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# Fatigue damage of materials and structures assessed by Wöhler and Gassner frameworks: recent insights about load spectra for the automotive

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## Abstract

High cycle fatigue behaviour of materials and structures is widely assessed with respect to constant amplitude loading, because of extreme simplification of experimental procedures and numerical computations. Nevertheless, fatigue strength is usually supposed to guarantee in-service integrity face to variable amplitude loading. As the fatigue response may be very sensitive to the load time history, the gap between the former two points of view results in a long-lasting debate among the fatigue community, implying the use of a damage model and the choice of a framework. The pioneering Wöhler model is the natural reference when dealing with constant amplitude loading, whereas standard load spectra are widely applied since the seminal work of Gassner.

This paper takes advantage from fruitful discussions between German & French automotive researchers. It presents an appreciation of the Gassner model, usually applied by German engineers since at least 50 years, with the point of view of French engineers, more used to the Wöhler model.

Neglecting any sequence effect of the load time history, thus accepting the Palmgren-Miner's rule as an assumption, we focus on the dual representation of high cycle fatigue behaviour expressed by Wöhler & Gassner, respectively. This necessary calls for the use of load spectra (in terms of amplitude, not frequency), which are here discussed with particular attention to the automotive field.

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## Nomenclature

$B$	material constant of Basquin model
$K$	material constant of Gassner model
$H$	cumulative load cycle number
$H_0$	overall load spectrum length
$N$	load cycle number
$R$	load ratio
$S$	load cycle amplitude (i.e. half of the cycle range)
$S_{min}$	minimum load cycle amplitude of a spectrum
$S_{max}$	maximum load cycle amplitude of a spectrum
$SSF$	spectrum shape factor
$b$	Basquin slope
$k$	Gassner slope
$r$	amplitude load ratio
$\nu$	load spectrum parameter or shape exponent

## 1. Introduction

This paper addresses fatigue phenomenon applied to automotive passenger vehicles. In particular, we focus on the chassis system, i.e. front and rear axles as highlighted in figure 1. The chassis system is composed of safety parts, whose reliability with respect to fatigue is expected to be proven, by numerical computations and experimental tests, respectively. Fatigue is not the only design concern, nevertheless it is one of the most relevant indeed: in terms of orders of magnitude, considering a life target of about  $10^5$  km, the customer usage analysis leads to count up to  $10^6$  braking and cornering events, thus  $10^6$  cyclic load occurrences, thus calling for high cycle fatigue. For the sake of simplicity, within this framework we pay attention on usual driving situations only, coming from everyday life customer usage, thus neglecting any overload related to special driving conditions or misuse.



Fig. 1. Automotive chassis system: front and rear axles of a passenger car, Peugeot 508 (left) and Peugeot RCZ (right).

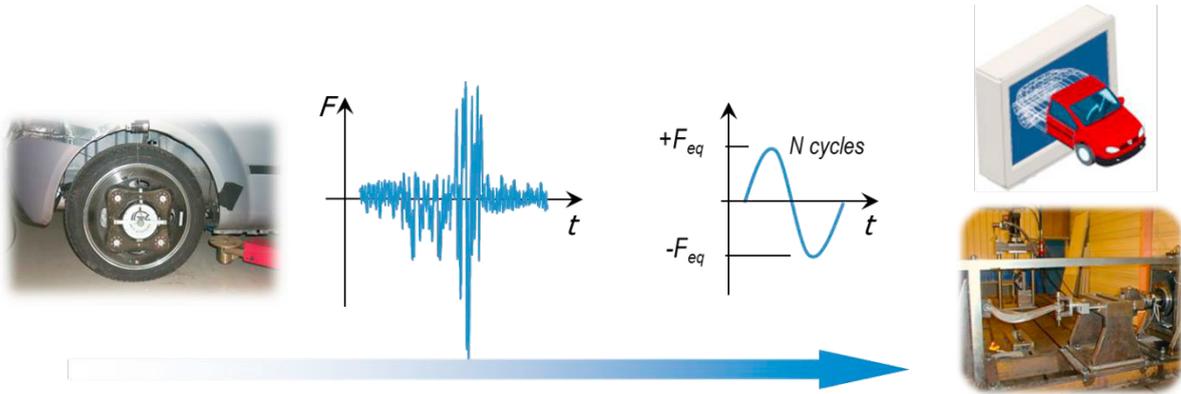


Fig. 2. Road load data: time series (left) towards an equivalent sinusoidal signal (right).

On one hand, in-service loading of chassis parts leads to complex time histories, e.g. recorded by force transducers applied to the wheel hub and active during driving sessions, see figure 2, left side. On the other hand, it may be really useful to reduce a long time history into a single sinusoidal signal of given amplitude (and frequency), the simplest to be managed by a testing rig or a FE solver, see figure 2, right side. This may be achieved thanks to a fatigue damage equivalence process, calling for at least three basic problems:

1. identify load cycles (i.e. sinusoidal patterns of defined mean, amplitude) from a time history
2. take into account for any sequence effect, i.e. the relevance of the cycle load order
3. compute a relevant fatigue damage induced by load cycles of different amplitude (and mean)

For the first step, there is a large consensus since the work of Endo & al. [1] and the application of the well-known “rainflow” count method, which has already been published in international standards. This paper simply accepts this state-of-the-art as an assumption. The second one is still under vigorous discussion: on one hand, there are the advantages and shortcuts of the simplest linear rule proposed by Palmgren and Miner [2,3] nearly 100 years ago; on the other hand, there exists several more recent and complex models for which the material parameters need for complex identifications as well. In this paper, we just assume and apply the simplest model: please remember that the in-service load history of a passenger car is very long (the order of magnitude is of about 10 years) and somewhat random, from customer to customer and with respect to any possible normal usage operating conditions.

At last, the fatigue response may be very sensitive to the load time history, the gap between variable and constant amplitude points of view results in a long-lasting debate among the fatigue community. In fact, there exists some international congress especially devoted to this very topic! This paper humbly presents some insights between two frameworks applied by the automotive industry in France and Germany, respectively. The pioneering Wöhler model [4] is the natural reference when dealing with constant amplitude loading and represents the historical point of view of French engineers, whereas standard load spectra are widely applied since the seminal work of Gassner [5] and applied by German engineers since at least 50 years.

The key features beyond this apparently nothing but theoretical discussion are practical indeed: it is matter of effective fatigue assessment of chassis parts through numerical computations and experimental tests, quotation of the relevance of a proving ground, acceleration of testing procedures ...

This paper is organized as follows. At first, we state all the assumptions and recall the basic models used (§2). Then, a historical overview on standard load spectra is recalled from the literature, with particular interest on chassis system application (§3). Thus, the very beginning of load spectra comparison between German and French standards is presented and commented (§4). At last, discussions and conclusions end the paper (§5).

## 2. Assumptions & Models

We consider the high cycle fatigue phenomenon [6,7], i.e. applying cyclic loads below the elastic to plastic behavior threshold of the materials used for chassis system parts. Nowadays it is of common practice to represent the load cycle amplitude,  $S$  with respect to the life cycle number,  $N$ , see figure 3. Actually, August Wöhler [4] did it for the first time by considering the  $S$  axis as the independent variable in ordinate, whereas the  $N$  axis was the dependent variable, as a result of his tests. Following Wöhler, constant amplitude fatigue tests provided up to final failure (or any other relevant stop criterion) lead to the so-called Wöhler curve, see figure 3. The most basic model used to express a relationship between the load cycle amplitude,  $S$  with respect to the life cycle number,  $N$  is that of Basquin [8], as expressed by the following equation:

$$NS^b = B \quad (1)$$

where the Basquin slope,  $b$  ( $b > 1$ ) and the Basquin constant,  $B$  ( $B \gg 1$ ) are material parameters to be identified. At least for metallic materials, this model is supposed to hold from  $10^4$  up to  $10^6$  cycles and leads to a linear relation in the  $\log S$ - $\log N$  plane, see figure 3. For some reference values of the Basquin slope and a more comprehensive statistical approach, please refer to [9].

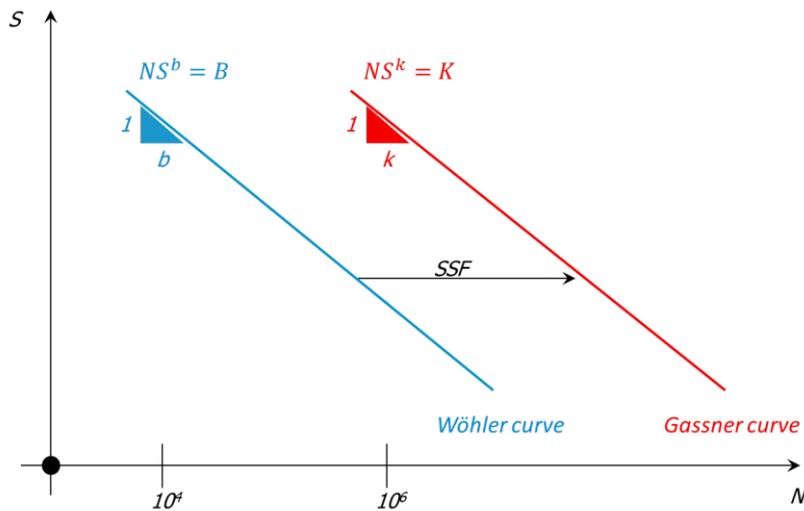


Fig. 3. Wöhler curve(left) & Gassner curve (right).

Moreover, Ernst Gassner is recognized as the pioneer of standard load spectra. His seminal work [5] introduces the concept of load spectrum in order to define a variable amplitude lifetime. Please note that the term “spectrum” does not rely here to any frequency analysis, but only deals with the occurrence of load cycles with different amplitudes within the same load sequence. Actually, the load frequency is not expected to have any effect in the range of the load time histories of interest. The Gassner block schedule, showed in figure 4, is composed of eight load amplitude levels,  $S_i$  linearly distributed from the lower,  $S_{min}$  up to the maximum,  $S_{max}$  and associated to cycle numbers  $N_i$  resulting from a Gaussian-like distribution. A cumulative occurrence,  $H$  may be introduced by the following equation:

$$H_i = \sum_{j=1}^i N_j \quad (2)$$

where the summation is usually started from the maximum amplitude load level (i.e.  $S_l = S_{max}$ ), which are expected to be associated to the smallest occurrence (i.e.  $N_l = N_{min}$  usually chosen to be equal to 1). Thus, applying the summation over the whole load sequence leads to the spectrum length,  $H_0$  which actually counts the overall cycle number of the spectrum, no matter of the load amplitudes. From the experimental point of view, once the load spectrum pattern is established by the choice of  $S_l = S_{max}$ , the load sequence is applied again and again until the specimen failure (or any other relevant stop criterion), leading to the overall lifetime  $H$ .

Within the previous  $S-N$  plane (actually  $\log S-\log N$ ), Gassner reports the experimentally achieved overall lifetime,  $H$  at the maximum load amplitude,  $S_{max}$  of the load spectrum considered, leading to the so-called Gassner curve, see figure 3. Another Basquin-like relation may be used to model Gassner results:

$$HS_{max}^k = K \tag{3}$$

where the slope,  $k$  and the constant,  $K$  are material parameters to be identified.

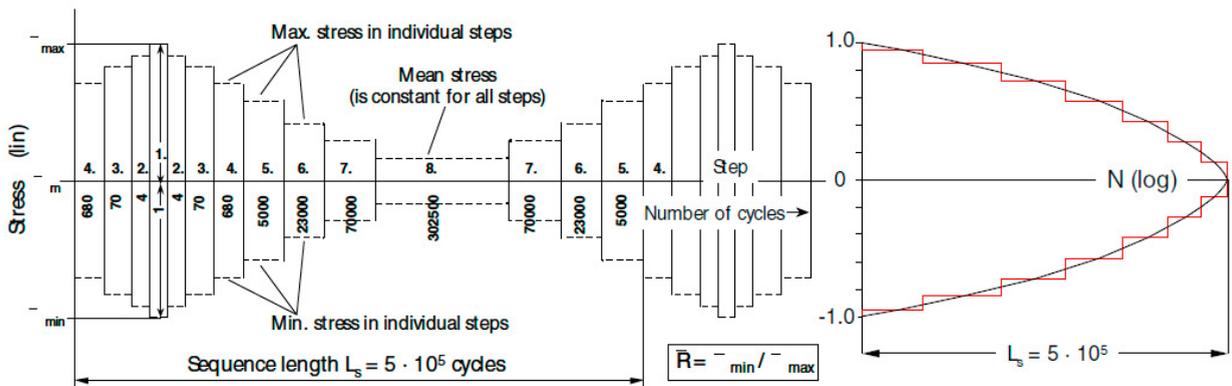


Fig. 4. Gassner load spectrum: load sequence (left), and cumulative cycle distribution (right). – Courtesy from LBF laboratory [12].

Several assumptions need to be stressed in regard to the Gassner framework. At first, the entire spectrum is supposed to be applied under the elastic to plastic behavior threshold of the materials used. Secondly, the load sequence, i.e. the way all the load cycles of different amplitudes are mixed before to be applied to the specimen, is expected to minimize any history effect: since the Gassner original block sequence, see figure 4, both increasing and decreasing load amplitudes are considered within the basic load schedule. This in order to let us apply the linear Palmgren-Miner’s damage rule [2,3]. Once this framework is assumed, it may be proven [10] that the exponents of the Basquin and Gassner laws are expected to be exactly the same:

$$b = k \tag{4}$$

Actually, if no history effects are considered, the only material parameters in use are the pairs  $(B,b)$  and  $(K,k)$ . Whereas the intercepts  $B$  and  $K$  are associated to the choice of plotting options ( $S$  or  $S_{max}$ ), the slopes  $b$  and  $k$  are expected to describe the same physical behavior: thus they must have the same value.

Some more remarks shall be addressed. In case of constant mean load equal to zero, the spectrum is associated to constant load ratio  $R = -1$ ; conversely, if the mean load is different from zero, complementary hypothesis are needed. Moreover, coming back to any possible history effect, the spectrum length,  $H_0$  is expected smaller than the overall

lifetime  $H$  in order to accept the concept of an elementary spectrum as a relevant metrics. If it were not the case, this would imply that rare maximum load levels are not guaranteed to be applied with the same statistical representation and fatigue test results would miss an important damage source. At last, plotting Gassner lines ( $S_{max} ; H$ ) in the Wöhler plane ( $S ; N$ ) gives the illusion that the Gassner framework offers longer lifetime than Wöhler does. This is nothing but a graphical mirage, due to the choice of  $S_{max}$  as the relevant load amplitude for the Gassner plot. According to [11], it may be proven [10] that the horizontal shift of the Gassner line with respect to the Wöhler one is provided by the following formula:

$$SSF = \log \left[ \frac{\sum N_i}{\sum N_i \left( \frac{S_i}{S_{max}} \right)^{1/b}} \right] \quad (5)$$

where SSF stands for spectrum shape factor, which solely depends on the shape of the spectrum and the Basquin slope. Please note that if the spectrum collapses to a constant amplitude load (i.e.  $S_i = S_{max}$ ), the Gassner framework reduces to the original Wöhler one:  $SSF = 0$ . For any consistent variable amplitude spectrum,  $SSF$  is definite positive, because of  $b > 1$  and  $S_i / S_{max} < 1$ . It may be shown that  $SSF < 1$  for spectra with many large and few small cycles, whereas  $SSF > 1$  for spectra with larger occurrences for small amplitude cycles. For example, for  $b = 5$  and a Gaussian-like load spectrum as the original Gassner one, one has  $SSF \sim 2.5$ , which means an horizontal shift of more than 2 decades on the right with respect to the Wöhler curve.

### 3. Standard load spectra

A comprehensive review of the standard load spectra applied in the industrial field has been provided by Heuler & al. [11,12]. Both European and US working groups have been providing reference design load spectra or standard load time-histories (SLH) since the 1970s, respectively for the aeronautics, the energy industry, the automotive ... along with experimental test results and fatigue criteria. Each example is collected with its own purpose, the intended structural application, a brief qualification, and some more technical details. To go further with the comparison, it is necessary to provide a general formulation. According to [11], we make use of the following equation:

$$\frac{\log H_i}{\log H_0} = 1 - \left( \frac{S_i}{S_{max}} \right)^\nu \quad (6)$$

where the pattern of the spectrum is based on an exponential law and  $\nu$  is the sole spectrum parameter, called the shape exponent. This equation may be plotted in a dimensionless field, see figure 5, considering the amplitude load ratio  $r_i = S_i / S_{max}$  on the Y-axis and the occurrences ratio  $\log H_i / \log H_0$  on the X-axis.

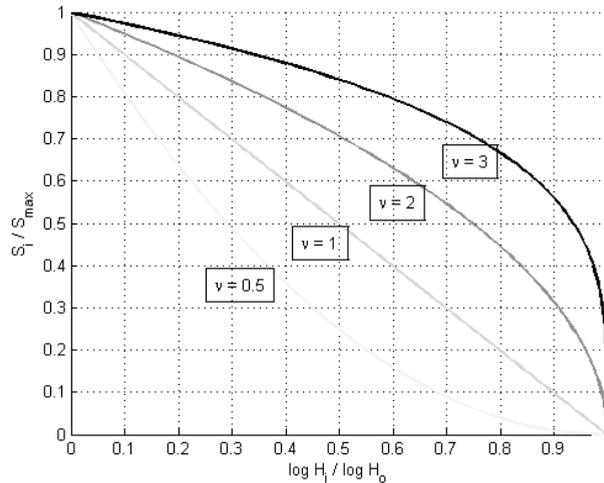


Fig. 5. Standard load spectra according to [11].

Please note that, once the load spectrum pattern is chosen, equation (6), the shape exponent is not expected to depend on the material fatigue parameters like the Basquin slope, because of it results from the cycle count operation without any damage ponderation.

Thus, the identification of the shape exponent,  $\nu$  may be a key to compare load spectra coming from independent sources, histories and practices. The German automotive industry has been applying the CARLOS – CAR Loading Standard since the 1990s. There exist several versions of this standard [11,12]: here we focus on the very first [13], specially devoted to suspension parts and providing load spectra for vertical, lateral and longitudinal loads, respectively. These three uniaxial sequences are expected representative of about 40 000 km driving length and built with  $H_0 \sim 10^5$  and SSF  $\sim 2.6$ , which means an almost Gaussian-like pattern with an expected cumulative occurrence of the maximum load level of about 10 for an overall lifetime of about  $10^6$  cycles.

In the following section, we will start to compare the CARLOS framework with the one applied by PSA Groupe, and coming from recent on-board measurements provided on the historical Belchamp proving ground.

#### 4. Comparison of chassis system load spectra

As announced in the previous section, the purpose of the present one is to start a comparison between standardized load spectra for chassis system parts. The reference one, CARLOS is widely applied by German carmakers and is supposed to correspond to German proving grounds with respect to fatigue damage. French carmakers are not yet used to standard load spectra within the Gassner framework, but indeed have their own proving grounds, built over a huge historical experience of the automotive product.

Thanks to some recent measurements on PSA Groupe proving ground in Belchamp, reference load time histories have been recorded for vertical, lateral and longitudinal directions, respectively. Time series coming from wheel force transducers have been processed by rainflow count method in order to provide a fatigue damage point of view of the proving ground, for each principal load direction. Thus, calling for the chosen load spectrum pattern, equation (6), it is possible to identify the most reliable shape exponent,  $\nu$  by applying a basic least squares minimization procedure. The same procedure is applied to a sample of the original CARLOS standard data [14], leading to a direct comparison of the load spectra, for each principal direction. Main results are condensed in the following table:

$\nu$	vertical	lateral	longitudinal
CARLOS	1,0	1,4	1,8
PSA	0,7	0,3	0,4

Table 1. Shape exponent of load spectra applied to automotive chassis system.

Please note that CARLOS data concern a unique sample of the standard, whereas PSA measurements cover several on-board measures dealing with different passenger car classes (i.e. different vehicle architectures in terms of base wheel, track, gravity center localization ...) and different axle architectures (e.g. front Mac Pherson, rear transverse beam, multi-link ...). Thus, data provided on PSA line of table 1 are to be considered as mean values around which some influence of the vehicle and chassis system architectures is checked. Conversely, an almost perfect symmetry between right-hand and left-hand wheels is noted, as expected. At last, before any relevant comment, all the identification data comes with a standard  $R^2$  statistics of at least 0.9, which means a reliable representation of the measures by the chosen model.

Table 1 analysis may be organized in two steps. First of all, comparing column values for the same line: it is not surprising to see that each principal direction has a specific  $\nu$  value. Actually, passenger car lifecycle has to be considered with respect to a huge list of driving events (e.g. braking, cornering, obstacle crossing ...) and each of them has its own peculiar load signature. Secondly, comparing line values for the same column: PSA shape exponents are significantly lower than CARLOS ones. Coming back to figure 5, this means that PSA load spectra are concave, whereas CARLOS ones are convex. From a more empirical point of view, assuming that the load range is common, PSA spectra are rather hollow in the middle.

## 5. Conclusions & Outlooks

In this paper we discuss and compare two different frameworks dealing with high cycle fatigue applied to chassis system in the automotive industry. From a theoretical point of view, German engineers are well accustomed to the variable amplitude approach proposed since Gassner, whereas the French counterpart usually applies the basic constant amplitude Wöhler approach. Taking advantage of recent discussions between German & French automotive researchers, PSA Groupe started a comparative analysis of load spectra applied for chassis system fatigue design.

Such an approach requires at first French engineers to get accustomed to the Gassner framework: chapters 2 and 3 should help to fill the gap. Thus, bilateral exchange of road load data would be useful in order to assess a relevant comparison. Chapter 3 is a first step towards a complete load spectra assessment: despite limited available data, it shows that different choices are in use, from the point of view of the so-called shape parameter.

Pretending to compare the complexity of a reference load using a sole parameter may be noted as ambitious. Actually, this work also aims at complete other existing metrics already and widely used when talking about fatigue, e.g. minimum and maximum load extrema, damage spectrum, equivalent load at a given lifecycle with respect to damage ... . Within this task, an essential point is not to confuse metrics based only on the evidence of a spectrum (e.g. the shape exponent here discussed), from those calling for a damage computation, which requires at least one more parameter depending on the material, e.g. the Basquin slope.

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