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New standard XP X50-144 related to mechanical environment strength proof. Implementation and first feedback

Jean-Yves Disson^a, Bruno Colin^b

^aMetravib ACOEM Group, 200 ch des Ormeaux, Limonest 69578, France ^bNexter Systems, 11 allees des Marronniers Versailles Cedex 78022, France

Abstract

The new standard of strength proof related to mechanical environment – design and implementation of environmental tests - includes new methods of assessment and synthesis of mechanical stresses.

This paper proposes to remind the innovations and to constitute a constructive approach, based on their implementation and on the first experiences.

The new standards contain new estimators of fatigue damage spectra and shock response spectra. In particular, the major novelty includes a statistical estimation for a given risk of overtaking of the stresses extrema and the damage extrema, which are computed from the measures performed on a duration much shorter than the in service real one. The purpose of these standards is to insure the mechanical design taking into account severe environments which

The purpose of these standards is to insure the mechanical design taking into account severe environments which may not have been measured.

The authors propose to recall the fundamentals and to perform an analysis based on comparative results. Especially, will its use make the products more reliable? Will the use of these standards become widespread?

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1. Introduction

The new standard AFNOR XP X50-144 « Demonstration of the resistance to environmental conditions — Design and Carrying of environmental tests" offers a synthesis and a guideline of all the tools and methods used to specify

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an accelerated mechanical endurance test of a specimen. Some of these methods have already been presented in military guidelines as NORMDEF 0101 [1]. The purpose of these methods is to specify an accelerated test performed on usual test bench (vibrating table) based on an equivalent damage between the real measured mechanical environment and the test. This is why these methods are also commonly named "tailoring process for mechanical environment", in opposition to the historical standards based on fallback approach that do not take into account the measured environment and which often produces a test too severe.

In particular, these methods take into consideration two mechanisms of damage: the first one for which a threshold stress value is exceeded, as the yield strength for metallic specimen. The second one, based on accumulated fatigue damage due to stress cycles.

The equivalence of damage related to the first mechanism is treated with the Extreme Response Spectrum (ERS), as the equivalence of the second mechanism is reached with the Fatigue Damage Spectrum (FDS).

However, the booklet XP X50-144-3 [2] of the new standard XP X50-144 [3] offers improved methods and tools. In particular:

- The standard insists on the relevance to calculate ERS and FDS in time domain rather than in frequency domain. Indeed, time domain is in accordance with all types of signals as frequency domain is mainly suitable only for stationary signals [4] & [5].
- The standard proposes improved ERS named XRS which introduces a stochastic approach in time domain: as the ERS deals with the deterministic measured stresses, XRS considers the largest extreme stress that should occur for a long duration and a given exceedance risk [6].
- On an equivalent principle, the standard proposes improved FDS named XFS which considers the cumulative damage for a long period and a given exceedance risk [7].

All the illustrations in the paper are related to one example. We have chosen an electronic housing, fixed on a lifting gear. Indeed, electronic components are particularly sensitive to mechanical environment.

The environment is mainly composed with 4 usage classes, named situations: two types of transport, the barrier crossing and the lifting. Note that the two transports are situations in parallel insofar as transport 1 or transport 2 is performed.



Electronic housing

Nomenclature

- C, b Parameters of "Basquin" law: $N.\sigma^b = C$
- K Constant coefficient between stress and relative displacement of the SDOF oscillator model
- CG Coefficient of Guarantee
- ERS Extreme Response Spectrum, also called Maximum Response Spectrum (MRS)
- **ERS** Mean value of ERS
- f() Density probability function

F()	Cumulative distribution function
f_0	Natural frequency of SDOF oscillator model
FDS	Fatigue Damage Spectrum
FDS	Mean value of FDS
FOH	First Order Hold
GEV	Generalized Extreme Value
М	Extrapolation factor over time
DBM	Disjoint Block Method or MBD for French abbreviation (Méthode des Blocs Disjoint)
PSD	Power Spectral Density
Q	Modal overtension factor ($Q = 10$ here)
SDOF	Single Degree Of Freedom
TF	Test Factor
Т	Duration of the environment situation of the life profile
T _{acq}	Acquisition time of the real environment $\ddot{x}(t)$
T _b	Duration of one block, associated to DBM approach
XFS	Fatigue Damage Spectra at a given exceeding risk
XRS	Extreme Response Spectra at a given exceeding risk
α	Up-crossing risk retained for equipment considered
х̈(t)	Measured acceleration environment introduces at the base of SDOF oscillator model
z(t)	Relative displacement of the SDOF oscillator model
Z _{max}	$\max(z(t))$
Z(-)	$\min(\mathbf{z}(\mathbf{t}))$
Z ₍₊₎	$\max(z(t))$
$\sigma(t)$	Stress environment generated in the SDOF oscillator model

2. Recalling about the method according to the standard

As described in the standard, the method is composed of four steps which we recall briefly.

- Step 1: establishment of the life profile where the usage conditions are identified, in particular those which can produce a significant mechanical environment. Each usage named situation is defined by its type, its duration or occurrence.
- Step 2: characterization of the life profile. Each situation environment is defined by measurements at the base of the equipment to be tested (acceleration, generally measured for three axis OX, OY, OZ), in the absence of measurements by previous measurements or standard documents. In the example, ten minutes of transport has been measured, ten barrier crossings and ten liftings, with a sampling frequency equal to 20 kHz to calculate XRS and XFS for f₀ between 1 Hz to 1 kHz with a good precision.



- Step 3: synthesis of the mechanical environment which consists to define a "simplified environment" to be simulated during the test, having the same severity that all the environments in operational condition.
 - The equivalence of severity is based on two damage mechanisms: exceeding a threshold stress value (ERS) and accumulated fatigue damage (FDS). ERS synthesis is the maximum of all ERSs averages. FDS synthesis is the sum of all FDSs averages according to the principle of linear cumulative damage, except situations in parallel (as transport 1 or transport 2) for which the max of each FDS is calculated.
 - O Coefficients of Guarantee (CG) are multiplied to the mean value of ERS and FDS, taking into account the variability of the environment, the variability of the strength of material and the probability of failure retained. The variability of the environment is generally assessed from the measurements as the one of the strength is provided by the experience or the supplier.

The following diagram illustrates the process for the selected example.



Fig 1b: process of step 3 as defined in the standard

• Step 4: definition of the test program. For each axis, the synthetized environment is increased by multiplying it with a Factory Experiment (FE) taking into account the number of specimens tested and the certainly level retained.

$$\begin{aligned} & ERS_{syntRaised} = FE \cdot ERS_{synt} & (3) \\ & FDS_{syntRaised} = FE \cdot FDS_{synt} & (4) \\ & For mechanism 1, a "shock test" is performed such that its ERS equals the ERS_{syntRaised} \\ & ERS_{syntRaised} = ERS_{test} & (5) \\ & For mechanism 2, the test is generally defined with a PSD such that its FDS equals the FDS_{synt} \\ & FDS_{syntRaised} = FDS_{test} & (6) \end{aligned}$$

3. Focus on ERS, XRS, FDS and XFS approach

3.1. ERS and FDS approach

ERS and FDS account for the effect of mechanical environment related to the two damage mechanisms. The effect is calculated on a "reference system" rather than computing the exact effect on the specimen. The reference system is an oscillator with a single degree of freedom whose natural frequency noted f_0 varies within a range covering the natural frequencies of the specimen excited by the environment and whose base is excited by the measured acceleration of environment. As mentioned in the standard, "two vibrations producing the same damage effects on the standard system will also have the same effects on the actual system".

ERS (or MRS) is the maximum displacement of each oscillator whose natural frequency f_0 is plotted in abscissa. FDS is the damage produced on each oscillator whose natural frequency f_0 is plotted in abscissa. The acceleration environment $\ddot{x}(t)$ measured during a short acquisition time (T_{acq}) is applied on the base of each oscillator. This environment $\ddot{x}(t)$ produces a relative displacement z(t) proportional to the stress $\sigma(t)$ applied in the reference system: $\sigma(t) = K.z(t)$ (7)



Assuming that the signal of environment is stationary and Gaussian, the displacement can be estimated in frequency domain with analytics calculus [8]

For 95 % of cases, the characteristics of signals do not match with these assumptions and a time domain calculus is necessary. The standard makes clear this important point which has often been reported in the literature [5] & [8]. Time domain is all the more relevant since it is suitable for any type of signal. The only disadvantage which tends to disappear with the increasing computing power is a higher calculation time than frequency analytic approach.

In time domain, the displacement z(t) is calculated from the measured signal $\ddot{x}(t)$ and the response of the reference oscillator with the FOH method, faster than the classical convolution method [7], [9]:

$$z(i)=-a_1.z(i-1)-a_2.z(i-2)+b_0.\ddot{x}(i)+b_1.\ddot{x}(i-1)+b_2.\ddot{x}(i-2)$$
(8)
where a_i and b_i are coefficients related to the displacement response of the oscillator [7]
 $ERS(f_0) = (2.\pi, f_0)^2 . \max(|z(t)|)$ where $0 < t < T_{aca}$ (9)

As mentioned, the equivalence of fatigue is reached with the Fatigue Damage Spectra (FDS). The damage is calculated from the assessed stress of each oscillator (equation 1) as follows:

- A rainflow counting of each stress cycle is performed, which is a time domain process, providing n_i cycles for each class (i) of stress amplitude σ_i = K.z_i
 (10)
- For each class (i), the damage D_i is calculated:

$$D_i = \frac{n_i}{N_i} = C.n_i.K^b.z_i^b$$
(11)

C and b are the parameters of the damage law, respectively the constant and the exponent of the law. The standard provides useful recommendations for the values of the parameters of the damage law in relationship with the type of material to be tested (electronic components, welding, steel). In our studies case, the b parameter is b = 4, suited to electronic housing.

Then, the damage D_c is cumulated according the linear Miner law: $D_c = \sum D_i$ and $FDS(f_0) = D_c(f_0)$ (12)

ERS and FDS are calculated from the measured signal during a relative short acquisition time (named T_{acq} in the standard, in general one hour maximum) in comparison with the duration of the "life situation", named $T \gg T_{acq}$

For ERS, what would have been max(|z(t)|) if $\ddot{x}(t)$ had been measured during T?

For FDS, what about the cumulative damage if $\ddot{x}(t)$ had been measured during T?

In Land Armament Industry, a new method has been proposed by B. Colin [5], [6] named MBD for "méthodes des blocs disjoints" in French allowing to process non-stationary and non-Gaussian environments cases.

In order to make secure the definition of the test specification, the standard proposes a statistical approach to assess max(|z(t)|) and the cumulative damage for the duration T and for a given exceedance risk named α . The value of the accepted risk α depends on the function of the tested component. The following table provides recommendations from the standard.

Risk α	Characteristic of the equipment
$\alpha = 10 \%$	Equipment whose function is of secondary importance
$\alpha = 1 \%$	Standard equipment
$\alpha = 0.1 \%$	Equipment for which the function is important, or safety material

3.2. XRS approach

In order to illustrate this new concept, we have chosen an electronic equipment (b = 4) assessed as "safety equipment" by its function on the lifting gear, such that $\alpha = 0.1$ %. The method proposed in the standard to assess max(|z(t)|) is described as following:

Note: for the following, max(|z(t)|) is noted Z_{max} as in the standard.

- The "local maxima" Z_{max}(i=1 to N) are extracted from the measured data by cutting the signal z(t=0 to T_{acq}=500 s) into N disjoints blocs (called disjoints blocs method).
- The distribution of "measured" Z_{max} (i=1 to N) is characterized by its mean $\overline{Z_{max}}$ and its standard deviation σ_{Zmax}
- The distribution function of the largest maxima of z(t) for the total duration T of the "life situation", named Y_M in the standard, is assumed to reach the asymptotic Gumbel model (GEV type 1) relying on the theory of extreme values for parent laws with exponential decrease [10]:

$$F_{Y_{M}}(y) \xrightarrow[M \to \infty]{} exp[-exp[-\alpha_{M}.(y \cdot u_{M})]]$$
(13)
where $M = T/T_{b} =$ number of blocks for the total duration T (extrapolation factor over time)
 $u_{M} = F_{Z_{max}}^{-1}(1 - \frac{1}{M})$ (14)
 $\alpha_{M} = M.f_{Z_{max}}(u_{M})$ (15)

 $f_{Zmax}(z)$ and $F_{Zmax}(z)$ are respectively the density probability and the cumulative distribution functions of the local maxima Z_{max}

This asymptotic model refers to the theory of generalized extreme value (GEV) when the density probability function of the local maxima Z_{max} (i=1 to N) is characterized with a thin tail (Weibull law with 2 parameters). The same approach is used for example to predict the "extreme wave height" for a 100 years period of observation [11].

For DBM approach with $T_b = 5$ s, the density probability function of $Z_{max}(i=1 \text{ to } N=100)$ can be assumed to be a Weibull model: see figure 2a showing z(t = 0 to 500 s) & figure 2b showing the histogram of $Z_{max}(i=1 \text{ to } N=100)$ for oscillator $f_0 = 50$ Hz. The Weibull probability density function is calculated analytically from \overline{Z}_{max} and σ_{Zmax} . We have check that all other oscillators have the same behavior. We have also checked that the density probability functions of $Z_{(-)}$ and $Z_{(+)}$ (i=1 to N=100) fit with Weibull models: see figures 2c and 2d.

- The value of the largest maxima of |z(t)| for the total duration T for a given exceedance risk α is then easily calculated from the Gumbel distribution function F as $y_{\alpha} = F_{Y_M}^{-1}(1-\alpha)$ (16)
- Then, XRS is calculated as: $XRS(f_0) = (2.\pi, f_0)^2 \cdot y_{\alpha}(f_0)$ (17) See figure 3a & 3b, calculated for an extended period of T = 1 hour corresponding to the average duration of each transport (in that case, XRS is named XRS_{1h}). When Tb = 5 s, the number of blocks to reach this duration is M = 720, which is a sufficiently significant value to validate the asymptotic approach of Gumbel.



Fig 2a: z(t=0 to 500 s) of oscillator at $f_0 = 50 \text{ Hz}$ Cut into N=100 blocks (red boundaries)



Fig 2c: histogram of $Z_{(-)}(i)$ for $f_0 = 50$ Hz / N = 100 blocks Weibull probability density function defined from $Z_{(+)}(i=1 \text{ to } N)$ $\beta = 1.09, \text{ eta} = -6.8\text{E-5}$



Fig 3a: XRS_{1h} with $\alpha = 0.1$ % and M = 720 (red line) ERS for each of the N=100 blocks (black dotted lines) Envelope curve = maxi of all ERS (black curve) Mean of all ERS (blue curve) Lin scale for f₀ axis



Fig 2b:histogram of $Z_{max}(i)$ for $f_0 = 50$ Hz / N = 100 blocks Weibull probability density function defined from $Z_{max}(i=1 \text{ to } N)$ $\beta = 1.16$, eta = 8.2E-5



Fig 2d: histogram of $Z_{(+)}(i)$ for $f_0 = 50$ Hz / N = 100 blocks Weibull probability density function defined from $Z_{(-)}(i=1 \text{ to } N)$ $\beta = 1.14$, eta = 8.17E-5



Fig 3b: XRS_{1h} with $\alpha = 0.1$ % and M = 720 (red line) ERS for each of the N=100 blocks (black dotted lines) Envelope curve = maxi of all ERS (black curve) Mean of all ERS (blue curve) Log scale for f₀ axis

Figures 3a and 3b show that the XRS with $\alpha = 0.1$ % is an envelope curve of the N individual ERS spectrum of each block. Moreover, the ratio of XRS_{1h} divided by the ERS of the signal (see figure 4a) is between 1.5 and 2.2 according to the frequency f₀.

We notice XRS_{1h}/ERS is the larger as the coefficient of variation CV_{ERS} of "measured ERS" (*i.e.* the standard deviation divided by the mean) becomes high: see figure 4b where we see that CV_{ERS} is in the range 0.6 to 0.9 depending on f₀. Indeed, the frequencies of the peaks on XRS_{1h} are similar to those of the coefficient of variation CV_{ERS} .

All these results make credible the XRS approach obtained from the DBM method, as illustrated by analytic model presented in [6].

DBM method logically depends on the T_b value to calculate XRS, for f_0 analysed between 1 Hz to 1 kHz, with Q = 10. For this reason, XRS_{1h} is also calculated with $T_b = 1$ s (N = 500 and M = 3600), like shown in figure 4c which compares the two ratio related to $T_b = 5$ s and 1 s.

 XRS_{1h} decreases slightly when T_b is set to 1 s, compared to XRS_{1h} with T_b set to 5 s, as presented in comments in § 5.2.3 of [6].

However, the use of XRS and XFS has to be defined in the standard and some parameters as the duration of each block has to be specified insofar as this paper shows their influence on XRS as on XFS.



Fig 4a: XRS_{1h} divided by ERS of the signal measured on $T_{acq} = 500$ s - $T_b = 5$ s

Fig 4b: Coefficient of Variation CV_{ERS} of $Zmax - T_b = 5$ s



Fig 4c: XRS_{1h} divided by ERS of the signal measured on $T_{acq} = 500 \text{ s} - \text{Tb} = 1 \text{ s}$

3.3. XFS approach

The method proposed in the standard is described as following:

- The "local damages" D_i (i=1 to N) are calculated from the measured data by cutting the signal z(t=0 to T_{acq} = ٠ 500s) into N disjoints blocs. The Basquin coefficient is set to b = 4 as recommended in the standard for electronic equipment.
- The distribution of "measured" damages D_i (i=1 to N) is characterized by its mean \overline{D} and its standard deviation σ_D , allowing to calculate variation of coefficient of measured damages : С

$$W(D) = \frac{\sigma_D}{\bar{D}}$$
(18)

Assuming all variables D_i are independent and identically distributed, the "central limit" theorem is used in order to define the probability density function of the cumulative damage D_c reached to the duration of the transport situation, *i.e.* $T = M.T_{b}$

$$f_{Dc}(y) \xrightarrow[M \text{ large}]{} \frac{1}{\sqrt{2.\pi}.\sigma_{Dc}} \cdot \exp\left[-\frac{(y-\overline{D_c})^2}{2.\sigma_{Dc}^2}\right]$$
(19)
with $\overline{D_c} = M.\overline{D}$ and $\sigma_{D_c} = \sqrt{M}.\sigma_D$
 $\overline{D_c}$ and σ_{D_c} : mean and standard deviation of cumulative damage D_c

From equation (20), it is easy to show that the cumulative damage $D_{c,\alpha}$ for an exceedance risk α is:

$$XFS(f_0) = \overline{D_c} \cdot [1 + \sqrt{2} \cdot CV \quad (D_c) \cdot erf^{-1}(1 - 2, \alpha)]$$

$$CV(D_c) = \sigma_{D_c} / \overline{D_c} = CV(D) / \sqrt{M}$$
(20)

M is assessed in order to get one hour of transport, which is the average duration of each transport. When $T_b = 5$ s, the number of blocks to reach this duration is M = 720.

Results for one transport are shown in figures 5 to 7, for $T_b = 5$ s and b = 4. To clarify, XFS and FDS related to one transport of 1 hour are named XFS_{1h}, FDS_{1h}.

 XFS_{1h} is calculated from equation (21) with M = 720.

$$FDS_{1h} = \frac{T}{T_b} \cdot \overline{FDS_{Tb=5s}}$$
(21)

The comparison of the FDS_{Tb} of each block and the "damage growth rate" related to XFS_{1h} and FDS_{1h} is shown in figure 5. The damage growth rate means that XFS_{1h} & FDS_{1h} are standardized to the duration T_b of each block, in order to be comparable with the FDS_{Tb} of each block. That means XFS_{1h} is divided by the number of extended blocks (M = 720), as SDF_{1h} is divided by the number of measured blocks (N = 100).

 FDS_{1h} is calculated from the entire signal, *i.e.* with one block with $T_b = 500$ s, and multiplied by 7.3 in order to match with the duration of one transport.

 XFS_{1h} is higher than FDS_{1h} of each measured block and than FDS_{1h} from the entire measured signal. The ratio XFS_{1h}/FDS_{1h} is in the range of 1.3 to 1.65 as shown on figure 6. In other words, it means that the damage will occur 1.3 to 1.65 times faster when assessed from XFS than from FDS_{1h} .

We notice XFS_{1h}/FDS_{1h} is the larger as the coefficient of variation of "measured FDS" (*i.e.* the standard deviation divided by the mean) becomes high. Indeed, the frequencies of the peaks on XFS are the same as those of the coefficient of variation: see figure 6 and 7.

The histogram of damages shown in figure 8 indicates that the distribution is a lognormal one.

As for XRS, figure 9 shows the influence of T_b : The ratio XFS_{1h}/FDS_{1h} decreases in the range 1.2 to 1.4 when T_b is set to 1 s.

10⁻⁵





Fig 5a: red: XFS_{1h} with M = 720, $\alpha = 0.1$ % for one transport black dotted line: FDS_{Tb} of each block linear scale for f₀ axis





Fig 6: XFS_{1h} divided by FDS_{1h} of the signal - Tb = 5 s



Fig 8: histogram of damage for $f_0 = 50 \text{ Hz} / \text{N} = 100 \text{ blocks Tb} = 5 \text{ s}$



Fig 7: coefficient of variation of damages of the measured bocks – Tb = 5 s



Fig 9: XFS_{1h} divided by FDS_{1h} of the signal - Tb = 5 s & Tb = 1 s

4. Conclusions

The new AFNOR XP X50-144 standard applied to the mechanical environment (XP X50-144-3) is an innovative method allowing to take into account the non-stationary and non-Gaussian nature of the environments measured on complex products emanating from the military sector (developed in 2010 by B. Colin and proposed in the framework of AFNOR in 2012-2013) or the civil sector, as presented in the main core of this article.

In contrast to the past, the DBM method proposed by this new standard recommends to characterize the nonstationary and non-Gaussian vibratory environments in the temporal domain and not in the spectral domain anymore, by evaluating the variability of this environment in terms of ERS and FDS, whose philosophical concepts remain unchanged in relation to the past spectral methods.

However, due to its ability to evaluate the variability of damage effects, the DBM method developed within the Land Armament Industry enabled the development of new tools for characterization of this very general environment type, called XRS and XFS. This allows characterizing the threshold stress overrun and fatigue damage generated on a mechanical structure whose behavior is modeled by a decoupled 1 DOF resonator assembly. These scientific concepts based on the EVT (Extreme Value Theory) for the XRS approach [6] and the TCL (Central Limit Theorem) for the XFS approach [7] have been introduced in the AFNOR XP X50-144 standard and evaluated through a software implementation developed in the Matlab environment to identify their advantages and disadvantages. In order to achieve this, the authors relied on a vibratory environment measured over a relatively short period of time (500 seconds) with the objective of evaluating the impact on the reliability of an electronic equipment for a solicitation duration of 1 hour (i.e. an extrapolation in the time domain of the order 7), taking into account a risk of overtaking in relation to the equipment criticality to be tested or to be developed.

The treated example clearly shows the method ability to experimentally evaluate the environment variation coefficients in terms of maximum stress or fatigue damage, thus making it possible to adjust by the technique of order 1 and 2, a Weibull-type statistical model, which is propagated in the time domain from the duration of a T_b block (5 seconds) to the duration of the real solicitation time T (1 hour in this case.). Moreover, it is clearly shown that the duration T_b has an impact on the assessment of the environment variability (particularly in terms of CV), whether in terms of maximum stress or fatigue damage, by passing from $T_b = 5$ s to $T_b = 1$ s. As a result, futures works are being studied at AFNOR level to define the optimization criteria to be taken into account in order to evaluate the optimal value of Tb, associated with this DBM method for characterizing environments vibrations.

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