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## A Contribution To Study The Tooth Flank Fracture (TFF) In Cylindrical Gears

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### Abstract

The Tooth Flank Fracture, TFF, is a failure mode observed in the surface-hardened of gear tooth flanks. There is a fatigue phenomenon which initiates from a subsurface crack that occurs in the transition zone between the hardened layer and the core structure of the tooth. It is substantially initiated close to the mid-height of the tooth in the single pair of contact area from a crack that progresses in a direction normal to the active surface flanks and also spreads in the thickness of the tooth up to the breakage of the non-active flank, slightly above the root fillet.

The two main modes of fatigue damages for gears are: (i) contact fatigue and (ii) tooth bending root fatigue, are well established and addressed in the international standards such as ISO 6336 part 2 and 3. However, up to now, there is no detailed calculation method to determine the load capacity related to the Tooth Flank Fracture, TFF.

The main objective of this work is to develop an efficient and a standardized methodology to identify the occurrence risk of the Tooth Flank Fracture (TFF) in cylindrical gears. Then, to estimate the risk of appearance of TFF, we have studied two fatigue criteria which are introduced in a developed numerical model. The developed approach, which is based on the Hertz theory with the half-space approach, has been validated by finite elements simulations, revealing a good compliance of the “simplified” criteria proposed initially by Dang Van to characterize this failure mode. Good agreement of this criterion is also observed after a comparative study relative to some experimental tests carried out in literature. An advanced parametric study is carried out to properly identify the real impact of the case hardening depth on this damage mode.

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*Keywords:* Gears; Fatigue; TFF.

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## 1. Introduction

A better control during the last decades of the quality of the materials, the surface finishing and the heat-treatment process has prevented the two main and classical modes of fatigue damages for gears: (i) flank surface contact fatigue and (ii) tooth bending root fatigue. These two classical damage modes are well established and addressed in the international standards such as ISO 6336 part 2 and 3 [1-2]. However, up to now, there is no detailed calculation method used to determine the load capacity related to the Tooth Flank Fracture, TFF.

The TFF, is a failure mode observed in the surface-hardened of gear tooth flanks. There is a fatigue phenomenon which initiates from a subsurface crack that occurs in the transition zone between the hardened layer and the core structure of the tooth [3-7]. It is substantially initiated close to the mid-height of the tooth in the single pair of contact area from a crack that progresses in a direction normal to the active surface flanks and also spreads in the thickness of the tooth up to the breakage of the non-active flank, slightly above the root fillet.

The main objective of this work is to develop a methodology to identify the occurrence risk of the fatigue failure by the tooth flank fracture (TFF) in cylindrical gears. Indeed, an accurate characterization of the fatigue strength of a gearing requires taking into account the cyclic nature of the multiaxial stresses that occur at each point of the tooth. Then, to estimate the risk of appearance of the TFF, we have evaluated a set of multiaxial fatigue criteria.

## 2. Modelling of the contact stresses

Recently, Weber et al. [7] have presented certain limitations, on the application of the classical Hertz theory under the half-space approach (plain strain state), especially for gear geometries which exhibit a relatively low thickness (depth at the normal contact point). Indeed, their method predicted for ratios ( $H/b_H$ ) which are less than 40 (ratio of the total normal depth ( $H$ ) with respect to the half-width of the contact band ( $b_H$ )), the appearance of a subsurface tensile stresses which could reach 0.2 times the maximum pressure of contact.

To validate and control the applicability of the Hertz theory in the contact between gear teeth, we have conducted a comparison between results based on the half-space approach with those obtained by a numerical method. The contact stresses between teeth are determined by a finite element Method (FE). The geometry of the teeth is generated by a "home" code and imported into the FE software. The calculations are performed with ComSol 4.4 by simulating three consecutive teeth. The load on the tooth is imposed as a hertzian contact pressure and the mesh is refined in the loading area (see Fig. 1).

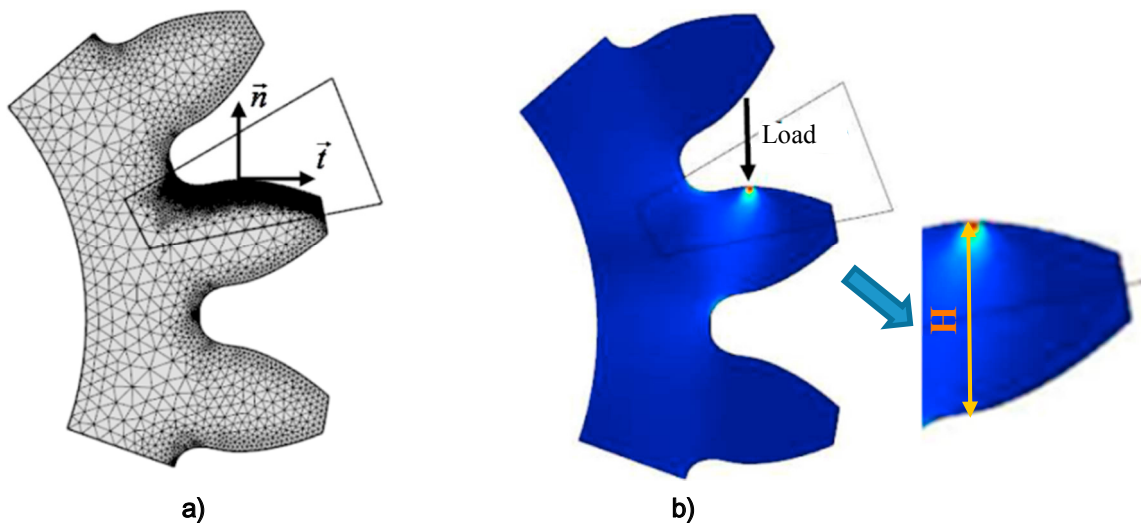


Fig. 1. (a) Finite elements Model; (b) Von Mises Stresses associated to a "punctual" applied load.

The Von Mises stresses are presented in the following figure in which the left distribution corresponds to the FE model of the tooth and the right one is associated to the Hertz Theory with a half-space approach. The difference is small and is located mainly at the level of the transverse stress (stress parallel to the tooth profile) where the gap is about 50 MPa (see Figures 2 and 3). This difference is mainly due to the integration into the FE model of the bending structure stresses.

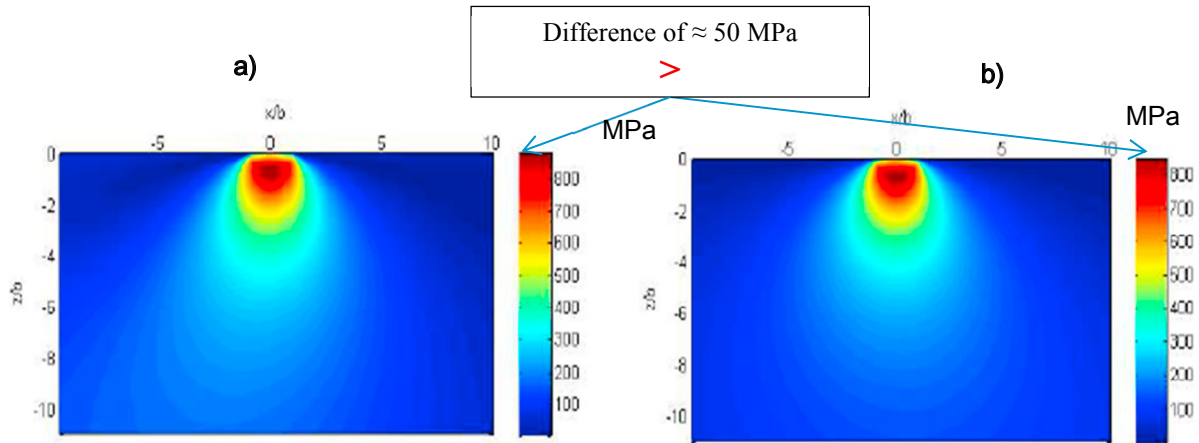


Fig. 2. Von Mises stress distribution (a) Finite elements Model; (b) Hertz theory.

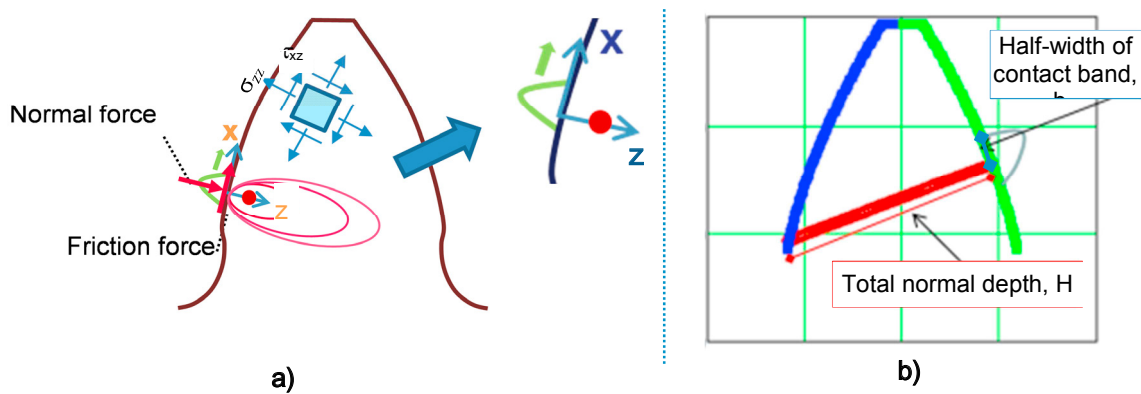


Fig. 3. (a) Stresses orientation and (b) Definition of parameters:  $b_H$  and  $H$ .

Figure 4 shows the evolution of the transverse stresses ( $\sigma_{xx}$ , parallel to the tooth profile) for two values of the normal depth to the active surface, as shown by Weber [7]. It is thus observed the presence of a tensile stress which takes place quite close to those found in our case study (50 MPa). Note that the studied application in this section has substantially the same characteristics as those shown in the paper [7] with ratios ( $H/b_H$ ) which reach values less than 25 along the tooth profile.

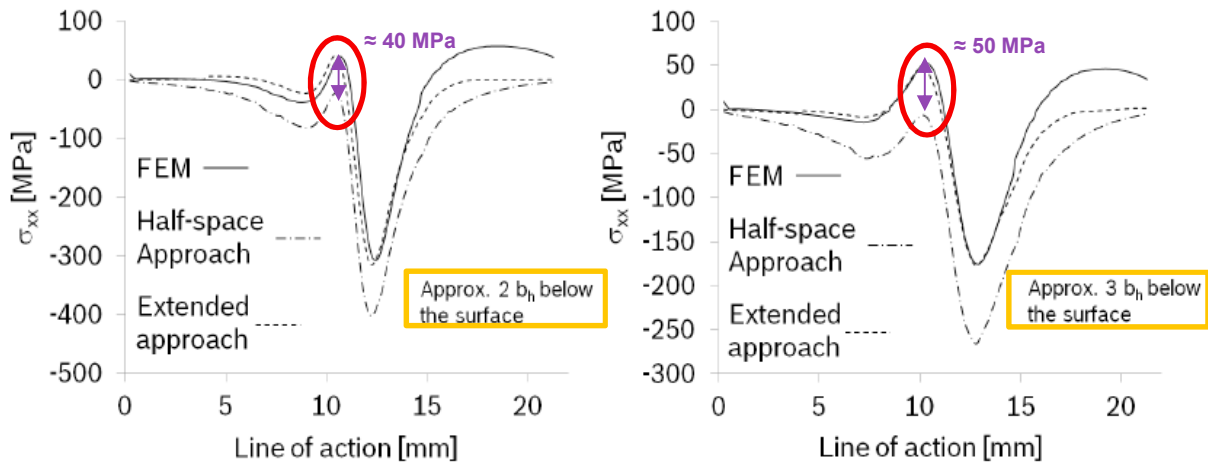


Fig. 4. Evolution of  $\sigma_{xx}$  stresses as obtained by Weber et al. [9].

### 3. Multiaxial Fatigue criterion

#### 3.1. Definition

A proper characterization of the fatigue strength of a gearing transmission requires to take into account the cyclic nature of the multiaxial stresses that occur at each point of contact. Thus to estimate the fatigue of such component subjected to a complex service loading, the characterization method consists in defining an equivalent stress in a multiaxial stress state.

Most criteria are using approaches derived from Von Mises or Tresca criteria in theory of elasticity. These criteria are based on the fact that the fatigue behaviour of a material is given in a plane containing the most solicited point of the structure.

The severity of the cycle of the local stresses is then expressed by some linear combinations of quantities related to the normal stress and shear stress acting on a given plane.

A fatigue criterion allows estimating whether the fatigue limit to  $N$  cycles of the material is reached for a serie of stress solicitations defining a multiaxial stress cycle. These criteria are more complex than the usual resistance criteria (Tresca, Von Mises) due to the variable nature of the stresses in the cycle. Thus, a fatigue criterion can be depicted using a fatigue function ( $D_f$ ) that involves the multiaxial stress cycle and the different fatigue characteristics of material. When the function ( $D_f$ ) takes a value greater or equal than the unity, the criterion estimates a crack initiation at the considered point.

The identification process, raised by these criteria involves two types of parameters:

- i) the local material fatigue limit
- ii) and the stress fields which includes mainly the hertzian contact stresses and the residual stresses due to the surface hardening treatment.

The prediction of the evolution of the residual stress along the depth below the surface is very little studied in literature (especially at a depth greater than the effective hardening depth due to the difficulty to provide measurements). For this reason, some realistic assumptions are used to estimate this variation which are mainly based on the balance of stresses i.e. the compressive residual stresses which have to be balanced by a tensile stresses in the core (for more details, please refer to a previous author's paper [5]).

Depending on the used assumptions and methods, the literature shows a large number of multiaxial fatigue criteria. The classification of the various fatigue criteria can be done according to the introduced combination on the fatigue function.

The multiaxial fatigue criteria considered in this study are based on the global approach. With this approach, the fatigue function is a linear combination of the second invariant (J2) of the deviatoric stress tensor and the first invariant of the stress tensor. The most famous criterion with this approach is the Crossland’s criterion [8]. It considers the path of the macroscopic loading. The risk of fatigue failure is expressed as a linear combination of the amplitude of the second invariant of the deviatoric stress tensor and the maximum value of the hydrostatic stress during the loading cycle.

$$\sigma_{eq,CRO} = \sqrt{J_{2a}} + \alpha_{CRO} P_{H,max} \leq \beta_{CRO} \tag{1}$$

with,

$$\begin{aligned} \sqrt{J_{2,a}} &= \max_{t \in T} \sqrt{J_2(t)} ; \quad \sqrt{J_2(t)} = \sqrt{1/6 [(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{yy})^2] + \sigma_{xz}^2} \\ \alpha_{CRO} &= \frac{3\tau_w}{\sigma_w} - \sqrt{3} ; \quad \beta_{CRO} = \tau_w \end{aligned}$$

$\sigma_w$  : The fatigue limit under alternating bending (fully reversed bending).

$\tau_w$  : The fatigue limit under alternating torsion (fully reversed torsion).

Dang Van has also proposed a simplified version of his plane critical criteria that will be called in this study “Dang Van 2” (DV2) criterion [9-10]. This criterion combines simultaneously the shear and the hydrostatic stresses.

$$\sigma_{eq,DV2} = \max_{t \in T} (\sqrt{J_2(t)} + \alpha_{DV2} P_H(t)) \leq \beta_{DV2} \tag{2}$$

with,

$$\alpha_{DV2} = 3 \left( \frac{\tau_w}{\sigma_w} - \frac{1}{2} \right) ; \quad \beta_{DV2} = \tau_w$$

Damage fatigue (Df) involved in both considered criteria are defined by the ratio of the equivalent stress ( $\sigma_{eq,CRO}$  et  $\sigma_{eq,DV2}$ ) with respect to the allowable limit fatigue of the material ( $\tau_w$  for both studied criteria). A value very close or greater than unity of these ratios indicates an advanced risk of a fatigue failure.

$$D_{f,CRO} = \frac{\sigma_{eq,CRO}}{\beta_{CRO}} ; D_{f,DV2} = \frac{\sigma_{eq,DV2}}{\beta_{DV2}} \tag{3}$$

#### 4. Parametric study

##### 4.1. Influence of the calculation model

A gear application has been studied to identify the influence of the calculation model: Hertzian calculation with a half-space approach and the developed finite element model. The following figure shows the evolution of the two criteria: Crossland and Dang Van (simplified criterion), following the normal depth from the surface at a critical contact point on tooth flank. This corresponds to a dimensionless hardness evolution which is based on the Thomas’s formulation [11] with a case hardening depth (CHD<sub>550HV</sub>, for which Hardness=550HV) equal in this case to twice the contact half-width (b<sub>H</sub>).

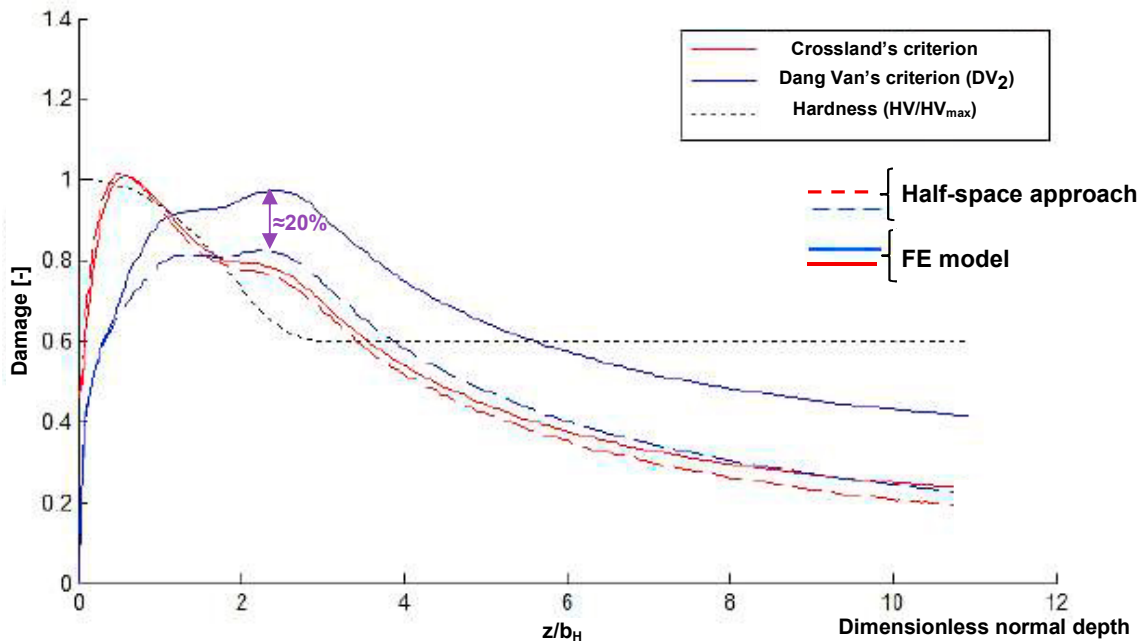


Fig. 5. Evolution of the damage criteria, ( $CHD_{550HV} = 2 \cdot b_H$ ).

By examining the set of curves shown in Figure 5, the following main observations can be drawn:

- The Crossland criterion leads to peaks that correspond substantially to the depth of the maximum variation of the shear stress obtained by applying the Hertz theory (i.e within the  $0,5 \cdot b_H$  to  $0,78 \cdot b_H$  interval of the normal depth). These peaks coincide with a criterion magnitude greater than one. This translates into a high risk of failure of the teeth by pitting or spalling. The amplitude of these criteria decreases gradually by moving towards greater depths. For this application, these amplitudes remain relatively large in the transition zone between the hardened surface layer and the core material .
- The other introduced criteria (Dang Van, DV2), is less sensitive to the presence of the maximum variation of the Hertzian shear stress. The peak amplitude with this criterion corresponds to a depth situated just below the case depth; this is a characteristic location of the initiation of failure by TFF.
- With the Crossland's criterion, both studied models led substantially to the same results (evolution shape, amplitude). However, the Dang Van's criterion is more effective to take into account the effect of the structure bending along the profile. Indeed, the criterion calculated with the FE model leads to an amplitude, at a depth situated just below the case depth, which is almost 20% higher than that obtained with the Hertz theory with the half-space approach. Therefore, the small difference on the transverse stresses,  $\sigma_{xx}$ , (in order of 50 MPa due to the introduction of the bending tensile stresses, see section 2) has an effect on the level of the damage risk according to the Dang Van's criterion. This observation confirms the interest to consider properly the tensile residual stresses in the teeth. It also justifies the definition of a limit value equal to 0,8 of the Dang Van's criterion to assess the tooth capacity with respect to a TFF failure, when the "simplified" hertzian model is applied.

#### 4.2. Influence of the case hardening depth

To study the impact of the case hardening depth, we have conducted further simulations, with another gear application, by keeping the simplified model which is based on the Hertz theory. Figure 6 shows the evolution of the two criteria following the normal depth of a critical contact point, by considering three value of the case hardening

depth ( $CHD_{550HV}$ ). The corresponding hardness evolution, for each  $CHD_{550HV}$ , is deduced by applying the Thomas's formulation [11]. A reference case without introduction of the hardening treatment was also treated (constant hardness, see Figure 6).

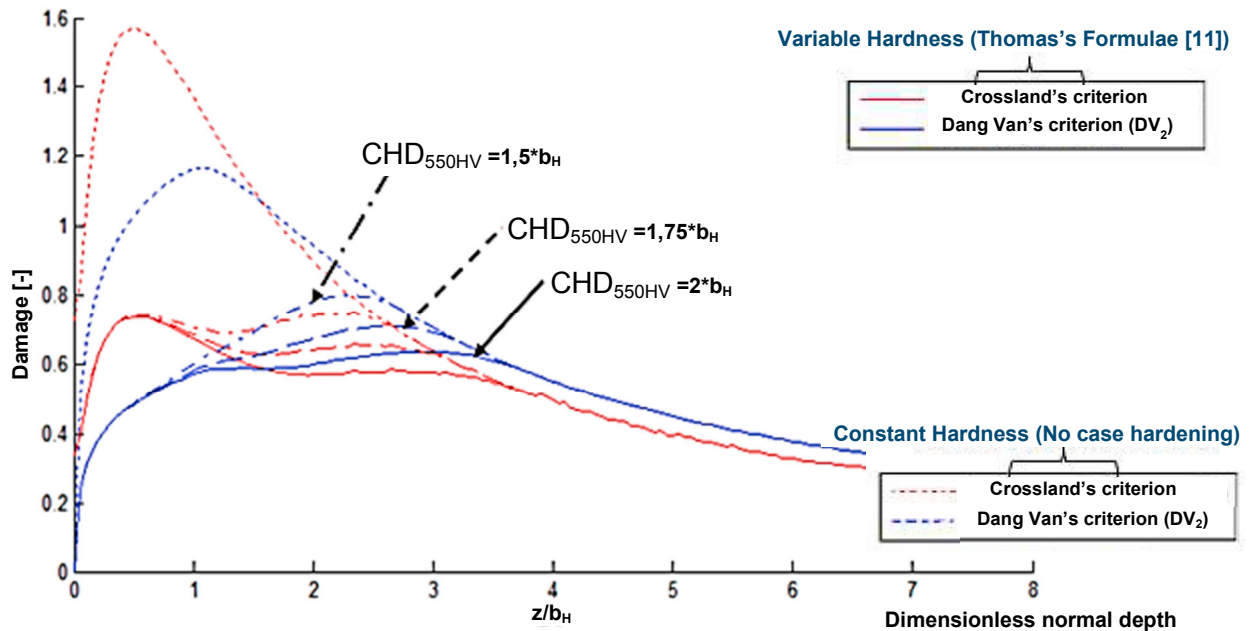


Fig. 6. Evolution of the damage criteria, for different value of  $CHD_{550HV}$ .

By examining the set of curves shown in Figure 6, the following main observations can be drawn:

- We notice the beneficial effect of increasing the case hardness depth ( $CHD_{550HV}$ ), by applying the surface heat treatment, which significantly decreases the risk of damage according to both criteria within the contact zone. We also observe that the maximum damage risk as given by Dang Van's Criteria is now located (after hardening) at the junction of the area affected by the carburizing heat treatment and the rest of the tooth area.
- We can see clearly that if the hardening is not sufficiently deep ( $CHD_{550HV} \leq 2*b_H$ ), the risk of damage, according to the Dang Van's criterion, becomes maximum at the transition zone. This is more pronounced in the presence of a rather high contact stresses.

## 5. TFF risk assessment: Developed method

### 5.1. Overview and description

In this study, we have developed a calculation tool, relatively easy to implement, to identify the risk of occurrence of TFF in the cylindrical gears. The identification procedure is divided practically in two parts:

- local strength of material
- contact model and stress fields

The flowchart shown in Figure 7 illustrates the main resolution steps introduced in this calculation method.

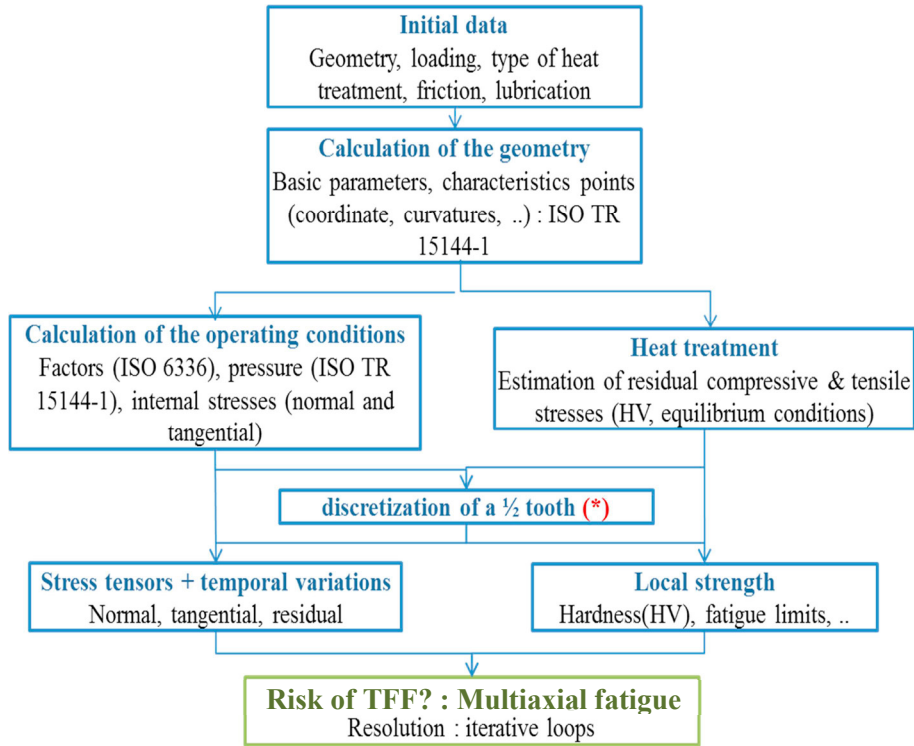


Fig. 7. Proposed approach to assess the risk of TFF.

It should be noted that to initialize the calculations, for the load and contact pressure distribution along the profile we have used the method B of ISO 15144-1 [12] which allows to determine for the gear pair of teeth in contact during a period of mesh:

- The geometric parameters characterizing the contact
- The load distribution along the tooth flank profile
- The contact pressure distribution along the tooth flank profile
- The mean friction coefficients.

5.2. Experimental validation: example developed by Witzig [6]

Table 1. Gear data as defined by Witzig [6].

-	Symbol	Description	Unit	Pinion	Wheel
				Comb.	
Geometry	$z$	number of teeth	-	67	69
	$m_n$	normal module	°	3	
	$\alpha_n$	normal pressure angle	°	20	
	$\beta$	helix angle (right-hand on pinion)	mm	Spur gear	
	$b$	face width	mm	18	
	$a$	center distance	mm	200	
	$x$	addendum modification factor	-	-0,61	-0,6169
	$d_a$	tip diameter of pinion	mm	201,2	207,16
	-	tooth flank modifications: linear tip relief	-	C <sub>a1</sub> = 30 μm, d <sub>End1</sub> = 199,62 mm C <sub>a2</sub> = 30 μm, d <sub>End2</sub> = 205,58 mm	
	$Q$	gear quality	-	6	
$R_a$	arithmetic mean roughness value	μm	0,3		
Material	-	material	-	20MnCr5	
	-	modulus of elasticity	N/mm <sup>2</sup>	206000	
	-	Poisson's ratio	-	0,3	



<b>Load</b>	$T_1$	nominal torque at the pinion	Nm	1673,5
	$n_1$	rotation speed of the pinion	min <sup>-1</sup>	1000
<b>Application</b>	$K_A$	application factor	-	1,00
	$K_V$	dynamic factor	-	1,00
	$K_{H\alpha}$	transverse load factor	-	1,00
	$K_{H\beta}$	face load factor	-	1,00
<b>Heat treatment</b>	$y_{eff}$	Case hardening depth at 550HV	mm	0,48
	$HV_s$	Surface hardness	HV	690
	$HV_{max}$	Maximum hardness	HV	690
	$HV_c$	Core hardness	HV	410

Figure 8 shows the distributions, in one half tooth thickness of the Dang Van proposed criteria (DV2) for estimating the risk occurrence of the TFF. A fine discretization is introduced with 45\*150 points and 46 time steps along the line of action or the active tooth flank profile. These simulations are carried out with taking into account the residual compressive stresses in the hardened layer by applying the Lang’s formulas [13]. The Tobie’s formulation [14] was retained to estimate the fatigue limit under alternating torsion ( $\tau_w$ ), from the hardness profile deduced by the Thomas’s formulation. The alternating bending limit ( $\sigma_w$ ) introduced in the formulation of the fatigue criteria; is deduced directly by applying a multiplicative factor of 1/0.6 to the alternating torsion limit ( $\tau_w$ ).

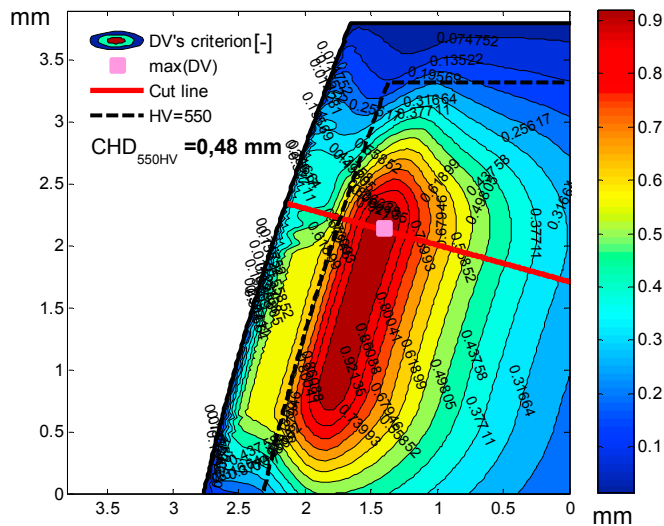


Fig. 8. Contour maps of the TFF Risk according to the Dang Van’s criteria.

The Figure 9 shows the evolution of the both criteria: Crossland and Dang Van, according to the cut line defined in Figure 8. This cut is performed along the tooth depth from the contact point that leads to the maximum value of the Dang Van’s criterion. Thus, in this figure, the x-axis corresponds to the normalized depth.

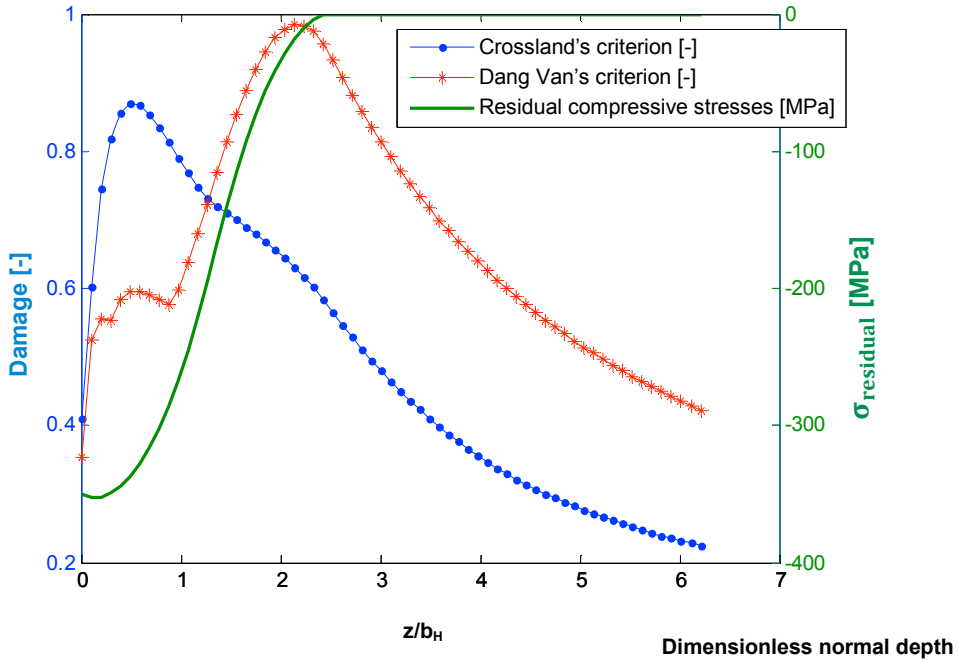


Fig. 9. Evolution of the damage criteria and the compressive residual stresses, according to the cut line defined in Fig. 8.

The distribution obtained from the Dang Van's simplified criterion shows an advanced risk of developing a Tooth Flank Fracture since it has amplitudes greater than the limit value (equal to 0,8, see section 3). An entire critical area is observed under layer at a normal depth between  $1.2 \cdot b_H$  ( $\approx 0.6$  mm) and  $3 \cdot b_H$  ( $\approx 1.4$  mm). This risk is greatest at the single highest contact point. The beneficial effects of the residual compressive stresses in the hardened layer are confirmed through these criteria. All of these findings are confirmed by the experimental results obtained by Witzig [6] in his dissertation (see Figure 10).

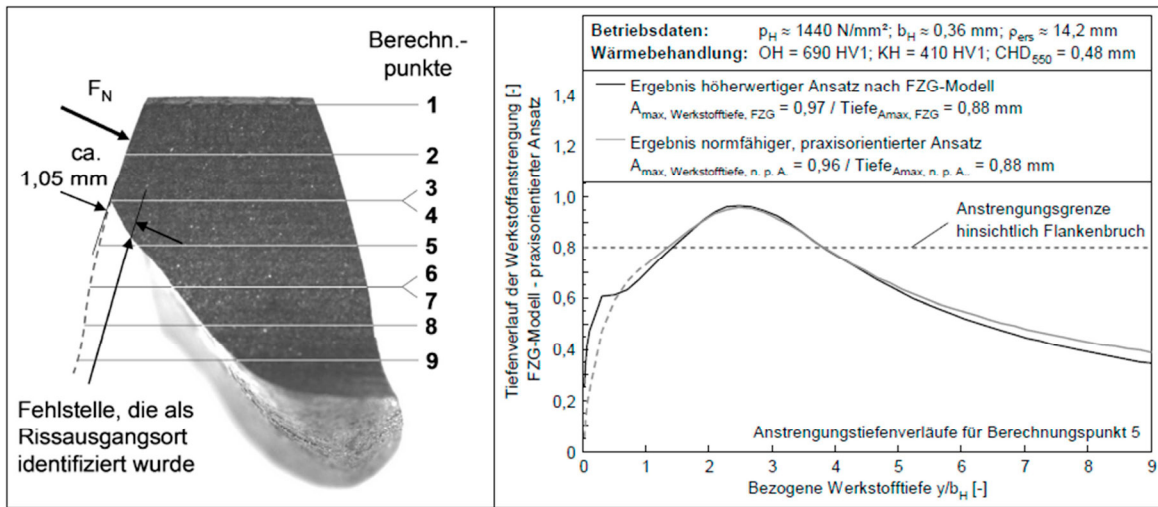


Fig. 10. Experimental validation as presented by Witzig [6].

#### 4. The future of this method in ISO standard

In the framework of the ISO Technical Committee TC 60 dedicated for gears, the working group SC2/WG6 is discussing a Technical Specification ISO/DTS 6336-4 "Tooth Flank Fracture (subsurface initiated fatigue)" in order to evaluate the risk of TFF.

A simplified approach calculation is proposed with formulae extrapolated from experimental investigations in which it is difficult to identify the impact of physical phenomena as the formulations contain several coefficients and factors depending from other ones not easy to understand.

This is why French delegation decided to propose in complement of this simplified approach the present detailed calculation method in which it is easier to keep the understanding of physical phenomena. This method should be included in Annex of this future ISO Technical Specification.

In order to be more accessible the 2 approaches in ISO have been implemented by CETIM and TFZG (TUM Munchen Germany) under Matlab with input and output EXCEL files in order to simplify the use for ISO TC60/SC1/WG6 experts.

At the moment several industrial examples are in discussion in ISO TC60/SC1/WG6 in order to finalize and to push to ballot the ISO/DTS 6336-4 document in the beginning of 2018.

#### 4. Summary

This paper presents the works done to answer to the objective of the study: characterization of damage mode of gears by the TFF failure. In this context, a comparative study was conducted to ensure the applicability of the Hetzr theory with the half-space approach at the contact between teeth. To do this, a suitable Finite Element model was developed. A small deviations in the transverse stresses, according to the profile, were observed. However, because of its formulation the Dang Van's simplified criterion is very sensitive to such a gap, which is not the case of Crossland criterion. A parametric study was carried out to properly identify the influence of the case depth hardening on the evolution of the two considered criteria.

In order to define an effective method, a numerical model was developed to predict the multiaxial fatigue criteria and the most probable localization of fatigue crack initiation. The model permits a satisfactory forecast of the TFF risk, using especially the Dang Van's simplified criterion DV2 which is easier to implement than the criteria based on the integral approach while keeping the physical phenomena.

It appears from the simulation results that the TFF remains mainly controlled by 2 parameters: the hardness profile, the contact and the compressive residual stresses in the hardened layer of tooth flanks. A sufficient hardened depth with a progressive profile (without an abrupt gradient) helps to protect the gear teeth from this failure mode. A recent study [15] has highlighted the significant effect of material imperfections (non-metallic inclusions) in explaining some failure by TFF. The resulting defects according to their sizes and density can significantly alter the hardness profile and the compressive residual stresses.

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