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Rateless Coded Multi-User Downlink Transmission in Cloud Radio Access Network

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Abstract
In this paper, we propose a rateless coded transmission scheme for the multi-user downlink of Cloud Radio Access Network (C-RAN). In the network, multiple users are served by a cluster of remote radio heads (RRH), which are connected to the building baseband units (BBU) pool through the fronthaul links with limited capacity. In the proposed transmission scheme, the precoder and compressor at the BBU pool, and the decoding algorithm at the users are designed. To further improve the performance of the proposed transmission scheme, we investigate the joint optimization of the precoder and the degree profiles of the rateless codes implemented at the BBU pool to maximize the sum throughput of the network. Explicitly, the optimization problem is formulated according to the extrinsic information transfer (EXIT) function analysis on the decoding process at the users. We give simulation results on the BER and throughput performance achieved by the proposed rateless coded transmission scheme, which verify the effectiveness of the joint optimization on the degree profiles and the precoder.

Keywords Cloud radio access networks · Rateless code · Precoding · EXIT analysis

1 Introduction

The trends of the next generation communication system are heterogeneous network and smaller cellular [1], which lead to the issues of severe inter-cell interference and cell association, and requirements for more sophisticated multi-cell coordination [2]. Cloud Radio Access Network (C-RAN) is a revolutionary mobile network architecture and has the potential to solve the above challenges [1, 3]. In C-RAN, the baseband processing units are migrated from the base stations to the building baseband units (BBU) pool where the signals from/to multiple cells are jointly processed. Therefore, inter-cell interference can be effectively eliminated and the network throughput can be improved. Moreover, the C-RAN is able to adapt to non-uniform traffic and make a rational use of resources [4]. Due to the fact that the expansion of C-RAN only requires the installation of new remote radio heads (RRH) and fronthaul links, it becomes possible for mobile operators to reduce the cost of network construction and upgrade [5].

In this work, we consider the multi-user downlink scenario in C-RAN. The BBU pool performs multi-antenna precoding on the messages intended for the users which are then compressed and delivered to each RRH via the capacity-limited fronthaul links. The resource management, multi-antenna precoding optimization and signal compression design in C-RAN and distributed relay/distributed antenna systems have been widely studied, e.g., in [6–25]. (Note that C-RAN has an intrinsic relation to distributed relay/antenna systems). Authors in [6–9] investigated the criterion for the optimization of the precoding matrix which assume the knowledge of global channel states information (CSI). The authors in [10–12] studied the compression scheme with the aim at lowering the effect of the compression noise which exploits the correlations of signals for RRHs. The precoding design for relay and multi-antenna systems was investigated in [13, 14].

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References [15–22] focused on wireless resource management for C-RAN and multi-antenna/relay systems.

As for the coded transmission in the downlink of C-RAN, the design and practical implementations with fixed-rate channel codes can be found in [23, 24]. With the fixed-rate channel codes such as Low Density Parity Check (LDPC) codes and Turbo codes, hybrid automatic repeat request (HARQ) is usually employed. However, note that in C-RAN, the fronthaul links with considerable lengths will cause additional signaling delay, which imposes stringent requirements on the decoding time at the BBU pool and users to insure HARQ work properly. Considering this challenge, in this work we refer to the rateless code for coded transmission in C-RAN. With rateless code, HARQ is not needed. The transmitter can send codewords of infinite length until the receiver decodes the message successfully and feedbacks only an ACK. Besides, rateless codes with optimized degree profiles can also approach the channel capacity like the fixed-rate channel code [23]. The rateless coded transmission have been studied for various communication systems including wireless broadcast systems [25], relay systems [26, 27] and distributed antenna systems [28, 29]. To the best of the authors' knowledge, rateless coded transmission for C-RAN downlink has not been considered so far.

1.1 Contributions

We consider the multi-user downlink of C-RAN network where the RRHs and users are all equipped with a single antenna. Each RRH is constrained by the individual peak transmit power.[1] The main contributions of this work can be summarized as follows:

(i) We propose the rateless coded transmission scheme for multi-user downlink of C-RAN. For the BBU pool, we adopt the Raptor code for rateless encoding on the messages for the users. Then a precoder based on zero-forcing (ZF) structure [30] is used to do precoding on the modulated signal, which is then followed by a fixed scalar quantizer to perform signal compression. For the users, assuming that they are aware of the ZF precoder,[2] we approximate the compression noise as Gaussian to derive the soft information from the received signals. Then the belief propagation algorithm is adopted to recover the messages.

(ii) To further improve the performance of the proposed downlink rateless coded transmission scheme, we consider the joint optimization of two sets of parameters. The first are the power coefficients of the basis vectors in the ZF precoder. The second are the output-symbol degree profiles of Raptor code used for downlink data of each user. We formulate the optimization problem according to the extrinsic information transfer (EXIT) function analysis on the decoding process at the users. Then, an algorithm is proposed to solve the optimization problem, which is validated through computer simulations.

The remainder of this paper is organized as follows. In Section 2, we introduce the system model. The rateless coded transmission scheme for the downlink of C-RAN is described in Section 3. The joint optimization problem formulation and the solution are given in Section 4. Simulation results are presented in Section 5 and Section 6 concludes the paper.

2 System model

The considered C-RAN downlink system is illustrated in Fig. 1, where K users are served by M RRHs which are connected to the BBU pool through fronthaul links, each with an individual rate constraint C_F bits/channel use (bpcu). The number of RRHs is assumed to be no less than the users, i.e., $M \geq K$. Each RRH has the same individual peak transmit power P_{max}. We assume the channel between RRHs and users experience block fading, and the channel coefficients keep invariant during the entire transmission of the codewords for all the users. The global channel matrix between the RRHs and users are denoted by $\boldsymbol{H} = [h_{ij}]_{K \times M} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_K]^{\mathrm{T}}$, where h_{ij} is the channel coefficient from the jth RRH to ith user, and $\boldsymbol{h}_k^{\mathrm{T}}(k = 1, \ldots, K)$ represents the kth row.

For the downlink transmission, Fig. 2 shows the operations at the BBU pool. The message $\boldsymbol{m}_k$ for the kth user is first encoded by Raptor code and then modulated. The modulated

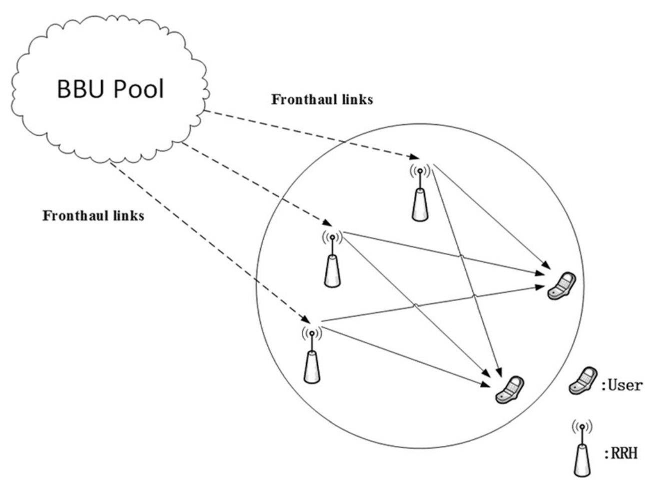

Fig. 1 Downlink scenario in the C-RAN

[1] Peak transmit power constraint is practical. It is due to the fact that the amplifier at the transmitter is linear only for input voltage within a given range. Besides, the extension of this work to an average power constraint is straightforward.

[2] In slow fading scenario, the resulted overhead is affordable.

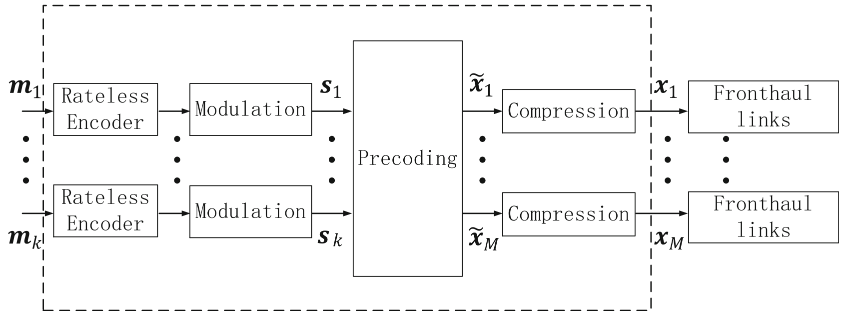

Fig. 2 Signal processing at the BBU pool

signal is denoted by $\boldsymbol{s}_k$. Then the signals $\boldsymbol{s} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K]^{\mathrm{T}}$ undergo precoding and compression.

The signals after precoding can be written as:

$$\tilde{\boldsymbol{x}} = \boldsymbol{W}\boldsymbol{s} \tag{1}$$

where $\tilde{\boldsymbol{x}} = [\tilde{\boldsymbol{x}}_1, \ldots, \tilde{\boldsymbol{x}}_M]^T$ and $\boldsymbol{W}$ is the M-by-K precoding matrix. Each $\tilde{\boldsymbol{x}}_m \,(m = 1, \ldots, M)$ is then compressed and sent to the mth RRH, respectively. With the assumption that the global channel matrix $\boldsymbol{H}$ is known at the BBU pool, we employ zero-forcing (ZF) precoding which can completely eliminate the inter-user interference. The general form of a ZF matrix $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_K]$ can be given as follows [30].

$$\boldsymbol{w}_k = a_k \boldsymbol{w}_{k,inv} + \sum\nolimits_{\boldsymbol{e}_i \perp \boldsymbol{w}_{k,inv}, \mathcal{N}(\boldsymbol{H})/\{\boldsymbol{h}_k{}^T\}} a_i^k \boldsymbol{e}_i, k = 1, 2, \ldots K \tag{2}$$

where $\boldsymbol{e}_i$ is the ith basis vector[3] in the subspace orthogonal to $\boldsymbol{w}_{k,inv}$ and $\mathcal{N}(\boldsymbol{H})/\{\boldsymbol{h}_k{}^T\}$, and $\mathcal{N}(\boldsymbol{H})/\{\boldsymbol{h}_k{}^T\}$ represents the set of all the row vectors of $\boldsymbol{H}$ except the kth one. Also, $\boldsymbol{w}_{k,inv}$ is the kth column of the Pseudo-inverse of channel matrix $\boldsymbol{H}$, i.e., $\boldsymbol{H}^{\mathrm{H}}(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})$. a_k, a_i^k are the power coefficients for $\boldsymbol{w}_{k,inv}$ and $\boldsymbol{e}_i$, respectively.

The precoded signal $\tilde{\boldsymbol{x}}_m$ is individually compressed to meet the rate restriction of the corresponding fronthaul link and then sent to the mth RRH. Denote the compressed signals as $\boldsymbol{x}_m$. The mth RRH then transmits $\boldsymbol{x}_m$ to the users. Under the individual peak power constraint at each RRH, we have

$$|\boldsymbol{x}_m|^2 \leq P_{max}, \forall m = 1, \ldots, M \tag{3}$$

The signal received at the kth user is given by

$$y_k = \boldsymbol{h}_k{}^T \boldsymbol{x} + z_k \tag{4}$$

where $\boldsymbol{x} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M]^{\mathrm{T}}$ and the noise z_k is assumed to be additional white Gaussian (AWGN) with a variance of σ^2.

[3] The number of basis vectors for the summation depends on the channel matrix $\boldsymbol{H}$. For example, when M = 3, K = 2 and $\boldsymbol{H}$ has full rank, there is only one basis vector for each k.

3 Rateless coded downlink transmission scheme

In this section we explicitly discuss the proposed rateless coded downlink transmission scheme in C-RAN. First we briefly introduce the encoding procedure of Raptor code, then we give the compression scheme implemented at the BBU pool. At last the decoding at the user is discussed.

3.1 Rateless coding at the BBU pool

We use Raptor code at the BBU pool. With Raptor code, each message is first precoded by a LDPC code and then encoded by a Luby Transform (LT) code. A Raptor code is parameterized by $(n, C, \Omega(x))$ where n is the information block to be encoded, C is the codebook of the LDPC code and $\Omega(x)$ is the LT code output-symbol degree profile. The characteristic of Raptor code is mainly determined by the output-symbol degree profile $\Omega(x)$, where $\Omega(x) = \sum_{d=1}^{D} \Omega_d x^d$ and Ω_d is the probability that an output symbol is of degree d.

In the LDPC precoding, the n-bit information vector is mapped to a n'-bit codewords according to the codebook C which are usually referred to as the input symbols (or bits) of the LT encoder. In the encoding process, a LT output symbol (or bit) is firstly assigned with a degree of d with the probability of Ω_d. Then the output bit is generated through the operation of XOR of d randomly chosen input symbols. With the above process repeated, an infinite stream of bits will be produced. Then the encoded bits are modulated. For simplicity, BPSK modulation is adopted, i.e., each entry of $\boldsymbol{s}$ in (1) is either 1 or − 1.

3.2 The compression scheme at the BBU pool

At the BBU pool, we adopt a scalar quantizer with the uniform step to compress the precoded signal $\tilde{\boldsymbol{x}}_m \,(m = 1, \ldots, M)$ individually. Under the individual peak power constraint P_{max} at each RRH, the range of quantizer is $[-F_{max}, F_{max}]$, where

$$F_{max} = \sqrt{P_{max}} \tag{5}$$

The quantizer thresholds can be expressed as follows

$$u_\ell = \left(\frac{-L}{2}+\ell\right)\Delta, \ell = 0, 1, ..., L \tag{6}$$

where $\Delta = 2 \times F_{max}/L$ is the step-size of the quantizer and $L = 2^b$ is the number of quantizer intervals for b bits. Note the b is determined by the capacity of the fronthaul C_F, and we have $b = C_F$. The quantizer outputs are given by

$$\boldsymbol{x}_m = v\left(\tilde{\boldsymbol{x}}_m\right) = \left(\frac{-L+1}{2}+\ell\right)\Delta, \text{if } \tilde{x}_m \in [u_\ell, u_{\ell+1}) \tag{7}$$

which is the corresponding transmit signal $\boldsymbol{x}_m$ of mth RRH. Although the design of the quantizer is simple, but it is suitable for practical implementation.

3.3 Iterative decoding at the users

In this subsection, we discuss the decoding scheme at the users.

3.3.1 Compression noise approximation

Firstly, we write the transmit signal $\boldsymbol{x}_m$ from the mth RRH in the following form

$$\boldsymbol{x}_m = \tilde{\boldsymbol{x}}_m + q_m \tag{8}$$

where $\tilde{\boldsymbol{x}}_m$ is the precoded signal in (1) and q_m is the compression error from the quantizer given in previous subsection. Explicitly, q_m equals $v(\tilde{\boldsymbol{x}}_m) - \tilde{\boldsymbol{x}}_m$ according to (7). Substitute (1) and (8) into (4), the received signal $\boldsymbol{y}_k$ for the kth user is given by:

$$\boldsymbol{y}_k = \boldsymbol{h}_k{}^T(\boldsymbol{W}\boldsymbol{s}+\boldsymbol{q})+z_k \tag{9}$$

where $\boldsymbol{q} = [q_1, q_2, ..., q_M]^{\mathrm{T}}$ is the set of compression error. According to (2), we have

$$\boldsymbol{y}_k = a_k \boldsymbol{s}_k + \boldsymbol{h}_k{}^T\boldsymbol{q} + z_k \tag{10}$$

We assume that the each user is notified the ZF precoder used at the BBU pool, and knows channel vector $\boldsymbol{h}_k{}^{\mathrm{T}}$ which is from each RRH to itself. Then the variance of the compression noise term $\boldsymbol{h}_k{}^{\mathrm{T}}\boldsymbol{q}$ can be calculated by averaging on all the possible signals in (1) (Note that each entry $\boldsymbol{s}_i$, *for* $i = 1, ..., K$ in $\boldsymbol{s}$ can only be −1 or 1 due to binary modulation)

$$Var\left(\boldsymbol{h}_k{}^T\boldsymbol{q}\right) = \frac{1}{2^K}\sum_{\boldsymbol{s}_i \in \pm 1, i=1,...,K}\left(\sum_{m=1}^M h_{km}q_m(S)\right)^2 \triangleq \sigma_k{}^2 \tag{11}$$

where

$$q_m(S) = v\left(\sum_{j=1}^K \boldsymbol{w}_{j,m}\boldsymbol{s}_j\right) - \sum_{j=1}^K \boldsymbol{w}_{j,m}\boldsymbol{s}_j \tag{12}$$

recalling that $\boldsymbol{w}_{j,m}$ is the jth entry of the vector $\boldsymbol{w}_m$ of the precoder $\boldsymbol{W}$.

We approximate the distribution of the compression noise $\boldsymbol{h}_k{}^{\mathrm{T}}\boldsymbol{q}$ as Gaussian with mean of zero and variance of $\sigma_k{}^2$. Denote c_k as each bit of the rateless codeword for kth user. Then according to (11), at the kth user, the Log Likelihood Ratio (LLR) of c_k can be calculated from the received bits y_k as follows

$$\mathrm{LLR} = \mathrm{In}\frac{\Pr(y_k|c_k=0)}{\Pr(y_k|c_k=1)} = \mathrm{In}\frac{\frac{1}{\sqrt{2\pi(\sigma^2+\sigma_k{}^2)}}e^{-\frac{(y_k-a_kc_k)^2}{2(\sigma^2+\sigma_k{}^2)}}}{\frac{1}{\sqrt{2\pi(\sigma^2+\sigma_k{}^2)}}e^{-\frac{(y_k+a_kc_k)^2}{2(\sigma^2+\sigma_k{}^2)}}} = \frac{2a_ky_k}{\sigma^2+\sigma_k{}^2} \tag{13}$$

3.3.2 Belief propagation based decoding scheme

With the LLR of Raptor coded bits as input, each user perform decoding based on belief propagation (BP) algorithm on the factor graph shown in Fig. 3, where the input symbols represent the codewords of a LDPC code and output symbols represent the codewords of a LT code. Here we briefly discuss the decoding procedure. The details can be found in, e.g., [31]. The messages (i.e., the posterior LLR of the input symbols) are passed along each edge between the input and output symbols and check nodes. The decoding process takes two steps. In the first step, messages follow the schedule on the factor graph involving both the LT code and the LDPC code. When the average LLR of the input symbols exceeds a certain threshold m_{th}, we move to the second step which the decoding iteration is performed independently on the LDPC code to eliminate the residual error. The specific process for the first step in round of l is give in the following.

(i) The message from input symbol i to LDPC check node c

$$m_{ic}^{(l)} = \sum_o m_{oi}^{(l-1)} \tag{14}$$

(ii) The message from check node c to input symbol i

$$\tanh\left(\frac{m_{ci}^{(l)}}{2}\right) = \prod_{i' \neq i}\tanh\left(\frac{m_{i'c}^{(l)}}{2}\right) \tag{15}$$

where the product is over all input symbols adjacent to c other than i.

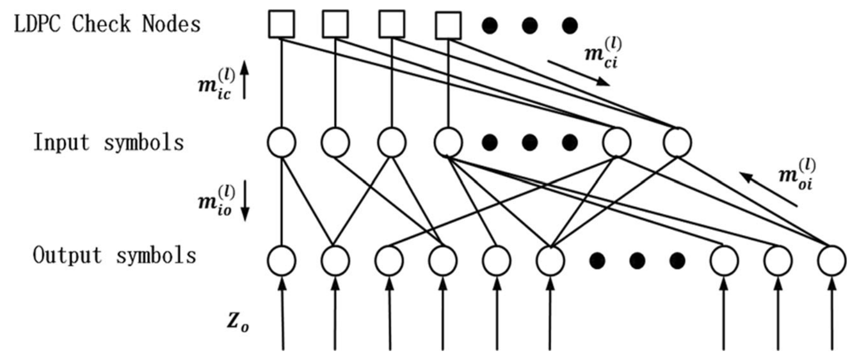

Fig. 3 BP decoding on the factor graph

(iii) The message from input symbol i to output symbol o

$$m_{io}^{(l)} = \sum_{o' \neq o} m_{o'i}^{(l-1)} + \sum_{c} m_{ci}^{(l)} \tag{16}$$

where the first summation is over all LT output symbols adjacent to i other than o.

(iv) The message from output symbol o to input symbol i

$$\tanh\left(\frac{m_{oi}^{(l)}}{2}\right) = \tanh\left(\frac{Z_o}{2}\right)\prod_{i' \neq i} \tanh\left(\frac{m_{i'o}^{(l)}}{2}\right) \tag{17}$$

where Z_o is the LLR calculated from (13) according to the corresponding received bits and the product is over all input symbols adjacent to o other than i.

(v) The LLR updates at input symbol i

$$m_i^{(l)} = \sum_{o} m_{oi}^{(l)} \tag{18}$$

When the average LLR of input symbols exceeds the threshold m_{th}, we then gather these LLRs and perform decoding iteration on the LDPC code where the specific process is the same as (i). (ii). Note that the threshold m_{th} can be derived according to the method mentioned in [31].

After the iteration is done on the LDPC code, we make a decision on the input symbols. If the decoding is successful, an ACK will be feedback to the BBU pool. Otherwise, the BBU pool continues to send the codewords to the user.

4 Joint optimization of the precoder and degree profiles

In this section, when given the fronthaul capacity C_F, and transmit power constraint P_{max}, we jointly optimize the precoder (or explicitly, the power coefficient a_k, a_i^k in (2)) and the degree profiles of Raptor codes at the BBU pool to maximize the sum rate of all the users which can be formulated as $\max \sum_{k=1}^{K} R_k$, subject to successful decoding at each user.

To solve this problem, firstly, given a_k and a_i^k, the extrinsic information transfer (EXIT) function is used to analyze the decoding process. With this, we can maximize the design rate of the kth user ($k = 1, ...K$) while guaranteeing the successful decoding, i.e., $R_k = \left(\beta_k \sum_{d=1}^{D}\left(w_d^k/d\right)\right)^{-1}$, where β_k is the average input-symbol degree and w_d^k is the part of edges in the LT code connecting to an output symbol of degree d. The output-symbol edge degree profile for Raptor code used for the kth user can be presented as $w^k(x) = \sum_{d=1}^{D} w_d^k x^{d-1}$. The relationship between the output edge degree profile and the output-symbol degree profile is $w^k(x) = \Omega^{k'}(x)/\Omega^{k'}(1)$ [32]. Secondly, we search for the optimal a_k and a_i^k.

4.1 Extrinsic mutual information evolution analysis on decoding

In this subsection, we give the explicit condition for successful decoding at the users. Specifically, we refer to the EXIT analysis on the decoding process. According to [31], the LLR messages (e.g., the $m_{ic}^{(l)}$ in Eq. (14)) passed between the neighboring symbols can be considered as random variables satisfying symmetric Gaussian distribution with a mean of τ and a variance of 2τ. The extrinsic information (EI) carried by the LLR messages can be calculated in the following form [31].

$$J(\tau) = 1 - \frac{1}{\sqrt{4\pi\tau}}\int_{-\infty}^{+\infty} \log_2\left(1 + e^{-v}\right)\exp\left(-\frac{(v-\tau)^2}{4\tau}\right)dv \tag{19}$$

For the decoding process mentioned in Section 3.3, we define $x_u^{(l)}$ (resp. $x_v^{(l)}$) as the EI of the LLR messages passed from the output symbols to the input symbols (resp. the input symbols to the output symbols) in round of l. In addition, we define $x_{ext}^{(l)}$ (resp. $T\left(x_{ext}^{(l)}\right)$) as the extrinsic information passed from the input symbols to the check nodes (resp. the check nodes to the input symbols) in round of l. We consider that the EI of LLR messages is passed between the LDPC code and the LT code in each round as Fig. 4 shows. The specific process in round of l is as follows.

(i) EI from input symbols to the LDPC check nodes

$$x_{ext}^{(l)} = \sum_i I_i\, J\left(i \cdot J^{-1}\left(x_u^{(l-1)}\right)\right) \tag{20}$$

where I_i denotes the probability that a randomly chosen input symbol is of degree i, which is determined by the amount of LT output symbols and the output-symbol degree profile $\Omega(x)$.

(ii) EI from the LDPC check nodes to input symbols

$$T\left(x_{ext}^{(l)}\right) = \sum_{i=1}^{d_v} \Lambda_i\, J\left(i \cdot J^{-1}\left(1 - \sum_{j=2}^{d_c} p_j J\left((j-1) J^{-1}\left(1 - x_{ext}^{(l)}\right)\right)\right)\right) \tag{21}$$

where Λ_i denotes the edge distribution of the input-symbol, i.e., the probability that a randomly chosen edge is connecting to a degree i variable node of the LDPC code (or equivalently, the input symbol of the LT code), and p_j denotes the edge distribution of the LDPC check node. The above parameters are determined by the LDPC codebook.

(iii) EI from input symbols to output symbols

$$x_v^{(l)} = \sum_{i=1}^{d_v} l_i \cdot J\left((i-1) J^{-1}\left(x_u^{(l-1)}\right) + J^{-1}\left(T\left(x_{ext}^{(l)}\right)\right)\right) \tag{22}$$

where l_i is the probability that a randomly chosen edge in the LT code connected to an input symbol of degree i.

(iv) EI from output symbols to input symbols

$$x_u^{(l)} = 1 - \sum_{d=1}^{d_c} w_d^k \cdot J\left((d-1) J^{-1}\left(1 - x_v^{(l)}\right) J^{-1}\left(1 - J\left(f_o\right)\right)\right) \tag{23}$$

where w_d^k is the fraction of edges in the LT code connecting to an output symbol of degree d and $f_o = 2a_k^2/(\sigma^2 + \sigma_k^2)$.

According to (20)–(23), we can give the recursive equation that describes the evolution of the EI at the input symbols (i.e, $x_u^{(l)}$)

$$\begin{aligned} x_u^{(l)} &= \Phi\left(x_u^{(l-1)}, a_k, \sigma^2 + \sigma_k{}^2, w_d^k, T\left(x_{ext}^{(l)}\right)\right) \\ &= 1 - \sum_{d=1}^{d_v} w_d^k \cdot J\Big\{(d-1) J^{-1}\Big(1 - \sum_{i=1}^{d_c} l_i \cdot J\Big((i-1) J^{-1}\left(x_u^{(l-1)}\right) \\ &\qquad + J^{-1}\left(T\left(x_{ext}^{(l-1)}\right)\right)\Big)\Big) + J^{-1}\left(1 - J\left(f_o\right)\right)\Big\} \end{aligned} \tag{24}$$

Obviously Eq. (24) has a linear relationship with the edge degree profile w_d^k. To guarantee a successful decoding, the following conditions should be satisfied: (1) $x_u^{(l+1)} > x_u^{(l)}$; (2) $x_u^{(l_{max})} > x_u^{th}$, where l_{max} is the largest decoding iterations. Condition (1) means that the EI transfers from output symbols should increase as the decoding iterates, or equivalently, the BER decreases in each iteration. In Condition (2), x_u^{th} is the minimum input EI required by the LDPC decoder to completely remove the residual error. x_u^{th} can be derived according to m_{th} (see Section 3.3).

4.2 Formulation and solution for the optimization problem

According to the analysis in the previous subsection, the optimization problem can be given by

$$\max_{a_k, a_i^k, w_d^k, \beta_k} \sum_{k=1}^{K} \left(\beta_k \sum_{d=1}^{D} \left(w_d^k / d\right)\right)^{-1} \qquad s.t. \tag{25}$$

$$\forall l = 1, \dots, l_{max}, x_u^{(l+1)} > x_u^{(l)}, k = 1, \dots, K \tag{25a}$$

$$\sum_{d=1}^{D} w_d^k = 1, k = 1, \dots, K \tag{25b}$$

$$\forall d = 2, \dots, D : w_d^k \geq 0, k = 1, \dots, K \tag{25c}$$

$$w_1^k > \varepsilon, k = 1, \dots, K \tag{25d}$$

Remark 1 The constraints of (25a) is from (24). Note that (24) is a continuous condition, which is difficult to treat in practice. Instead, we take N discrete points ($x_{u,\ i}$, $i = 1, \dots, N$) from $\left[0, x_u^{th}\right]$, and require (25a) to be satisfied when $x_u^{(l)}$ equals each $x_{u,\ i}$. Therefore, the (25a) is equivalent to the following set of N constraints:

$$x_u^{(l)} = \Phi\left(x_u^{(l-1)}, a_k, \sigma^2 + \sigma_k{}^2, w_d^k, T\left(x_{ext}^{(l)}\right)\right) > x_{u,i} \tag{26}$$

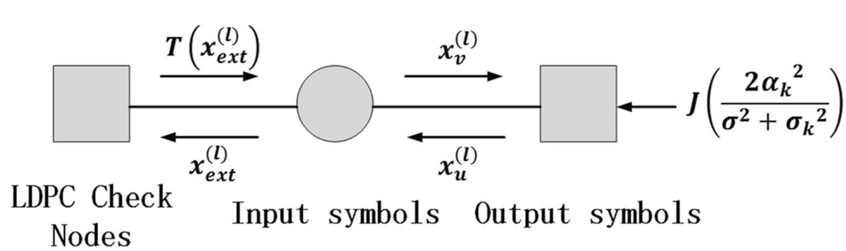

Fig. 4 EXIT analysis on each decoding iteration

(25b) and (25c) are the common constraints for the edge degree profile of linear codes [31]. (25d) is due to the fact that, to start the decoding algorithm, a small number of output-symbols with the degree of 1 are required.

Remark 2 In this optimization problem, we have two sets of variables to be jointly optimized. The first set are the output-symbol degree profiles and the average input-symbol degree of the Raptor code implemented for each user, which determine how close the code approaches the channel capacity. The second set are the power coefficients $\left(a_k, a_i^k\right), \forall k = 1, 2, \ldots, K$ of the precoder $\boldsymbol{W}$ used at the BBU pool. The effects of power coefficients lie in two folds. Firstly, according to (10), a_k determines the received signal power for the kth user. Secondly, a_k and a_i^k affects the compression noise according to the discussion in Section 3.3. Note that the optimal degree profile w_d^k depends on a_k and a_i^k since they has influence on the convergence of the decoding process, i.e., condition (25a).

Note that the optimization problem (25) is non-convex. We refer to a scheme with a two-loop structure to solve this problem. The algorithm is depicted in Table 1. In the inner loop, given by the fixed power coefficients set, i.e., $\left(a_k, a_i^k\right), \forall k = 1, 2, \ldots, K$, we optimize the output-symbol degree profiles $\Omega^k(x)$ (or equivalently, w_d^k) and the average input-symbol degree β_k for the kth user. Note that the problem is transferred to linear optimization problem when β_k is fixed, since both the objective function and the constraints are linear on w_d^k. Then we exhaustively search the best β_k. In the outer loop, we exhaustively search the best a_k and a_i^k within the range limited by the peak-power constraints P_{max} to maximize the objective function.

5 Simulation results

In this section, we present the numerical results to show the advantage of the proposed rateless code downlink transmission strategy with the optimized degree profiles of Raptor code and power coefficients in the precoder. We consider a C-RAN downlink system with 3 RRHs and 2 users. Each RRH is subjected to the same peak power constraints $P_{max} = 2\text{W}$ and each fronthaul link has the same capacity of C_F. The range of quantizers can be calculated from (5) as $[-1.4142, 1.4142]$. We simulate the bit error rate (BER) of the first user and sum throughput of the network.

Table 1 The proposed algorithm for problem (25)

Step1.Initialize the $\left(a_k, a_i^k\right), \forall k = 1, 2, \ldots, K$ within the range limited by P_{max}.

Step2.Initialize β_k for $k = 1, \ldots, K$.

Step3.For each user, we solve the following problem

$$\max_{w_d^k} \left(\beta_k \sum_{d=1}^{D} \left(w_d^k/d\right)\right)^{-1} s.t. \forall l = 1, \ldots, l_{max}, x_u^{(l+1)} > x_u^{(l)} \qquad (27)$$

$$\sum_{d=1}^{D} w_d^k = 1 \quad \forall d = 2, \ldots, D : w_d^k \geq 0 \quad w_1^k > \varepsilon$$

Step4.Back to step 2. Search β_k for $k = 1, \ldots, K$ to maximize the objective function of (27), respectively.

Step5.Back to Step 1. Search (a_k, a_i^k) to maximize the objective function of (25).

In the simulation we set the channel matrix $\boldsymbol{H} = \begin{vmatrix} 1.395 & 1.731 & 1.419 \\ 0.362 & 1.579 & 0.971 \end{vmatrix}$ and we use a LDPC code with a rate of 0.95 as the precoder of the Raptor code. The successful decoding threshold x_u^{th} is set to 0.983 and the maximum degree of output-symbol D is set to 60. The length of message for each user is 9500 bits.

5.1 BER performance

We examine the BER performance of the user 1. For BER simulation, the noise variance at each user is set to $\sigma^2 = 1.8^2$. According to the optimization algorithm discussed in Section 4, we derive the optimal power coefficients in the precoder (2) as $a_1 = 1.031$, $a_2 = 0.548$, $a_1^1 = 0.629$ and $a_1^2 = -0.611$ when the fronthaul $C_F = 10$(bpcu), i.e., the BBU pool applies a 10-bit scalar quantizer. The optimized degree profile $\Omega(x)$ of user 1 is:

$$\Omega_1 = 0.0051, \Omega_2 = 0.3903, \Omega_3 = 0.3054, \Omega_6 = 0.0619$$
$$\Omega_7 = 0.1561, \Omega_{19} = 0.0377, \Omega_{20} = 0.0343, \Omega_{60} = 0.0093$$

Then we consider the $C_F = 5$(bpcu) and we derive the optimal power coefficient in the precoding matrix as $a_1 = 1.028$, $a_2 = 0.543$, $a_1^1 = 0.638$ and $a_1^2 = -0.607$, and we have the optimized degree profile $\Omega(x)$:

$$\Omega_1 = 0.0051, \Omega_2 = 0.3895, \Omega_3 = 0.3069, \Omega_6 = 0.0496$$
$$\Omega_7 = 0.1694, \Omega_{20} = 0.0698, \Omega_{54} = 0.0071, \Omega_{55} = 0.0026$$

For comparisons, we also simulate the BER achieved by the optimal degree profile under BEC [32] and the degree profile $\Omega'(x)$ which is optimized for different capacity of the fronthaul link but without considering compression noise, that is, we set ${\sigma_k}^2 = 0$ in (24).

We simulate BER under the different overhead which is defined as

$$overhead = \frac{C(\gamma) \cdot N}{k} - 1 \qquad (28)$$

where $C(\gamma)$ is capacity of a BiAWGN channel with SNR of $\gamma = {a_1}^2/\sigma^2$, N is the length of the Raptor code k is the length of information block. A smaller overhead means a high rate of the code and overhead of 0 means that the theoretical limit is reached. Here we use $C(\gamma)$ as the theoretical limit due to the fact that according to (10), the

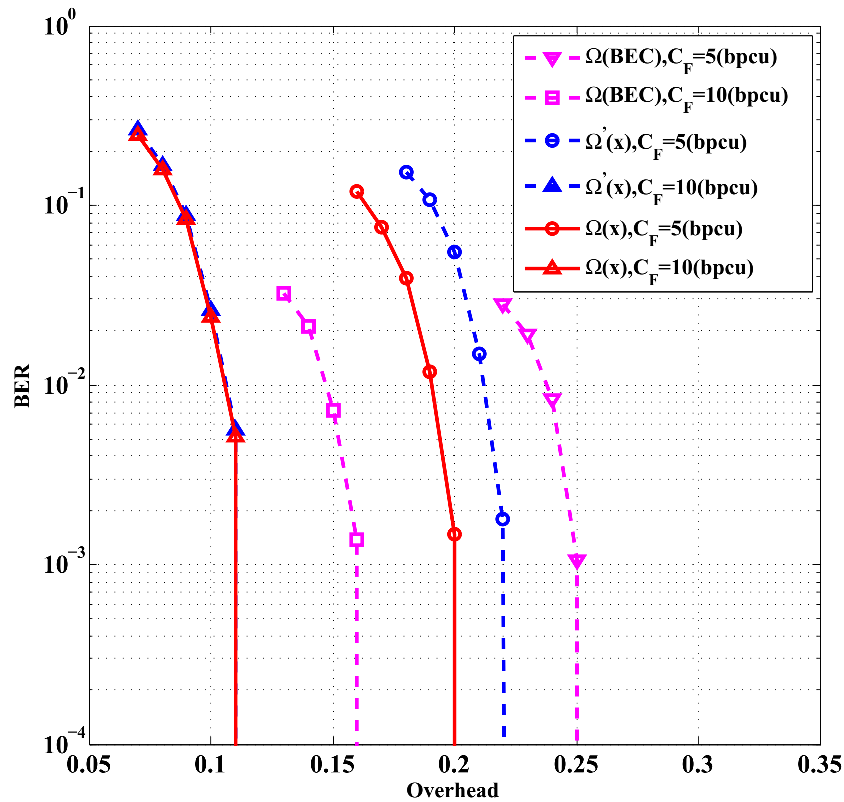

Fig. 5 BER performance for different profiles

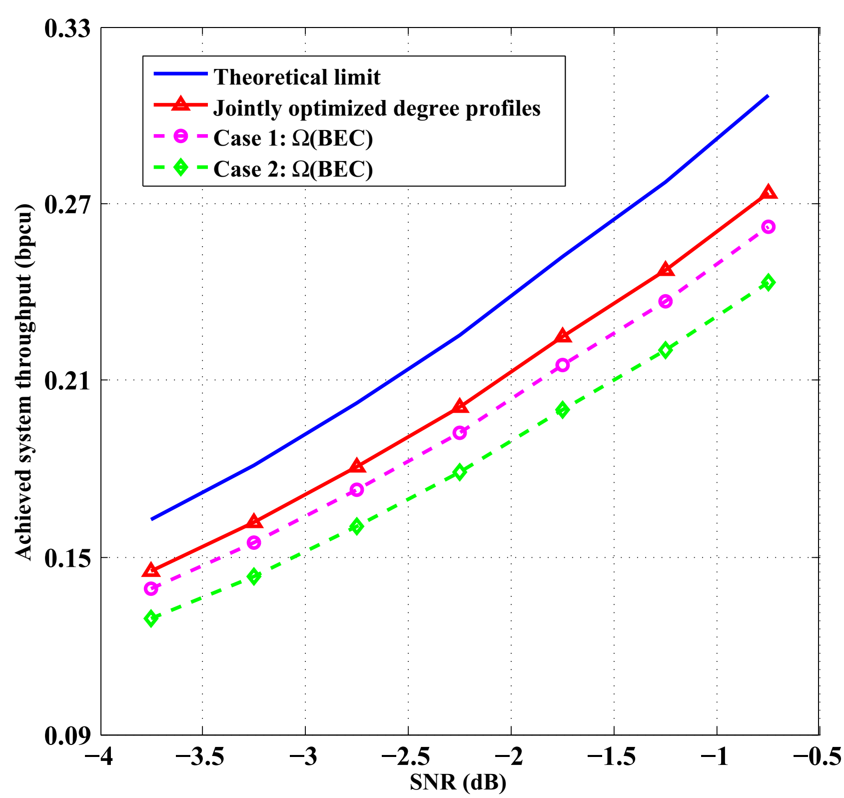

Fig. 6 Throughput performance of the system

$C(a_k{}^2/\sigma^2)$ is the capacity of the channel when the compression noise $\sigma_k{}^2 = 0$ for the kth user.

The expression of $C(\gamma)$ is [32]:

$$C(\gamma) = 1 - \frac{1}{2\sqrt{2\pi\gamma}} \int_{-\infty}^{\infty} \log_2(1+e^{-x}) \cdot e^{\frac{(x-2\gamma)^2}{8\gamma}} dx \tag{29}$$

As shown in Fig. 5, the optimized profiles achieves a BER under 10^{-4} at a smaller overhead comparing with other two profiles. Moreover, it is observed that with the capacity of the fronthaul growing larger, the optimized profile achieves a lower BER. The observation is straightforward since a higher fronthaul capacity allows the quantizer with higher output bits which produces a lower compression noise.

5.2 Throughout performance of the system

Then we simulate the sum throughput of the network under different signal to noise ratio (SNR), i.e., P_{max}/σ^2. Note that for convenience, in the simulation we change the value of σ while keep P_{max} invariant. To evaluate the throughput, we make the BBU pool keep sending coded symbols to all users until the ACKs from all the users are received.

In the simulation, we set the fronthaul capacity as $C_F = 10$(bpcu). We first optimize the degree profiles and precoding power coefficients under each SNR and simulate the sum throughput. For comparisons, the following cases are also simulated.

Case 1: BEC degree profile with the same power coefficients as in the optimized case;
Case 2: BEC profile with the power coefficients of $a_1 = 0.95$, $a_2 = 0.65$, $a_1^1 = 0.55$ and $a_1^2 = -0.5$.

Note that here the choice of the suboptimal alpha insures that the precoded symbols in $\tilde{\boldsymbol{x}}$ lie in the range of the scalar quantize, i.e., $[-F_{max}, F_{max}]$, in order for a fair comparison. The theoretical limit is calculated by $C(a_1{}^2/\sigma^2) + C(a_2{}^2/\sigma^2)$, where a_1 and a_2 are the same as those in the optimized case.

It can be observed in Fig. 6 that under the same SNR, the optimized degree profile achieves a higher throughput comparing with the BEC profile with the same optimized power coefficients, which is within 11% away from the theoretical limit. Moreover, we can find the second case has the worst throughput performance, which is straightforward since the power coefficients and degree profiles used in this case are suboptimal.

6 Conclusion

In this paper, we proposed the downlink rateless coded transmission scheme in C-RAN where the number of RRHs is more than the users. At the BBU pool, we adopted the Raptor code for rateless coding and designed the precoder and the quantizer. Since the capacity of the fronthaul links is limited, the precoded signals must be compressed. The scalar quantizer design is according to the peak power constraint of the RRHs. At the users, we calculated the variance of the compression noise and use Gaussian approximation to derive the soft information from the received signals. With this, we gave the decoding algorithm based on belief propagation. Besides, we proposed a joint optimization scheme for the precoder and the degree profiles of Raptor code in order to maximize the sum rate of the network.

Note that there are several possible extensions for this work. For example, one can consider a more sophisticated compression scheme and the joint design of the degree profiles, precoder, and compressor. Moreover, note that in this work, the optimization of the rateless code profiles is under a fixed channel matrix. In the future work, we can investigate the universally optimal rateless code profiles under fading channel scenario.

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