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Procedia Engineering 213 (2018) 192–206

Procedia Engineering

www.elsevier.com/locate/procedia

7th International Conference on Fatigue Design, Fatigue Design 2017, 29-30 November 2017, Senlis, France

Rapid Determination of the Fatigue Limit by the Simulation of Self-Heating Test by the Collaborative Model Based On the Fractional Derivative Approach

Alina Krasnobrizha[a], Laurent Gornet[a]*, Patrick Rozycki[a], Pascal Cosson[a]

[a]*Ecole Centrale de Nantes, Institut de Recherche en Génie Civil et Mécanique (GeM), 1 rue de la Noë, Nantes 44321, France*

Abstract

This paper proposes a method for on the fast fatigue limit estimation for composite materials using a simulation of a self-heating test. The experimental method based on the material self-heating was successfully applied for composite materials and allows to determine a material's endurance limit in a few hours. In order to increase the potential of this experience, the thermomechanical modelling is proposed with the precise description of the intrinsic energy dissipation. The rise of material's temperature is linked with development of plastic strains, material damage, viscoelastic proprieties of polymer matrix and intro-crack friction. Therefore, the collaborative behaviour model including the hysteresis loops is used to represent the visco-elastoplastic damage composite behaviour. The collaborative model consists of two sub-models. The first one describes an envelope of the loading curves and insures the computation of elastic and the in-elastic strains as well as the in-ply damage. The second part deals with modelling of hysteresis loops during unloading path using a fractional derivative approach. Just a few parameters are required to represent the hysteresis loops. Using a proposed modelling, the dissipation due to the in-ply damage propagation, the material hardening and the viscoelastic effects can be precisely calculated. The fatigue limit is determined by using thermodynamic simulation the quasi-static cyclic test with the dissipation measurements as an equivalent to self-heating test. The method is validated for composite materials with thermosetting and thermoplastic matrixes.

Peer-review under responsibility of the scientific committee of the 7th International Conference on Fatigue Design.

Keywords: Composite ; Fatigue ; Self-heating ; Dissipation ; Behaviour modelling ; Fractional derivative ; Hysteresis loop

* Laurent Gornet. Tel.: +33 2 40 37 25 82
E-mail address: laurent.gornet@ec-nantes.fr

Peer-review under responsibility of the scientific committee of the 7th International Conference on Fatigue Design.
10.1016/j.proeng.2018.02.021

1. Introduction

Nowadays, carbon fiber reinforced plastics (CFRP) are widely used in the different industrial fields. Their lightweight, high strength and long durability make them superior to classical metallic materials in the complex structures design. In order to increase the structure's safety and their economic potential, the validation phase has to be made including the mechanical test and numerical simulations of the material behaviour. In the present work, the fatigue loading is concerned as the essential point in the certification of the industrial structures.

The experimental measurement of fatigue limit for composite materials required a significant number of tests in the different orthotropic directions. The classical method consists in the applying of the cyclic loading for the given laminate during a substantial period of time (usually several weeks) in order to obtain Wöhler curve. The alternative method allows to determine the fatigue limit in several hours using the material self-heating effect. This method has been initially proposed for isotropic materials such as metal alloys [1], elastomers [2] and more recently for the short fiber composite materials [3]. In the case of carbon fiber composites, the self-heating method consists of applying a sequence of constant amplitude cyclic loading blocks. The stabilized temperature of the composite specimen is measured during each block. When the value of the stabilized temperature increases significantly, it is considered that the fatigue limit is attainted. Tomographic studies and the traditional fatigue tests allow to justify the developed method. The experimental results of the self-heating method are in good agreement with the conventional fatigue tests (Wöhler curve) for thermosetting and thermoplastic composite materials [4], [5].

The potential of the experimental method can be increased by using the numerical simulation of self-heating test proposed in this paper. In order to calculate the variation of material's temperature, a full thermo-mechanical material analysis is required. The collaborative model [6] is a suitable tool to calculate the intrinsic dissipation including the hysteresis loops which determine the quantity of dissipated energy dominating in the material self-heating. The proposed model is composed of the elastoplastic damage behaviour law [7] with possible strain-rate sensitivity [8], [9] and including a newly proposed fractional derivative approach.

A fractional derivative theory is a promising technique to describe the history-dependent phenomena as hysteresis loops. The viscoelastic equation is one of the first applications of fractional calculus. The fractional derivatives provide the formulation of the behaviour law in the integral form thus ensure the contribution of the past loading history. Considering that the material is pristine in the initial state $\varepsilon(0) = 0$, the hereditary stress-strain relation for polymer materials can be represented by the integral law (1) [10], [11]:

$$\sigma(t) = \frac{G}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{\varepsilon(\tau)}{(t-\tau)^\alpha} d\tau = G D_0^\alpha \varepsilon(t) \tag{1}$$

where G is the material parameter, Γ is the gamma function defined by equation (3) and D_0^α signifies a fractional derivative of order α (4).

In the rheological sense, the fractional derivative operator D_0^α can be represented by the spring-pot element [12]. This element is an asymptotic representation of the assembly of the elastic (spring) and viscoelastic (dash-pot) elements connected in the series and parallel (Figure 1). The spring-pot element is able to capture different types of the mechanical behaviour by the variation of order α. If the fractional derivative order tends to zero: $\alpha \to 0$, the behaviour of the element tends to the elastic spring and if the fractional derivative order tends to one: $\alpha \to 1$, the viscoelasticity increases.

From a mathematical point of view, the fractional operators have several definitions. In this work the classical Riemann-Liouville definition is used [13]. The fractional Riemann-Liouville integral of order α is defined as following:

$$(I_a^\alpha f)_{RL}(t) \overset{\text{def}}{=} \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad \alpha > 0 \tag{2}$$

where $(I_a^\alpha f)_{RL}(t)$ is the fractional integral of order α.

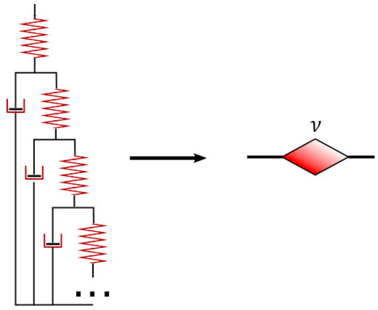


Figure 1. Spring-pot element

$$\Gamma(z) = \int_0^{+\infty} e^{-x} x^{z-1} dx, \quad z \in \mathbb{R}_+^* \tag{3}$$

The fractional derivative is an inverse operator to the fractional integral. Within the viscoelastic theory the order α belongs to the open interval $(0,1)$ and the fractional derivative of order α is equal to the fractional integral of order $1-\alpha$ derived one time:

$$(D_a^\alpha f)_{RL}(t) \stackrel{\text{def}}{=} \frac{d}{dt}(I_a^{1-\alpha} f)_{RL}(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_a^t \frac{f(\tau)}{(t-\tau)^\alpha} d\tau, \quad 0 < \alpha < 1 \tag{4}$$

where $(D_a^\alpha f)_{RL}(t)$ is the fractional derivative of order α in the sense of Riemann- Liouville. Henceforth in the article only Riemann-Liouville fractional operators will be used and subscript will be neglected.

As the fractional derivatives are suitable to take into account the past loading history, they are widely used to describe the hereditary behaviour of heterogeneous materials and fractal structures [14], [15], [16], [17], [18], [19], [20], [21], [22]. In the present work we use fractional derivatives to model the hysteresis loops which are associated with the viscoelastic proprieties of polymer matrix and its coupling with the damage propagation. We will show that with the precise description of dissipated energy it is possible to represent self-heating test using the cyclic quasi-static data.

This paper is organized as follows. In the section 2, the experimental data is presented. The material behaviour modelling is developed in the section 3. The section 4 concerns the parameter identification procedure. The validation of the collaborative model and its application to thermodynamic problems is demonstrated in the section 5. The section 6 concludes this paper and notes the various engineering application of this model.

Nomenclature

$(O,\vec{x},\vec{y})$	global coordinate system
$(O,\vec{1},\vec{2})$	coordinate system of a simple composite ply
α	order of fractional derivative
A	fractional order parameter
B	fractional order parameter
β	material parameter associated with material hardening
C	heat capacity
$\boldsymbol{C}$	stiffness matrix
$\boldsymbol{C^0}$	stiffness undamaged matrix

Δ	intrinsic dissipation
$\tilde{\Delta}$	approximated intrinsic dissipation
Δ^{load}	intrinsic dissipation calculated by elastoplastic damage model
Δ^{unload}	intrinsic dissipation by fractional model
Δt	time-step
d	damage internal variable
D_0^{α}	fractional derivative of order α
ε	unidirectional strain variable
$\boldsymbol{\varepsilon}^t$	total strain vector
$\boldsymbol{\varepsilon}^e$	elastic strain vector
$\boldsymbol{\varepsilon}^p$	plastic strain vector
$\boldsymbol{\varepsilon}^v$	viscous strain vector
ε_{12}^{ve}	viscoelastic strain vector
f	yield function
Γ	gamma function
G_{12}^0	undamaged shear modulus
G_{12}^i	shear modulus of damaged material
I_0^{α}	fractional integral of order α
k	material parameter associated with material hardening
p	cumulative plasticity
$\vec{q}$	heat flux
ρ	material density
R	isotropic hardening function
R_0	yield stress
σ	unidirectional stress variable
$\tilde{\sigma}$	effective stress
σ_{12}	stress variable in shear direction
σ_{12}^f	fatigue limit in shear direction
σ_{12}^R	failure stress
Θ	mean temperature changes of laminate
T	temperature variable
t	time variable
ψ	thermodynamic potential
W_e^d	elastic strain energy of damaged material
W_p	irreversible strain energy
$\boldsymbol{Y}$	thermodynamic force associated with damage internal variables
$\sqrt{\overline{\boldsymbol{Y}}_{12}^c}$	speed of damage propagation
$\sqrt{\overline{\boldsymbol{Y}}_{12}^0}$	initial damage threshold
$\sqrt{\overline{\boldsymbol{Y}}_{12}^R}$	failure-damage threshold

2. Experimental data

The material which are used in this work are: a thermosetting carbon TR50 fiber epoxy R367-2 matrix composite and thermoplastic carbon T700 fiber polyamide PA66 matrix composite.

The self-heating test was performed under ambient temperature by using the fatigue test systems MTS 880 with hydraulic jaws and 100 kN load cell. The test was made with mean stress value $\sigma_{12} = 25$ MPa and with the loading frequency at 5 Hz. Thermal infrared camera Cedip is used with 25 Hz acquisition frequency allowed to get 5 images per loading cycle and to measure the evolution of mean sample's temperature.

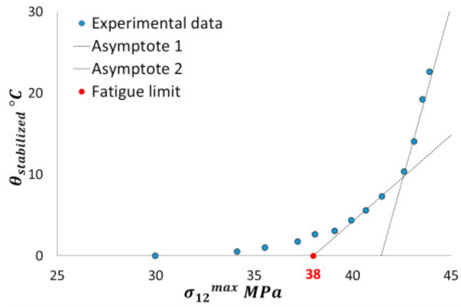


Figure 2. Self-heating test results for $[\pm 45]_{2S}$ carbon/epoxy laminate

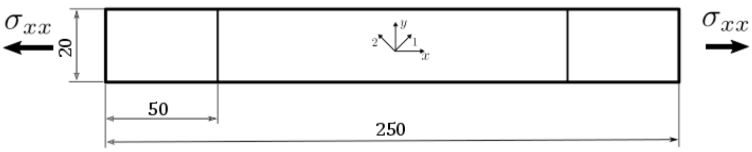


Figure 3. Specimen geometry for $[\pm 45]_{2S}$ carbon/epoxy laminate

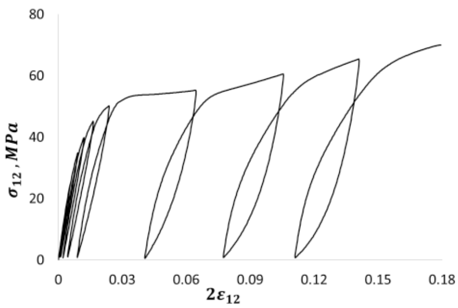


Figure 4. Shear stress-strain curve for the carbon/epoxy composite

The fatigue limit was determined by rapid self-heating method for $[\pm 45]_{2S}$ composite laminate and it is equal to $\sigma_{12}^{f} = 38$ MPa for thermosetting composite [4] and $\sigma_{12}^{f} = 63$ MPa for thermoplastic composite [23]. These results are in good agreement with classical *S-N* fatigue test results. The resulting self-heating test curve is illustrated in the Figure 2 for thermosetting composite material.

We propose to replace a self-heating test by the quasi-static cyclic tensile-tensile test with increasing stress level. The internal dissipation that causes the material to heat, depends on materials behaviour. In order to characterize the mechanical behaviour of single composite ply, the quasi-static traction cyclic test is realised for $[\pm 45]_{2S}$ composite laminate with hydraulic test systems MTS 801. The loading velocity is fixed at 2 mm/min. The displacement field is determined by bidirectional gauge. The specimen's geometry and its dimension are shown in the Figure 3. The nominal thickness of the samples is 2.2 mm. Subscripts 1 and 2 represent the local (material) coordinate system and subscripts x and y represent global (loading) coordinate system.

The experimental result of the tensile cyclic test for the carbon/epoxy composite in shear direction is illustrated in the Figure 4. Typically, the experimental tests show a non-linear behaviour in shear direction where the polymer matrix is subjected by significant amount of loading. The irreversible strains, the in-ply damage (which is observed from a regular decrease of shear modulus) and the hysteresis loops are presented in the stress-strain curve. All these phenomena make their contribution to the internal energy dissipation leading to material heating. The coherent behaviour modelling is proposed in following section.

3. Behaviour modelling

The experimental tests (Figure 4) show the appearance of hysteresis loops in shear direction. This phenomenon is associated with viscoelastic proprieties of composite laminate and friction inside cracks. Its modelling is particularly important in thermodynamic analysis. Moreover, the damage propagation and material hardening had to be taken into account to obtain the accurate behaviour modelling.

Hence, we propose to use a collaborative modelling [6] in order to represent the visco-elastoplastic damaged material behaviour under the cyclic loading. This model consists of two sub-models. The first sub-model deals with the modelling of material damage and inelastic deformations appearing with increasing of stress level. This type of behaviour is described by the classical elastoplastic damage model [7] (sub-model 1). On the other hand, there is neither damage nor plastic flow development during unloading path and material is considered to have the viscoelastic response due to the appearance of the hysteresis loops. This type of behaviour is represented by the fractional derivative approach which allows to take into account the loading history (sub-model 2). The collaborative model is developed in mesoscale under a state of plane stress ($\sigma_{33} = 0$). The constitutive equations are deduced within the framework of irreversible thermodynamics processes using the local state method [24]. The hypotheses of small perturbation is assumed then

$$\dot{\boldsymbol{\varepsilon}}^t = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p + \dot{\boldsymbol{\varepsilon}}^v \tag{5}$$

where $\dot{\boldsymbol{\varepsilon}}^t$, $\dot{\boldsymbol{\varepsilon}}^e$, $\dot{\boldsymbol{\varepsilon}}^p$ and $\dot{\boldsymbol{\varepsilon}}^v$ are the total, elastic, plastic and viscous strain rate vectors.

3.1. Thermodynamic aspects

The thermodynamic state of given composite specimen under self-heating test loading is defined by following equation:

$$div(\vec{q}) - \Delta = \rho C \frac{\partial T}{\partial t} \tag{6}$$

where $\vec{q}$ is a heat flux, Δ is a heat source associated with the material behaviour, ρ is material density, C is heat capacity and T is a mean temperature of specimen.

The local state method is used to determine the internal dissipation source Δ. Within the framework of irreversible thermodynamics, the behaviour law for the given mechanical system can be determined from Helmholtz potential $\rho\psi$ as a function of internal variables:

$$\rho\psi = \rho\psi(\boldsymbol{\varepsilon}^e, d_i, p) \tag{7}$$

where $\boldsymbol{\varepsilon}^e$, d_i, p are internal variables associated with elastic strain, damage in the orthotropic directions and cumulated plasticity, respectively.

The Clausius-Duihem inequality expresses the second principals of thermodynamics and for isothermal processes for small deformations it gives:

$$\boldsymbol{\sigma}: \dot{\boldsymbol{\varepsilon}} - \rho\dot{\psi} \geq 0 \tag{8}$$

where $\boldsymbol{\sigma}$ is the stress vector.

Taken into account equation (5), the Clausius-Duihem inequality (8) can be written in the following way:

$$\left(\boldsymbol{\sigma} - \rho\frac{\partial\psi}{\partial\boldsymbol{\varepsilon}^e}\right): \dot{\boldsymbol{\varepsilon}}^e + \boldsymbol{\sigma}: \dot{\boldsymbol{\varepsilon}}^p + \boldsymbol{\sigma}: \dot{\boldsymbol{\varepsilon}}^v - \rho\frac{\partial\psi}{\partial d}\dot{d} - \rho\frac{\partial\psi}{\partial p}\dot{p} \geq 0 \tag{9}$$

3.2. Elastoplastic damage modelling

The continuum damage mechanics theory is used to describe the in-ply damage by using the effective stresses notation $\tilde{\sigma} = \frac{\sigma}{(1-d)}$ [24, 25, 26, 27]. Then the degraded stiffness matrix $\boldsymbol{C}$ is introduced as follows:

$$\boldsymbol{C} = \boldsymbol{C}^{\mathbf{0}}(1 - \boldsymbol{d}) \tag{10}$$

In order to deduce the constitutive equations, the Helmholtz potential $\rho\psi$ (10) is presented as a sum of elastic strain energy of damaged material W_e^d and irrecoverable energy W_p:

$$\rho\psi = W_e^d + W_p = \frac{1}{2}\boldsymbol{\varepsilon}^e : \boldsymbol{C}^{\mathbf{0}}(1 - \boldsymbol{d}) : \boldsymbol{\varepsilon}^e + R(p) \tag{11}$$

The constitutive equations deduced from the equation (11) are following:

$$\begin{cases} \boldsymbol{\sigma} = \rho \dfrac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e} = \boldsymbol{C}^{\mathbf{0}}(1 - \boldsymbol{d}) : \boldsymbol{\varepsilon}^e; \\ \boldsymbol{Y} = -\rho \dfrac{\partial \psi}{\partial \boldsymbol{d}} = \dfrac{1}{2}\boldsymbol{\varepsilon}^e : \boldsymbol{C}^{\mathbf{0}} : \boldsymbol{\varepsilon}^e; \\ R = \rho \dfrac{\partial \psi}{\partial p}\ ; \end{cases} \tag{12}$$

where $\boldsymbol{Y}$ is thermodynamic force associated with damage internal variables $\boldsymbol{d}$ and R is material hardening function.

In this work we are particularly interested in strongly non-linear behaviour of the composite in the shear direction. For most of composite, hardening function is approximated by a power law. Hence, the constitutive equations in shear are deduced from the equation (12) as following:

$$\begin{cases} \sigma_{12} = (1 - d_{12}) G_{12}^0 (2\varepsilon_{12}^e); \\ Y_{12} = \dfrac{1}{2} G_{12}^0 (2\varepsilon_{12}^e)^2; \\ R = \beta p^m\ ; \quad \text{with } p = \displaystyle\int_0^{\varepsilon_{12}^p} (1 - d_{12})\, d\varepsilon_{12}^p \end{cases} \tag{13}$$

where β and m are isotropic hardening material parameters to be determined from experimental data.

The associated thermodynamic forces Y_{12} characterize the damage propagation in shear direction. The state of damage can only grow [7, 24] and therefore, the threshold of undamaged zone is defined as a maximal thermodynamic force for all previous time (τ) up to the current time (t) [8]:

$$\overline{Y}_{12} = sup_{\tau \le t}\big(Y_{12}(t)\big) \tag{14}$$

Equation (14) corresponds to damage criteria. The coupling between the internal damage variables and the associated thermodynamic forces is made by the experimental data fitting. Generally, the damage propagation can be described by polynomial function (15).

$$d_{12} = \sum_{n=1}^{N} a_{12}^n \left(\sqrt{\bar{Y}_{12}} - \sqrt{\bar{Y}_{12}^0} \right)^n \text{ if } d_{12} < 1 \text{ and } \bar{Y}_{12} < \bar{Y}_{12}^R;\ \text{otherwise } d_{12} = 1 \tag{15}$$

where $\bar{Y}_{12}^{0}$ is initial damage threshold and $\bar{Y}_{12}^{R}$ is failure-damage threshold. These constants are the material parameters.

The irreversible strains are taken into account by yield plastic function *f:*

$$f = |\tilde{\sigma}_{12}| - R(p) - R_0 \quad (16)$$

where R_0 is a yield stress.

Yield function f (16) allows to distinguish the elastic domain ($f < 0$) from plastic flow ($f = 0$).

The Figure 5 represents resulting stress-strain shear curve obtained by the elastoplastic damage model [7] for the carbon/epoxy composite (using the material parameter identification in the next sections). The simulation provides a good representation of loading curve except the hysteresis loops. The fractional derivative modelling of hysteresis loops is proposed in the section 3.3.

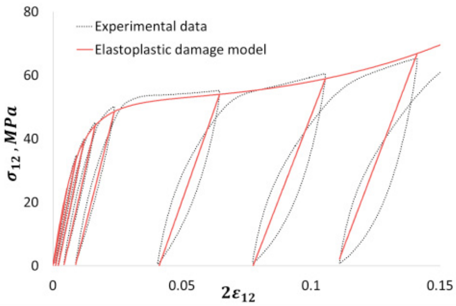


Figure 5. Experimental and numerical behaviour comparison in shear for the carbon/epoxy composite

3.3. Fractional derivative modelling to represent hysteresis loops

In this section we introduce the viscoelastic fractional sub-model. According to the elastoplastic damage model, the levels of plastic strain and material damage stay constant and the elastic strain is a linear function of time within each hysteresis loop. However, a non-linearity of strain is observed in the experimental curve during the unloading/reloading path (Figure 4). Since this non-linearity is associated with viscoelastic matrix properties, the fractional derivatives are introduced in the governing law in order to model hysteresis loops. We consider that the total strain within one hysteresis loop is equivalent to viscoelastic strain ε_{12}^{ve} and is defined as a non-linear function of elastic strain:

$$\varepsilon_{12}^{ve}(t) = A + BD_0^{\alpha}(2\varepsilon_{12}^{e})(t) \quad (17)$$

where ε_{12}^{e} is the elastic strain in shear determined by the elastoplastic damaged model, D_0^{α} is the Riemann-Liouville fractional derivative (4) and A, B and α are fractional model parameters.

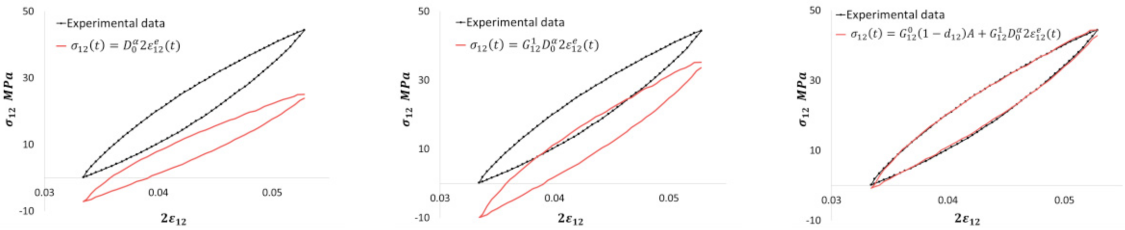
As the plastic flow stays constant, the stress is expressed by the elastic law:

$$\sigma_{12}(t) = G_{12}^{0}(1 - d_{12})\, \varepsilon_{12}^{ve}(t) \quad (18)$$

By substitution the equation (18) in the equation (17), the constitutive law for unloading path is:

$$\sigma_{12}(t) = G_{12}^{0}(1 - d_{12})A + G_{12}^{1}(D_0^{\alpha}2\varepsilon_{12}^{e})(t) \quad (19)$$

with $G_{12}^{1} = G_{12}^{0}(1 - d_{12})B$.



a) b) c)

Figure 6. (a) Graphical representation of the fractional derivative $\sigma_{12}(t) = 2D_0^\alpha \varepsilon_{12}^e(t)$, (b) Graphical representation of the term $\sigma_{12}(t) = 2G_{12}^1 D_0^\alpha \varepsilon_{12}^e(t)$, (c) Graphical representation of the fractional model $\sigma_{12}(t) = G_{12}^0(1-d_{12})A + 2G_{12}^1 D_0^\alpha \varepsilon_{12}^e(t)$.

The equation (19) is a non-standard form of Kelvin-Voight fractional model. The parameters A, B and α are considered to be the functions of damage internal variable d_{12}. In order to signify meanings of the fractional model parameters, the influence of each term in the equation (19) is considered.

- The fractional derivative of the elastic strain $D_0^\alpha \varepsilon_{12}^e(t)$ provides a strain non-linearity. Hence, the hysteresis loop appears (Figure 6a). The fractional order α adjusts the loop size. The growth of α leads to the increasing of loop area and consequently the energy dissipation increases
- The factor G_{12}^1 governs the slope of the loop (Figure 6b). The damage level is adjusted in respect to the experimental data.
- The term $G_{12}^0(1-d_{12})A$ is a “connecting” stress that links two sub-models: the fractional and the elastoplastic damage models (Figure 6c). Thus the position of the loop is adjusted and the stress becomes a continuous function.

In order to implement fractional derivative in a numerical code, a simple methodology is further proposed. At first, the Riemann-Liouville fractional integral (2) of order $1-\alpha$ can be rewritten in the alternative form:

$$(I_0^{1-\alpha}\varepsilon_{12}^e(t))_{M1} = \frac{t^{1-\alpha}}{\Gamma(2-\alpha)}\int_0^1 \varepsilon_{12}^e\left(t\left(1-v^{\frac{1}{1-\alpha}}\right)\right)dv \tag{20}$$

The integral in formula (20) can be easily computed numerically by Gaussian quadrature if the analytical expression of the linear elastic function $\varepsilon_{12}^e(t)$ is known. According to the expression (4), the fractional derivative of order α is the first derivative of the fractional integral of order $1-\alpha$ (20) which can be calculated numerically by a central difference scheme:

$$(D_0^\alpha \varepsilon_{12}^e(t))_{M1} = \frac{(I_0^{1-\alpha}\varepsilon_{12}^e(t+\Delta t))_{M1} - (I_0^{1-\alpha}\varepsilon_{12}^e(t-\Delta t))_{M1}}{2\Delta t} \tag{21}$$

The proposed method is simple in realisation especially in case of quasi-static loading where the number of loading cycles is limited. Note that the fractional derivative model is calculated only during the unloading path. The more detailed information about the implementation of fraction sub-model in numerical code is described in the work [6].

3.4. Coupling of two sub-models

In this part, the coupling between two sub-models is considered. The collaboration between the elastoplastic damage model and fractional model is ensured by the yield function f (15). There are two possible cases:

- If the plastic yield function f with its derivative f' are zero ($f = f' = 0$), we consider the development of the plastic flow and damage in the material's behaviour. In this case, the elastoplastic damage model [7] is used to represent the envelope of the stress-strain curve as in the Figure 5.
- If the plastic yield function f is negative or if the yield function f is zero and its derivative f' is nonzero ($f < 0$ or $f = 0$ and $f' < 0$ or $f = 0$ and $f' > 0$), then a viscoelastic unloading is to be considered. The plasticity and damage levels remain a constant, thus the fractional derivative model is used to capture the hysteresis loops which was not possible using the elastoplastic damage model as shown in the Figure 5.

3.5. Material energy calculus

Using the developed modelling and the Clausius-Duhem inequality (8) it is possible to calculate precisely the dissipated energy associate with visco-elastoplastic damage behaviour. The total dissipate energy is split into a sum in order to take into account the contribution of each sub-model.

$$\Delta = \Delta^{load} + \Delta^{unload} \tag{22}$$

We consider that during loading path the level of viscous strain ε_{12}^{v} stay constant then $\dot{\varepsilon}_{12}^{v} = 0$ and $\dot{\varepsilon}_{12}^{t} = \dot{\varepsilon}_{12}^{e} + \dot{\varepsilon}_{12}^{p}$. Hence, the dissipated material energy during the loading path Δ^{load} is deduced from the equation (9) as following:

$$\Delta^{load} = \sigma_{12} 2\dot{\varepsilon}_{12}^{p} - R\dot{p} + Y_{12}\dot{d}_{12} \geq 0 \tag{23}$$

Otherwise, with respect to the developed model, the damage propagation and the plastic flow stay constant during unloading: $\dot{\varepsilon}_{12}^{p} = 0$ and $\dot{d}_{12} = 0$ and the total strain coincides with the non-linear viscoelastic strain ($\varepsilon_{12}^{t} = \varepsilon_{12}^{ve}$) defined by the equation (17). So, the dissipated energy during unloading-loading path, which is the cause for the material self-heating, is defined such as:

$$\Delta^{unload} = \sigma_{12}\dot{\varepsilon}_{12}^{ve} \tag{24}$$

On the other hand, the dissipated energy during unloading is equal to the area of hysteresis loop defined by equation (25). The last expression allows us to control the computation of the dissipated energy and the material self-heating by the collaborative model.

$$\Delta^{unload} = \oint \sigma_{12} d\varepsilon_{12}^{ve} \tag{25}$$

4. Material characterisation in shear direction

In this section the material characterisation procedure in shear direction is described. The identification of the model parameters is illustrated using the experimental data for the carbon/epoxy composite (Figure 4). The in-ply damage, the isotropic strain hardening and the hysteresis loops modelling are taken into consideration.

The elastic behaviour is characterized by the undamaged shear modulus G_{12}^{0} and the yield stress R_0. These parameters are identified from the linear regression of the elastic part in the experimental data. The failure stress σ_{12}^{R} is the maximum stress reached during the test.

Typical cyclic shear tests show a reduction in the material stiffness. Therefore, the internal damage variable d_{12} has been introduced in order to model the material degradation. This variable can be measured through the reduction in the shear modulus during cycling test:

$$d_{12} = 1 - \frac{G_{12}^{i}}{G_{12}^{0}} \tag{26}$$

where G_{12}^{i} is a degraded shear modulus associated which each loading cycle.

The internal damage variable is a function of associated thermodynamic force $\sqrt{\bar{Y}_{12}}$. The experimental data shows that damage grows linearly (Figure 7) in the carbon/epoxy composite in accordance with the equation (15).

The cumulative plastic strain p can be calculated since the damage is identified. The cumulative plasticity makes a link between irreversible deformations and material damage. Then, the strain hardening function $R(p)$ is calculating by the equation (27). The experimental data are fitted using the power law. The approximation is illustrated in the Figure 8.

The material parameters identified by elastoplastic damage sub-model are presented in the Table 1

$$R(p) = |\tilde{\sigma}_{12}| - R_0 \tag{27}$$

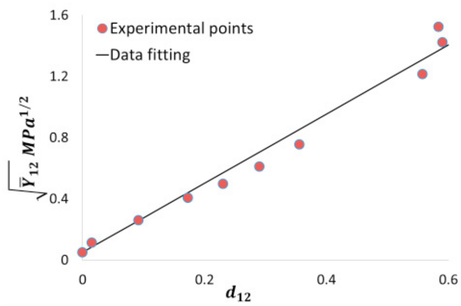


Figure 7. Shear damage function for the carbon/epoxy composite

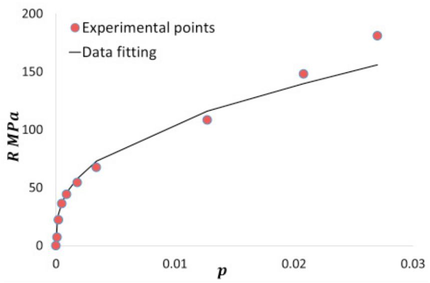


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Figure 8. Hardening function $R(p)$ for the carbon/epoxy composite

Table 1. Parameters identification in shear for the carbon/epoxy composite

Phenomenon	Parameter	Value	Units
Elastic parameters	G_{12}^{0}	5100	MPa
	R_0	11	MPa
	σ_{12}^{R}	70	MPa
Damage parameters	$\sqrt{\bar{Y}_{12}^{C}}$	0.0504	$MPa^{1/2}$
	$\sqrt{\bar{Y}_{12}^{0}}$	1.5226	$MPa^{1/2}$
	$\sqrt{\bar{Y}_{12}^{R}}$	2.2594	$MPa^{1/2}$
Placticity	β	578	MPa
	k	0.362	

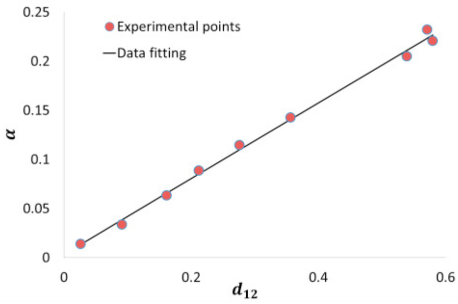

Figure 9. Fractional parameter α evaluation with damage for the carbon/epoxy composite

The fractional model parameters A, B and α in the equation (17) are identified by solving a non-linear optimization problem with constrains. The main constrain is imposed on the fractional derivative order α: $0 < \alpha < 1$. The objective function is defined as a relative error:

$$\delta = \frac{\sqrt{\sum_{i=1}^{N}(\varepsilon_{12}^{exp}(t_i) - \varepsilon_{12}^{ve}(t_i))^2}}{N} \quad (28)$$

where ε_{12}^{exp} is an experimental strain determined by the equation (29), ε_{12}^{loop} is a shear strain calculated by the fractional model (17) and *N* is a number of time-points inside the considered interval.

$$\varepsilon_{12}^{exp} = \frac{\sigma_{12}^{exp}}{G_{12}^{0}(1 - d_{12})} \quad (29)$$

where σ_{12}^{exp} is the experimental stress values.

Three fractional parameters A, B and α are defined for each hysteresis loop. Their values stay constant within the hysteresis loop but they are altered in every loop. Like it has been mentioned above, the fractional parameters are the functions of damage. For carbon/epoxy composite we assume that parameter B stays constant over all loading and $B = 1$. The evaluation of fractional parameters A and α is linear with respect to damage. How it was mentioned in the sub-section 3.3, parameter α determines the quantity of dissipate energy. Its evolution is illustrated on the Figure 9.

5. Results

The collaborative model is applied to represent the shear stress-strain curve using identified parameters from section 4. The resulting curve illustrated in the Figure 10 is more close to the experimental test compare to elastoplastic damage model (Figure 5).

The precise representation of loading curve allows to determine accurately the intrinsic dissipation. The individual contribution of each phenomenon such as damage propagation, plastic flow and hysteresis behaviour into dissipation is illustrated in the Figure 11. These results show that the hysteresis behaviour has a dominant role in energy dissipation compare to elastoplastic damage behaviour. Moreover, the fractional derivative sub-model provides a good prediction of hysteresis loops what is proved by coincident of curve determined by equations (24) and (25).

Further, the fatigue limit is determined using an evaluation of total material dissipation (Figure 12).

A generalized methodology is further proposed to identify the point of intersection of the tangent with the dissipation curve and then a fatigue limit. The internal energy dissipation is increases exponentially with applied stress and it can be approximated by following function:

$$\tilde{\Delta} = y_0(e^{x_0 \cdot \sigma} - 1) \tag{30}$$

where $\tilde{\Delta}$ is approximation of dissipated energy and x_0, y_0 are fitting parameters.

Firstly, two tangents (1 and 2 in the Figure 12) are built for extreme values of approximated dissipation function . Then, from the intersection point of these tangents (point N°1 in the Figure 12) a normal curve to the dissipation graph is plotted. The intersection point of normal and a dissipation curve is also a point of curve intersection with desired tangent (point N°2 and tangent 3 in the Figure 12). The fatigue limit corresponds to intersection of tangent 3 and stress axis. This methodology allows to determine the fatigue limit for different composite materials in the common way. The resulting fatigue limit is in good agreement with experimental data obtained by self-heating test or classical S-N curve (Table 2).

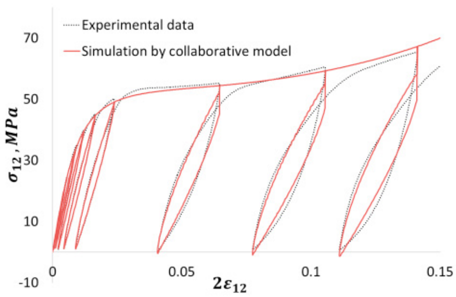

Figure 10.Experimental and numerical behaviour comparison in shear for the carbon/epoxy composite

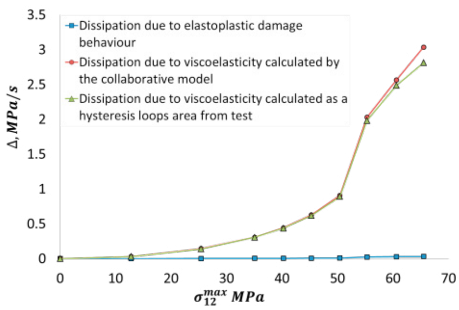

Figure 11. Contribution in energy dissipation by different models

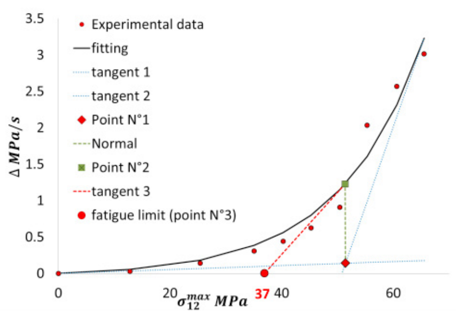

Figure 12. Contribution in energy dissipation by different models

Table 2. Resulting fatigue limit values

Fatigue limit	S-N curve	Self-heating experimental test	Self-heating test modelling
σ_{12}^{f}	35 MPa	38 MPa	37 MPa

6. Conclusions

This paper deals with the modelling of self-heating test in order to simplify the fatigue limit identification. When a composite material has reached its yield limit, the plasticity and damage occur. Moreover, composite material with polymer matrix shows some hysteresis behaviour behind the elastic limit due to viscoelastic resin properties and micro-damage propagation. The inter-crack friction causes a significant temperature rise. Therefore, an accurate dissipation calculus is required to determine the evaluation of material temperature.

The collaborative model is used to represent visco-elastoplastic damage behaviour as well as the hysteresis loops which occur under cyclic loading. The behaviour modelling is developed within the framework of irreversible thermodynamics using a local state method. The classical elastoplastic damage model is used to represent the loading envelope. On the other hand, a viscoelastic fractional model representing the hysteresis loops is applied during unloading path. It has been shown on the example of carbon/epoxy $[\pm 45]_{2S}$ composite laminate, that collaborative model provides a realistic behaviour modelling in shear direction.

Further, the intrinsic dissipated energy has been easily calculated by the proposed material model. The obtained results have shown that the energy dissipation in the hysteresis loops has the dominating effect on the material self-heating. The fatigue limit has been predicted successfully as an intersection of tangent to calculated total energy dissipation with the stress axes. The results obtained by proposed modelling are in good agreement with the experimental self-heating test and with classical S-N method.

References

[1] S. Giancane, A. Chrysochoos, V. Dattoma, B. Wattrisse, Deformation and dissipated energy for high cycle fatigue of 2024-T3 aluminium alloy, Theoretical and Applied Fracture Mechanics. 2 (2009) 117-121.

[2] V. Le Saux, Y. Marcoa, S. Callocha, C. Doudarda, P. Charrierb, An energetic criterion for the fatigue of rubbers: an approach based on a heat build-upp protocol and microtomography measurements, Procedia Engineering. 2 (2010) 949-958.

[3] L. Jegou, Y. Marco, V. Le Saux, S. Calloch, Fast prediction of the Wöhler curvefrom heat build-up measurements on Short Fiber Reinforced Plastic, International Journal of Fatigue. 47(2013) 259-267.

[4] L. Gornet, O. Wesphal, C. Burtin, J.-C. Bailleul, P. Rozycki, L. Stainier, Rapid Determination of the High Cycle Fatigue Limit Curve of Carbon Fiber Epoxy Matrix Composite Laminates by Thermography Methodology: Tests and Finite Element Simulations, Procedia Engineering. 66(2013) 697–704.

[5] O. Westphal, L. Gornet, L. Stainier, P. Rozycki, Thermomechanical analysis of fatigue degradations in carbon/epoxy laminates, 15th European Conference on Composite Materials-ECCM15, Venice, 2012.

[6] A. Krasnobrizha, P. Rozycki, L. Gornet, P. Cosson, Hysteresis behaviour modelling of woven composite using a collaborative elastoplastic damage model with fractional derivatives, Composite Structures. 158 (2016) 101–111.

[7] P. Ladevèze, E. Le Dantec, Damage modelling of the elementary ply for laminated composites, Composites Science and Technology. 43 (1992) 257-267.

[8] A. Johnson, A. Pickett, P. Rozycki, Computational methods for predicting impact damage in composite structures, Composites Science and Technology. 61(2001) 2183-2192.

[9] S. Marguet, P. Rozyck, L. Gornet, A rate dependence constitutive model for carbon-fiber reinforced plastic woven fabrics, Mechanics of Advanced Materials and Structures. 14 (2007) 619-631.

[10] P. G. Nutting, A new general law deformation, Journal of the Franklin Institute. 191 (1921) 678–685.

[11] A. Gerasimov, Generalization of linear laws of deformation and its application to problems with internal friction, Prikladnaya Matematika i Mechanika (PMM), 12 (1948) 251-260 (in Russian).

[12] R. Koeller, Applications of fractional calculus to the theory of viscoelasticity, Journal of Applied Mechanics. 51 (1984) 299–307.

[13] S. G. Samko, A. A. Kilbas, O. I. Marichev, Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach Science, Amsterdam, 1993.

[14] P. Cosson, J. Michon, Identification by a non-integer order model of the mechanical behaviour of an elastomer, Chaos, Solitons & Fractals, 7 (1996) 1807-1824.

[15] G. Scott Blair, J. E. Caffyn, An application of the theory of quasiproperties to the treatment of anomalous stress-strain relations, Philosophical Magazine. 40 (1949) 80–94.

[16] F. Gorenflo, F. Mainardi, Fractals and Fractional Calculus in Continuum Mechanics, springer ed., Wien and New York, 1997.
[17] H. Holmgren, Om differentialkalkylen: med indices af hvad natur som helst, Kongliga Svenska Ventenkaps-Akademiens Handlingar, 5 (1866) 1-83.
[18] C. Friedrich, Relaxation and retardation functions of the Maxwell model with fractional derivatives, Rheologica Acta. 30 (1991) 151–15.
[19] R. Bagley, P. Torvik, A theoretical basis for the application of fractional calculus to viscoelasticity, J. of Rheology. 27 (1983) 1–10.
[20] R. Bagley, P. Torvik, On the fractional calculus model of viscoelastic behavior, J. of Rheology. 30 (1986) 133-155.
[21] Caputo, J. Carcione, Hysteresis cycles and fatigue criteria using anelastic models based on fractional derivatives, Rheologica Acta. 50 (2011) 107-115.
[22] M. H. Mateos, L. Gornet, P. Rozycki, L. Aretxabaleta, Hysteretic shear behavior of fiber-reinforced composite laminates, 16th European Conference on Composite Materials – ECCM16, Sevilla, 2014.
[23] C. Peyrac, T. Jollivet, N. Leray, F. Lefebvre, O. Westphal, L. Gornet, Self-heating Method for Fatigue Limit Determination on Thermoplastic Composites, Procedia Engineering. 133 (2015), 129-135.
[24] J. Lemaitre, J. Chaboche, A. Benallal, R. Desmorat, Mécanique des Matériaux Solides, Dunod, Paris, 2009.
[25] L. Kachanov, Time of the rupture process under creep conditions, Izvestia Akademii Nauk SSSR, Otdelenie tekhnicheskich nauk. 8 (1958) 26-31,.
[26] L. Kachanov, Introduction to continuum damage mechanics, Springer Science & Business Media, Brookline, USA, 1986.
[27] N. Rabotnov, Introduction to damage mechanics, Nauka, Moscow, 1987 (in Russian).