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A mathematical programming approach for full coverage hole optimization in Cloud Radio Access Networks

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ABSTRACT

This paper proposes a Branch-and-Cut algorithm for network operators and providers to propose a full coverage hole in the context of Cloud Radio Access Networks (C-RAN). In this context, and to optimize the network coverage when reducing interferences, network operators need new algorithms that enable to consolidate and re-optimize the antennas radii. This paper provides an NP-Hardness complexity proof of the full coverage hole problem and proposes a deep Branch-and-Cut algorithm based on the description of new cutting planes to accelerate the convergence time even for large problem sizes. Simulation results and comparison to the state of the art highlight the efficiency and the usefulness of our approach.

1. Introduction and motivation

With the growth of traffic demands in mobile networks, Telecommunications Service Providers (TSPs) are investigating new approaches to increase the density of existing cells by deploying new antennas to enlarge the network spectrum. Nevertheless, this comes with important drawbacks such increasing interferences causing serious degradations of the provided networks' quality of service. Hence, proposing efficient solutions of interferences management is a key challenge to meet and maintain a full network coverage with minimum interferences.

1.1. Background and definitions

With the era of programmable networks, TSPs are motivated by deploying more antennas to provide new network services with enhanced network coverage and connectivity. Indeed, this is feasible when embracing Cloud Radio Access Networks (C-RAN) technology described in the sequel (see Fig. 1).

In this context, C-RAN is seen as a key enabler for the next generation mobile networks to handle the diverse service requirements. The main functionality of C-RAN (depicted in Fig. 1) consists in decoupling the BaseBand processing Unit (BBU) from the Remote Radio Head (RRH) to increase network coverage and reduce both the network CAPEX (CAPital EXpenses) and OPEX (OPerating EXpenses). Moreover, in C-RAN, network operators or service providers will propose more services to end users and this requires guaranteeing the network connectivity leveraging new approaches to reduce network holes (see Fig. 2) and to fulfill end-users requirements jointly. Thus, network operators have to investigate new approaches to rapidly reach a good tradeoff between interference elimination/reduction and coverage hole detection when embracing C-RAN technology.

This work focuses on optimizing the full network coverage in C-RAN by reducing the number of coverage holes and minimizing the inter-cell interferences. Indeed, to cope with this problem, numerous schemes have been proposed in different networks (e.g. sensor networks, …) using different approaches. Traditionally, and to guarantee the global coverage of a network, geometrical methods can be used under some conditions and constraints (see [1,2] for instance). These methods are based on the generalization of graphs to more generic combinatorial objects known as simplicial complexes and are made up of vertices, edges, triangles, tetrahedra and their $n-$dimensional counterparts. We define by $k-$simplex an unordered subset of $k+1$ vertices. Thus, a $2-$simplex is a triangle. In addition, and for sake of clarity, we note the existence of two approaches noted by Cĕch complex and Rips complex which are based on the verification of intersection between cells, to detect holes and connectivity problems (the detailed description of these complexes is not in the scope of this work, but more information can be found in [3]). These approaches are using time consuming methods to characterize the number of $0-$dimensional holes (i.e. $k-$dimensional holes with $k=0$) (noted by β_0) that represents the

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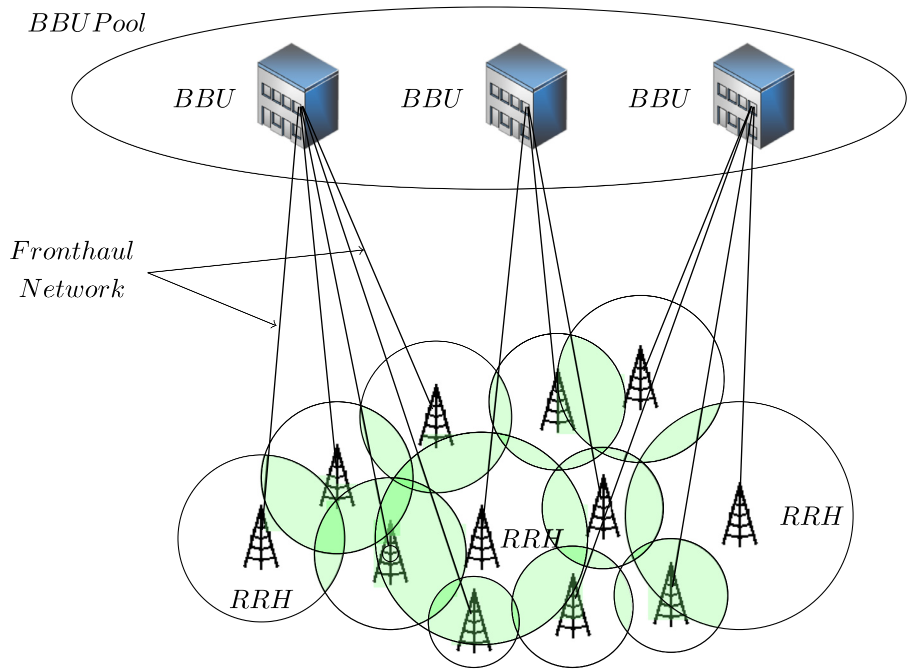
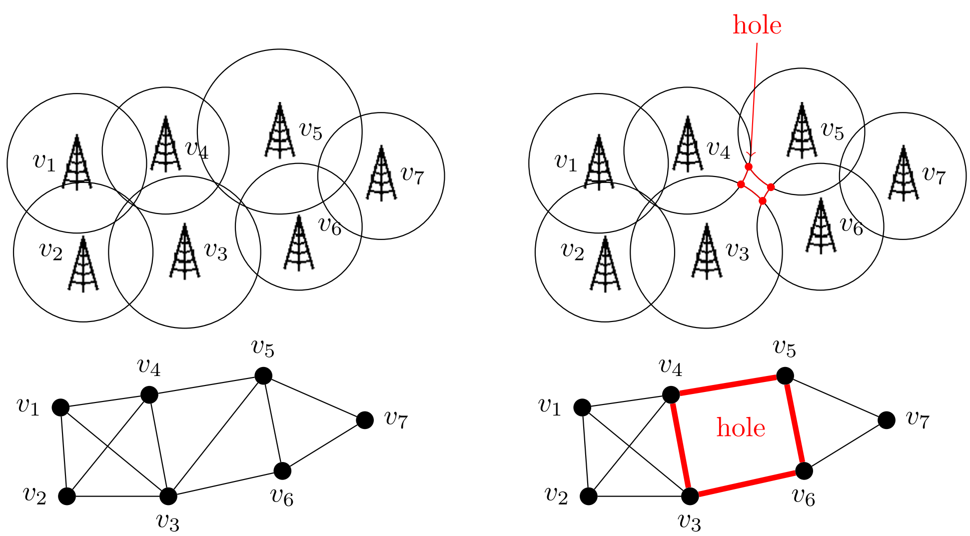


Fig. 1. C-RAN Architecture and Components.



(a) Example of **Full Covered** Network ($\beta_0 = 1$ **and** $\beta_1 = 0$)

(b) Example of Network Containing a **Coverage Hole** ($\beta_0 = 1$ **and** $\beta_1 = 1$)

Fig. 2. Coverage Network Examples.

number of connected components in a network. Moreover, we note by β_1 the number of holes in the plane. β_0 and β_1 are called Betti numbers and they indicate if all cells are connected in one component with no holes (see Fig. 2).

Other methods based on the triangulation of a graph are already proposed in the literature. The objective of these methods is to construct a 2−simplex (triangles) to ensure the full coverage of a network according to the definition of Rips complex (see [4] for instance). Our proposal is close to these approaches, and we describe in the following, the Delaunay triangulation (see [5,6]) to construct 2−simplexes when supposing that antennas have the same coverage radius.

Before providing the Delaunay triangulation definition, it is worth noting that in our approach, the coverage area of each antenna is modeled by circles with variable coverage radius. Nevertheless, in real life, cells have random shape of coverage area which depends on geographic, environmental and network parameters (base station location, transmission power, terrain and artificial structures properties, …). In the literature and for sake of representation and analytical simplicity, approximate approaches are often adopted to design and model the cells' coverage area in cellular networks. In particular, [7–9] used hexagons to model the cells coverage area with no overlap between cells. This approximation is frequently employed in planning and analysing of wireless networks due to its flexibility and convenience. However, since the hexagons are only an idealization of the irregular cell shape, a simpler approach, called circular-cell approximation, is used to model the cell coverage area by circles (see [10–12] for example). The circular-cell approximation is reasonable and very used in the modeling of cellular networks due to its low computational complexity. Hence, some references (see [13,14]) are using this approach to address the network coverage problem, and the authors proposed methods and algorithms that do not converge in acceptable times and do not provide good solutions. Our optimization is using circles to represent antennas coverage areas, and we propose an exact formulation that always provide optimal solutions in negligible times.

Definition 1.1. A triangulation for a set S of points in a plane is a **Delaunay triangulation** if no point in S is inside the circumcircle of any triangle.

In [6], and to guarantee a full coverage hole in a network, a new mechanism is provided and consists in identifying a necessary and sufficient condition for the full coverage hole of a triangle that has no other nodes inside its circumcircle. The time complexity of this method is close to $O(bn)$ where n is the total number of nodes and b represents the number of adjacent nodes in the vinicity of each node. Nevertheless, this proposal is based on a simple mathematical formula that do not address all of the possible scenarios.

Similarly to the Delaunay triangulation, and from a given graph G, we extract a subgraph G_Δ composed by adjacent triangles. The set of nodes of the graph G are the antennas and the edges are constructed as follows:

- There is an edge e_{ij} in the graph G if and only if the Eq. (1) holds.

$$r_i + r_j \geq d_{ij} \tag{1}$$

where r_i and r_j are the radii of antennas i and j respectively, and d_{ij} is the Euclidean distance between i (with the coordinates (x_i, y_i)) and j (with the coordinates (x_j, y_j)) and provided by:

$$d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{2}$$

Let δ_{ij} be the overlapping (interferences) caused by two antennas i and j.

$$\delta_{ij} = r_i + r_j - d_{ij} \tag{3}$$

This overlapping is causing interferences that should be reduced or totally eliminated when considering full network coverage and connectivity. Fig. 3 is illustrating our system model.

The triangulation method used to construct a new graph G_Δ with adjacent triangles can be assimilated to a well known problem noted by the Minimum Weight Triangulation (MWT) problem (see [15] for more details). Our full coverage hole problem is similar to this former when considering connectivity constraints. More details and definitions can be found in Section 2.

1.2. Objectives and contributions

Our contribution consists in proposing a Branch-and-Cut algorithm to solve efficiently the global coverage hole problem in the context of C-RAN. Thus, we propose a mathematical description modeling the problem according to the Delaunay representation which considers that each tuple of three antennas constituting a triangle allows a complete coverage of the area around the three antennas. Our mathematical model describes the convex hull of the discussed problem and allows to reach optimal solutions even for large problem instances. This description is enlarged and specified by adding new valid inequalities and cutting planes to better precise the polytope containing the optimal solution. Thus, the main contributions of our paper are summarized in the sequel:

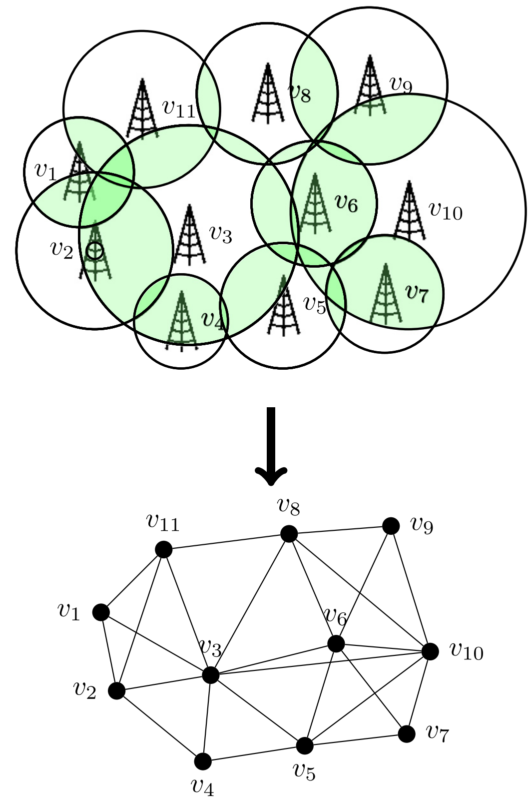


Fig. 3. System Model: Graph construction based on antennas positions and interferences.

- Minimize the number of coverage holes in our cellular network.
- Reduce or eliminate the interferences by adjusting the coverage radius of antennas without creating a coverage hole.
- Rapid (polynomial time) detection of coverage holes.

For the best of our knowledge, this contribution based on polyhedral approaches optimization is new and has never been addressed in the literature to cope with the full coverage hole problem in C-RAN.

1.3. Paper's organisation

The rest of this paper is organized as follows: Section 2 is dedicated to the definition of the MWT problem and its correlation with related works addressing the full coverage hole problem when proposing heuristic approaches that do not attend optimal solutions. In Section 3, we describe the problem statement and discuss network topologies that will be used and then propose a complexity study of our problem. In Section 4, we provide a Branch-and-Cut approach describing the convex hull of the full coverage hole problem and will reinforce this formulation by proposing new families of valid inequalities to accelerate the convergence time to the optimum. Numerical results and performance assessment can be found in Section 5. We conclude this paper in Section 6.

2. Related work

Before addressing and summarizing relevant and closest related works compared to our considered problem, it is important to highlight some papers in the context of Cloud Radio Access Networks when addressing multi-dimensional resource optimization issues. To better get a grasp of resource optimization problems in the context of C-RAN, authors of [16] proposed a Cloud-Based Radio over Optical Fiber Network (noted by C-RoFN) architecture with multi-stratum resources optimization using Software Defined Networks paradigm. In this architecture, optical spectrum and BBU processing resources are optimized jointly to maximize radio coverage when meeting quality of service.

A deep study on multi-dimensional resources integration for service provisioning in cloud radio over fiber networks is provided in [17]. Indeed, the authors of this reference proposed a global optimization when considering together radio frequency, optical network and processing resources leading to maximize radio coverage. A mathematical modeling is then provided and an experimental test bed is used to confirm the efficiency and the feasibility of proposed C-RoFN architecture.

As mentioned before, our problem is close to the MWT problem which consists in finding in a graph G a set of edges of minimum total weight that triangulates the total nodes of G. Moreover, the MWT problem is NP-Hard (see [18] for instance) as we can easily deduce a linear reduction from another NP-Complete problem noted by PLANAR-1-IN-3-SAT (see [19]). Nevertheless, and under certain conditions, the MWT problem has some polynomial variants. In fact, in [20], the authors discussed a polynomial case where the set of points to be triangulated are on a constant number of nested convex hulls. Thus, it is important to describe the total convex hull of the MWT problem to characterize all of the possible solutions. For this, authors in [21] proposed a Branch-and-Cut description of the MWT problem using efficient facets and valid inequalities. This Branch-and-Cut formulation is finally an Integer Linear Programm (ILP) that converges rapidly to optimal solutions only for small and medium instances of the problem. For scalability issues, there are many works proposing approximation algorithms to cope with large MWT problem sizes (see [22], for example).

Many works tackled the coverage hole problem for different communication networks. Authors in [6] proposed a mathematical model based on Delaunay triangulation to detect coverage holes in Wireless Sensor Network (WSN) and found the shortest paths for node movement to heal the holes. The proposed algorithm requires that cells will be identical with the same coverage radius. In C-RAN, cells' radii should be different depending on density of users. In [23] and [24], the authors used a probability method to detect the coverage holes and calculated the smallest size of cells which allows full coverage of the considered network. They also suppose that all cells have the same coverage radius which is not realistic. The authors in [25] proposed an ILP model that maximizes the coverage in WSN. The main limitation is that they discretize the area to be covered into several cells, and each cell is discretized into several points. In [26], the authors proposed a heuristic algorithm to turn off the minimal number of cells without generating coverage holes. They used the simplicial complex which is introduced in [27] and [28] as a representation of coverage topology. In this algorithm, at every time a cell is turned-off, we need to compute the Betti numbers to ensure that the network coverage is maintained. Moreover, turning off cells can not optimize neither network coverage nor overlapping region. In [4], the authors proposed a homology based algorithm to minimize the total consumed power for wireless networks. They used a heuristic approach noted by Simulated Annealing (SA) to find sub-optimal solutions instead of investigating rapid and efficient approaches to attend optimal solutions. The SA algorithm considers all of the cells and adjusts their coverage radii cell by cell. This approach requires to construct the Čech complex and compute their Betti numbers every time the coverage radius of cells has been modified. The authors of this paper do not consider the problem of locating the coverage holes, they only addressed the energy saving problem in wireless networks. In our paper, we propose an optimization model considering multiple objectives as described in Section 1.

3. Problem statement

Before investigating a new mathematical formulation of the global coverage hole problem under interference constraints, we start our analysis by describing, in the following, the network model considered in our optimization and a deep analysis of the problem's complexity.

3.1. Network topology

We consider a cellular network deployed in a large area represented by a set of antennas denoted by $\mathcal{A}$. Each antenna i is defined by its position on the plane and its coverage radius r_i which varies in the range $\left[r_i^{\min}; r_i^{\max}\right]$, where $i = 1, \ldots, |\mathcal{A}|$. We represent our network using an undirected graph denoted by $G = (\mathcal{A}, \mathcal{E})$ where $\mathcal{A}$ and $\mathcal{E}$ are the sets of available nodes and edges, respectively. We also associate to each antenna i an initial value of its coverage radius, noted by r_i^{init}. There is an edge (i, j) between two antennas i and j if condition (1) is met.

The lower part of Fig. 3 shows the undirected graph obtained from the network topology in the upper part of the same Figure.

3.2. Problem complexity

In the following, we discuss the full coverage hole problem's complexity before investigating a Branch-and-Cut solution based on the description of the convex hull of our problem. Thus, we propose the following theorem.

Theorem 3.1. *For an instance of the optimal full coverage hole problem defined above, deciding whether a solution with no violations exist is NP-Complete.*

Proof. To prove this theorem, we will proceed according to the following steps: It is important to recall that to cope efficiently with our problem, we investigated a polyhedral approach describing a set of valid inequalities to attend optimal solutions using a Branch-and-Cut strategy. This polyhedral approach will lead to find an optimal minimum weight triangulation when eliminating totally network interferences represented by the set of edges intersections. This family of valid inequalities is illustrating the main difference between the full coverage hole problem we are addressing in our work and the MWT problem. For sake of clarity, and for a given instance of a weighted graph $G = (\mathcal{A}, \mathcal{E})$, let φ^*_{mwt} be the optimal value of the MWT problem, and $\psi^*_{hole\_cov}$ the optimum found when solving the full coverage hole problem. As our problem is more constrained compared to the MWT problem, then we deduce that $\varphi^*_{mwt} \leq \psi^*_{hole\_cov}$. This implies that, the relaxation of the constraints which consists in eliminating existing interferences (edges intersections) will hold to retrieve an instance of the MWT problem. Indeed, the **optimal** solution of the minimum weight triangulation problem is a **feasible** (not necessarily optimal) solution in the full coverage hole problem instance.

In addition, in 2006, W. Mulzer and G. Rote (see [18]) have proven the NP-Hardness of the MWT problem. Thus, by using the previous linear reduction from our problem to the MWT problem, we conclude that the full coverage hole problem is also NP-Hard. This implies that the decision formulation concerning the existence of no violation solutions of the full coverage hole problem is NP-Complete. □

Our problem is then NP-Complete, and we need rapid approaches to attend optimal solutions in acceptable times. Our proposal is based on the construction of a complete description of the convex hull of the incidence vectors characterizing the optimal solution of the full coverage hole problem.

4. A new efficient optimization algorithm

To solve the full coverage hole problem, we propose a Branch-and-Cut algorithm based on the description of the convex hull of the problem's incidence vectors. This description consists in various families of valid inequalities leading to attend the optimal solution in acceptable times.

4.1. Mathematical formulation

Before introducing our mathematical formulation, we start by providing the variables and parameters that will be used in the sequel.

- We consider our initial graph $G = (\mathcal{A}, \mathcal{E})$ representing the network topology as illustrated by the lower part of Fig. 3. $\mathcal{A}$ is the set of antennas and $\mathcal{E}$ is the set of edges between antennas. Each antenna i has a coverage radius r_i initially instantiated to r_i^{init}. According to formula (1), we populate the graph G (see lower part of Fig. 3 from the upper part of the same Figure).
- Each antenna $i \in \mathcal{A}$ can operate with its own coverage radius r_i which varies in the range $\left[r_i^{\min}; r_i^{\max}\right]$.
- Let x_{ij} be a binary variable indicating if the edge (i, j) is considered in the final solution ($x_{ij} = 1$), or not ($x_{ij} = 0$).
- Let $\mathcal{N}(i)$ be the set of neighborhood nodes/antennas of i. A node j is a neighbor of i only if the condition (1) used with the maximum radii values, is verified.
- Let $\mathcal{I}(i, j)$ be the set of all edges (i, j) that don't intersect with any other edge (l, k).

The objective of the full coverage hole problem is to detect rapidly holes in the network and reduce considerably interferences represented by the overlapping regions measured mathematically by formula (3). These objectives will be reached by optimizing the radii values of the network antennas. This is equivalent to select in the final solution, only couple of antennas with a minimum Euclidean distance guaranteeing the graph connectivity and the network coverage which allows to intuitively reduce the interferences. This objective is given by:

$$\min \Gamma = \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{N}(i)} d_{ij} \times x_{ij} \tag{4}$$

The global or full coverage hole problem has to comply with a number of constraints which will be summarized and mathematically expressed in the following.

Constraints (5) guarantees that each node i has at least two neighbors in the graph (we recall that the objective is to obtain a triangulation meeting connectivity and reduced interferences).

$$\sum_{j \in \mathcal{N}(i)} x_{ij} \geq 2, \forall i \in \mathcal{A} \tag{5}$$

Constraints (6) impose that if an edge (i, j) do not have any intersection in the initial graph (see solid line edges in Fig. 4), then $x_{ij} = 1$ in the final graph (the solution graph). These constraints are mathematically provided by:

$$x_{ij} = 1, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall (i, j) \in \mathcal{I}(i, j) \tag{6}$$

In order to avoid any intersection in the final graph between any two edges (i, j) and (k, l) (see Fig. 5 in which i, j, k, l can be represented by v_1, v_6, v_4, v_5 respectively), we propose the following nonlinear inequality:

$$x_{ij} \times x_{il} \leq x_{jl} + \sum_{k \in \mathcal{N}(i)} x_{ik}, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \tag{7}$$

Constraints (7) are nonlinear as we used the product of two decision variables. We replace them (constraints (7)) by new family of linear inequalities when introducing a new binary variable z_{ijl},

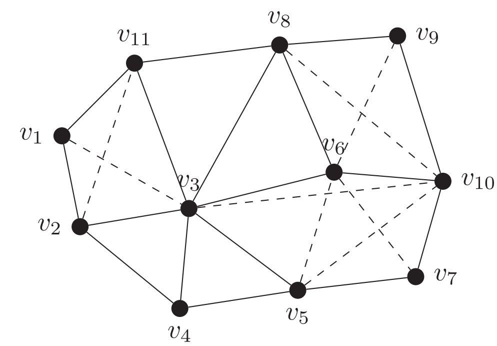


Fig. 4. Each solid line edge (i, j) is necessary in the final graph/solution.

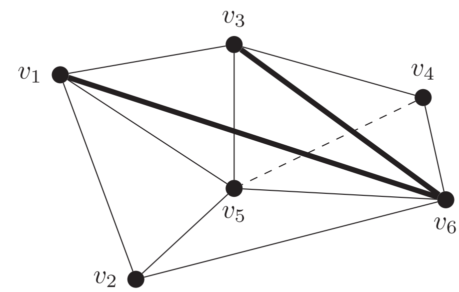


Fig. 5. Example of edge intersection (interference).

such that $z_{ijl} = x_{ij} \times x_{il}$, $\forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j)$. Thus, we obtain the constraints in (8), (9) and (10).

$$z_{ijl} \leq x_{ij} \tag{8}$$

$$z_{ijl} \leq x_{il} \tag{9}$$

$$z_{ijl} \geq x_{ij} + x_{il} - 1 \tag{10}$$

By summing (8) and (9), we obtain:

$$z_{ijl} \leq \frac{1}{2}\left(x_{ij} + x_{il}\right)$$

We finally have three new valid inequalities for the full coverage hole problem, and they are provided by:

$$z_{ijl} \leq x_{jl} + \sum_{k \in \mathcal{N}(i)} x_{ik}, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \tag{11}$$

$$z_{ijl} \leq \frac{1}{2}(x_{ij} + x_{il}), \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \tag{12}$$

$$z_{ijl} \geq x_{ij} + x_{il} - 1, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \tag{13}$$

Our mathematical model is hence characterized by the following Integer Linear Programming:

$$\begin{aligned}
&\min \Gamma = \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{N}(i)} d_{ij} \times x_{ij} \\
&S.T.: \\
&\sum_{j \in \mathcal{N}(i)} x_{ij} \geq 2, \forall i \in \mathcal{A} \\
&x_{ij} = 1, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall (i, j) \in \mathcal{I}(i, j) \\
&z_{ijl} \leq x_{jl} + \sum_{k \in \mathcal{N}(i)} x_{ik}, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \\
&z_{ijl} \leq \frac{1}{2}(x_{ij} + x_{il}), \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \\
&z_{ijl} \geq x_{ij} + x_{il} - 1, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j) \\
&x_{ij}, z_{ijl} \in \{0, 1\}, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j);
\end{aligned} \tag{14}$$

To address larger problem instances and to better describe the convex hull of the full coverage hole problem, we need to investigate new valid inequalities and facets allowing to accelerate convergence time and to find optimal solutions jointly. Thus, we propose to investigate new families of inequalities that are valid for our problem.

4.2. Chordless cycles inequalities

Solving the mathematical formulation provided in (14) allows to obtain optimal solutions for the full coverage hole problem. Nevertheless, and for some initial graph instances, the described convex hull in (14) is missing some solutions that do not contain holes. Thus, to integrate holes detection in our mathematical formulation, we investigate a new facet or valid inequality based on holes detection that should be added to our optimization. These inequalities are based on detecting *Chordless Cycles*, that will be defined in the following.

Definition 4.1. [29] Let G be an undirected graph and let $v_0, v_1, \ldots, v_{k-1}$ be a sequence of k distinct vertices such that there is an edge between v_i and $v_{(i+1) \bmod k}$ $(\forall i = 0, \ldots, k-1)$, and no other edge between any two of these vertices. Then, this sequence is a chordless cycle on k vertices. A hole may be a chordless cycle on four or more vertices.

According to the previous definition, we would like to optimally solve the full coverage hole problem when detecting all the existing holes in the initial graph. For this, we propose the following result.

Theorem 4.1. *For any initial graph G, and for each chordless cycle C in G, such that $|C| \geq 4$, the following inequality (15) is valid for the global coverage hole problem:*

$$x(E(C)) \leq 3 \tag{15}$$

Proof. Let G be an undirected graph and $v_0, v_1, \ldots, v_{k-1}$ (with $k \geq 4$) the set of vertices making a chordless cycle (i.e. a hole) noted by C. Our objective is to detect chordless cycles (holes) and then eliminate them using our optimization. Using the absurd reasoning, we suppose that $x(E(C)) = \sum_{i=0}^{k-2} x_{v_i, v_{i+1}} \geq 4$ which means that our optimization should keep at least 4 edges in C leading to obtain one of the two following cases:

1. A solution with a chordless cycle noted by $\{v_0, v_1, \ldots, v_{k-1}, v_0\}$ which represents a hole
2. A solution with at least two intersecting edges that represent an interference in the final solution

The case 1 is not feasible as our optimization is focusing on eliminating all the existing holes. The second case 2 cannot hold thanks to constraints (7) eliminating intersections and interferences efficiently.

To better understand the proof, we propose a simple and clear example based on the graph of Fig. 6 that contains a chordless cycle $C = \{v_2, v_3, v_5, v_8, v_2\}$ of size 4. We can easily remark that (v_2, v_3), (v_3, v_5) and (v_5, v_8) do not intersect with any other edge in the graph. So, by applying constraints (6), we obtain a solution with $x_{v_2 v_3} = 1$, $x_{v_3 v_5} = 1$ and $x_{v_5 v_8} = 1$. Hence, we discuss two cases on the status of the edge (v_2, v_8) of the chordless cycle C:

- $x_{v_2 v_8} = 1$: In order to eliminate intersections in the final graph/solution, the edges (v_1, v_3), (v_1, v_5), (v_3, v_7) and (v_5, v_7) will be removed ($x_{v_1 v_3} = 0$, $x_{v_1 v_5} = 0$, $x_{v_3 v_7} = 0$ and $x_{v_5 v_7} = 0$) using constraints (7). This means that our final solution has a coverage hole, and this is not desirable.
- $x_{v_2 v_8} = 0$: There is no coverage hole in the final graph/solution (full coverage) while totally eliminating network interferences. In this case $x_{v_2 v_3} + x_{v_3 v_5} + x_{v_5 v_8} + x_{v_8 v_2} \leq 3$, leading to $x(E(C)) \leq 3$. □

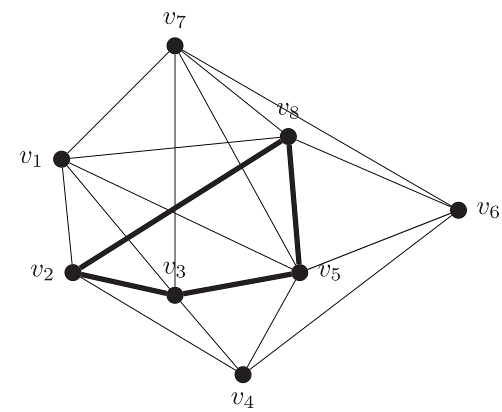


Fig. 6. Example of a chordless cycle (v_2, v_3, v_5, v_8, v_2) of size 4.

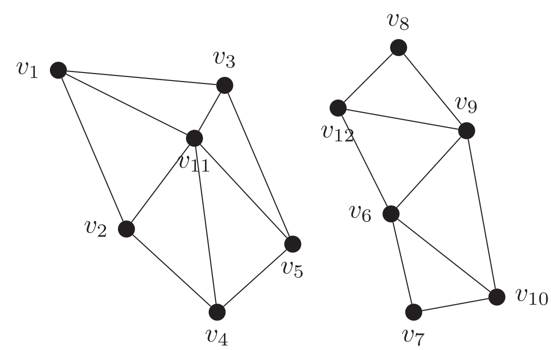


Fig. 7. Example of two connected components (triangulations) creating holes in the final graph.

Separation of chordless cycles inequalities (15)

Thanks to inequalities (15), we guarantee the non existence of holes in our final and optimal solution. This is due to the generation and implication of (15) in the mathematical model (14). Nevertheless, as the number of chordless cycles can be exponential, then we cannot explore all of the existing chordless cycles as this can be time consuming for our optimization.

The separation of inequalities (15) consists in finding chordless cycles C^* violating constraints (15). This is a well known NP-Hard problem (see [19]). Thus, we only explore finding few number of chordless cycles using a heuristic approach (see [29], for instance) that converges in acceptable times when providing sufficient number of violated chordless cycles constraints. We add these facets to our final optimization to eliminate possible holes in the final graph.

4.3. Connectivity inequalities

In addition to the constraints eliminating holes described above, the new mathematical formulation (14) + (15) can lead to find optimal triangulations in a non connected graph (see Fig. 7 representing a triangulation of two connected components). Moreover and by definition, two connected components in the final graph are creating a hole. Thus, we investigate new valid inequalities (facets) to obtain a unique optimal triangulation without holes. This consists to guarantee the connectivity of the final graph. We introduce the following constraints that will be integrated to our mathematical formulation.

Theorem 4.2. *For any initial graph $G = (\mathcal{A}, \mathcal{E})$, and for each subset $S \subseteq \mathcal{A}$, the following inequality (16) is valid to guarantee the network* *connectivity for the global coverage hole problem:*

$$x(\delta(S)) \geq 1 \quad (16)$$

where $\delta(S)$ represents the set of edges with exactly one extremity (or end-point) in S and the other one in the complement set of S (i.e. $\bar{S}$).

Proof. Let a and b two nodes or antennas in the final graph that contains at least two connected triangulations components. For instance, we can imagine that $a = v_3$ and $b = v_8$ as illustrated in Fig. 7. Then, it is clear that the maximum flow or the minimum cut between a and b in the graph of Fig. 7 is zero, as they are separated into two connected components. The objective in our problem is to construct one triangulation with a minimum weight when guaranteeing the connectivity. Thus, we impose a and b to be in the same component. To do this, we simply have to impose that the maximum flow or the minimum cut between this couple of nodes should be greater than 1 (the value 1 is selected to guarantee that there exists at least **one** edge between a and b). Recall that the considered weights in this graph are the actual solution $x_e, e \in \mathcal{E}$ of the full coverage hole problem. By applying connectivity constraints (16) we guarantee that all separated couples of nodes will be jointly on the same and unique connected component. □

Separation of connectivity inequalities (16)

The separation problem of (16) consists in finding the optimal set of nodes (antennas) S^* that violates these constraints. Thus, we investigate all of the possible couple of nodes a and b that are not in the same connected component in the final graph. Next to that, we identify a minimum cut (set of edges) separating a and b, and then impose that the value of this minimum cut (using the weights x) will not exceed 1. Exploring all of the possible sets S violating (16) is NP-Hard as there is an exponential number of possibilities. Thus, we propose to explore only few number of sets that can be found polynomially when solving the minimum cut or maximum flow problem using a well known algorithm such Ford-Fulkerson [30]. In fact, few generations of (16) may be sufficient to guarantee the connectivity of the final graph. To summarize, and by considering all of the described constraints leading to find optimal solutions for the full coverage hole problem, our mathematical formulation is then provided by:

$$\min \Gamma = \sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{N}(i)} d_{ij} \times x_{ij}$$

$$S.T.:$$

$$\sum_{j \in \mathcal{N}(i)} x_{ij} \geq 2, \forall i \in \mathcal{A}$$

$$x_{ij} = 1, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall (i,j) \in \mathcal{I}(i,j)$$

$$z_{ijl} \leq x_{jl} + \sum_{k \in \mathcal{N}(i)} x_{ik}, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j)$$

$$z_{ijl} \leq \frac{1}{2}(x_{ij} + x_{il}), \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j)$$

$$z_{ijl} \geq x_{ij} + x_{il} - 1, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j)$$

$$x(E(C)) \leq 3, \forall C \subseteq \mathcal{A}, |C| \geq 4, C \ \ is \ \ a \ \ chordless \ \ cycle$$

$$x(\delta(S)) \geq 1, \forall S \subseteq \mathcal{A}$$

$$x_{ij}, z_{ijl} \in \{0,1\}, \forall i \in \mathcal{A}, \forall j \in \mathcal{N}(i), \forall l \in \mathcal{N}(i) \cap \mathcal{N}(j); \quad (17)$$

Finally, and to cope with the full coverage hole problem, we use the complete mathematical formulation provided by (17) to propose the Algorithm 1.

5. Numerical results

The performance evaluation of our algorithm, coded in Java, is conducted using an Intel Core CPU at 2.40 GHz with 8 GB RAM. Each initial network represented by a graph comprises a random number of antennas following a Poisson process with a parameter $\Lambda = \lambda \times space\_dimensions$, where λ is varying in the range [0.1; 1], and $space\_dimensions$ are generated according to two essential spaces (5×5 and 10×10). Each antenna, represented by a vertex of this graph, has a radius value initialized to $r_{\max} = 1$ km. The simulation considers the generation of 100 feasible instances for each run. For sake of clarity, we summarize the simulation settings and parameters in Table 1.

For our simulation and experiments, we will use the optimization solver [31] to solve the exact mathematical model (17), and we also compare and benchmark our algorithm to other existing approaches.

Algorithm 1 Full Coverage Hole Algorithm: Branch and Cut approach.

Input: A real telecommunications network (cells with antennas) with given interferences and coverage holes
Output: A full coverage network (no holes) with no interference
- Graph transformation of the real network G: each antenna is a node
- There is an edge between two nodes (antennas) i and j if $r_i + r_j \geq d_{ij}$
- An interference is represented by an intersection of two edges
- Run the Branch and Cut optimization model (17)
- The optimized/obtained network has no holes and no interference

Table 1
Simulation settings and parameters.

Parameters	Values
Density of Antennas	$\lambda \in [0.1; 1]$
Space Dimensions	5×5; 10×10;...
Poisson Parameter	$\Lambda = \lambda \times space\_dimensions$
Number of Antennas	Poisson Distribution $\mathcal{P}(\Lambda)$
Antenna Coordinates	Uniform Distribution $\mathcal{U}(0, space\_dimensions)$
Min Coverage Radius	$r_{\min} = 0.1\,km$
Max Coverage Radius	$r_{\max} = 1\,km$

5.1. Performance metrics and analysis

The algorithm performance assessment is based on the following metrics:

- **Convergence time**: is the time needed by the proposed exact algorithm to find an optimal solution.
- **Interference elimination rate**: is the rate of eliminated interferences such that, and with no loss of generality, we consider an interference as an intersection of two edges in the final graph (the triangulation graph).
- **Coverage hole**: is the network coverage in terms of existing holes in the final solution. Hence, zero holes leads to a **full** coverage hole.

To assess performance of the proposed approach of the full coverage hole problem using the described metrics, we considered a real trace and random instances described as follows:

1. **Random instances**: Networks with an average number of antennas ranging in [7; 100] interval, and an average number of edges in the [18; 456] interval according to the formula (1).
2. **Real trace**: We used a real trace from a coverage cell 4G-LTE of the network operator Orange, in a small area in Paris [32]. This topology is in an area containing 26 4G-LTE antennas with their given geographical positions (coordinates), 94 edges and a maximum radius value of 0.4 km for each antenna.

Table 2
Exact algorithm performance: convergence time to the optimum.

Space	Density	#Antennas	#Edges	#(15)	#(16)	Convergence Time (s)
5 × 5	0.3	7.5	18.81	1	1	0.016
	0.5	12.5	34.63	1.13	1	0.0677
	0.8	20	70.21	5.79	3.45	9.43
	1.0	25	105.4	11.92	10.25	34.31
10 × 10	0.3	30	96.16	6.08	5.81	4.28
	0.5	50	186.21	17.79	12.64	64.47
	0.8	80	298.24	32.64	5.71	67.13
	1.0	100	456.35	63	14.59	139.4

Table 3
Performance comparison : ILP Vs Rips approach.

Space	Density	#Antennas	#Edges	Convergence Time (s)		Interference elimination (%)	
				ILP	RIPS	ILP	RIPS
5 × 5	0.3	7.5	18.81	0.016	0.67	100	88.89
	0.5	12.5	34.63	0.0677	2.51	100	96.42
	0.8	20	70.21	9.43	12.36	100	97.91
	1.0	25	105.4	34.31	34.76	100	**98.01**
10 × 10	0.3	30	96.16	4.28	11.83	100	96.15
	0.5	50	186.21	64.47	**57.38**	100	95.61
	0.8	80	298.24	67.13	**63.91**	100	94.96
	1.0	100	456.35	139.4	**74.9**	100	97.97

5.2. Algorithm's performance evaluation and comparison to the state of the art

Our performance evaluation starts by assessing the execution time needed by the Branch-and-Cut algorithm to find the optimal solution to the full coverage hole problem.

In Table 2, the average convergence time to the optimum remains bellow 35 s and 140 s in the worst case for the scenario with average networks size of 100 antennas. The Branch-and-Cut algorithm scales reasonably well with problem size (number of antennas and edges) and this is due to the efficiency of the added cutting planes provided by formulas (15) and (16) guaranteeing an optimal result with $\beta_0 = 1$ and $\beta_1 = 0$ (a covered network without holes and with a unique connected component). Moreover, this average execution time depends on the average number of added constraints (15) and (16) to our global optimization. As illustrated in Table 2, running and adding these constraints is requiring negligible times thanks to the heuristics approaches deployed to separate them in polynomial time, as mentioned in Section 4.

In the following, we assess the convergence time and interference elimination rate of our algorithm and benchmark them with the solution provided by RIPS complex, a well known method to cope with the network coverage hole problem. As our approach is based on an exact mathematical formulation leading to always attend the optimum, then RIPS approach can be considered as an upper bound to our algorithm.

For small graphs or networks (at most 25 antennas in Table 3), the Branch-and-Cut algorithm has negligible average convergence time compared to the necessary convergence time required by RIPS method. The worst case for these graphs concerns the scenario with 25 antennas in which the ILP necessitates 34.31 s (to converge to the **optimal solution**) compared to 34.76 s for RIPS method to converge to a **feasible solution**.

For larger graphs (between 50 and 100 antennas in Table 3) our algorithm is consuming little more time compared to the convergence time of RIPS method, as we spent time to reach the optimum in the contrary of RIPS method looking only for a feasible solution, and not necessary optimal ones. This is confirmed by the interference elimination rates provided in Table 3.

The interference elimination rate metric is reported in Table 3 and confirms that the Branch-and-Cut algorithm performs better than RIPS approach even for large networks. In fact, our approach is eliminating **totally** the interferences for all the considered scenarios compared to RIPS algorithm that proposes final solutions with remaining interferences and holes. Thus, our proposed approach guarantees the optimality of the found solution in terms of interference elimination and full coverage hole jointly. RIPS method is providing weak network coverage hole and **partial** interference elimination (98.01% as the best result when considering small graphs). Hence, by considering jointly the expected convergence time, the total interference elimination and the full network coverage hole solutions, the Branch-and-Cut approach can be used online by network providers offering connectivity and mobile services to end-users.

In other words, constraints (7) are dedicated to eliminating intersections and they are violated when two edges in the new graph, have a common point of intersection in the graph representation. Constraints (7) are stronger when combined with the other valid inequalities described in the mathematical model (17) which finds an optimal graph with only adjacent triangles (a full covered zone according to the Delaunay definition). Thus, the joint optimization leads to eliminate totally these intersections (interferences) as the used Branch and Cut approach is guaranteeing the optimality (zero interference and no holes).

5.3. Algorithm's performance evaluation for real traces and scalability

The performance assessment would not be complete without addressing the scalability for very large problem instances, and also by applying our algorithm to a real network or graph as shown in Fig. 8. This network is in a small area in Paris, containing 26 antennas, 94 edges and an interference rate equivalent to 80.85%. Note that in our work, and with no loss of generality, an interference is the intersection of two edges in the graphic representation of the network. In this experimentation, we would like to apply our Branch-and-Cut algorithm on the map of Fig. 8 when assessing the three metrics cited above (i.e. Coverage hole, Interference elimination, and Convergence time).

Fig. 9 reveals for the topology in Fig. 8 of reasonable size, the obtained covered network when applying our Branch-and-Cut algorithm which has the advantage of exploring the entire network space at once during optimization. The obtained triangulation in

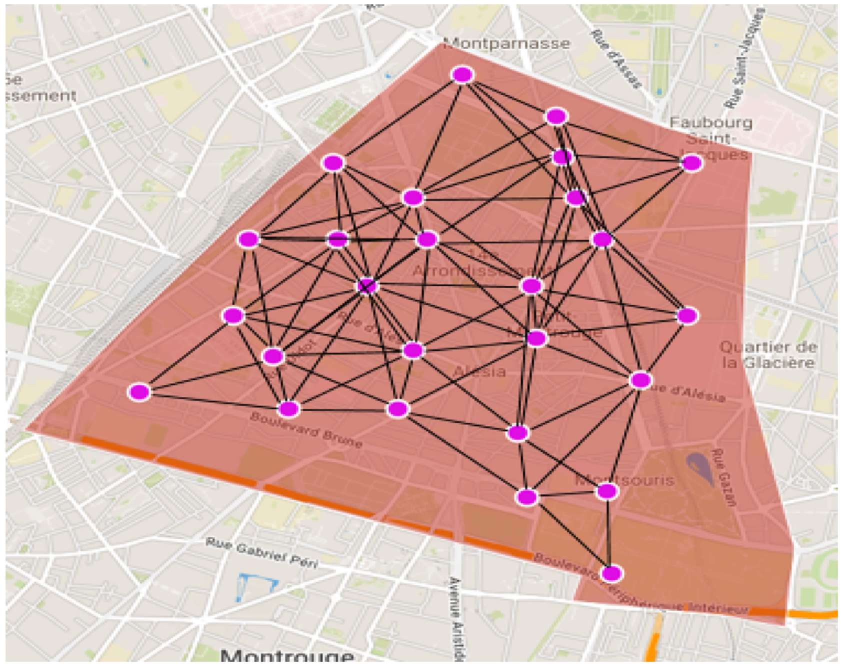


Fig. 8. An Orange 4G-LTE Cell Map: **Before Branch & Cut Optimization**.

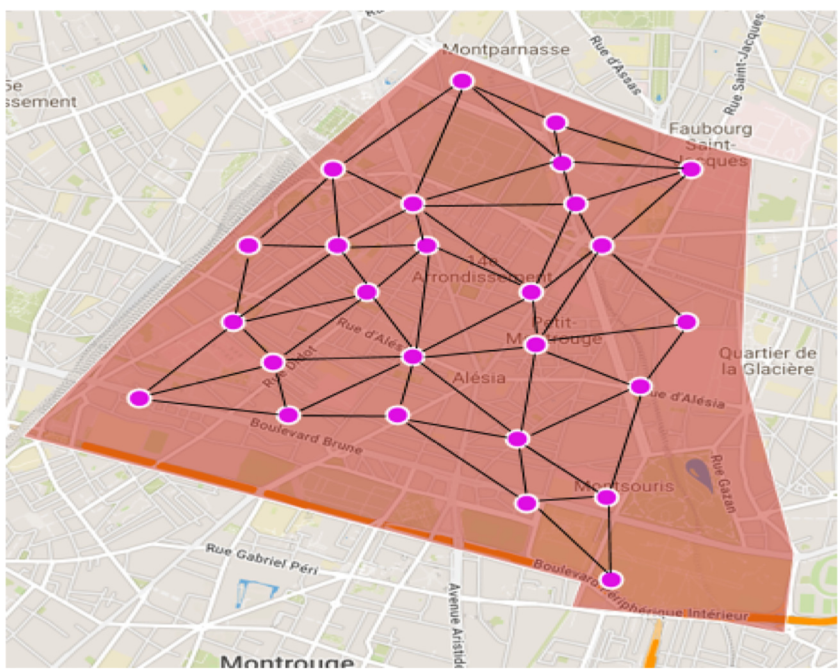


Fig. 9. An Orange 4G-LTE Cell Map: **After Branch & Cut Optimization**.

Fig. 9 is optimal and with no holes leading to a network with a full coverage hole. Our exact algorithm has totally eliminated the existing interferences and reached the optimal solution in less than 1 sec (or exactly in 0.006 s). This real network instance is in fact “easy” to solve using our Branch-and-Cut approach.

To discuss the scalability analysis of our approach, we propose a network instance of 1000 antennas generated as described in Section 5.1. We apply our mathematical formulation provided by (17) and the obtained convergence time is close to 116.88 s for an optimal solution with no holes (full coverage) and no interferences. Note that the selected network/graph instance do not contain chordless cycles and the obtained result is a connected graph (i.e. without many connected components). This allows to avoid generating chordless cycles inequalities (15) and connectivity constraints (16) which can be time consuming when added to our optimization. Indeed, the generation of these cutting planes can be time consuming even if their used separation algorithms are converging in polynomial time, as we can observe it in the previous simulations in Table 2. This explains the necessary convergence time (for a network of 1000 antennas) which is less than the necessary time for a network with 100 antennas in which 63 chordless cycles constraints and 15 connectivity constraints are used to attend the optimum (see Table 2).

6. Conclusion

This paper proposes a Branch-and-Cut algorithm dealing with the full coverage hole problem in the C-RAN context. The proposed algorithm can scale even for large problem instances thanks to the new cutting planes added to our optimization. The performance evaluation is conducted using simulations and a real network map to confront our algorithm to different infrastructures and network topologies. The results of these evaluations reveal the efficiency of our approach that performs consistently well across all evaluations and metrics. This confirms the reliability of our algorithm and the quality of the found solutions.

In the future, we will address two new issues described as follows:

1. We considered in our model circular antenna coverage area. However, in real life, antennas do not have regular shapes and their coverage area depends on geographic, environmental and network parameters. We aim to extend our model by taking into account the irregularity of antenna coverage area to better evaluate the performance of our approach when considering real life constraints.
2. We evaluated the performance of our algorithms using various experimentations, and then, we identified some network variants (cliques with at most four edges) that can be solved optimally in negligible times when the integrity constraints are relaxed. Hence, we will investigate new valid inequalities to characterize polynomial time variants of the coverage network problem.

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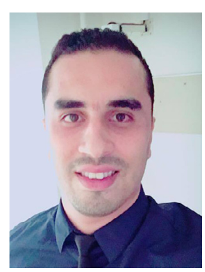

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