Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 213 (2018) 270–281

Procedia Engineering

www.elsevier.com/locate/procedia

7th International Conference on Fatigue Design, Fatigue Design 2017, 29-30 November 2017, Senlis, France

Influence of the modal damping on the estimated fatigue life

Fred Kihm[a], Arnaud Langelier[b],Kurt Munson[c]*

[a]*HBM Prenscia, 95700 Roissy-en-France, France*
[b]*Donaldson Filtration, 50420 Domjean, France*
[c]*HBM Prenscia, Southfield MI 48076, USA*

Abstract

The durability assessment of a component or structure is often assessed using Finite Element Analysis (FEA)-based structural simulations. Fatigue is a progressive failure mode that is related to the stress cycle's range in a power law such that the fatigue damage increases exponentially as the stress range increases.
When the frequency content of the loading gets closer to the natural frequencies of the structure, resonance effects are activated. In this case, the dynamic properties of the structure must be included in the analysis in order to better account for the increased stress response. A dynamic FEA produces therefore more realistic results compared to a static analysis, but requires knowledge of additional characteristics such as mass and damping. In a dynamic FEA, whether transient or steady state frequency response, the damping property governs indeed the magnitude of the dynamic stress response and hence the durability of the component.
Unfortunately a default damping value is sometimes erroneously assumed for all modes leading to errors in the stress response, which in turn leads to significant errors in the fatigue life estimates. The modal parameters – including damping ratios - for the structure's modes of vibration can be extracted using experimental modal analysis techniques.
The purpose of this article is first, to explain the critical role that damping plays in fatigue damage; and second, to recommend best practices for determining modal damping experimentally.
A simple example of an automotive component will be used to illustrate the importance of using experimental modal analysis to adjust the properties of a FE-based dynamic structural analysis. An experimental modal test followed by a modal analysis will be described. The mode shapes obtained experimentally and analytically will be compared to validate the mass, stiffness and boundary conditions used in the modelling. Then, it will be shown that using realistic values for the damping ratios will help to obtain stress results that correlate with the measured stress.

* Corresponding author. Tel.: +33 130182020; fax: =33 130182019.
E-mail address: Fred.kihm@hbmprenscia.com

Peer-review under responsibility of the scientific committee of the 7th International Conference on Fatigue Design.
10.1016/j.proeng.2018.02.028

Finally, this example will illustrate the very high sensitivity of the fatigue life estimate to the damping ratios.

Peer-review under responsibility of the scientific committee of the 7th International Conference on Fatigue Design.

Keywords: fatigue, modal, damping, dynamic FEA, virtual strain gage.

1. Introduction

Fatigue is a progressive failure mode caused by stress cycles. These stress cycles can be driven by dynamics and resonance under some conditions, so prediction of fatigue life requires an understanding of resonance.

FEA is often used for the prediction of product durability and can account for dynamics. However, these analytical stress results need to be correlated to real structural behaviour in order to accurately predict fatigue life.

Often a damping ratio is assumed when performing the FE structural analysis. This can lead to errors in the fatigue life estimation. Experimental techniques can be used to properly quantify the damping and therefore improve the accuracy of the durability analysis.

This paper recommends best practices for improving FEA-based stress and durability analysis via experimental techniques for determining modal damping and correlating analytical results with the real world.

2. Vibration, Dynamics, and FE simulation

Mechanical structures can oscillate elastically when exposed to dynamic excitation. For example, the response motion of a single degree of freedom (SDOF) system, where a single lumped mass is linked with a flexible connection to the ground can be modelled by by Equation 1 [1]:

$$F(t) = m\ddot{x}(t) + c\dot{x}(t) + kx(t) \qquad (1)$$

All physical structures have natural frequencies at which they freely respond if they are disturbed from equilibrium [2]. This is the eigensolution to Equation 1, and is called a resonant frequency. The response can be represented in the frequency domain as the frequency response function (FRF), which has both gain and phase. Gain is the ratio of response to excitation, while phase shows any lead or lag between excitation and response. Figure 1 shows an example FRF for a lightly damped SDOF system, with gain (in red) and phase (in blue).

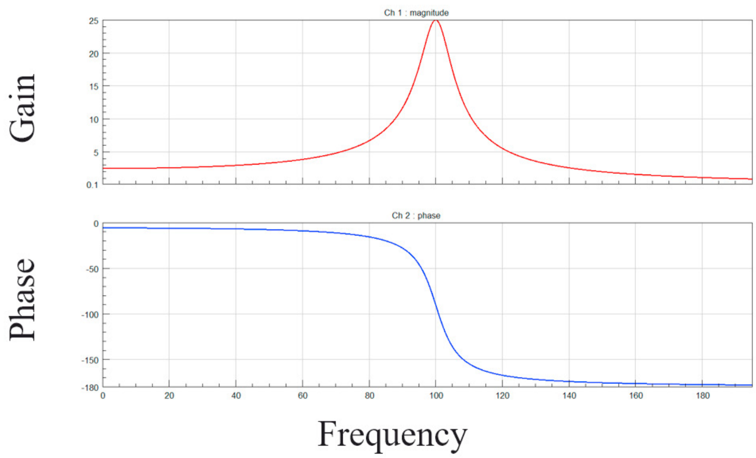

Fig. 1. Frequency response function (FRF) of a lightly damped SDOF system.

For a damped system, the natural frequency of the response is dependent on stiffness k, mass m, and damping ratio ξ in Equation 2:

$$\omega_n = \sqrt{k/m}\ \sqrt{1-\xi^2} \qquad (2)$$

Note that for lightly damped systems, the natural frequency is not heavily influenced by the damping ratio.

When excited at this resonant frequency, the magnitude of the response is dependent on stiffness and damping. As damping changes, the natural frequency changes very little, but the gain's magnitude changes significantly. Figure 2 shows the gain of the FRF for 2% and 5% damping:

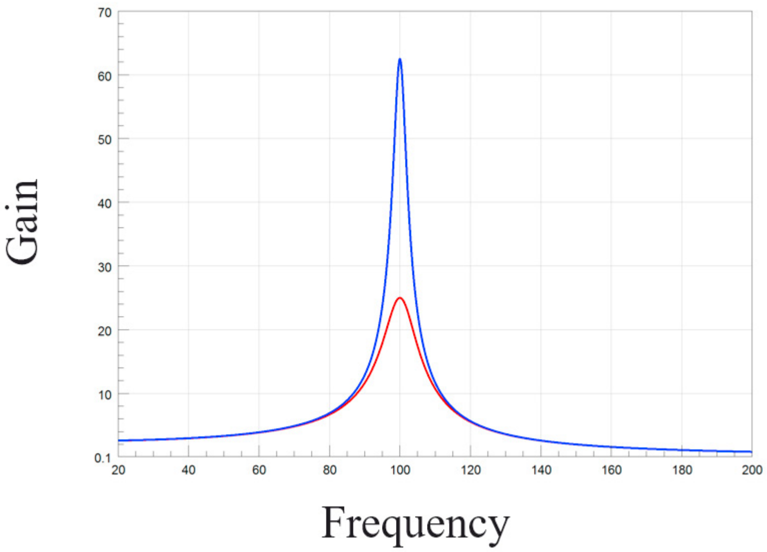

Fig. 2. FRF gain for two different damping ratios: 2% (blue) vs 5% (red).

Each natural frequency has a corresponding damping ratio. Damping values are empirical values that must be obtained experimentally [2]. Damping is proportional to the rate at which the FRF's peak decreases away from the natural frequency [3]. Mass and stiffness govern the frequency at which resonance occurs, while damping controls the magnitude of the response.

In reality, nearly all structures have multiple natural frequencies [2] and they are referred to as multiple degree of freedom (MDOF) systems, in which multiple masses can move in different amounts and directions under excitation. These relative motions at a given natural frequency create a deformed shape, also known as the eigenvector or mode shape. MDOF systems have more complicated FRFs that contain multiple resonant frequencies or modes of vibration, one for each degree of freedom. Each vibration mode has its own natural frequency, mode shape, and damping ratio.

If the modes are not closely spaced and are not heavily damped, then at a given resonant frequency, the structure behaves predominantly as an SDOF system. This means the modal parameters can be determined for each mode separately, one after another. If the modes are close, more sophisticated methods that account for modal interaction can be used to extract modal parameters [4].

Structural simulation via FEA is often used to assess the strength and durability of components and structures. This finite element stress analysis can be done with a number of different techniques: linear or nonlinear, static or dynamic, transient or steady state, time or frequency domain. These predicted stresses from FEA can then be used in conjunction with fatigue life prediction algorithms to estimate product durability.

Dynamic FEA is typically used instead of quasi-static FEA if the frequencies of excitation align with the frequencies of resonance. If the highest frequency of excitation remains less than 1/3 of the system's lowest natural frequency, the response can be approximated as quasi-static and static FEA can suffice. Up to about 1/3 of the first natural frequency, the gain is not really magnified due to the resonance (this can be observed in Figure 1). Beyond this limit, dynamic FEA is needed.

In this case, dynamic FEA is expected to be more realistic than static analysis, but it requires knowledge of additional characteristics such as mass distribution and damping. Mass's effect are important but will be ignored in this paper in order to focus on the critical role that damping plays.

3. Important factors in fatigue analysis

Fatigue is a progressive failure mode that is driven by stress cycles. If numerous enough, these stress cycles may cause structural failure over time even though the stress magnitude is not large enough to cause immediate failure. It is the combination of stress cycle size and number of repetitions of those cycles that drive fatigue failure. The size of these stress cycles is often described by the total change of stress, called stress range. The number of cycles seen in service can be counted many ways, including rainflow cycle counting [5].

3.1. The Stress Life (SN) Curve

A stress-life (SN) fatigue curve describes for a given material how the stress range and number of cycles to failure are related. The SN curve is experimentally determined by subjecting material coupons to known cyclic loading and recording failure times. Typically 10-20 material coupons are broken in order to understand the relationship between stress and life. An example SN curve is shown in Figure 3.

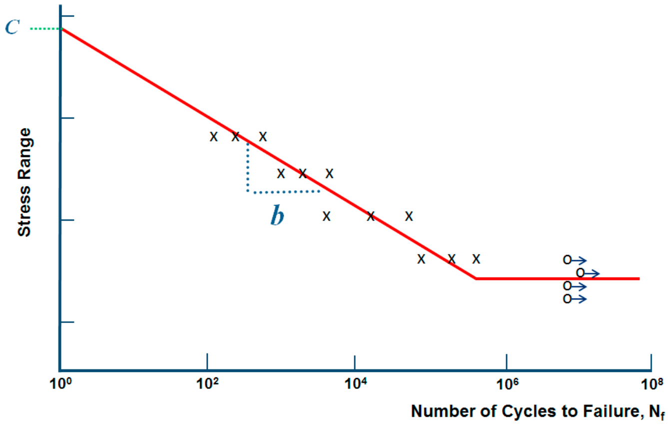

Fig. 3. Stress-Life (SN) curve.

The relationship between stress range and number of cycles to failure is often described mathematically using Basquin's formulation [6], in Equation 3:

$$S_R = C \cdot N_f^b \qquad (3)$$

where S_R is the range of the applied stress cycle, N_f is the number of cycles to failure, C is the Basquin coefficient (intercept of the SN curve with the stress range axis), and b is the Basquin exponent (slope of the SN curve).

This power law can be rearranged to solve for fatigue damage, which is the inverse of number of cycles to failure. Equation 4 illustrates the nonlinear relationship between stress range and damage:

$$Damage = \left(\frac{S_R}{C}\right)^{\left(-\frac{1}{b}\right)} \qquad (4)$$

Fatigue damage per cycle increases exponentially as stress range increases. This is mathematically related to b, the slope of the SN curve. The negative reciprocal of this is sometimes called the damage exponent, which is useful for illustrating how sensitive fatigue damage is to stress range.

3.2. Small change in stress, large change in damage

Small changes in stress range lead to large changes in fatigue damage and life. Typical SN slopes range from -0.2 to -0.1, meaning the damage exponent is 5 to 10. This means that fatigue damage is proportional to stress range to the 5-10th power, as shown in Equation 5.

$$Damage \propto {S_R}^{\sim 5-10} \qquad (5)$$

To illustrate this, assume a component is made of a material with SN slope b=-0.2 (a damage exponent of 5). Further assume that cyclic stresses increase by 10%. Equation 5 shows that this 10% increase in stress results in a 60% increase in damage, highlighting the nonlinear relationship between stress and damage.

This is why vibration and resonance both play important roles in fatigue life.

First, vibration can lead to a large number of accumulated cycles. Excitation at 40 Hz means the potential response of 40 stress cycles per second, or 144,000 cycles per hour. This potential accumulation of a large number of cycles often means that allowable stress targets must be reduced in order to withstand this accumulation of fatigue damage.

Second, and more importantly, resonance leads to increased stress response. This in turn leads to high damage accumulation due to the exponential relationship between stress range and damage. Further, even modest increases in cyclic stress lead to significant increases in damage, as shown by the SN curve. Recall that damping governs the magnitude of the FRF and hence the stress response.

Unfortunately the importance of dynamics and damping is often overlooked. If dynamics are important in the structure's response, a dynamic FEA solution must be employed to calculate the stresses for subsequent fatigue life estimation.

Sometimes a default damping value such as 2% or 5% is assumed for all modes. This may not be correct or even consistent for all of the structure's significant modes. Different modes may exhibit different damping ratios as damping can occur due to a number of different physical phenomena like viscous interaction, hysteresis, friction, and others.

4. Experimental modal analysis

Damping can be determined experimentally from physical tests, in which the instrumented component is excited over a broad frequency range [4,7]. This can be done as the component hangs in a free state while subjected to an impulsive excitation from hammer impact. This impact hammer is typically instrumented with a load cell so the magnitude and frequency content of its impulse is known. Excitation can also be provided by mounting the component on a shaker table, in which case excitation comes from the shaker's harmonic or random input. Again the magnitude and frequency content of excitation are known via measurement.

Component responses are measured during excitation. These responses are usually triaxial accelerations or displacements measured in orthogonal directions at a variety of locations spaced over the vibrating system. Noncontact acceleration and displacement methods such as a laser Doppler vibrometer can also be used. Noncontact methods offer the advantage that no mass is added to the tested system, whereas traditional contact transducers' mass may influence the system dynamics. Through this instrumentation, the input to the system and its response are then known.

The FRF is calculated by comparing the system response to the input excitation. Since typical components involve many degrees of freedom, the component usually has multiples modes of vibration, as seen in the FRF in Figure 4, where a number of FRF are overlaid. Each FRF was calculated from a different sensor positioned at a different place on the same component:

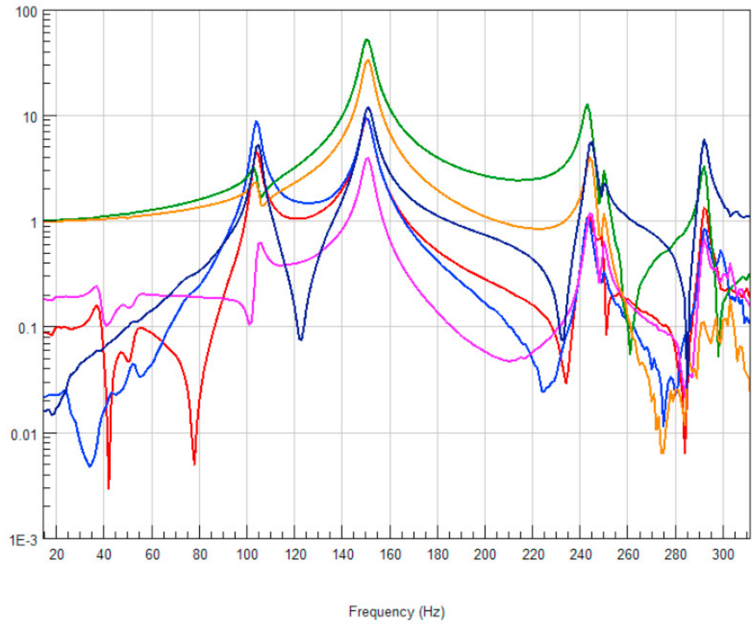

Fig. 4. Experimentally determined FRFs.

Peaks in the FRF correspond to vibration modes. Each mode has its own frequency, mode shape, and damping ratio, all of which can be calculated using experimental modal analysis. The mode shape can be visualized on a wireframe or solid model to better understand the component's modal displacements.

Damping is proportional to the height of each peak and the rate of change away from that peak, as shown in Figure 5:

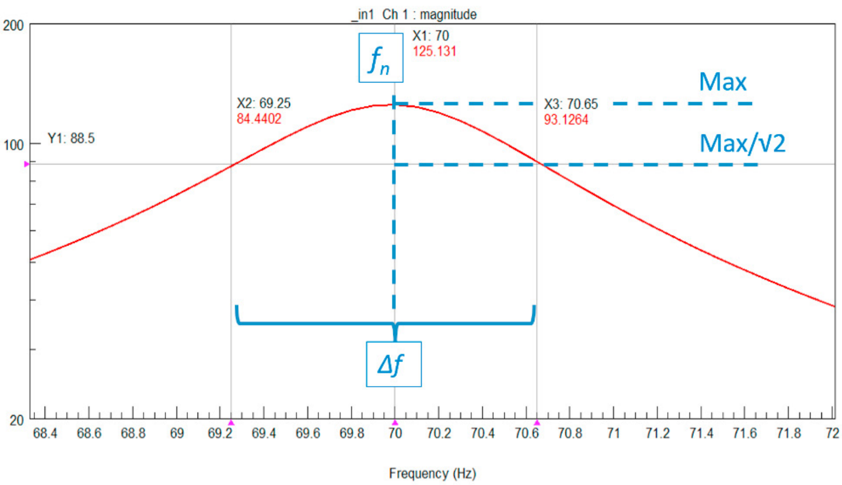

Fig. 5. Experimentally determined FRFs.

For a particular mode, the damping ratio ξ can be calculated from Equation 7 [4,7]:

$$\xi = \frac{\Delta f}{2 f_n} \qquad (7)$$

where Δf is the frequency bandwidth between the two half power points and f_n is the resonance frequency.

Example results from this experimental modal analysis are shown in Table 1. Notice that damping ratios can vary from mode to mode as different damping mechanisms may be present for each. It is not necessarily correct to assume that all modes exhibit the same amount of damping.

Table 1. Example experimental modal analysis results.

Mode	Frequency (Hz)	Damping Ratio (%)
1	172	2.5
2	255	4.3
…	…	…
N	f_n	d_n

5. Improving Fatigue Analysis through Experimental and CAE Correlation

FEA-based stress and durability calculations need to account for the system dynamics if the excitation causes resonance. Predicted stresses are dependent on modal characteristics like damping. Fatigue lives are even more dependent on damping due to the exponential relationship of the SN curve. Therefore it is important to accurately determine the damping in order to account for dynamics in fatigue life estimates.

The accuracy of the FEA-based fatigue analysis can be improved through experimental modal analysis and measured strain responses. Three steps are suggested:

- Step 1: Comparison of modal frequencies and shapes
- Step 2: Calculation of modal damping
- Step 3: Comparison of measured vs. virtual strain response

It is indeed recommended to validate the modal behaviour of the FE model by comparing the analytical natural frequencies and mode shapes with those measured experimentally. This first check may lead the FE analyst to modify boundary conditions or to check the stiffness and mass characteristics of the model in order to obtain a better match. By doing so, any further dynamic analysis is done with a greater degree of confidence. Example mode shapes comparisons between experimental modal and FEA results for an exhaust muffler are presented in the case study section (see figure 7).

This damping ratio can be quantified by experimental modal analysis. The experimental FRF is curve fitted and modal parameters are extracted. This is illustrated in the case study section (see figure 8, where the Experimental FRF is represented in blue and the curve fitted FRF is in red)

The last correlation step consists of validating the FE-predicted stresses under loading. In order to gain confidence in the analytical stresses, one can use a ‘virtual strain gage’ to extract the FE-based response and correlate them with measured strain gage data. Note that the virtual strains can be recovered in the exact orientation and location of the physical gages to aid in this correlation exercise (see figure 9 in the case study section).

If the model has been properly meshed, constrained, and loaded, the predicted virtual strain will correlate with the measured strain. Various techniques have been developed to assess the degree of correlation between the measured and predicted strain values [8].

6. Case study

The engineering staff at a company that produces filtration and exhaust systems realised how dependent FEA-based fatigue life predictions were on valid FE modelling. Their goal was to increase the fatigue prediction’s accuracy through experimental modal analysis and measured strain responses.

A muffler was instrumented with 24 triaxial accelerometers positioned at various places on the outer skin of a muffler to capture its overall motion. Figure 6 shows the test article with a few accelerometers mounted. Additional accelerometers were installed after this picture was taken. Their location is shown by the visible mounting pads. The muffler was positioned on the electrodynamic shaker and was excited with random PSD excitation.

Fig. 6. Instrumented test article on shaker.

Both the excitation and the response signals were captured and the FRFs were calculated between the excitation signal and the various responses.

A digital wireframe representing the exhaust muffler's geometry was created. The measured accelerations were mapped onto the appropriate locations and directions on the wireframe muffler. The FRFs were used to animate the wireframe at the various modal frequencies.

In parallel to this experimental activity, an FEA-based modal analysis was performed using a more refined 3D model in order to identify the theoretical natural frequencies and mode shapes. This enabled the comparison of the FE model's analytical modal behaviour with the experimentally determined natural frequencies and mode shapes. Figure 7 illustrates the mode shape comparisons between FEA and experimental results for the exhaust muffler.

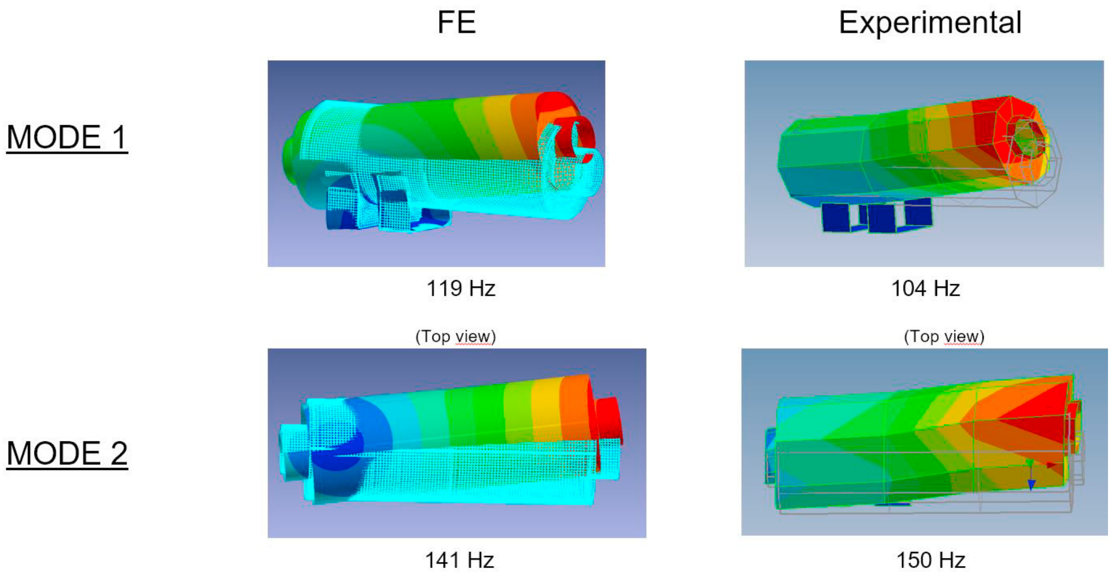

Fig. 7. Example comparison of the first mode shapes between FEA and experimental results at around 110 Hz and 140 Hz.

Mode shapes 1, 2, and 3 corresponded to first order vertical bending, first order horizontal bending, and bracket bending, respectively. This comparison showed very good agreement for the first three modes shapes and frequencies. The fourth mode shape was difficult to compare because it was a local mode concerning the muffler's internal baffles, which were not instrumented during testing. However, the measured external displacements are consistent with deformation resulting from an internal baffle mode.

After this modal comparison, the FE model was considered to correctly represent the boundary conditions, stiffness and mass characteristics of the physical muffler.

A further analysis was needed to quantify the damping and to make sure the FEA-based forced response analysis computed realistic stress levels. Experimental modal analysis was used to extract the damping ratios for each mode from the frequency response function. To do so, the FRF was curve fitted and a mathematical model incorporating modal damping was found. Figure 8 shows an example curve fit of an FRF. An MDOF curve fitting algorithm based on least squares was used to account for the closed modes [7]. The red line represents the measured FRF and the blue line is the obtained curve fit.

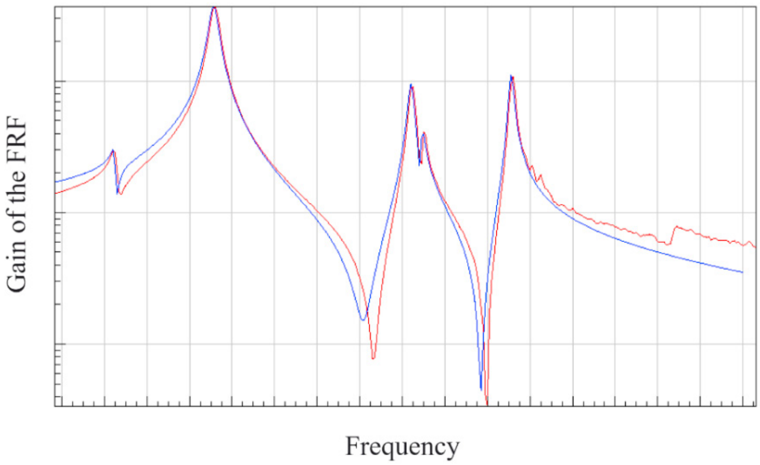

Fig. 8. MDOF Curve fitting an FRF to quantify damping.

The exhaust muffler's damping ratios were found experimentally to be between 1.5% and 2.5% for the various modes in the frequency band of interest. These empirical damping values were used for the dynamic FEA stress analysis, instead of relying on assumed or default damping values.

Two different types of stress analyses were then conducted. First, a transient FE analysis was performed to obtain time domain stress response signals. This analysis was primarily done to validate the calculated mechanical stress. Such a validation is particularly useful when the model is loaded with simultaneous multiaxial excitations, where the calculated stresses are a combination of the stresses obtained from each of the simultaneously applied loads, just as they are in the real world.

Second, in order to validate the stress results obtained analytically, virtual strain gages were used to extract the FE-predicted results and to correlate them with the measured strain gage data. Virtual strain gages enabled this comparison as the virtual strains were recovered in user-defined locations and orientations that matched strain gage placement on the physical part.

Figure 9 illustrates the use of virtual strain rosettes positioned on the brackets of the meshed FE model.

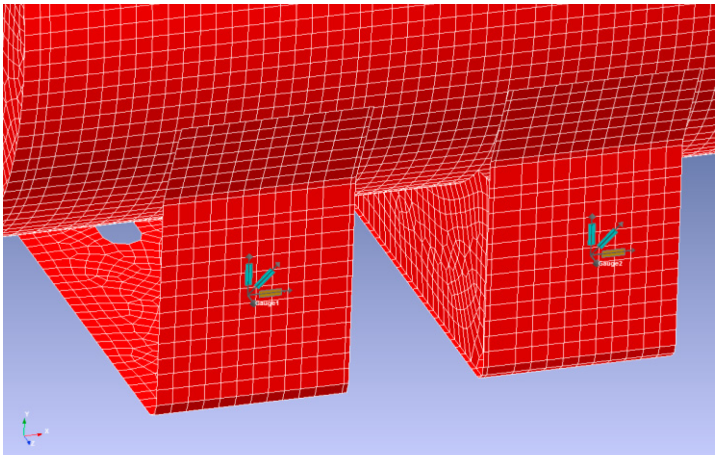

Fig. 9. Placement of virtual strain gages.

Figure 10 shows a comparison between the calculated stress and the measured stress (converted from strain).

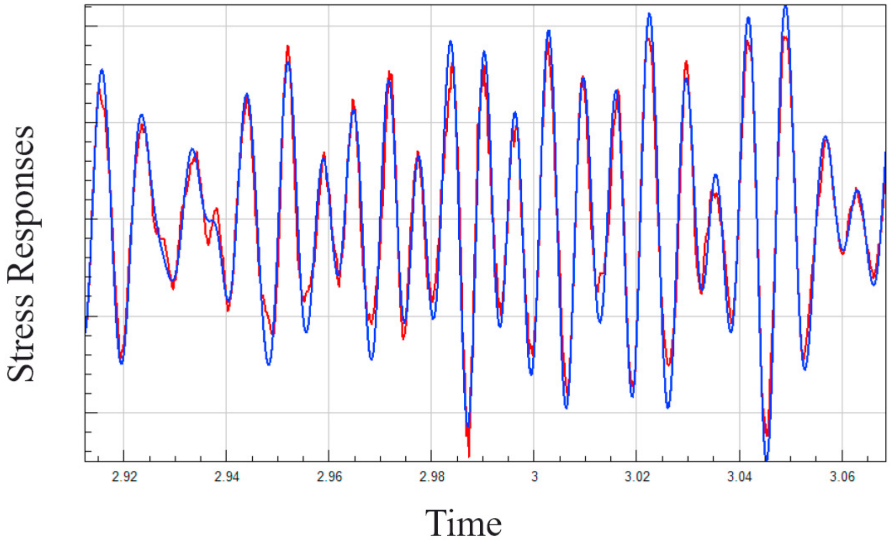

Fig. 10. Comparison between measured stress (red) and analytical stress (blue).

There was a very good correlation between the virtual strain and the measured strain, so that the FE model was considered properly meshed, constrained, and loaded.

6.1. Stress and Fatigue Analysis

A forced frequency response FE analysis was performed by exciting the structure with a unitary sinusoidal base acceleration input over the frequency range of interest. The FRF between the loading and the response was obtained. Assuming a linear structural response, the response power spectral density (PSD) G_{YY} can be calculated for any loading PSD G_{XX} as in Equation 8:

$$G_{YY}(\omega) = \left|H(\omega)\right|^2 \cdot G_{XX}(\omega) \quad (8)$$

Finally, a random vibration fatigue analysis was conducted to analytically predict the fatigue life. For the fatigue analysis, stress cycles are derived statistically from the stress response PSD GYY. There are a number of expressions typically used to determine the fatigue cycles' probability density function (PDF) from a PSD of arbitrary bandwidth. Dirlik [9] proposed an empirical closed form solution to estimate the PDF of rainflow cycle stress range. Rice [10] proposed a solution to estimate the PDF of stress peaks, which Lalanne [11] used as a basis for determining the PDF of cyclic stress range.

Stress analysis and fatigue life prediction were done twice, with damping ratios set to 2% and 5% to assess sensitivity. For this example, this change in damping ratio lead to life estimates varying over an order of magnitude. To illustrates this, figure 11 shows the range-only Rainflow cycle count histograms for 2% and 5% damping. It is clear that there are far more cycles with high stress ranges in the case with the lowest damping ratio.

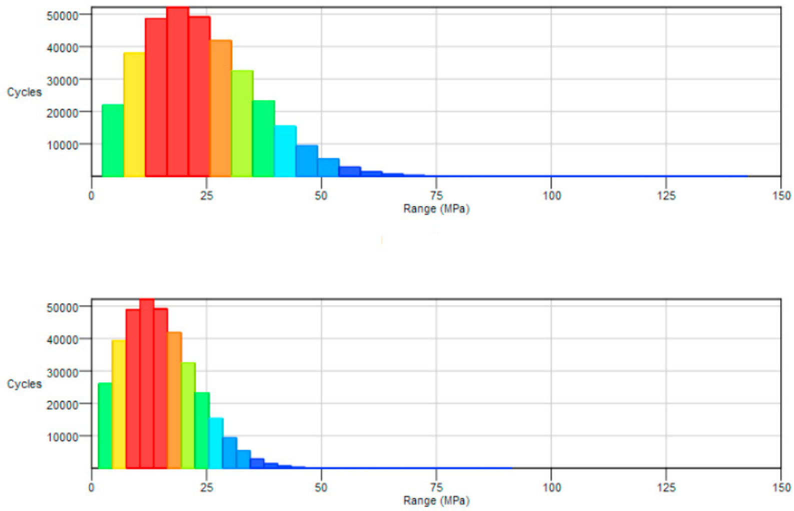

Fig. 11. Rainflow histogram for two different damping ratios: 2% (top) vs 5% (bottom).

When the test was continued to failure, the experimental life was much closer to the estimated life obtained with the correct damping ratio (2%). Actual fatigue results are not available for discussion in this paper.

This case study highlights the fact that accurate fatigue life predictions rely heavily on realistic damping ratios. The use of experimentally determined modal characteristics and measured stress or strain responses dramatically improved the accuracy of the fatigue life estimation.

7. Conclusions

This paper discusses the important role that dynamics plays in fatigue life estimates, and how to improve fatigue life estimates by using experimental modal analysis.

It discussed the importance of damping in predicting component stress response. Since the fatigue life estimation is heavily dependent on the stress response, assumed or wrong damping ratios can lead to inaccurate fatigue life predictions. Experimental modal analysis techniques may be used to calculate damping ratios from laboratory tests.

The use of experimentally determined modal characteristics and measured strain response can dramatically improve the accuracy of the fatigue life estimation. The basic concepts introduced in this paper can be summarized in the following three steps:

- Step 1: Comparison of modal frequencies and shapes to validate the boundary conditions, the stiffness and the mass distribution.
- Step 2: Calculation of modal damping to correctly predict the magnitude of the stress response.
- Step 3: Comparison of measured vs. virtual strain response to validate the meshing and the loading.

References

[1] D. Newland, Random Vibrations, Spectral & Wavelet analysis (3rd edition), New York: Dover Publications, 1993.
[2] T. Irvine, The Steady-state Response of a Single-degree-of-freedom System Subjected to a Harmonic Force, Vibrationdata.com Publications, 1999.
[3] S. Rao, Mechanical Vibrations 5th edition, Pearson Publications, 2010.
[4] D.J. Ewins, Modal Testing: Theory and Practice, Letchworth Research Studies Press, 1995.
[5] M. Matsuishi and T. Endo, Fatigue of Metals Subjected to Varying Stress, Japan Society of Mechanical Engineers, Fukuoka Japan, 1968.
[6] O.H. Basquin, The exponential law of endurance tests, Proceedings of ASTM, Vol. 10(II), 1910.
[7] nCode VibeSys Theory Guide, HBM Prenscia, 2016.
[8] J. Spadola, Best Practices for Strain Gage Correlation, nCode user group meeting, 2016.
[9] T. Dirlik, "Application of computers to fatigue analysis," PhD Thesis, Warwick University, 1985.
[10] S.O. Rice, “Mathematical analysis of random noise”, Selected papers on noise and stochastic processes, Dover, New York, 1954
[11] C. Lalanne, "Mechanical vibration & shock (volume 4)," Hermes Penton Ltd, London, 2002.