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Abstract

The design of composite structures relies on the accurate determination of design allowables, which are statistically based material parameters that take into account manufacturing, geometrical and microstructure variability. The accurate determination of these design parameters requires extensive experimental testing, which makes the certification process of a composite material extremely costly and time consuming. To increase the efficiency of the design process, there is the need to develop alternatives to the mostly experimental material characterization process, ideally based on accurate and quick modelling analysis combined with powerful statistical tools.

In this work an analytical model to compute the notched strength of composite structures based on three ply-based material properties (elastic modulus, unnotched strength and $\mathcal{R}$-curve) is combined with an uncertainty quantification and management (UQ&M) framework to compute the B-basis design allowables of notched configurations of CFRP laminates. The framework is validated with open-hole tension experimental results for the IM7/8552 material. Given the analytical nature of the developed framework and consequent computational efficiency, the UQ&M methodology is applied to the generation of design charts for notched geometries, whose generation would otherwise be impractical, using experimental test based methods.

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1. Introduction

The design and certification of composite structures is based on the building block approach [1]. This approach relies on the accurate determination of design allowables that drive the design of structures at larger scales. These design allowables are statistically based material parameters that define an acceptable stress value for a material and, therefore, ensure their safe and efficient use. Design allowables have to account for the variability of the material properties and of the manufacturing process, and are a function of the structural details and loading conditions [2] and, consequently, their experimental determination is an extremely costly and time-consuming process. The standard design allowable used in the aeronautical industry for fail safe structures is the B-basis [1, 3], which is defined as the 95% lower confidence bound on the tenth percentile of a specified population of measurements. This is a conservative allowable that ensures with 95% confidence that 90% of the population will have a given property, e.g. strength, higher than the B-value allowable.

It is of key importance to accurately determine these design allowables, however, time consuming processes are not ideal during preliminary design. For this reason, alternatives to fully experimental material characterization have been proposed, namely, the use of statistically based numerical and analytical models [4, 5, 6, 3]. These models include the influence of the uncertainty related to the determination of the input parameters and their intrinsic variability on the global response of the model. A convenient way to describe these uncertain quantities is to describe them using a probability distribution which can be defined through experimental measurements or assumed based on empirical evidence.

The stochastic finite element method [7, 5] is a powerful tool to address the influence of the uncertainty related to the determination of the material and geometrical properties and loading conditions on the global response of composite structures. Nam et al. [7] proposed a methodology to determine the design allowables of composite laminates using lamina level test data and finite element analysis and validate the proposed methodology for both un-notched and open hole strength. However, stochastic finite element method solutions rely on computationally expensive procedures, which makes the consideration of the variability of the input parameters an extremely time consuming and, therefore, impracticable process quick design.

Quick analytical prediction tools are therefore desirable, specially for preliminary design and material selection. Furtado et al. [8] proposed an analytical framework to estimate the notched strength of multi-directional carbon-epoxy laminates based on three ply properties (the longitudinal Young's modulus, the longitudinal strength, and the longitudinal crack resistance curve) and concluded that the framework was able to provide good predictions for the open-hole tensile and compressive strengths of general balanced carbon/epoxy laminates. Since the model uses ply-level properties as building blocks, it is ideal for preliminary design, since the notched strength of different layups and geometries can be quickly estimated.

The authors validated the analytical framework for the nominal values of the material properties and the geometrical parameters. However, the uncertainty associated to the material properties and dimensions may be taken into account in an attempt to define design allowables for the notched strength.

In this work, a methodology to predict the B-value of notched composite laminates using the analytical framework proposed by Furtado et al. [8] is proposed by taking the variability of the material properties that dominate failure and the effect of geometrical imperfections into account. The proposed Uncertainty Quantification and Management (UQ&M) methodology is validated against available experimental data and is applied to generate practical engineering design tools.

2. Methodology

2.1. Description of the analytical framework

Furtado et al. [8] proposed an analytical framework to estimate the notched strength of multidirectional carbon-epoxy laminates based on three ply properties: the longitudinal Young's modulus, E_1, the longitudinal strength, X, and the longitudinal crack resistance curve, $\mathcal{R}$-curve. The framework combines the finite fracture mechanics model proposed by Camanho et al. [9] with the invariant-based approaches to estimate stiffness and strength proposed by Tsai and Melo [10, 11] and with an analytical model based on linear elastic fracture mechanics to estimate the laminate fracture toughness proposed by Camanho et al. [12].

The coupled Finite Fracture Mechanics (FFMs) model proposed by Camanho et al. [9] is used to predict the notched strength of open-hole laminate specimens (Fig 1) with fibre dominated failure. Both a stress-based and energy-based criteria must be satisfied during crack propagation:

$$\begin{cases} \frac{1}{l}\int_R^{R+l} \sigma_{xx}(0,y)dy = X^L \\ \int_R^{R+l} \mathcal{G}_I(a)da = \int_0^l \mathcal{R}(\Delta a)d\Delta a \end{cases} \tag{1}$$

where R is the hole radius, $\sigma_{xx}(0,y)$ is the stress distribution along the ligament section perpendicular to the loading direction (along the transverse axis), X^L is the laminate unnotched strength, $\mathcal{G}_I(a)$ is the mode I energy release rate (ERR) of a laminated plate with a central circular hole of radius R and two symmetric cracks propagating from the hole edge, $\mathcal{R}(\Delta a)$ is the $\mathcal{R}$-curve of the laminate and l is the crack extension at failure.

The first equation corresponds to the average-stress criterion while the second represents an energy balance. Therefore, a stress equilibrium between the average stress in the narrowest critical section with the hole and the maximum admissible strength of the laminate, and an equilibrium between the energy released and the maximum admissible fracture energy of the laminate in a finite length must be satisfied during crack propagation. The model only requires two independent laminate properties: the laminate unnotched strength X^L and the laminate $\mathcal{R}$-curve.

The FFMs model is therefore based on properties at laminate level, which need to be determined each time the layup changes. To determine the laminate unnotched strength X^L, Furtado et al. [8] proposed the use of the invariant-based approach to estimate stiffness and strength proposed by Tsai and Melo [10, 11]. This approach is based on the Unit Circle failure envelope, which was proposed by Tsai and Melo [11] as a conservative simplification of the last ply failure Omni Strain Failure Envelope. The Unit Circle envelope is defined by the uniaxial tensile and compressive strains-to-failure. Following Tsai and Melo [11], the laminate unnotched strength under uniaxial loading is estimated by a simple maximum strain criterion:

$$X^L \approx \frac{X}{E_1} \times E^L \tag{2}$$

where X is the laminate unnotched strength, E_1 is the longitudinal Young's modulus and E^L is the laminate longitudinal Young's modulus, which can be estimated using the Trace theory and Master Ply concept [10]. Tsai and Melo [10] defined a Master Ply for CFRPs based on the finding that the normalised UD stiffness components of several CFRP systems (normalized by the trace) is almost constant. The authors concluded that the stiffness of CFRPs along the fibre direction is responsible for about 88% of the value of trace, which means that the value of trace can be estimated from the longitudinal stiffness E_1 as

$$Tr \approx \frac{E_1}{0.88} \tag{3}$$

The Young's modulus of a given laminate can be determined as a product of the value of trace and a laminate factor, which can be determined using laminate plate theory and the Master Ply presented in table 1:

$$E^L \approx E_x/Tr \times \frac{E_1}{0.88} \tag{4}$$

Table 1: Universal Laminate Factors of the Master Ply.

Lay-up	E_x/Tr	E_y/Tr	G_{xy}/Tr	ν_{xy}
Master Ply	0.880	0.052	0.031	0.320

To estimate the laminate $\mathcal{R}$-curve, the analytical model proposed by Camanho et al. [12] can be used. The model is based on a combination of linear elastic fracture mechanics and laminate plate theory and can be used to estimate the fracture toughness of balanced multidirectional laminates, G_{Ic}, using the fracture toughness of the 0° ply, G_{Ic}^0.

Furtado et al. [8] concluded that the framework is able to provide good predictions for the open-hole tensile and compressive strengths of general balanced carbon/epoxy laminates with fibre dominated failure using only the lay-up, the geometry of the specimen (the radius, R, and the width, W) and three ply properties as inputs: the longitudinal Young's modulus, E_1, the longitudinal strength, X, and the longitudinal crack resistance curve, $\mathcal{R}$-curve. Since the model uses only three ply level parameters as building blocks, the framework can be particularly useful for preliminary design and optimization, as the number of elementary tests needed to characterize the composite system is drastically reduced. In addition, due to the computational efficiency of the model it can be used to perform uncertainty quantification and management (UQ&M) analysis, allowing not only the analysis of the effects of the mean parameters on the response, but also the analysis of the influence of their variability.

2.2. Uncertainty quantification of the model parameters

The analytical framework [8] summarized in the previous section requires three ply material parameters to estimate the strength of a multidirectional notched laminate: the longitudinal Young's modulus, the longitudinal strength and the $\mathcal{R}$-curve of the 0° plies. The model was validated using the mean ply properties determined experimentally, resulting in the prediction of a nominal notched strength for a given nominal dimension (hole radius and specimen width). In this work, the variability associated with the determination of the ply properties and the geometry of the specimens is accounted for. The variability associated with the geometrical parameters (notch radius and specimen width) is directly related to the manufacturing process, namely the cutting methodology and respective tolerances. Since direct measurements were not available, the dimensions of the specimen were assumed to follow a uniform distribution.

Accounting for the variability of the longitudinal Young's modulus and the longitudinal strength is straightforward since these properties are obtained directly from the experimental tests and have an associated standard deviation. It is assumed here that these two properties follow a normal distribution with known mean and standard deviation, corresponding to the values obtained experimentally.

Accounting for the variability of the $\mathcal{R}$-curve is less clear since the $\mathcal{R}$-curves are generally not measured directly but determined from notched strengths measured experimentally. Thus, it is of key importance to define a methodology to randomly generate statistically representative $\mathcal{R}$-curves. Such methodology is proposed in section 2.2.1.

2.2.1. Mode I crack resistance curve in the fibre direction

Catalanotti et al. [13, 14] proposed a methodology to determine the $\mathcal{R}$-curve of polymer composites reinforced by unidirectional fibres based on the size effect law, i.e the relation between the size of the specimens and their notched strength $\overline{\sigma}^{\infty}(w)$. The size effect law can be determined by experimentally testing geometrically similar double edge notch specimens, i.e. with the same width-to-crack length ratio $2w/a$ and different widths $2w$. The size effect law can be determined by finding a fitting regression that best approximates the experimental data [15] and the $\mathcal{R}$-curve parameters (length of the fracture process zone, l_{fpz}, and the fracture toughness at propagation $\mathcal{R}_{ss}$) can then be obtained as a function of these fitting parameters [15, 13, 14]. Catalanotti et al. [13] also suggested to express the $\mathcal{R}$-curve analytically. In this work, the following analytical expression is proposed to represent the $\mathcal{R}$-curve:

$$\begin{cases} R(\Delta a) = R_{ss}\left[1-(1-\Delta a/l_{fpz})^{\beta}\right] \quad \text{if} \quad \Delta a < l_{fpz} \\ R(\Delta a) = R_{ss} \quad \text{if} \quad \Delta a \geq l_{fpz} \end{cases} \tag{5}$$

where β is a parameter determined to obtain the best fit of the $\mathcal{R}$-curve. The proposed equation guarantees that the steady state value of the fracture toughness is reached when $\Delta a = l_{fpz}$.Since the mean $\mathcal{R}$-curve is determined from the mean experimental notched strengths of the double edge notch specimens, accounting for the variability of the $\mathcal{R}$-curves implies accounting for the variability of the size effect law. Two methodologies to determine the variability of the $\mathcal{R}$-curves are proposed in this section.

Method 1. The variability is obtained by generating a large number of $\mathcal{R}$-curves accounting for the variability of the notched strength ($\bar{\sigma}^{\infty}$) of the specimens with different geometries by:

1. Randomly generating N_i strengths per each specimen geometry following a statistical distribution determined experimentally for each specimen geometry.
2. Fitting the data to one of the fitting regressions proposed in Ref. [15].
3. Determining the $\mathcal{R}$-curve parameters (l_{fpz} and $\mathcal{R}_{ss}$) as proposed in Ref. [15, 13, 14].
4. Fitting the $\mathcal{R}$-curve to the analytical expression proposed in Equation (5) .
5. Repeat 1-4, N times obtaining a large number of R-curves and the distribution of the fitting parameters.

Using this methodology, a set of statistically representative crack resistance curves is obtained. With the generated $\mathcal{R}$-curves it is possible to determine the mean values and standard deviation of the three $\mathcal{R}$-curve fitting parameters (l_{fpz}, $\mathcal{R}_{ss}$ and β). However, due to the nature of the crack resistance curves, the fitting parameters cannot be treated independently as that would lead to unrealistic and potentially non-continuous $\mathcal{R}$-curves. For this reason, a relation between the parameters should be established as a function of $\mathcal{R}_{ss}$, i.e $l_{fpz} = f(\mathcal{R}_{ss})$ and $\beta = g(\mathcal{R}_{ss})$. These functions can vary and should be analysed for each material system considered. A more detailed analysis is given in section 3.3.

Method 2. The variability is obtained from the determination of the 95% prediction bounds of the linear regression used to fit the size effect law measured experimentally. Either the whole set of experimental points or the mean strengths per specimen geometry can be used, however, the confidence intervals will be generally narrower if only the mean size effect law is used. This process allows the determination of the mean $\mathcal{R}$-curve and the two 95% confidence $\mathcal{R}$-curves. The three $\mathcal{R}$-curve parameters and the respective standard deviations can also be determined.

This method provides only three sets of $\mathcal{R}$-curve parameters and therefore, $\mathcal{R}_{ss}$, l_{fpz} and β are considered independent. This second method is simpler to apply and less computationally expensive, however, the relation between $\mathcal{R}_{ss}$ and the remaining parameters has to be assumed, so caution is required when applying this method.

2.3. Estimation of the B-basis value

In the design of a composite structure it is important to take into account the variability of the design parameters, namely the material properties. According to the Composite Materials Handbook (CMH17) [1], variability should be taken into account in the design of composite structures by using the B-basis for the design allowables. The B-basis (B-value) is a statistically-based design allowable defined as the 95% lower confidence bound on the tenth percentile of a specified population of measurements [1].

By taking the variability of the input parameters (material and geometrical) and using the proposed analytical model, it is possible to propagate the uncertainty of the input parameters to the notched strength, i.e. a statistical distribution of the notched strength can be obtained, based on the variability of the input parameters, which can then be used to compute the statistical design allowables. To obtain the B-value for the open hole strength, two methodologies have been used: (i) the CMH-17 approach and (ii) a Monte Carlo based approach.

Both approaches rely on the set of material and geometrical properties and respective statistical distribution and differ in how the strength data is dealt with to determine the B-value. Nevertheless, for a given run of the analytical model the geometrical and material properties are considered deterministic.

CMH-17 approach. The CMH-17 [1] defines different methods to determine the B-value depending on the distribution that best fits the data. As summarised in Figure 2, for unstructured data, the CMH-17 suggests to successively test if the Weibull, normal and lognormal distributions are adequate fits to the data. If any of these distributions fits the data then the respective methods to calculate the B-basis should be used. If none of these three distributions can be assumed, nonparametric procedures should be used to determine the B-value.

To find the best fitting distribution, the CMH-17 suggests the use of the Anderson-Darling test. This test compares the Cumulative Distribution Function (CDF) of the distribution of interest with the CDF of the data, which allows the determination of a Observance Significance Level (OSL). If the calculated OSL is greater than 0.05, it is concluded that the distribution analysed fits the data. Otherwise, the analysed distribution does not fit the data and the subsequent distribution is analysed. Once a fitting distribution has been found, the B-value can be computed according to the procedures in the CMH-17 for that statistical distribution [1]. If none of these distributions fit the data, nonparametric procedures are used. These procedures depend on the sample size, being the Hanson-Koopmans method used for small sample sizes ($n < 28$). For large sample sizes the B-value can be computed from tabulated data in the CMH-17. For more information on these procedures, the reader is referred to the CMH-17 [1].

Monte Carlo simulations . The Monte Carlo Methods (MCS) rely on the repeated random sampling to obtain numerical results. To determine the B-value using this approach it is necessary to run the analytical model a large number of times to determine an Empirical Cumulative Distribution Function (ECDF) for the parameter in study, namely the notched strength. For each set of n results, where n is the sample size that should be large enough to be representative of the population, it is possible to determine the ECDF and extract the 10th percentile value, $P_{10,j}$. This process is repeated N times, determining a distribution for the 10th percentile. From this distribution the B-value can be computed by considering the 95% lower confidence bound [16], which corresponds to the 5$^{\text{th}}$ percentile of the ECDF. The MCS based methodology to calculate the B-value can be summarised as follows (see Figure 3):

1. Design of the experiments (DOE). The material properties and geometrical parameters are distributed according to their associated statistical distributions to define the uncertainty quantification and management matrix. Using the current analytical framework, the dimensions of the matrix are n x 5 where n are the different cases to be analysed and 5 are the model input parameters (E_{11}, X_T, $\mathcal{R}_{ssT}$, R and W).
2. Notched strength computation. For each case i the notched strength ($\bar{\sigma}_i^\infty$) is calculated using the analytical model described in Section 2.1.
3. Determination of the 10th percentile. Once all the cases have been computed ($\bar{\sigma}_{i:1\rightarrow n}^\infty$) the ECDF of the notched strengths is used to determine the $P_{10,j}$.
4. Computation of the B-basis allowable. Steps 1, 2 and 3 are repeated N times to obtain the ECDF of the $P_{10,j:1\rightarrow N}$ and to determine the 5$^{\text{th}}$ percentile which corresponds to the B-basis value.

If the sample size (n) is large enough, then the 10$^{\text{th}}$ percentile of the population can be directly approximated by the 10$^{\text{th}}$ percentile of the sample, as the variability between the samples will be minimal. This will be explored in more detail in Section 5.1.

3. Case study

3.1. Description of the case

To exemplify and validate the methodology proposed to calculate the B-value of the notched strength, IM7/8552 $[90/0/\text{-}45/45]_{3s}$ quasi isotropic laminate with a central circular hole loaded in tension was used. Hole diameter-to-width ratios of $0.05 < 2R/W < 0.6$ and hole diameters of 2, 4, 6, 8 and 10mm were used.

As explained in section 2.2, the variability associated with the material parameters and with the geometry of the specimens is considered to calculate the B-value. The input parameters used are presented hereafter.

3.2. Uncertainty quantification associated with the geometry of the specimens

The variability associated with the geometry of the specimens is directly related to the manufacturing process, namely the cutting methodology and respective allowed tolerances. The specimen dimensions were assumed to follow a uniform distribution with a maximum deviation of ±2% of the nominal value of the width and hole diamater.

Table 2: Variability of the geometry of the specimen [17].

Geometry	W [mm]	R [mm]
tol	$\pm 2\% \times W$	$\pm 2\% \times R$

3.3. Uncertainty quantification associated with the determination of the material properties

In this work, it is assumed that the material properties follow a normal distribution with known mean and standard deviation. These properties are summarised in Table 3. The uncertainty related to the longitudinal Young's modulus and strength is directly related to the the mean values ($\bar{x}$) and respective standard deviation (s) determined experimentally [18] while the variability of the $\mathcal{R}$-curve is determined as explained in section 2.2.

Table 3: Value of the material properties used for the analysis [18].

IM7/8552	E_1 [GPa]	X_T [GPa]	$\mathcal{R}_{ssT}$ [N/mm]
$\bar{x}$	171.42	2323.47	206.75
s	2.38	127.45	23.64

The determination of the $\mathcal{R}$-curve is based on the size effect law which can be determined from the strengths of geometrically similar double edge notched specimens with different widths. Table 4 shows the notched strengths and respective standard deviations of the double edge notch tension specimens that were used to determine the longitudinal crack resistance curve of IM7/8552 material system [13].

Table 4: Double Edge Notched Tension Strength for IM7/8552 $[90/0]_{8s}$ [13].

Ref.	**w [mm]**	$\bar{x}$ **[MPa]**	s **[MPa]**
B	7.5	309	9
C	10	289	16
D	12.5	269	11
E	15	256	10

Using Method 1 described in Section 2.2.1, a set of statistically representative crack resistance curves, with a known mean and standard deviation of the three fitting parameters (l_{fpz}, $\mathcal{R}_{ss}$ and β) is obtained, as shown in Figure 4.

As explained in section Section 2.2.1, the fitting parameters of the crack resistance curves cannot be treated independently as that would potentially lead to non admissible $\mathcal{R}$-curves. For this reason, a dependence between the parameters was established as a function of $\mathcal{R}_{ss}$. As shown in figure 5, it was found that l_{fpz} varies linearly with $\mathcal{R}_{ss}$ and β is almost constant for the case analysed. Therefore, the crack resistance curves can be defined as a function of $\mathcal{R}_{ss}$. $\mathcal{R}_{ss}$ is generated randomly following a normal distribution with a know mean (206.75 N/mm) and standard deviation (23.64 N/mm) and the other two parameters are estimated as:

$$l_{fpz} = 2.7776 \times 10^{-2} \times \mathcal{R}_{ss} - 3.0598 \qquad \text{[mm]}$$
$$\beta = 2.9027 \qquad \text{[-]}$$

Using Method 2 the variability is obtained from the determination of the 95% prediction bounds of the fitting of the size effect law. Either the whole set of experimental points or the mean strengths per specimen geometry can be used. In this study only the mean strengths were used since the full set of results was not available.

Since this method provides only three sets of $\mathcal{R}$-curve parameters, the relation between l_{fpz}, $\mathcal{R}_{ss}$ and β is undefined. However, using method 1, it was shown that a linear functions can be used to relate $\mathcal{R}_{ss}$ to l_{fpz} and β, and so the fitting parameters of the curves can be easily determined as a function of $\mathcal{R}_{ss}$ as shown in figure 5. Using this method, $\mathcal{R}_{ss}$ is generated randomly following a normal distribution with a know mean (205.26 N/mm) and standard deviation (14.83 N/mm) and the other two parameters are estimated as:

$$l_{fpz} = 2.8654 \times 10^{-2} \times \mathcal{R}_{ss} - 3.2701 \qquad \text{[mm]}$$
$$\beta = 2.9024 \qquad \text{[-]}$$

As shown in figure 5 the fitting curves obtained with both methods show similar trends. Figure 6 shows the normal distribution and the corresponding average and 95% IC $\mathcal{R}$-curves obtained with both methods. Only a 1.5 N/mm difference in the mean $\mathcal{R}_{ss}$ using methods 1 and 2 was found. However, since the standard deviation obtained using method 2 is around 40% lower than the one measured using method 1 because the confidence bounds were determined using the mean double edge notch strengths, the normal distribution of $\mathcal{R}_{ss}$ is significantly narrower if method 2 is used. Using the whole set of data would be preferred in method 2. Therefore, in, method 1 was used to characterize the distribution of the crack resistance curve parameters.

4. Sensitivity analysis

Due to the analytical nature of the model, it is possible to run a large number of simulations within a reasonable time frame, enabling the performance of numerical analysis that would not be possible via experimental characterization or finite element simulations.

The proposed framework depends on three material properties and two geometrical properties. It is interesting to understand their influence on the expected notched strength of the laminate selected for this study. To do so, a sensitivity analysis was performed on these five parameters. The sensitivity analysis is performed by considering that the parameter in study varies while the remaining are kept constant and with a value equal to the nominal one. Here the material and layup considered are the ones presented in Section 3 and an open-hole tension specimen with width equal to 36 mm and hole radius of 3 mm is considered. For each material property a range from $\bar{x}_i - 3s_i$ to $\bar{x}_i + 3s_i$ was considered. For the geometrical parameters a variation of $\pm 2\%$ was considered. The results of the sensitivity analysis are shown in Figure 7.

From the sensitivity analysis, it is possible to conclude that, as expected, the material properties have a larger influence on the notched strength than the geometrical properties. For the Young's modulus the variation of the OH strength is linear, being lower for lower elastic moduli. Both the tensile strength and toughness of the material have a more complex influence on the open hole strength of the material. In addition, both have a higher influence on the open hole strength of the material, therefore, it is essential to accurately characterize these properties to ensure accurate predictions of the notched strength of composite laminates.

5. UQ&M framework validation

In this section the sample sizes required to accurately take into account geometrical and material variability within the UQ&M framework is analysed and the results are validated against available experimental data.

5.1. Effect of the sample size on the mean notched strength and on B-basis value using the MCS method

To validate the proposed UQ&M methodology, it is important to analyse the number of simulations required to ensure an accurate determination of the output parameters. The fact that the framework used is fully analytical, allows a very large number of simulations to be performed, however, it is of key importance to ensure that the open-hole strength (mean and B-basis) are determined efficiently, i.e. performing the minimum number of simulations required to obtain accurate and statistically consistent results.

The methodology to determine the B-basis using MCS is described in Section 2.3. This methodology requires the computation of $n \times N$ number of simulations to determine the B-value. This may lead to a very high number of simulations, rendering the methodology computationally expensive. However, it is possible to determine the B-basis based on a smaller number of simulations if we consider $N = 1$ and have a sample size (n) sufficiently large to be representative of the population of results. With this methodology, the B-basis can be approximated by the 10^{th} percentile of the sample, therefore, reducing the number of simulations to be performed.

To determine the minimum sample size that ensures this representativeness, the sample size was varied between 10 and 100,000. For each sample size 10 random samples were obtained to compute both the average and standard deviation of the mean open hole strength ($\bar{\sigma}^{\infty}$) and the respective B-basis (P_{10}). Figure 8 shows the convergence analysis of both the average OH strength and B-basis.

Table 5: Mean value and variance of the average OH strength ($\bar{\sigma}^{\infty}$) and B-value (P_{10}) according to the number of samples when $N = 10$.

Samples, n	$\bar{\sigma}^{\infty}$		P_{10}	
	$\overline{\sigma^{\infty}}$ [MPa]	$s_{\sigma^{\infty}}$ [MPa]	$\overline{P_{10}}$ [MPa]	$s_{P_{10}}$ [MPa]
10	450.77	3.11	436.83	6.15
50	452.61	0.62	440.15	2.90
100	452.54	0.91	441.33	2.52
1000	452.68	0.24	441.86	0.53
10000	452.63	0.11	441.65	0.21
30000	452.62	0.04	441.64	0.13
100000	452.62	0.03	441.64	0.05

Analysing the data, it is possible to conclude that the variability of both the mean OH strength and B-basis is reduced with increasing sample size, however, the computational cost increases. It is possible to conclude that for a sample size of 10,000 the Coefficient of Variation (CoV) of both the mean OH strength and B-basis is very low, 0.02% and 0.05%, respectively. Therefore, a sample size of 10,000 can be considered as representative of the population of results and be used to obtain the average OH strength and respective B-basis. If we consider a sample size of 30,000, which has a three times increase in computational time, there is an insignificant reduction in the CoV for the mean strength and B-basis (to 0.01% and 0.03% respectively). Therefore, it is concluded that a sample size of 10,000 is large enough and ensures a good compromise between the accuracy and computational cost.

To summarize, the calculation of the B-basis allowable using MCS can be done in a computationally efficient way by running 10,000 simulations ($N = 1$) and determining the 10^{th} percentile of the sample as this number of samples is considered representative of the whole population. This methodology will be considered for the determinantion of the B-basis allowables using Monte Carlo simulations in the following sections.

5.2. Effect of the sample size on the B-basis using the CMH-17 approach

In Section 2.3 the methodology to determine the B-basis allowable based on the CMH-17 was presented. In this section a comparison between the B-basis determined using this methodology is compared with the results obtained using Monte Carlo simulations. The CMH-17 approach is useful since it takes into account the size of the population and the distribution that most accurately represents the data to determine the B-basis and therefore, a good estimate of this parameter can be obtained using a small number of data points.

In Figure 9 the B-basis allowable for OH strength determined using the CMH-17 methodology for different sample sizes is shown and is compared with the value obtained using MCS. For each sample size, 100 simulations were performed based on different randomly generated samples, to get not only an average value for the B-basis but also to determine its dispersion for each sample size.

As the sample size increases, the B-value determined with the CMH-17 approach becomes less conservative and the confidence interval is reduced, as bigger samples are considered more representative of the population. In Table 6 the results of the B-value are also shown for different sample sizes. In addition, the methodologies from the CMH-17 that were applied for each sample are shown, as different distributions were seen to best fit the data depending on the sample considered.

For a sample size of 30 it is seen that the variability of the calculated B-basis increases. This increase in variability with increased sample size can be justified with the fact that for the mentioned sample size, there was an increase in the number of samples that could not be represented by a Weibull distribution (see Table 6) and, therefore, a different distribution had to be used, or even the non-parametric methodology, which increased the dispersion in the determination of the B-basis.

In the remainder of this study, a sample size of 25 is considered when determining the B-basis with the CMH-17 methodology, as it is seen to be a reasonable sample size, which might be used in experimental campaigns, that ensures a good B-basis estimation.

5.3. Validation of the UQ&M framework

A comparison between the experimental results presented in Ref. [18] and the predictions using the proposed framework is shown in Figure 10. Both the OH strengths computed using the nominal values of the material and geometrical properties and the results obtained when these properties are considered stochastic are included. The latter methodology allows not only to obtain the average value for OH strength for each geometry but also the expected variability.

As expected, using the nominal values of the geometrical and material parameters results in approximately the same open hole strength as the average of the stochastic results, ensuring the consistency of the uncertainty quantification framework developed. The results shown in Figure 10 indicate that the proposed

Table 6: Results for the B-basis determination using the CMH-17 methodology.

Samples, n	Weibull	Normal	Lognormal	Non parametric	$\bar{B}$ [MPa]	s_B [MPa]	IC_B [± MPa]	error %
5	92	8	0	0	411.35	17.992	3.5696	6.858
10	93	3	0	0	428.77	9.5808	1.9008	2.9152
15	88	3	0	0	431.84	8.0774	1.6026	2.2193
20	94	2	0	0	434.78	4.2685	0.84686	1.553
25	94	2	0	0	435.92	4.3825	0.86948	1.2953
30	91	4	0	5	438.17	7.051	1.3989	0.78675
40	94	3	0	3	437.74	3.4001	0.67458	0.88285
50	92	5	0	3	437.24	3.2488	0.64456	0.99621
100	78	5	0	17	438.76	2.7482	0.54525	0.65224
150	91	1	0	8	439.43	1.5205	0.30167	0.50112

framework is capable of accurate predictions of the open-hole tension strength. The maximum error obtained for this case study was 12% which, taking into account that this is an analytical formulation with very reduced computational cost, is very reasonable.

As the developed framework is aimed at the determination of the B-basis allowable for open hole strength, the comparison between the B-basis obtained analytically, with the two presented methods, and experimentally is shown in Figure 11. For consistency, as the experimental sample size used was 5 specimens [18], the same sample size was considered when computing the B-value with the CMH-17 approach. This allows a direct comparison between the experimental B-basis and the one obtained numerically. Nevertheless, the results with a sample size of 25 are also shown. To ensure that the results obtained did not result in outliers, 10 B-basis calculations were performed for each geometry. For the Monte Carlo simulations approach a larger number of simulations is always required to ensure the representativeness of the population, therefore, the sample size was kept at 10,000.

Observing the previous results it is concluded that the B-value determined with the CMH-17 approach is similar to that obtained experimentally, for the same sample size ($n = 5$), which reflects not only the ability of the framework to accurately compute the open hole strength of a given configuration, but also its ability to propagate the uncertainty of the input parameters to the open hole strength. The B-basis obtained with the MCS approach is always less conservative than the one obtained with the CMH-17 approach due to the larger sample size, which is reflected in the results of Figure 11 and was also obtained in the numerical comparison provided in Figure 9. The same sample size effect can be observed comparing the CMH-17 approach with $n = 5$ and $n = 25$. Nevertheless, the results obtained are consistent with the experimental ones.

6. Applications

6.1. Design charts for open hole tension

Taking into account that the analytical UQ&M framework developed enables the quick estimation of the notched strength of laminated composites and the respective B-basis allowables, it can be used to generate design charts and compare the performance of different layups and materials in a preliminary stage of the design process.

Following Camanho et al. [9], design charts that relate the diameter-to-width ratio to the notched tensile strength of specimens with diameters 2, 6 and 10mm were generated. Monte Carlo simulations with $n = 10,000$ were used to generate the average notched strength distribution of each point and compute the mean value and respective B-basis allowable, as defined in Section 5.1 (Fig. 12). To calculate the B-value, the CMH-17 approach could also have been used without significant loss of accuracy as shown in Fig. 13 for a specimen with a hole radius of 6mm, however, given the computational efficiency of the model, performing Monte Carlo simulations is not a particularly limiting approach.

Experimentally generating statistically representative design charts is unreasonable given the number of specimens, specimen configurations, layups and materials required to populate them. The analytical UQ&M framework here proposed can help overcome this limitation and assist engineers during the design process given its simplicity and efficiency.

6.2. Influence of the load direction on the open hole strength

The framework was developed to work as a fast design tool that is capable to predict the notched strength of a laminate in the most varied cases. In this section, the variation of the loading direction and its effect on the open hole tensile strength is explored. The design of a laminate for a given structure is usually optimized for a given load direction, however, it is not acceptable to have a laminate whose strength is very high in one direction but any misalignment in the load, which most certainty occurs in real usage, leads to a high reduction of its strength. Therefore, being able to rapidly predict the notched strength in a multitude of loading directions is an useful design tool. The variation of the mean open hole strength as a function of the load direction and the respective 95% confidence interval and predicted B-basis value based on MCS ($n = 10,000$) and on the CMH-17 ($n = 25$) are shown in Figure 14. This analysis was done for the baseline configuration of a width of 36 mm and a radius of 6 mm.

Due to the fact that the laminate in study is quasi isotropic (Section 3), the notched strengths at 0, 45 and 90° are equal. However the strength is reduced for any other load direction. From the shown results it is possible to conclude that with the given laminate the reduction of strength due to changing the load direction is small, being the lowest value equal to 377.1 MPa, while the maximum (for 0, 45 and 90°) is equal to 455.0 MPa. Additionally, it is observed that small variations around the principal load direction (0°) have only a small effect on the notched strength. Regarding the B-basis allowable it is seen that for the analysed cases the results from the CMH-17 and MCS approach are similar. It is interesting to note that the difference between the B-basis and the mean value for the open hole strength is not constant throughout the angle space. This difference is highest when the average strength is lowest, which creates a wider span of the B-basis allowable between its maximum and minimum. This can be explained by the fact that at these load angles the variability of the material and geometrical parameters leads to a higher variability of the notched strength and, therefore, a reduced B-basis allowable.

6.3. Large damage capability

The proposed framework was developed with the aim of predicting the open-hole strength of laminate structures, however, it is general enough to be able to predict the strength of different notched geometries, provided the stress distribution and energy release rate are known for those geometries and loading conditions. As it is well known, the tensile strength of composite laminates in the presence of through-the-thickness notches is significantly affected by size, being the smallest geometries strength-dominated and large ones toughness-dominated [19]. Therefore, the analysis tools must be able to account this distinct material behaviours when computing the notched strength. Following Arteiro et al. [19] the developed framework is used to predict the large damage capability of the laminate in study, considering a centre notched plate under tension loading (Figure 15).

In Figure 16, the mean notched strength and respective B-basis allowable of centre notched plates with a constant plate width-to-notch lentgth ratio ($W/2a$) equal to 7.5 with different notch sizes are shown. The notches were considered to have a constant tip radius of 0.5 mm ($h = 1$ mm). For the smaller geometries the traditional methods that only consider the steady state value of the fracture toughness in their formulation are able to predict the notched strength, however, for larger specimens and large damage capability analysis the introduction of the R-curve in the modelling strategy is of utmost importance [19]. This is taken into account in the present framework, which increases the reliability of the modelling strategy. It is possible to see in Figure 16 that both the mean notched strength and its respective B-basis allowable follow the same trends, being the difference between both parameters similar throughout the analysed space.

In this study, two notched geometries are analysed, open-hole tension and centre-notched tension, however the framework is generic enough to take into account other geometries such as open-hole compression and bolted joints failing by net-tension [20], given that the stress concentration factors and energy release rates of the configuration in study are known.

7. Conclusions

The current approach to determine the design allowables in the aeronautical industry relies in extensive testing based on the building block approach, which makes the selection and certification of composite materials expensive and time consuming. To increase the efficiency of material and laminate selection during preliminary design, there is a need to reduce the number of experimental tests required during this process, and replace or complement them with accurate modelling strategies coupled with the statistical tools to account for material, manufacturing and geometrical variability.

In this work an UQ&M framework was developed to estimate the B-basis design allowable for notched components. This framework is based on the analytical model developed by Furtado et al. [8] that only requires three lamina level material properties to estimate the notched strength of a laminate, given that the stress distribution and energy release rate are known for the geometries and loading conditions in study. This model is coupled with the statistical tools required to take into account the variability of both the material and geometrical parameters and propagate this uncertainty to the notched strength, therefore allowing the quick estimation of B-basis allowable.

The developed framework allows the computation of the B-basis allowable based on Monte Carlo simulations and on the the approach proposed in the CMH-17, which requires a lower number of samples. Both approaches are compared and it is concluded that the CMH-17 gives a more conservative estimation of the B-basis allowable due to the lower number of samples usually used. Given that the current modelling strategy is computationally efficient, the usage of Monte Carlo simulations allows the estimation of a less conservative B-basis as a large number of samples can be computed in a reasonable time frame. This makes the proposed framework specially interesting in the preliminary design and selection of materials and layups.

The proposed framework is validated successfully with the open-hole tension experimental campaign for the IM7/8552 material [18], ensuring a maximum error around 10%, which is very reasonable given the analytical formulation of the model.

Additionally, the framework is used to develop design charts for notched specimens, tools that are useful for design engineers and would otherwise be infeasible to attain as they require a large number of testing or time consuming simulations to be performed.

Note that in this paper, the methodology is applied to open hole tension and center notched specimens, but the framework can be enriched with other notched configurations, provided the stress distribution and energy release rate are known for those geometries and loading conditions.

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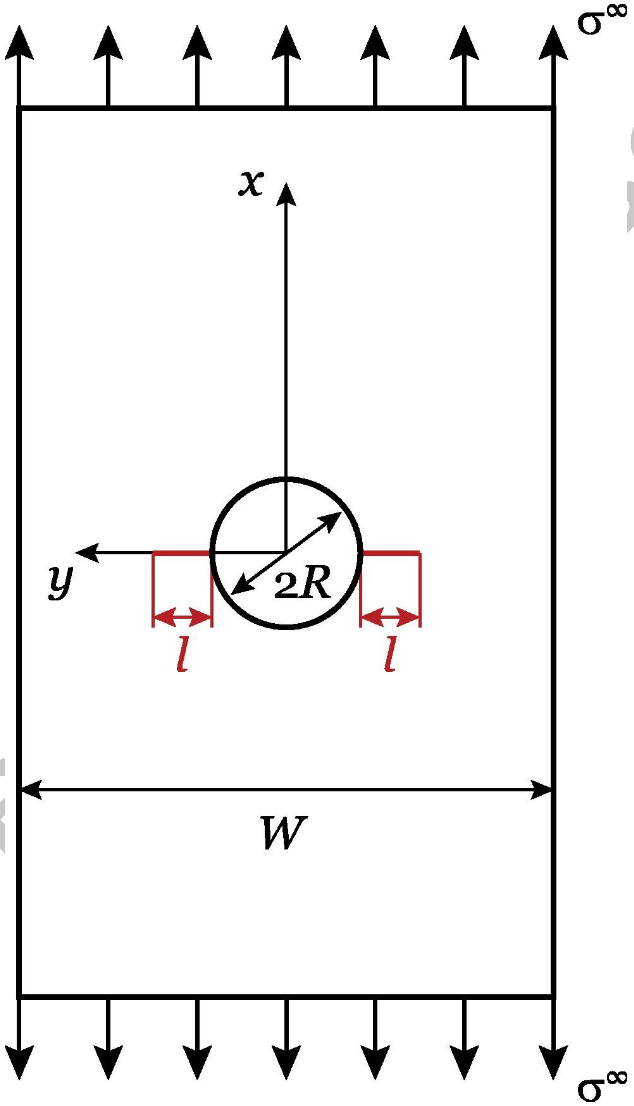


Figure 1: Notched laminate with central circular open hole [8].

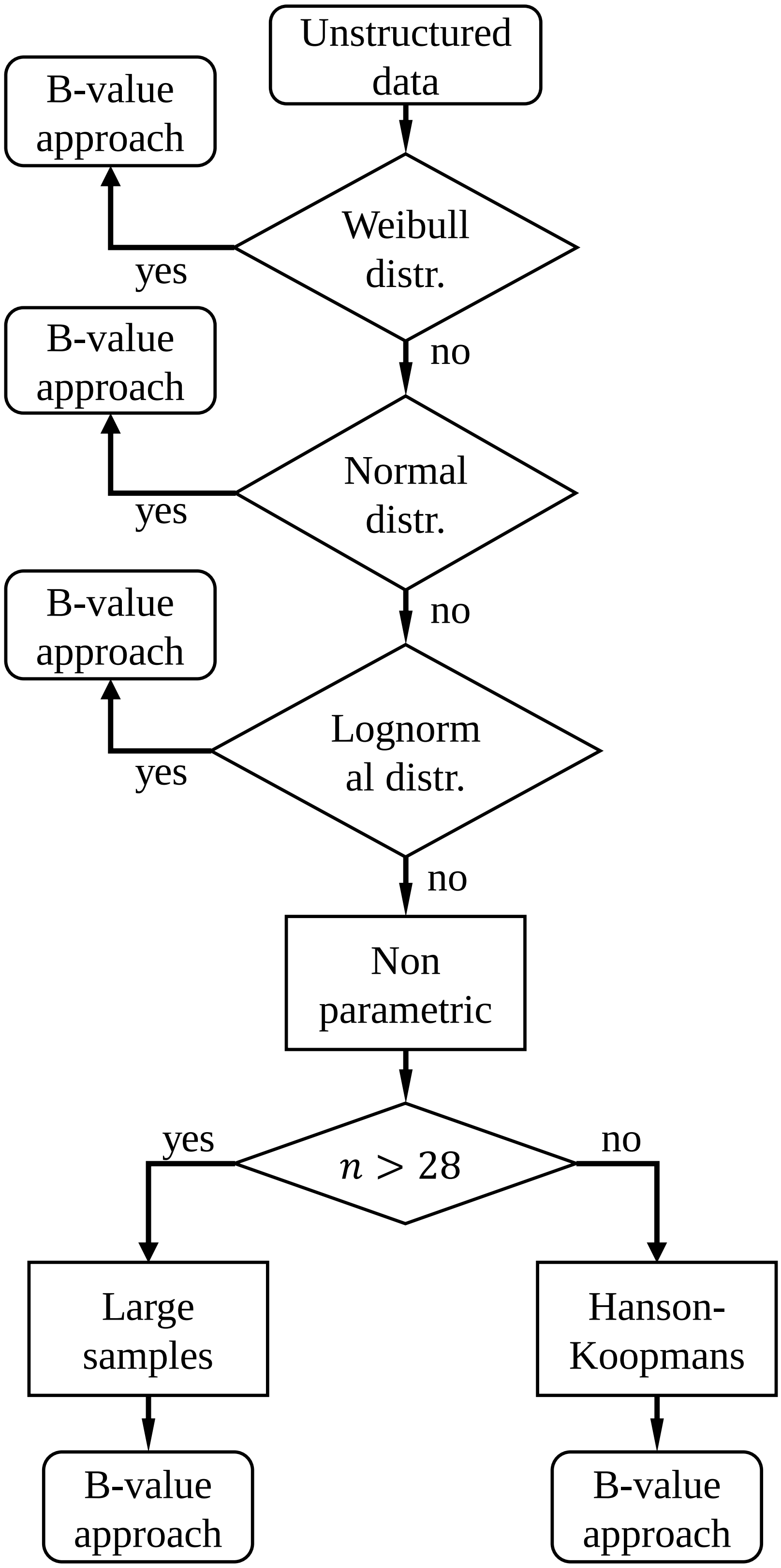
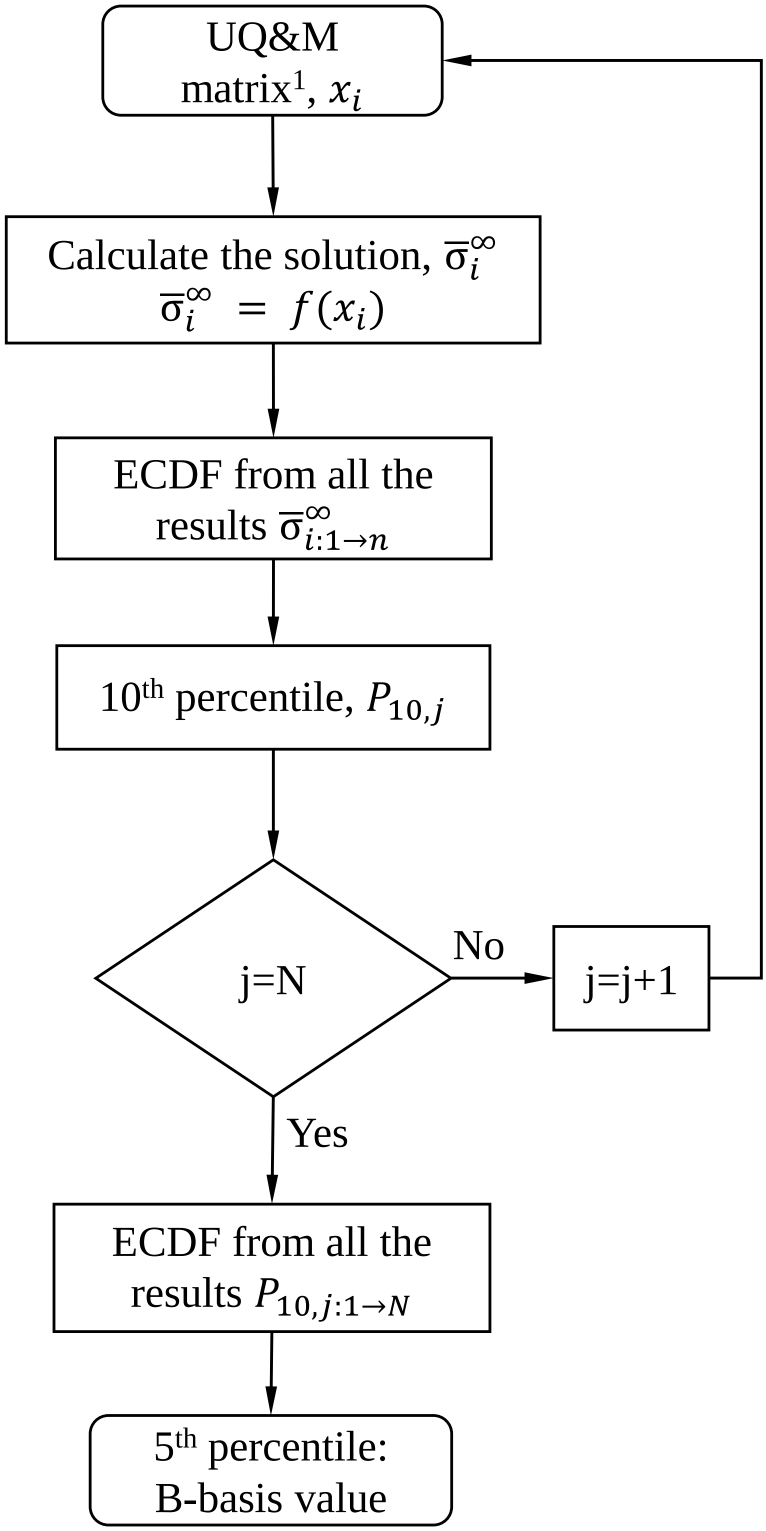


Figure 2: Schematic representation of the steps to calculate the B-value using the CMH-17 methodology.



[1] UQ&M matrix dimensions are: n different cases per 5 input variables (x_i); in the first iteration $j = 1$.

Figure 3: Schematic representation of the steps to calculate the B-value using the Monte Carlo based methodology.

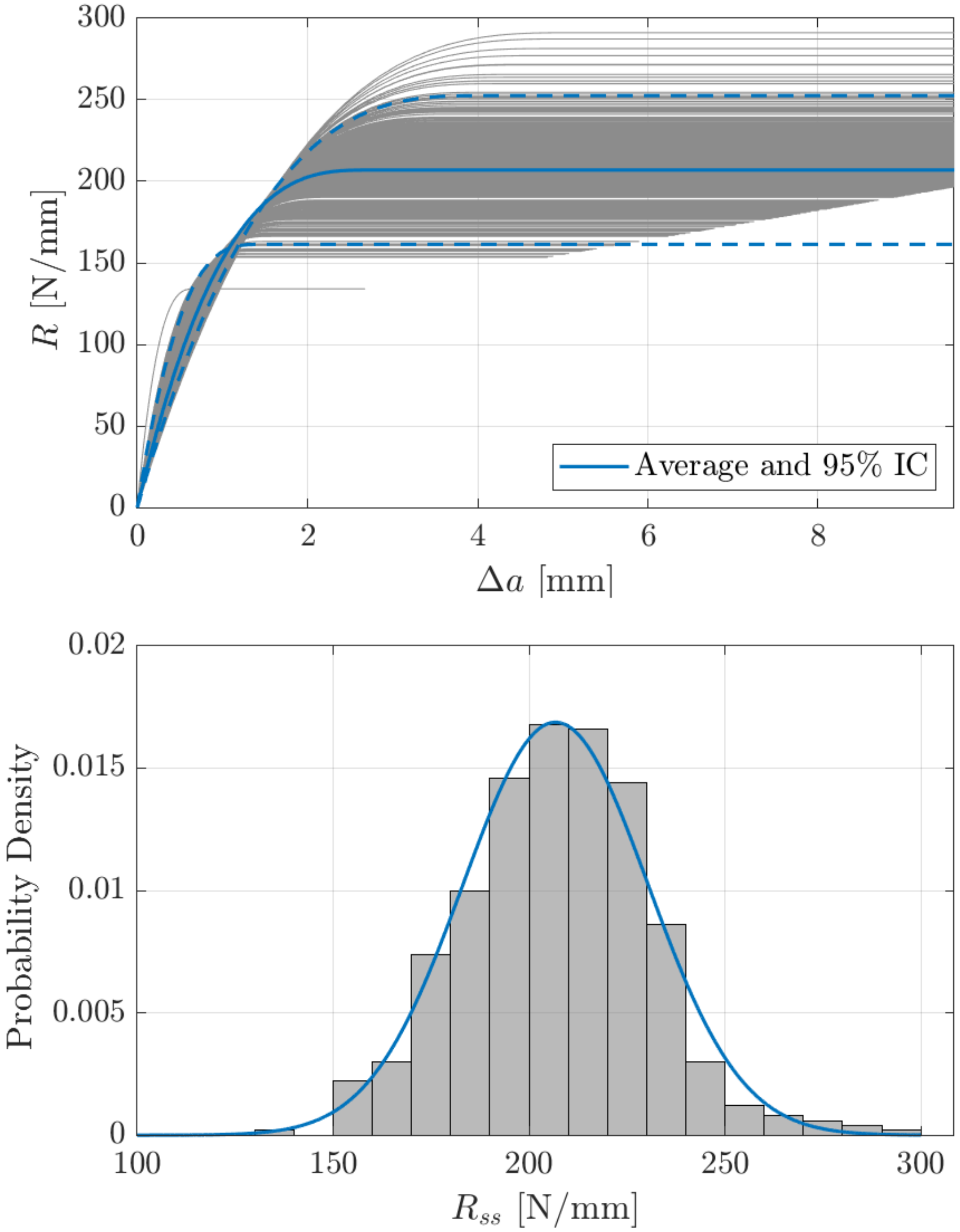


Figure 4: Schematic representation of randomly generated $\mathcal{R}$-curves using method 1 (top) and distribution of the steady state fracture toughness R_{ss} (bottom).

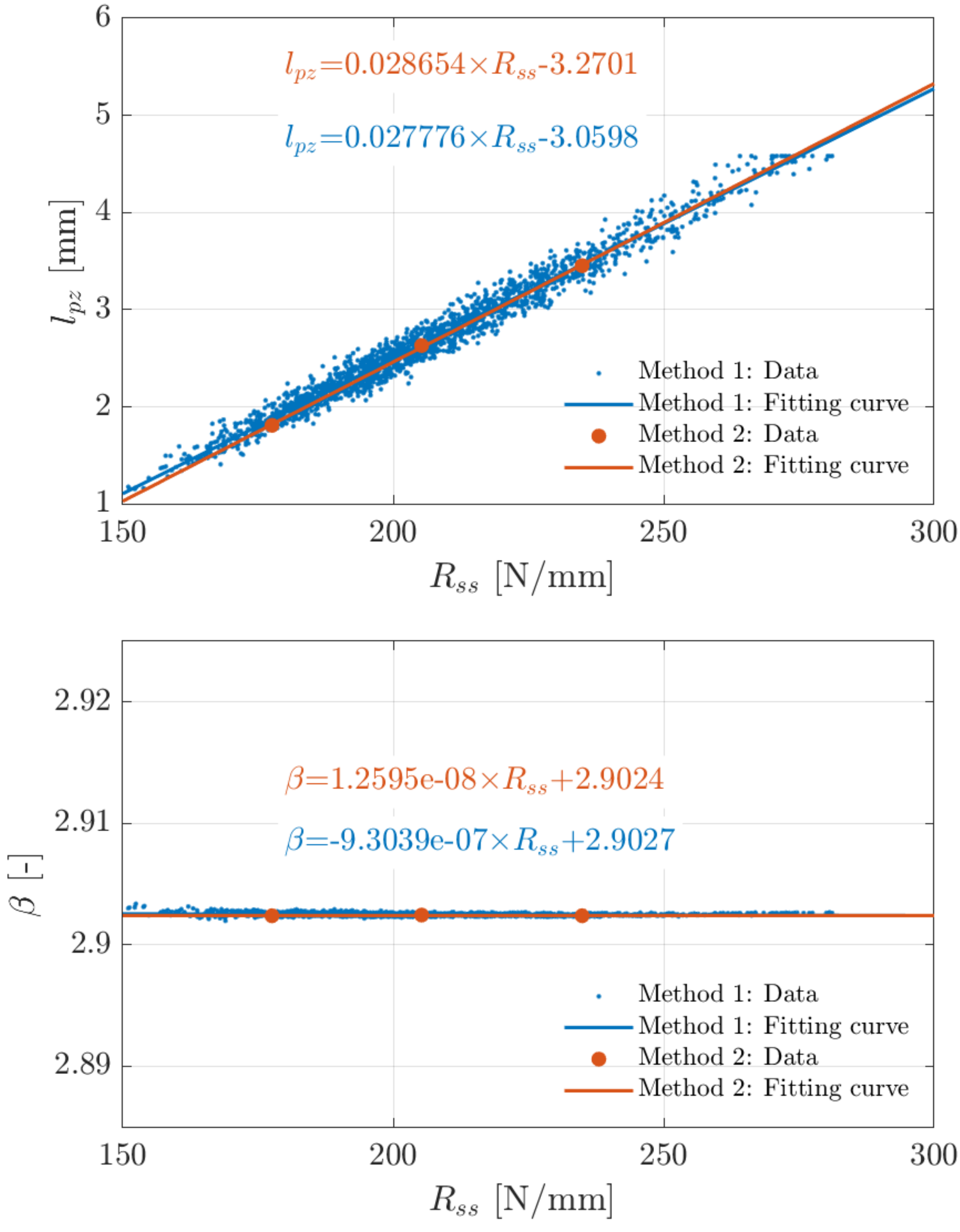


Figure 5: l_{fpz}= f($\mathcal{R}_{ss}$) (top) and β= g($\mathcal{R}_{ss}$) (bottom) obtained with method 1 and method 2.

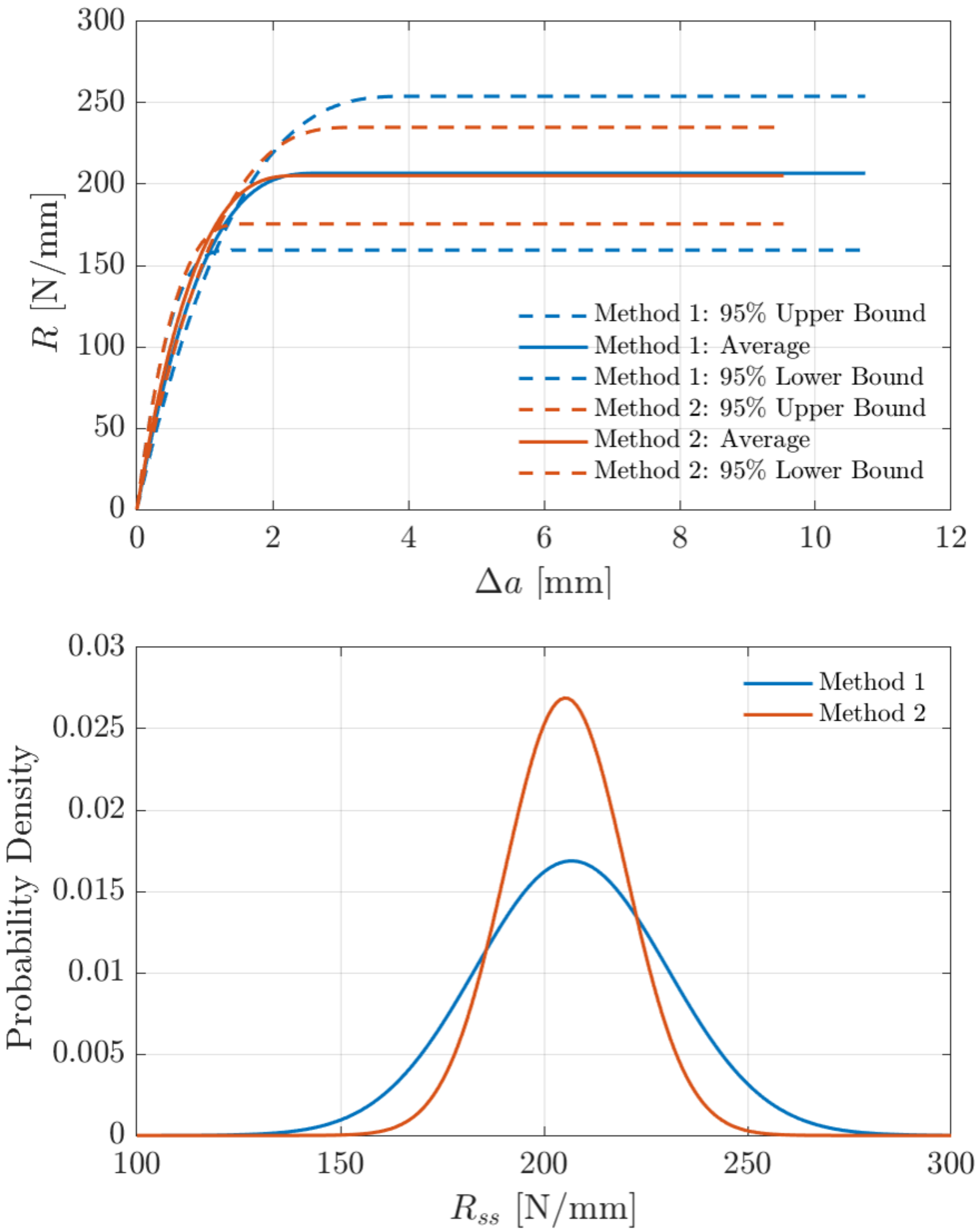


Figure 6: Average and 95% confidence bounds $\mathcal{R}$-curves (top) and predicted normal distribution of $\mathcal{R}_{ss}$ using method 1 and method 2 (bottom).

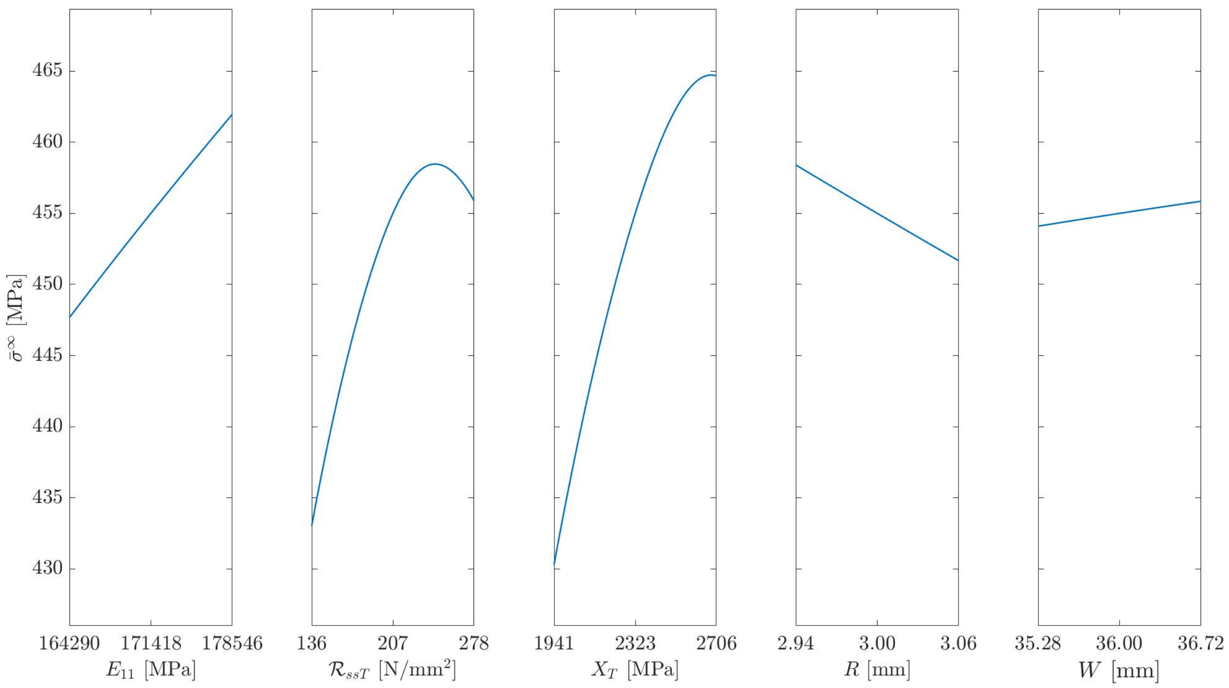


Figure 7: Sensitivity analysis on the notched strength for W = 36 mm and R = 3 mm.

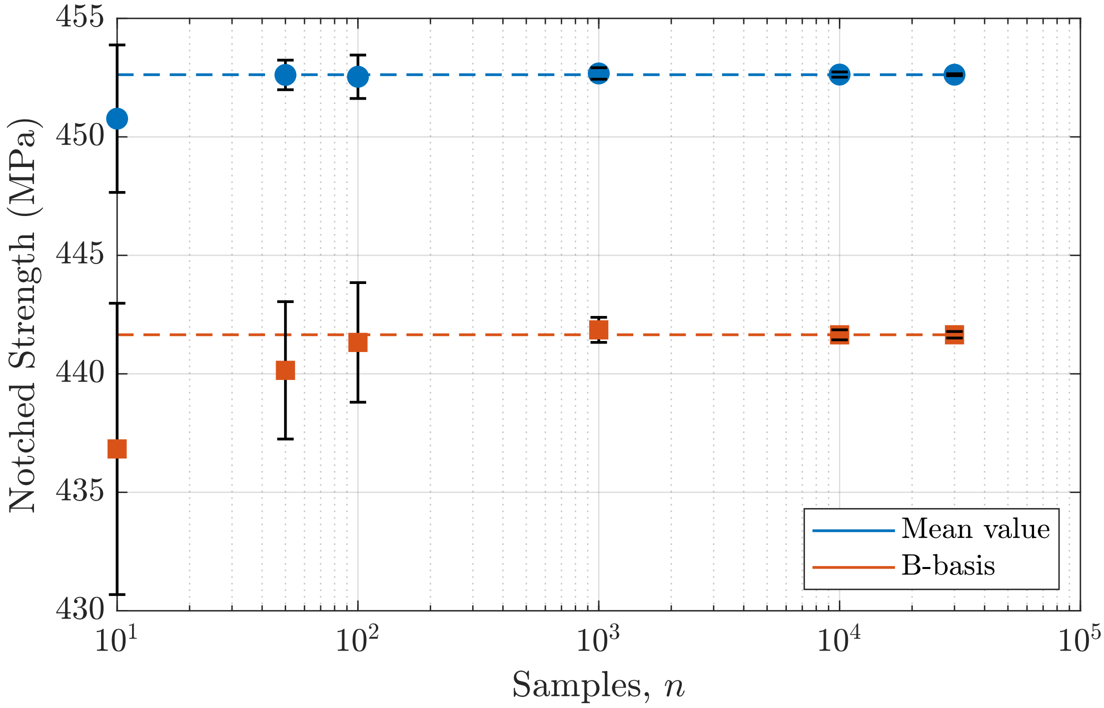


Figure 8: Average OH strength and 10th percentile from $N = 10$ simulations determined from different number of samples n.

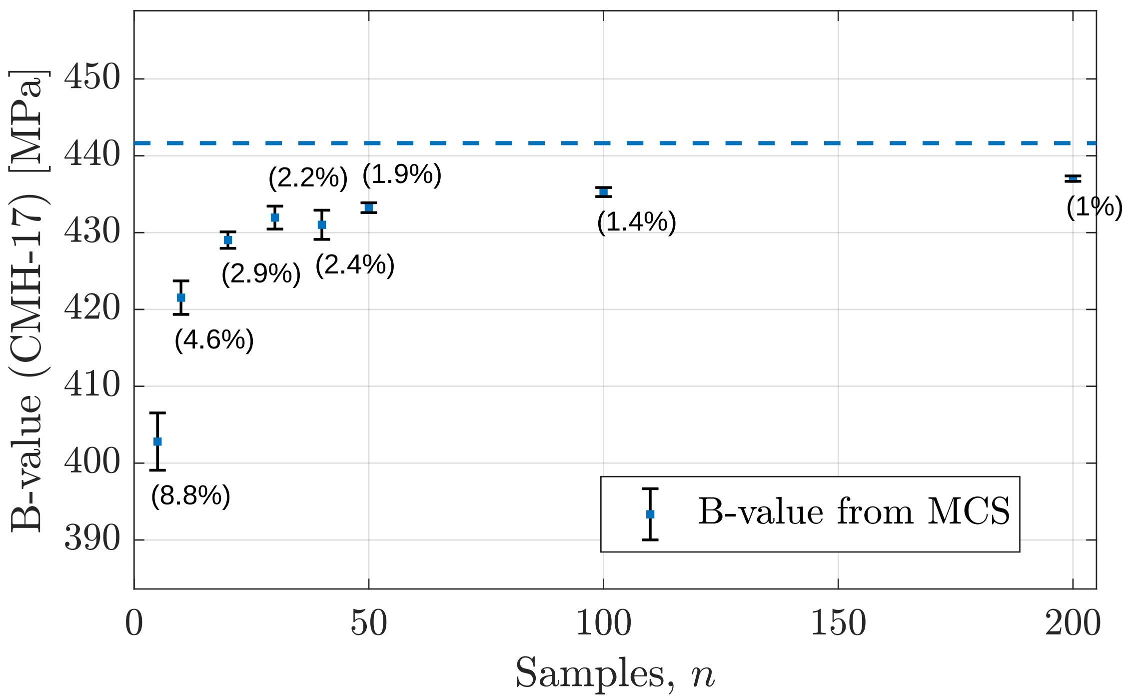


Figure 9: Comparison of the b-value obtained from the CMH-17 approach with its 95% interval of confidence, for different sample sizes (n) and the B-value obtained from MCS (dashed line).

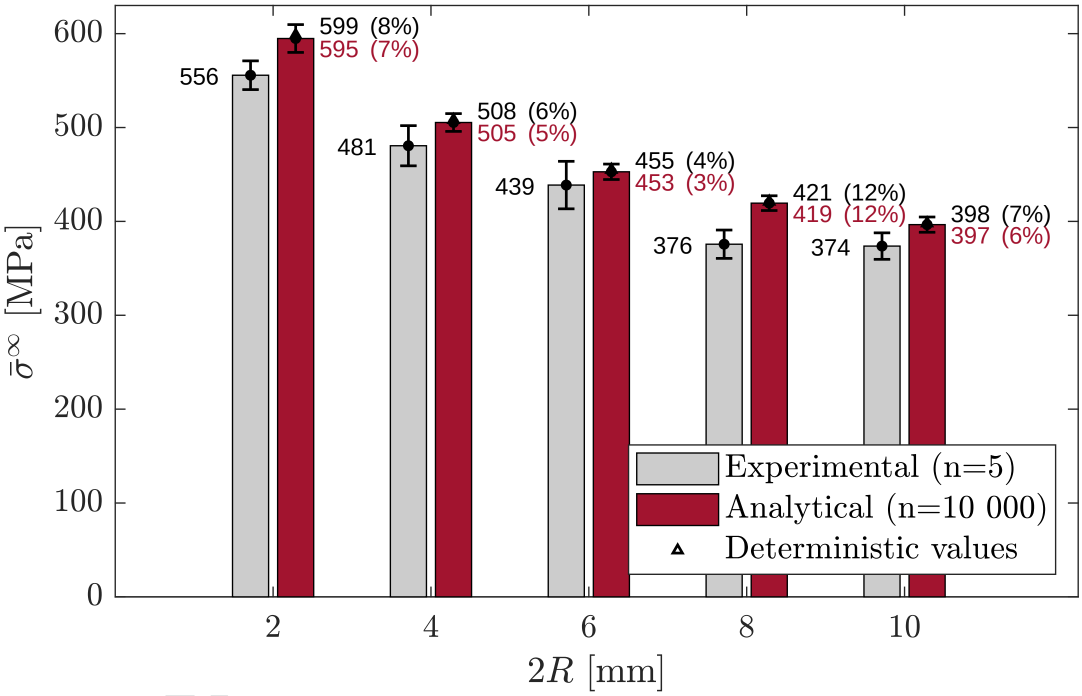


Figure 10: Comparison between the mean open hole strength of experimental results [18] and the analytical results of five different $2R$ and a fixed ratio $2R/W = 1/6$, where $\bar{\sigma}^{\infty}$ is the notched strength, R the radius of the hole and W the width.

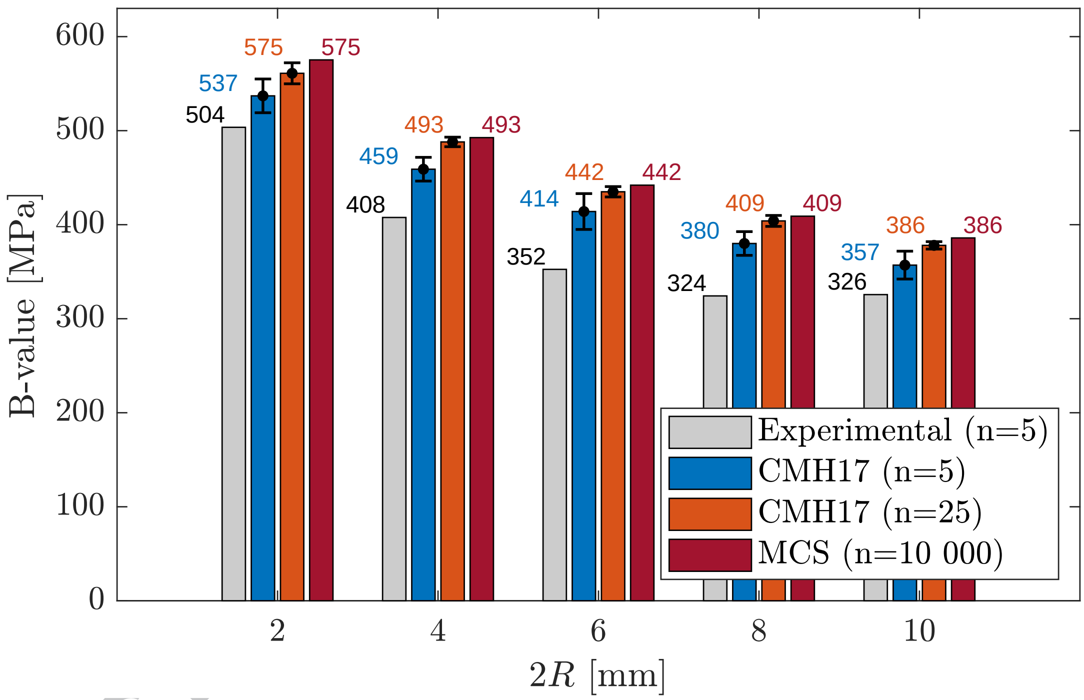


Figure 11: Comparison between the B-value obtained experimentally ($n = 5$), with the CMH-17 ($n = 5$ and $n = 25$) and with the MCS method ($n = 10,000$).

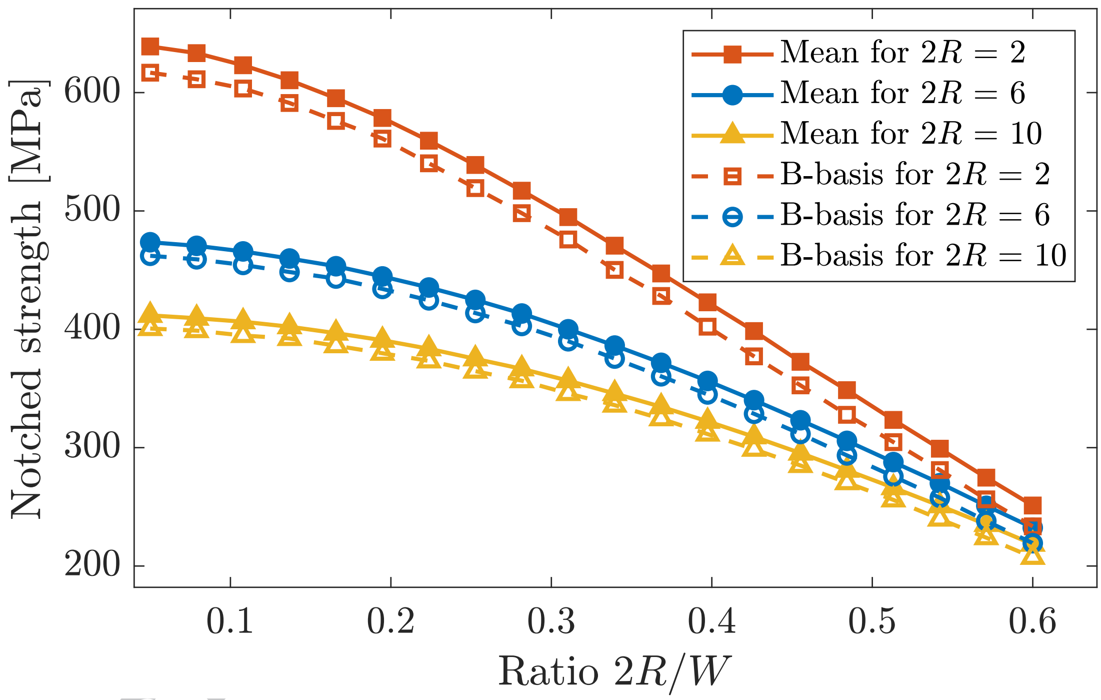


Figure 12: Design chart of the mean and B-basis value of the open hole strength calculated by means of MCS for different $2R$ and $2R/W$ ratios.

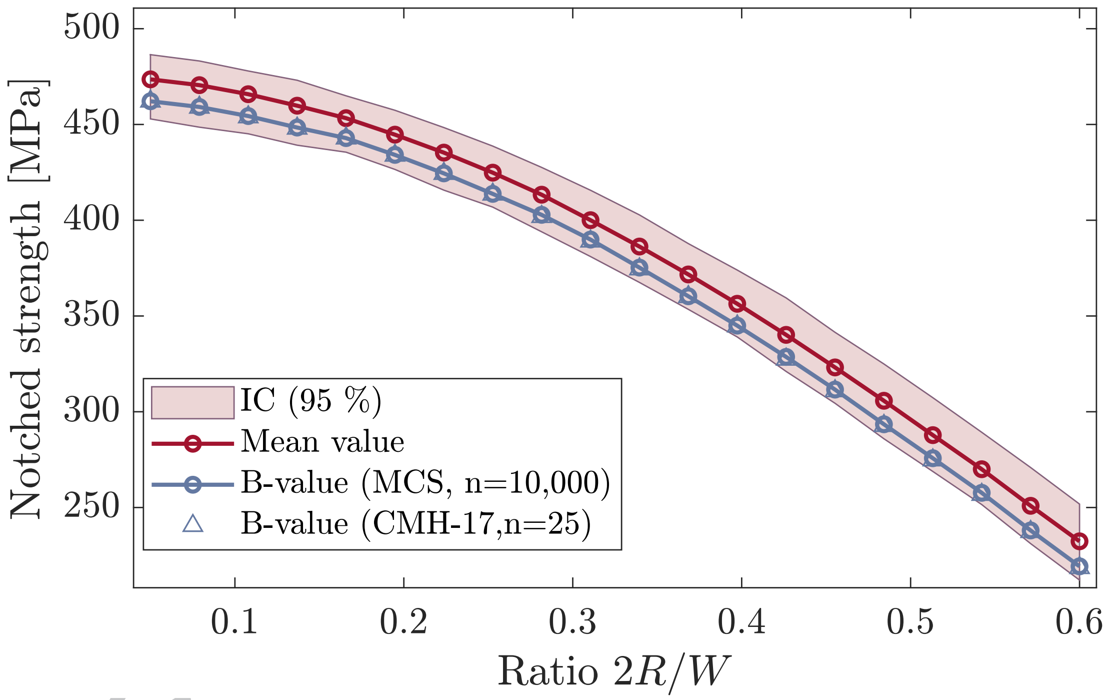


Figure 13: Design chart of the notched strength for $2R = 6$ mm.

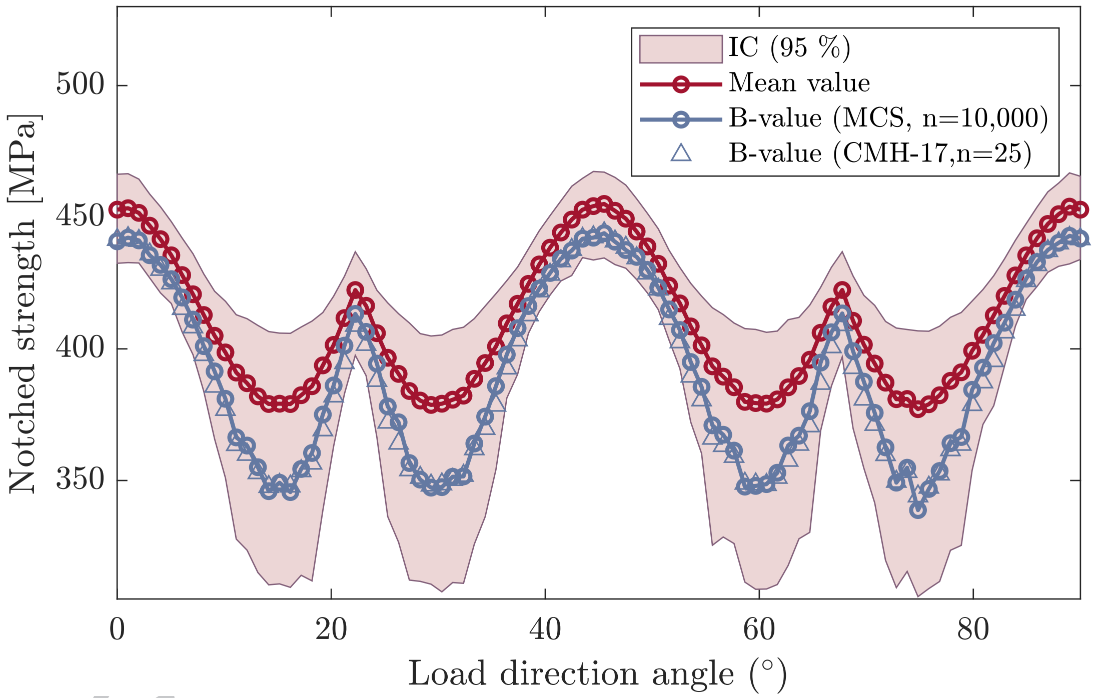


Figure 14: Notched strength variation with the load direction.

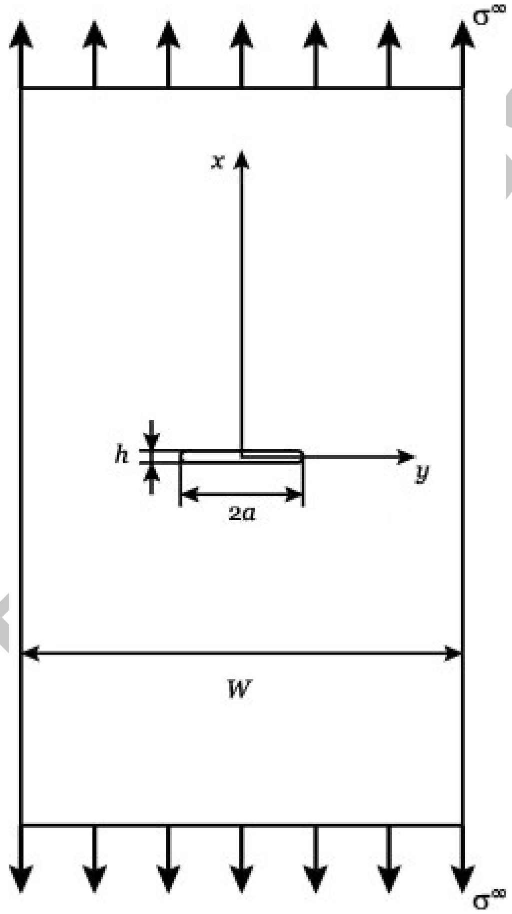


Figure 15: Centre notched plate configuration [19].

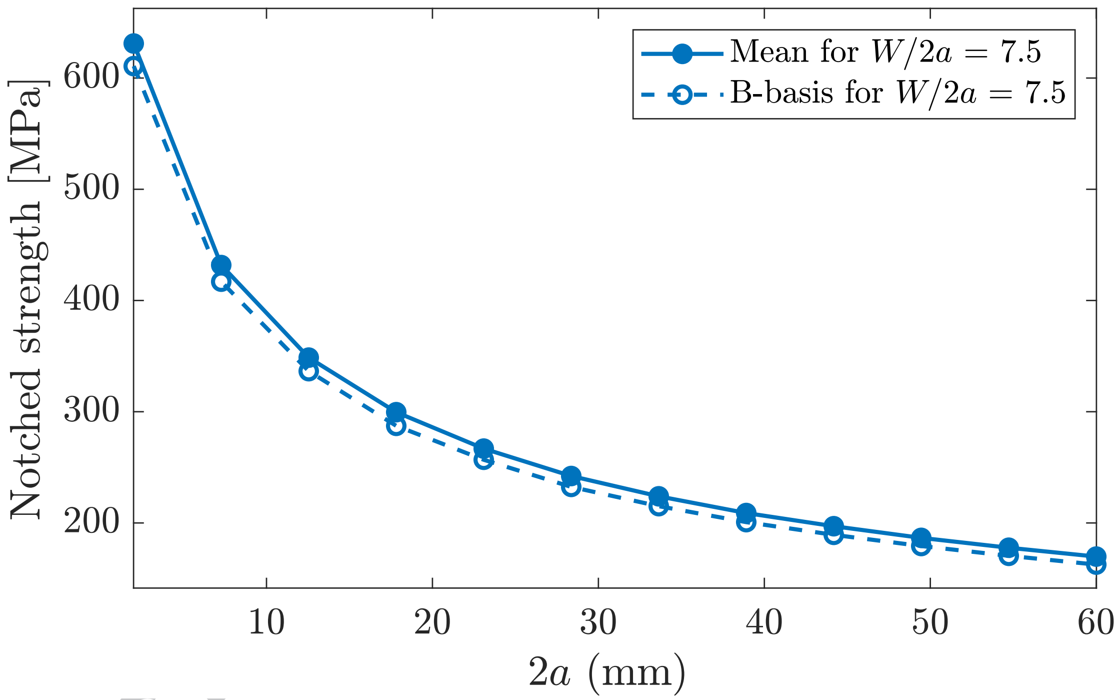


Figure 16: Design chart of the mean and B-basis value of the notched strength calculated by means of MCS ($n = 10,000$) for centre notched plates.