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Recovery of Compressed Sampling in Shift-Invariant Spaces Base on L1 Norm

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Abstract

Compressed sampling in shift-invariant spaces (SI) is an effective method for sampling of sparse signals. But, reconstruction of compressed sampling may be unstable. In the paper, the possibility of stable reconstruction under a sufficient sparsity is proven. Further, we consider the situation where the minimal L1 norm is used to recover sparse signals from the noisy data. The result shows that they are stable. Finally, we show that the minimal L1 norm through the simulation, and explain the applicability of our algorithm to sampling systems.

Keywords: SI; compressed sampling; L1-norm; reconstruction.

I. Introduction

The sampling in SI has been discussed in recent years. As discussed by A. Aldroubi in [1], a selection of the generator eliminates some problems which relate to the classical sampling. The model contains the signals which can be used to signal processing. For example, the band-limited signal is SI with Sinc[2], [3] and pulse modulation in signal processing. The signals can be described using multiple generators, multiband signal [4], [5], [6]. Therefore, we give a Si generated by the L function, which shift with the period T .

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$$W = \left\{ \sum_{j=1}^{L} \sum_{n \in Z} c_i[k] \varphi_i(t - kT) : c_j[k] \in l^2 \right\}$$

The $\{\varphi_j(t)\}_{j=1,2,\cdots,L}$ are generators. In other words, the $\{\varphi_j(t-kT)\}_{j=1,2,\cdots,L,k\in Z}$ is the basis of W. In the Fourier domain

$$X(\omega) = \sum_{j=1}^{L} C_j(\omega) \psi_j(\omega)$$

$C_i(\omega)$ and $\psi_i(\omega)$ are the Fourier transform of $c_i(n)$ and $\varphi_i(t)$.
Since x(t) can be given by SI $\{\varphi_i(t)\}$, sampling x(t) is used by L functions s(-t), as Fig. 1. The sampling system is showed by

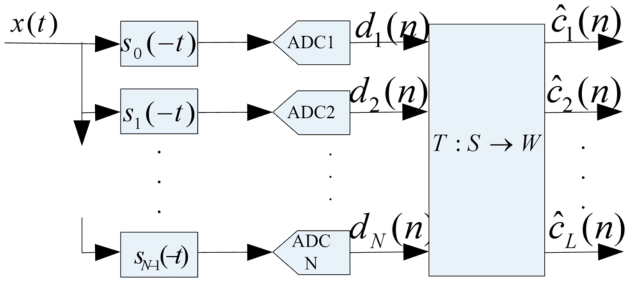

Fig. 1. Sampled and reconstruction in SI

$$d_j[k] = \mathrm{S}x = \left\langle x(t), s_j(t-kT) \right\rangle \quad \mathrm{j} = 1,2, \cdots, N$$

We obtain

$$H(\omega)C(\omega) = D(\omega) \tag{1.1}$$

Where

$$H(\omega) = \begin{pmatrix} h_1(\omega) & h_2(\omega) & \cdots & h_L(\omega) \end{pmatrix}$$

$$h_i(\omega) = \begin{pmatrix} h_{1,i}(\omega) & h_{2,i}(\omega) & \cdots & h_{N,i}(\omega) \end{pmatrix}^T$$

$$h_{i,k}(\omega) = \sum_{n=-\infty}^{+\infty} S_i(\omega + 2\pi n) \psi_k(\omega + 2\pi n)$$

$$D(\omega) = \begin{pmatrix} D_1(\omega) & D_2(\omega) & \cdots & D_N(\omega) \end{pmatrix}^T$$

$$C(\omega) = \begin{pmatrix} C_1(\omega) & C_2(\omega) & \cdots & C_L(\omega) \end{pmatrix}^T$$

From (1.1), if n > k, the reconstruction is underdetermined. The sampling is belong to the compressed sensing (CS). To reconstruct vector x from N < L [7]. Many reconstruction algorithms have been given in [8], [9], [10], [11], [12], [13].We reconstruct signals from CS.

In this article ,we study algorithms which can in the field of CS, and derive rigorous bounds that show the algorithms locally stable.

The paper is organized as follows. The minimizing the L1-norm problems are introduced in Section II. Section III presents some simulations.

2 Minimizing the L1-Norm

We show $\|C(\omega)\|_1$

$$\|C(\omega)\|_1 = \sum_{i=0}^{L-1} \frac{1}{2\pi} \int_{-\pi}^{\pi} |C_i(\omega)| d\omega$$

The problem:

$$\text{L1-norm:} \quad \min \|C(\omega)\|_1 \quad D(\omega) = H(\omega)C(\omega)$$

In most practical situations, we can observe $Y(\omega) = D(\omega) + Z(\omega)$, $\|Z(\omega)\|_2 < \xi$ of noisy version. Another , we can show a strategy of minimizing n L1-norm.

$$\text{L1}_\delta\text{-norm:} \quad \min \|C(\omega)\|_1 \quad \|D(\omega) - H(\omega)C(\omega)\|_2 \le \delta \, (2.1)$$

3. Uniqueness in L1-Norm Minimization

We calculate the solution from L1-norm ,which $C(\omega)$ is a solution from L1-norm. Here $C_i(\omega)$ is the ith element of matrix $C(\omega)$. $H_S(\omega)$ is a matrix which belong to $H(\omega)$ with indices from S. We write $H_S(\omega)C_S(\omega) = D(\omega)$, $C_S(\omega)$ is nonzero elements of $C(\omega)$. So, we assume that $H_S(\omega)$ is full rank; otherwise, The generalized inverse of $H_S(\omega)$ is $H_S(\omega)^+ = \left(H_S(\omega)^H H_S(\omega)\right)^{-1} H_S(\omega)^H$. So We present a sufficient condition of $C(\omega)$ in the theorem 2.1.

Theorem 2.1: A sufficient condition for $C(\omega)$ to be the unique solution to L1-norm is that

$$\max_{h_i(\omega)\in\Psi_{opt}(\omega)} \left(\max_{-\pi\le\omega<\pi} \left| H_s^+(\omega) h_i(\omega) \right|_1 \right) < 1$$

$$\text{Where,} \quad \Psi_{opt}(\omega) = H(\omega) \setminus H_S(\omega)$$

Proof. Suppose there are two representations: $H(\omega)C_{opt}(\omega) = H(\omega)C_{alt}(\omega) = D(\omega)$, $C(\omega) \ne C_{alt}(\omega)$. Then,

$$\begin{aligned} \|C(\omega)\|_1 &= \left\| H_s^+(\omega) H_{alt}(\omega) C_{alt}(\omega) \right\|_1 \\ &= \left\| H_s^+(\omega) D(\omega) \right\|_1 \\ &= \left\| H_s^+(\omega) H_{alt}(\omega) C_{alt}(\omega) \right\|_1 \\ &< \max_{h_i(\omega)\in\Psi_{opt}(\omega)} \left(\max_{-\pi\le\omega<\pi} \left| H_s^+(\omega) h_i(\omega) \right|_1 \right) \|C_{alt}(\omega)\|_1 \end{aligned}$$

$$\text{If} \max_{h_i(\omega)\in\Psi_{opt}(\omega)} \left(\max_{-\pi\le\omega<\pi} \left| H_s^+(\omega) h_i(\omega) \right|_1 \right) < 1 \text{ . Then}$$

$$\|C_S(\omega)\|_1 < \|C_{alt}(\omega)\|_1$$

Known

$$\left\|C(\omega)\right\|_0 < \frac{1}{2}\left(M^{-1}+1\right)$$

Where

$$M = \max_{j\neq k}\left(\max_{-\pi\leq\omega<\pi}\left|h_j^H(\omega)h_k(\omega)\right|\right)$$

$\max_{h_i(\omega)\in\Psi_{opt}(\omega)}\left(\max_{-\pi\leq\omega<\pi}\left|H_S^+(\omega)h_i(\omega)\right|_1\right)<1$ is a sufficient condition of $\left\|C(\omega)\right\|_1$ atoms in its optimal representation.

4. Stability of the Reconstruction

As the introduction, the signal $Y(\omega)=D(\omega)+Z(\omega)$, $\left\|Z(\omega)\right\|_2<\xi$ is a noise. We apply ($L1_\delta$-norm) to this signal; i.e. we obtain a solution $\hat{C}(\omega)$ from (2.1).

Theorem 2.2: The noiseless signal $H(\omega)C(\omega)=D(\omega)$ is satisfied $\left\|C(\omega)\right\|_0<\frac{1}{2}\left(M^{-1}+1\right)$ .The deviation of the $L1_\xi$-norm from $C(\omega)$ can be bounded by

$$\left\|\left(\hat{C}(\omega)-C(\omega)\right)\right\|_2^2 < \frac{\Delta^2}{1-M\left(m-1\right)}$$

$$\text{Where, } \Delta^2 = \left\|Z(\omega)\right\|_2^2+\delta^2$$

Proof: Suppose $C'(\omega)$ is solution of $\left\|D(\omega)-H(\omega)C(\omega)\right\|_2\leq\delta$. Let

$$V(\omega)=C'(\omega)-C(\omega)$$

Suppose $W(\omega)=\hat{C}(\omega)-C(\omega)$, then

$$W(\omega)=\arg\min_{V(\omega)}\left\|C(\omega)+V(\omega)\right\|_1$$

Because

$$\left\|H(\omega)V(\omega)-Z(\omega)\right\|_2^2\leq\delta^2$$

Then

$$\left\|H(\omega)V(\omega)\right\|_2^2\leq\Delta^2$$

We can compute

$$\|H(\omega)V(\omega)\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} V(\omega)^H H(\omega)^H H(\omega)V(\omega)d\omega$$
$$\geq \frac{1}{2\pi}\int_{-\infty}^{+\infty} V(\omega)^H V(\omega) - V(\omega)^H \left(H(\omega)^H H(\omega) - E\right)V(\omega)d\omega$$
$$\geq \frac{1}{2\pi}\int_{-\infty}^{+\infty} V(\omega)^H V(\omega) - MV(\omega)^H \left(1-E\right)V(\omega)d\omega$$
$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left(1+M\right)V(\omega)^H V(\omega) - MV(\omega)^H 1V(\omega)d\omega$$

$$\geq \frac{1}{2\pi}\int_{-\infty}^{+\infty} \left(1+M\right)V(\omega)^H V(\omega)d\omega - M\|V(\omega)\|_1^2$$

However

$$\|V(\omega)\|_1 \geq \|V(\omega)\|_2 \geq \frac{\|V(\omega)\|_1}{\sqrt{N}}$$

Returning to (3.3), obtain

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} \left(1+M\right)V(\omega)^H V(\omega)d\omega - M\|V(\omega)\|_1^2$$
$$\geq \frac{1}{\pi}\int_{-\infty}^{+\infty} \left(1+M\right)V(\omega)^H V(\omega)d\omega - MN\|V(\omega)\|_2^2$$

Hence

$$\|V(\omega)\|_2^2 \leq \frac{\Delta}{1-M\left(N-1\right)}$$

5. Simulation

We consider L=4. The anlysis filters are:

$$s_1(-t) = \delta(t)\,; \qquad s_2(-t) = \delta(t - T_s/3)\,;$$

$$s_3(-t) = \delta(t - T_s/4)\,; \qquad s_4(-t) = \delta(t - T_s/5)\,.$$

The generating functions are :

$$\varphi_0(t) = \sin c\left(\frac{t}{T_s}\right);\quad \varphi_1(t) = \sin c\left(\frac{t}{T_s}\right)e^{-j2\pi t/T_s};\quad \varphi_2(t) = \sin c\left(\frac{t}{T_s}\right)e^{j2\pi t/T_s};\quad \varphi_3(t) = \sin c\left(\frac{t}{T_s}\right)e^{-j4\pi t/T_s};$$
$$\varphi_4(t) = \sin c\left(\frac{t}{T_s}\right)e^{j4\pi t/T_s};\quad \varphi_5(t) = \sin c\left(\frac{t}{T_s}\right)e^{-j6\pi t/T_s};\quad \varphi_6(t) = \sin c\left(\frac{t}{T_s}\right)e^{j6\pi t/T_s}$$

T_s : sampling period.

6. Exact recovery of L1-norm

In the experiment, $H(\omega)$'s dimensions are L=4 and r=7. The $D(\omega)$, from $H(\omega)C'(\omega) = D(\omega)$, $C'(\omega)$ are randomly chosen, and M nonzero elements. The M is change from 1 to 7. $C'(\omega)$ is calculated from L1-norm. If $C'(\omega) = C(\omega)$, the reconstruction. is given. From 1000 times of simulation, the exact recoveries are reported as 'empirical probabilities of exact recovery' in Figure 2.

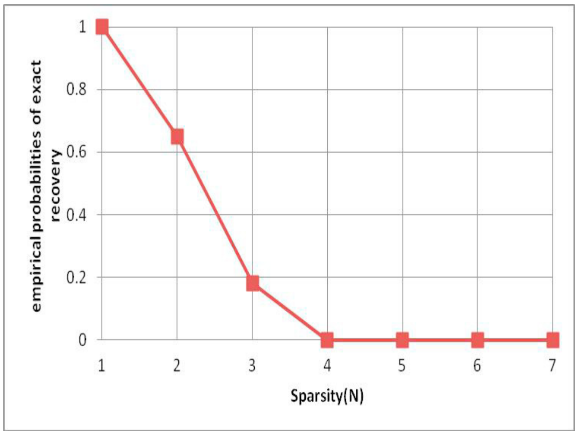

Fig.2. Probabilities of exact recovery

7. Error of Recovery L1-Norm

In the subsection, We randomly generated the ideal $C(\omega)$ which satisfy to the conditions of Theorem 2.1. Since $\left(M^{-1}+1\right)/2<2$ we use $\|C(\omega)\|_0 = 1$. The ideal noiseless signal $H(\omega)C(\omega) = D(\omega)$ was contaminated to zero-mean white Gaussian noise $\|Z(\omega)\|_2 < \xi$, obtaining $\hat{D}(\omega) = D(\omega) + Z(\omega)$. We numerically solved from L1-norm ,and calculated the reconstruction.error $\left\|\hat{C}(\omega) - C(\omega)\right\|_2^2$.

Figure 3 refers to the case $\xi = 12/4^n$, where $n = 5,6,\ldots,12$, which gives ADC-bits, and displays bounds and errors. L1-norm is stabile from Figure 3. Moreover, the errors are less than the upper bounds provided in Theorem 2.2. Note that in this experiment we have used L1-norm.

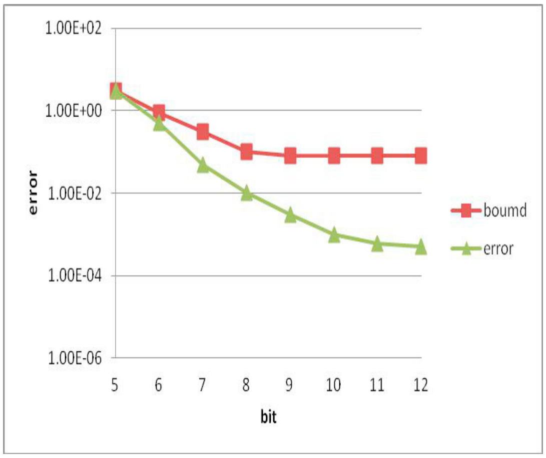

Fig. 3. Error of recovery

8. Conclusion

We showed the impact of sparsity constraints the reconstruction of CS in SI, and algorithm which can be used to certain sparse signals from overcomplete dictionary. We derived rigorous bounds from a su ciently sparse representation.

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