15 year olds, with students in PISA 2012 ranging in age from 15 years, 3 months to

16 years, 2 months. The average student age across OECD countries was 15 years, 9 months with country means varying by a little more than 2 months (OECD, 2013b, p. 265).[1]

PISA produces five plausible values for each student for eight types of scores: an overall ML score; four mathematics content scores—quantity, uncertainty and data, change and relationship, space and shape; and the three mathematical processes scores—formulate, employ and interpret. These represent the key aspects in the PISA ML model (see Figure 1). In the 2012 cycle, PISA also administered two computer-based assessments. One, computer mathematics, was designed to assess the same ML as that in the main assessment. This computer-based assessment was conducted in 32 countries/economies, 23 of which were OECD countries. Another computer-based assessment, Problem Solving, occurred in 43 countries/economies, 27 of which were OECD countries. This was not conceived as a mathematics assessment but rather one that addressed 'general cognitive processes involved in problem solving, rather than on the ability to solve problems in particular school subjects' (OECD, 2014a, p. 29).

### *OTL measures*

Four items probed student experience at school with different kinds of mathematics problems. Each item had two examples of a particular type of mathematics. Students were not asked to solve any of these problems but rather were asked to indicate how often they had 'encountered these types of problems in your mathematics lessons' (OECD, 2013a, pp. 235-236). Note that no time frame is suggested for students to consider other than 'in your mathematics lessons'. PISA assesses all 15-year-old students who are enrolled in school whether or not they are currently taking a mathematics course. Consequently, the implied time frame for students to consider is cumulative of their experience in mathematics lessons in school whenever they may have seen the particular type of item. Students responded to each item selecting 'frequently', 'sometimes', 'rarely' or 'never'. We scored these on a 0–3 scale with 0 corresponding to 'never' and 3 corresponding to 'frequently'. The first item 'dressed up' a purely mathematical problem in the words of everyday life, i.e. a purely computational problem is presented in verbal form in a somewhat contrived context such as 'Judy has 3 apples. If Bobby gives her 2 of his apples, how many apples will Judy have?' (Q44a, Blum & Niss, 1991; OECD, 2013a). As an example of word problems often found in mathematics instruction, we included this in our analyses as 'Word Problems'.

Two items contained examples of mathematics in specific contexts, e.g. determining dimensions of a pyramid, calculations involving a person's heart rate and interpreting a bar graph. Students mean response to these two items is included in our analyses as 'Applied Mathematics' (Q46a and Q47a, OECD, 2013a). The definitions used here for both 'Word Problems' and 'Applied Mathematics' are those included in the first PISA 2012 report (OECD, 2013b).

Another item (ST62) listed 13 different mathematical concepts and asked students to indicate how familiar they were to each one using a 5-point scale (Q39, OECD, 2013a, p. 234). This item formed the basis for 'School Mathematics'. These concepts are ones typically encountered in school at some point between grades 3 and 12 such as division and complex numbers. In addition, we weighted those students' responses (scaled from 0 to 4 with 0 representing the category, 'never heard of it') with an estimate of the concepts' curricular rigor. The concept weights, termed the 'international grade placement' or IGP for a topic, had been developed from the 1995 TIMSS cross-national curriculum data. The IGP weight reflects the 'typical' or average grade level at which the specific topic received its greatest instructional focus, taking into account the grade at which it was first introduced (Schmidt et al., 2001). Although the IGP's concept-to-grade-level associations have been reviewed and deemed valid by an Advisory Board of mathematicians for a mathematics education research project, the purpose of employing the IGP weights to the concepts listed was not to assign concepts to grade levels but rather to distinguish between concepts likely covered earlier in a student's schooling experience from those most likely to be covered later. Given the hierarchical nature of mathematics, it seems most likely that those topics receiving instructional focus in later grades across most countries are likely to be more advanced than those receiving focus in earlier grades. Consequently, summing then across all the IGP-weighted experiences provides an estimate of the cumulative rigor or content-related difficulty of the mathematics curriculum that students have experienced in school up to and including the age of 15. The empirically derived IGP index of topic rigor has been shown to have strong face validity as well as content and predictive validity and has proved valuable in previous research on student learning (Achieve, 2004; Cogan, Schmidt, & Wiley, 2001; Schmidt & Cogan, 2009; Schmidt, Cogan, Houang, & McKnight, 2011; Schmidt, Cogan, & McKnight, 2010; Schmidt & McKnight, 2012).

Thus 'School Mathematics' was operationalized using the following equation:

$$\mathit{Content\ Rigor\ of\ Student\ OTL} = \sum_{i}^{13} (\mathit{ConceptFamiliarity}_i \times \mathit{ConceptIGP}_i)$$

This weighted familiarity variable is a multifaceted measure that is based on three distinct aspects of OTL: (1) the mathematics content itself (topic experienced—yes/no), (2) degree of familiarity with each topic, and (3) rigor or content difficulty (IGP, estimated from international curriculum data). Therefore, the IGP measure of the mathematics students have been exposed to in their classrooms over their academic career up to age 15 is a measure of content-specific OTL defined at the student level which can be related to student achievement. The variable was standardized with values ranging between 0 (no familiarity with any of the 13 School Mathematics concepts listed) and 12 (highest degree of familiarity for all 13).[2]

PISA listed three terms in the ST62 item that were not true mathematics concepts such as 'declarative fraction'. These were included as 'measures to guard against over claiming' (OECD, 2013a, p. 13). We used two of the terms to adjust the familiarity students expressed to those concepts with an IGP value above grade 7. Only extreme responses to these non-existent concepts (i.e. students claimed to 'know it well' or had 'heard it often') were used to identify over-claimers (Paulhus, Harms, Bruce, & Lysy, 2003). If for either of these concepts (topics) students claimed to 'know it well' or had 'heard it often' their responses to the 10 items with an IGP above grade 7 were reduced three or two points, respectively. In this manner, typically around 10% of the students were identified as over-claimers. Reducing the familiarity claims for these students resulted in a small difference (typically around 0.1) in country means for the School Mathematics variable.

To address the research questions identified at the end of the introduction, we analysed country means and conducted multilevel hierarchical regression analyses with the OTL measures together with SES and student grade to predict the various PISA literacy scores. In conducting all these analyses, we used the macros made available by PISA. These macros use the balanced repeated replicate (BRR) weights which provide more precise standard errors of the estimated statistics. Since the BRR weights take into account the various sampling strata countries have used to draw their sample of schools, the standard errors generated in this manner are typically smaller than those estimated by the standard procedures available with most statistical software and weighted simply by the final student weight. In addition, the macros provided by PISA for use with the plausible values conduct analyses with each plausible value and then process these to determine the final means or regression estimates and their appropriate standard errors (OECD, 2009).

## Results

### *OTL and grade level*

The means for the three OTL variables, SES, the overall ML measure, the literacy sub-scales and the computer-related test of literacy and problem solving are given in Table 1.[3] There was substantial variation across countries for all three OTL measures. For example, School Mathematics in Korea had an average of 8.2 compared to 2.7 for Sweden and 5.8 for Germany. This was also the case for the two applied OTL measures. However, most of the variation in OTL was found to be within, as opposed to between, countries. Furthermore, for most of the countries the within-school variation

**Table 1.** Student-level variable means for OECD and Partner countries/economies.[a]

| | | OTL indices | | | Literacy scales | | | | | | | | Computer scales | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | SES | Applied Math | Word Problems | School Math | Literacy | Change | Quantity | Space | Data | Employ | Formulate | Interpret | Problem Solving | Computer Math |
| Australia | 0.2 | 2.0 | 1.8 | 5.7 | 504 | 509 | 500 | 497 | 508 | 500 | 498 | 514 | 523 | 508 |
| Austria | 0.1 | 1.8 | 2.1 | 5.4 | 506 | 506 | 510 | 501 | 499 | 510 | 499 | 509 | 506 | 507 |
| Belgium | 0.1 | 1.9 | 1.9 | 6.2 | 515 | 513 | 519 | 509 | 508 | 516 | 512 | 513 | 508 | 512 |
| Canada | 0.4 | 2.1 | 2.0 | 6.6 | 518 | 525 | 515 | 510 | 516 | 517 | 516 | 521 | 526 | 523 |
| Chile | −0.6 | 2.1 | 2.0 | 6.1 | 423 | 411 | 421 | 419 | 430 | 416 | 420 | 433 | 448 | 432 |
| Czech Republic | −0.1 | 1.6 | 1.6 | 6.6 | 499 | 499 | 505 | 499 | 488 | 504 | 495 | 494 | 509 | |
| Denmark | 0.4 | 2.0 | 1.9 | 5.7 | 500 | 494 | 502 | 497 | 505 | 495 | 502 | 508 | 497 | 496 |
| Estonia | 0.1 | 1.8 | 1.8 | 7.3 | 521 | 530 | 525 | 513 | 510 | 524 | 517 | 513 | 515 | 516 |
| Finland | 0.4 | 1.7 | 2.1 | 5.1 | 519 | 520 | 527 | 507 | 519 | 516 | 519 | 528 | 523 | |
| France | 0.0 | 2.0 | 2.1 | 6.5 | 495 | 497 | 496 | 489 | 492 | 496 | 483 | 511 | 511 | 508 |
| Germany | 0.2 | 2.0 | 2.0 | 5.8 | 514 | 516 | 517 | 507 | 509 | 516 | 511 | 517 | 509 | 509 |
| Greece | −0.1 | 1.9 | 1.3 | 7.1 | 453 | 446 | 455 | 436 | 460 | 449 | 448 | 467 | | |
| Hungary | −0.3 | 1.9 | 2.0 | 7.0 | 477 | 481 | 476 | 474 | 476 | 481 | 469 | 477 | 459 | 470 |
| Iceland | 0.8 | 2.0 | 2.4 | 3.9 | 493 | 487 | 496 | 489 | 496 | 490 | 500 | 492 | | |
| Ireland | 0.1 | 1.9 | 1.8 | 4.9 | 501 | 501 | 505 | 478 | 509 | 502 | 492 | 507 | 498 | 493 |
| Israel | 0.2 | 1.8 | 1.7 | 6.0 | 466 | 462 | 480 | 449 | 465 | 469 | 465 | 462 | 454 | 447 |
| Italy | −0.1 | 1.8 | 1.7 | 6.9 | 485 | 477 | 491 | 487 | 482 | 485 | 475 | 498 | 510 | 499 |
| Japan | −0.1 | 1.7 | 1.6 | 7.2 | 536 | 542 | 518 | 558 | 528 | 530 | 554 | 531 | 552 | 539 |
| Korea | 0.0 | 1.8 | 1.7 | 8.2 | 554 | 559 | 537 | 573 | 538 | 553 | 562 | 540 | 561 | 553 |
| Luxembourg | 0.1 | 1.9 | 2.0 | 4.8 | 490 | 488 | 495 | 486 | 483 | 493 | 482 | 495 | | |
| Mexico | −1.1 | 2.2 | 1.8 | 5.6 | 413 | 405 | 414 | 413 | 413 | 413 | 409 | 413 | | |
| Netherlands | 0.2 | 2.1 | 1.6 | 4.9 | 523 | 518 | 532 | 507 | 532 | 518 | 527 | 526 | 511 | |
| New Zealand | 0.0 | 2.0 | 1.6 | 5.0 | 500 | 501 | 499 | 491 | 506 | 495 | 496 | 511 | | |
| Poland | −0.2 | 2.0 | 2.0 | 6.2 | 518 | 509 | 519 | 524 | 517 | 519 | 516 | 515 | 481 | 489 |
| Portugal | −0.5 | 2.2 | 1.5 | 6.4 | 487 | 486 | 481 | 491 | 486 | 489 | 479 | 490 | 494 | 489 |
| Slovak Republic | −0.2 | 1.9 | 2.0 | 6.0 | 482 | 474 | 486 | 490 | 472 | 485 | 480 | 473 | 483 | 497 |
| Slovenia | 0.1 | 1.9 | 2.1 | 7.0 | 501 | 499 | 504 | 503 | 496 | 505 | 492 | 498 | 476 | 487 |
| Spain | −0.2 | 2.0 | 2.2 | 6.8 | 484 | 482 | 491 | 477 | 487 | 481 | 477 | 495 | 477 | 475 |
| Sweden | 0.3 | 1.7 | 1.9 | 2.7 | 478 | 469 | 482 | 469 | 483 | 474 | 479 | 485 | 491 | 490 |
| Switzerland | 0.2 | 1.9 | 2.1 | 4.9 | 531 | 530 | 531 | 544 | 522 | 529 | 538 | 529 | | |
| Turkey | −1.5 | 2.0 | 1.3 | 6.9 | 448 | 448 | 442 | 443 | 447 | 448 | 449 | 446 | 454 | |
| UK | 0.3 | 1.9 | 1.9 | 5.5 | 494 | 496 | 494 | 475 | 502 | 492 | 489 | 501 | 517 | |
| USA | 0.2 | 2.0 | 1.8 | 6.6 | 481 | 488 | 478 | 463 | 488 | 480 | 476 | 490 | 508 | 498 |

(*Continued*)

**Table 1.** (Continued).

| | | OTL indices | | | Literacy scales | | | | | | | | Computer scales | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Country | SES | Applied Math | Word Problems | School Math | Literacy | Change | Quantity | Space | Data | Employ | Formulate | Interpret | Problem Solving | Computer Math |
| *OECD average* | *0.0* | *1.9* | *1.9* | *6.0* | *494* | *493* | *495* | *490* | *493* | *494* | *492* | *497* | *500* | *497* |
| Brazil | −1.2 | 2.0 | 1.5 | 4.9 | 389 | 368 | 389 | 378 | 400 | 385 | 373 | 398 | 425 | 418 |
| Bulgaria | −0.3 | 1.9 | 1.5 | 6.4 | 439 | 434 | 443 | 442 | 432 | 439 | 437 | 441 | 402 | |
| China-Shanghai | −0.4 | 1.6 | 1.3 | 8.9 | 613 | 624 | 591 | 649 | 592 | 613 | 624 | 579 | 536 | 562 |
| Chinese Taipei | −0.4 | 1.7 | 1.5 | 7.5 | 560 | 561 | 543 | 592 | 549 | 549 | 578 | 549 | 534 | 537 |
| Colombia | −1.3 | 2.2 | 1.9 | 5.7 | 376 | 357 | 375 | 369 | 388 | 368 | 375 | 388 | 399 | 397 |
| Croatia | −0.3 | 1.8 | 2.0 | 7.0 | 471 | 468 | 480 | 460 | 468 | 478 | 453 | 477 | 466 | |
| Hong Kong-China | −0.8 | 1.8 | 1.4 | 6.4 | 561 | 564 | 566 | 567 | 553 | 558 | 568 | 551 | 540 | 550 |
| Macao-China | −0.9 | 1.6 | 1.2 | 7.6 | 538 | 542 | 531 | 558 | 525 | 536 | 545 | 530 | 540 | 543 |
| Malaysia | −0.7 | 2.0 | 1.8 | 4.5 | 421 | 401 | 409 | 434 | 422 | 423 | 406 | 418 | 422 | |
| Montenegro | −0.2 | 1.9 | 2.0 | 6.0 | 410 | 399 | 409 | 412 | 415 | 409 | 404 | 413 | 407 | |
| Russian Federation | −0.1 | 2.0 | 2.0 | 6.9 | 482 | 491 | 478 | 496 | 463 | 487 | 481 | 471 | 489 | 489 |
| Serbia | −0.3 | 1.8 | 1.5 | 6.7 | 449 | 442 | 456 | 446 | 448 | 451 | 447 | 445 | 473 | |
| Singapore | −0.3 | 2.0 | 1.6 | 7.2 | 573 | 580 | 569 | 580 | 559 | 574 | 582 | 555 | 562 | 566 |
| United Arab Emirates | 0.3 | 2.1 | 1.8 | 7.0 | 434 | 442 | 431 | 425 | 432 | 440 | 426 | 428 | 411 | 434 |
| Uruguay | −0.9 | 1.7 | 1.3 | 5.7 | 409 | 401 | 411 | 413 | 407 | 408 | 406 | 409 | 403 | |

[a]Norway, an OECD country, is not listed as it did not include the School Math OTL item on the student background questionnaire. Only Partner countries that participated in at least one of the computer-based assessments are listed here.

in OTL was greater than the between-school variation; the OECD average indicated within-school variation to account for around 80% of the total OTL variation.

The nature of PISA sampling affords an opportunity to explore one source for the large variation in the OTL measures. One of the consequences of the PISA age-based (15-year-old) student population definition is that students within a country were not all necessarily in the same grade and as a result had been exposed to different courses and hence different mathematics content. In the USA, for example, most PISA students were in the 10th grade (about 71%), with an additional 17% in 11th grade and about 12% in 9th grade. Less than 1% of the US sampled students was from any of grades 7, 8 or 12 (OECD, 2013b). All PISA students were in a single grade in only two OECD countries, Iceland and Japan.

The modal grade in which most PISA students were found also varied across countries. This reflects differences in the structure and organization of education and as a result 15-year-old students in some PISA countries had not experienced the same number of years of schooling as those in another. To capture these distinctions, PISA defined a variable based on the modal grade for 15 year olds in each country resulting in three categories: the modal grade, the grade before the modal grade and the grade after the modal grade.[4]

Twenty of the 33 OECD countries were like the USA in that most PISA students were in 10th grade. Ninth grade was the modal grade for 11 countries including Germany and Finland. Eleventh grade was the modal grade only for New Zealand and the UK.

Figure 2 plots the 33 OECD country means for each of the three OTL variables at each of the three grades which are 1 year apart centred around the modal grade for that country. The substantial variation for each OTL variable among OECD countries is evident within each of the three grades. That variation was especially large for School Mathematics at the modal grade. In addition, the patterns for each OTL variable across the three grades for four countries—the Netherlands, the USA, Germany and Canada— are identified in order to illustrate the relationship between grade and OTL within these exemplar countries.

For most of the countries, the shift in OTL across the three grades for Word Problems and Applied Mathematics was relatively small, with the exception of Germany for Applied Mathematics. Since these OTL variables reflect the instructional context and not the content itself, small differences based on grade level do not seem unusual. By contrast, for virtually all countries the School Mathematics means increase for each successive year as illustrated by the four countries

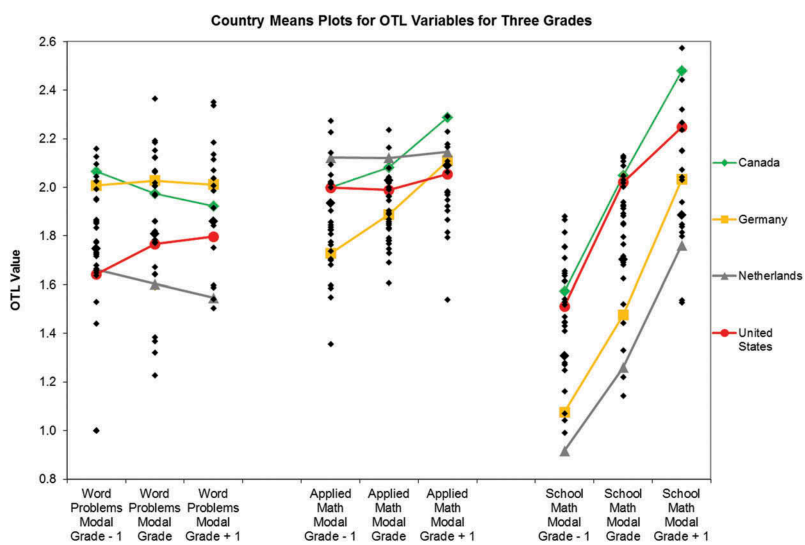


Figure 2. Plots of the 33 OECD country means for three OTL variables by grade category.

identified in Figure 2. This OTL variable exhibits a strong impact of grade which one would expect in mathematics given the hierarchical nature of the discipline as it develops over 3 years of schooling.

To more formally explore the nature of the relationship between these OTL variables and grade level, a series of one-way multivariate ANOVAs (one for each of the 33 OECD countries) on these means was conducted. Four distinct patterns—illustrated by the four countries highlighted in Figure 2—of statistical significance were found. School Mathematics had a statistically significant grade effect for all countries. In four countries (Austria, Israel, the Netherlands and the UK), this was the only OTL variable to have a significant grade-level effect. Both School Mathematics and Word Problems demonstrated a statistically significant grade effect in five countries, typified by the USA in Figure 2; the others were Chile, France, Poland and Spain. Six countries had profiles like that of Germany for which both School Mathematics and Applied Mathematics demonstrated significant grade effects: Estonia, Finland, Hungary, New Zealand and South Korea. The remaining 18 countries are represented in Figure 2 by Canada in which all three OTL variables demonstrated significant grade effects.[5]

Nonetheless, for all 33 OECD countries the greatest grade-based mean differences were found for School Mathematics. Thus it appears that the OTL variable measuring students' content opportunities is different from the two OTL variables measuring the (instructional) context in which students' encountered the content. This is consistent with the typical curricular structure in which mathematics content increases in rigor and substance from one grade to the next in effect providing still another measure of OTL. Indeed, grade has often been used as a surrogate for content coverage in educational research. On the other hand, the instructional context does not typically vary to any great extent over the grades.

Returning to the issue of the cross-country variation in OTL, Figure 3 provides a visualization of the extent of that variation. Word Problems and School Mathematics (scale converted to same 4-point scale 0–3 to facilitate comparisons across the three OTL variables) demonstrated substantial variation among the means ranging over a considerable range (1.2–2.3). Applied Mathematics demonstrated somewhat less variation. Nevertheless, the average values across the PISA 2012 participants for all three indices ranged between 'rarely' and 'sometimes' providing such opportunities.

Figure 4 divides the OECD countries into four groups related to the amount of opportunities to learn in both School and Applied Mathematics. The OECD averages are used to define the four quadrants. As one might expect, this graph suggests that there may be a trade-off between the two types of opportunities. Although there were countries above the OECD average on both indices, only Canada was in the top five both in the amount of School and Applied Mathematics.

### *Relating literacy and OTL at the country level*

In this and the following sections, we explore the relationship of School Mathematics and Applied Mathematics to ML as measured in PISA 2012. Such a relationship can manifest itself both between and within countries. In this section, we examine the cross-country relationship of ML to these two OTL indices.

Putting both OTL indices in the same country-level regression analysis resulted in a statistically significant relationship for both School and Applied Mathematics. The estimate of the magnitude of the relationship between performance and the School Mathematics index was linear and the size of the coefficient indicated about a two-thirds of a standard deviation effect size (66.9; $p < 0.001$). The mean frequency of encounters with Applied Mathematics—in terms of problems in students' lessons and on the assessments they have taken—was also related to average performance on the literacy assessment ($p < 0.001$). This relationship however was quadratic (the coefficient was negative indicating an inflection point at which more Applied Mathematics OTL was not positively but negatively related to performance).

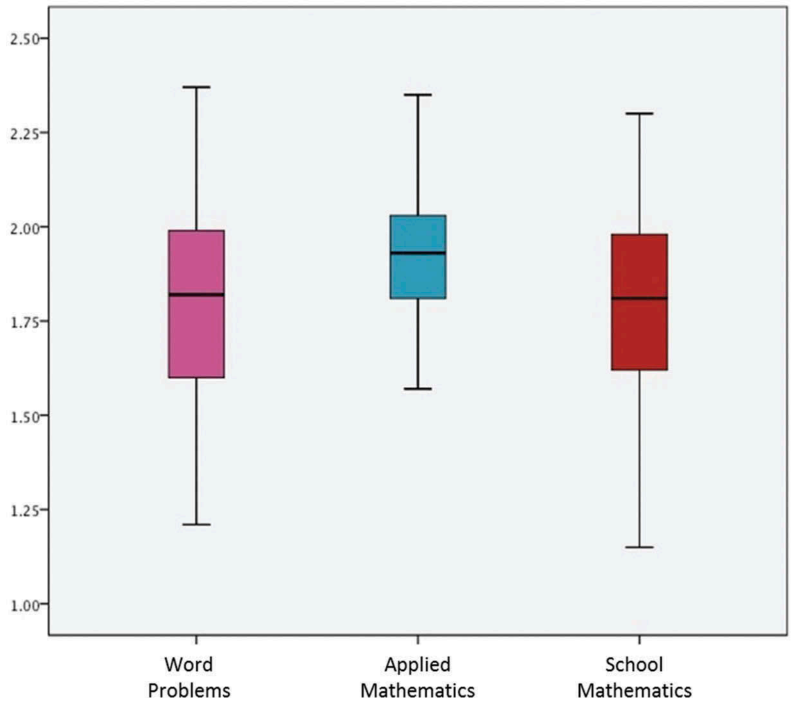


**Figure 3.** Box plot of country means for all OECD and other participating countries in PISA 2012.

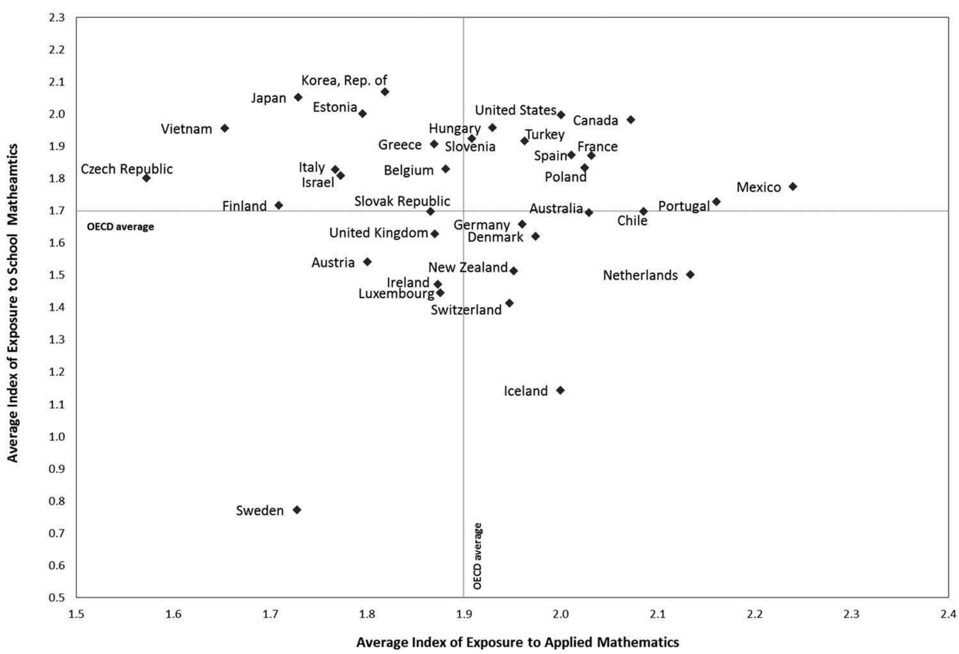


**Figure 4.** Average index of exposure to School Mathematics for OECD countries by average index of Applied Mathematics exposure.

Students in 43 countries participated in a computer-based problem-solving competency assessment as a special option. The PISA 2012 framework defined problem-solving competence as,

> an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen. (OECD, 2014a, p. 30)

Problems were not specific to mathematics but rather were characterized by the crossing of two dimensions: technology versus a non-technology context and either a personal or a social context. Although problem-solving competency was more related to ML than reading literacy it was not designed to be a mathematics problem-solving test (see OECD, 2014a, p. 101). Conducting a parallel regression analysis relating the two OTL variables to performance on this test produced some intriguing results. It was interesting and thought-provoking to find that School Mathematics was not statistically significantly related ($p < 0.14$) to problem-solving competency but Applied Mathematics OTL was statistically significantly related to performance across the participating 43 countries. However, the nature of the relationship was again quadratic. Why there was a relationship of Applied Mathematics to performance on a general problem-solving assessment awaits further analysis. Perhaps at this point, at most, it can be pointed out that these results are consistent with those who have suggested that mathematics is an area of schooling that helps to support the development of problem-solving competencies more generally.

### *Relating literacy and OTL within each country*

The cross-country relationships described in the previous section may not reflect the nature of these relationships within any country. In this section, we examine within-country relationships by fitting a multilevel linear regression model using School Mathematics, Applied Mathematics and Word Problems (as a more traditional version of Applied Mathematics) together with the PISA measure of SES to predict PISA performance. In this way, we explore both the between- and the within-school relationship of OTL to performance.[6]

Examining differences in OTL across schools within a country may reflect an organizational policy that defines different types of schools for 15 year olds together with policies that determine which students can attend which type of school. This type of tracking is often associated with indicators of student ability; certain types of students are tracked into different types of schools. Typically, these different types of schools also yield different post-secondary options for their students. Another source of school differences may stem from social class segregation in the housing patterns that define school attendance boundaries. Nonetheless, whatever the source of such school differences, differences in content exposure are not likely to be inconsequential.

In addition to the grade-level differences observed in PISA, within-school differences in OTL may also reflect a different set of policies and structures. Such differences might reflect within-school tracking and/or student/parent choices regarding a course of study. They may also reflect teacher difference as to what content they cover even under the same course designation (Cogan et al., 2001).

Table 2 provides the results of the two-level regression analyses of OTL on ML controlling for SES (ESCS) and students' grade level. Several different relationship patterns emerged between the OTL variables and the overall PISA literacy score. The UK, Canada, Spain, Chinese Taipei, the Russian Federation and the Netherlands demonstrated significant relationships for all three OTL variables at the within-school level. The quadratic form of the relationship for Applied Math—the only type of significant relationship except in the Russian Federation and Uruguay—suggests that after a certain amount of exposure to instruction and testing that embeds mathematics in practical, real-life situations, on average student performance begins to decline (see Figure 5). Applied Mathematics was not significantly related to performance in Australia, Japan, Germany, the USA and a few other countries at either the within or the between levels. Word Problems demonstrated

**Table 2.** Within country analyses relating OTL and SES to performance.[a]

| | Within schools | | | | | | Between schools | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Applied Math | | | | | Applied Math | | | |
| Countries | Grade | SES | Linear | Quadratic | Word Problems | School Math | SES | Linear | Quadratic | Word Problems | School Math |
| Australia | 15.3 | 11.7 | | | 5.7 | 70.7 | 42.8 | −36.7 | | 7.3 | 101.7 |
| Austria | 26.0 | 10.4 | | | | 36.1 | 28.1 | | −26.7 | 21.6 | 99.9 |
| Belgium | 41.1 | 6.9 | | −4.9 | 2.0 | 46.4 | 42.0 | | | −4.5 | 83.9 |
| Canada | 22.6 | 14.1 | | −3.3 | 4.6 | 56.6 | 32.6 | | −17.0 | 31.2 | 78.4 |
| Chile | 23.8 | 7.0 | | −4.6 | 5.0 | 38.5 | 26.4 | | | 19.3 | 72.1 |
| Czech Republic | 24.8 | 8.1 | | | | 50.0 | 77.4 | | −17.6 | 15.4 | 92.8 |
| Denmark | 13.2 | 21.1 | | | 2.5 | 54.9 | 41.2 | | −10.9 | | 66.0 |
| Estonia | 20.1 | 13.7 | | | 5.8 | 50.6 | 41.5 | | −19.6 | 20.5 | 32.0 |
| Finland | 27.0 | 17.8 | | −3.3 | 8.4 | 69.4 | 35.2 | | | | 47.7 |
| France | 44.1 | 15.3 | | | | 37.5 | 47.7 | | −12.3 | | 79.8 |
| Germany | 29.7 | 6.7 | | | 2.5 | 33.4 | 39.7 | | | 17.0 | 101.2 |
| Greece | 12.0 | 14.6 | | | −2.5 | 39.2 | 33.0 | | −29.4 | −8.1 | 96.0 |
| Hungary | 17.8 | 3.3 | | | | 34.2 | 48.6 | | | 13.8 | 84.1 |
| Iceland | 0.0 | 18.0 | | −11.8 | 15.8 | 46.5 | 29.2 | | −32.0 | 22.7 | 32.8 |
| Ireland | 6.2 | 15.9 | | | | 60.0 | 51.2 | | | | 79.0 |
| Israel | 14.7 | 17.2 | | | | 51.9 | 85.3 | | −47.6 | 10.2 | 64.6 |
| Italy | 25.4 | 4.4 | | −3.6 | 2.1 | 33.1 | 33.1 | | −20.6 | 10.0 | 97.0 |
| Japan | 0.0 | | | | 6.7 | 44.0 | 68.5 | | | 39.1 | 120.1 |
| Korea | −60.6 | 6.5 | | −4.5 | 3.8 | 80.7 | 15.0 | | | | 194.8 |
| Luxembourg | 37.1 | 8.8 | | −7.2 | 6.2 | 38.1 | 30.7 | | | 41.8 | 78.0 |
| Mexico | 15.8 | 4.0 | | −5.0 | 7.9 | 28.1 | 16.4 | | −20.6 | 16.6 | 61.0 |
| Netherlands | 26.2 | 4.6 | | −4.4 | −3.8 | 40.7 | 24.1 | | −36.2 | | 150.1 |
| New Zealand | 21.4 | 19.5 | | −6.1 | 6.9 | 73.1 | 63.4 | | | | 81.8 |
| Poland | 54.5 | 21.8 | | | 9.6 | 52.7 | 41.1 | −76.5 | | 14.8 | 97.1 |
| Portugal | 42.6 | 12.3 | | −4.5 | 2.6 | 46.9 | 22.0 | | −28.6 | −9.0 | 62.1 |
| Slovak Republic | 22.3 | 18.0 | | | 4.3 | 54.9 | 45.6 | | | 24.5 | 100.4 |
| Slovenia | 17.4 | 4.2 | | | | 26.1 | 59.9 | | | 8.7 | 102.3 |
| Spain | 43.0 | 9.6 | | −4.6 | 2.8 | 39.1 | 25.6 | | | 11.1 | 45.3 |
| Sweden | 65.4 | 23.8 | | −11.9 | 11.5 | 21.0 | 55.9 | | | 15.0 | |
| Switzerland | 37.1 | 17.2 | | −6.4 | 2.8 | 53.8 | 46.2 | | −40.0 | 13.6 | 43.9 |
| Turkey | 27.5 | 4.8 | | | | 23.7 | 34.7 | | | | 96.1 |
| UK | 6.7 | 11.3 | | | 3.8 | 72.8 | 57.2 | | −36.9 | 8.2 | 94.6 |
| USA | 24.2 | 13.2 | | | 4.1 | 61.1 | 33.1 | | | | 91.8 |
| *OECD average* | *22.6* | *12.0* | | *−5.8* | *4.8* | *47.5* | *41.7* | *−56.6* | *−26.4* | *15.0* | *85.3* |
| Brazil | 20.6 | 4.2 | | −3.0 | | 27.3 | 19.7 | | −6.3 | | 69.2 |
| Bulgaria | 13.9 | 8.4 | | | | 32.3 | 25.0 | | −54.9 | | 92.8 |

(*Continued*)

**Table 2.** (Continued).

| | Within schools | | | | | | Between schools | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Applied Math | | | | | Applied Math | | | |
| Countries | Grade | SES | Linear | Quadratic | Word Problems | School Math | SES | Linear | Quadratic | Word Problems | School Math |
| China-Shanghai | 21.2 | 7.2 | | | −4.1 | 39.1 | 51.0 | | 20.3 | −50.0 | 107.3 |
| Chinese Taipei | | 18.3 | | −10.3 | 4.2 | 66.9 | 67.4 | | −33.4 | | 146.2 |
| Colombia | 16.6 | 5.5 | | −4.1 | 3.9 | 28.2 | 24.1 | | −20.2 | 14.5 | 57.3 |
| Croatia | | 9.2 | | −3.3 | 2.9 | 43.5 | 37.7 | | | 7.3 | 154.1 |
| Hong Kong-China | 20.4 | | | | | 40.1 | 69.0 | | −47.0 | | 85.3 |
| Macao-China | 38.0 | | | | | 30.5 | | | −37.4 | 44.0 | 139.6 |
| Malaysia | 75.0 | 11.1 | | −4.1 | 7.4 | 40.9 | 40.7 | | −18.0 | | 119.4 |
| Montenegro | | 9.0 | | −7.8 | 4.0 | 39.5 | 42.1 | | | | 146.1 |
| Russian Federation | 26.9 | 18.7 | −16.3 | | 2.9 | 46.9 | 34.2 | | | 16.2 | 80.3 |
| Serbia | 15.3 | 7.5 | | −3.9 | | 41.4 | 51.4 | | −18.6 | 26.1 | 127.1 |
| Singapore | | 9.7 | | | −3.5 | 69.7 | 29.0 | | | −31.5 | 146.6 |
| United Arab Emirates | 20.9 | 8.4 | | | | 42.2 | 43.1 | | −22.4 | 32.8 | 59.0 |
| Uruguay | 27.2 | 10.0 | −12.1 | | | 34.0 | 32.1 | | | | 50.3 |

[a]Norway, an OECD country, is not listed as it did not include the School Math OTL item on the student background questionnaire. Only Partner countries that participated in at least one of the computer-based assessments are listed here.

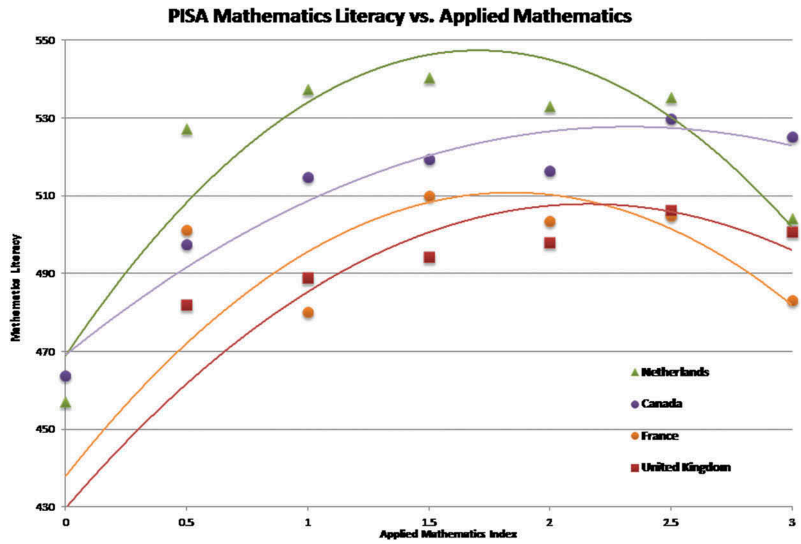


**Figure 5.** The quadratic relationship of Applied Mathematics to performance illustrated for four countries.

no statistically significant relationships in France, Ireland, Hong Kong and eight other countries. Neither Applied Mathematics nor Word Problems were statistically significantly related to performance at either the within- or the between-school level in France, Ireland, Hong Kong and three other countries. Most countries demonstrated a significant relationship to performance for both School Mathematics and at least one form of mathematics in an applied context. However, School Mathematics was the only OTL variable to have had a significant relationship in all countries and this significant relationship was evident at both the within- and the between-school levels for the vast majority of countries. The only exception to this strikingly consistent relationship was in Iceland and Sweden where School Mathematics was not significant at the between-school level, an indication of the consistency learning opportunities from one school to another in these countries.

In another paper (Schmidt, Burroughs, Zoido, & Houang, 2015), the strong and consistent relationship of School Mathematics to performance was found to be resilient even after controlling for SES, grade level and other factors known to be related to performance including the presence of tracking and/or ability grouping. Such a robust relationship provides strong evidence for the hypothesis that School Mathematics is strongly related to ML, just as it has been found to be with assessments of school academics such as the TIMSS studies (Schmidt et al., 2001). Both suggest the importance of schooling. Yet with the PISA results it can be said that even for ML, where it might be argued that exposure to Applied Mathematics would be most important, exposure to rigorous mathematics even after controlling for SES, grade and the two types of applied OTL was still significant with strikingly large estimated effect sizes.

No such consistent relationship for Word Problems or Applied Mathematics was evident across all OECD countries. Nonetheless, 76% of the OECD countries did have a statistically significant relationship ($p < 0.01$) between exposure to Word Problems and performance but primarily at the within-school level. For Applied Mathematics, this was true for 45% of OECD countries at the within-school level. The fact that the *only* significant relationship among OECD countries was the non-linear (quadratic) one between exposure to real-life problems in school and literacy performance is very striking. Even after controlling for School Mathematics, there was an additional relationship for such OTL exposure. However, at some point that exposure was no longer positive.

In some OECD countries such as Canada, the Netherlands and the UK, we found such a relationship but in others such as Japan, Germany and the USA we did not. The fact that these relationships, when present, were always non-linear implies a limiting constraint on the amount of such exposure, a significant finding in light of current debates about the way in which mathematics should be taught (e.g. de Lange, 2003).

Table 3 summarizes the estimated effects for the same two-level regression model in Table 2 for all 10 PISA tests and subtests across all OECD countries and the Non-OECD countries/economies that participated in the computer-based Problem-Solving assessment. The summaries include the mean estimated effects and per cent of statistically significant relationships at both the between- and the within-school levels as well as an estimate of the mean of the various regression coefficients over those countries in which they were statistically significant.

## *A closer look at the USA*

As Table 2 illustrates, after controlling for SES at the within-school level, School Mathematics and Word Problems were statistically significantly related to PISA performance. For School Mathematics, the magnitude of the estimated relationship was large—nearly two-thirds of a standard deviation (61.1). In fact, the size of the estimated effect was among the 10 highest values across all PISA

**Table 3.** Average significant estimates and percentage of OECD and Partner countries/economies with statistically significant relationships of OTL to PISA performance.[a]

| | Effect | Literacy | Change | Quantity | Space | Data | Employ | Formulate | Interpret | Problem Solving |
|---|---|---|---|---|---|---|---|---|---|---|
| | Grade | 23.6 | 27.2 | 25.8 | 21.7 | 24.3 | 24.1 | 24.7 | 24.6 | 23.7 |
| | | *92%* | *90%* | *90%* | *83%* | *94%* | *90%* | *92%* | *92%* | *93%* |
| Within-school level | SES | 11.4 | 11.8 | 11.9 | 12.2 | 11.0 | 11.2 | 12.1 | 11.6 | 9.6 |
| | | *94%* | *94%* | *88%* | *92%* | *96%* | *92%* | *96%* | *90%* | *88%* |
| | Applied Math | −13.1 | −16.8 | −14.0 | −13.4 | −12.1 | −5.8 | −18.7 | −14.0 | −13.9 |
| | | *4%* | *2%* | *8%* | *6%* | *8%* | *6%* | *4%* | *4%* | *7%* |
| | Appl. Math Quadratic | −5.6 | −5.9 | −6.3 | −5.8 | −5.9 | −5.5 | −6.2 | −6.2 | −5.6 |
| | | *46%* | *40%* | *38%* | *31%* | *38%* | *42%* | *35%* | *42%* | *31%* |
| | Word Problems | 4.2 | 5.2 | 5.2 | 4.3 | 4.6 | 4.2 | 4.8 | 4.9 | 30.5 |
| | | *69%* | *63%* | *56%* | *50%* | *60%* | *69%* | *56%* | *58%* | *62%* |
| | School Math | 45.6 | 48.9 | 45.4 | 48.2 | 41.7 | 44.7 | 49.4 | 43.2 | 38.9 |
| | | *100%* | *100%* | *100%* | *100%* | *100%* | *100%* | *100%* | *100%* | *98%* |
| Between-school level | SES | 41.3 | 44.4 | 41.4 | 40.2 | 41.7 | 39.5 | 43.9 | 44.0 | 39.7 |
| | | *98%* | *96%* | *98%* | *100%* | *98%* | *98%* | *100%* | *96%* | *98%* |
| | Applied Math | −56.6 | 6.9 | −49.5 | −75.3 | −51.5 | −40.8 | −27.6 | −80.1 | −9.5 |
| | | *4%* | *6%* | *4%* | *2%* | *6%* | *6%* | *8%* | *2%* | *7%* |
| | Appl. Math Quadratic | −25.4 | −24.4 | −25.5 | −20.9 | −22.1 | −26.5 | −26.0 | −25.2 | −23.3 |
| | | *52%* | *58%* | *56%* | *69%* | *58%* | *54%* | *52%* | *58%* | *71%* |
| | Word Problems | 13.1 | 14.4 | 14.9 | 14.8 | 11.3 | 15.1 | 15.1 | 12.2 | 11.1 |
| | | *67%* | *73%* | *71%* | *56%* | *79%* | *69%* | *65%* | *75%* | *76%* |
| | School Math | 91.7 | 100.9 | 91.1 | 97.2 | 86.2 | 92.0 | 97.8 | 89.0 | 76.0 |
| | | *98%* | *98%* | *98%* | *98%* | *96%* | *98%* | *96%* | *96%* | *98%* |
| Either level | Grade | 92% | 90% | 90% | 83% | 94% | 90% | 92% | 92% | 93% |
| | SES | 98% | 98% | 98% | 100% | 100% | 98% | 100% | 96% | 98% |
| | Applied Math | 8% | 6% | 10% | 8% | 15% | 10% | 13% | 6% | 14% |
| | Appl Math Quadratic | 73% | 71% | 71% | 75% | 77% | 69% | 65% | 77% | 81% |
| | Word Problems | 86% | 88% | 88% | 79% | 86% | 91% | 88% | 88% | 95% |
| | School Math | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 100% | 98% |

[a]Norway, an OECD country, is not included as it did not include the School Math OTL item on the student background questionnaire. Only Partner countries that participated in at least one of the computer-based assessments are included here.

countries/economies. There was also a large estimated effect size present at the school level so clearly School Mathematics was strongly related to performance on the literacy test in the USA. Put another way, the greater the exposure to more advanced mathematics such as algebra and geometry, the higher the predicted performance of 15 year olds in the USA. This relationship is similar in pattern to what is typical in OECD countries but the magnitude of the relationship is more than the OECD average. This would certainly offer a word of caution to those in the USA who are suggesting mathematics should be taught almost exclusively in applied contexts. Even on a ML test with problems presented in real-world contexts, it seems that a solid foundation in School Mathematics is a necessary foundation.

For the USA, the confusing results have not to do with the nature of the mathematics content but with the context in which it is presented. Both Applied Mathematics and Word Problems are indicators of the instructional context in which the mathematics content is presented but this is especially the case for Applied Mathematics. Traditional Word Problems are a type of application but are not well situated in a genuine real-world context, merely dressing up a mathematics problem in an artificial everyday context. For the USA, Word Problems was statistically significantly related to performance but not Applied Mathematics. This pattern is seen in only three other countries in Table 2: Germany, Japan and the Slovak Republic. The lack of a significant relationship to literacy for Applied Mathematics needs a further examination as it is somewhat of a surprising result since Applied Mathematics was significantly related to performance in either the linear or quadratic form at either the student (within-school) or school level in most countries (see Table 3). Why not in the USA and these others?

Figure 6 shows the nature of the non-significant relationship of Applied Mathematics to literacy for the USA. Canada is also represented in the figure for contrast but in that case the relationship was statistically significant. For Canada, the predicted gain in performance as the amount of Applied Mathematics varies between none (0) and sometimes (2) was around 60 points. For the USA, the quadratic pattern was the same but the corresponding predicted gain from the model was only around 20 points. Ignoring the fact that the estimated effect was not significant, the magnitude suggests a relatively weak relationship in the USA.

To examine this issue further, we look at the results for the three states that participated in PISA 2012, Connecticut, Massachusetts and Florida. We again found a strong statistically significant

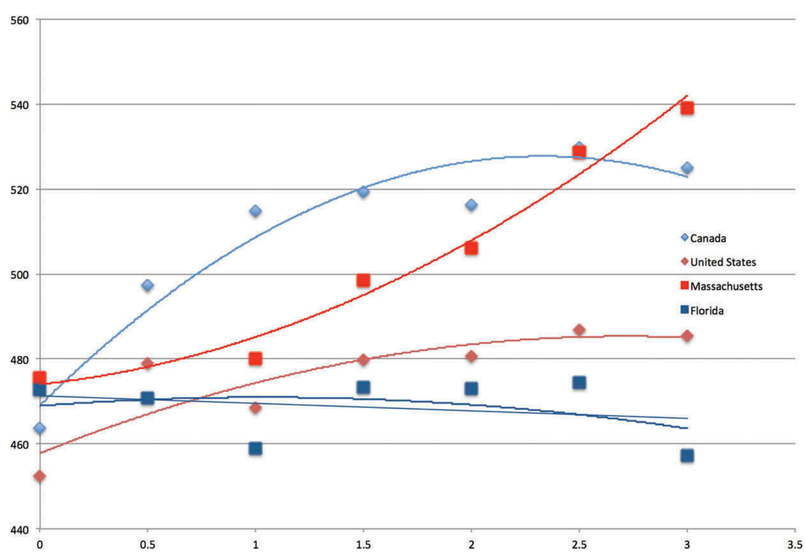


**Figure 6.** Illustration of the nature of the relationship of Applied Mathematics to performance for the USA, Canada, and two states.

relationship between School Mathematics and performance at both the within- and the between-school levels in all three states. The estimated within-school effect sizes varied from 0.62 for Florida to over two-thirds of a standard deviation for both Massachusetts and Connecticut. As to the context of instruction, Word Problems was not significant in any of the three states and neither was Applied Mathematics. In Massachusetts, there was at best a significant relationship between Applied Mathematics and performance. The nature of the relationship is portrayed in Figure 6 along with the USA and Canada.

The non-significant pattern for Florida and Connecticut was essentially the same as the USA, but for Massachusetts the pattern was different where more exposure to Applied Mathematics in their classroom instruction was related to higher performance. Perhaps the different patterns for different states have a 'washing out' effect for the USA as a whole. This is at least one conjecture but with no way of validating it. In addition, it could be that what is considered by US students as 'applied' is not like what is called 'applied' in many other countries.

Given the absence of a significant relationship for Applied Mathematics, the question arises as to whether this might help to explain why the USA does poorly on ML tests such as those in PISA. Perhaps what was noted earlier suggesting that School Mathematics (OTL) is a necessary condition for good performance even on a literacy test, needs to be modified to read 'but not sufficient'. The overall results for OECD would support this as exposure to Applied Mathematics was significant at least at one of the two levels in a majority of the countries adding an additional effect to performance even after controlling for School Mathematics. The reason this was not the case in the USA is not clear but future studies using an even more robust Applied Mathematics index may provide greater insight.

In 2016, the newest PISA study results were released. This study focused on science but, as is the PISA design, it also included test results in mathematics but did not include the OTL data that were available in 2012. The US performance was once again statistically significantly below the OECD average in mathematics, as it was in 2012.

At the same time, the results of the 2015 Trends in Mathematics and Science Study (TIMSS) study were released providing a unique opportunity in which to view the results presented in this paper. Since each study's data represent a random sample of students—8th graders (TIMSS) or 15 year olds (PISA, most of which were in 10th grade)—we have in the same year the performance of US students on two different types of tests; something which is totally unique. Earlier in the background section we noted that OTL had been related to performance on school curriculum-based tests such as TIMSS but had not been measured in relation to literacy tests until PISA 2012. The results presented in this paper show that for the USA, OTL related to Applied Mathematics is not related to performance on PISA.

We hypothesized that the absence of such a relationship might be related to why the US performance on PISA would be below the OECD average. Assuming that the 2012 portrayal of OTL in the USA was still reasonably accurate in 2015 and that our hypothesis is correct, we would have predicted that the same result would occur in PISA 2015, and it did. But we also have the results on a more school-based test for the country's eighth-grade students (2 years earlier for most 15-year-old students). For the USA, on a test where Applied Mathematics would play less of a role, and given the positive relationship of School Mathematics, we would have predicted a stronger performance on TIMSS, which was the case—US eighth-grade performance was statistically significantly above the TIMSS international average.

What this suggests is that the absence of a relationship of Applied Mathematics to literacy could be constraining overall performance on PISA for 15 year olds, but that on a more curriculum-based test given in the same year to eighth-grade students where School Mathematics would play a stronger role, the US performance was above the international average. These are demographically the same US students, typically 2 years apart in school (for most) and yet they perform differently on the two types of tests. The best performance occurs on the assessment for which School

Mathematics would likely play the larger role. This seems logical but will have to remain a hypothesis as the causal structure cannot be investigated with the available data.

## Discussion and conclusion

The grounding of the PISA ML assessment in authentic real-world contexts provides an important window on how ready students are to tackle the situations and problems that await them upon completion of their secondary schooling whether they intend to pursue further education beyond high school or intend to go directly into the labour force. As evidenced by the released PISA items, both the mathematics content and the mathematical processes required to correctly solve these authentic problems represent the type of competencies that are in demand and required both in the workplace and in higher education.

Furthermore, in real life, no one hands a well-defined problem consisting only of numbers and some operation to a worker who is then told to solve the problem. Today we have calculators and computers to handle such situations. The real challenge is to discern from the situation what the relevant parameters or numbers are and to figure out how to formulate them to arrive at a suitable solution. The overall PISA literacy score together with the seven literacy-oriented sub-scores provides a nuanced indication of how ready students are to successfully complete the real-world authentic problems they may well encounter in their personal, social or occupational contexts.

The analyses presented here suggest that the rigor of the mathematics curriculum students' encounter in their K-12 schooling is related to their literacy performance. School Mathematics was the only OTL variable that was consistently statistically significantly related to all the literacy measures at both the within and the between-school levels in virtually every OECD country. Furthermore, as all the OTL variables were in the same metric, a comparison of their estimated parameters as evidenced in Tables 2 and 3 reveals that the estimated School Mathematics effect is far greater than all the other OTL factors combined.

Also, evident in more than two-thirds of PISA countries (economies), having opportunities to encounter challenging mathematics in an applied context was significantly related to PISA performance. The nature of the relationship, however, was quadratic implying that beyond a certain point further exposure to Applied Mathematics was negatively related to performance. Thus, the instructional context, how students encounter and practise mathematics in their schooling, is also important and if better measured (given the limited number of OTL items) it might be important even in those countries where these context OTL factors failed to demonstrate a statistically significant relationship. Some may want to apply a compositional effect explanation to the observed quadratic Applied Mathematics effect, i.e. only the poorest students receive the most Applied Mathematics instruction. However, the nature of the regression analysis used here controls for such an alternative explanation. In the model, the effect of Applied Mathematics is computed while holding constant values for other variables in the model. In other words, the quadratic effect of Applied Mathematics holds for classrooms with low values for School Mathematics and, hence lower literacy scores, as well as for classrooms with high values for School Mathematics and higher literacy scores. In any case, we caution that although the Applied Mathematics index is the same as that included in the PISA 2012 report, it is based on only two items in the student background survey (pages 149–151, OECD, 2013b) and should not be over interpreted. Future studies may well be able to provide greater insight into the non-linear relationship that this OTL index demonstrated with performance if more items were used to measure the Applied instructional context.

The strong relationship between School Mathematics and ML is particularly relevant to the USA as so many states have adopted the CCSSM in an effort to equip more students to be both college- and career-ready by the time they complete high school. One of the consequences of the adoption of the CCSSM in the USA is that states have become more focused on students completing high school being both college- and career-ready. This focus on career readiness in addition to college readiness makes PISA quite pertinent for the USA. The PISA focus on ML is readily applicable to a range of career paths that do not require a higher education degree but also to those that do but which do not require advanced mathematics. The substantial relationship between students' learning opportunities, particularly with School Mathematics, and their ML performance underscores the importance of students' schooling. Yet policy and education efforts cannot wait until the end or near the end of secondary education to focus attention or effort on the lack of students' mathematics competencies. The competencies of interest do not develop solely as a result of secondary education but grow out of the foundational competencies developed beginning in the early years of schooling.

Mathematics is a coherent discipline, perhaps uniquely so, with concepts and topics having important logical and conceptual connections. These connections are essential for students to grasp in order to truly understand and be able to flexibly apply the mathematics needed to address the situations, problems and challenges they encounter in their everyday world as workers, citizens and consumers. Essential to developing this understanding is a quality mathematics curriculum, one that reflects the coherence of the discipline, building concept upon concept, competency upon competency, from one year of schooling to the next (Schmidt, Wang, & McKnight, 2005).

The consequence of a curriculum that does not do this or does not ensure that students master one year's competencies before tackling the next year's has grave repercussions for students. Small deficiencies in mathematics competence quickly accumulate over the years. This is evidenced by the great need for remedial education in the USA after students have completed high school. One state's report indicated that 37% of the students completing high school required some remedial education and that most often they required remedial mathematics education (Colorado Department of Higher Education, 2014). The report also included longitudinal data that indicated this was down from the 41.4% of students requiring remedial education 2 years earlier. These findings are not atypical for the USA, and they come from one of the few states that has instituted a concerted effort to reduce this phenomenon.

The large and consistent significant relationship of School Mathematics, a measure of the extent to which students have encountered rigorous mathematics, to all of the various aspects of ML as measured by PISA suggests that schooling in the form of a rigorous mathematics curriculum is related to being college- or career-ready, i.e. mathematically literate. Since School Mathematics measures not only the number of mathematics topics to which students have been exposed but also the rigor of the mathematics as defined by the IGP index, this result is particularly striking. In a follow-up analysis, we found that the complexity or rigor of the topics was equally important to the number of topics in predicting literacy performance. This is somewhat surprising given that the PISA literacy test is mostly made up of items where the mathematics needed for their solution does not typically require anything beyond basic algebra or geometry. Perhaps, and this can only be taken as a hypothesis, it is the underlying structure and logic of mathematics that is related to PISA performance and having exposure to more advanced mathematics such as exponential and quadratics functions, vectors and trigonometry provides an even more challenging context in which to use that structural knowledge. Such experiences may well help to develop a greater fluency for formulating, employing and interpreting quantitative challenges encountered in the real world. Verifying this hypothesis through additional research has important implications for classroom instruction and the development of students' ML.

## Notes

1. Full details on the PISA sampling design and the corresponding sampling weights are presented in Chapters 4 and 8 of the PISA 2012 Technical Report (OECD, 2014b).
2. The PISA report (OECD, 2013b) provides statistics for all PISA participants including the OTL indices but not for the IGP-weighted version of the School Mathematics variable used here. Reliabilities of the OTL indices are reported in Table 16.20 for all OECD countries in the PISA 2012 Technical Report (OECD, 2014b, p. 325). Applied Math (EXAPPLM) reliabilities ranged from 0.69 to 0.82. The OECD median reliability was 0.77. The reliabilities for School Mathematics (FAMCON in Table 16.20) ranged from 0.81 to 0.91. The OECD median

reliability was 0.85. Since the School Mathematics used here is a linear transformation of FAMCON in Table 16.20, the reliabilities are the same. No reliability is reported for Word Problems as this index consists of a single item.
3. Only the weighted means for the variables used in this paper are listed in Table 1. Standard errors and other statistical measures are included in the first report volume and the PISA 2012 Technical Report volume (OECD, 2013b, 2014b). All OECD countries are listed except for Norway as Norway did not include the School Math item on their student questionnaire. For brevity's sake, only those non-OECD countries that participated in at least one of the computer-based assessments are included in Tables 1–3.
4. The percentage of the PISA student sample in each grade is presented in Table A2.4a on page 274 of the PISA 2012 Technical Report (OECD, 2104b). The PISA student sample design which includes a discussion of grade level is on page 267. Page 261 explains the creation and use of the PISA modal grade variable.
5. The term 'effect' is used here consistent with its classical ANOVA meaning. It is not meant to be interpreted in a causal way.
6. The PISA SES index is made up of three other indices: home possessions (itself a composite of four other scales), highest parental education, and highest parental occupation. See page 351 and following of the PISA 2012 Technical Report for a full discussion of this (OECD, 2014b).

## Disclosure statement

No potential conflict of interest was reported by the authors.

## Funding

This work was supported by the Thomas J. Alexander Fellowship.

## Notes on contributors

***Leland S. Cogan*** is a Senior Researcher with the Center for the Study of Curriculum in the College of Education at Michigan State University. His research interests focus on mathematics and science curricula and the preparation of teachers to teach these subjects in schools.

***William H. Schmidt*** is a University Distinguished Professor of statistics and education at Michigan State University. His current work is focused on educational policy related to mathematics, science and testing in general.

***Siwen Guo***, is a PhD candidate in measurement and quantitative methods at Michigan State University. Her research interests focus on the application of quantitative methods to large-scale datasets and policy research, and the effect of mathematics curriculum on student performance.

## ORCID

William H. Schmidt http://orcid.org/0000-0002-7933-5457

## References

Achieve. (2004). *Do graduation tests measure up? A closer look at state high school exit exams*. Washington, DC: Author.

Blum, W., & Niss, M. (1991). Applied mathematics problem solving, modelling, applications, and links to other subjects—state, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, *22*(1), 37–68.

Carroll, J. B. (1963). A model of school learning. *Teachers College Record*, *64*(8), 723–733.

Cheema, J. R. (2015). The private–public literacy divide amid educational reform in Qatar: What does PISA tell us? *International Review of Education*, 61(2), 173–189.

Cogan, L. S., & Schmidt, W. H. (2015). The concept of opportunity to learn (OTL) in international comparisons of education. In K. Stacey & R. Turner (Eds.), *Assessing Mathematical Literacy: The PISA Experience* (pp. 207–216). New York: Springer.

Cogan, L. S., & Schmidt, W. H. (in press). Measuring opportunity: Two perspectives on the "black box" of school learning. In L. E. Suter, B. Denman, & E. Smith (Eds.), *The SAGE Handbook of Comparative Studies in Education*. London: Sage.

Cogan, L. S., Schmidt, W. H., & Wiley, D. E. (2001). Who takes what math and in which track? Using TIMSS to characterize US students' eighth-grade mathematics learning opportunities. *Educational Evaluation and Policy Analysis*, 23(4), 323–341. doi:10.3102/01623737023004323

Colorado Department of Higher Education. (2014). (2013 Legislative Report on Remedial Education). (pp. 67). Denver, CO: Author.

de Lange, J. (1996). Using and applying mathematics in education. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education, Part 1* (pp. 49–97). Dordrecht: Kluwer.

de Lange, J. (2003). Mathematics for literacy. In B. L. Madison & L. A. Steen (Eds.), *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges* (pp. 75–89). Princeton, NJ: The National Council on Education and the Disciplines.

Donoghue, E. F. (2003). Algebra and Geometry Textbooks in Twentieth-Century America. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A History of School Mathematics* (Vol. 1, pp. 329–398). Reston, VA: National Council of Teachers of Mathematics.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan & J. D. Bransford (Eds.), *How Students Learn: History, Mathematics, and Science in the Classroom* (pp. 217–256). Washington, DC: Committee on How People Learn, A Targeted Report for Teachers, Division of Behavioral and Social Sciences and Education, National Academy Press.

Gelman, R., & Greeno, J. G. (1989). On the nature of competence: Principles for understanding in a domain. In L. B. Resnick (Ed.), *Knowing, Learning, and Instruction: Essays in Honor of Robert Glasser* (pp. 125–186). Hillsdale, NJ: Lawrence Erlbaum.

Ginsburg, H. P., Cannon, J., Eisenband, J., & Pappas, S. (2008). Mathematical Thinking and Learning. In K. McCartney & D. Phillips (Eds.), *Blackwell Handbook of Early Childhood Development* (pp. 208–229). Oxford, UK: Blackwell Publishing Ltd.

James, W. (1983). *Talks to Teachers on Psychology and to Students on Some of Life's Id*eals. Cambridge, MA: Harvard University Press. Original work published 1904.

Jürges, H., Schneider, K., Senkbeil, M., & Carstensen, C. H. (2012). Assessment drives learning: The effect of central exit exams on curricular knowledge and mathematical literacy. *Economics of Education Review*, *31*(1), 56–65.

Kliebard, H. M., & Franklin, B. M. (2003). The ascendance of practical and vocational mathematics, 1893-1945: Academic mathematics under Siege. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A History of School Mathematics* (Vol. 1, pp. 399–440). Reston, VA: National Council of Teachers of Mathematics.

Lappan, G., & Wanko, J. J. (2003). The changing roles and priorities of the Federal Government in Mathematics Education in the United States. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A History of School Mathematics* (Vol. 2, pp. 897–930). Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2005). *How students learn: history, mathematics, and science in the classroom*. Washington, DC: Committee on How People Learn, A Targeted Report for Teachers, Division of Behavioral and Social Sciences and Education, National Academy Press.

Nonoyama-Tarumi, Y., & Willms, J. D. (2010). The relative and absolute risks of disadvantaged family background and low levels of school resources on student literacy. *Economics of Education Review*, *29*(2), 214–224.

OECD. (1999). Measuring student knowledge and skills (*A New Framework for Assessment)*. (pp. 85). Paris: Author.

OECD. (2009). PISA Data Analysis Manual: SAS® (2nd edn). p. 470. Paris: Author.

OECD. (2013a). PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy (pp. 264). Paris: Author.

OECD. (2013b). PISA 2012 Results: What Students Know and Can Do – Student Performance in Mathematics, Reading and Science (Vol. I). Paris: Author.

OECD. (2014a). PISA 2012 Results: Creative Problem Solving: Students' Skills in Tackling Real-Life Problems (pp. 254). Paris: Author.

OECD. (2014b). (*PISA 2012 Technical Report)* (pp. 469). Paris: Author.

Paulhus, D. L., Harms, P. D., Bruce, M. N., & Lysy, D. C. (2003). The over-claiming technique: Measuring self-enhancement independent of ability. *Journal of Personality and Social Psychology*, *84*(4), 890–904. doi:10.1037/0022-3514.84.4.890

Schmidt, W. H., Burroughs, N., Zoido, P., & Houang, R. T. (2015). The role of schooling in perpetuating educational inequality: An international perspective. *Educational Researcher*, *44*, 371–386. doi:10.3102/001389X15603982

Schmidt, W. H., & Cogan, L. S. (2009). The myth of equal content. *Educational Leadership*, 67(3), 44–47.

Schmidt, W. H., Cogan, L. S., Houang, R. T., & McKnight, C. C. (2011). Content coverage differences across districts/states: A persisting challenge for U.S. education policy. *American Journal of Education*, 117(3), 399–427.

Schmidt, W. H., Cogan, L. S., & McKnight, C. (2010). Equality of educational opportunity: Myth or reality in US schooling. *American Educator*, 34(4), 12–19.

Schmidt, W. H., & Maier, A. (2009). Opportunity to learn. In G. Sykes, B. Schneider, & D. N. Plank (Eds.), *Handbook of Education Policy Research* (pp. 541–559). New York: American Educational Research Association, Rutledge.

Schmidt, W. H., & McKnight, C. (2012). Inequality for all: The challenge of unequal opportunity in American schools. New York: Teachers College Press.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H. A., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. (2001). *Why schools matter: A cross-national comparison of curriculum and learning*. San Francisco: Jossey-Bass.

Schmidt, W. H., Wang, H. A., & McKnight, C. C. (2005). Curriculum coherence: An examination of U.S. mathematics and science content standards from an international perspective. *Journal of Curriculum Studies*, 37(5), 525–559.

Smith, J. P., III. (1999). Tracking the mathematics of automobile production: Are schools failing to prepare students for work? *American Educational Research Journal*, *36*(4), 835–878.

Stacey, K. (2015). The real world and the mathematical world. In K. Stacey & R. Turner (Eds.), *Assessing Mathematical Literacy: The PISA Experience* (pp. 57–84). New York: Springer.

Stacey, K., & Turner, R. (2015). The evolution and key concepts of the PISA mathematics frameworks. In K. Stacey & R. Turner (Eds.), *Assessing Mathematical Literacy: The PISA Experience* (pp. 5–33). New York: Springer.

Thien, L. M. (2016). Malaysian students' performance in mathematics literacy in PISA from gender and socioeconomic status perspectives. *The Asia-Pacific Education Researcher*, *25*(4), 657–666.

Thorndike, E. L. (1913). *Educational Psychology: The Psychology of Lea*rning (Vol. 2). New York: Teachers College.

U.S. Department of Education. (2010). College- and Career-Ready Standards and Assessments (Excerpts from *A Blueprint for Reform: The Reauthorization of the Elementary and Secondary Education Act (ESEA*), p. 16). Washington, DC: Author. Retrieved from www2.ed.gov/policy/elsec/leg/blueprint/faq/college-career.pdf.