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A modified firefly algorithm for global minimum optimization

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ABSTRACT

The Firefly algorithm is a population-based optimization algorithm. It has become popular in the field of optimization and has been applied to engineering practices. Recent works have failed to address how to find the global minimum because their algorithm was trapped in the local minimum. Also, they were not able to provide a balance between exploration and exploitation. In this paper, the Tidal Force formula has been applied to modify the Firefly algorithm, which describes the effect of a massive body that gravitationally affects another massive body. The proposed algorithm brings a new strategy into the optimization field. It is applied by using exploitation (Tidal Force) and keeping a balance between the exploration and exploitation on function suitability. Plate shaped, Steep Ridges, Unimodal and Multimodal benchmark functions were used to compare experimental results. The study findings indicate that the Tidal Force Firefly algorithm outperforms the other existing modified Firefly algorithms.

1. Introduction

Population-based optimization techniques have become widespread in the last two decades. Optimization problems are examined in a variety of fields, which consist of highly nonlinear, multimodal, multidimensional, and differentiable functions. However, traditional optimization techniques have not been able to solve these problems. The population-based techniques, with their own robustness and flexible behavior in solving optimization problems, bring novel insights in order to solve the problems instead of using traditional optimization techniques. The PSO algorithm is inspired by birds' behavior [1], which defines that all birds move towards the best bird. The PSO works with two populations, such as best position and current positions. Diversity solutions are one of the advantages of the PSO, which performs better than the single point algorithms. An ant colony is another popular algorithm that was designed for optimization problems. The algorithm is based on the behavior of ants and was proposed by Dorigo. Ants search for a best path solution between their colony and a food source. Each ant randomly moves towards a destination ant. The paths are followed by ants, based on the probability of pheromones [2]. Differential evolution based on individual's differences (called the DE algorithm) is similar to the GA algorithm that defines specified crossover, mutation and selection [3]. It computes parallels and takes the best result in a few dimensions. The Harmony Search (HS) algorithm was modeled by taking inspiration from the improvisation of musicians [4]. The algorithm uses musical Pitch Range, Harmony, Aesthetics, Practice and experience in the algorithm, which links to decision variables, iteration concepts and so on. The harmony (solution) is produced randomly and checks with stored solutions to place better solutions in their place. Biogeography-Based Optimization (BBO) was proposed using the concept of species immigration and emigration. Each species is moved to another place that is based on its own decision to take a feature of each other. One of the main advantages of this method is a critical distinction in the strategy that aids optimal performance in optimizing problems [5]. Gravitational Search Algorithm (GSA), based on Newtonian gravity and laws of motion, was also proposed. Bodies or mass may connect and move to each other based on the mentioned rule [6]. In order to improve the GSA, a new operator was proposed by [7] using black holes. The new operator is employed to avoid premature convergence phenomena and also balances exploration and exploitation. In their work, a star is stated as a black hole. One of the characteristics of black holes is Tidal Force, which uses a strong force (Tidal Force) to slice an object into smaller parts [7].

Swarm Intelligence is a family of population-based optimization techniques. Swarm Intelligence (SI), which is a branch of artificial intelligence (AI), is referred to as the design of intelligent multi agent systems. Multi-agent systems are inspired by the behavior of animal societies such as Ants, Termites, Bees, and Wasps, flocks of birds, schools of fishes [8], e.g., ABC [9], Ant and PSO. Some algorithms are categorized with a different class such as Swarm intelligence, Bio inspired, Physics chemistry-based algorithm and

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Firefly algorithm

Objective function f(x), x=(x_1,...,x_d)^T
Generate initial population of fireflies x_i (i = 1,2,...,n)
Light intensity I_i at x_i is determined by f(x_i)
Define light absorption coefficient γ
While (t<MaxGeneration)
  For i=1:n all n fireflies
    For j=1:n all n fireflies(inner loop)
      If (I_i < I_j), Move firefly i towards j; end if
      Vary attractiveness with distance r via exp(-γr)
      Evaluate new solutions and update light intensity
    end for j
  end for i
Rank the firefly and find the current global best g_*
End While
Postprocess results and visualization
```

Fig. 1. Pseudo code of firefly1 [18].

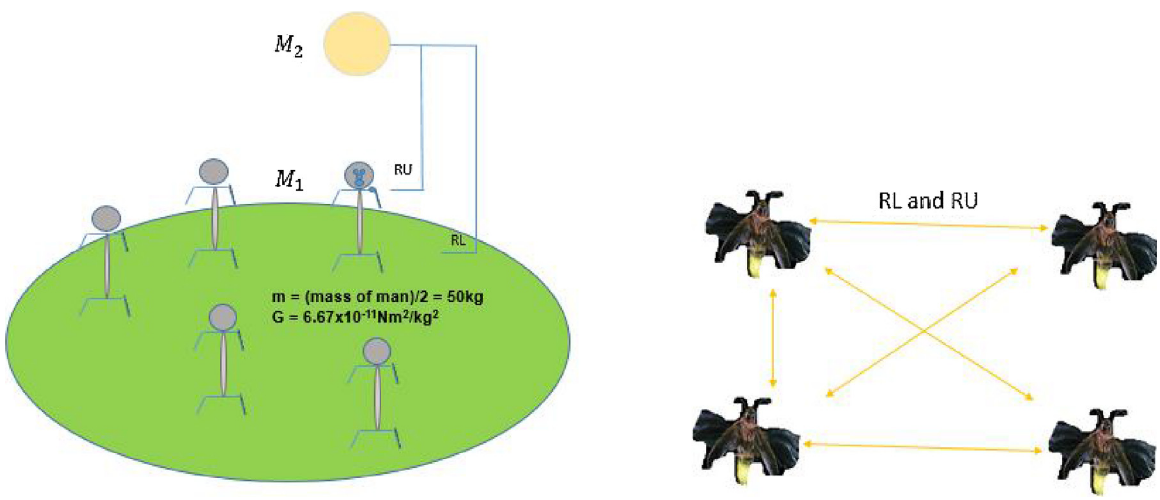

Fig. 2. Tidal Force.

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Firefly Tidal algorithm

Objective function f(x), x=(x_1,...,x_d)^T
Generate initial population of fireflies x_i (i = 1,2,...,n)
Calculate opposite of population in fireflies x̃_i (see 15,16)
Evaluate the fitness of both population x̃_i and x
Select the one of x̃_i and x based on fitness function
Define G,m_1,m_2,alpha, MaxGeneration, nVar, β
Tidal Force I_i at x_i is
Define light absorption coefficient γ
  While (t<MaxGeneration)
    For i=1:n all n fireflies
      For j=1:n all n fireflies(inner loop)
        If (I_i ≠ I_j)
        If (I_i < I_j), Move firefly i towards j; end if
        For k=1:nVar all n fireflies(inner loop) (variables mapped to vector in more dimensions)
        Vary attractiveness with distance r (see the formula 7,8,9,10,11,12)
        Evaluate new solutions and update Tidal Force (see the formula 13,14 or 15)
        End for k
        end if
      end for j
    end for i
Rank the firefly and find the current global best g_*
End While
Postprocess results and visualization
```

Fig. 3. Pseudo code of Tidal Force.

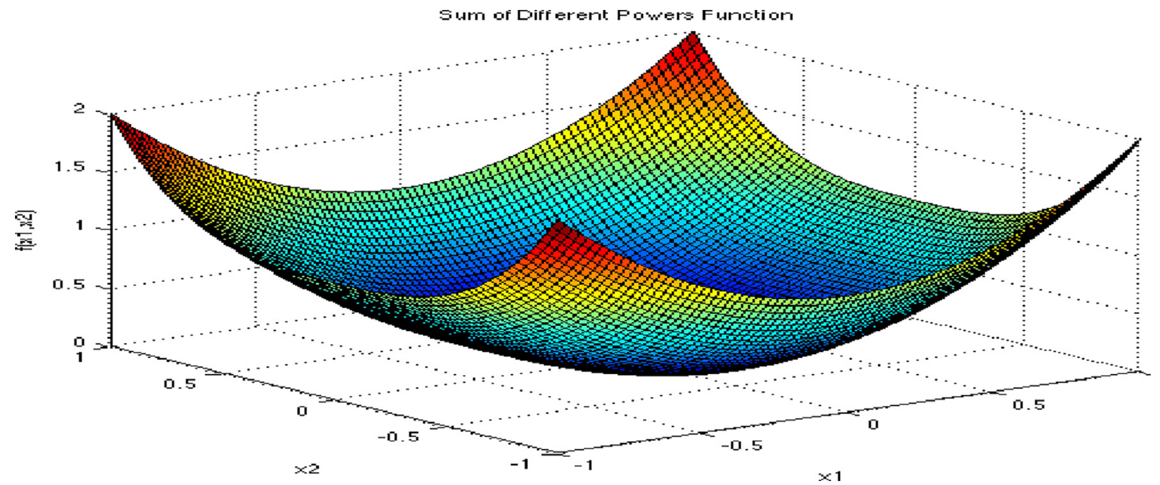

Fig. 4. Sum of Different Power functions [34].

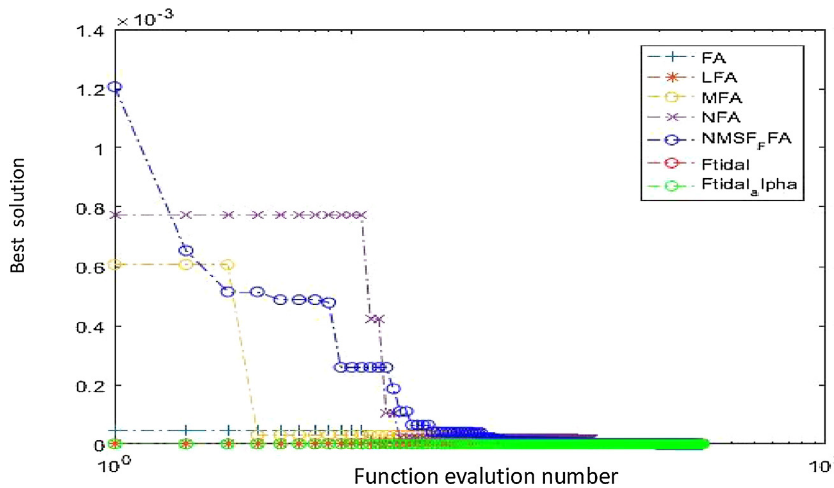

Fig. 5. Convergence curve of algorithms for F3 Sum of Different Powers function in 2 dimensions.

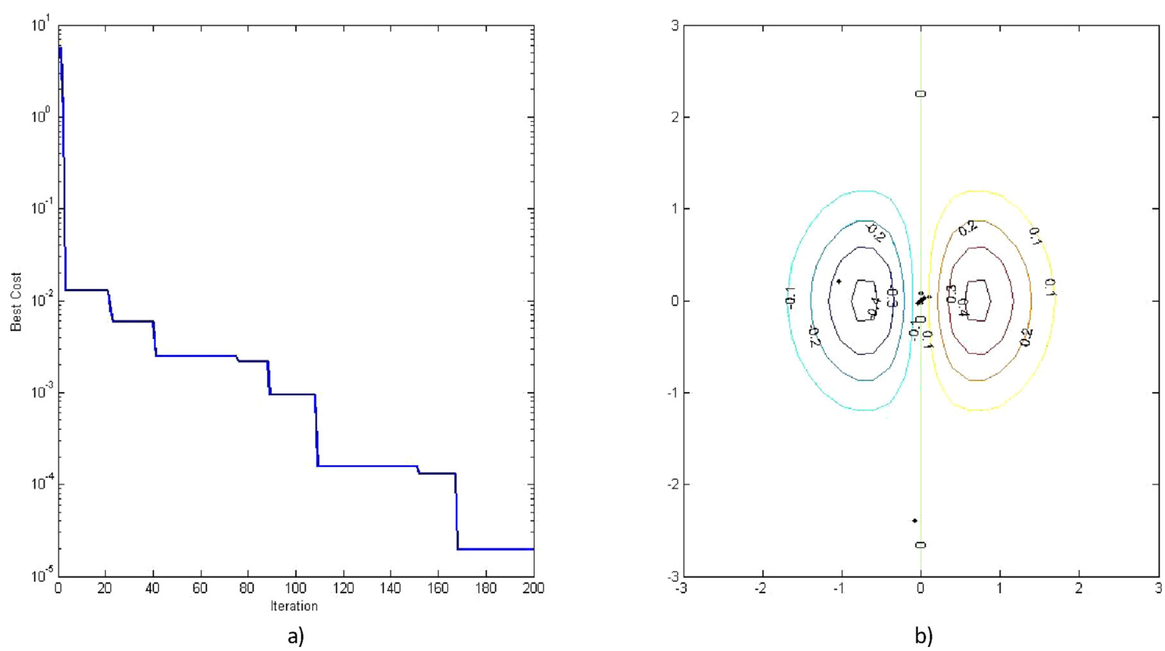

Fig. 6. Sample of iteration on sphere function by running FAtidal a) divergence of movement b) contour space (0,0) is global minimum.

Table 1
Benchmark Functions.

Function/Characteristic	Equation	Range x_i	Global Optimum
Many Local Minima/Multimodal			
Schaffer	$f_1(x) = \frac{sin^2\sqrt{\sum_{i=1}^{D} x_i^2 - 0.5}}{\left[1+0.001\left(\sum_{i=1}^{D} x^2\right)\right]^2} + 0.5$	[−100, 100]	**0**
Ackley/ nearly flat outer region and a large hole at the center	$f_2(x) = 20 - 20exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + e$	[−32.768, 32.768]	**0**
Schawefel	$f_3(x) = 418.9829d - \sum_{i=1}^{d} x_i sin\left(\sqrt{\lvert x_i\rvert}\right)$	[−500,500]	**0**
Langermann/ unevenly distributed local minima(A and C for 2 dimensions was available in this study)	$f_4(x) = -\sum_{i=1}^{m} c_i exp\left(-\frac{1}{\pi}\sum_{j=1}^{d}\left(x_j - A_{ij}\right)^2\right) cos\left(\pi\sum_{j=1}^{d}\left(x_j - A_{ij}\right)^2\right)$	[0,10]	−**5.162125**
Levy	$f_5(x) = sin^2(\pi w_1) - \sum_{i=1}^{d-1}(w_i - 1)^2\left[1 + 10sin^2(\pi w_1 + 1)\right] + (w_d - 1)^2\left[1 + sin^2(2\pi w_d)\right], where w_i = 1 + \frac{x_i - 1}{4}, for all i = 1, \ldots, d$	[−10,10]	0
Rastrigin/ locations of the minima are regularly distributed	$f_6(x) = \sum_{i=1}^{D}\left[x_i^2 - 10\cos(2\pi x_i)\right]$	[−5.12,5.12]	0
Bowl Shaped			
Sphere/has d local minima except for the global one, continuous, convex	$f_7(x) = \sum_{i=1}^{D} x_i^2$	[−5.12,5.12]	0
Perm Function 0, d, Beta	$f_8(x) = \sum_{i=1}^{D}\left(\sum_{j=1}^{d}\left(j+\beta\right)\left(x_j^i - \frac{1}{j^i}\right)\right)^2$	[-d, d]	0
Rotated Hyper-Ellipsoid/continuous, convex	$f_9(x) = \sum_{i=1}^{d}\sum_{j=1}^{i} x_j^2$	[−65.536, 65.536]	0
Sum of Different Powers	$f_{10}(x) = \sum_{i=1}^{d}\lvert x_i\rvert^{i+1}$	[−1,1]	0
Sum Squares/ continuous, convex	$f_{11}(x) = \sum_{i=1}^{d} ix_i^2$	[−10,10]	0
Plate Shaped			
Zakharov/ has no local minima except the global one	$f_{12}(x) = \sum_{i=1}^{d} x_i^2 + \left(\sum_{i=1}^{d} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{d} 0.5ix_i\right)^4$	[−5,10]	0
Dixon-Price	$f_{13}(x) = (x_1 - 1)^2 + \sum_{i=2}^{d} i\left(2x_i^2 - x_{i-1}\right)^2$	[−10,10]	0
Rosenbrock/ global minimum lies in a narrow, parabolic Valley	$f_{14}(x) = \sum_{i=1}^{D-1}\left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	[−5,10]	0
Steep Ridges			
Michalewicz/ has d! local minima	$f_{15}(x) = -\sum_{i=1}^{d} sin(x_i) sin^{2m}\left(\frac{ix_i^2}{\pi}\right)$	[0, π]	-1.8013(d=2) -9.6601(d=10) -99.6201 (d=100)

others [10]. In recent years, there has been considerable attention paid to the Firefly algorithm. The Firefly algorithm was formulated by Yang and was inspired by a firefly's behavior. A firefly's flash is a signal system used to attract other fireflies. The most distinctive features of the Firefly algorithm include discovering the mechanism, the social behavior of fireflies and communication.

A comprehensive review of the Firefly algorithm, proposed by [11], shows that the FA needs to be modified and hybridized to different situations. In the literature, FA usually refers to the Firefly algorithm. The review is divided into three questions, which are as follows: what part of the Firefly strategy is modified, how is it modified, and what is the scope of the modification [11]. There are more than twenty Firefly algorithm varieties, including Multi objective FA, Lagrangian FA, Chaotic FA and so on [11]. The FA is also extended to multimodal [12] and hybrid [13], and it is applied to Clustering fields [14–16], etc. A Firefly algorithm based on History_driven [17] was proposed for dynamic optimization, which was used in an uncertain environment to track the changing optimum. The nature of the Firefly algorithm is that the communication with each other is pairwise. The attractiveness of the Firefly algorithm is associated

Table 2
Benchmark Functions from CEC.

Function/Characteristic	Equation	Range x_i	Global Optimum
Unimodal			
Shifted Sphere Function/ separable, scalable	$f_{16}(x) = \sum_{i=1}^{d} z_i^2 + f_{bias_1}, z = x - o, x = [x_1, x_2, \ldots, x_D]$	[−100, 100]	**−450**
Shifted Schwefel's Problem 1.2/non-separable, scalable	$f_{17}(x) = \sum_{i}^{d} \left(\sum_{j=1}^{i} z_j \right) + f_{bias_1}, z = x - o, x = [x_1, x_2, \ldots, x_D]$	[−100, 100]	**−450**
Multimodal			
Shifted Rosenbrock/ non-separable, scalable	$f_{18}(x) = \sum_{i=1}^{d-1} \left(100 \left(z_i^2 - z_{i+1} \right)^2 + (z_i - 1)^2 \right) + f_{bias_1}, z = (x - o) + 1, x = [x_1, x_2, \ldots, x_D]$	[−100, 100]	**390**
Shifted Rotated Griewank/ non-separable, scalable and no bound for variable x	$f_{19}(x) = \sum_{i=1}^{d} \frac{z_i^2}{4000} - \prod_{i=1}^{d} \cos\left(\frac{z_i}{\sqrt{i}} \right) + f_{bias_1}, z = (x - o) * M, x = [x_1, x_2, \ldots, x_D]$	**[0,600]**	**−180**
Shifted Rotated Ackley non-separable, scalable and global optimum on the optimum	$f_{20}(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{d} \sum_{i=1}^{d} z_i^2} \right) - \exp\left(\frac{1}{d} \sum_{i=1}^{d} \cos(2\pi z_i) + 20 + e \right) + f_{bias_1}, z = (x - o) * M, x = [x_1, x_2, \ldots, x_D]$	[−32,32]	−140
Expanded Functions			
Expanded Rotated Extended Schaffer/ non-separable, scalable	$f_{21}(x, y) = 0.5 \frac{\left(sin^2 \left(\sqrt{x^2+y^2} \right) - 0.5 \right)}{\left(1+0.001(x^2+y^2)\right)^2}$ $f_{22}(x) = EF(z_1, z_2, \ldots, z_d) = F(z_1, z_2) + F(z_2, z_3) + \ldots + F(z_{d-1}, z_d) + F(z_d, z_1) + f_{bias_1}, z = (x - o) * M, x = [x_1, x_2, \ldots, x_D]$	[−100, 100]	−300

with its considered function and decrease of its own value during each iteration. The Tidal Force formula was used instead of the fireflies' attractiveness, including the distance and mass of a body. The absorption coefficient of the Firefly algorithm does not work in the Tidal force formula because of the nature of its formula. The simple formula that was taken from Tidal Force ignores the additional phenomena that are so adaptive to the Firefly strategy. It causes the individual to behave sensitively towards their behavior movement. Then, it aids the achievement of the global minimum, avoids premature convergence, and coordinates with the exploration and exploitation.

The structure of this paper is as follows. In Section 2, we outlined the modified Firefly algorithm. Section 3 describes the Tidal Force Firefly Algorithm. The proposed algorithm is applied to 21 benchmark functions with different classifications. The experimental results and evaluation are shown in Section 4. Section 5 states some concluding remarks and suggests further work.

2. Related work

2.1. Modified firefly and firefly algorithm

Another FA developed to solve nonlinear design problems demonstrated that the modified FA is better than other algorithms at solving NP-hard optimization. The Levy-flight FA combined with Firefly outperformed a well-known algorithm called the PSO, which has some drawbacks in the FA. The drawbacks are the premature convergence and movement of fireflies toward the best solution in long distances [18–20]. Another modified Firefly, called LFA, changed the light intensity as was proposed by Wang and others. The variation of the light intensity can be used in this situation since it is self-adaptive and adjusts to any problem during its application [21]. In contrast to the above-mentioned work, it is dependent on the parameters selected. The other FA modified work was proposed by Verma and others using the Opposition-based methodology and dimensional-based approach. In their work, there was an attempt to find the global best value and spend less time in solving a highly dimensional problem [22]. NaFA is also one of the FA variants that is applied to overcome time complexity and improve the results. The strategy is proposed by randomly selecting the population and is used for better exploration with the Cauchy distribution. It uses neighbors of each population in the framework [23]. In MFA [24], an intermediate population is extracted from the original population and then sorted ascendingly for providing a suitable search. To maintain an elite population, the best individuals are selected in each iteration. The MSA_FFA [25] proposes the new selection population. It is able to search the promising region and three parameters such as alpha, attractiveness and absorption coefficient defined as self-adaptive. This restrains bad sets in initialization and defines specific fitness diversity to balance exploration and exploitation. It solved the discrete problem.

Another modified Firefly algorithm that is extended from MSA_FFA is called NMSA_FFA [26]. The algorithm includes self-adaptation of control parameters, a population model, and a local search procedure that attempts to balance the exploration and exploitation with an elite population and self-adaptive parameter alpha. The FA and modified FA have the disadvantage of premature convergence when the fireflies or individuals are faced with a local minimum. When an individual (or firefly) moves towards the best individual, they have a behavioral fluctuation during movement. Therefore, setting the absorption coefficient is important. Another disadvantage of it is finding the optimal solution in multimodal and unimodal problems. Our contributions are as follows:

1. The absorption coefficient is no longer used in the formula.
2. Individuals behave sensitively to their behavior movement.
3. Some previous works have been conducted using specific alpha operator for balancing exploration and exploitation in more

Table 3
Comparing Algorithms in 2 dimensions refers to Table 1.

Table 3 Comparing Algorithms in 2 dimensions refers to Table 1

Algorithms	Many Local Minima						bowl_shaped					Plate_shaped			Steep Ridges
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
BBO(m)	0.00E+00	2.22E-14	2.55E-05	-5.16E+00	6.43E-32	0.00E+00	1.22E-31	9.81E-16	2.75E-31	3.35E-34	8.28E-30	6.82E-27	1.04E-16	9.53E-10	-1.80E+00
BBO(M)	0.00E+00	4.00E-10	2.37E+02	-1.60E+00	1.16E+00	3.98E+00	7.12E-22	2.50E-09	1.05E-20	3.63E-18	3.64E-21	1.22E-20	1.23E-13	2.06E+00	-1.21E+00
BBO(A)	0.00E+00	1.78E-11	4.74E+01	-4.19E+00	3.87E-02	1.23E+00	2.58E-23	2.56E-10	4.12E-22	1.22E-19	2.40E-22	1.13E-21	1.93E-14	8.85E-02	-1.78E+00
BBO(V)	0.00E+00	5.27E-21	4.45E+03	1.38E+00	4.49E-02	7.29E-01	1.68E-44	2.89E-19	3.66E-42	4.40E-37	6.82E-43	7.77E-42	7.92E-28	1.48E-01	1.15E-02
BEE(m)	4.35E-14	7.73E-07	2.55E-05	-5.16E+00	8.07E-14	1.02E-12	1.50E-14	3.85E-08	1.01E-13	1.35E-18	9.98E-15	1.49E-13	1.34E-12	1.99E-05	-1.80E+00
BEE(M)	1.47E-03	3.53E-05	2.55E-05	-5.16E+00	2.54E-12	1.63E-09	3.11E-12	2.73E-07	4.78E-10	2.00E-14	9.05E-12	1.59E-11	1.65E-09	3.46E-02	-1.80E+00
BEE(A)	2.87E-04	1.16E-05	2.55E-05	-5.16E+00	7.34E-13	3.03E-10	5.71E-13	1.39E-07	8.90E-11	2.21E-15	2.37E-12	1.77E-12	3.65E-10	6.97E-03	-1.80E+00
BEE(V)	2.02E-07	6.60E-11	9.64E-19	8.32E-23	5.10E-25	1.98E-19	5.75E-25	4.50E-15	8.87E-21	1.73E-29	5.51E-24	8.75E-24	1.65E-19	8.88E-05	2.75E-24
CU(m)	0.00E+00	8.88E-16	2.55E-05	-5.16E+00	1.50E-32	0.00E+00	1.34E-119	5.16E-17	5.08E-123	1.21E-52	3.57E-222	1.15E-121	3.70E-32	3.85E-05	-1.80E+00
CU(M)	2.14E-02	5.94E+00	3.36E+02	-2.21E+00	2.37E-06	3.98E+00	4.81E-06	3.80E-01	4.58E-07	9.45E-12	1.05E-06	2.45E-05	1.27E-02	1.18E+01	-1.00E+00
CU(A)	1.20E-03	2.42E-01	1.14E+02	-4.30E+00	8.29E-08	7.96E-01	1.61E-07	4.33E-02	1.54E-08	4.03E-13	3.93E-08	8.25E-07	7.21E-04	1.22E+00	-1.77E+00
CU(V)	1.91E-05	1.22E+00	9.99E+03	1.40E+00	1.87E-13	7.10E-01	7.72E-13	8.65E-03	7.00E-15	3.02E-24	3.67E-14	2.00E-11	7.49E-06	9.46E+00	2.14E-02
GA(m)	0.00E+00	5.37E-11	2.55E-05	-5.16E+00	4.26E-28	0.00E+00	1.30E-24	2.35E-14	1.64E-21	7.64E-29	1.21E-27	1.60E-19	1.38E-23	1.54E-07	-1.80E+00
GA(M)	2.64E-03	4.71E-03	1.18E+02	-2.95E+00	1.95E-07	1.23E-05	1.45E-07	1.32E-02	1.10E-05	7.92E-09	9.10E-08	9.17E-07	1.13E-04	6.91E-01	-1.80E+00
GA(A)	3.86E-04	4.17E-04	3.55E+01	-4.29E+00	1.07E-08	1.13E-06	7.00E-09	2.81E-03	8.15E-07	3.57E-10	1.02E-08	6.18E-08	1.67E-05	1.32E-01	-1.80E+00
GA(V)	5.26E-07	1.05E-06	3.05E+03	1.05E+00	1.33E-15	7.38E-12	7.17E-16	1.55E-05	7.41E-12	2.22E-18	5.49E-16	2.74E-14	5.72E-10	2.51E-02	9.44E-16
HS(m)	0.00E+00	9.33E-14	2.55E-05	-5.16E+00	3.39E-29	0.00E+00	6.23E-29	3.70E-22	2.49E-26	3.03E-33	6.77E-29	3.82E-28	1.30E-26	5.31E-06	-1.80E+00
HS(M)	5.33E-03	5.07E-12	2.55E-05	-5.16E+00	1.18E-25	0.00E+00	3.00E-26	1.85E-03	7.27E-24	2.97E-28	1.31E-25	1.32E-25	1.35E-23	1.36E-01	-1.80E+00
HS(A)	7.57E-04	1.09E-12	2.55E-05	-5.16E+00	1.10E-26	0.00E+00	5.08E-27	8.43E-05	1.02E-24	4.62E-29	2.35E-26	2.55E-26	4.23E-24	1.17E-02	-1.80E+00
HS(V)	1.86E-06	9.09E-25	0.00E+00	3.26E-30	6.46E-52	0.00E+00	4.29E-53	1.24E-07	1.91E-48	4.50E-57	7.33E-52	1.07E-51	1.51E-47	6.27E-04	4.59E-31
ICA(m)	0.00E+00	8.88E-16	2.55E-05	-5.16E+00	1.50E-32	0.00E+00	2.55E-60	7.89E-31	1.52E-54	1.34E-68	3.64E-55	4.09E-51	3.70E-32	6.15E-15	-1.80E+00
ICA(M)	0.00E+00	4.44E-15	1.18E+02	-3.00E+00	1.50E-32	0.00E+00	5.22E-45	3.48E-23	4.87E-45	3.20E-52	1.15E-41	7.89E-40	5.03E-30	2.11E-07	-1.80E+00
ICA(A)	0.00E+00	1.24E-15	4.74E+01	-4.15E+00	1.50E-32	0.00E+00	4.57E-46	2.93E-24	1.83E-46	1.07E-53	3.85E-43	2.97E-41	3.98E-31	1.55E-08	-1.80E+00
ICA(V)	0.00E+00	1.18E-30	3.48E+03	1.20E+00	7.75E-96	0.00E+00	1.71E-90	6.61E-47	7.90E-91	3.42E-105	4.42E-84	2.08E-80	9.23E-61	2.41E-15	4.59E-31
FA(m)	6.54E-03	8.88E-16	1.72E-01	-5.16E+00	1.50E-32	0.00E+00	7.18E-130	0.00E+00	7.91E-126	2.08E-22	4.53E-129	3.35E-129	3.70E-32	0.00E+00	-1.80E+00
FA(M)	2.39E-01	4.14E+00	1.21E+02	-3.20E+00	1.50E-32	1.99E+00	1.65E-127	7.89E-31	9.55E+00	4.23E-14	2.17E-127	2.05E-127	4.93E-32	7.01E-02	-1.80E+00
FA(A)	7.37E-02	1.40E+00	1.79E+01	-5.05E+00	1.50E-32	5.15E-01	2.61E-128	1.31E-31	2.13E+00	1.53E-15	9.90E-128	7.07E-128	3.86E-32	4.87E-03	-1.80E+00
FA(V)	2.88E-03	2.25E+00	7.01E+02	1.48E-01	7.75E-96	4.15E-01	1.17E-255	8.94E-62	6.22E+00	5.94E-29	3.50E-255	2.88E-255	1.82E-65	2.46E-04	4.59E-31
LFA(m)	8.71E-06	6.36E-09	8.34E+01	-5.16E+00	4.97E-09	3.31E-09	8.92E-19	3.72E-06	8.74E-21	4.27E-11	1.05E-14	1.90E-12	8.87E-31	5.31E-04	-1.80E+00
LFA(M)	2.96E-01	9.02E+00	4.83E+02	-1.08E+00	9.28E-01	3.98E+00	3.41E-01	4.30E+01	1.97E+02	1.27E-02	2.80E+00	4.01E+00	3.24E+00	8.88E+00	-1.00E+00
LFA(A)	6.43E-02	1.39E+00	2.71E+02	-2.71E+00	9.41E-02	1.29E+00	4.30E-02	2.58E+00	1.21E+01	6.42E-04	1.55E-01	4.95E-01	1.85E-01	1.34E+00	-1.61E+00
LFA(V)	6.94E-03	4.35E+00	1.32E+04	1.42E+00	3.28E-02	1.17E+00	5.86E-03	7.56E+01	1.37E+03	5.86E-06	2.71E-01	1.09E+00	3.59E-01	4.13E+00	6.89E-02
MFA(m)	1.45E-12	2.00E-05	2.55E-05	-5.16E+00	3.79E-13	1.64E-10	4.58E-13	4.32E-11	8.76E-10	7.20E-17	2.02E-11	7.48E-13	3.05E-11	6.30E-03	-1.78E+00
MFA(M)	4.03E-11	2.83E-04	1.18E+02	-3.00E+00	2.08E-10	7.56E-08	2.08E-10	6.44E-09	5.14E-08	4.05E-14	9.71E-10	1.45E-09	2.06E-09	1.84E+01	-9.96E-01
MFA(A)	1.52E-11	1.31E-04	2.37E+01	-5.09E+00	6.38E-11	1.67E-08	5.82E-11	1.00E-09	1.64E-08	8.14E-15	3.04E-10	2.60E-10	8.38E-10	2.83E+00	-1.52E+00
MFA(V)	1.10E-22	4.52E-09	2.32E+03	1.56E-01	2.62E-21	3.98E-16	2.34E-21	1.41E-18	2.05E-16	9.55E-29	6.06E-20	1.01E-19	3.13E-19	1.80E+01	8.07E-02
NMSA_FFA(m)	0.00E+00	8.88E-16	5.93E+02	-5.16E+00	7.13E-09	0.00E+00	3.63E-55	5.90E-06	2.61E-53	5.26E-58	1.70E-55	1.09E-50	2.48E-07	6.34E-05	-1.93E+00
NMSA_FFA(M)	1.36E-02	1.48E+00	6.61E+02	-5.16E+00	1.64E-06	9.95E-01	4.67E-04	6.76E-03	1.25E-01	1.92E-06	6.60E-03	7.12E-03	9.70E-05	5.99E-02	-1.80E+00
NMSA_FFA(A)	3.11E-03	5.97E-01	6.19E+02	-5.16E+00	2.48E-07	1.06E-01	3.04E-05	2.08E-03	7.05E-03	1.33E-07	4.39E-04	2.45E-04	2.12E-05	9.01E-03	-1.81E+00
NMSA_FFA(V)	1.37E-05	3.80E-01	5.77E+02	1.09E-10	1.63E-13	6.15E-02	1.05E-08	4.67E-06	5.91E-04	1.83E-13	1.77E-06	1.69E-06	5.83E-10	1.44E-04	1.08E-03
NaFA(m)	3.62E-13	2.97E-05	2.55E-05	-5.16E+00	7.13E-13	5.76E-11	2.69E-12	8.43E-11	4.06E-10	2.19E-17	8.87E-13	2.19E-12	1.71E-11	3.24E-11	-1.80E+00
NaFA(M)	3.27E-11	2.30E-04	1.18E+02	-3.00E+00	1.66E-10	2.51E-08	8.98E-11	2.24E-09	2.95E-08	1.20E-14	6.70E-10	2.71E-10	2.66E-09	4.71E-09	-1.80E+00
NaFA(A)	8.59E-12	1.00E-04	7.90E+00	-5.02E+00	4.31E-11	6.17E-09	2.17E-11	6.40E-10	6.64E-09	4.02E-15	1.27E-10	9.14E-11	4.59E-10	9.73E-10	-1.80E+00
NaFA(V)	4.77E-23	2.24E-09	9.03E+02	3.01E-01	2.02E-21	3.93E-17	5.63E-22	2.29E-19	4.17E-17	1.10E-29	2.01E-20	5.27E-21	2.55E-19	9.54E-19	6.19E-21
FAtidal(m)	0.00E+00	8.88E-16	2.55E-05	-5.16E+00	1.50E-32	0.00E+00	2.27E-54	0.00E+00	4.18E-51	3.39E-28	5.99E-53	1.58E-53	3.70E-32	0.00E+00	-1.80E+00
FAtidal(M)	0.00E+00	8.88E-16	4.57E+02	-2.42E+00	1.50E-32	3.98E+00	6.47E-52	0.00E+00	1.70E-49	1.60E-17	4.47E-51	4.02E-51	3.70E-32	2.27E-01	-1.21E+00
FAtidal(A)	0.00E+00	8.88E-16	1.21E+02	-5.01E+00	1.50E-32	1.13E+00	2.27E-52	0.00E+00	5.32E-50	1.03E-18	1.14E-51	8.22E-52	3.70E-32	7.56E-03	-1.76E+00
FAtidal(V)	0.00E+00	1.01E-62	1.57E+04	3.62E-01	7.75E-96	1.01E+00	3.97E-104	0.00E+00	1.61E-99	8.71E-36	8.18E-103	6.63E-103	2.79E-94	1.72E-03	2.22E-02

dimensions, however we have come out with reliable results using only General alpha damping due to the flexibility of the proposed algorithm.

4. We have the capability of finding the global minimum.

2.2. Definition of firefly algorithm

This algorithm was proposed by X.S. Yang [18]. The FA was inspired by the flashing activity of fireflies and is utilized in optimization. This algorithm is a specific class of PSO that is simple for comprehension and implementation in optimization. The FA consists of three rules:

1. Irrespective of the sex of a firefly, they are attracted to each other.
2. Attractiveness is based on brightness. A dimmer firefly move towards a brighter firefly. The brightness (I) and attractiveness (B) of fireflies are inversely proportional to the distance (r).
3. Here, the landscape of the fitness value is the brightness of fireflies [18].

Table 4
Comparing Algorithms in 2 dimensions refers to Table 2.

Table 4 Comparing Algorithms in 2 dimensions refers to Table 2

Algorithm	Unimodal Functions		Multimodal Functions			Expanded Function
	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}	f_{21}
FA(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.17E+02	-3.00E+02
FA(M)	-3.92E+02	-4.34E+02	4.79E+02	-7.78E+01	-1.15E+02	-2.77E+02
FA(A)	-4.44E+02	-4.46E+02	3.96E+02	-1.28E+02	-1.16E+02	-2.97E+02
FA(V)	1.26E+02	2.10E+01	2.86E+02	2.18E+02	5.09E-01	3.51E+01
LFA(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.37E+02	-1.17E+02	-3.00E+02
LFA(M)	-8.08E+01	-3.76E+02	1.41E+03	6.45E+01	-1.09E+02	9.70E+02
LFA(A)	-4.05E+02	-4.34E+02	5.50E+02	-5.64E+01	-1.13E+02	-1.85E+02
LFA(V)	1.09E+04	6.02E+02	6.96E+04	2.68E+03	5.09E+00	9.58E+04
MFA(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.17E+02	-3.00E+02
MFA(M)	-4.50E+02	-4.50E+02	3.90E+02	-1.40E+02	-1.17E+02	-3.00E+02
MFA(A)	-4.50E+02	-4.50E+02	3.90E+02	-1.40E+02	-1.17E+02	-3.00E+02
MFA(V)	3.26E-16	4.14E-16	4.66E-16	4.00E-01	7.25E-18	2.84E-16
NMSA_FFA(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.17E+02	**-3.00E+02**
NMSA_FFA(M)	-4.50E+02	-4.50E+02	3.90E+02	-1.40E+02	-1.17E+02	-3.00E+02
NMSA_FFA(A)	-4.50E+02	-4.50E+02	3.90E+02	-1.40E+02	-1.17E+02	-3.00E+02
NMSA_FFA(V)	6.96E-17	1.28E-16	7.34E-17	4.50E-01	2.25E-18	7.85E-17
NaFA(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.40E+02	-3.00E+02
NaFA(M)	-4.50E+02	-4.50E+02	4.03E+02	-1.40E+02	-1.30E+02	-3.00E+02
NaFA(A)	-4.50E+02	-4.50E+02	3.92E+02	-1.40E+02	-1.36E+02	-3.00E+02
NaFA(V)	3.72E-07	3.08E-07	8.17E+00	1.78E-01	8.48E+00	9.75E-07
FAtidal(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.17E+02	-3.00E+02
FAtidal(M)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.17E+02	-3.00E+02
FAtidal(A)	-4.50E+02	-4.50E+02	3.90E+02	-1.41E+02	-1.17E+02	-3.00E+02
FAtidal(V)	4.56E-23	4.98E-23	3.04E-23	8.36E-28	7.52E-27	1.13E-22

Table 5
wilcoxon signed rank test on Fatidal and others in 2 dimension refers to Table 1.

Table 5 wilcoxon signed rank test on Fatidal and others in 2 dimension refers to Table 1

Function	FA(p)	(h)	LFA(p)	(h)	MFA(p)	(h)	NMSA_FFA(P)	(h)	NaFA(p)	(h)
f1	1.73E-06	1	1.73E-06	1	1.73E-06	1	8.30E-06	1	1.73E-06	1
f2	2.42E-06	1	1.73E-06	1	1.73E-06	1	2.70E-05	1	1.73E-06	1
f3	4.68E-03	1	2.22E-04	1	1.38E-03	1	1.73E-06	1	5.87E-05	1
f4	6.33E-01	0	1.73E-06	1	4.65E-03	1	3.59E-04	1	7.50E-02	0
f5	1.00E+00	0	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f6	4.24E-03	1	3.71E-01	0	2.60E-05	1	4.54E-05	1	2.60E-05	1
f7	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.13E-05	1	1.73E-06	1
f8	6.25E-02	0	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f9	2.35E-06	1	1.73E-06	1	1.73E-06	1	2.60E-05	1	1.73E-06	1
f10	4.11E-03	1	1.73E-06	1	1.73E-06	1	6.73E-01	0	1.73E-06	1
f11	1.73E-06	1	1.73E-06	1	1.73E-06	1	5.75E-06	1	1.73E-06	1
f12	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f13	1.25E-01	**0**	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f14	2.15E-02	1	1.73E-06	1	1.73E-06	1	3.11E-05	1	3.11E-05	1
f15	7.80E-02	0	1.89E-04	1	1.36E-04	1	1.65E-01	0	5.00E-01	0

Table 6
wilcoxon signed rank test on Fatidal and others in 2 dimension refers to Table 2.

Table 6 wilcoxon signed rank test on Fatidal and others in 2 dimension refers to Table 2										
Function	FA(p)	(h)	LFA(p)	(h)	MFA(p)	(h)	NMSA_FFA(P)	(h)	NaFA(p)	(h)
f16	1.22E-05	1	3.77E-06	1	5.00E-01	0	1.73E-06	1	1.00E+00	0
f17	8.27E-06	1	5.58E-06	1	1.56E-02	1	1.73E-06	1	1.00E+00	0
f18	2.56E-06	1	2.55E-06	1	6.25E-02	0	1.73E-06	1	1.00E+00	0
f19	1.72E-06	1	1.73E-06	1	1.78E-06	1	4.18E-07	1	3.62E-05	1
f20	2.70E-05	1	1.73E-06	1	1.00E+00	0	1.73E-06	1	1.00E+00	0
f21	8.83E-05	1	5.58E-06	1	5.00E-01	0	1.73E-06	1	1.00E+00	0

Table 7
Comparing Algorithms in 10 dimensions refers to Table 1.

Table 7 Comparing Algorithms in 10 dimensions refers to Table 1														
Algorithms	Many Local Minima						bowl shaped				plate shaped			Steep Ridges
	f1	f2	f3	f5	f6	f7	f8	f9	f10	f11	f12	f13	f14	f15
FA(m)	2.58E-14	1.96E-03	4.74E+02	1.12E-07	9.95E-01	4.97E-08	6.39E-06	6.35E-05	4.78E-11	1.35E-06	2.54E-07	1.80E-05	2.01E+00	-9.58E+00
FA(M)	1.04E-10	3.63E-03	1.64E+03	3.87E-07	2.39E+01	1.93E-07	1.70E+00	2.17E-04	1.35E-09	3.97E-06	1.22E-06	6.67E-01	9.22E+00	-7.34E+00
FA(A)	1.99E-11	2.80E-03	1.02E+03	2.49E-07	7.79E+00	1.20E-07	3.76E-01	1.15E-04	4.69E-10	2.49E-06	6.21E-07	6.44E-01	3.79E+00	-8.66E+00
FA(V)	4.69E-22	1.75E-07	8.89E+04	5.54E-15	2.57E+01	1.69E-15	3.87E-01	1.56E-09	9.88E-20	5.36E-13	5.33E-14	1.48E-02	2.90E+00	3.70E-01
LFA(m)	6.66E-16	1.16E+01	2.80E+02	4.39E+00	5.94E+01	1.97E+00	1.53E+02	3.64E+03	1.69E-03	8.49E+01	2.04E+01	1.68E+02	1.97E+03	-5.33E+00
LFA(M)	3.40E-02	1.87E+01	3.06E+03	3.37E+01	1.10E+02	2.79E+01	1.41E+05	2.51E+04	1.86E-01	4.96E+02	2.94E+03	5.40E+04	1.15E+05	-3.09E+00
LFA(A)	1.63E-03	1.62E+01	2.42E+03	1.54E+01	8.55E+01	1.75E+01	1.41E+04	1.21E+04	5.23E-02	2.98E+02	1.58E+02	1.68E+04	3.15E+04	-4.13E+00
LFA(V)	4.06E-05	3.36E+00	2.80E+05	5.04E+01	1.50E+02	4.30E+01	7.33E+08	2.75E+07	2.75E-03	1.58E+04	2.77E+05	1.86E+08	6.10E+08	3.13E-01
MFA(m)	3.57E-14	3.46E-03	2.07E+03	6.99E-07	8.95E+00	3.79E-07	1.44E-04	5.54E-04	7.07E-10	1.76E-05	2.87E-06	6.67E-01	1.42E+01	-1.68E+01
MFA(M)	5.06E-11	5.75E-03	4.34E+03	2.72E-06	4.38E+01	8.77E-07	1.05E+02	3.97E-03	9.85E-09	1.22E-04	9.65E-06	8.43E-01	1.93E+01	-1.26E+01
MFA(A)	1.69E-11	4.43E-03	2.99E+03	1.50E-06	2.44E+01	6.66E-07	1.49E+01	1.66E-03	3.01E-09	3.81E-05	6.10E-06	6.81E-01	1.71E+01	-1.49E+01
MFA(V)	1.94E-22	3.40E-07	2.39E+05	2.04E-13	8.76E+01	1.59E-14	7.02E+02	7.40E-07	3.22E-18	3.68E-10	2.84E-12	1.96E-03	1.69E+00	1.44E+00
NMSA_FFA(m)	2.66E-15	1.33E+00	3.51E+03	1.10E+01	4.98E+00	7.42E-19	1.33E-02	1.68E-17	7.16E-23	3.62E-18	1.30E-07	6.67E-01	6.31E+00	-9.80E+00
NMSA_FFA(M)	1.47E-08	2.00E+01	3.57E+03	4.42E+01	5.47E+01	1.93E-03	2.70E+01	1.01E-02	4.32E-08	1.11E-02	5.26E-04	6.82E-01	8.88E+01	-6.50E+00
NMSA_FFA(A)	3.36E-09	1.66E+01	3.52E+03	2.10E+01	2.89E+01	1.99E-04	4.26E+00	8.59E-04	2.90E-09	8.67E-04	7.44E-05	6.68E-01	1.32E+01	-9.02E+00
NMSA_FFA(V)	1.70E-17	4.94E+01	1.92E+02	8.43E+01	1.42E+02	1.85E-07	6.16E+01	4.73E-06	1.12E-16	7.66E-06	1.34E-08	1.06E-05	3.55E+02	5.01E-01
NaFA(m)	1.11E-14	1.59E-03	5.72E+02	5.37E-08	4.97E+00	3.11E-08	5.64E-04	3.05E-05	2.19E-09	6.43E-07	9.75E-08	6.43E-01	5.79E-01	-9.39E+00
NaFA(M)	1.44E-11	8.40E-01	1.69E+03	1.56E-07	2.29E+01	8.39E-08	5.75E+00	6.40E-05	2.75E-08	1.67E-06	3.08E-07	6.67E-01	2.48E+00	-6.66E+00
NaFA(A)	3.69E-12	2.99E-02	9.88E+02	1.15E-07	1.14E+01	5.77E-08	1.17E+00	4.64E-05	1.31E-08	1.19E-06	2.20E-07	6.66E-01	1.48E+00	-7.88E+00
NaFA(V)	1.14E-23	2.34E-02	8.78E+04	6.15E-16	1.32E+01	2.17E-16	2.67E+00	1.05E-10	3.85E-17	6.68E-14	2.74E-15	1.87E-05	1.70E-01	4.30E-01
Fatidal(m)	0.00E+00	4.44E-15	2.25E+03	1.26E-31	2.98E+00	9.66E-44	7.68E-03	4.19E-41	2.32E-09	1.13E-42	3.45E-43	6.67E-01	9.57E-01	-9.28E+00
Fatidal(M)	0.00E+00	1.51E-14	3.37E+03	1.50E-30	1.59E+01	3.00E-43	1.03E+01	2.45E-40	1.97E-07	6.39E-42	1.37E-42	6.67E-01	7.72E+01	-7.68E+00
Fatidal(A)	0.00E+00	7.40E-15	2.86E+03	7.24E-31	7.20E+00	2.05E-43	1.05E+00	1.66E-40	6.44E-08	3.83E-42	8.99E-43	6.67E-01	9.93E+00	-8.80E+00
Fatidal(V)	0.00E+00	4.42E-30	1.00E+05	1.44E-61	1.01E+01	3.02E-87	4.80E+00	2.01E-81	2.42E-15	1.35E-84	5.89E-86	3.19E-31	2.52E+02	1.71E-01

The distance between any two fireflies x_i and x_j is the Cartesian distance.

$$r_{ij} = \sqrt{\sum_{m=1}^{D} \left(x_{i,m} - x_{j,m}\right)^2} \tag{1}$$

where $x_{i,m}$ is the m_{th} dimension of the spatial coordinate and i_{th} firefly.

In the general implementation, the attractiveness function can be decreased monotonically and is shown in the following formula:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \quad (m \geq 1) \tag{2}$$

when a firefly moves towards firefly j due to the higher attractiveness of firefly j. The movement formula is defined as follows:

$$x_i^{new} = x_i^{old} + \beta_0 e^{-\gamma r_{ij}^2} \left(x_i^{old} - x_j\right) + \alpha \left(rand - \frac{1}{2}\right) \tag{3}$$

Schematically, the pseudo code of the Firefly algorithm is summarized in Fig. 1 [18].

3. Proposed work

3.1. Definition of tidal force in our proposed algorithm

In the celestial mechanics field, the expression “Tidal Force” is defined as a situation in which a body (e.g., tidal water) is dependent on the gravitational influence of a second body/mass (e.g., the Earth) as well as a third body/mass (e.g., the Moon). The described force in such cases is known as a Tidal Force [27]. Newton’s law of universal gravitation defines the attraction of two bodies for each other and the magnitude of it is proportional to the product of their body and inversely proportional to the square of the distance between the bodies [28,29]. Tidal Forces is one class of gravitational attractive forces that has only one difference between them.

Table 8
Comparing Algorithms in 10 dimensions refers to Table 2.

Table 8 Comparing Algorithms in 10 dimensions refers to Table 2						
Algorithms	f16	f17	f18	f19	f20	f21
FA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
FA(M)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
FA(A)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
FA(V)	2.47E-10	2.08E-10	1.99E-10	2.32E-02	4.93E-03	1.05E-10
LFA(m)	3.90E+03	4.69E+03	5.68E+03	2.82E+04	1.41E+04	3.18E+03
LFA(M)	1.94E+04	2.62E+04	2.23E+04	6.33E+05	2.27E+04	2.12E+04
LFA(A)	1.21E+04	1.25E+04	1.38E+04	3.52E+05	1.82E+04	1.31E+04
LFA(V)	1.06E+07	2.78E+07	1.61E+07	4.61E+10	5.29E+06	2.74E+07
MFA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
MFA(M)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
MFA(A)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
MFA(V)	1.58E-10	2.77E-10	2.84E-10	1.87E-02	1.12E-03	1.14E-10
NMSA_FFA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
NMSA_FFA(M)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
NMSA_FFA(A)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
NMSA_FFA(V)	4.35E-11	3.59E-11	2.71E-11	1.20E-09	1.90E-13	2.91E-11
NaFA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.09E+03	-1.20E+02	2.95E+02
NaFA(M)	-4.50E+02	-4.50E+02	3.90E+02	1.09E+03	-1.20E+02	3.17E+02
NaFA(A)	-4.50E+02	-4.50E+02	3.90E+02	1.09E+03	-1.20E+02	3.12E+02
NaFA(V)	1.64E-05	2.96E-05	1.81E-05	2.14E-25	4.19E-03	4.47E+01
Fatidal(m)	-4.50E+02	-4.50E+02	3.90E+02	-1.80E+02	-1.40E+02	-3.00E+02
Fatidal(M)	-4.50E+02	-4.50E+02	3.90E+02	-1.80E+02	-1.40E+02	-3.00E+02
Fatidal(A)	-4.50E+02	-4.50E+02	3.90E+02	-1.80E+02	-1.40E+02	-3.00E+02
Fatidal(V)	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00

Table 9
wilcoxon signed rank test on Fatidal and others in 10 dimension refers to Table 1.

Table 9 wilcoxon signed rank test on Fatidal and others in 10 dimension refers to Table 1										
Function	FA(p)	(h)	LFA(p)	(h)	MFA(p)	(h)	NMSA_FFA(P)	(h)	NaFA(p)	(h)
f1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f2	1.73E-06	1	1.73E-06	1	1.73E-06	1	3.35E-07	1	1.73E-06	1
f3	1.73E-06	1	1.60E-04	1	2.10E-01	0	1.73E-06	1	1.73E-06	1
f5	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.55E-06	1	1.73E-06	1
f6	1.00E+00	0	1.73E-06	1	2.55E-06	1	1.92E-06	1	2.92E-04	1
f7	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f8	2.29E-01	0	1.73E-06	1	3.16E-03	1	2.25E-01	0	5.58E-01	0
f9	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f10	1.73E-06	1	1.73E-06	1	1.92E-06	1	1.73E-06	1	6.98E-06	1
f11	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f12	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f13	1.00E+00	0	1.73E-06	1	6.25E-02	0	3.13E-02	1	1.00E+00	0
f14	2.22E-04	1	1.73E-06	1	3.59E-04	1	2.61E-04	1	1.73E-06	1
f15	4.59E-01	0	1.73E-06	1	1.73E-06	1	1.57E-02	1	8.18E-05	1

The gravitational attractive force is inversely proportional to the square of the distance from the body, but Tidal Forces are inversely proportional to the cube of the distance from the body [29].

As seen in the left part of Fig. 2, we can simply imagine (ignoring spring and neap tidal phenomena) all things on the surface of the earth when (M2=moon) is close to the earth and causes the appearance of Tidal Force phenomena. We used Tidal Force shown in the left side of Fig. 2 instead of the intensity of fireflies as shown on the right side. The Tidal Force equation is a derivative of Newton's Law of Gravitation:

Table 10
wilcoxon signed rank test on Fatidal and others in 10 dimension refers to Table 2.

Table 10 wilcoxon signed rank test on Fatidal and others in 10 dimension refers to Table 2										
Functions	FA(p)	(h)	LFA(p)	(h)	MFA(p)	(h)	NMSA_FFA(P)	(h)	NaFA(p)	(h)
f16	1.00E+00	0	1.73E-06	1	1.00E+00	0	1.00E+00	0	1.00E+00	0
f17	1.00E+00	0	1.73E-06	1	1.00E+00	0	1.00E+00	0	1.00E+00	0
f18	1.00E+00	0	1.73E-06	1	1.00E+00	0	1.00E+00	0	1.00E+00	0
f19	4.32E-08	1	1.64E-06	1	4.32E-08	1	4.32E-08	1	4.32E-08	1
f20	4.32E-08	1	1.73E-06	1	4.32E-08	1	4.32E-08	1	4.32E-08	1
f21	1.00E+00	0	1.73E-06	1	1.00E+00	0	1.00E+00	0	1.43E-06	1

Table 11
Comparing Algorithms in 100 dimensions refers to Table 1.

Table 11 Comparing Algorithms in 100 dimensions refers to Table 1													
Algorithms	Many Local Minima					bowl_shaped				plate shaped			Steep Ridges
	f1	f2	f3	f5	f6	f8	f9	f10	f11	f12	f13	f14	f15
FA(m)	3.69E-13	1.44E-02	2.32E-02	8.57E-05	1.66E-04	3.64E-05	1.41E+00	1.02E-09	1.89E-01	8.93E+01	7.89E-01	9.60E+01	-5.98E+01
FA(M)	3.05E-11	1.42E-01	1.85E+05	2.14E+04	2.49E+02	1.21E+02	9.65E+01	1.14E-08	1.16E+01	2.07E+02	1.76E+01	9.73E+02	-3.90E+01
FA(A)	1.01E-11	2.43E-02	2.28E+04	1.50E+03	1.46E+02	4.05E+00	2.16E+01	6.61E-09	2.59E+00	1.56E+02	4.39E+00	3.24E+02	-5.31E+01
FA(V)	7.84E-23	4.98E-04	9.80E+08	2.94E+07	3.57E+03	4.91E+02	4.89E+02	9.56E-18	6.87E+00	1.25E+03	1.67E+01	1.10E+05	2.64E+01
LFA(m)	8.75E-14	1.51E+01	9.54E-05	1.47E+02	3.98E+02	1.41E+01	4.66E+05	2.81E-03	1.51E+03	3.98E+02	5.82E+03	2.70E+04	-3.05E+01
LFA(M)	6.65E-04	2.14E+04	3.77E+04	4.06E+02	1.23E+03	3.07E+02	2.26E+06	2.15E-01	5.04E+04	1.58E+08	3.12E+09	7.69E+06	-2.40E+01
LFA(A)	3.05E-05	4.56E+03	2.41E+04	2.62E+02	1.07E+03	2.25E+02	1.67E+06	3.54E-02	3.75E+04	5.28E+06	1.07E+08	1.24E+06	-2.70E+01
LFA(V)	1.48E-08	7.04E+07	2.58E+08	4.96E+03	3.69E+04	2.90E+03	1.90E+11	1.67E-03	1.64E+08	8.35E+14	3.23E+17	3.61E+12	2.44E+00
MFA(m)	3.29E-14	3.30E-07	2.02E+01	9.64E-05	1.06E+02	3.69E-05	1.39E+00	1.70E-09	5.32E-02	9.62E+01	7.58E-01	9.22E+01	-6.29E+01
MFA(M)	9.56E-11	3.99E-07	2.01E+01	2.01E+02	1.99E+02	6.84E-05	9.02E+01	1.80E-08	8.25E+00	2.40E+02	1.66E+01	9.87E+01	0.00E+00
MFA(A)	1.71E-11	4.73E-07	2.01E+01	3.50E+01	1.47E+02	5.12E-05	2.11E+01	6.85E-09	2.32E+00	1.62E+02	4.15E+00	9.70E+01	-5.29E+01
MFA(V)	4.73E-22	9.17E-07	2.01E+01	4.42E+03	5.30E+02	6.97E-11	5.26E+02	1.61E-17	5.26E+00	1.47E+03	1.71E+01	1.48E+00	1.22E+02
NMSA_FFA(m)	2.49E-10	1.60E+01	1.99E+01	1.18E+01	2.32E+02	2.06E-02	3.51E+00	5.79E-10	2.98E+00	6.69E+03	3.13E+00	9.74E+01	-5.04E+01
NMSA_FFA(M)	3.10E-06	3.63E+04	3.63E+04	9.81E+02	5.75E+02	1.20E-01	1.63E+02	2.51E-06	2.12E+02	3.27E+04	3.56E+02	4.68E+02	0.00E+00
NMSA_FFA(A)	1.46E-07	8.49E+03	1.10E+04	2.65E+02	3.73E+02	5.40E-02	2.43E+01	2.12E-07	3.41E+01	1.47E+04	4.95E+01	1.56E+02	-3.56E+01
NMSA_FFA(V)	3.15E-13	2.44E+08	2.85E+08	7.13E+04	6.57E+03	5.51E-04	1.29E+03	2.38E-13	2.31E+03	2.50E+07	8.24E+03	6.55E+03	8.56E+01
NaFA(m)	1.24E-13	3.10E-03	4.64E+00	4.35E-07	3.12E+01	2.08E-07	5.26E-02	5.08E-04	1.89E-08	1.01E-05	1.45E-06	6.44E-01	9.81E+00
NaFA(M)	1.74E-11	2.89E+03	2.01E+04	5.27E+01	1.92E+03	4.31E-07	3.29E+01	8.39E-04	6.20E-07	1.92E-05	2.98E-06	7.98E-01	1.71E+01
NaFA(A)	4.58E-12	5.38E+02	2.78E+03	1.10E+01	5.08E+02	3.38E-07	6.02E+00	6.59E-04	9.05E-08	1.51E-05	2.34E-06	6.71E-01	1.35E+01
NaFA(V)	1.70E-23	1.01E+06	1.13E+07	3.51E+02	2.99E+05	2.95E-15	7.66E+01	8.52E-09	1.09E-14	5.40E-12	1.17E-13	6.00E-04	1.70E+00
FAtidal(m)	0.00E+00	2.93E-14	3.60E+04	2.06E-12	8.95E+01	8.56E-42	1.86E+01	2.16E-08	1.25E+00	2.89E+02	7.96E-01	9.60E+01	-7.76E+01
FAtidal(M)	0.00E+00	4.35E-14	3.89E+04	4.46E+00	2.16E+02	1.14E-41	1.61E+03	1.26E-06	3.69E+01	6.23E+02	4.97E+01	3.12E+02	-5.92E+01
FAtidal(A)	0.00E+00	3.42E-14	3.75E+04	4.50E-01	1.36E+02	9.78E-42	5.20E+02	3.34E-07	1.17E+01	4.54E+02	8.57E+00	1.24E+02	-6.99E+01
FAtidal(V)	0.00E+00	2.48E-29	4.35E+05	8.31E-01	6.59E+02	6.25E-85	2.20E+05	1.01E-13	7.39E+01	8.16E+03	1.04E+02	2.83E+03	2.27E+01

F=total Tidal Force, G=gravitational Constant, M_2 = M = Moon, M_1=m= mass of affected object, d=distance, r= radius of Moon

$$F_{tidal} = \frac{GMm}{d^2} - \frac{GMm}{(d+\Delta r)^2} \tag{4}$$

$$F_{tidal} = \frac{GMm(d+\Delta r)^2 - GMmd^2}{d^2(d+\Delta r)^2} \tag{5}$$

After rearranging and simplification:

$$F_{tidal} = \frac{2GMmr}{d^3} \equiv F_{near} - F_{far} \equiv \left(G * m2 * m/Ru^2\right) - \left(G * m2 * m/Rl^2\right) \tag{6}$$

$Rl \neq Ru \neq 0$

Generally, Tidal Force is defined as Eq. (6) [30], and we used this equation for the firefly intensity in a new formula. In the next section, it will be described in detail.

3.2. Our proposed algorithm

By using the Tidal Force in the framework of the Firefly algorithm, we can achieve the global minimum value with less generation. As described in section 2, if the distance r decreases as I $\propto 1/r^2$, the light intensity at a distance r from the light source is equal to the increased light intensity. Let the population x= $(x_1, \ldots, x_n)$ and n equal the population. Each individual in the population is

Table 12
Comparing Algorithms in 100 dimensions refers to Table 2.

Table 12 Comparing Algorithms in 100 dimensions refers to Table 2						
Algorithms	f16	f17	f18	f19	f20	f21
FA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.01E+04	8.08E+03	-3.00E+02
FA(M)	-4.49E+02	-4.48E+02	4.91E+02	1.34E+05	8.21E+06	-3.00E+02
FA(A)	-4.50E+02	-4.50E+02	4.33E+02	4.29E+04	9.64E+05	-3.00E+02
FA(V)	3.38E-02	1.18E-01	2.50E+03	3.05E+09	6.08E+12	5.23E-04
LFA(m)	2.79E+04	3.88E+03	2.82E+05	1.10E+04	2.55E+05	2.69E+04
LFA(M)	3.56E+05	3.01E+06	2.16E+11	5.71E+06	2.67E+07	3.11E+06
LFA(A)	2.99E+05	3.50E+05	8.22E+10	7.46E+05	1.71E+06	4.81E+05
LFA(V)	2.98E+09	2.64E+11	9.45E+21	3.22E+12	2.34E+13	6.73E+11
MFA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.01E+04	8.15E+04	-3.00E+02
MFA(M)	-4.50E+02	-4.49E+02	4.90E+02	1.34E+05	8.60E+06	-3.00E+02
MFA(A)	-4.50E+02	-4.50E+02	4.50E+02	3.07E+04	4.16E+05	-3.00E+02
MFA(V)	6.58E-04	6.74E-02	2.44E+03	2.19E+09	2.43E+12	2.89E-04
NMSA_FFA(m)	-4.49E+02	-4.49E+02	3.91E+02	2.60E+04	8.10E+04	-2.99E+02
NMSA_FFA(M)	-4.48E+02	-4.48E+02	1.05E+03	2.96E+06	9.29E+05	-2.98E+02
NMSA_FFA(A)	-4.49E+02	-4.48E+02	6.49E+02	5.22E+05	1.73E+05	-2.98E+02
NMSA_FFA(V)	6.42E-02	4.63E-02	8.06E+04	6.78E+11	6.45E+10	1.87E-02
NaFA(m)	-4.50E+02	-4.50E+02	3.90E+02	1.02E+04	8.21E+04	-3.00E+02
NaFA(M)	-4.50E+02	-4.50E+02	7.40E+04	1.36E+05	8.59E+05	-3.00E+02
NaFA(A)	-4.50E+02	-4.50E+02	4.06E+03	3.96E+04	2.83E+05	-3.00E+02
NaFA(V)	3.72E-03	3.26E-03	2.00E+08	2.91E+09	1.14E+11	1.22E-03
FAtidal(m)	-4.50E+02	-4.50E+02	4.87E+02	5.98E+03	-1.20E+02	-2.55E+02
FAtidal(M)	-4.50E+02	-4.50E+02	8.17E+05	9.06E+03	-1.19E+02	-2.52E+02
FAtidal(A)	-4.50E+02	-4.50E+02	2.08E+05	7.08E+03	-1.19E+02	-2.53E+02
FAtidal(V)	0.00E+00	8.48E-05	5.09E+10	5.14E+05	5.26E-03	4.33E-01

x= $(x_{11}, \ldots, x_{1d})$, where d is the number of dimensions or number of variables. In each iteration, fireflies compute pairwise. If one of the pair is less than the other, it would be selected for new movement in all dimensions.

To calculate Tidal Forces in less dimensions problem, we compute the distance between the same elements of the ith column (e.g., x_{11}, x_{21}) of the pairwise fireflies, on the other hand the distance is computed directly from the pairwise fireflies (e.g., x_1, x_2) in more dimensions problem. It is imagined that body (m_1) on the left of Fig. 2, after taking the amount of x_{11} and x_{21}, would then be inverted in accordance with I $\propto 1/r^2$. Generally, the strategy will be started as follows. First, one is divided by all values. A matrix shows the generated value, and B matrix is 1 divided by A matrix.

$$A=\begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n} \end{bmatrix} \Rightarrow B=\begin{bmatrix} \frac{1}{x_{1,1}} & \frac{1}{x_{1,2}} & \cdots & \frac{1}{x_{1,n}} \\ \frac{1}{x_{2,1}} & \frac{1}{x_{2,2}} & \cdots & \frac{1}{x_{2,n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{x_{m,1}} & \frac{1}{x_{m,2}} & \cdots & \frac{1}{x_{m,n}} \end{bmatrix} \quad (7)$$

Each vector shows one Firefly that has n dimensions.

$$V = \left[\frac{1}{x_{1.1}} \frac{1}{x_{1,2}} \cdots \frac{1}{x_{1,n}} \right] \quad (8)$$

Gaining the far and near distances in each dimension with two different vectors/fireflies,

$$R_L = w = \mathrm{Max}(V) \quad (9)$$

$$R_U = z = \mathrm{Min}(V) \quad (10)$$

Computing the Tidal force in each dimension,

$$\beta = \left(G * m2 * m/{R_U}^2\right) - \left(G * m2 * m/{R_L}^2\right) \equiv \left(G * m2 * m/z^2\right) - \left(G * m2 * m/w^2\right) \quad (11)$$

Table 13
wilcoxon signed rank test on Fatidal and others in 100 dimension refers to Table 1.

Table 13 wilcoxon signed rank test on Fatidal and others in 100 dimension refers to Table 1										
Function	FA(p)	(h)	LFA(p)	(h)	MFA(p)	(h)	NMSA_FFA(P)	(h)	NaFA(p)	(h)
f1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f2	1.73E-06	1	1.72E-06	1	1.73E-06	1	1.16E-06	1	1.73E-06	1
f3	3.10E-05	1	1.56E-05	1	1.73E-06	1	2.01E-06	1	1.73E-06	1
f5	4.28E-01	0	1.73E-06	1	3.31E-04	1	1.73E-06	1	3.31E-04	1
f6	9.98E-02	0	1.73E-06	1	8.99E-03	1	1.73E-06	1	8.99E-03	1
f8	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f9	2.13E-06	1	1.73E-06	1	1.73E-06	1	2.88E-06	1	1.73E-06	1
f10	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.96E-02	1	1.73E-06	1
f11	1.13E-05	1	1.73E-06	1	1.73E-06	1	5.45E-02	0	1.73E-06	1
f12	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f13	5.71E-02	0	1.73E-06	1	1.73E-06	1	2.96E-03	1	1.73E-06	1
f14	1.58E-02	1	1.73E-06	1	1.72E-06	1	2.50E-02	1	1.72E-06	1
f15	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1

It is worth pointing out that in the derivation of Tidal Force, which is the step value for movement of fireflies, dz and dw are stepped values, and when subtracted, it yields dx.

$$\text{ddx} = 2G * m_1 * m_2 * z - 2G * m_1 * m_2 * w$$

$$\text{dz} = 2G * m_1 * m_2 * z \quad (12)$$

$$\text{dw} = 2G * m_1 * m_2 * w$$

For example: $x_{11} = \frac{1}{x_{11}}, x_{21} = \frac{1}{x_{21}}$

Here, R_L and R_U are the same as drawn in the left of Fig. 2.

$$R_L = \max(x_{11}, x_{21}), \; R_U = \min(x_{11}, x_{21})$$

$$F_i = m_1 * m_2 * G/{R_L}^2, \; F_u = m_1 * m_2 * G/{R_U}^2$$

$$\text{dx} = F_i - F_u$$

In general, flashing fireflies can be formulated in Tidal Force, with regard to the objective function. For the direction of firefly movement, we used a random uniform distribution in less dimensions and more dimensions, which is suitable to coordinate with the amount of Tidal Force due to the starchy body.

The feature of starchy body aids in balancing the firefly movement in each dimension between two fireflies. The speed of firefly movement is controlled with m_1, m_2 and G. To get rid of singularity events in the R_L and R_u denominator, it is faced with the zero problem in the denominator. This is controlled by the equation $R_L \neq R_u$ in the program and one (1) is added to R_L and R_u.

For exploration/diversification, the algorithm searches the minimum value in all possible regions. This means that fireflies move towards the best firefly in each generation with a combined normal uniform distribution and general alpha operator. In some algorithms, including Firefly and its varients, a new equation alpha has been defined for more dimensions, to produce better exploration/diversification, but in our proposition, the new operator is not required, in the place we just used general alpha operator.

$$(\text{alpha} = \text{alpha} * 0.99). \quad (13)$$

Exploration tends to reduce the search's convergence rate. Alpha is also considered with a decreasing damping operator. In this study, we used the continuous Alpha damping and uniform distributions in less and more dimensions. In general, for keeping a balance between exploration and exploitation, the exploration was divided into two sections, one for less dimensions and the other for more dimensions.

$x_i = (x_{i1}, x_{i2}, \ldots, x_{id}, x_{in})$, D refers to the number of variables.

$$\begin{matrix} \textit{pairwisefirefly} & \text{D} \geq 10 \\ \textit{pairwisevariable (dimensions) offirefly} & \text{D} < 10 \end{matrix} \quad (14)$$

$$x_{ij} = x_i + dx_{ij} * (x_i - x_j) + a * \left(rand - \frac{1}{2}\right) \quad (15)$$

This algorithm should avoid premature convergence and coordinate the exploration and exploitation. One of the characteristics of Tidal Force is that it depends on the distance, and we can use the amount of m_1 and m_2 with the damping operator for greater exploitation. Another characteristic of the Tidal Force is its adaptability with the firefly strategy. To get rid of randomization at the initialization point, which can cause worse results, the Opposition-based learning (OBL) method was used. This was introduced by Tizhoosh [31]. Suppose x is a real number that lies in $x \in [m, n]$. The opposite number will be as below:

$$\tilde{x} = m + n - x \quad (16)$$

In D-dimensional space, the formula can be as follows:

$$\tilde{x}_i = m_i + n_i - x_i \quad \text{i} = 1 \ldots, \text{D}. \quad (17)$$

For opposition-based optimization, let f(.) be the fitness function, x be a candidate solution in D-dimensional space and $\tilde{x}$ the opposition point of x. The replacement continues with $\tilde{x}$ if $(\tilde{x}) \geq$ f(x), or else with x. See Fig. 3, which is the pseudo code of Tidal Force.

4. Comparative study

We statistically compared the proposed algorithm with modified Firefly algorithms and ABC algorithm to demonstrate its efficiency for both lower and higher dimensions. However, for the other classified algorithms (swarm intelligence and evolutionary algorithm), we only showed their efficiency in the two dimensions without statistical tests because they are categorized in different classes and some in multiple classes. For example, in this study, the PSO, FA and BEE are swarm intelligence and BBO is Bioinspired. Some of the Bioinspired algorithms can be called swarm intelli-

Table 14
wilcoxon signed rank test on Fatidal and others in 100 dimension refers to Table 2.

Table 14 wilcoxon signed rank test on Fatidal and others in 100 dimension refers to Table 2										
Functions	FA(p)	(h)	LFA(p)	(h)	MFA(p)	(h)	NMSA_FFA(P)	(h)	NaFA(p)	(h)
f16	5.00E-01	0	1.71E-06	1	1.00E+00	0	1.01E-07	1	1.00E+00	0
f17	1.00E+00	0	1.73E-06	1	5.00E-01	0	2.57E-07	1	1.00E+00	0
f18	1.92E-06	1	6.32E-05	1	1.92E-06	1	1.92E-06	1	1.92E-06	1
f19	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f20	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1
f21	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1	1.73E-06	1

Table 15
wilcoxon signed rank test and optimum valueABC in 2 dimension refers to Table 1.

Table 15 wilcoxon signed rank test and optimum value ABC in 2 dimension refers to Table 1															
ABC	Many Local Minima						bowl_shaped					Plate_shaped			Steep Ridges
	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10	f11	f12	f13	f14	f15
(min)	2.65E-07	2.69E-04	5.97E-04	-5.16E+00	1.41E-08	1.20E-03	1.26E-09	3.84E-06	2.66E-06	2.21E-12	4.57E-08	6.11E-08	2.25E-06	1.10E-04	-1.80E+00
(max)	3.42E-04	2.40E-02	1.70E+00	-5.15E+00	7.68E-07	1.84E-02	4.08E-07	7.72E-04	4.66E-04	5.64E-10	5.23E-06	7.37E-06	2.62E-04	6.87E-03	-1.80E+00
(Average)	5.21E-05	1.12E-02	3.23E-01	-5.16E+00	2.16E-07	6.93E-03	1.20E-07	1.79E-04	1.22E-04	7.95E-11	1.21E-06	2.36E-06	5.95E-05	1.10E-03	-1.80E+00
(variance)	7.04E-09	3.75E-05	1.56E-01	3.86E-06	3.84E-14	2.72E-05	9.95E-15	3.08E-08	1.41E-08	1.21E-20	1.18E-12	5.30E-12	4.02E-09	1.66E-06	2.09E-13
(P)	1.73E-06	1.73E-06	1.48E-03	6.25E-01	1.73E-06	2.60E-05	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.11E-05	5.00E-01
(h)	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0

* here, the bolded value is dominated by Ftidal algorithm

gence, such as FA and PSO [10]. HS is also categorized in physics and chemistry.

4.1. Experimental setting

The experiment includes the comparison of the FAtidal algorithm with Firefly, modified Firefly algorithms and ABC algorithm. The well-known population-based algorithms are presented only to illustrate the status of the functions and were not further investigated due to the framework. All algorithms with proper settings were adjusted. In each experiment, they were executed 30 times and each performed until the defined function evaluation number (FEN). In this experiment, the FEN for two dimensions was set to 100,000 and for ten and hundred dimensions were set to 200,000 and 300,000 respectively. The population size was set to 20,40 and 60 respectively.

- nPop refers to the population size.
- n is a head of the word and usually refers to the number.
- Co refers to Coefficient, and Mu also refers to mutation.

GA settings: Mu is probability= 0.05, crossover probability= 0.8, gene length = 10.

ABC settings: nOnlooker=nPop*1/2, Abandonment Limit, Parameter=round (0.5* number of variable * population size) Abandonment Limit Parameter, colony size=20, Acceleration Co Upper Boun=1.

BBO settings: KeepRate=0.2, nKeep=round(KeepRate*nPop), nNew=nPop-nKeep, α=0.9, pMutation=0.1, sigma=0.02*(Maximum Variable – Minimum Variable).

BEE settings: nSelected Sites =round(0.5*nScoutBee), Number of Selected Elite Sites =round(0.4*nSelectedSite), nRecruited Bees for Selected Sites= round(0.5*nScoutBee), Number of Recruited Bees for Elite Sites=nEliteSiteBee=2*nSelectedSiteBee, Neighborhood Radius=0.1*(Maximum Variable – Minimum Variable), rdamp=0.99.

CU settings: Acceptance Ratio =0.35, nAccepted =round(pAccept*nPop), Accepted Individuals, α=0.25, β=0.5.

HSsettings: Memory Size =20, nNew Harmonies =20, Harmony Memory Consideration Rate =0.5, Pitch Adjustment Rate=0.1, Fret Width (Bandwidth)=0.02*(Maximum Variable – Minimum Variable), Fret Width Damp Ratio=0.995.

ICA settings: nEmpires and nImperialists=10, Selection Pressure=1, Assimilation Co =2, Revolution Probability =0.1, Revolution Rate =0.05, Colonies Mean Cost Co Share Settings =0.1.

FA settings: β0 = γ = 1, α = 0.2, α _damp=0.97

LFA settings: γ = 1, γ0 = 4, η1 = 0.3, η2 = 0.1.

MFA settings: α =0.5, betamin=0.2, γ0 =1.

NMSA_FFA settings: α=0.01, γ0 =1, betamin=0.2.

NaFA settings: α =0.5, β0 = γ = 1

Ftidal: Uniform Mu Range =0.05*(Maximum Variable – Minimum Variable), α =0.9, α _damp=0.99, m1=0.1, m2=0.1, G=0.667.

4.2. Benchmark function for testing

As seen in Table 1 (taken from [34]) and Table 2 (taken from CEC 2005 competition), twenty-one benchmark functions were considered for the experiment with different shapes and characteristics. There are four classes in Table 1, including Many Local Minima, Bowl Shaped, Plate Shaped and Steep Ridges. Table 2 includes three classes, such as Unimodal, Multimodal and expanded functions. The first column describes the characteristics of the functions.

4.3 Results and Discussion

The divergence of FAtidal on the sum of different power functions in Fig. 4 is presented in Fig. 5 in which the Sphere function (as mentioned earlier) has been applied with its divergence (a) and contour space (b). FAtidal shows its exploitation and moves stepwise in a unimodal space with suitable divergence. The sum of

Table 16
wilcoxon signed rank test and optimum value ABC in 2 dimension refers to Table 2.

Table 16 wilcoxon signed rank test and optimum value ABC in 2 dimension refers to Table 2						
ABC	f16	f17	f18	f19	f20	f21
(min)	-4.50E+02	-4.50E+02	3.90E+02	1.37E+03	6.37E+02	-3.00E+02
(Max)	-4.50E+02	-4.50E+02	3.90E+02	1.37E+03	6.37E+02	-3.00E+02
(Average)	-4.50E+02	-4.50E+02	3.90E+02	1.37E+03	6.37E+02	-3.00E+02
(variance)	5.10E-09	3.05E-09	1.10E-09	1.34E-15	1.20E-25	2.25E-09
(P)	1.00E+00	1.00E+00	1.00E+00	4.32E-08	4.32E-08	1.00E+00
(h)	0	0	0	1	1	0

* here, the bolded value is dominated by Ftidal algorithm

Table 17
wilcoxon signed rank test and optimum value ABC in 10 dimension refers to Table 1.

Table 17 wilcoxon signed rank test and optimum value ABC in 10 dimension refers to Table 1														
ABC	Many Local Minima						bowl shaped				plate shaped			Steep Ridges
	f1	f2	f3	f5	f6	f7	f8	f9	f10	f11	f12	f13	f14	f15
(min)	0.00E+00	3.11E-05	8.28E+02	1.95E-10	1.38E+01	9.81E-13	8.30E-01	1.50E-08	1.63E-21	2.67E-11	1.22E+00	6.67E-01	6.71E-02	-8.38E+00
(Max)	0.00E+00	3.69E-04	1.32E+03	3.67E-09	2.58E+01	1.04E-11	1.75E+01	4.03E-07	1.72E-18	8.27E-10	4.15E+00	6.67E-01	5.49E+00	-7.43E+00
(Average)	0.00E+00	7.83E-05	1.11E+03	1.41E-09	2.07E+01	4.22E-12	7.55E+00	1.18E-07	3.11E-19	2.79E-10	2.77E+00	6.67E-01	3.47E+00	-7.83E+00
(Variance)	0.00E+00	3.89E-09	1.66E+04	8.04E-19	7.51E+00	6.57E-24	1.47E+01	8.63E-15	1.94E-37	3.57E-20	6.68E-01	1.96E-10	2.91E+00	9.17E-02
(P)	1.00E+00	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	4.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.00E+00	1.73E-06	2.60E-06
(h)	0	1	1	1	1	1	1	1	1	1	1	0	1	1

* here, the bolded value is dominated by Ftidal algorithm

Table 18
wilcoxon signed rank test and optimum value ABC in 10 dimensions refers to Table 2.

Table 18 wilcoxon signed rank test and optimum value ABC in 10 dimensions refers to Table 2						
ABC	f16	f17	f18	f19	f20	f21
(min)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
(Max)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
(Average)	-4.50E+02	-4.50E+02	3.90E+02	1.60E+04	8.36E+03	-3.00E+02
(variance)	9.23E-18	1.04E-17	4.97E-18	4.64E-15	1.05E-18	9.89E-18
(P)	1.00E+00	1.00E+00	1.00E+00	4.32E-08	4.32E-08	1.00E+00
(h)	0	0	0	1	1	0

* here, the bolded value is dominated by Ftidal algorithm

Table 19
wilcoxon signed rank test and optimum value ABC in 100 dimensions refers to Table 1.

Table 19 wilcoxon signed rank test and optimum value ABC in 100 dimensions refers to Table 1													
ABC	Many Local Minima					bowl_shaped				Plate_shaped			Steep Ridges
	f1	f2	f3	f5	f6	f8	f9	f10	f11	f12	f13	f14	f15
(min)	0.00E+00	1.59E+01	2.76E+04	3.54E+02	1.09E+03	1.03E+02	5.01E+05	5.32E-01	1.17E+04	1.39E+03	2.81E+06	4.33E+05	-2.47E+01
(Max)	0.00E+00	1.72E+01	3.24E+04	5.18E+02	1.23E+03	1.38E+02	6.43E+05	1.58E+00	1.45E+04	1.62E+03	4.87E+06	4.12E+07	-2.18E+01
(Average)	0.00E+00	1.67E+01	3.12E+04	4.51E+02	1.17E+03	1.20E+02	5.83E+05	9.78E-01	1.30E+04	1.49E+03	4.05E+06	2.27E+06	-2.29E+01
(variance)	0.00E+00	8.45E-02	1.14E+06	1.85E+03	1.17E+03	6.98E+01	9.94E+08	8.31E-02	5.58E+05	3.55E+03	1.94E+11	5.52E+13	4.40E-01
(P)	1.00E+00	1.65E-06	1.71E-06	1.73E-06	1.73E-06	1.72E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.732E-06
(h)	0	1	1	1	1	1	1	1	1	1	1	1	1

* here, the bolded value is dominated by Ftidal algorithm

different power functions was selected to show the convergence of the algorithms indicated in Fig. 6. As shown, the initialization of the proposed algorithm outperforms the others.

On the other hand, this comparison is not suitable for convergence, stability, and robustness. For this, we used the Wilcoxon signed-rank test, which is performed to statistically validate the results. The test is done using a pairwise method, where p is the significance value and h is the logic value based on whether the defined hypothesis is rejected. In here, Test is done with the hypothesis at the a = 0.05 (95%) level.

$$H_0 : \mu_1 = \mu_2 \text{Twodatasetsarenotstatisticallydifferent}$$

Table 20
wilcoxon signed rank test and optimum value ABC in 100 dimensions refers to Table 2.

Table 20 wilcoxon signed rank test and optimum value ABC in 100 dimensions refers to Table 2						
ABC	f16	f17	f18	f19	f20	f21
(min)	1.08E+04	1.16E+04	1.25E+04	1.35E+05	8.15E+04	1.22E+04
(Max)	1.61E+04	1.73E+04	1.69E+04	1.37E+05	8.15E+04	1.66E+04
(Average)	1.35E+04	1.40E+04	1.50E+04	1.36E+05	8.15E+04	1.38E+04
(variance)	1.68E+06	1.41E+06	9.21E+05	1.54E+05	3.55E+01	1.01E+06
(P)	1.72E-06	1.72E-06	9.32E-06	1.73E-06	1.73E-06	1.73E-06
(h)	1	1	1	1	1	1

* here, the bolded value is dominated by Ftidal algorithm

$H_1 : \mu_1 \neq \mu_2$Twodatasetsarestatisticallydifferent

The running algorithms are performed 30 times with adjusted settings, and their statistical results are shown on Tables 5,6,9,10,13 and 14. The bolded value indicates that the Proposed algorithm outperforms the considered algorithm, and underlined values indicate no significant difference. Tables 3,7,11 and 4,8,12 refer to the benchmark function of Tables 1 and 2 respectively. Tables 3,4 and 7,8 and 11,12 indicate the achieved values in 2-,10- and 100- dimension spaces respectively. As it is shown in the tables the bolded values are the best or minimum value achieved by algorithms on the considered function. It is compared to other algorithms for each function. In the first column, there are 4 terms for describing the statistical data. m, M, A and V refer to the Maximum, Minimum, Average value and variance on achieved values, respectively.

Table 3 shows that, in the first experiment (2 dimensions), FA and FAtidal has better convergence capabilities than the others. But the statistical test shows that FAtidal is more robust than the FA. whereas in Table 4 NAFA has better convergence and statistical test shows the best performance. (Tables 3,4,5,6). Tables 7,8 show that, In the second experiment for 10 dimensions, FAtidal considering the minimum value and result of statistical tests the performance is better than the others. (Tables 7–10). Table 11 shows that, In the third experiment for 100 dimensions, FAtidal and NAFA algorithms obtained the best minimum value, but in the statistical test only MFA is better than the FAtidal. (Tables 11–14). Compared to the classical ABC algorithm, the proposed algorithm yields best results. (Tables 15–20). In general, The proposed algorithm in all experiments (2-,10- and 100- dimensions spaces) showed its robustness in finding global minimum and persistence behavior in all different functions.

In this article, the time complexity of algorithms is considered informally and formally. $G_{\max}$ is the maximum of generation, N^2 is population size, f is considered the function that the algorithm conducts on it, and a includes its own formula and damping alpha in each generation. As can be seen, the NaFA algorithm is more efficient than the other algorithms. Ftidal, FA, MFA, and LFA possess similar complexity times in the second group. Luus Jaakola and Opposition-based learning, since they conduct only one initialization at a time (Table 21). We have run the F1 function 30 times with all the algorithms for 2-,10- and 100- dimensions (Table 22 and Fig. 7). We notice that when going from 2 to 10 and 100 dimensions the results are becoming better. All experiments are implemented on Intel(R) Core (TM i5-4210u CPU 1.70 GHz and 2.40 GHz with 8.00 GB memory and Computer model is Toshiba Skull candy. Our proposed algorithm and its variants are coded in MATLAB R2016a and the operating system is Windows 10.

Table 21
Time Complexity with formal .

Algorithms	Time complexity
FA	O($G_{\max}$. N^2 . f . a)
LFA	O ($G_{\max}$. N^2 . f . a)
MFA	O($G_{\max}$. N^2 . f . a)
NaFA	O ($G_{\max}$. N . k . f . a)
	K is neighborhood which should be much less than $\frac{N-1}{2}$
NMSF_FFA	O ($G_{\max}$. N^2 . f . $a + o + L$)
	L is Luus_jaakola Function for initialization
FATIDAL	O ($G_{\max}$. N^2 . f . $a + o$)
	O is Opposition-based learning for initialization

Table 22
Time Complexity Comparison on F1 function.

Algorithms	2 dimensions	10 dimensions	100 dimensions
FA	4.95E-01	1.55E+01	7.34E+01
LFA	1.49E+00	9.96E+00	3.34E+01
MFA	5.28E-01	1.42E+01	7.02E+01
NMSF_FFA	6.08E-01	2.29E+01	4.62E+01
NaFA	7.60E-01	1.70E+00	2.75E+00
FAtidal	3.30E+00	1.15E+01	3.29E+01

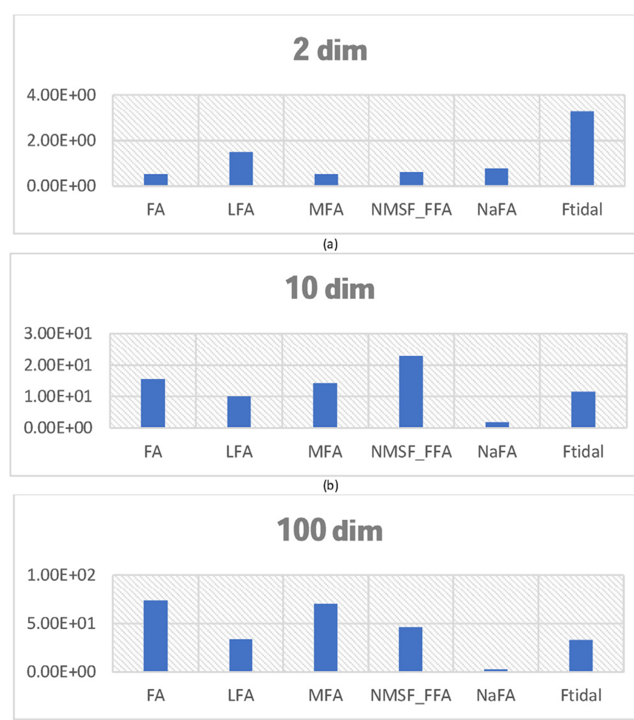

Fig. 7. Time complexity comparison: a) 2 dimensions b)10 dimensions c)100 dimensions.

5. Conclusion

This study, inspired by Tidal Force, was combined with the framework of the Firefly algorithm. It was flexible to less and increased dimensional function problems. One of the contributions to the algorithm is the work with simple Alpha operator owing to its flexibility, which improved the balance between exploration and exploitation. FAtidal provided new insight to increased exploitation by using the variables of formulae, e.g., m_1, m_2, which are Internal damping operators. It can also be utilized to achieve the optimum minimum value, since there is no absorption coefficient in the formula. This paper provides an account of the proposed algorithm suitable for the achievement of the tradeoff between exploration and exploitation. It possesses the best Wilcoxon Signed Rank test results for the global optimum and overcomes the premature convergence from the unimodal, multi-modal, expanded function, Plate shaped and Steep Ridges functions. The compared studies show that the proposed algorithm is highly competitive with others. The proposed algorithm can be applied to NP Hard problems and adapted to other existing population-based algorithms.

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