Big Data Aided Vehicular Network Feature Analysis and Mobility Models Design

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Abstract

Vehicular networks play a pivotal role in intelligent transportation system (ITS) and smart city (SC) construction, especially with the coming of 5G. Mobility models are crucial parts of vehicular network, especially for routing policy evaluation as well as traffic flow management. The big data aided vehicle mobility analysis and design attract researchers a lot with the acceleration of big data technology. Besides, complex network theory reveals the intrinsic temporal and spatial characteristics, considering the dynamic feature of vehicular network. In the following content, a big GPS dataset in Beijing, and its complex features verification are introduced. Some novel vehicle and location collaborative mobility schemes are proposed relying on the GPS dataset. We evaluate their performance in terms of complex features, such as duration distribution, interval time distribution and temporal and spatial characteristics. This paper elaborates upon mobility design and graph analysis of vehicular networks.

Keywords Big data · Vehicular network · Complex network · Mobility models

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1 Introduction

Nowadays, large population and heavy traffic in urban areas lead to traffic congestion and automobile exhaust, reducing people's travel experience by a large scale [1]. To address these issue, many schemes have been proposed, and vehicular networks are one of them [2]. Furthermore, vehicular networks also play a pivotal role in intelligent transportation system (ITS) and smart city (SC) construction [3], which means a lot in terms of traffic flow management [4], urban planing [5], location based recommendation service [6], etc. The arrival of 5G era provide vast potential for future development for vehicular network. In other words, there will be a greater bandwidth, higher carrier frequency, extreme base station and device densities [7, 8]. Therefore, how to design and optimize vehicular networks has been concerned by many scholars and researchers.

Mobility models are of great importance to vehicular network with the consideration that vehicular network is dynamic [9]. Actually, mobility models determine its spatial and temporal characteristics of the network topology. Hence, a practical mobility model is important for assessing



relevant algorithms and systems, especially for routing policy evaluation as well as traffic flow management [10]. An inappropriate mobility model may even lead to erroneous conclusions [11]. In fact, we give the way of combining the two classical mobility model construction methods. The classic mobility model is based on math, such as RWP. Pure data driven approach does not have a certain of universality. Our method aims to combine the two methods.

A single vehicle's trajectory seems to be a random motion but actually has its inherent pattern, which inspires us to use the existing data to dig the pattern. Big data technology, like parallel computing, machine learning and deep learning, is booming, which makes the vehicular network analysis and design based on big data quite noticed [12, 13].

On the basis of the vehicle GPS data set, relevant researches are conducted and original contributions are as follows:

- Complex network feature verification: We first verify that the vehicular network is essentially a complex network. This inspires us to analyze and evaluate the mobility model with the relevant characteristics of complex networks theory.
- Mobility model design: Under the inspiration of the ecommerce recommended system, we design some latest vehicular mobility models, which approximates actual data and is easy to explain.
- Time-varying network characteristics analysis: In view of the fact that vehicular network is a dynamic network, we compare the temporal and spatial characteristics of different mobility models as well as their relevance with real networks.
- Real-world dataset evaluation: Relying on the vehicle GPS data in Beijing, we evaluate our scheme in terms of complex features, duration distribution, interval time distribution and temporal and spatial characteristics.

The remainders of this article are outlined as follows. We commence with the related work review in Section 2. Then, we introduce the vehicle GPS dataset and verify that the vehicular network is essentially a complex network in Section 3. In Section 4, we introduce the mobility models and relevant indicators. Section 5 establishes the performance comparison in complex features, duration distribution, interval time distribution and temporal and spatial characteristics, followed by the conclusion in Section 6.

2 State-of-the-art

Complex network studies become popular with the founding of the "small world" characteristics in 1999 [14] and the the

"scale-free" feature in 2000 [15, 16]. Thanks to the truth that the complex network theory has a certain degree of universality, coupled with the complex network feature of many real world networks, the complex network theory is widely used in network analysis and optimization. Wang et al. analyzed the complex features of vehicular network based on some static parameters, focusing on their value on clustering algorithm [17, 18]. Relying on complex network, Ruela et al. designed a genetic algorithm for designing a wireless sensor network [19]. At present, the combination of complex networks and big data analysis is a trend of complex network research [20, 21]. In our works, we exploit the characteristics of complex networks to explore the temporal and spatial characteristics of vehicular networks and evaluate different mobility models based on static and dynamic characteristics.

As vehicular network is a dynamic network, the mobility models are essential to evaluate the performance of upperlayer protocol, which means incorrect mobility model can result in wrong conclusions [11]. Nowadays, GPS data is quite popular in order to analyze the mobility model. And as for vehicular network, four categories can be roughly incorporated into mobility models [22]: synthetic models, survey-based models, traffic simulatorbased models and the trace-based models. Synthetic models, relying on mathematical models, are used for reflecting a realistic physical effect, such as random way point [23] and weighted way point model [24]. As the name goes, surveybased models get the models property through surveys, such as the agenda-based mobility model [25]. And the traffic simulator-based models are generated from traffic simulator obviously, such as SUMO [26], VISSIM [27]. However, the big data focus more on the trace-based models [28].

Actually, the big data for mobility model analysis includes not only the trace-based models but also the first category [22]. For instance, the model based on social network belongs to the first category [22]. Specifically, Gonzalez et al. [25] discovered that most users traveled around their familiar places, on the analysis of the data collected through tracing mobile phone users. Song et al. [29] researched and analyzed the distribution of time interval, i.e, how long a user stayed at a certain place, and sought out the truncated power law distribution. On account of real trajectory data, Musolesi et al. [30] modeled social relationships using interaction matrix, and the value of matrix elements represented the relationship between two specific users.

3 Complex Network Feature Verification

In this section, we first introduce the GPS dataset in Section 3.1, which is the basis of our analysis and evaluation afterwards.

Then, several typical complex network metrics will be introduced in Section 3.2. Finally, relying on this dataset, we verify the complexity of the network, that is, the "scale-free" characteristics.

3.1 Dataset Introduction

Ten thousand vehicles' GPS data in Beijing are the research objects and each one of them is recorded for a few days. The latitudes of taxi are distributed from 39.8 to 40.05 and longitudes are distributed from 116.25 to 116.5. The edge distribution is relatively sparse in remote area, and dense in the middle area. This is consistent with the actual traffic system. So we will adopt this dataset for analyzing our models.

The whole vehicle distribution in a specific time in Beijing is reflected clearly in Fig. 1. However, The function of Fig. 1 is far beyond a good refection of the spatial distribution of vehicles. It also presents the characteristics of Beijings road network structure, i.e., the grid network topology.

3.2 Complex Network Indicators and Feature Verification

In the first place, we propose some key parameters depending on the complex network theory, which will be utilized in this subsection and the Section 5. Then, the "scale-free" characteristics of the vehicular network will be verified.

Node Degree Distribution: The node's degree of the vehicle *i* in the vehicular network, denoted by k_i , is defined as the number of its neighborhood. It describes the quantity of links between the nodes in the network which is directly connected with it [17]. Moreover, p(k) is the probability



Fig. 1 A whole vehicle distribution in a specific time

that a randomized node's degree of k. Then, the whole of all vehicles' degree distribution contribute to the node degree distribution [31].

$$p_k = \frac{num_k}{N},\tag{1}$$

where num_k is the number of nodes with k neighborhoods and N is the total number of nodes.

Clustering Coefficients: The neighbors of a specific node can also connect with each other, which is measured by the clustering characteristic, showing the tightness of the network [17, 32]. And its specific definition is:

$$C_{i} = \frac{E_{i}}{k_{i}(k_{i}-1)/2}.$$
(2)

In which k_i indicates the node degree of vehicle *i* and E_i is the number of links among its neighbors. Further more, the general clustering coefficient of the entire network is the average of C_i .

Betweenness Centrality: The normalized betweenness centrality *B* reflects the importance of nodes in all the path [17, 33], i.e.,

$$B_i = \frac{2}{(N-1)(N-2)} \sum_{s \neq i \neq t} \frac{n_{st}^i}{g_{st}},$$
(3)

where g_{st} is the number of the shortest path from *s* to *t*, and n_{st}^{i} notes the number of the shortest path via *i* from *s* to *t*.

Connected Component: The connected component describes the connectivity of the network, which is a very important parameter for the communication network. Especially in the application of information broadcasting, the maximum connected component determines the upper bound of the information coverage ratio. It contains the node index of the largest connected piece in the network [34].

Core Number: The core number is an effective index to analyze the hierarchical structure of network. We can get rid of the outer layer of the network nodes iteratively to get the core number of nodes [35].

Due to different communication schemes, the communication distances may be different. We set up 3 sets of experiments, the communication distances are 200 m, 500 m and 1000 m. Based on the above communication distance and the vehicle distribution at a particular time, we establish the corresponding topological graph. Specifically, if the distance between the two vehicles is less than the preset communication distance, we think that the corresponding node in the topology graph has a directly connected edge. Relying on this topology graph, we analyze the degree distribution and the betweenness distribution, as shown in the Fig. 2.

As shown in the Fig. 2, the degree distribution basically satisfies the scale-free property, especially when the communication distance is small. As for the probability density distribution of betweenness, all the communication distances are satisfied with the scale-free property. In fact,

200m

500m

- 1000m



Fig. 2 Degree and Betweenness distribution comparison

the density of the network increases because of the increase of the communication distance. And the increase of the network density makes the network nodes converge, which leads to the uniform distribution of the network in terms of the degree distribution. This proves that the vehicle network belongs to the complex network, and we can use the complex network theory to analyze and optimize the vehicle network.

4 System Model

In this section, we commence with introducing the mobility model designed in Section 4.1. Then, in Section 4.2, we specify the evaluating indicator, which is utilized for mobility scheme comparison in this paper.



Fig. 3 Moving track of a vehicle



Explanation of the reasons for introducing social attributes comes first. The positions of a certain vehicle are plotted in the given area, which are painted in the Fig. 3.

3000

4000

Betweenness

(b) Betweenness

5000

6000

7000

8000

We can reach a conclusion from the chart that the driver favors some locations, which is considered as random walk traditionally. Therefore, We can not draw such conclusion from mobility model. By contrast, using this preference property, we may get a more realistic mobility model, which will be reflected by social attributes.

Interaction matrix, denoted as R, is in demand a typical social features which are for mirroring the connections between two vehicles or a vehicle to a position. What's



Fig. 4 Duration comparison



Fig. 5 Interval time comparison

more, distance matrix, i.e., D, is the distance relationship of the vehicles. Hence, the relationship and the distance of i and j is on behalf of R_{ij} and D_{ij} . Markov process is capable of modeling vehicles movement, which can be defined as:

$$Q_{ij} = \frac{R_{ij}D_{ij}}{\sum_{j=1}^{m} R_{ij}D_{ij}}$$
(4)

The RIS (random initialization scheme), the GCLAS (global critical location assessment scheme) and the PCLAS (personalized important location assessment scheme) are included in the related solutions that is on account of the diversities of social properties in R.

The initialization scheme are affected by the values in R. Particularly, the initialization scheme are random when the values in R are random, and the R_{ij} is given as:

$$R_{ij} = rand(1). \tag{5}$$

In which rand(1) is defined as a random number between 0 and 1.

GCLAS is achieved, relying on the GCLA (global critical location assessment). For example, the frequency of each site which is under assessment is in proportion to its probability. In GCLAS, the R_{ij} is given by:

$$R_{ij} = \frac{\sum_{v} \sum_{t} transit_{i \to j}}{\sum_{v} \sum_{t} \sum_{k} transit_{i \to k}},$$
(6)

In which v is defined as a vehicle, t stands for a period of time and k means any location connecting location i. In view of the diversities of the vehicle, the significance of

the location is assessed by the specific vehicle history data, which results in the PCLAS. And the R_{ij} is defined as:

$$R_{ij}^{k} = \frac{\sum_{t} transit_{i \to j}^{k}}{\sum_{t} \sum_{k} transit_{i \to j}^{k}}$$
(7)

And the Q_{ij} should be modified as follows:

$$Q_{ij}^{k} = \frac{R_{ij}^{k} D_{ij}}{\sum_{j=1}^{m} R_{ij}^{k} D_{ij}}$$
(8)

What's more, real track data, simplified trajectory data and random walk are regarded as comprising group. STD (simplified trajectory data) is achievable by mapping RTD (real track data) into a 25 * 25 grid network. The RWS (random walk scheme) takes the whole vehicles' movements as random.

4.2 Evaluating Indicator

Particularly, we consider VANETs as the time variant graph $\mathcal{G} = (No, E, \mathcal{T}, \rho)$. No set is composed by vehicles. *E* set stands for the relationship between No set, which means a communication link in this research. In this paper, the relationship represents a communication link. In dynamic network, this relationship may chance over time. Therefore, we regard *T* as the survival time and \mathcal{T} as the time domain, which meets the $T \subseteq \mathcal{T}$. $\rho : E \times \mathcal{T} \rightarrow \{0, 1\}$ manifests survival function, presenting in a specific time whether a given edge exists or not. The duration of the connection and the length of time internal are of great significance to the time vary graph. That is to say:

- The duration of the connection: beginning with the entity *i* and entity *j* connection, until the first breaking time point appears. In this time horizon, $\rho_{ij}(T) = 1$ is maintained;
- The length of time internal: beginning with the entity *i* and entity *j* break, until the first connection time point appears. In this time horizon, $\rho_{ij}(T) = 0$ is maintained.

The 2 parameters mentioned above will be discussed later to assess the mobility model.

5 Scheme Comparison

In this section, we conduct some simulation analysis on mobility models of the vehicular networks relying on the dataset introduced in Section 3.1. We consider three typical indicators, i.e., duration distribution, interval time distribution, degree distribution and temporal and spatial characteristics, in mobility model assessment. And the



Fig. 6 Degree distribution comparison

relevant results and analysis are presented in Sections 5.1 and 5.2, 5.3 and 5.4, respectively.

5.1 Duration Comparison

People may lay emphasis on the latter 2 markers in the research of network communication, leading to the exploration in their property in duration chart.

The Fig. 4 above compares duration distribution of 6 schemes. It's clear from the diagram that 4 schemes gather close while the STD and RTD converge. Although the 4 schemes gather close, GCLAS and PCLAS draw closer to RTD.

5.2 Interval Time Comparison

In this part, we take interval time of the 6 schemes into comparison, which is shown in Fig. 5.

Figure 5 reveals that the interval time of GCLAS and PC LAS are similar to STD, which lays its dominant position.

5.3 Degree Distribution Comparison

The mobility models are under evaluation in this section, in the matter of their degree distribution. In fact, the degree distribution play a vital role in complicated communication networks. Its superiority embodies in distinguishing network types.

Figure 6 manifests the 5 mobility schemes degree distribution which are specified in Section 5.3 and the realistic data. We can draw a conclusion from the figure that the real data reflects the property of scale-free. However, we can also discover the fact that the STD and 4 mobility schemes are similar to a Gaussian network.

5.4 Time-Varying Network Characteristics Analysis

In this subsection, we compare the temporal and spatial characteristics of different mobility models and the relevance of real networks.

Key parameters	Time slot	RTD	STD	RWS	RIS	GCLAS	PCLAS
Edges	[1-100]	1056.5	3170.2	1905.3	1903.9	2134.4	1931.9
	[101-200]	1041.5	3110.3	1850.8	1900.4	2069.9	1930.5
	[201-300]	970.0	3363.1	1871.3	1825.1	2053.6	1906.7
	[301-400]	872.7	3456.4	1851.5	1844.2	1970.5	1949.6
Degree	[1-100]	3.3809	10.1447	6.0969	6.0924	6.8300	6.1822
	[101-200]	3.3329	9.9531	5.9225	6.0814	6.6236	6.1777
	[201-300]	3.1042	10.7620	5.9883	5.8404	6.5714	6.1013
	[301-400]	2.7926	11.0605	5.9248	5.9014	6.3055	6.2387
Clustering Correlation	[1-100]	0.4413	0.5767	0.5455	0.5470	0.5433	0.5529
	[101-200]	0.4278	0.5833	0.5423	0.5411	0.5394	0.5520
	[201-300]	0.4182	0.5983	0.5394	0.5416	0.5362	0.5503
	[301-400]	0.3893	0.5975	0.5423	0.5389	0.5374	0.5477
Betweenness	[1-100]	609.9	4518.9	5883.8	5559.9	4912.0	6027.5
	[101-200]	567.8	4375.5	5469.7	5515.2	4501.9	5732.2
	[201-300]	464.2	4545.1	5403.3	5486.1	4691.1	5823.9
	[301-400]	237.5	4177.0	5479.9	5282.7	4745.0	5590.5
Connected Component	[1-100]	178.4400	38.7700	29.9900	31.1400	38.2900	24.9500
	[101-200]	181.6800	36.5300	34.3000	33.5300	41.8000	26.3400
	[201-300]	195.2700	38.5600	35.8200	35.2800	42.0300	26.9100
	[301-400]	218.4100	36.8300	34.8100	37.9800	43.7800	26.8500
Core Number	[1-100]	2.5164	8.3040	4.0936	4.0880	4.5234	4.1609
	[101-200]	2.4643	8.0443	3.9989	4.0870	4.3773	4.1511
	[201-300]	2.3474	9.0084	4.0230	3.9380	4.3520	4.1076
	[301-400]	2.1120	9.4184	3.9938	3.9762	4.2060	4.1755

Table 1 The value of some key parameters of vehicular complex network

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We split the whole time into 400 time periods, i.e, [1 - 400]. We counted the relevant indicators of 6 models, shown in the Table 1. For the sake of intuition, we averaged 100

points in time [1 - 100], [101 - 200], [201 - 300] and [301 - 400]. From the Table 1, we may also find that real world network always have fewer edges, degrees, clustering



Fig. 7 The Average Degree and Betweenness Over Time

correlation, core number and betweenness, especially the betweenness, but its connected component is much more larger. Based on this finding, we can infer that the real network has a large number of weakly connected edges. A large number of weakly connected edges, i.e.,weak tie [36], make the sparse clusters in the graph connected with each other. Most of our simulations have lost this feature.

The Fig. 7 shows the change of the average and betweenness of the network at all times. It can be seen that the network has a certain degree of stability over time.

6 Conclusion

To sum, relying on the GPS dataset, we first verify that the vehicular network is essentially a complex network. This inspires us to analyze and evaluate the mobility model using the relevant characteristics of complex networks. Besides, we propose the corresponding vehicle to location collaboration scheme in vehicular network based on user to product collaboration scheme in the e-commerce recommended system. Based on degree distribution comparison, duration distribution, interval time distribution and temporal and spatial characteristics, the performances of the vehicle to location collaboration scheme are verified.

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