Journal of Curriculum Studies

ISSN: 0022-0272 (Print) 1366-5839 (Online) Journal homepage: http://www.tandfonline.com/loi/tcus20

Curriculum spaces for connecting to children's multiple mathematical knowledge bases

Tonia J. Land, Tonya G. Bartell, Corey Drake, Mary Q. Foote, Amy Roth McDuffie, Erin E. Turner & Julia M. Aguirre

JOURNAL OF CURRICULUM STUDIES, 2018
https://doi.org/10.1080/00220272.2018.1428365

Curriculum spaces for connecting to children's multiple mathematical knowledge bases

Tonia J. Land[a], Tonya G. Bartell[b], Corey Drake[c], Mary Q. Foote[d], Amy Roth McDuffie[e], Erin E. Turner[f] and Julia M. Aguirre[g]

[a]Department of Teaching and Learning, Drake University, Des Moines, IA, USA; [b]Department of Curriculum, Instruction, and Teacher Education, Michigan State University, East Lansing, MI, USA; [c]Department of Teacher Education, Michigan State University, East Lansing, MI, USA; [d]Department of Elementary and Early Childhood Education, Queens College, CUNY, Flushing, NY, USA; [e]Department of Teaching and Learning, University of Washington, Tri-Cities, Richland, WA, USA; [f]Department of Teaching, Learning and Sociocultural Studies, University of Arizona, Tucson, AZ, USA; [g]School of Education, University of Washington-Tacoma, Tacoma, WA, USA

ABSTRACT
Elementary mathematics curriculum materials can serve as a lever for instructional change. In this paper, we promote a particular kind of instructional change: supporting teachers in learning to integrate children's multiple mathematical knowledge bases (MMKB), including children's mathematical thinking and children's home and community-based mathematical funds of knowledge, in instruction. A powerful means of supporting pre-service teachers in integrating children's MMKB in instruction may be to scaffold teachers' noticing of potential spaces in elementary mathematics curriculum materials for connecting to children's MMKB and then developing practices for leveraging these spaces during instruction. We focus on existing and potential spaces in written curriculum materials, or curriculum spaces, so as to better support teachers in enacting curriculum that opens spaces for connecting to children's MMKB.

KEYWORDS
Elementary education; elementary mathematics; curriculum materials; children's mathematical thinking; children's funds of knowledge

Introduction

Mathematics classrooms should provide opportunities for *all students* to learn and participate in mathematics (Civil, 2012; National Council of Teachers of Mathematics [NCTM], 2014). Long-standing patterns document, however, that historically marginalized K-12 (ages 5–18) students have fewer opportunities to learn and participate in mathematics due to dominant cultural and school structures (Berry, Pinter, & McClain, 2013; Civil & Planas, 2004; Flores, 2007). Research in contexts within and outside of the U.S. demonstrates that classrooms engaged in pedagogy that draws on students' family and community funds of knowledge (knowledge gained from being a member in a community, described in greater detail below) can support more students, and particularly those historically marginalized, with greater opportunity to learn mathematics (e.g. Andrews & Yee, 2006; Borden, 2013; Civil, 2012; Ewing, 2012; Hogg, 2016; Lipka, 2002; Poirier, 2005; Turner & Celedon-Pattichis, 2011). In other words,

CONTACT Tonia J. Land tonia.land@drake.edu

'students' social, cultural, and linguistic backgrounds are valued, reflected, and key to their academic advancement' (Civil, 2012, p. 43).

At the same time, commercial curriculum materials are a prominent feature of most mathematics classrooms. Ball and Cohen (1996) argued curriculum materials can serve as a lever for large-scale change, given their ubiquitous use and the centrality of the materials in most teachers' instructional planning. Their argument is supported by research that establishes curriculum can support students' learning of rigorous mathematics (Stein, Remillard, & Smith, 2007; Tarr et al., 2008); teachers rely heavily on curriculum materials as novices (Grossman & Thompson, 2008); and curriculum materials can be educative (Davis & Krajcik, 2005; Remillard, 2005). Curriculum materials can, however, be constraining in that they are written for a general audience (Ebby et al., 2011), which could further privilege dominant views.

Little research exists that considers these ideas in tandem—either in mathematics or in other content areas. That is, how might curriculum materials support teachers' development and enactment of instruction that builds on students' family and community-based funds of knowledge? The goal of TEACH Math is to support teachers in learning to integrate into their teaching their knowledge of (1) mathematics, (2) children's mathematical thinking, and (3) children's family and community-based funds of knowledge. We refer to (2) and (3) as children's *multiple mathematical knowledge bases* (MMKB; Aguirre et al., 2012; Turner et al., 2012). As noted above, research documents that to support students' learning, teachers need to build connections with their students, families and communities and draw on these connections in mathematics teaching (Civil, 2007; Ewing, 2012; Lipka et al., 2005; Meaney & Evans, 2013). Research *also* documents the importance of drawing on children's mathematical thinking to support their learning (e.g. Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). MMKB aims to integrate these foci and is defined as 'the understandings and experiences that have the potential to shape and support children's mathematics learning—including children's mathematical thinking, and children's cultural, home, and community-based knowledge' (Turner et al., 2012, p. 67). We argue that a powerful means of integrating children's MMKB into instruction is to scaffold teachers in (a) identifying the range of spaces in elementary mathematics curriculum materials for connecting to children's MMKB and (b) developing practices for leveraging these spaces.

One challenge to this argument, however, is that schools and districts across the United States use a wide range of mathematics curriculum series with different designs and pedagogical approaches. Further, there is little predictability in which curriculum series teachers will be able to access, as decisions about the use of curriculum materials are most often made at the district level. Thus, supporting teachers in learning to notice and leverage curriculum spaces is only a productive approach if these spaces exist across a range of curriculum series and if teachers have strategies that are 'curriculum proof' (Taylor, 2013). That is, teachers must have strategies to notice and leverage spaces across curriculum series as opposed to being limited to strategies that work for just one particular series. Given this broader goal, our purpose in this study was to analyse a wide range of elementary (5- to 12-year olds) mathematics curriculum materials to understand (a) whether these spaces exist across all materials and (b) the nature of those spaces.

Theoretical framework

We ground our study in the theoretical construct of third space (Moje et al., 2004). Using hybridity theory as a frame (Bhabha, 1994; Soja, 1996), Moje and colleagues (2004) argued that people use multiple resources or funds to make sense of their world and that being 'in-between' different funds of knowledge can be both productive and limiting to individuals. Moje and colleagues (2004) denoted this third space as a place where individuals integrate multiple funds of knowledge. In third space, knowledge and discourses drawn from an individual's home, community and peer networks are merged with knowledge and discourses drawn from more formalized institutions, such as work, school and church (Moje et al., 2004). Moje and colleagues discussed three different perspectives on third space. First, third space is 'a way to build bridges from knowledges and Discourses often marginalized in school settings to the learning of conventional academic knowledges and Discourses' (Moje et al., 2004, p. 43). Second, third spaces can be navigation spaces where students can work between and succeed in both their first and second spaces. Last, third space can be space where 'competing knowledge and Discourses of different spaces are brought into 'conversation' to challenge and reshape both academic content literacy practices and the knowledges and Discourses of youths' everyday lives' (Moje et al., 2004, p. 44).

We draw primarily from the first perspective of third space as a bridge for students to use their MMKB to learn formal mathematics. In prior work, we have argued for the importance of teachers learning to connect to children's MMKB (Aguirre et al., 2012; Turner et al., 2012). In particular, we draw on research documenting that teachers need to provide children with opportunities to make sense of mathematics and develop their own strategies for solving mathematical problems (Carpenter, Fennema, & Franke, 1996; Carpenter et al., 1989). This work has shown that increased teacher knowledge about children's strategies for solving different types of mathematical problems supports increased student learning and achievement. Similarly, work related to 'funds of knowledge' (González, Moll, & Amanti, 2005) has demonstrated the ways in which a strengths-based approach to understanding and connecting to children's 'life experiences' (González, Moll, & Amanti, 2005, p. 72) can support children's learning. More specifically, research on teachers' connections to children's mathematical funds of knowledge (e.g. Civil, 2002; Foote, 2009; González, Andrade, Civil, & Moll, 2001) supports the claim that connecting to children's home and community-based mathematical experiences, interests and practices can support children's learning of school mathematics. This claim is particularly important when considering the learning of children from underrepresented groups, as their mathematical knowledge is especially unlikely to be prominently represented in, or connected to, the school mathematics curriculum.

In this paper, we focus on spaces in curriculum materials. We call these designed spaces in curriculum materials *curriculum spaces* and conceptualize them as potential third spaces. That is, we examine *curriculum spaces* in written curriculum materials that have the potential to serve as a bridge for students to draw on their MMKB in accessing rigorous mathematics. In our analyses, we identified the range of ways in which real-world connections and other design features were used in curriculum materials to create spaces for children's MMKB and we conjectured about the range of effectiveness, or openness, of these spaces in accomplishing that goal.

Literature review

To further inform our work, we draw from the literature around teachers' curriculum use, which includes activities associated with curriculum use and the nature of the teacher–curriculum relationship. Second, we use equity frameworks (Civil, 2012; Guitierrez, 2007) to discuss the learning opportunities required so *all* students have access to rigorous mathematics.

Teachers' use of curriculum materials

Sherin and Drake (2009) developed a *curriculum strategy framework* outlining three primary ways teachers interact with curriculum: reading, evaluating and adapting. Teachers *read* to understand the information in the curriculum materials; they *evaluate* these materials in relation to their goals, both in planning and during a lesson-in-progress; and they *adapt* materials based on their reading and evaluation. Further, there are regular patterns in the ways in which teachers engage in these practices, suggesting that, 'teachers' use of reform-based materials, even in their first year, is not haphazard' (p. 490).

Brown (2009) focused his analysis of teacher–curriculum interactions on the construct of 'pedagogical design capacity', defined in part as teachers' capacity to 'perceive and mobilize' instructional resources. Pedagogical design capacity has implications for professional development, which 'should help teachers link their instructional goals to the specific features and affordances of curriculum materials, and should support teachers in making the necessary design modifications required to achieve this alignment' (Brown & Edelson, 2003, p. 6).

Remillard's (2005) work has been particularly influential in helping us understand the nature of the relationship between teachers and curriculum materials and significant subsequent work has investigated the various teacher characteristics that both influence and are influenced by teachers' interactions with curriculum materials (Lloyd, 2008; Sherin & Drake, 2009). Much less research, however, has attended to the contributions of the design features of the materials themselves to these interactions, despite the following caution from Brown (2002):

> Reforms that focus just on the design of curriculum materials … overestimate the capacity of curriculum materials to communicate and convey the means for accomplishing classroom innovations, and reforms that focus just on the development of teacher capacity … underestimate the capacity of curriculum materials to do the same. (p. 26)

Similarly, little research has attended to the ways in which curriculum materials can be used to support the enactment of a particular kind of instruction—i.e. instruction integrating children's MMKB. We addressed these needs by closely analysing the features of elementary mathematics curriculum materials teachers might 'perceive and mobilize' to design and enact instruction connecting to children's MMKB.

Equitable access to rigorous curriculum

Long-standing focus on equity in mathematics education can be traced back to at least the 1980s (e.g. Powell, 2012). Little progress, however, has been made to address the inequitable distribution of access to mathematics instruction (Lee, 2012; McGraw, Lubienski, & Strutchens, 2006). Instead of framing the disparities in mathematics education in terms of an 'achievement gap', we join others in reframing disparities as the unfair distribution of opportunities to learn, or access, mathematics (e.g. Esmonde, 2009; Gresalfi & Cobb, 2006). Addressing equity in this way shifts the focus of mathematics education away from the remediation of particular groups towards supporting mathematics learning for these groups (Martin, 2009; Moschkovich, 2010).

A key component of students' opportunity to learn is *access*, which relates to the resources and practices that enable students to participate in mathematics (Guitierrez, 2007). Access for students includes opportunities to engage in 'high level tasks' (Smith & Stein, 1998) with multiple entry points that support students in reasoning, explaining and justifying their mathematical thinking. Furthermore, because some students experience the need to 'down play some of their personal, cultural, [and] linguistic capacities to participate in the classroom or the math pipeline', it is imperative that students 'see themselves in the curriculum' (Guitierrez, 2007, p. 3). Seeing oneself in the curriculum goes beyond 'real world' connections as often defined by textbooks or teachers to include mathematics meaningful to students' lives, such as students drawing upon their cultural and linguistic resources and forms of reasoning and problem-solving in out-of-school contexts (Civil, 2012; Guitierrez, 2007).

Drawing on students' out-of-school experiences and/or creating hybrid spaces is a pedagogical strategy used across disciplines and international community groups with a focus on equity. For example, Barton and Tan (2009), working with a teacher and four students in a low-income urban middle school, designed science instruction around food and nutrition. Through this process, the researchers found that students drew from their knowledge of family life, nutritional habits and food preparation to make sense of the scientific concepts. Ramnarain and de Beer (2013) found that ninth grade students in South Africa were able to create hybrid spaces when provided an opportunity by their science teacher to conduct an open science investigation. For instance, one student (Susan) merged her identities as AIDs activist and novel scientist to answer the question, 'How can fruits and vegetables be preserved for AIDs sufferers in rural areas?' (Ramnarain & de Beer, 2013). In New Zealand, the education ministry strives to honour the indigenous Māori people by having a bicultural curriculum for young children (Hedges & Cooper, 2016) where teachers are invited to 'construct curriculum pertinent to the children, families, and communities of each centre' (Hedges & Cooper, 2016, p. 304). Research in this context has documented teachers' application of students' funds of knowledge to support student learning (e.g. Hogg, 2016). Further, research in the United States integrating Yu'pik culture and everyday experience to support mathematics learning (Lipka, 2002; Lipka et al., 2005), research in Canada demonstrating how understanding of Mi'kmay language supports Mi'kmaw learners in mathematics (Borden, 2013) and research in the UK (Andrews & Yee, 2006) as well as Torres Strait Islands (Ewing, 2012) aimed at recognizing and valuing funds of knowledge in traditionally marginalized communities to support student learning demonstrates a myriad of ways connecting to students funds of knowledge and/or creating hybrid spaces in instruction can support student learning.

Given these arguments, work identifying how curriculum materials can support teachers in providing spaces for students to draw on MMKB is vital. Recently, Hoover, Mosvold, Ball, and Lai (2016) observed that most equity studies focused on arguments for equitable teaching and that 'few studies focused directly on specific practices of equitable mathematics teaching or knowledge for equitable mathematics teaching' (p. 26). The researchers argued that an increased focus around equity, and in particular equitable practices, is crucial. We agree with these arguments and suggest that understanding how these practices can be enacted in the context of using curriculum materials is also critical. The following research questions guided our work:

(1) Do curriculum spaces for connecting to children's MMKB exist across a range of elementary mathematics curriculum materials?
(2) If so, what is the nature of these spaces?

Research methods

Data collection

Three sets of lessons from 11 different curriculum series, chosen to represent a range of material designs and approaches, comprised our data-set. We chose one lesson (grades 2–4, ages 7–9) from each series' textbook focused on introduction of fractions, multi-digit addition and single-digit multiplication. We chose lessons pertaining to the introduction of fractions because this topic has been identified as a potentially rich place where students can use their home-based knowledge of fractions, and particularly of fair-sharing situations, to develop strategies (Empson & Levi, 2011). We chose multi-digit addition lessons because it comprises a large component of elementary mathematics, and significant research exists around students' solutions for multi-digit addition that can be integrated into curriculum series (e.g. Carpenter, Fennema, Franke, Levi, & Empson, 2014). Finally, single-digit multiplication was chosen, as it is a prominent topic within elementary mathematics.

Code development

The first and third authors began coding by reading and analysing the fraction lessons and identifying possible curriculum spaces. In doing this, we considered research related to children's MMKB and looked primarily for spaces that might support students in developing and making use of their MMKB, including spaces that presented potential opportunities to bridge the learning of textbook mathematics with outside of school experiences. Through this process, three types of spaces were identified, and became first level codes: (1) spaces for real-world connections, (2) spaces for student strategies and (3) spaces for student explanations.

Spaces for real-world connections are places that refer to real-world contexts to support mathematics learning. Spaces for student strategies are places where students develop and/or use strategies to solve a task. Spaces for student explanations are instances in which students are asked to describe and/or explain a strategy. The three primary codes are listed in Table 1. Secondary codes, listed below each primary code, are also defined.

Spaces for real-world contexts code

We used a process of open and emergent coding (Strauss & Corbin, 1998) to develop secondary codes for real-world contexts because we noticed that real-world contexts were treated differently across lessons and curriculum series—as others have also found (e.g. Meyer, Dekker, & Querelle, 2001). For instance, in one lesson, party cups were used to illustrate an array, a rather superficial use of a real-world context of a party because prior experiences with parties or party cups are unlikely to provide access to the mathematics of arrays; whereas in another lesson, students were explicitly asked to use their prior experiences with fair sharing contexts (elicited through prompting and visualization) to find fractional parts.

Table 1. Coding scheme for curriculum spaces.

1. Real-World Connections—places within the materials that refer to real-world contexts to support students in learning mathematics• Replace—Real-world objects replace another manipulative
- Single Space—A single real-world connection is made by the textbook
- Open Space—Children have space to make their own real-world connections to the mathematics (that is, they are prompted to draw on their MMKB)
- No Math—No math is discussed in the connection (e.g. a connection to a social studies concept)

Each of the above codes could occur before (B) or after (A) a solution strategy had been presented by the teacher and/or textbook

2. Space for Students' Strategies—an opportunity for students to develop strategies for solving mathematical tasks and/or make sense of mathematics
- Before—Space occurs before teacher/textbook presents a strategy, supporting students in drawing on their MMKB
- After—Space occurs after teacher/textbook presents a strategy
- With teacher support
- Without teacher support

3. Space for Student Explanations—an opportunity for students to explain/describe a strategy
- Open space for students to discuss/explain their own strategy, developed by drawing on their MMKB
- Space for students to discuss/explain a strategy presented by the teacher/textbook.
- With teacher support
- Without teacher support

Secondary codes for real-world contexts were refined during several data passes for a final list of: *replace, single space, open space* and *no math* (Table 1). A *replace* code indicated that there was an object that simply replaced another manipulative (like the party cups described above) and was not used in a way that was likely to help students access or make more sense of mathematics than another manipulative. *Single space* denoted a single real-world connection. A contextualized word problem was the most common single space (e.g. story problem about baseball). Some lessons had one contextualized word problem while others had several either using the same context or not. In either case, the lesson was coded as having a single space because it was same type of space no matter how many times it occurred. While such word problems might connect to some children's MMKB, such as those students with knowledge about the mathematical practices of baseball, *only* the baseball context was presented. A real-world connection was considered an *open space* if there was an opportunity for students to make their own real-world connections. For instance, in one lesson, students were asked to draw on fair sharing experiences to develop strategies for finding fractional parts (TERC, 2008). *No math* was used as a code for instances where there was a real-world connection that did not involve mathematics—like finding facts about Brazil and its capital (UCSMP, 2007).

Spaces for students' strategies

Spaces for student strategies were coded as *before* and *after*. If the space for a student-developed solution strategy occurred *before* the textbook presented a solution strategy, the space was coded as *before*; the space was coded as *after* if student development and use of strategies occurred *after* a solution strategy was presented in the textbook. If the space for students to develop and use strategies to solve problems comes *after* the teacher/textbook has presented a strategy, then students will most likely use the textbook strategy instead of developing their own. Conversely, if the space for a student strategy occurs *before* (or instead of) a teacher/textbook strategy, students are more likely to create and/or develop their own strategies. Additionally, we coded for the presence of teacher supports related to facilitating and making sense of students' strategies.

Spaces for student explanations

For *student explanations* spaces, we created secondary codes of *open* and *closed*. We considered the space *open* if students were provided an opportunity to discuss or explain the strategies that they developed and used. The space was *closed* if students were prompted to explain the strategy given by the textbook. This distinction was again important for students' opportunities to access and use their MMKB. If students are required to explain a textbook strategy, there is limited space for students to access their MMKB. Again, we also coded these spaces for the presence of teacher supports.

Focused coding

After final codes were established (Table 1), the first three authors coded all 33 lessons using focused coding (Saldana, 2013). Two researchers coded each lesson so that each researcher could check for consistency and to ensure all curriculum spaces were identified. Each researcher also coded two of the three lessons from each series. After all lessons were independently coded, coders met to resolve differences. For these identified spaces, we analysed *where* the spaces occurred; and *what* kinds of spaces, open or closed, were provided. To determine *where*, we recorded whether the spaces occurred in the main lesson or the peripherals (e.g. teaching notes, differentiation activities, homework). To determine *what* kind of space, open or closed, we noted the secondary codes, but also noted activities before and after as well as text features related to the space that seemed to contribute to the nature of the space.

Findings

While curriculum spaces exist in all the curriculum series, we found that the nature of those spaces differed across the series in terms of the specific features of the spaces, the sequences of spaces within lessons, the locations of spaces and supports provided to teachers. Specifically, we had three major findings: (1) almost all (97%) lessons provided real-world connections; however, the connections were not always a means to connect to children's MMKB; (2) there were three types of curriculum spaces (open curriculum spaces, conflicting curriculum spaces and closed curriculum spaces) for students to access their MMKB when making sense of and explaining mathematics; and (3) significant differences existed among the curriculum spaces found in the main lessons and peripheries.

Real-world connections

Of the 33 lessons, all but one (97%) involved real-world connections—either *for* students (textbook made the connections) or *with* students (students contributed their own connections) in four different ways: (1) with an open space at the beginning of the lesson; (2) with a single space at the beginning of a context-focused lesson; (3) with single spaces occurring after a strategy, model or procedure had been introduced; and 4) with replacement manipulatives.

Spaces opened in beginning of lesson

In some lessons, we identified open curriculum spaces for real-world connections in the beginning of the lesson (versus application tasks at the end of a lesson). These open spaces at the beginnings of lessons are notable because they frontload students' possible use of MMKB, particularly in the form of cultural, family and community knowledge. Frontloading MMKB provides students with opportunities to think about and share how they engage in mathematical practices and activities in their out-of-school and in-school lives. For example, in *Writing Multiplication Stories* (Charles et al., 2012—See Appendix A for a list of the curriculum series and authors.), students are expected to write and share their own stories for 4×5. It is only after this opportunity to connect to personal experiences that students are asked to use formal solution procedures provided by the textbook. Similarly, in *Pizza Problems* (University of Illinois at Chicago [UIC], 2008), students are asked, 'Is this a fair way to share a pizza (p. 43)?' regarding a circle separated into two unequal spaces. This question affords students opportunities to draw on their understanding of fairness and to consider this understanding in relation to others.

Context-focused lessons

Context-focused lessons begin with a problem situated in a real-world context. The difference between these lessons and the lessons described above is that, in context-focused lessons, the textbook provides the context and there is no prompt for students to make personally relevant connections. In most cases, the context is presented in story problem form, providing space for students to use and share any method; then returning to the context to establish meaningful connections between strategies and contexts. For example, in *More Than Ten Ones* (TERC, 2008), students are first asked: 'Ari has 63 baseball cards. His mother gave him her collection of 26 cards. How many cards does he have now?' After some discussion to establish student understanding (e.g. How many baseball cards did Ari have to start? How many more did his mother give him?), students are asked to solve using any method.

The fact that students are asked to solve problems using their own methods within the given context is significant (Carpenter, Fennema, Franke, Levi, & Empson, 1999), as students can leverage their MMKB around how they might add these sums based on their own experience counting or combining object quantities. At the same time, the context is provided, making it such that students with experiences collecting baseball cards (or any other item) may leverage their personal experiences with this real-world context and associated mathematical practices. The lesson then asks the teacher and students to examine the equations generated (e.g. $63 + 10 = 73$, $73 + 10 = 83$, $83 + 6 = 89$) and connect them back to the context, asking students, for example, 'Where in these equations do you see the 26 cards that Ari's mother gave him?'

Strategy-, model- or procedure-focused lessons

These lesson types provide contextual real-world problems within the lesson, but the contextual spaces are presented *after* a strategy, model or procedure is introduced. The use of a particular model or strategy is the focus, and the contexts provide a place for application. In these lessons, students *may* leverage their MMKB related to the provided contexts, but it would not be necessary, and is definitely not encouraged or elicited, as the lesson implies students need only apply the previously learned model or strategy.

For example, in *Two-Digit Addition* (Altieri et al., 2009), the lesson begins by showing students how to use base-ten blocks to model and solve 28 + 7. Next, two 'real-world examples', one of which is, 'Gaspar has 8 game tokens. His brother has 24 tokens. How many tokens do they have in all (p. 78)?' are provided and then *solved*. Students are then asked to practice individually by solving bare number addition problems like 27 + 2, 'using models if needed' and, following such bare-number questions, students are asked questions based in real-world contexts similar to the game token situation. The focus in this lesson is on using the provided model to solve problems, whether these problems are contextually based or not. It is possible the contextual problems could support students in leveraging their MMKB. For example, students with experiences combining tokens with a sibling to win a prize may think about how they engaged in that mathematics and bring those experiences to bear. On the other hand, a student who did not have these experiences could also likely solve this problem by applying the demonstrated method—and, as mentioned above, connections to personal experiences are not elicited or encouraged in these lessons.

Replacement manipulatives

We found real-world connections made with *replacement manipulatives*. In *Arrays and Multiplication* (Altieri et al., 2009), a teacher is instructed to begin by arranging counters into 4 rows of 6 counters, asking students how many rows are shown, how many counters are in each row and total counters in all. Next, the teacher is to do the same with 3 rows of 5 counters, which matches the picture of 'party cups' on the student page. Students are instructed to 'open their books and read the information[al]' paragraph that explains, 'The cups are arranged in equal rows and equal columns. This arrangement is an **array**. Arrays can help you **multiply**. The numbers you multiply are **factors**. The result is the **product** (p. 139)'. In this instance, the cups are simply a real-world object put in place of the mathematical counters used to form an array. While students could leverage their MMKB to make sense of this context, perhaps drawing on a party planning experience, the lesson is written to support students in simply understanding the party cups as interchangeable with the counters. The lesson focuses on students using a provided model to solve problems and the context becomes immaterial. A picture or vague connection to a real-world context does not leverage children's MMKB in the same ways as connections to mathematical experiences in a setting do (Aguirre et al., 2012).

Open, conflicting and closed curriculum spaces for students' strategies and explanations

Using secondary codes, we identified open, conflicting and closed curriculum spaces for students to draw on their MMKB. *Open spaces* were defined as those aspects of curriculum materials that explicitly prompt and/or support students in accessing and/or using their MMKB—both their mathematical thinking and their community and family-based funds of knowledge—to make sense of mathematics. We defined *closed spaces* as spaces that do no provide explicit opportunities for making connections to MMKB. We also identified *conflicting curriculum spaces* in which the lessons had both open and closed curriculum spaces along with design features that had implications for those spaces.

Open spaces for student strategies and explanation

In open spaces for student strategies and explanation, the textbook presents no strategy and, therefore, the space for student strategy development is opened by the absence of a textbook strategy. For example, in *The 500 Hats* (UIC, 2008), the teacher is prompted to pose word problems that relate to the story (e.g. 'When he had taken off 275 hats, how many more would eventually appear on his head (p. 29)?'). Teachers are not directed to give students a strategy, providing an open space for students to develop and use a strategy of their own, leveraging their MMKB particularly with respect to children's mathematical thinking.

Supports for maintaining open curriculum spaces

In coding for supports, we identified four text features that could potentially support teachers in maintaining open curriculum spaces: (1) possible student solutions, (2) sample unpacking questions, (3) ongoing assessment sections, and (4) supports for students' strategy explanations in large group discussions.

We found possible student solutions provided across several series. These solutions can help teachers anticipate and plan responses for student solutions (Davis & Krajcik, 2005) and communicate that multiple solutions should be accepted and/or explored. In *Parts-and-Total Number Stories* (UCSMP, 2007), three possible solution strategies are given for finding the value of a hot dog (45¢) and orange (25¢). For example, students could 'count up from the larger addend by using the values of dimes and nickels' (p. 255). Helping teachers anticipate solutions can support opening the space for students to develop and discuss various strategies.

Ongoing assessment sections (TERC, 2008; UCSMP, 2007) provide guidance in assessing students while working. In *Parts-and-Total Number Stories* (UCSMP, 2007), one section states, 'Children are making adequate progress if they correctly find the total using the number grid or manipulatives. Some children may be able to find the total without the use of manipulatives' (p. 256). *Investigations* (TERC, 2008) provides questions for teachers to consider while monitoring student work. For example, in *More Than Ten Ones*, teachers are asked, 'Can students write equations that accurately represent the problem (p. 49)?' and 'What addition strategies do students use (p. 49)?'

With regard to supporting students' explanations, the most common support was follow-up questions to student explanations. In *More Than Ten Ones* (TERC, 2008), the materials first provide a possible student strategy for 63 + 26: 'First I added 60 and 20 and got 80. Then I added the 3 and 6 and got 9. I added the 80 and 9 and got 89 (p. 45)'. The materials then suggest follow-up teacher questions including, 'How can we describe what Deondra did to solve this problem? What did she add first? Where did the 60 and 20 come from? What did she do next? Where did she get the 3 and 6 (p. 45)?' These questions provide opportunities to open a space by allowing students to talk specifically about solutions and to socially construct strategies.

Conflicting spaces for connecting to children's MMKB

We identified conflicting curriculum spaces when the materials (1) presented opportunities to develop or explain strategies only after a textbook strategy was presented, (2) presented an open space and then immediately closed it or (3) provided both open and closed spaces. In an example of the first conflicting space, a contextualized word problem is given *after* the textbook presents a strategy. In *Fractions of Whole Numbers* (Fuson, 2009), students are asked, 'Carlos saw 35 fish at the aquarium. 1/5 of them were clownfish. How many fish were clownfish?' (p. 974). This problem and others like it come after the textbook presents a 'fraction times a whole number formula'. Seemingly, the message is that students should use the textbook strategy.

We also identified lessons in which the main lesson focused on a single textbook-provided strategy, but the peripherals suggested opportunities for using other strategies. In *Fractions of Whole Numbers* (Fuson, 2009), an alternate option of using counters is explained and illustrated in the peripheral material. Also, a directive is given for how to have students explain strategies. Conflicting messages within single lessons about students' opportunities to develop and explain their own strategies versus using, practicing and explaining the strategy provided by the textbook can close spaces for students' MMKB.

Similarly, curriculum spaces for students to explain a strategy were often found only after the lesson provided a strategy to explain. In the addition lesson from *Engage NY* (NYSED, n.d.), 'Student Debrief' is intended for reflecting and active processing after students are directed to use one particular procedure. In *Fractions of Whole Numbers* (Fuson, 2009), students are shown the 'fraction of a whole number formula' and then asked to solve 12 problems. The last task of the lesson directs students to 'Explain in your own words how to find a fraction of a whole number. Give an example' (p. 974). Our interpretation was that students describe the 'fraction of the whole number formula' in their own words. It is worth noting that the teacher's guide states that, 'Answers will vary'. It seems that the curriculum developers would like students to develop multiple ways of explaining the textbook method. Asking for explanations, but restricting those explanations to one method, is conflicting. There are certainly potential benefits for students in explaining a textbook-provided strategy, but the space is less open for connecting to children's MMKB than it would be if students were given opportunities to develop and explain their own strategies.

In the second type of conflicting curriculum space, students were asked to solve a problem, which provided an open space to generate a solution strategy. However, the very next part of the lesson presented a single strategy and students were expected to use that strategy to solve the remaining problems. In *Fractions of a Set* (UCSMP, 2007), students are asked to find ½ of 20 pennies in an opening routine called *Math Message*. After students are given time to explain their strategies, the teacher is directed to model a specific strategy and then be given a problem set. By giving students the directive to model solutions, this lesson limits students' opportunities to use the strategies they might have used and shared in the opening routine.

In the third type of conflicting space, lessons had both open and closed spaces for students' explanations. In a *Saxon Math* lesson, students are asked to fold a square so that two corners match the other two corners, and then are asked to state how many rectangles were made (2) and if they are the same size (yes) (Larson et al., 2012). After providing answers, students are to explain how they knew the rectangles were the same size. Later, students are to explain how to find each answer on a worksheet in which a solution strategy is given. On one hand, students are to generate and share their own solution methods. On the other, students are expected to know and practice the textbook solutions. One might interpret these instances to mean that students could use either strategy. We speculate, however, that conflicting spaces will more often be enacted and experienced as closed spaces.

Closed spaces for connecting to and using children's MMKB

Finally, we found lessons with only closed spaces. These were procedure-focused lessons that were low in cognitive demand (Smith & Stein, 1998). In *Addition Without Regrouping* (Kheong, Ramakrishnan, & Wah, 2009) there is a picture of a place value chart depicting 1482 + 7516. Along with the picture are the following steps: 'Step 1: Add the ones. Step 2: Add the tens. Step 3: Add the hundreds. Step 4: Add the thousands'. Under each step is a vertical equation being built by the execution of each step. For Step 2, the equation looks like Figure 1.

After the teacher displays and models this procedure, students are to solve eight more similar problems. We described this lesson as procedure-focused due to its primary focus on the steps of adding, and as having low cognitive demand because each place is treated as digits rather than as values (8 + 1 = 9 rather than 80 + 10 = 90). Because mathematics and mathematical thinking are stripped from the task with the exception of adding single-digit numbers and there are no connections to children's home- and community-based knowledge, either through context or eliciting student experiences, this lesson provides only a closed space for accessing and using MMKB.

Peripheral curriculum spaces

Last, we found that significant differences existed among the curriculum spaces in the main lesson and lesson peripherals. Peripheral spaces could be defined differently across series, but we found that peripherals typically included opening routines/messages, ideas in the margins or end of the lesson for adapting and/or differentiating the lesson, and homework. When comparing space location (main lesson vs. peripheral), we found three curriculum structures: (1) curriculum spaces primarily in the opening routine; (2) curriculum spaces primarily in differentiation activities; and (3) curriculum spaces evenly distributed across main lessons and peripheral components.

Curriculum spaces primarily in the opening routine

Here, curriculum spaces existed in the opening routine, but not in the main lesson. In the *Saxon* lessons (Larson et al., 2012), spaces for children's MMKB were primarily in the 'in the morning' section of the lesson (time, temperature, etc.) and part of 'The Meeting' section (e.g. calendar, Number of the Day, patterns). This particular structure also existed in other curriculum series. In two of the *Everyday Mathematics* lessons (UCSMP, 2007), the *Math Message* and *Follow-up* are open to multiple strategies and provide a space for students to explain strategies. Then, in the main lessons, students are directed to follow and practice a particular strategy.

Second, we found lessons in which curriculum spaces existed primarily in differentiation activities. For the majority of these cases, the focus was procedural in the main lesson. Differentiation activities, however, included many more opportunities for eliciting and

$$\boxed{\begin{array}{r} 1482 \\ \underline{+\,7516} \\ 98 \end{array}}$$

Figure 1. Step 2 for adding 1482 and 7516.

building on students' MMKB. In *Multiplication as Repeated Groups* (Fuson, 2009) students are asked in a differentiation activity, 'Suppose you know $8 \times 15 = 120$. How could you use this information to find the product of 7×15? Explain your thinking' (p. 473). In the same series, another lesson asks students to 'Write about a real-world situation in which you might need to multiply a whole number by a fraction' (Fuson, 2009, p. 975). Both examples are opportunities for students to use and build their MMKB (mathematical knowledge in the former and possibly cultural, family or community knowledge in the latter) and were found only in differentiation sections.

Third, open curriculum spaces existed in both peripherals and main lessons. In *Sharing Among Friends* (TERC, 2008), students are asked to solve a series of equations that involve 15 (e.g. 40 – ____ = 15, 35 – _____ = 15) and explain their thinking. While this task does not promote use of a context, it does allow students to use mathematical knowledge gained in the first equation to solve the second. In the main lesson, students are asked to solve problems that involved sharing a group of objects among friends, providing students with opportunities to connect to their knowledge of sharing situations.

Homework as a space for children's MMKB

We were also interested in the kinds of spaces in homework (and other family communication) that would allow students and/or teachers to directly elicit and build on children's family and community-based knowledge. We were not examining the role of homework in this context, but rather analysing the features of homework as one of the components of curriculum materials. While research documents the effectiveness of parent involvement in student homework for students' learning of mathematics (e.g. Sheldon, Epstein, & Galindo 2010), our focus here was on whether spaces for connecting to children's MMKB in homework were similar to or different from spaces in the main textbook lessons. Most often, homework problems looked similar to those completed in class. However, we did find some instances in which homework assignments created spaces for connecting to children's MMKB, most often through asking students to find real-world objects (or stories) at home or in the community that connected to a particular mathematical topic. In *Multiplication as Equal Groups* (Fuson, 2009), students are to 'make drawings of familiar objects that can show repeated groups. For example, have them draw 3 flowers with 5 petals per flower, and then label the total number of petals' (p. 474).

Family notes or directions for students to share with their families potentially open spaces for mathematical communication and therefore potentially open spaces for children to draw on or connect to their family's mathematical knowledge and practices. Some spaces (e.g. *Multiplication Stories* [UIC, 2008] and *Buying at the Stock-Up Sale* [UCSMP, 2007]) were more open in that they encouraged children to share mathematics stories or strategies with families, with an explicit expectation that there might be multiple 'correct' responses. Others, such as *Parts-and-Total Number Stories* (UCSMP, 2007) and *Explore Multi-Digit Addition* (Fuson, 2009), were less open as the primary purpose was to teach families about a particular textbook-developed strategy.

Saxon (Larson et al., 2012) had a homework structure that was designed to provide support for families who may be unsure of how to solve particular kinds of problems or lack confidence. But, it also served to close spaces to elicit or build on families' mathematical understandings and practices. In *Saxon*, the homework was almost identical to a 'Guided

Class Practice' sheet meant to be completed in class the same day. Therefore, it was a *support* for families to learn about school-based mathematics, without being a *space* for eliciting or building on families' mathematical understandings or practices.

Discussion

Our analysis of a wide range of elementary mathematics curriculum materials determined that *curriculum spaces* do exist for connecting to children's MMKB and described the range of *curriculum spaces*. *Spaces Opened in the Beginning of the Lesson* seem to have the most potential for students to draw on their MMKB as they allow opportunities for students to consider their own contexts for doing mathematics and thus are open curriculum spaces. Several studies, unrelated to the use of curriculum materials, have identified the benefits of students drawing from lived experiences in problem-solving situations (e.g. Simic-Muller, Turner, & Varley, 2009; Walkington & Bernacki, 2015). Simic-Muller and colleagues (2009) documented students drawing from their community-based knowledge to solve problems about area businesses. In another study, students were able to pose problems about lived experiences such as sporting events and social networking (Walkington & Bernacki, 2015). We argue that the enactment of open curriculum spaces for real-world connections could produce outcomes similar to these study results and possibly allow students to see themselves. Additionally, the enactment of open curriculum spaces could increase teachers' understanding of students' MMKB as these spaces provide opportunities for teachers to listen to children and gain information about students' lives, which many claim is necessary in order to teach successfully (e.g. Ball, Lubienski, & Mewborn, 2001; Ladson-Billings, 1994). In their literature review, Llopart and Esteban-Guitart (2016), pointed out that recent research around utilizing students' funds of knowledge in instruction is 'linked to social transformation by the manner in which it understands the relationship between under-represented students and educational practice and culture' (p. 12). Thus, we contend that understanding students' MMKB could lead to social transformations.

Context-Focused Lessons provided students with opportunities to use their own strategies for completing a task within a textbook-given context, but prior research on real-world connections suggests that these curriculum spaces will provide mixed opportunities for students to draw on their MMKB. Boaler (1993), for instance, found students' responses to tasks varied considerably depending on the context implying that 'students' perceptions of the contexts were individually constructed' (p. 341). Kazemi (2002) recognized that children draw on life experiences when the experiences are salient. Because of the uncertainty around how these types of real-world contexts will support all students, we identified them as conflicting—sometimes supportive and sometimes not. Real-world connections in *strategy-, model- or procedure-focused lessons* as well as real-world connections in the form of *replacement manipulatives* left little room for children to access their MMKB, and thus were closed curriculum spaces. They were closed because children were taught how to solve a problem for a context instead of accessing and using their MMKB to solve the same problem.

Like real-world connections, the nature of the curriculum spaces for children to make sense of and explain mathematics also differed. Curriculum spaces for making sense of and explaining mathematics ranged from *open* to *conflicting* to *closed*. Open curriculum spaces provided the most potential for drawing on MMKB, particularly with respect to mathematical thinking, as students could explore strategies with autonomy. Text features, in some cases, provided teacher support to maintain open spaces. Teachers taking up these curriculum spaces could increase their knowledge base around children's mathematical thinking, which then could support student achievement.

Text features that provide teacher support in maintaining open spaces are limited, however, because they tend to focus on students' school-based strategies and explanations and not on the other knowledge bases students might have related to mathematics such as home- and community-based knowledge. While unpacking and large-group discussion questions are fundamental to a mathematics classroom, questions focusing on students' out-of-school lives might be equally beneficial. In *More Than Ten Ones* (TERC, 2008) where students are asked to add baseball cards together, students could be also asked questions like: 'Do you ever have to keep track of how many or how much of something you have when you are playing with friends, or doing things with your family? If so, what do you keep track of and how?' In this way, students could be supported in drawing on their lived experiences to gain access to school mathematics.

Locations of curriculum spaces differed across series in that some were in the main lesson to be easily noticed by the teacher while others were located in peripherals. This finding is important because if most of the curriculum spaces—including many of the most open spaces—exist in the peripherals, then there is a less of a chance that students will be able to access and use their MMKB within lessons, as research has found that teachers primarily attend to main lesson components (Remillard & Ciganik, 2013). Thus, lesson components or text features that entail open spaces or supports for open spaces (e.g. 'Teacher Notes') will likely not be enacted.

We focused on curriculum spaces written into the materials, which are subject to teacher enactment. A curriculum space written into the materials does not ensure children's access to open spaces. Several studies (Collopy, 2003; Remillard, 2005; Sherin & Drake, 2009) have documented that teachers read, evaluate and adapt curriculum materials differently, implying that access to open spaces is influenced by the teacher. Conversely, conflicting and closed spaces could be adapted by the teacher to make them open. Given the myriad ways that teachers and curriculum interact in the enactment of instruction, our framework provides a means by which teachers can structure their use of curriculum materials—reading for curriculum spaces, evaluating materials for the presence of open, conflicting and closed spaces and making productive adaptations to open spaces.

Teachers' knowledge of children's mathematical thinking can increase through curriculum materials (Choppin, 2011; Empson & Junk, 2004). Empson and Junk (2004) recognized that *Investigations in Number, Space and Time* (TERC, 2008) contributed to teacher learning, but noticed that learning differed depending on mathematical content, which the researchers attributed to differences in the materials. We conjecture that the teachers in Empson and Junk's study increased their knowledge about children's mathematical thinking because open spaces existed and teachers leveraged those spaces in productive ways. With these conjectures in mind, we have developed a curriculum analysis tool based on the curriculum spaces framework and have begun working with prospective teachers on using the tool to identify potential spaces and make productive adaptations (Drake et al., 2015).

Given that we analysed three lessons from each curriculum series and that our purpose was to identify and analyse the nature of curriculum spaces, we cannot make any claims about any one series. We did notice, however, that some series had particular kinds of spaces that reoccurred in all three lessons. For example, in all three lessons from *Saxon* (Larson et al., 2012), spaces for children's MMKB were primarily in the routines and not in the main lessons, leading us to infer that this structure may be consistent. We did not, however, find this kind of consistency in all series. *Go Math!* (Burger et al., 2014), for instance, had an open space for a real-world connection in the beginning of one lesson, but not the other two, suggesting that this type of space is not consistent. An additional study that analysed each or selected curriculum further would be needed in order to make claims about particular series.

Implications

Here, we respond to Brown's (2002, 2009) caution of not underestimating the role of curriculum design in the teacher–curriculum interaction and discuss implications for curriculum designers. These implications include considerations for how to provide more open spaces, where to locate the open curriculum spaces and how to design curriculum materials without conflicting spaces. These implications are drawn from the context of our analysis of mathematics curriculum, but are not limited to mathematics curriculum designers. As such, teachers have roles in identifying open curriculum spaces and/or adapting curriculum materials to include them. For example, teachers could recognize the adaptable nature of real-world contexts and choose other contexts (e.g. community event rather than zoos in various countries) that are more relatable (Drake et al., 2015).

First, how might curriculum designers embed structured encouragement for teachers to adapt curriculum to open spaces to connect to children's MMKB and/or provide more open spaces overall? In a few instances, the materials (e.g. *Everyday Mathematics*) encourage teachers to make adaptations according to student needs. It may be beneficial to include encouragements to adapt repeatedly throughout the curriculum, or make the materials clearly adaptable. In *500 Hats* (UIC, 2008), the text suggests using other stories with 'contexts for addition and subtraction (p. 29)' if the teacher prefers and provides 'example' word problems. We interpret 'example' to mean that other problems could be created. However, curriculum materials are generally designed in ways that do not allow much flexibility (e.g. student workbook pages) to make adaptations. Curriculum designers could consider giving teachers guidance in making productive adaptations. This guidance might include sample questions to elicit children's lived experiences, suggestions for children to draw on or create their own stories or connections, or specific reference to students doing the mathematical thinking prior to learning new methods or strategies.

Second, curriculum designers could consider whether and how their current designs locate open curriculum spaces in the main lesson versus peripheral spaces. Explicit and purposeful movement of sections that make connections to home and community experiences may be moved to the main lesson. Explicit and purposeful movement of sections that model a particular strategy may be moved to the peripherals. We concede that there may be times where students need to be directed to use a particular strategy, but strategy modeling does not need to be presented as the lesson focus, which is often the case. Supports, in the form of example student responses and discussion questions, could be provided across series.

Third, how might curriculum designers revise their designs so as to avoid conflicting messages? In our analysis of materials, we often found unclear expectations concerning which strategies students were supposed to use (the textbook's or their own) or lessons that included both open and closed spaces. We hope that the framework provided here can support curriculum designers in identifying spaces that are initially open, only to be quickly closed by providing an isolated context or specific strategy to follow. Curriculum designers might include questions to maintain open spaces—questions that focus on eliciting and building on students' mathematical thinking and having students discuss connections between the mathematics and their personal experiences. We suggest either making the message consistent that students use their own strategies and/or provide teachers with support in how to negotiate the integration of teacher/textbook and student strategies in ways that promote student autonomy.

Our work suggests the importance of attending to children's MMKB not only in instruction, but also in curriculum design. In what ways are curricula attending to both children's thinking and their community and cultural funds of knowledge? How are these spaces opened and leveraged to support student thinking? What might closed or conflicting spaces look like? Our results provide some initial guidance for the field—a first step to supporting teachers in integrating a focus on children's MMKB in their instruction is identifying the ways in which curricula provide support to do so and beginning to define what those supports (or spaces) entail.

Acknowledgements

Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work is based in part on research supported by the National Science Foundation (NSF) [grant number 0736964], [grant number 1228034].

Notes on contributors

Tonia J. Land is an associate professor of Elementary Mathematics and Technology Education at Drake University. Her recent interests include supporting pre-service teachers in their reading of curriculum materials and developing teachers' capacity to be responsive to children's mathematical thinking.

Tonya G. Bartell is an associate professor in the College of Education at Michigan State University. Her research focuses on issues of culture, race, and power in mathematics teaching and learning, with particular attention to teachers' development of mathematics pedagogy for social justice and pedagogy integrating a focus on mathematics, children's mathematical thinking, and children's community and cultural knowledge.

Corey Drake is the director of Teacher Preparation and associate professor of Elementary Mathematics Education at Michigan State University. Her research interests include the preparation of teachers to use mathematics curriculum materials to support the learning of diverse groups of students. She is also interested in the design and implementation of teacher preparation programs and courses.

Mary Q. Foote is an associate professor in the Department of Elementary and Early Childhood Education at Queens College-CUNY. Her research interests fall broadly within issues of teacher knowledge. More specifically her interests are in teachers' understanding of students' cultural and community knowledge and practices, and how they might inform equitable mathematics teaching practice.

Amy Roth McDuffie is a professor of Mathematics Education and associate dean for Research and External Funding for the College of Education at Washington State University. Her interests focus on teaching practices with regard to using curriculum materials and enacting equitable instruction for diverse learners. She is the series editor for the 2014–2016 volumes of Annual Perspectives in Mathematics Education (National Council of Teachers of Mathematics), and has published recently in Journal of Mathematics Teacher Education and Mathematics Teacher Educator.

Erin E. Turner is an associate professor in mathematics education in the Teaching, Learning and Sociocultural Studies department at the University of Arizona. Her research and teaching interests focus on issues of equity and social justice in mathematics education, and on preparing and supporting teachers for effective mathematics teaching with students from diverse cultural and linguistic backgrounds.

Julia M. Aguirre is an associate professor of education at the University of Washington Tacoma. Her work examines issues of equity in mathematics teaching and learning, teacher education, and culturally responsive mathematics pedagogy.

References

Aguirre, J. M., Turner, E. E., Bartell, T. G., Kalenic-Craig, C., Foote, M. Q., McDuffie, M. R., & Drake, C. (2012). Making connections in practice: How prospective elementary teachers connect to children's mathematical thinking and community funds of knowledge in mathematics instruction. *Journal of Teacher Education, 64*(2), 178–192.

Altieri, M. B., Balka, D. S., Day, R., Gonsalves, P. D., Grace, E. C., Krulik, S., ... Vielhaber, K. (2009). *Math Connects*. Chicago, IL: McGraw Hill.

Andrews, J., & Yee, W. C. (2006). Children's 'funds of knowledge' and their real life activities: Two minority ethnic children learning in out-of-school contexts in the UK. *Educational Review, 58*(4), 435–449.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is – or might be – The role of curriculum materials in teacher learning and instructional reform? *Educational Researcher, 25*(6), 8–14.

Ball, D. L., Lubienski, S., & Mewborn, D. (2001). Research on teaching mathematics: The unsolved problem of teacher's mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed.). (pp. 433–456). New York, NY: Macmillian.

Barton, A. C., & Tan, E. (2009). Funds of knowledge and discourses and hybrid space. *Journal for Research in Science Teaching, 46*(1), 50–73.

Berry, R. Q., III, Pinter, H., & McClain, O. L. (2013). A critical review of American K-12 mathematics education, 1900-present: Implications for the experiences and achievement of black children. In J. Leonard & D. B. Martin (Eds.), *The brilliance of black children in mathematics: Beyond the numbers and toward new discourse* (pp. 23–54). Charlotte, NC: Information Age Publishing, Incorporated.

Bhabha, H. K. (1994). *The location of culture*. London: Routledge.

Boaler, J. (1993). Encouraging the transfer of school mathematics to the real world through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*(4), 341–373.

Borden, L. L. (2013). What's the word for…? Is there a word for…? How understanding Mi'kmaw language can help support Mi'kmaw learners in mathematics. *Mathematics Education Research Journal, 25*(1), 5–22.

Brown, M. W. (2002). *Teaching by design: Understanding the intersection between teacher practice and the design of curricular innovations*. Unpublished doctoral dissertation, Northwestern University, Evanston, IL.

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). New York, NY: Routledge.

Brown, M., & Edelson, D. (2003). *Teaching as design: Can we better understand the ways in which teachers use materials so we can better design materials to support changes in practice?* (Research Report, Center for Learning Technologies in Urban Schools). Chicago, IL: Northwestern University.

Burger, E. B., Dixon, J. K., Kanold, T. D., Larson, M. R., Leinwand, S. J., Sandoval-Martinez, M. E. (2014). *Go Math!* Boston, MA: Houghton Mifflin Harcourt.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C. P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499–531.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3–20.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2014). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Charles, R., Fennell, F., Caldwell, J. H., Cavanagh, M., Sammons, K.B., Schielack, J. F., ... Murphy, S. J. (2012). *Envision Math*. Glenview, IL: Pearson Scott Foresman.

Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*(5), 331–353.

Civil, M. (2002). Everyday mathematics, mathematicians' mathematics, and school mathematics: Can we bring them together? In M. Brenner & J. Moschkovich (Eds.), *Everyday and academic mathematics in the classroom. Journal of Research in Mathematics Education Monograph #11* (pp. 40–62). Reston, VA: NCTM.

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105–117). New York, NY: Teachers College Press.

Civil, M. (2012). Opportunities to learn in mathematics education: Insights from research with "non-dominant" cultures. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–43). Taipei: PME.

Civil, M., & Planas, Nuria (2004). Participation in the mathematics classroom: Does every student have a voice? *For the Learning of Mathematics, 24*(1), 7–12.

Collopy, R. (2003). Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. *Elementary School Journal, 103*(3), 287–311.

Davis, E., & Krajcik, J. (2005). Designing educative curriculum materials to promote teacher learning. *Educational Researcher, 34*(3), 3–14.

Drake, C., Land, T. J., Bartell, T. G., Aguirre, J. M., Turner, E. E., Foote, M. Q., & McDuffie, A. R. (2015). Three strategies for opening curriculum spaces: Building on children's mathematical thinking while using mandated curriculum materials. *Teaching Children Mathematics, 21*(6), 346–353.

Ebby, C., Vivian, L., Reinke, L., Remillard, J., Magee, E., Hoe, N., & Cyrus, M. (2011). Community based mathematics project: Conceptualizing access through locally relevant mathematics curricula. *Perspectives on Urban Education, 8*(Spring), 12–18.

Empson, S. B., & Junk, D. L. (2004). Teacher's knowledge of children's mathematics after implementing a student-centered curriculum. *Journal of Mathematics Teacher Education, 7*(2), 121–144.

Empson, S. B., & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals*. Portsmouth, NH: Heinemann.

Esmonde, I. (2009). Ideas and identies: Supporting equity in cooperative mathematics learning. *Review of Educational Research, 79*(2), 1008–1043.

Ewing, B. (2012). Mathematics funds of knowledge: Sotmaute and Sermaute fish in a Torres Strait Islander community. *Australian Journal of Adult Learning, 52*(1), 134.

Flores, A. (2007). Examining disparities in mathematics education: Achievement gap or opportunity gap? *The High School Journal, 91*(1), 29–42.

Foote, M. Q. (2009). Stepping out of the classroom: Building teacher knowledge for developing classroom practice. *Teacher Education Quarterly, 36*(3), 39–53.

Fuson, K. (2009). *Math expressions*. Boston, MA: Houghton Mifflin.

González, N., Andrade, R., Civil, M., & Moll, L. C. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk, 6*(1–2), 115–132.

González, N., Moll, L. C., & Amanti, C. (2005). *Funds of knowledge: Theorizing practices in households and classrooms*. Mahwah, NJ: Lawrence Erlbaum.

Gresalfi, M. S., & Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. *Pedagogies: An International Journal, 1*(1), 49–57.

Grossman, P., & Thompson, C. (2008). Learning from curriculum materials: Scaffolds for new teachers? *Teaching and Teacher Education, 24*(8), 2014–2026.

Guitierrez, R. (2007). Context matters: Equity, Success and the future of mathematics education. In T. Lamberg & L. R. Wiest (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Stateline (Lake Tahoe)* (pp. 1–18). Reno: University of Nevada.

Hedges, H., & Cooper, M. (2016). Inquiring minds: Theorizing children's interests. *Journal of Curriculum Studies, 48*(3), 303–322.

Hogg, L. M. (2016). Applying funds of knowledge theory in a New Zealand high school: New directions for pedagogical practice. *Teachers and Curriculum, 16*(1), 49–56.

Hoover, M., Mosvold, R., Ball, D. L., & Lai, Y. (2016). Making progress on mathematical knowledge for teaching. *The Mathematics Enthusiast, 13*(1&2), 3–34.

Kazemi, E. (2002). Exploring test performance in mathematics: The questions children's answers raise. *Journal of Mathematical Behavior, 21*(2), 203–224.

Kheong, F. H., Ramakrishnan, C., & Wah, B. L. P. (2009). *Singapore Math*. Boston, MA: Houghton Mifflin Harcourt.

Ladson-Billings, G. (1994). *The Dreamkeepers: Successful teachers of African American children*. San Francisco, CA: Jossey-Bass.

Larson, N., Hake, S., Wrigley, P., & Wang, F. (2012). *Saxon Math*. Boston, MA: Houghton Mifflin Harcourt.

Lee, J. E. (2012). Prospective elementary teachers' perceptions of real-life connections reflected in posing and evaluating story problems. *Journal of Mathematics Teacher Education, 15*(6), 429–452.

Lipka, J. (2002). *Schooling for self-determination: Research on the effects of including native language and culture in the schools*. Charleston, WV: ERIC Digest.

Lipka, J., Hogan, M. P., Webster, J. P., Yanez, E., Adams, B., Clark, S., & Lacy, D. (2005). Math in a cultural context: Two case studies of a successful culturally based math project. *Anthropology & Education Quarterly, 36*(4), 367–385.

Llopart, M., & Esteban-Guitart, M. (2016). Funds of knowledge in 21st century societies: Inclusive educational practices for under-represented students. A literature review. *Journal of Curriculum Studies, 14*, 1–17.

Lloyd, G. M. (2008). Curriculum use while learning to teach: One student teacher's appropriation of mathematics curriculum materials. *Journal for Research in Mathematics Education, 39*(1), 63–94.

Martin, D. B. (2009). Does race matter? *Teaching Children Mathematics, 16*(3), 134–139.

McGraw, R., Lubienski, S. T., & Strutchens, M. E. (2006). A closer look at gender in NAEP mathematics achievement and affect data: Intersections with achievement, race/ethnicity, and socioeconomic status. *Journal for Research in Mathematics Education, 37*(2), 129–150.

Meaney, T., & Evans, D. (2013). What is the responsibility of mathematics education to the Indigenous students that it serves? *Educational Studies in Mathematics, 82*(3), 481–496.

Meyer, M. R., Dekker, T., & Querelle, N. (2001). Context in mathematics curricula. *Mathematics Teaching in the Middle School, 6*(9), 522–527.

Moje, E. B., Ciechanowski, K. M., Kramer, K., Ellis, L., Carillo, R., & Collazo, T. (2004). Working toward third space in content area literacy: An examination of everyday funds of knowledge and discourse. *Reading Research Quarterly, 39*(1), 38–70.

Moschkovich, J. N. (2010). Language(s) and learning mathematics: Resources, challenges, and issues for research. In J. N. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research* (pp. 1–27). Charlotte, NC: Information Age Publishing.

National Council of Teachers of Mathematics. (2014). *Principles to action: Ensuring mathematical success for all*. Reston, VA: Author.
New York State Education Department (n.d.). *Engage NY*. Retrieved at www.engageny.org.
Poirier, L. (2005). Teaching mathematics to Inuit children in Nunavik: Taking into account the environment, the culture, and the language. In *Canadian Mathematics Education Study Group*. Proceedings of the Annual Meeting, Ottawa, Canada, May 27–31.
Powell, A. (2012). The historical development of Critical mathematics education. In A. Wager & D. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with educators* (pp. 21–34). Reston, VA: NCTM.
Ramnarain, U., & de Beer, J. (2013). Science students creating hybrid spaces when engaging in an expo investigation project. *Research in Science Education, 43*(1), 99–116.
Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246.
Remillard, J. T., & Ciganik, S. (2013). Do elementary mathematics curriculum materials support teacher decision making? Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA, April 27–May 1.
Saldana, J. (2013). *The coding manual for qualitative researchers*. Thousand Oaks, CA: Sage.
Sheldon, S. B., Epstein, J. L., & Galindo, C. L. (2010). Not just numbers: Creating a partnership climate to improve math proficiency in schools. *Leadership and Policy in Schools*, *9*(1), 27–48.
Sherin, M., & Drake, C. (2009). Curriculum strategy framework: Investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies, 41*(4), 467–500.
Silver Burdett Ginn. (1998). *Mathematics*. Glenview, IL: Pearson.
Simic-Muller, K., Turner, E. E., & Varley, M. C. (2009). Math club problem-posing. *Teaching Children Mathematics, 16*(4), 206–212.
Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344–350.
Soja, E. W. (1996). *Thirdspace: Journeys to Los Angeles and other real-and-imagined places*. Oxford: Basil Blackwell.
Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 319–369). Charlotte, NC: Information Age Publishing.
Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Grounded theory procedures and techniques*. Thousand Oaks, CA: Sage.
Tarr, J. A., Reys, R. E., Reys, B. J., Chávez, Ó., Shih, J., & Osterlind, S. J. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education, 39*(3), 247–280.
Taylor, M. W. (2013). Replacing the 'teacher-proof' curriculum with the 'curriculum-proof teacher: Toward more effective interactions with mathematics textbooks. *Journal of Curriculum Studies, 45*(3), 295–321.
TERC. (2008). *Investigations in number, data, and space*. Glenview, IL: Pearson Scott Foresman.
Turner, E. E., & Celedon-Pattichis, S. (2011). Mathematical problem solving among Latina/o kindergartners: An analysis of opportunities to learn. *Journal of Latinos and Education, 10*(2), 146–169.
Turner, E. E., Drake, C., McDuffie, A. R., Aguirre, J., Bartell, T. G., & Foote, M. Q. (2012). Promoting equity in mathematics teacher preparation: A framework for advancing teacher learning of children's multiple mathematics knowledge bases. *Journal of Mathematics Teacher Education, 15*(1), 67–82.
UCSMP. (2007). *Everyday mathematics: The University of Chicago School Mathematics Project*. Chicago, IL: McGraw Hill.
University of Illinois at Chicago. (2008). *Math Trailblazers*. Dubuque, IA: Kendall/Hunt Publishing.
Walkington, C., & Bernacki, M. (2015). Students authoring personalized "algebra stories": Problem-posing in the context of out-of-school interests. *Journal of Mathematical Behavior, 40*, 171–191.

Appendix A

- Engage NY (New York State Education Department, n.d.).
- EnVision Math (Charles et al., 2012).
- Everyday Mathematics (UCSMP, 2007).
- GoMath! (Burger et al., 2014).
- Investigations (TERC, 2008).
- Math Connect (Altieri et al., 2009).
- Math Expressions (Fuson, 2009).
- Math Trailblazers (UIC, 2008).
- Mathematics (*Silver Burdett Ginn Mathematics*, 1998).
- Saxon (Larson et al., 2012).
- Singapore Math (Kheong, Ramakrishnan, & Wah, 2009).