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Controlling Default Contagion Through Small-World Networks Analysis

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Abstract

In our increasingly connected business environment, an enterprise's financial downturn can spread to its economic associates, a phenomenon defined as default contagion. In academic research, the default contagion is usually measured by the default dependence degree. This paper utilizes the Copula metric method to capture the default dependence degree, and studies the default dependence degree of listed companies in China through quantitative analysis. Related properties of default dependence of the listed companies are then explored through a simulation method. Results find that the default dependence degrees of listed companies in China present the remarkable characteristics of a small-world network, which are a high cluster and a short path. This indicates that once the core company defaults, the adverse effect will quickly spread to other companies through the network diffusion effect of the financial market. Hence, this study can provide some theoretical basis for the design of a mechanism to prevent and control large-scale default events.

Keywords: Copula; default contagion; default correlation; small-world networks

1. Introduction

Default contagion is the impact that the default of one debtor has on other creditors. With the rapid development of the financial industry, enterprises are becoming closely linked in unprecedented ways. Due to the increasing correlation between modern enterprises, an enterprise's financial downturn can spread to its economic related

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partners. Once a firm defaults, the financial influence can spread rapidly to other firms, not only through the financial network, but more often through the amplification effect produced by the clustered network. The subprime mortgage crisis in 2008 was the result of the default contagion, increasing scholarly attention to the default problem.

In general, academic research uses default dependence degree to measure default contagion. Statistically, when two default events are independent, the probability of their occurrence can be expressed as the product of the probability of each event. However, the occurrence of default events among enterprises is usually related to each other. Therefore, the most commonly used measurement method is the linear correlation analysis. In addition to linear correlation, other methods including non-linear correlations and more complex time-relations, such as causal effect, are also used to measure default contagion.

2. Previous studies

Davis and Lo[1] first introduced default contagion in the credit risk portfolio model. Their studies confirmed that a firm's bankruptcy affects the stock returns of associated non-default firms. There are two views as to why default spreads between firms. One is the causal effect, for example, the counterparty risk. A corporate default will lead to the occurrence of a series of defaults, which is more distinct during an economic down turn. for a more in-depth analysis, reference the discussion between Allen and Gale[2]. Another view is the information effect, for example, belief revision. The information effect has a small initial impact, only affecting limited financial institutions and economic industries, but with deteriorating credit quality and subsequent rises in infection, other industries will be affected, Schobucher[3] provides a detailed demonstration of this process).

Currently, research on default contagion modeling is relatively absent, and is typically measured by linear correlations and the Copula nonlinear correlation. The method of measurement based on linear correlation is usually directly described by Pearson correlation coefficients, such as studies by Lucas[4], Kealhofer[5], Hull[6], White, and Zhou[7], which adopt this kind of method. This metric is simple and easy to calculate, but the analysis of Embrechts, Llindskog and Mcneil[8], notes some limitations, including the ability to describe only the part of the linear correlation, leaving the part of the nonlinear correlation powerless. Thus, domestic and foreign scholars are now turning to study default dependence based on the Copula function measurement, and link marginal distribution with the default-related structure using the Copula function. Since Li[9] first applied Copula to the research of default correlations, this method has absorbed the attention of many scholars. He proves that in the Credit Metrics model, when defining the dependent structure of the return on assets, the correlation coefficient of asset return is typically used as the dependence parameter of the normal Copula. He also applies normal Copula to pricing the swaps and contract of credit default.

Subsequently, many scholars have studied the default dependency problem and discussed the dependency structure of default risk models through Copula. Mashal and Naldi[10] use a t-Copula to model the structure of default dependency. Giesecke[11] use the Copula approach to study issues with incomplete default-related information among enterprises. He uses Gumble Copula and Clayton Copula to study the measurement of default risk of buyers and suppliers, and greatly simplifies the modeling process by separating the models of dependency structure and marginal distribution. Wong[12] discussed the general theory of correlated default.

3. Measurement of default contagion

In the Merton model, if the company value $V_i(t)$ is less than the default threshold $D_i(t)$, a default will occur. The probability of default is

$$DP_i(t) = P(V_i(t) \le D_i(t)) \text{ , and } DP_i(t) = \phi(d_{2,i}(t))$$

and $\phi(\bullet)$ obey the standard normal distribution. Assuming that n assets obey joint normal distribution with the correlation coefficient matrix R, the distribution function of the joint probability of default is

$$DP(t) = P(V_1(t) \le D_1(t), \ldots, V_n(t) \le D_n(t)) = \phi_R(-d_{21}(t), \ldots, -d_{2n}(t))$$

By bringing $d_{2,i} = \phi^{-1}(DP_i(t))$ into the distribution function, we can obtain

$$DP(t) = \phi_R\left\{\phi^{-1}(DP_1),\ldots,\phi^{-1}(DP_n)\right\},$$

which means that the joint probability of the default is the function of the marginal distribution of the default. It can be expressed by the Copula function as

$$C(u_1,\ldots,u_n) \triangleq \phi_R\left\{\phi^{-1}(u_1),\phi^{-1}(u_2),\ldots,\phi^{-1}(u_n)\right\}.$$

The common Copula functions are the multivariate normal Copula function, the multivariate T-Copula function and the Archimedes Copula function. The expression of the multivariate normal Copula distribution function and the density function are, respectively: $C(u_1,\ldots,u_n) \triangleq \phi_R\left\{\phi^{-1}(u_1),\phi^{-1}(u_2),\ldots,\phi^{-1}(u_n)\right\}$ and

$$C(u_1,u_2,\ldots,u_n;\rho) = |\rho|^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\varsigma'(\rho^{-1}-I)\varsigma\right),$$

where ρ is the symmetric positive definite matrix whose elements of the diagonal are 1; $|\rho|$ represents the value of the determinant of the matrix ρ, $\phi_\rho(\bullet,\bullet,\cdots,\bullet)$ represents the multivariate standard normal distribution function whose correlation coefficient matrix is ρ; $\phi^{-1}(\bullet)$ is the inverse function of the standard normal distribution function $\phi(\bullet)$; $\zeta = \left(\zeta_{1,}\zeta_{2,\ldots,}\zeta_N\right)'$; $\zeta_n = \phi^{-1}(u_n)$, n=1,2,…,N; I is the identity matrix.

The calculation process will be demonstrated as follows, utilizing Fenglezhongye, TongJunGe and Zhonghejidian (stock codes, respectively, are 000591, 000713and 000925) as examples.

Firstly, the GARCH model is used to establish the marginal distribution estimation function of the asset value of the sample enterprises. Second, the appropriate Copula function is selected. Generally, the multivariate Copula normal function is used for the Metorn model, based on the above introduction. Next, parameters of the Copula function are estimated, most commonly using the Kernel density estimation method. Kernel is a kind of function, which, due to its smoothing attribute, is often used as a structural unit to obtain the amount to be estimated. It is able to estimate the Copula function smoothly and differentially, and need not assume a priori parameter when establishing the dependence structure among the marginal distribution. Kernel density function k_{ij} is bounded and symmetric on R, and satisfies $\int_R k_{ij}(x)dx = 1$. Its original function is

$$K_i(x,h) = \sum_{j-1}^{n} k_{ij}\left(\frac{x_j}{h_j}\right),$$

where $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$; h represents the step size or smoothing parameter. It is a diagonal matrix with the elements $\{h_j\}_{j=1,2,\ldots,n}$ whose determinant is $|h|$, where h_j is the positive function of T, when satisfying

$$T \to \infty, \quad |h| + \frac{1}{T|h|} \to \infty.$$

Then, the marginal probability density function of Y_{jt} at the point y_i is

$$\hat{f}_j\left(y_j\right) = \frac{1}{Th_j}\sum_{t=1}^{T} K_j\left(\frac{y_j - Y_{jt}}{h_j}\right),$$

and the estimator of the joint probability density function of Y_t at the point

$$y=(y_1,y_2,...,y_n)' \text{ is } \hat{f}_j(y_j)=\frac{1}{Th}\sum_{t=1}^{T}K(y-Y_t,h)=\frac{1}{T|h|}\sum_{t=1}^{T}\prod_{j=1}^{n}k_j\left(\frac{y_j-Y_{jt}}{h_j}\right).$$

Hence, the estimator of the marginal cumulative distribution function of Y_{jt} at different points y_{ij} is $\hat{F}_j(y_{ij})=\int_{-\infty}^{y_{ij}}\hat{f}_j(x)dx$, and the estimator of the joint cumulative distribution function of Y_t at the point

$$y=(y_1,y_2,...,y_n)' \text{ is } \hat{F}(y)=\int_{-\infty}^{y_1}\int_{-\infty}^{y_2}...\int_{-\infty}^{y_n}\hat{f}(x)dx.$$

Given the single Gauss kernel function $k_{ij}(x)=\frac{1}{\sqrt{2\pi}}exp\left(-\frac{x^2}{2}\right)$,

we can then get

$$\hat{F}_j(y_j)=\frac{1}{T}\sum_{t=1}^{T}\Phi\left(\frac{y_j-Y_{jt}}{h_j}\right),\ \hat{F}(y)=\frac{1}{T}\sum_{t=1}^{T}\prod_{j=1}^{n}\Phi\left(\frac{y_j-Y_{jt}}{h_j}\right),$$

where $\phi(\bullet)$ represents the cumulative distribution function of the random variable, obeying the standard Gaussian distribution.

Secondly, at the point u_i, which represents the asset value of the specific listed corporations, the estimator of the Copula function can be obtained with the insertion method. Inserting the estimator of the marginal cumulative distribution inverse function $\hat{F}_j^{-1}(u_{ij})$ into the joint cumulative distribution function

$$\hat{F}(\zeta_i), \text{ i.e. } \hat{C}(u_i)=\hat{F}(\hat{\zeta}_i),\ \hat{\zeta}_i=\hat{F}_j^{-1}(u_{ij}), \text{ thus generates } \hat{C}(u_i)=\hat{F}(\hat{F}_j^{-1}(u_{ij})).$$

Then, the estimator $u(\bullet)$ of the marginal distribution function of asset value at the point

$$V_n \text{ is } u_n=\frac{1}{T}\sum_{t=1}^{T}\Phi\left(\frac{V_n-V_n^{(t)}}{\hat{h}}\right).$$

Thus, the marginal distribution function of asset value is shown in Figure 1.

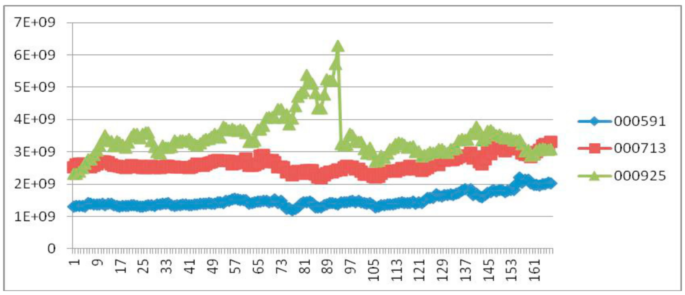

Fig. 1. The marginal distribution function of asset value

As shown, the overall trend of the assets of the three firms u_1,u_2,u_3 remains consistent, and overall maintains a high degree of correlation, although there are some divergences in the middle and upper parts.

Then, the optimal Copula function should be chosen. Through nonparametric kernel density estimation, the estimator of the Gaussian kernel Copula function is calculated as

$$\hat{C}(u_i) = \frac{1}{T\,|\,h\,|}\sum_{t=1}^{T}\prod_{j=1}^{n}\Phi\left(\frac{\hat{F}^{-1}(u_{ij}) - Y_{jt}}{h_j}\right).$$

The joint distribution function of asset value can be reckoned from the assets of the three firms, which is

$$\hat{F}(V) = \frac{1}{T}\sum_{t=1}^{T}\prod_{n=1}^{N}\Phi\left(\frac{V_n - V_n^{(t)}}{\hat{h}}\right),$$

where $T = 243$ and $N = 3$. Parameters of the common Copula function can be estimated by maximumizing the likelihood estimation and calculating the corresponding Copula function value C_k. Then, calculate the distance

$$d_k = \sqrt{\sum_{t=1}^{T}\left(\hat{F} - C_k\right)^2}$$

between the joint density functions of $\hat{F}(y)$ and C_k.

Finally, according to the principle of minimum distance, the function with a minimal distance d_k should be selected as the optimal Copula function. The estimation results of the three firms above by five kinds of Copula functions, Gaussian Copula, T-Copula, Clayton-Copula, Gumbel-Copula and Frank-Copula, are shown in Table 1 below.

Table 1. The estimation of Copula

Type of Copula	Parameter estimation	Maximum likelihood estimation	d_k
Gaussian	$\begin{bmatrix}1,0.948,0.878\\0.948,1,0.895\\0.878,0.895,1\end{bmatrix}$	403.27	6.376
T	$\begin{bmatrix}1,0.950,0.882\\0.950,1,0.898\\0.882,0.898,1\end{bmatrix}_{,34}$	414.78	6.654
Clayton	5.843	562.57	4.112
Gumbel	0.342	348.65	9.847
Frank	45.372	270.69	16.004

As is shown in Table 1, Clayton is the optimal Copula function, and therefore the Clayton-Copula was selected to establish the relationship of estimation between the default dependence coefficients of the three firms. Due to the equivalence of the asset value of the Copula structure and default Copula structure, the optional Copula structure maybe utilized to describe the default dependence structure according to the asset value, and to calculate the joint default probability of the firms

$$p_{12} = C(p_1, p_2),\ p_{13} = C(p_1, p_3),\ p_{23} = C(p_2, p_3).$$

Therefore, the default dependence degree can be calculated by

$$\rho = \frac{p_{12} - p_1 p_2}{\sqrt{p_1(1-p_1)\,p_2(1-p_2)}}.$$

4. Networks analysis of default contagion

Taking a single enterprise as a node, the edges between the each node represent the relationship of default between those enterprises. In calculation, if the default dependence degree of two companies is negatively correlated, the default of one company will do not affect the default of another company. Similarly, if the default dependence degree of two companies is 0, their default contagions will not affect each other either. So, only the mutually reinforcing effect of default between companies, positive correlations, is considered here. This study is conducted under the assumption that the links between each enterprise are bidirectional and symmetrical, that is, that the edges of the network are undirected and the weights of all connections are 1.

According to the discussion about the default dependence degree above, we can conclude that the Copula default dependence degree reflects both the linear and nonlinear relationships of default, and thus the probability of the reconnection of broken bonds between each node can be mapped through the Copula default dependence degree, where it is assumed that $P = 1 - |\rho|$.

According to the assumption above, we can construct a network model of enterprise default. Here, we do a simulation experiment on its properties, based on the network model. Firstly, we take the listed companies of a financial industry, excluding 100 companies, as the nodes, to study the property of evolution. Take $P = 1 - |\rho| = 0$ as the probability of the original reconnection of a broken bond, where the connection between two companies is established only when the default dependence degree is over 1. Then the original network can be seen as 100 isolated points, with an average path length of $L \to \infty$ and the cluster coefficient $C \to 0$ .Then, gradually increase the length by 0.001 each time. The average lengths of path and cluster coefficients of the different probabilities of broken bond reconnections are shown in Figure 2.

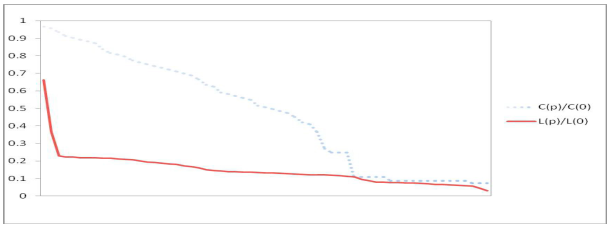

Fig. 2. The change process of broken bond reconnections

The fully connected network can be constituted only when $P >= 0.39$, $L \approx 38$, and $C \approx 0.003$, that is, when the average length of path is high and the clustering coefficient is low, consistent with the properties of the regular network. From Figure 2, we can see that with an increase in the probability of the reconnection of a broken bond, L drops quickly, while the C drops relatively slowly, which is in accordance with the small-world properties.

Take $P = 1 - |\rho| = 1$ as the probability that broken bond reconnection is terminated, that is, the connection of two companies is established as long as the default dependence degree is over 0. Through calculations, the average length of path of the network is $L \approx 1.14$ and the cluster coefficient is $C \approx 0.007$ .It can be observed that, generally, almost every two companies are connected with each other and the central degree is poor, which is consistent with the property of a random network.

Finally, we continue to study the relationships between the default contagions of all enterprises. Based on the default dependence degree of 1697 listed companies, excluding financial enterprises in 2009, we constructed the default network with the method of network construction mentioned above. According to this method of network construction, we took 97 companies as the original nodes, put $P = 1 - |\rho| = 0.7$ as the probability of critical reconnection of the broken bonds, and randomly added 50 enterprises each time as new nodes.

We can find that $L \sim \ln(N)$ through the examination of x^2, which better verifies that the default contagion of listed companies represent the properties of a small-world network. Hence, we can conclude that the default between listed companies is highly correlated. The default of one company may quickly spread to another company through the financial market, and this kind of spread accords with the properties of a small-world network.

5. Conclusion

The current study discusses the default contagion problem from a brand-new angle of view, and studies the default contagion among enterprises, with the method of analyzing the dependent network. Through a method of simulation, results verify that the default contagion among enterprises satisfies the property of small-world networks. This supplies a new method of research for the study of the default contagion of listed corporations, and provides some theoretical basis for a mechanism design to prevent and control large-scale default events.

The research and demonstration of this paper are based on the sample data of listed corporations, and lacks a discussion about the default contagion of small and medium-sized enterprises, due to data limitations. The equivalent metric method of asset dependence is still used to measure the default contagion in this paper, but the direct causes of default contagion are spread throughout the capital chain. The ability of data of funds flow to be obtained between enterprises directly would construct a network of default contagion, and lead to great break-through in the study of property.

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