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Simplified formulas for the seismic bearing capacity of shallow strip foundations

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ABSTRACT

The seismic bearing capacity of shallow foundations is affected by inertia forces acting both on the structure and in the supporting soil. Even though the former have been recognised to play often the major role, by increasing the horizontal load and the overturning moment transferred to the foundation, both of them must be taken into account in the seismic design of foundations. Using a pseudostatic approach and based on the upper bound theorem of limit analysis, a comprehensive set of formulas is derived for the computation of the seismic bearing capacity of strip footings resting on cohesive-frictional and purely cohesive soils. Results are given in terms of: (i) reduction coefficients for the Terzaghi's equation of the vertical bearing capacity and (ii) ultimate failure envelopes in the space of normalised loading variables. These formulas extend to more general conditions other literature results, allowing to take into account easily the effects of inertia forces acting both on the superstructure (load inclination and eccentricity) and into the foundation soil. The reliability of the proposed equations, suitable for the design practice, is verified through a thorough comparison with other rigorous and approximate solutions.

1. Introduction

Many studies on the seismic bearing capacity of shallow foundations have shown that inertia forces acting on the structure and in the supporting soil tend to reduce the bearing capacity under seismic conditions. Most works on this topic have been carried out with a pseudostatic approach [18,22,23,25,3,29,30,32,34,35,5,6,9], by adopting: (i) different methods (numerical or theoretical) and theories (limit equilibrium, limit analysis or method of characteristics); (ii) different constitutive assumptions for the soil (purely frictional, purely cohesive or cohesive-frictional); (iii) different hypotheses on the inertia forces on the soil (with or without the vertical component) and (iv) on the structure (equal to or a fraction of those acting on the soil).

Despite the fact that structure inertia has been recognised to play often the major role in reducing the seismic bearing capacity of shallow foundations, recent studies have highlighted possible situations in which even the effects associated to soil inertia can have a significant relevance, in the case of either frictional [22,6] or purely cohesive [24] soils. Moreover, most design codes recommend to take into account the effects of soil inertia in the seismic design of such systems (*e.g.*: [10]).

Going to the design practice, the bearing capacity of shallow foundations under general loading is usually evaluated by means of simple approaches, neglecting any possible soil-structure interaction effect. In this context, codes and guidelines make use of closed form expressions for the bearing capacity, given in the form of either the classical Terzaghi's formula [1,17] or complete three-dimensional failure envelopes [10]. With this respect, only few works in the literature provide empirical formulas including inertia forces both on the structure and into the soil.

As far as spread footings on cohesive-frictional soils are concerned, Budhu and Al-Karni [3], Paolucci & Pecker [23] and Cascone et al. [5] provide reduction factors for the vertical bearing capacity. However, Budhu and Al-Karni [3] consider the same accelerations into the soil and the structure; Paolucci & Pecker [23] do not contemplate the effects of the structure inertia on the N_c and N_q bearing capacity factors, while Cascone et al. [5] refer only to the effects of the seismic action on the N_γ term, thus resulting in a limited applicability of the proposed formulas. Only very recently, Cascone & Casablanca [6] proposed empirical expressions for the reduction coefficients, derived from the best fit of numerical results.

On the other hand, no reduction coefficients are available for the case of shallow foundations on purely cohesive soils, while, in this case, an approximate equation of the failure envelope was proposed by Faccioli et al. [11], based on results of limit analysis [24,25].

This work aims to provide a comprehensive set of empirical equations for the evaluation of the seismic bearing capacity of shallow strip foundations resting on a homogeneous layer of either cohesive-frictional or purely cohesive soil. Moreover, the relative merits of structure

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and soil inertia in the reduction of the bearing capacity are discussed. To this end, following a pseudostatic approach, the upper bound theorem of limit analysis is used, by modelling the soil as an elasto-plastic material with a Mohr-Coulomb and Tresca yield criterion respectively.

Given the inherent uncertainties in the definition of the parameters involved in a bearing capacity calculation, related to both the geotechnical soil model and the earthquake input motion, the simplicity of the empirical equation is by all means a key ingredient when suggesting formulas to be used in the design practice. This is indeed one of the underlying ideas of this work, where, after a thorough comparison of the upper bound results with other literature data, simple formulas are proposed for the reduction coefficients of the Terzaghi's equation, partly incorporating the empirical equations provided by Hansen [14], widely used in the static design practice. Moreover, the same reduction coefficients are used to construct three-dimensional failure envelopes for shallow strip foundations under pseudostatic loading.

Neither the effects of pore water pressure nor the reduction of the shear strength of the soil due to seismic effects are taken into account. Different inertia forces are considered on the structure and into the soil.

2. Problem definition and theoretical framework

Fig. 1 shows the problem under examination, consisting of a shallow strip foundation (width B, embedment depth D) resting on a homogeneous soil (unit weight γ, friction angle ϕ, cohesion c). The foundation is subjected to an inclined and eccentric load, including both the static and inertia forces transmitted by the superstructure. The load is defined by its vertical component V, its horizontal component $H = V \tan\beta$ and an overturning moment $M = V\,e$, where β and e are the angle of inclination and the eccentricity, respectively. The inertia forces into the soil are introduced through the pseudostatic coefficients k_h and k_v, acting in the horizontal and vertical direction, respectively. The soil above the foundation level is replaced by a shear and normal stress distribution proportional to the dead weight of the lateral soil, $q = \gamma D$.

The load eccentricity is taken into account only indirectly, by assuming a reduced effective width $B' = B\text{-}2e$, in agreement with the Meyerhof's suggestion [20]. This strategy, often adopted in the literature to reduce the complexity of the problem at hand, provides a good approximation of the collapse load for shallow footings resting both on sand [19] and clay [15,36].

According to limit analysis, an upper-bound of the exact collapse load can be obtained by equating the power of external forces (P^{ext}) to the power of internal dissipation (P^{int}), computed with reference to a kinematically admissible collapse mechanism. Following Dormieux & Pecker [9], a non-symmetrical Prandtl's mechanism is examined, characterised by two rigid wedges connected by a log-spiral plastic zone, the latter reducing to a circle for a pure cohesive material. The geometry of the failure mechanism, completely defined by the two angles ρ and ψ, together with the assumed kinematic field, is given in Fig. 1(c).

The reader may refer to Chen & Liu [7] for a thorough dissertation on the upper-bound theorem of limit analysis, while its application to the specific mechanism considered herein is detailed in Appendix A and B for a cohesive-frictional soil (M-C yield criterion) and a purely cohesive soil (Tresca yield criterion) respectively.

The average limit load corresponding to the assumed failure mechanism can be expressed as:

$$q_{\text{lim}}^{*}(\rho, \psi) = \frac{1}{2}\gamma B' N_{\gamma E}^{*} + cN_{cE}^{*} + qN_{qE}^{*} \tag{1}$$

where $N_{\gamma E}^{*}$, N_{cE}^{*} and N_{qE}^{*} are functions of the geometry of the failure mechanism, material properties, load inclination, and pseudostatic soil accelerations. The upper-bound estimate of the bearing capacity is given by:

$$q_{\text{lim}} = \min_{\mathbf{H}(\rho,\psi)\le 0} q_{\text{lim}}^{*}(\rho, \psi) \tag{2}$$

where $\boldsymbol{H}$(ρ,ψ) is the vector of physical and/or geometrical constraints. Eq. (2) can be solved by numerical minimization and the results given in standard form as:

$$q_{\text{lim}} = \frac{1}{2}\gamma B' N_{\gamma E} + cN_{cE} + qN_{qE} \tag{3}$$

where $N_{\gamma E}$, N_{cE} and N_{qE} are the seismic bearing capacity factors.

Based on the best fit of rigorous upper bound numerical solutions, the following sections provide a comprehensive set of simplified formulas for the seismic bearing capacity factors of shallow strip foundations. In order to simplify the structure of the empirical equations, the vertical pseudostatic coefficient is not taken into account in their derivation ($k_v = 0$), thus implicitly neglecting any contribution of the vertical soil acceleration. This assumption if often introduced when dealing with the seismic stability of geotechnical systems, including shallow foundations [16,29–31], based on the fact that the vertical acceleration is generally out of phase with and has a different frequency content than the horizontal component, with the corresponding peak

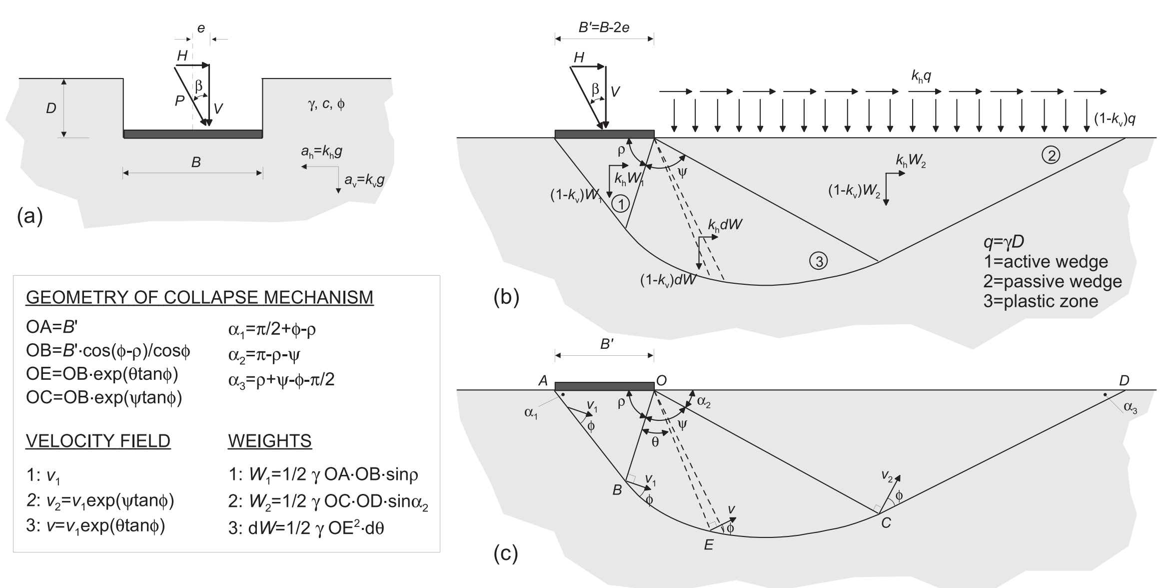

Fig. 1. Shallow strip foundation on homogeneous Mohr-Coulomb soil: (a) geometry and load configuration, (b) failure mechanism and (c) velocity field.

values never occurring simultaneously [12,16,26,8]. As a result, positive and negative contributions from the vertical acceleration, on average, have little effect on the seismic response of the system and can be reasonably overlooked without significant loss of accuracy. On the other hand, a thorough investigation on the effects of k_v on the pseudostatic calculation of the seismic bearing capacity of shallow foundations has been recently carried out by Cascone & Casablanca [6], at least with reference to cohesive-frictional soils, showing that positive (upward) and negative (downward) values of k_v tend to reduce and increase the bearing capacity factors, respectively, compared to the case of $k_v = 0$.

First, applications of the proposed method to static conditions ($k_h = 0$) will be discussed. Then, the results for the dynamic case will be given in terms of reduction factors with respect to the maximum static values ($N_{\gamma S}$, N_{cS}, N_{qS}), corresponding to a vertically loaded footing ($H = 0$).

3. Cohesive-frictional soil ($\gamma \neq 0$, $c \neq 0$, $\phi \neq 0$)

As already pointed out by Soubra [34], the proposed mechanism, applied to the case of a weightless M-C soil ($\gamma = 0$), provides the exact values of the N_{cS} and N_{qS} factors, given by Prandtl [27] and Reissner [28] respectively, *i.e.*:

$$N_{qS} = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)e^{\pi\tan\phi}$$
$$N_{cS} = (N_{qS} - 1)\cot\phi \tag{4}$$

On the other hand, when $\gamma \neq 0$ both the failure mechanism and the bearing factors depend not only on the soil friction angle, but also on the dimensionless coefficients $c/\gamma B$ and $q/\gamma B$. To simplify the calculation, an "all-minimum" procedure can be adopted, in which each factor is taken as the minimum of the corresponding term in the bearing capacity equation [21], *i.e.*:

$$N_{iE} = \min_{\mathbf{H}(\rho,\psi)\leq 0} N_{iE}^{*}(\rho,\psi) \qquad i = q,\ c,\ \gamma$$

In spite of referring to different failure mechanisms, this procedure provides a good estimate of the static bearing capacity factors, with small errors always on the conservative side [21,37]. Moreover, following an "all-minimum" approach, the bearing capacity factors become function of the soil friction angle only.

The upper bound values of $N_{\gamma S}$, referring to the static condition, are shown in Fig. 2 for typical values of the soil friction angle, together with the range of literature data and the approximate function proposed by Hansen [14]. In agreement with Paolucci & Pecker [23], Fig. 2 indicates that the upper bound theorem, combined with the non-symmetrical Prandtl's mechanism, can lead to highly unconservative results with respect to the Hansen's formula.

As far as the pseudostatic case is concerned, the bearing capacity load is given by Eq. (3), where the seismic bearing capacity factors can be written as:

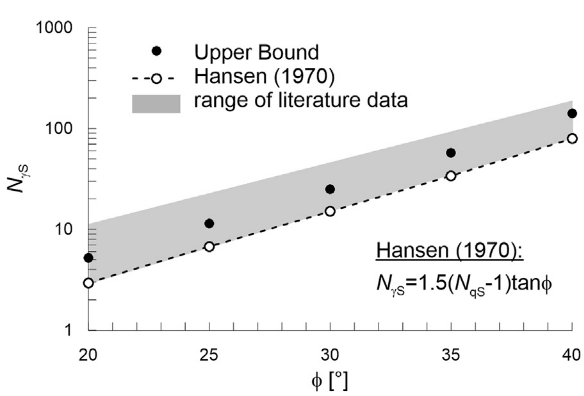

Fig. 2. M-C soil. Static bearing capacity factor $N_{\gamma S}$.

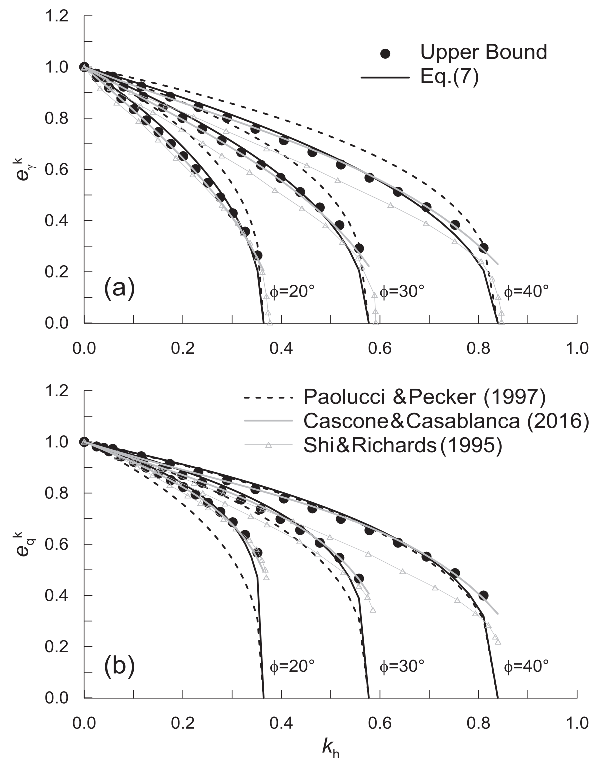

Fig. 3. M-C soil. Reduction coefficients due to soil inertia for: (a) $N_{\gamma S}$ and (b) N_{qS}.

$$N_{qE} = e_q^k e_q^\beta N_{qS}$$
$$N_{cE} = e_c^k e_c^\beta N_{cS}$$
$$N_{\gamma E} = e_\gamma^k e_\gamma^\beta N_{\gamma S} \tag{5}$$

The reduction coefficients in Eq. (5) are defined as:

$$e_i^k = \frac{N_{iE\,(\beta=0)}}{N_{iS}} \qquad i = q,\ c,\ \gamma$$
$$e_i^\beta = \frac{N_{iE\,(k_h=0)}}{N_{iS}} \tag{6}$$

describing the effect of soil inertia and the influence of the horizontal load transmitted by the superstructure, respectively.

As far as soil inertia effects are concerned, the all-minimum procedure provides exactly $e_c^k = 1$, as the assumption of weightless soil is introduced when $c \neq 0$. Nonetheless, this is in substantial agreement with the results by Shi & Richards [33], obtained for $\gamma \neq 0$, showing indeed a negligible dependence of N_{cE} on soil acceleration. On the other hand, Fig. 3 shows the upper bound values of e_γ^k and e_q^k, together with the empirical formulas obtained from the best-fit of numerical data, *i.e.*:

$$e_q^k = \left(1 - \frac{k_h}{\tan\phi}\right)^{(0.37\tan\phi^{0.5})}$$
$$e_\gamma^k = \left(1 - \frac{k_h}{\tan\phi}\right)^{0.47} \tag{7}$$

For the sake of completeness, Fig. 3 shows also other approximate solutions proposed in the literature [23,33,6]. Both coefficients depend strongly on k_h, vanishing for $k_h = \tan\phi$, corresponding to which soil fluidization occurs (Richards *et al.*, 1990). A general good agreement is observed between the upper bound results, the empirical equations proposed by Cascone & Casablanca [6] and those suggested herein (Eq. (7)), the latter having the same simple mathematical structure as the equations proposed by Paolucci and Pecker [23] and Cascone et al. [5], the difference being only in the exponents.

As far as load inclination effects are concerned, Fig. 4 shows the upper bound values of e_γ^β, e_q^β and e_c^β, together with the empirical formulas proposed by Hansen [14]. In agreement with Loukidis et al. [19],

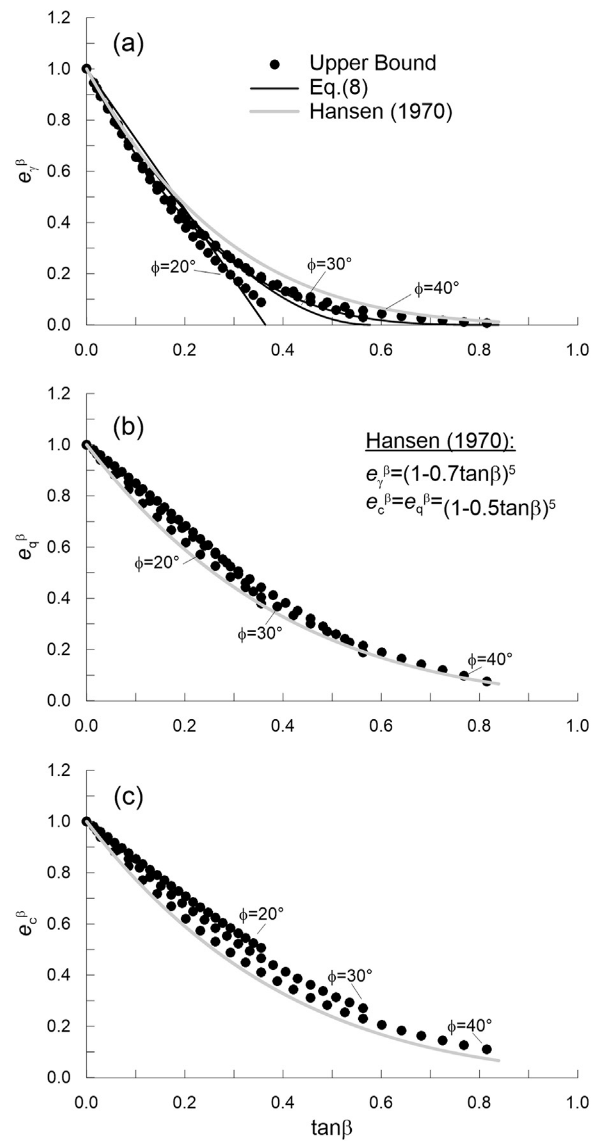

Fig. 4. M-C soil. Reduction coefficients due to load inclination for: (a) $N_{\gamma S}$, (b) N_{qS} and (c) N_{cS}.

soil friction angle does affect the e_γ^β coefficient, which tends to vanish for $\tan\beta = \tan\phi$. In this case, the equation:

$$e_\gamma^\beta = \left(1 - \frac{\tan\beta}{\tan\phi}\right)^{(4.1\tan\phi^{1.4})} \tag{8}$$

derived from the best-fit of numerical data, provides a better description of the actual trend exhibited by e_γ^β. This trend is less evident for the other coefficients, corresponding to which the empirical formulas proposed by Hansen [14] provide a reasonable and conservative estimate of the actual values. It is worth noting that the upper bound results are in very good agreement with those obtained very recently by Cascone & Casablanca [6] using the method of characteristics.

In order to check the reliability of the proposed formulas, Fig. 5 shows a comparison between the values of the bearing capacity load provided by the rigorous upper bound solution, the "all-minimum" procedure and the empirical equations, with reference to the case of a shallow strip foundation on a cohesive-frictional soil ($\gamma = 20$ kN/m^3, $c = 10$ kPa, $B = 3$ m, $D = 1$ m) and assuming the same pseudostatic accelerations into the soil and the structure ($k_h = \tan\beta$). As expected, the all-minimum procedure and the empirical formulas provide slightly conservative values of the bearing capacity, with a maximum difference of 15% with respect to the rigorous solution.

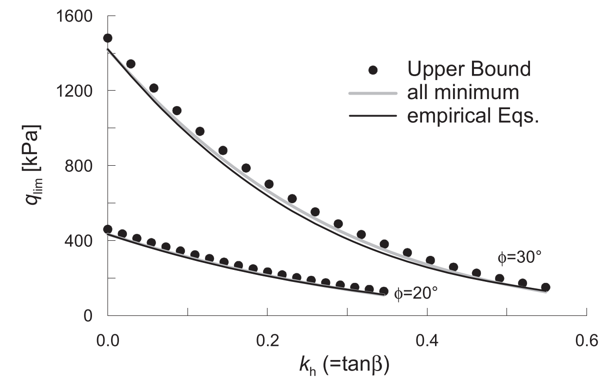

Fig. 5. M-C soil. Reduction of the seismic bearing capacity due to the combination of soil inertia and load inclination ($\gamma = 20$ kN/m^3, $c = 10$ kPa, $B = 3$ m, $D = 1$ m, $k_h = \tan\beta$).

4. Purely cohesive soil ($\gamma \neq 0$, $c = c_u$, $\phi = 0$)

Under static conditions ($k_h = 0$) and vertical load ($H = 0$), the upper bound theorem provides the exact values of the bearing capacity factors, that is:

$$N_{qS} = 1$$
$$N_{cS} = 2 + \pi$$
$$N_{\gamma S} = 0 \tag{9}$$

Moving to the pseudostatic case, the bearing capacity load is given by Eq. (3), where:

$$N_{qE} = e_q^k e_q^\beta N_{qS}$$
$$N_{cE} = e_c^k e_c^\beta N_{cS}$$
$$N_{\gamma E} = e_\gamma^k \tag{10}$$

and the reduction coefficients are defined as:

$$e_\gamma^k = N_{\gamma E(\tau=0)}$$
$$e_i^k = \frac{N_{iE(\tau=0)}}{N_{iS}} \qquad i = q,\ c$$
$$e_i^\beta = \frac{N_{iE(k_h=0)}}{N_{iS}} \tag{11}$$

where $\tau = H/B'$ is the shear stress at the foundation base.

Contrary to the case of M-C soil, the seismic bearing capacity factors were computed following a rigorous application of the kinematic theorem (Eq. (2)). On the one hand, in fact, when the sole horizontal loads are taken into account ($H \neq 0$, $k_h = 0$), the functions $N_{\gamma E}^*$ and N_{qE}^* are identically equal to zero and to one, respectively. On the other hand, when soil inertia is introduced ($k_h \neq 0$), the function $N_{\gamma E}^*$ is negative and the "all-minimum procedure" cannot be applied.

As far as load inclination effects are concerned, Fig. 6 shows a comparison between the upper bound values of e_c^β, the static solution provided by Bolton [2] and the simplified formula proposed by Hansen [14]. The upper and lower bound solutions are virtually the same, the results being in substantial agreement with the Hansen's equation (1970).

As far as soil inertia effects are concerned, the best-fit of numerical data is given by:

$$e_q^k = 1 - a_q\left(\frac{k_h}{k_{h,lim}}\right) - b_q\left(\frac{k_h}{k_{h,lim}}\right)^2 \qquad a_q = 0.75k_{h,lim} \quad b_q = 1.4k_{h,lim}$$
$$e_c^k = 1$$
$$e_\gamma^k = -a_\gamma\left(\frac{k_h}{k_{h,lim}}\right) - b_\gamma\left(\frac{k_h}{k_{h,lim}}\right)^2 \qquad a_\gamma = 1.75k_{h,lim} \quad b_\gamma = 1.4k_{h,lim} \tag{12}$$

where $k_{h,lim}$ is a dimensionless number depending both on the physical

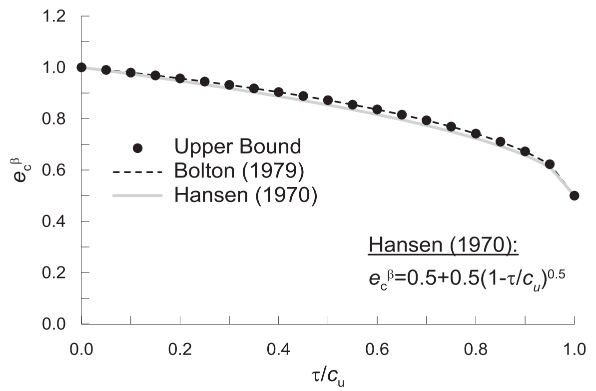

Fig. 6. Tresca soil. Reduction coefficient due to load inclination for N_{cS}.

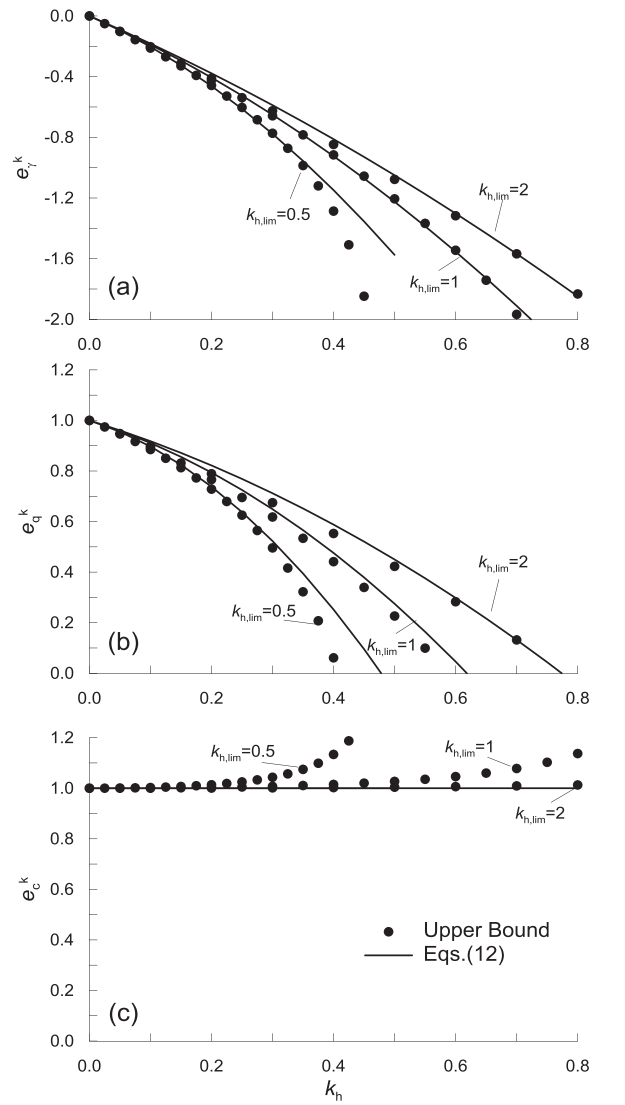

Fig. 7. Tresca soil. Reduction coefficients due to soil inertia for: (a) $N_{\gamma S}$, (b) N_{qS} and (c) N_{cS}.

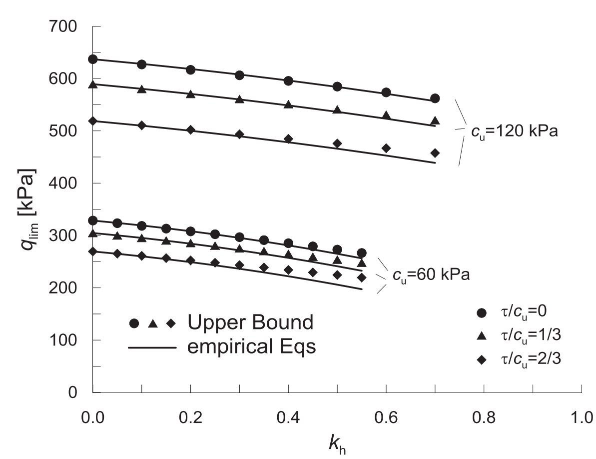

Fig. 8. Tresca soil. Reduction of the seismic bearing capacity due to the combination of soil inertia and load inclination (γ = 20 kN/m^3, B = 4 m, D = 1 m).

and mechanical properties of the soil and on the geometry of the foundation, *i.e.*:

$$k_{h,lim} = \frac{c_u}{\gamma(D + B/2)} \tag{13}$$

As pointed out by Pecker & Salençon [25], $k_{h,lim}$ corresponds to the actual limiting value of the pseudostatic acceleration, beyond which no further equilibrium is attained into the soil. Its value can be obtained by minimizing Eq. (2) under the assumption of $V = H = 0$.

Fig. 7 reports the upper bound values and the empirical formulas for the reduction coefficients due to soil inertia, showing a good agreement in terms of e_γ^k and e_q^k. Moreover, as for the case of M-C soil, the factor N_{cE} exhibits a negligible dependence on the pseudostatic soil acceleration, thus justifying the approximation for e_c^k in Eq. (12).

The reliability of the proposed formulas is illustrated in Fig. 8 with reference to the case of a shallow strip foundation on a purely cohesive soil (γ = 20 kN/m^3, B = 4 m, D = 1 m), assuming three different values of the ratio τ/c_u between the shear stress at the foundation base and the undrained shear strength of the soil. Specifically, Fig. 8 shows a comparison between the values of the seismic bearing capacity load provided by the upper bound solution and the empirical equations, the latter providing slightly conservative values of the bearing load, with a maximum difference of about 15% with respect to the upper bound solution.

5. Summary of results and ultimate failure envelopes

Based on the above results, Table 1 summarizes the complete set of formulas for the seismic bearing capacity of shallow strip foundations resting on either cohesive-frictional or purely cohesive soils, partly incorporating the simplified equations provided by Hansen [14]. Specifically, focusing on the case of purely frictional and purely cohesive soils, two limiting conditions can be identified for the inertia forces acting into the soil and the superstructure, *i.e.*: (i) soil fluidization (k_h = tanϕ or $k_h = k_{h,lim}$) and (ii) sliding along the foundation base (tanβ = tanϕ or $\tau = c_u$).

Using Eq. (3) together with the reduction coefficients given in Table 1, and focusing for simplicity on the case of purely shallow foundations (D = 0), it is possible to construct the three-dimensional failure envelopes of the foundation in the general space of the loading variables. This uncoupled procedure neglects any possible interaction between horizontal and eccentric loads. However, it provides a good approximation of more rigorous results for foundations on cohesionless soils, at least in the case of a positive load eccentricity-inclination combination [19], which is the most likely under seismic conditions. Moreover, it provides always slightly conservative results in the case of purely cohesive soils [13].

5.1. Cohesionless soils ($\gamma \neq 0$, $c = 0$, $\phi \neq 0$)

In static conditions, the ultimate bearing capacity of a strip foundation under a centered vertical load is $V_{max} = 0.5\gamma B^2 N_{\gamma S}$. After

Table 1
Reduction coefficients for the seismic bearing capacity of shallow strip foundations on a homogeneous layer of cohesive-frictional and purely cohesive soil.

	Cohesive-frictional soil ($\gamma \neq 0$, $\phi \neq 0$, $c \neq 0$)	Purely cohesive soil ($\gamma \neq 0$, $\phi = 0$, $c = c_u$)
seismic bearing capacity load	$q_{\lim} = \frac{1}{2}\gamma B' N_{\gamma E} + cN_{cE} + qN_{qE}$	
load eccentricity ($e = M/V$)	$B' = B - 2e$	
seismic bearing capacity factors ($k_h \neq 0$, $H \neq 0$)	$N_{qE} = e_q^k e_q^\beta N_{qS}$	$N_{qE} = e_q^k e_q^\beta N_{qS}$
	$N_{cE} = e_c^k e_c^\beta N_{cS}$	$N_{cE} = e_c^k e_c^\beta N_{cS}$
	$N_{\gamma E} = e_\gamma^k e_\gamma^\beta N_{\gamma S}$	$N_{\gamma E} = e_\gamma^k$
static conditions ($k_h = 0$, $H = 0$)	$N_{qS} = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)e^{\pi\tan\phi}$	$N_{qS} = 1$
	$N_{cS} = (N_{qS} - 1)\cot\phi$	$N_{cS} = 2 + \pi$
	$N_{\gamma S} = 1.5(N_{qS} - 1)\tan\phi$ (*)	$N_{\gamma S} = 0$
load inclination ($\tan\beta = H/V$, $\tau = H/B'$)	$e_q^\beta = (1 - 0.5\tan\beta)^5$ (*)	$e_q^\beta = 1$
	$e_c^\beta = e_q^\beta$	$e_c^\beta = 0.5 + 0.5\sqrt{1 - (\tau/c_u)}$ (*)
	$e_\gamma^\beta = \left(1 - \frac{\tan\beta}{\tan\phi}\right)^{(4.1\tan\phi^{1.4})}$	$e_\gamma^\beta = 0$
soil inertia ($k_h \neq 0$)	$e_q^k = \left(1 - \frac{k_h}{\tan\phi}\right)^{(0.37\tan\varphi^{0.5})}$	$e_q^k = 1 - a_q\left(\frac{k_h}{k_{h,lim}}\right) - b_q\left(\frac{k_h}{k_{h,lim}}\right)^2$
	$e_c^k = 1$	$e_c^k = 1$
	$e_\gamma^k = \left(1 - \frac{k_h}{\tan\phi}\right)^{0.47}$	$e_\gamma^k = -a_\gamma\left(\frac{k_h}{k_{h,lim}}\right) - b_\gamma\left(\frac{k_h}{k_{h,lim}}\right)^2$
		$a_q = 0.75k_{h,lim}$ $b_q = 1.4k_{h,lim}$
		$a_\gamma = 1.75k_{h,lim}$ $b_\gamma = 1.4k_{h,lim}$
		$k_{h,lim} = \frac{c_u}{\gamma(D + B/2)}$

(*) Hansen [14].

normalising the loading variables as $\overline{V} = V/V_{\max}$, $\overline{H} = H/V_{\max}$ and $\overline{M} = M/BV_{\max}$, and defining $\overline{E} = 1 - 2\overline{M}/\overline{V}$ and $\overline{K} = e_\gamma^k$ as the dimensionless variables taking into account the load eccentricity and the soil inertia, respectively, the equation of the failure locus is given by:

$$\Phi(\overline{V}, \overline{H}, \overline{E}, \overline{K}):= \overline{K}\overline{E}^2\left(1 - \frac{\overline{H}}{a\overline{V}}\right)^b - \overline{V} = 0 \tag{14}$$

with the constraint $0 < \overline{V} \le \overline{K}$, where a and b depend on soil friction angle ($a = \tan\phi$, $b = 4.1\tan\phi^{1.4}$).

Fig. 9 shows two sections of the failure envelope in: (a) the $\overline{M} - \overline{V}$ plane ($\overline{H} = 0$) and (b) the $\overline{H} - \overline{V}$ plane ($\overline{M} = 0$), computed under static conditions ($\overline{K} = 1$) and for $\phi = 30°$, together with other literature results [4,14,19,20,23]. It is apparent that Eq. (14) provides results in good agreement with other numerical and theoretical static solutions, with predictions being always slightly conservative.

To understand the relative merits of soil and superstructure inertia forces in the reduction of the bearing capacity, Fig. 10 shows the 3D failure envelopes computed for three different values of the pseudo-static soil acceleration ($k_h/\tan\phi = 0, 0.25, 0.5$), together with the iso-moment contours in the $\overline{H} - \overline{V}$ plane, for the case $\phi = 30°$. Clearly, while inertia forces in the superstructure can play a major role, by increasing both the inclination and the eccentricity of the load transferred to the foundation, soil accelerations also have a significant effect in reducing the size of the failure locus.

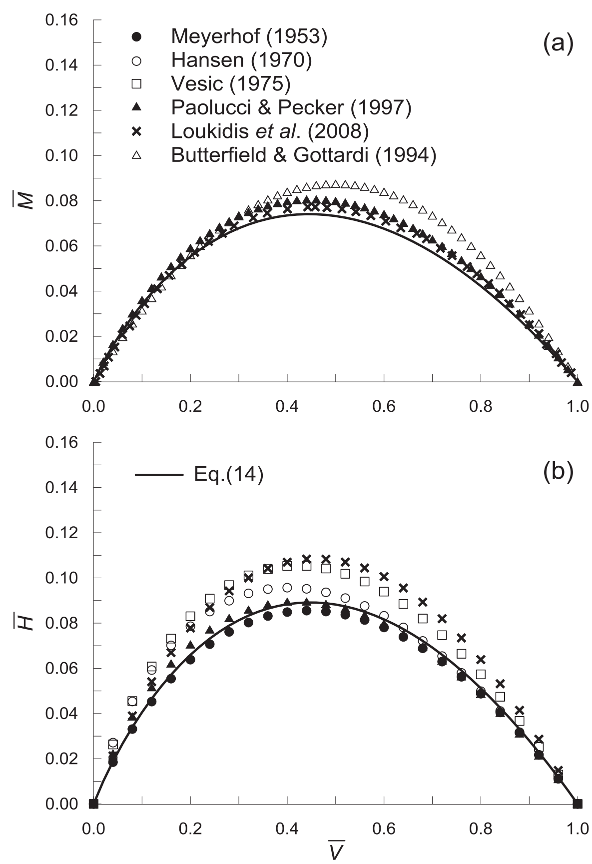

Fig. 9. M-C soil. Sections of the normalised failure locus in: (a) the $\overline{M} - \overline{V}$ plane ($\overline{H} = 0$) and (b) the $\overline{H} - \overline{V}$ plane ($\overline{M} = 0$), computed under static conditions ($\overline{K} = 1$) ($D = 0$, $c = 0$, $\phi = 30°$).

5.2. Purely cohesive soils ($\gamma \neq 0$, $c = c_u$, $\phi = 0$)

In this case the loading variables can be normalised as $\overline{V} = V/c_uB$, $\overline{H} = H/c_uB$ and $\overline{M} = M/c_uB^2$, while the other dimensionless loading variables are: $\overline{E} = 1 - 2\overline{M}/\overline{V}$ and $\overline{K} = \overline{\gamma}e_\gamma^k$, where $\overline{\gamma} = \gamma B/c_u$. The equation of the failure locus is given by:

$$\Phi(\overline{V}, \overline{H}, \overline{E}, \overline{K}):= \overline{E}N_{cS}\left(1 + \sqrt{1 - \frac{\overline{H}}{\overline{E}}}\right) + \overline{K}\overline{E}^2 - 2\overline{V} = 0 \tag{15}$$

with the constraints: $0 < \overline{V} \le (N_{cS} + 0.5\overline{K})$ and $\overline{H} \le \overline{E}$.

Fig. 11 shows two sections of the static failure envelope ($\overline{K} = 0$) in:

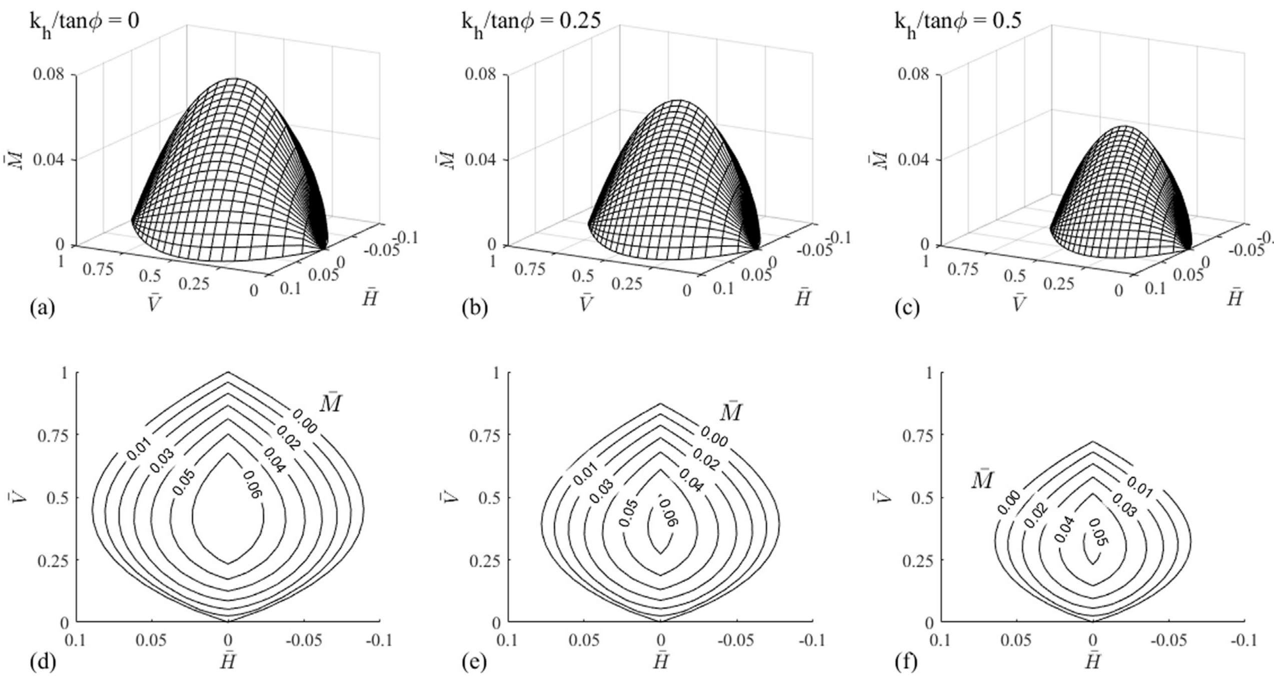

Fig. 10. M-C soil. Three-dimensional failure envelope ($\overline{H} - \overline{V} - \overline{M}$ space) and contours of moment capacity ($\overline{H} - \overline{V}$ plane) for: (a, d) $k_h/\tan\phi = 0$, (b, e) $k_h/\tan\phi = 0.25$ and (c, f) $k_h/\tan\phi = 0.5$ ($D = 0$, $c = 0$, $\phi = 30°$).

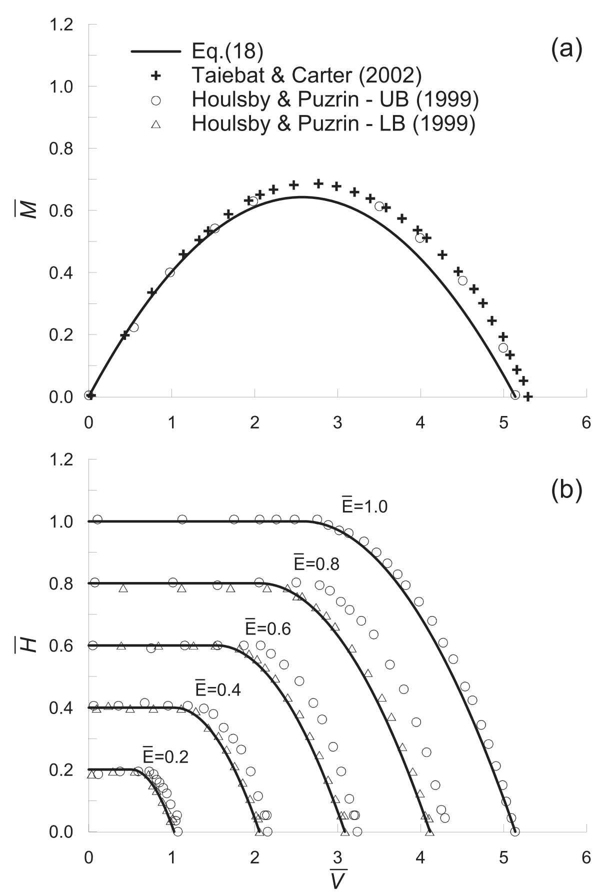

Fig. 11. Tresca soil. Sections of the normalised static ($\overline{K} = 0$) failure locus in: (a) the $\overline{M} - \overline{V}$ plane ($\overline{H} = 0$) and (b) the $\overline{H} - \overline{V}$ plane, the latter computed for different load eccentricities ($\overline{E} = 1.0,\ 0.8,\ 0.6,\ 0.4,\ 0.2$).

(a) the $\overline{M} - \overline{V}$ plane ($\overline{H} = 0$) and (b) the $\overline{H} - \overline{V}$ plane, the latter computed for different values of the load eccentricity ($\overline{E} = 1.0,\ 0.8,\ 0.6,\ 0.4,\ 0.2$). For comparison, the figure also shows the numerical results of Taiebat & Carter [36] and the limit upper and lower bound solutions given by Houlsby & Puzrin [15]. Again, the static limit locus given by Eq. (15) is in perfect agreement with the other results, with predictions always on the safe side.

Going to seismic conditions, the relative significance of soil and structure inertia is highlighted in Fig. 12, showing the 3D failure envelopes computed for three different values of the ratio $k_h/k_{h,\lim}$ (= 0, 0.25, 0.5), together with the iso-moment contours in the $\overline{H} - \overline{V}$ plane. The limit locus is substantially unaffected by the soil inertia for small values of the normalised vertical load ($\overline{V} \le 2$), *i.e.* for well-designed foundations (see *e.g.* [24]), while a sharp size reduction can occur for high values of $\overline{V}$. However, it is worth noting that the limiting value of the pseudostatic coefficients, $k_{h,\lim}$, is usually larger than one in standard applications, thus implying that a ratio of $k_h/k_{h,\lim} = 0.5$ would correspond to relatively large accelerations.

The effects of soil inertia is further highlighted in Fig. 13, referring to the case of a shallow foundation ($D = 0$) on a purely cohesive soil, subjected to an inclined centered load. The reduction of the horizontal limit load, $\overline{H}$, with respect to the corresponding static value, $\overline{H}_S$, is computed under given values of the vertical normalised load, $\overline{V}$, using both the upper bound theorem and the empirical equations in Table 1. The results of Paolucci & Pecker [24], obtained by applying the upper bound theorem to a different plastic mechanism, are also reported for comparison. A general good agreement is observed between the empirical formulas and the rigorous upper bound solution presented in this work, both providing conservative results with respect to those reported by Paolucci & Pecker [24]. Again, soil inertia has a negligible influence on the bearing capacity for $\overline{V} \le 2$, while inducing a sharp drop in the bearing resistance with increasing the vertical load.

6. Conclusions

Following a pseudostatic approach and based on the upper bound theorem of limit analysis, simple equations for the evaluation of the seismic bearing capacity of shallow strip foundations resting on both

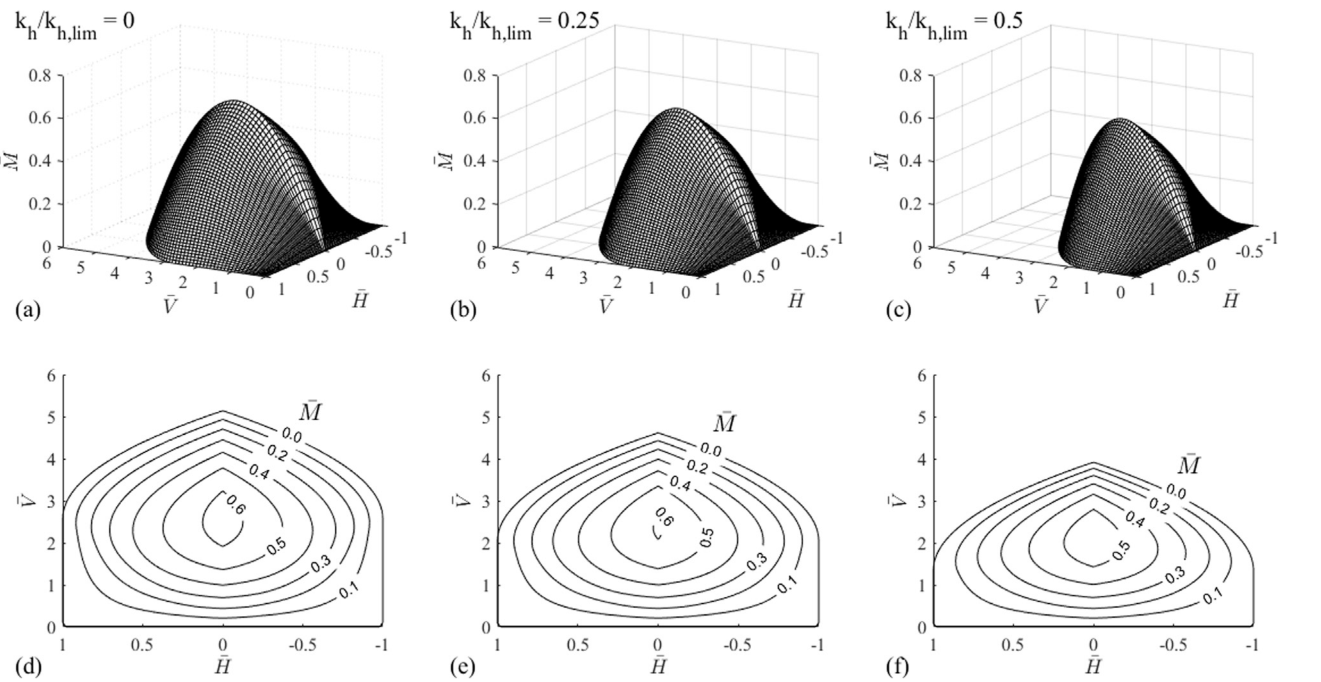

Fig. 12. Tresca soil. Three-dimensional failure envelope ($\overline{H} - \overline{V} - \overline{M}$ space) and contours of moment capacity ($\overline{H} - \overline{V}$ plane) for: (a, d) $k_h/k_{h,lim} = 0$, (b, e) $k_h/k_{h,lim} = 0.25$ and (c, f) $k_h/k_{h,lim} = 0.5$ ($D = 0$).

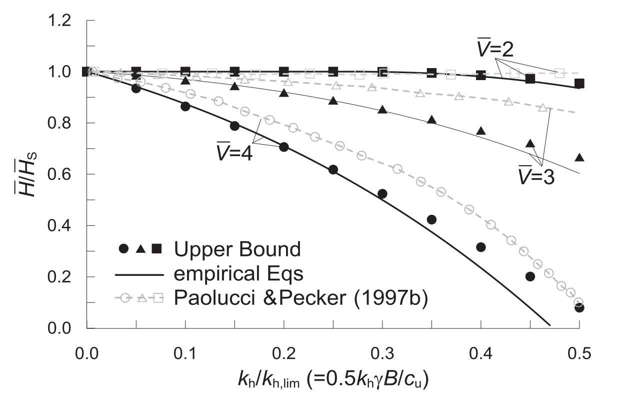

Fig. 13. Tresca soil. Reduction of the horizontal limit load as a function of $k_h/k_{h,lim}$, for different values of the normalised vertical load ($D=0$).

cohesive-frictional and purely cohesive soils were derived. An all-minimum procedure was used in the first case, always leading to an accurate and conservative estimate of the bearing capacity factors, while a rigorous application of the kinematic theorem was adopted for the case of purely cohesive soils.

Results were firstly given in terms of reduction coefficients for the Terzaghi's equation of the vertical bearing capacity, permitting to take into account easily the effects of inertia forces acting both on the superstructure (load inclination and eccentricity) and into the foundation soil.

Then, referring to the case of purely shallow foundations ($D = 0$), the classical equation for the vertical bearing capacity (Eq. (3)) and the reduction coefficients given in Table 1 were used to construct three-dimensional failure envelopes in the general space of the loading variables, extending to pseudostatic conditions other results presented in the literature for the static case. The availability of simple equations for both the Terzaghi's formula and the 3D failure domains provides definitely a better description of the bearing capacity of shallow foundations under the general loading induced by seismic actions, including soil inertia, horizontal shear and moment.

The reliability of the proposed equations, suitable for the design practice, has been verified through a thorough comparison with other rigorous and approximate solutions, referring to both static and pseudostatic conditions.

As far as cohesive-frictional soils are concerned, soil inertia leads to a significant reduction of the N_{qE} and $N_{\gamma E}$ terms, depending on the ratio $k_h/\tan\phi$, while the all-minimum procedure provides exactly $e_c^k = 1$, due to the assumption of weightless soil. The latter result is in substantial agreement with other works in the literature, taking into account soil inertia, showing indeed a negligible dependence of N_{cE} on k_h. On the other hand, the effects of load inclination due to structure inertia on the N_{qE} and N_{cE} terms can be conveniently described using the Hansen's equations (1970), widely used in the static design practice, while an *ad hoc* formula is introduced for the e_γ^β coefficient, depending on the soil friction angle. Finally, moving to the general space of the loading variables and referring for simplicity to the case of a shallow foundation on a cohesionless soil ($D = 0$, $c = 0$), it was shown that soil inertia can lead to a significant size reduction of the failure locus.

As far as purely cohesive soils are concerned, upper and lower bound solutions for the load inclination coefficient e_c^β are virtually the same and, again, in good agreement with the simplified equation proposed by Hansen [14]. Soil inertia effects are introduced by means of the reduction coefficients e_γ^k and e_q^k, both depending on the limiting acceleration $k_{h,lim}$, while N_{cE} shows a negligible dependence on soil acceleration. In agreement with the results by Paolucci & Pecker [24], it was found that the failure envelopes are substantially unaffected by soil inertia for $\overline{V}\leq 2$, while possibly exhibiting a significant reduction of the limit horizontal load otherwise, depending on the ratio $k_h/k_{h,lim}$.

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Appendix A. Cohesive-frictional soil

By referring to the case of a homogeneous soil layer obeying the Mohr-Coulomb yield criterion ($\gamma \neq 0$, $c \neq 0$, $\phi \neq 0$) and to the failure mechanism in Fig. 1, expressions are provided here for: (i) the power of the external forces; (ii) the power of the internal dissipation; (iii) the average limit load and (iv) the vector of constraints for the minimization problem.

Power of external forces

The power of external forces is given by:

$$P^{ext} = P_L + P_s + P_q \tag{A1}$$

where P_L is the power of the external load applied to the foundation, P_S is the power of body forces into the soil and P_q is the power of the lateral overburden. The three terms are given by:

$$\begin{aligned} P_L &= V\frac{\cos(\beta-\rho)}{\cos\beta}\cdot v_1 \\ P_s &= \frac{1}{2}\gamma B'^2\frac{\cos(\rho-\phi)}{\cos\phi}(b_1 + c_1 - d_1)\cdot v_1 \\ P_q &= -qB'd_3\cdot v_1 \end{aligned} \tag{A2}$$

where:

$$\begin{aligned} b_1 &= \sin\rho[(1-k_v)\cos\rho + k_h\sin\rho] \\ c_1 &= \frac{\cos(\rho-\phi)}{(1+9\tan^2\phi)\cos\phi} \\ &\quad\times\{(1-k_v)[e^{3\psi\tan\phi}(3\tan\phi\cos(\rho+\psi) + \sin(\rho+\psi)) - (3\tan\phi\cos\rho + \sin\rho)] \\ &\quad + k_h[e^{3\psi\tan\phi}(3\tan\phi\sin(\rho+\psi) - \cos(\rho+\psi)) - (3\tan\phi\sin\rho - \cos\rho)]\} \\ d_1 &= \frac{\cos(\rho-\phi)\sin(\rho+\psi)}{\cos(\rho+\psi-\phi)}e^{3\psi\tan\phi}[(1-k_v)\cos(\rho+\psi) + k_h\sin(\rho+\psi)] \end{aligned} \tag{A3}$$

and:

$$d_3 = \frac{\cos(\rho-\phi)}{\cos(\rho+\psi-\phi)}e^{2\psi\tan\phi}[(1-k_v)\cos(\rho+\psi) + k_h\sin(\rho+\psi)] \tag{A4}$$

Power of internal dissipation

The power of internal dissipation is given by:

$$P^{\text{int}} = cB'\cdot(b_2 + c_2 - d_2)\cdot v_1 \tag{A5}$$

where:

$$\begin{aligned} b_2 &= \sin\rho \\ c_2 &= \frac{\cos(\rho-\phi)}{\sin\phi}(e^{2\psi\tan\phi} - 1) \\ d_2 &= \frac{\cos(\rho-\phi)\sin(\rho+\psi)}{\cos(\rho+\psi-\phi)}e^{2\psi\tan\phi} \end{aligned} \tag{A6}$$

Average limit load

By equating $P^{\text{ext}} = P^{\text{int}}$ and defining $q^*_{\text{lim}} = V/B'$, we obtain Eq. (1), where:

$$\begin{aligned} N^*_{\gamma E}(\rho,\psi) &= a_1(d_1 - b_1 - c_1) \\ N^*_{cE}(\rho,\psi) &= a_2(b_2 + c_2 - d_2) \\ N^*_{qE}(\rho,\psi) &= a_3 d_3 \end{aligned} \tag{A7}$$

and

$$\begin{aligned} a_1 &= \frac{\cos\beta\cos(\rho-\phi)}{\cos\phi\cos(\beta-\rho)} \\ a_2 = a_3 &= \frac{\cos\beta}{\cos(\beta-\rho)} \end{aligned} \tag{A8}$$

Constraints for minimization

The two angles $\rho > 0$ and $\psi > 0$ must satisfy some constraints, emerging from the requirement that all the angles are positive and the external load must do positive work for the assumed velocity field. The vector of constraints is given by:

$$\mathbf{H}(\rho,\psi)=\begin{cases}\rho+\psi-\pi\\ \pi/2+\phi-\rho-\psi\\ \rho-\phi-\pi/2\\ \rho-\beta-\pi/2\end{cases} \tag{A9}$$

Appensix B. Pure cohesive soil

In this section, the case of a homogeneous soil layer obeying the Tresca yield criterion is addressed ($\gamma \ddagger 0$, $c = c_u$, $\phi = 0$). The failure mechanism and the velocity field are the same as those given in Fig. 1 for a M-C soil, provided a zero friction angle is assumed. Terms related to the external load are given in a different form with respect to the M-C case, more suitable for the analysis of a pure cohesive material.

Incremental external work

The power of external forces is given by Eq. (A1), where:

$$\begin{aligned}&P_L=(V\cos\rho+H\sin\rho)\bullet v_1\\ &P_s=\tfrac{1}{2}\gamma B'^2\bullet k_h\cos\rho\tfrac{\cos(\rho+\psi)-\cos\rho}{\cos(\rho+\psi)}\bullet v_1\\ &P_q=-qB'\bullet\tfrac{\cos\rho}{\cos(\rho+\psi)}[(1-k_v)\cos(\rho+\psi)+k_h\sin(\rho+\psi)]\bullet v_1\end{aligned} \tag{B1}$$

Incremental internal energy dissipation

The power of internal dissipation is given by:

$$P^{\text{int}}=c_uB'\cdot\left[\sin\rho-\frac{\cos\rho\cdot\sin(\rho+\psi)}{\cos(\rho+\psi)}+2\psi\cos\rho\right]\cdot v_1 \tag{B2}$$

Average limit load

By equating $P^{\text{ext}} = P^{\text{int}}$ and defining $q^*_{\text{lim}} = V/B'$ and $\tau = H/B'$, Eq. (1) is obtained, where:

$$\begin{aligned}&N^*_{\gamma E}(\rho,\psi)=k_h\tfrac{\cos\rho-\cos(\rho+\psi)}{\cos(\rho+\psi)}\\ &N^*_{qE}(\rho,\psi)=(1-k_v)+k_h\tfrac{\sin(\rho+\psi)}{\cos(\rho+\psi)}\\ &N^*_{cE}(\rho,\psi)=2\psi+\tfrac{\sin\rho}{\cos\rho}\left(1-\tfrac{\tau}{c_u}\right)-\tfrac{\sin(\rho+\psi)}{\cos(\rho+\psi)}\end{aligned} \tag{B3}$$

Constraints for minimization

The two angles $\rho > 0$ and $\psi > 0$ must satisfy the requirement that all the angles are positive. The resulting vector of constraints is given by:

$$\mathbf{H}(\rho,\psi)=\begin{cases}\rho+\psi-\pi\\ \pi/2-\rho-\psi\\ \rho-\pi/2\end{cases} \tag{B4}$$

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