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Research on Fuzzy Order Variable Precision Rough Set Over Two Universes and Its Uncertainty Measures

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Abstract

Aiming at the problem that the U×V-type double-domain rough set can't deal with fuzzy data in the past, this paper firstly established the variable precision rough set model on the basis of U×V-type double-domain rough set,and made an in-depth study on the upper approximation set and the lower approximation set of this model.Then the variable precision rough set model of U×V-type double-domain fuzzy order information system is established by introducing the U×V-type double-domain fuzzy dominance relation, and its related properties are discussed.The uncertainty measures are studied by defining the U×V-type double-domain fuzzy order precision rough set roughness and rough entropy. The conclusions are that as the precision threshold increases, its roughness and roughness entropy decrease gradually. These conclusions are verified by an example, which provides a theoretical basis for further revealing the uncertainty measure law of U×V-type double-domain fuzzy order variable precision rough set.

Keywords:U× V-type double-domain rough set; fuzzy ordered information system; rough entropy

1. Introduction

Rough set theory was proposed by Polish mathematician Pawlak in 1982, which was a mathematical tool that could analyze and process inaccurate, incomplete, ambiguous, hesitant and fuzzy data in practical applications [1-2].The core of its application is to derive a pair of approximation operators from the approximate space, namely the

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upper approximation set and the lower approximation set [3], and has been successfully applied in many aspects [4-5].

With the rapid development of the real society, the complex data acquired in many fields has made the Pawlak rough set insufficiency in many aspects. For example, the object in question is always on the same domain set, but the problem to be solved in reality is cannot be described by a domain.Another example is that the knowledge or concept that people come into contact with in practical problems is vague and uncertain, and knowledge or concept cann't be described with precise knowledge, such as medical diagnosis problems, one disease corresponds to multiple symptoms, and one symptom is manifested in multiple diseases. This problem obviously cannot be described by a domain domain, and a precise definition cannot be used to express the severity of diseases and conditions.Therefore, many scholars have established a rough set model of two universes by extending the single domain to the double-domain [6-9], and some scholars have extended the exact set to the fuzzy set and established the fuzzy rough set model [10- 11].With the rapid development of artificial intelligence ,machine learning and the continuous generation of complex data in many fields, the demand for double-domain and multi-disciplinary information systems and rough set theory has become increasingly prominent.

The uncertainty measures of information system and rough set are important measures to study its information structure from the perspective of information, including information entropy, rough entropy, information granularity and knowledge granularity. According to the size relation of the granularity in the classical rough set approximation space and the relations in different information systems have different thicknesses, many scholars have used the above metrics to study the uncertainty measurement of information systems.In [12], the uncertainty of the double-domain rough set of information system classification ability is studied. It mainly includes the definition of information entropy and information granularity and the proof of properties. The conclusion is that with the thinner of the binary relation R from U to V, the information entropy becomes smaller, and the information granularity becomes larger.Literature [13] proposed a definition of knowledge granularity for incomplete information systems in rough set theory, and studied the relation between knowledge granularity and information entropy, provided a feasible method for knowledge measurement and knowledge evaluation.In [14], the parameters are introduced to express the degree of superiority for the situation that the dominance relation of the disjunction-type set value order information system is too loose. The paper also discusses information entropy and knowledge granularity uncertainty.Literature [15] analyzed the theoretical model of rough set, fuzzy set and entropy space through the uncertainty measurement of knowledge in granular computing. The literature also described the knowledge granularity in a unified way, and proposed the relation between knowledge uncertainty measure and knowledge granularity.In addition to the above-mentioned methods for measuring the uncertainty of knowledge in information systems, the literature [16] constructed a distance space from the perspective of knowledge distance, and discussed the knowledge relation, knowledge roughness and rough entropy in the knowledge space of information systems. It provided a new understanding of the discovery of information system knowledge roughness.The literature [17] defined the concept of combined entropy and combined granularity based on the compatible rough set model, and studied the properties and laws between them.Rough entropy is effective methods to measure the uncertainty of information systems. The advantage is that they have easy understanding of information gain. However, the current literature reflects the uncertainty measurement of single-domain information system and rough set model.

The $U\times V$-type double-domain fuzzy order variable precision rough set is taken as the research objec in this paper. On the basis of in-depth study of the article model, the related properties and theorems are discussed.We also uses rough entropy to study the uncertainty.

The paper is organized as follows.Section 2 introduces the concept of $U\times V$-type double-domain variable precision rough sets.Section 3 defines the $U\times V$-type double-domain fuzzy order information system and its dominant relation, and defines the $U\times V$-type double-domain fuzzy order precision rough set. The model discusses its properties and theorems.In Section 4, the metrics of rough entropy is defined on the $U\times V$-type double-domain fuzzy order precision rough set model and its properties and conclusions are studied. Section 5 verifies the properties and theorems by an example; Section 6 summarizes the full text.

2. Preliminaries

In this section, we will introduce the equivalence relation on the double-domain, the $U\times V$-type double-domain rough set, and the $U\times V$-type double-domain variable precision rough set.

Definition **1.**[6,7] Let U and V be two non-empty finite universes, R_1 and R_2 are the equivalence relations on U and V, respectively.For $(x,y)R(x',y')$,only when xR_1x' and yR_2y' are established, R is said to be the equivale-nce relation of $U\times V$ induced by R_1 and R_2, let $[(x,y)]_R$ denotes the equivalence class of relation R that contai-ns (x,y).

Definition **2.**[6,7] Let U and V be two non-empty finite universes, R is said to be the equivalence relation of $U\times V$ induced by R_1 and R_2,and ($U\times V,R$) be approximate space of double-domain. For any $X\times Y\subseteq U\times V$,we can describe $X\times Y$ by a pair of lower and upper approximations defined as follows:

$$\overline{R}(X\times Y)=\cup\{[(x,y)]_R \mid [(x,y)]_R\cap X\times Y\neq\Phi\} \tag{1}$$
$$\underline{R}(X\times Y)=\cup\{[(x,y)]_R \mid [(x,y)]_R\subset X\times Y\} \tag{2}$$

Definition **3**. Let ($U\times V,R$) be a double-domain approximation space,for any $X\times Y\subseteq U\times V$, $[(x,y)]_R$ denotes the equivalence class of relation R that contains (x,y).If there is $(e,s)\subseteq X\times Y$ for each pair of $(e,s)\subseteq[(x,y)]_R$, we call $[(x,y)]_R$ contained in $X\times Y$, which is recorded as $X\times Y$.

Let $e([(x,y)]_R,X\times Y)=\begin{cases}1-\left|[(x,y)]_R\cap X\times Y\right|/\left|[(x,y)]_R\right|,\left|[(x,y)]_R\right|>0\\ 0,\left|[(x,y)]_R\right|=0\end{cases}$,

Where $|\sim|$ is the cardinality of the set, and $e([(x,y)]_R/X\times Y)$ is the relative misclassification rate of set $[(x,y)]_R$ on set $X\times Y$.The implication is that if the element of $[(x,y)]_R$ is divided into the set $X\times Y$, the proportion of the classification error is $e([(x,y)]_R/X\times Y)\times 100\%$. Let the precision control parameter $\beta\in[0,0.5)$, and most inclusion relations are defined as, $[(x,y)]_R\overset{\beta}{\supseteq}X\times Y\Leftrightarrow c(X,Y)\leq\beta$, and for any $X\times Y\subseteq U\times V$,the lower and upper approximations of $X\times Y$ about the approximation space ($U\times V,R$) are as follows, respectively:

$$\overline{R_\beta}(X\times Y)=\cup\{[(x,y)]_R\in U\times V/R \mid e([(x,y)]_R/(X\times Y))<1-\beta\} \tag{3}$$
$$\underline{R_\beta}(X\times Y)=\cup\{[(x,y)]_R\in U\times V/R \mid e([(x,y)]_R/(X\times Y))\leq\beta\} \tag{4}$$

Where $\overline{R_\beta}(X\times Y)$ denotes a set of $X\times Y$ whose misclassification rate is less than $1-\beta$, $\underline{R_\beta}(X\times Y)$ denotes that the misclassification rate of $X\times Y$ is not greater than β, and for $\overline{R_\beta}(X\times Y)=\underline{R_\beta}(X\times Y)$,and we call $X\times Y$ is accurate at the error classification level of β, otherwise it is rough.

3. *U*× *V*-Type Double-Domain Fuzzy Order Variable Precision Rough Set

Definition **4.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy information system,where

$F(U)=\{x_1,x_2\cdots,x_m\}$, $F(V)=\{y_1,y_2\cdots,y_n\}$ are two non-empty finite fuzzy sets, each $x_i(1\le i\le m)$ is an object of $F(U)$, and each $y_j\ (1\le j\le n)$ is an object of $F(V)$, $<x_i,y_j>\in F(U\times V)$. $AT=\{a_1,a_2\cdots,a_q\}$ is a non-empty finite set of attributes, $V_a\ \in[0,1]$ is a set of information system attribute values, $F=\{f:F(U\times V)\to V_a,a\in AT\}$,let $\tilde{R}=\{f<x_i,\ \ y_j>|<x_i,y_j>\in F(U\times V)\}$ then $\tilde{R}$ represents a fuzzy relation of $F(U\times V)$ to attribute set AT. we establish an dominant relation on some attribute values of the fuzzy information system, we classify the attribute as a criterion a-nd call the information system a fuzzy order information system.

Definition **5.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy order information system, $A\subseteq AT$,let the dominant relation of attribute set A as $\tilde{R}_A^{\ge}=\{(<x_i,y_j>,<x_m,y_n>)\in F(U\times V)\mid f(<x_i,y_j>,a)\ge f(<x_m,\ x_n>,\ a),\ \forall a\in A\}$, $f(<x_i,y_j>,a)$ denotes the degree of ambiguity of $<x_i,y_j>$ on attribute a , and the dominant class is $[<x_i,y_j>]_A^{\ge}=\{<x_i,y_j>\in F(U\times V)\mid(<x_i,y_j>,<x_m,y_n>)\in\tilde{R}_A^{\ge}\}$.Note $F(U\times V)/\tilde{R}_A^{\ge}$ is a classification of the set of information system objects on the attribute set A , $F(U\times V)/\tilde{R}_A^{\ge}=\ \{[<x_i,y_j>]_A^{\ge}\mid\ <x_i,\ y_j>\in\ F(U\times\ V)\}$ .For any $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$,the lower and upper approximations of $X\times Y$ about dominant relation $\tilde{R}_A^{\ge}$ are as follows, respectively:

$$\underline{\underline{\tilde{R}_A^{\ge}}}\ (\tilde{X}\times\tilde{Y})=\{<x_i,y_j>\in F(U)\times F(V)\mid[<x_i,y_j>]_A^{\ge}\subseteq(\tilde{X}\times\tilde{Y})\}\tag{5}$$

$$\tilde{R}_A^{\ge}\ (\tilde{X}\times\tilde{Y})=\{<x_i,y_j>\in F(U)\times F(V)\mid[<x_i,y_j>]_A^{\ge}\cap(\tilde{X}\times\tilde{Y})\ne\Phi\}\tag{6}$$

Theorem **1.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy order information system,for any $B_1,B_2\subseteq AT$,then the f-ollowing statements hold:

(1)For any $B_1,B_2\subseteq AT$,there are $\tilde{R}_{B_2}^{\ge}\supseteq\tilde{R}_{B_1}^{\ge}\supseteq\tilde{R}_{AT}^{\ge}$.

(2)For any $B_1,B_2\subseteq AT$,there are $[<x_i,y_j>]_{B_2}^{\ge}\supseteq[<x_i,y_j>]_{B_1}^{\ge}\supseteq[<x_i,y_j>]_{AT}^{\ge}$.

Proof**.** (1)For any $(<x_i,y_j>,<x_m,y_n>)\in\tilde{R}_{B_1}^{\ge}$,then for any $a\in B_1$, $f(<x_i,y_j>,B_1)\ge f(<x_m,y_n>,B_1)$,for $B_2\subseteq B_1$,then $f(<x_i,y_j>,B_2)\ge f(<x_m,y_n>,B_2)$, $(<x_i,y_j>,<x_m,y_n>)\in\tilde{R}_{B_2}^{\ge}$,so, $\tilde{R}_{B_1}^{\ge}\subseteq\tilde{R}_{B_2}^{\ge}$,the same can be pro-ved, when $B_1\subseteq AT$,then $\tilde{R}_{AT}^{\ge}\subseteq\tilde{R}_{B_1}^{\ge}$ .Therefore,for $B_2\subseteq B_1\subseteq AT$,then $\tilde{R}_{B_2}^{\ge}\supseteq\tilde{R}_{B_1}^{\ge}\supseteq\tilde{R}_{AT}^{\ge}$.

(2)The proof process is the same as (1).

Definition **6.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy order information system,for any $B_1,B_2\subseteq AT$,there is $[<x_i,y_j>]_{B_1}^{\ge}\subseteq[<x_i,y_j>]_{B_2}^{\ge}$,it is said that the set $F(U\times V)/\tilde{R}_{B_1}^{\ge}$ that satisfies the dominant relation class is thi-nner than $F(U\times V)/\tilde{R}_{B_2}^{\ge}$, and is recorded as $F(U\times V)/\tilde{R}_{B_1}^{\ge}\subseteq F(U\times V)/\tilde{R}_{B_2}^{\ge}$.

Definition **7.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy order information system, $A\subseteq AT$, $\tilde{R}_A^{\ge}$ is the dominant relation on the fuzzy order information system.For any $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$,let the precision control parameter $\beta \in [0,0.5)$, where the misclassification rate of dominant class $[<x_i,y_j>]_A^{\geq}$ on $X \times Y$ is $e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) = \begin{cases} 1-\sum_{(s,t)\in[<x_i,y_j>]_A^{\geq}} <\tilde{X}(s),\tilde{Y}(t)>/q\left|[<x_i,y_j>]_A^{\geq}\right|, \left|[<x_i,y_j>]_A^{\geq}\right|>0 \\ 0, \left|[<x_i,y_j>]_A^{\geq}\right|=0 \end{cases}$, q denotes the number of attributes.Fuzzy set $\overline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) = \bigvee_{(s,t)\in[<x_i,y_j>]_A^{\geq}}\{<\tilde{X}(s),\tilde{Y}(t)> \mid e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) < 1-\beta\}$, $\underline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) = \bigwedge_{(s,t)\in[<x_i,y_j>]_A^{\geq}}\{<\tilde{X}(s),\tilde{Y}(t)> \mid e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) \leq \beta\}$ are lower and upper approximations on fuzzy order information system of $X \times Y$ about β .For $\overline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) = \underline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y})$, we call $\tilde{X}\times\tilde{Y}$ is accurate at the error classification le-vel of β, otherwise it is rough.Its positive domain, negative domain, upper boundary domain, lower boundary dom-ain and boundary domain are:

$$pos\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) = \overline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) \cap \underline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) \tag{7}$$

$$neg\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) = \sim[\overline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) \cup \underline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y})] \tag{8}$$

$$ubn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) = \overline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) - \underline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) \tag{9}$$

$$lbn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) = \underline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) - \overline{\tilde{R}_{\beta}}(\tilde{X}\times\tilde{Y}) \tag{10}$$

$$bn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) = ubn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) \cup lbn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) \tag{11}$$

Theorem **2**. Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy order information system, $\beta\in[0,0.5)$,for any $\tilde{X}\times\tilde{Y} \subseteq F(U\times V)$,then the follwing relations are satisfied.

$$(1)\ \overline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) = pos\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) \cup ubn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y});$$

$$(2)\ \underline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) = pos\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y}) \cup lbn\,\tilde{R}_{\beta}^{\geq}(\tilde{X}\times\tilde{Y});$$

Proof. It can be obtained directly by definition 7.

Theorem **3.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be a fuzzy order information system, $\alpha,\beta\in[0,0.5)$,and $\alpha\leq\beta$, for any $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$,then the follwing relations are satisfied.

$$(1)\ \overline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) \subseteq \overline{\tilde{R}_{\alpha}^{\geq}}(\tilde{X}\times\tilde{Y});$$

$$(2)\ \underline{\tilde{R}_{\alpha}^{\geq}}(\tilde{X}\times\tilde{Y}) \subseteq \underline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y});$$

Proof. (1)According to the definition $\overline{\tilde{R}_{\alpha}^{\geq}}(\tilde{X}\times\tilde{Y}) = \bigvee_{(s,t)\in[<x_i,y_j>]_A^{\geq}}\{<\tilde{X}(s),\tilde{Y}(t)> \mid e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) < 1-\alpha\}$, $\overline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) = \bigvee_{(s,t)\in[<x_i,y_j>]_A^{\geq}}\{<\tilde{X}(s),\tilde{Y}(t)> \mid e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) < 1-\beta\}$, for $\alpha\leq\beta$, $1-\beta\leq1-\alpha$,then $\{e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) \leq 1-\beta\} \subseteq \{e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y}) \leq 1-\alpha\}$,therefore $\overline{\tilde{R}_{\beta}^{\geq}}(\tilde{X}\times\tilde{Y}) \subseteq \overline{\tilde{R}_{\alpha}^{\geq}}(\tilde{X}\times\tilde{Y})$.

(2)The proof process is the same as (1).

4. Uncertainty Measure of $U\times V$-Type Double-Domain Fuzzy Order Variable Precision Rough Set

4.1 Accuracy and roughness

Definition **8.** Let $S=(F(U\times V),AT,V_a,F)$ be an fuzzy order information system, $A\subseteq AT$, $\tilde{R}_A^{\geq}$ is the dominant relation on the fuzzy order information system, $\beta\in[0,0.5)$.For any $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$,then the β accuracy and roughness of knowledge A are as follows, respectively:

$$\alpha_A^{\geq}(\tilde{X}\times\tilde{Y})=|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})| \tag{12}$$

$$\rho_A^{\geq}(\tilde{X}\times\tilde{Y})=1-|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})| \tag{13}$$

If $\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})=0$,then $\alpha_A^{\geq}(\tilde{X}\times\tilde{Y})=1,\rho_A^{\geq}(\tilde{X}\times\tilde{Y})=0$.

Theorem **4.** Let $S=(F(U\times V),AT,V_a,F)$ be an fuzzy order information system, $A\subseteq AT$, $\alpha,\beta\in[0,0.5)$.For any $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$,if $\alpha\leq\beta$ then

(1) $\alpha_{A\alpha}^{\geq}(\tilde{X}\times\tilde{Y})\leq\alpha_{A\beta}^{\geq}(\tilde{X}\times\tilde{Y})$;

(2) $\rho_{A\alpha}^{\geq}(\tilde{X}\times\tilde{Y})\geq\rho_{A\beta}^{\geq}(\tilde{X}\times\tilde{Y})$;

Proof**.** (1)According to theorem 3,if $\alpha\leq\beta$,then $\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})\subseteq\overline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})$, $\underline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})\subseteq\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})$,so $|\overline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})|\geq|\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|$, $|\underline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})|\leq|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|$.For $\alpha_{A\alpha}^{\geq}(\tilde{X}\times\tilde{Y})=|\underline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})|$, $\alpha_{A\beta}^{\geq}(\tilde{X}\times\tilde{Y})=|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|/|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|$,then $|\underline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\alpha^{\geq}}(\tilde{X}\times\tilde{Y})|\leq|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|$,therefore $\alpha_{A\alpha}^{\geq}(\tilde{X}\times\tilde{Y})\leq\alpha_{A\beta}^{\geq}(\tilde{X}\times\tilde{Y})$.

(2)According to definition 8, $\rho_A^{\geq}(\tilde{X}\times\tilde{Y})=1-|\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|$,so $\rho_{A\alpha}^{\geq}(\tilde{X}\times\tilde{Y})\geq\rho_{A\beta}^{\geq}(\tilde{X}\times\tilde{Y})$.

4.2 Rough entropy

Since the rough entropy of the rough set is not only related to the roughness, but also related to its own order information system, the following definition of the double-domain fuzzy order variable precision rough set rough entropy is the product of the roughness of the double-domain fuzzy order variable precision rough set and the rough entropy of knowledge A.

Definition **9.** Let $S=(F(U\times V),AT,V_a,F)$ be an fuzzy order information system, $A\subseteq AT$, $\tilde{R}_A^{\geq}$ is the dominant relation on the fuzzy order information system, $\left|[<x_i,y_j>]_A^{\geq}\right|$ is the cardinality of the set of dominant classes in the fuzzy order information system. $\beta\in[0,0.5)$.According to the definition 7, the set $\tilde{X}\times\tilde{Y}$ has lower and upper ap-proximations of parameter β on the fuzzy order information system,now defines the β rough entropy of roug-h set $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$ on knowledge A is as follows:

$$E_A(\tilde{X}\times\tilde{Y}) = \rho_A^{\geq}(\tilde{X}\times\tilde{Y})\cdot[-\sum_{i=1,j=1}^{|F(U\times V)|}(\left|[<x_i,y_j>]_A^{\geq}\right|/\left|F(U\times V)\right|).log_2(1/\left|[<x_i,y_j>]_A^{\geq}\right|)]. \tag{14}$$

Theorem **5.** Let $S=(F(U\times V),AT,\ V_a,\ F)$ be an fuzzy order information system, $A\subseteq AT$, $\alpha,\beta\in[0,0.5)$.For any $\tilde{X}\times\tilde{Y}\subseteq F(U\times V)$,if $\alpha\leq\beta$ then $E_{A^\alpha}(\tilde{X}\times\tilde{Y})\geq E_{A^\beta}(\tilde{X}\times\tilde{Y})$.

Proof**.** (1)According to theorem 4,if $\alpha\leq\beta$,then $\rho_{A^\alpha}^{\geq}(\tilde{X}\times\tilde{Y})\geq\rho_{A^\beta}^{\geq}(\tilde{X}\times\tilde{Y})$, $E_{A^\alpha}(\tilde{X}\times\tilde{Y})=$ $\rho_{A^\alpha}^{\geq}(\tilde{X}\times\tilde{Y})\cdot$ $[-\sum_{i=1,j=1}^{|F(U\times V)|}(\left|[<x_i,y_j>]_A^{\geq}\right|/\left|F(U\times V)\right|).log_2(1/\left|[<x_i,y_j>]_A^{\geq}\right|)]$ $\geq$ $\rho_{A^\beta}^{\geq}(\tilde{X}\times\tilde{Y})\cdot$ $[-\sum_{i=1,j=1}^{|F(U\times V)|}(\left|[<x_i,y_j>]_A^{\geq}\right|/\left|F(U\times V)\right|).$ $.log_2(1/\left|[<x_i,y_j>]_A^{\geq}\right|)]$ $.=E_{A^\beta}(\tilde{X}\times\tilde{Y})$.

It is concluded from theorem 4 and theorem 5. When the β is smaller, the accuracy of the rough set is smaller,the value of roughness and rough entropy is larger. The rough entropy reflects the classification ability of the dominant relation. When the value of the rough entropy is larger, the uncertainty of knowledge is stronger and the distinguishing ability is weaker.

5. An illustrate Example

In this section,We illustrate the set model of $U\times V$-type double-domain fuzzy order information system by a med-ical diagnosis case.In disease diagnosis, a disease may have more than one symptom, and one symptom is shared by many diseases. The symptoms exhibited by each disease are vague, and the number between [0, 1] can be used to indicate the degr-ee of blurring.In the case of a disease diagnosis, a disease may occur as more than one symptom, and a symptom is shared by multiple diseases. The various symptoms of each disease are fuzzy, so we use the number between [0,1] to denote the degree of ambiguity.

Let U,V be the disease set and symptom set respectively, $U=\{x_1,x_2,x_3,x_4,x_5\}$ respectively denote five diseas-es of lung disease, cold, rhinitis, stomach disease and hepatitis, $V=\{y_1,y_2,y_3,y_4\}$ respectively denote 4 symptoms of night sweats, coughing, sneezing, vomiting, $AT=\{a_1,a_2,a_3,a_4,a_5\}$ denote the degree of symptoms during trea-tment, treatment effect, recovery status, medical expenses, recurrence probability, $V_a\in(0,1)$ denotes an evaluation of the patient's treatment process, 1 means the best , 0 means very bad.We give a amount of data on the disease, the degree of symptoms, and the possible evaluation during the treatment by certain methods, as shown in Table 1.

Table 1.Fuzzy evaluation data of diseases and diseases for the treatment process

$F(U\times V)$	Symptom level	treatment effect	recovery degree	Medical expenses	recurrence probability	$F(U\times V)$	Symptom le vel	treatment ef fect	recovery de gree	Medical expenses	recurrence probability
$<x\ y>$	0.3	0.8	0.4	0.7	0.3	$<x\ y>$	0.7	0.8	0.9	0.4	0.8
$<x\ y>$	0.4	0.6	0.4	0.2	0.6	$<x\ y>$	0.6	0.5	0.5	0.5	0.4
$<x\ y>$	0.6	0.8	0.3	0.8	0.4	$<x\ y>$	0.5	0.8	0.7	0.7	0.3
$<x\ y>$	0.8	0.4	0.1	0.8	0.2	$<x\ y>$	0.8	0.3	0.6	0.4	0.3
$<x\ y>$	0.7	0.9	0.4	0.3	0.7	$<x\ y>$	0.8	0.3	0.3	0.3	0.6

$< x \quad y >$	0.4	0.7	0.5	0.8	0.4	$< x \quad y >$	0.5	0.7	0.2	0.7	0.7
$< x \quad y >$	0.9	0.6	0.2	0.5	0.8	$< x \quad y >$	0.6	0.6	0.5	0.5	0.6
$< x \quad y >$	0.8	0.8	0.4	0.8	0.8	$< x \quad y >$	0.3	0.4	0.6	0.6	0.3
$< x \quad y >$	0.4	0.9	0.8	0.7	0.3	$< x \quad y >$	0.7	0.3	0.5	0.2	0.4
$< x \quad y >$	0.7	0.9	0.4	0.9	0.3	$< x \quad y >$	0.6	0.4	0.6	0.7	0.8

We now extract a part of the object set $(\tilde{X}\times\tilde{Y}) = \{ <x_1,y_4>, <x_2,y_1>, <x_2,y_3>, <x_2,y_4>, <x_3,y_1>, <x_3,y_3>, <x_4,y_1>, <x_4,y_2>, <x_5,y_1>, <x_5,y_4> \}$ as the research object, and calculate the values of $q\,|\,[x_i,y_i]^{\geq}\,|$, $\sum_{(s,t)\in[<x_i,y_i>]_A^{\geq}}[<\tilde{X}(s),\tilde{Y}(t)>]$, $e([<x_i,y_i>]_A^{\geq},\tilde{X}\times\tilde{Y})$ as shown in Table 2.

Table 2.Calculation results of fuzzy evaluation data

$F(U\times V)$	$q\,\vert\sum[<x_i,y_i>]_A^{\geq}\vert$	$\sum_{(s,t)\in[<x_i,y_j>]_A^{\geq}}[<\tilde{X}(s),\tilde{Y}(t)>]$	$e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y})$	$F(U\times V)$	$q\,\vert\sum[<x_i,y_i>]_A^{\geq}\vert$	$\sum_{(s,t)\in[<x_i,y_j>]_A^{\geq}}[<\tilde{X}(s),\tilde{Y}(t)>]$	$e([<x_i,y_j>]_A^{\geq},\tilde{X}\times\tilde{Y})$
$< x \quad y >$	30	13.1	0.56	$< x \quad y >$	5	3.6	0.28
$< x \quad y >$	25	13	0.48	$< x \quad y >$	15	3.5	0.77
$< x \quad y >$	20	7.1	0.65	$< x \quad y >$	10	6.5	0.35
$< x \quad y >$	10	5.9	0.41	$< x \quad y >$	5	2.4	0.52
$< x \quad y >$	5	3	0.4	$< x \quad y >$	10	3.6	0.64
$< x \quad y >$	5	0	1	$< x \quad y >$	10	3.6	0.64
$< x \quad y >$	5	3	0.4	$< x \quad y >$	5	3.1	0.38
$< x \quad y >$	5	3.6	0.28	$< x \quad y >$	20	9.6	0.52
$< x \quad y >$	5	3.5	0.3	$< x \quad y >$	15	7.1	0.53
$< x \quad y >$	10	3.5	0.65	$< x \quad y >$	5	3.1	0.38

(1)If $\beta = 0.3$,then $\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y}) = \{ <x_1,y_1>, <x_1,y_2>, <x_1,y_3>, <x_1,y_4>, <x_2,y_1>, <x_2,y_3>, <x_2,y_4>, <x_3,y_1>, <x_3,y_2>, <x_3,y_3>, <x_4,y_1>, <x_4,y_2>, <x_4,y_3>, <x_4,y_4>, <x_5,y_1>, <x_5,y_2>, <x_5,y_3>, <x_5,y_4> \}$; $\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y}) = \{ <x_2,y_4>, <x_3,y_1>, <x_3,y_3> \}$.The accuracy is $\alpha_A^{\geq}(\tilde{X}\times\tilde{Y})$ =| $\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})$ |/| $\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})$ |=1/6.The roughness is $\rho_A^{\geq}(\tilde{X}\times\tilde{Y}) = 1 - |\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|/|\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y})|$=5/6.

The roughness entropy is $E_A(\tilde{X}\times\tilde{Y}) = \rho_A^{\geq}(\tilde{X}\times\tilde{Y})\cdot [-\sum_{i=1,j=1}^{|F(U\times V)|}(\left|[<x_i,y_j>]_A^{\geq}\right|/\left|F(U\times V)\right|).log_2(1/\left|[<x_i,y_j>]_A^{\geq}\right|$ $= \rho_A^{\geq}(\tilde{X}\times\tilde{Y})\cdot(\sum_{i=1,j=1}^{|F(U\times V)|}(\left|[<x_i,y_j>]_A^{\geq}\right|/\left|F(U\times V)\right|)\cdot log_2\left|[<x_i,y_j>]_A^{\geq}\right|) \approx \frac{5}{6}\times 3.132 \approx 2.61$.

(2)If $\beta = 0.4$, $\overline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y}) = \{ <x_1,y_1>, <x_1,y_2>, <x_1,y_4>, <x_2,y_1>, <x_2,y_3>, <x_2,y_4>, <x_3,y_1>, <x_3,y_3>, <x_4,y_1>, <x_4,y_2>, <x_5,y_1>, <x_5,y_2>, <x_5,y_3>, <x_5,y_4> \}$; $\underline{\tilde{R}_\beta^{\geq}}(\tilde{X}\times\tilde{Y}) = \{ <x_2,y_1>, <x_2,y_3>, <x_2,y_4>, <x_3,y_1>, <x_3,y_3>, <x_4,y_1>, <x_5,y_1>, <x_5,y_4>$

}.Compared with $\beta=0.3$, it satisfies the theorem 3.

The accuracy is $\alpha_A^{\geq}(\tilde{X}\times\tilde{Y})$ =4/7.The roughness is $\rho_A^{\geq}(\tilde{X}\times\tilde{Y})=$ 3/7.Compared with $\beta=0.3$, it satisfies the theorem 4.

The roughness entropy is $E_A(\tilde{X}\times\tilde{Y})\approx 1.34$.Compared with $\beta=0.3$, it satisfies the theorem 5.

$U\times V$-type double-domain fuzzy order variable precision rough entropy is the product of the entropy and roughness of the dominance relation property.According to the theorem, when the value of the precision threshold β is smaller, the roughness and the roughness entropy are larger;The $U\times V$-type double-domain fuzzy set as an extension of the ordinary set is discussed in this paper, which abstracts the more objective and complex real world.

6. Conclusion

In this paper,we firstly defines the concept of U× V-type double-domain variable precision rough set on the equivalence relation,the approximate set and the lower approximate set on the model are described.Then,by defining the fuzzy order information system and the dominance relation, we propose a U× V-type double-domain fuzzy order variable precision rough set model and discuss its properties.Thus, the concept of accuracy and roughness of the rough set model is defined, and the concept of rough entropy is introduced in the rough set model according to the defined dominant relation and dominant class.For the roughness and rough entropy measures of U× V-type double-domain fuzzy order variable precision rough set,the results show that the roughness threshold β becomes larger, and the roughness and roughness entropy are monotonica-lly decreasing.Finally, they is verified by an example. We define the concept of U× V-type double-domain fuzzy order vari-able precision rough set, which lays a foundation for further research on the uncertainty and rule mining of double-domain rough sets.

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