



7th International Conference on Fatigue Design, Fatigue Design 2017, 29-30 November 2017,
Senlis, France

Strength of a pinion-motor shaft connection : computational and experimental assessment

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Abstract

Load capacity of keyway couplings is usually calculated according to standards (e.g. DIN6892, DIN743) based on nominal stress and simplifying assumptions dating back to several decades. Detailed modeling of keyway couplings is still a research topic, because of the complex mechanical behaviour involved. Moreover, the current standards apply only to usual geometries the designer sometimes needs to depart from, especially for the sake of compacity.

This is the case in gearmotors, where the input pinion is directly fixed on the electric motor shaft. This requires, for small diameter pinions, that the pinion shaft be inserted in the hollow motor shaft end.

In the design investigated hereunder, a special key is built in an opening in the hollow shaft wall. This design is substantially different from usual shaft-hub connections; it combines a geometrical notch with an interference fit, and is submitted to a peculiar stress distribution.

This article explains the detailed investigations made on such a connection. After summarizing the different possible failure modes on classical keyway connections, it explains how a simple interference fit behaves under an external load. Despite its inherent limitations, a FE model gives valuable insight into the connection behaviour : especially the influence of the interference fit, the load combination and the progressive stress stabilization after a few revolutions, due to the combination of friction and relative deformations.

Static test results are then presented, and the challenges of a realistic fatigue tests are analyzed.

A simplified dimensioning strategy is finally set out, which is more suited to practical application in the design office.

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Peer-review under responsibility of the scientific committee of the 7th International Conference on Fatigue Design.

Keywords: shaft-hub connections, coupling, pinion, keyway, interference fit, gearmotor

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1. Introduction

1.1. Industrial application : gearbox-motor connection

In order to connect the electric motor to the gearbox without intermediate coupling device, Leroy Somer has developed the so-called “Montage Intégré” (MI, i.e. compact mount), where the input pinion is directly fixed on the motor shaft.

This design requires a special shaft end. In the case of small pinions, the pinion shaft must be inserted in a hollow motor shaft end, with an additional feature for transmitting the torque: either a transverse pin, or a special key, as shown in Fig. 1.

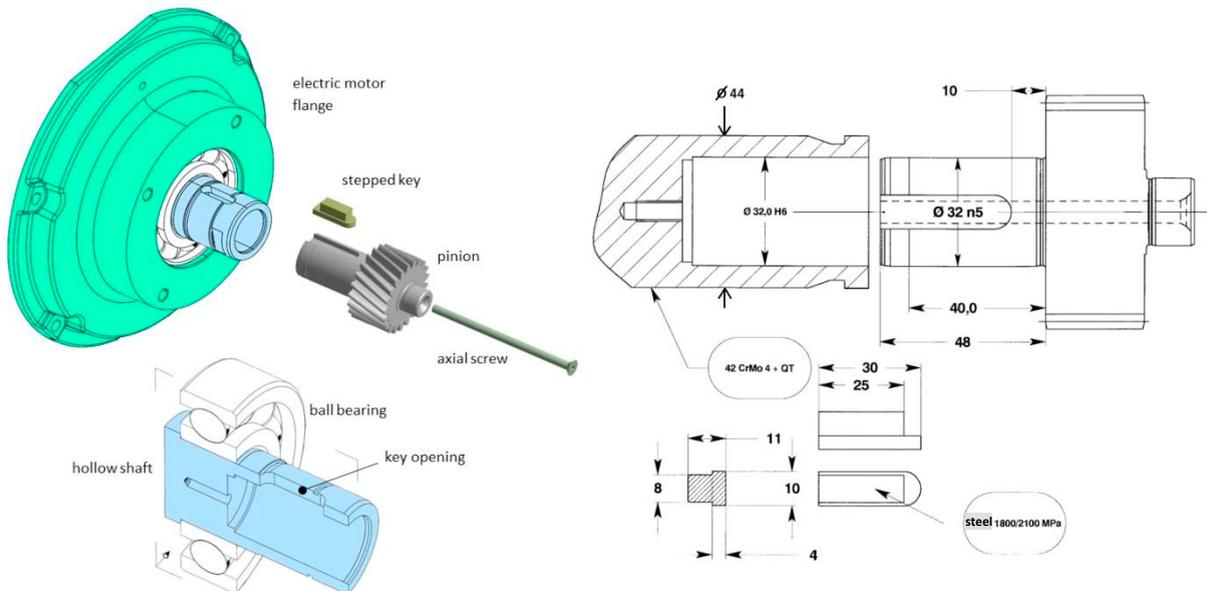


Figure 1 : hollow shaft and pinion assembly – main dimensions

With the evolution of electric motors over the years, and specifically with the introduction of the IE2 efficiency compliant motor range, the torque characteristics of the motors have changed. For the most heavily loaded connections, it is therefore necessary to assess their mechanical strength.

1.2. Aims of the study

The objective of this work is threefold :

1. Detailed strength assessment :

- What are the failure modes and where are the critical areas?
- What is the allowable torque? This is not quite the same as getting a safety margin for a given load, it is actually the inverse problem. Since no simple relationship can be given a priori between load and safety margin, solving the strength problem this way turn out to be significantly more complicated.

The estimated torque limit will depend on :

- the different load sequences (starting, nominal running conditions, stop, torque inversion)
- the unavoidable parameter variations (fit of the pinion in the hollow shaft, scatter of friction coefficient)
- the different pinion sizes (for different gearbox reduction ratios)

2. Can we use a simplified calculation model, and which one?
3. How to set up an experimental validation?

2. Failure modes of shaft-hub connection

Three main failure modes are associated with keyway connections. The first one is plastic deformation of the keyway, due to excessive contact pressure between the key and the keyway surface, when an excessive yielding of the parts leads to excessive backlash (Fig.2a). It is actually the only failure mode considered in the DIN6892 standard [2], which focuses on the contact pressure evaluation.

However, keyway connections are prone to a second failure mode, which is very often more critical than excessive contact pressure : the shaft breaks due to fretting fatigue (Fig.2b), in case of rotative bending or repeated torsion for thick-walled hubs ($Q_A < 0.7$, with $Q_A = \text{inner diameter} / \text{outer diameter}$) [5,6,7]. Therefore, ensuring enough interference between shaft and hub diameter is critical for the mechanical resistance of the connection (i.e. the larger the interference, the lower the fretting fatigue risk), and a fatigue assessment of the shaft must be done in any case.

A third failure mode may appear in case of repeated torsion (Fig.2c) ; it has been shown in [5] that the hub is always the weak point of the connection in case of Q_A being larger than 0.7 (thin-walled hubs), even for ductile materials, since the yield stress in the hub is rapidly exceeded.

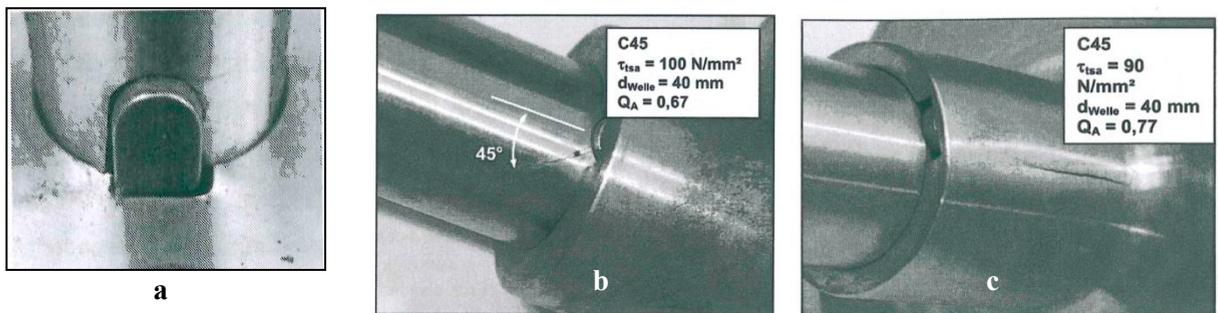


Figure 2 : failure modes of keyway connections : Sources a) Brůžek [1], b) and c) Forbrig [5]

In the references above, none of the investigated keyway connections have an opening in the hub similar to the design considered here. This opening weakens the hollow shaft considerably : as can be seen in Fig.4, the most stressed areas are the inner edges of the key opening ends, the highest stress being on the side farther from the pinion.

3. Torque transmission in interference fits under combined load

3.1. Simple analytical 2D-model

Analytical models rely on 2D continuum's mechanics, under plane stress assumption. They build the basis of standards like DIN7190 [3] or NF-E-22621 [10]. The transmittable torque is extrapolated from the contact pressure at the interface with Coulomb's friction law, under the assumption that full sliding occurs (so that : shear stress at the interface = pressure*friction coefficient). The German literature (e.g. [3,11]) interestingly introduces a distinction between friction coefficient in axial and peripheral direction.

3.2. 3D-Model under pure torque

3.2.1.1. Real geometry

When the hub is not axisymmetric, the pressure distribution at the interface is obviously modified. In the case of our connection, the key slot in the hub causes a pressure drop in its vicinity (Fig.3). The corresponding sliding torque (calculated by FEA) is 13% lower than the value given by the analytical formula above (taking into account the reduced contact surface, but with a uniform pressure).

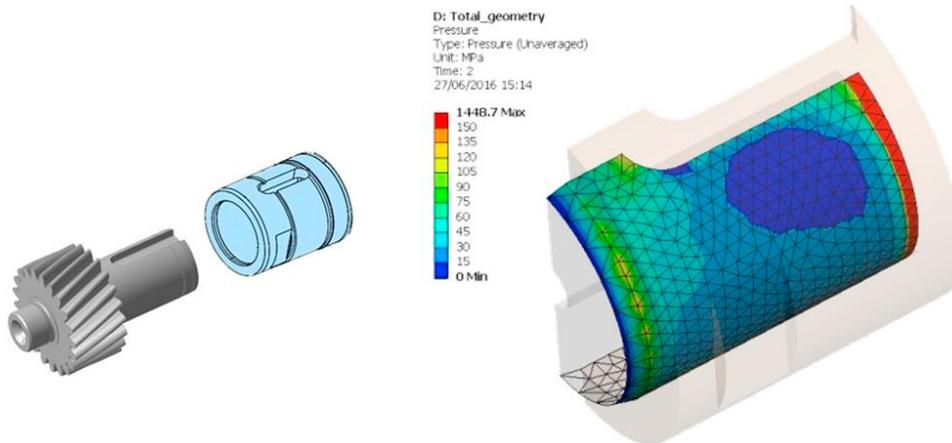


Figure 3 : contact pressure distribution due to interference

3.2.1.2. Finite length

Two additional effects are not considered in 2D models. The first one is the stress peak on the edge of the hub end, which is clearly visible in Fig.3 (top right red strip). This edge effect is also called “punching effect” (since a shorter part is pressed against a longer one).

The second one is the partial sliding occurring at the interface when enough torque is applied to the connection, even if it is still fully transmitted (so well below the full sliding torque) [6].

The combination of both is critical for fretting fatigue initiation.

3.3. Influence of a bending moment

Several authors have investigated the effect of external loads on an interference fit. Ref.[10] includes a calculation procedure to evaluate the modification of pressure distribution in case of external load. But the formulae, based on linear superposition, cannot easily take into account the possible local separation of the contact surfaces.

An external force not only modifies the pressure distribution, but can lead to loosening of the connection. As shown in [12], a bending moment decreases drastically the transmittable (torsion) torque (by 45% in the given example). Based on experiments on conical interference fits, the author suggests an ellipse-shaped curve for the torque-bending moment dependency. Loosen connections display helicoidal grooves on the shaft, evidence of the progressive “unscrewing” of the hub caused by the asymmetric axial force. However, the bending moment effect caused by a pure radial force seems not having been investigated.

3.4. Influence of a radial force

Without axial force, a pure radial force may not lead to an axial separation of the connection, in case of a cylindrical fit, but to the creeping of the hub on the shaft. This phenomenon is well known for interference fits of rolling bearing rings, and has been thoroughly investigated in [9] : deformation of the ring under the local radial load

applied by the rolling element makes the ring crawl in the bore with a slow rotation movement, causing eventually critical wear in the bearing seat.

The present work highlights this phenomenon on the FE model of the connection. Rotation of the radial gear meshing load lowers the torque transmitted by interference which is then partly transferred to the key. This is shown in Fig.4 : since the high stress area is clearly sensitive to the key pressure but very little to the torque transmitted by friction in the fit, this redistribution of the load increases the stress until balance is reached between the two load paths (interference fit and key).

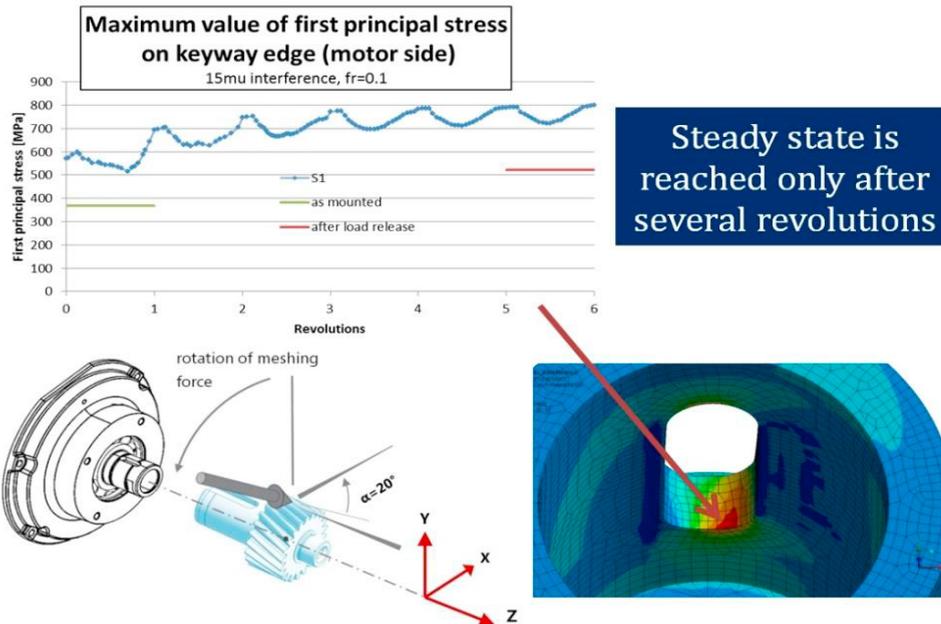


Figure 4 : progressive stress redistribution after several revolutions of the meshing force

4. Load analysis on hollow-shaft keyway connection

4.1. Load history

When dimensioning a gearmotor application, a critical point is the starting torque of the motor, especially when frequent start-stop cycles are associated with high inertia (e.g. gantry motorisation). An induction motor can deliver a starting torque more than 3 times its nominal torque. A typical load sequence on the connection is drawn in Fig.5. It is thus important to discriminate major load cycles (from one start to the next stop) from minor load cycles (each shaft revolution).

In order to address the large spectrum of applications (some of them requiring repetitive start-stop cycles), the connection is dimensioned for infinite life, i.e. stresses are related to the endurance limit.

4.2. Interference fit + Key pressure + Bending

The stress in the connexion is caused by the simultaneous combination of 3 different kinds of loads, whose variation with time differ. 1) Interference fit tends to enlarge symmetrically the keyway ; it is set once and for all by the assembly. 2) Key pressure, acting on one side of the keyway, increases during a few revolutions until reaching stabilization (see §3.4 and Fig.4), then drops when the transmitted torque is released. 3) Rotative bending comes from the gear meshing force, and will thus vary sinusoidally with rotation angle θ .

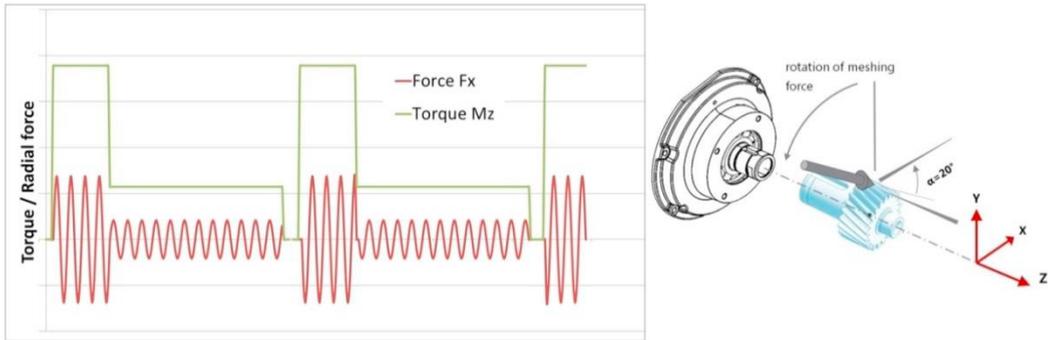


Figure 5 : typical load history (Fy is not pictured, because similar to Fx with a phase shift)

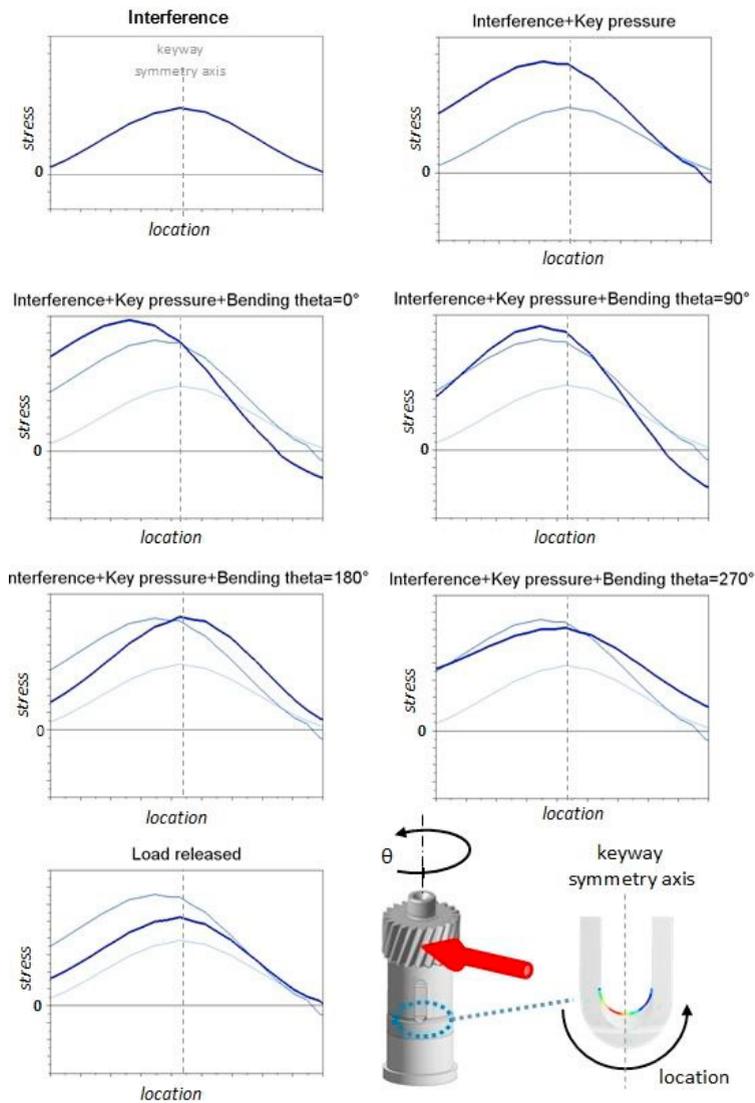


Figure 6 : first principal stress evolution in keyway end inner edge

The stress caused by these different loads in the keyway end inner edge (critical place) is displayed in Fig.6. One can see the lateral shifting of the peak stress when key pressure is applied; then the periodic shifting of the stress bump due to rotative bending (when running under torque). It is also remarkable that the stress after any load has been released does not fall back to the initial value (given by the interference fit). This is because friction in the fit maintains a residual stress in the connection.

4.3. Influence of interference and pinion diameter

Fig.7 outlines the stress pattern at keyway end depending on interference and pinion pitch diameter. The four possible combinations of interference and pinion diameter are shown, two of them giving a similar stress pattern (B and D).

In case B, C and D, the major stress cycles are given by the bending load, thus varying with rotation angle. In case A, however, the major stress cycle corresponds to a start-stop sequence. Its frequency is therefore much lower.

Increasing interference (case B and D) nullifies the torque transmitted by the key, whereas decreasing the pinion diameter expands the rotative bending load, whose compressive part may overbalance the key pressure (case C).

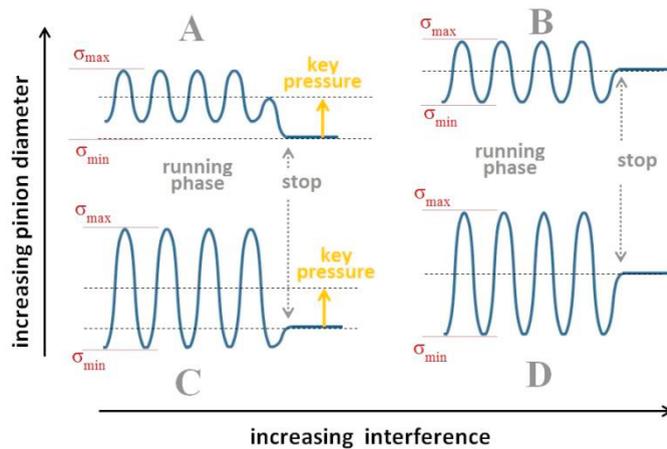
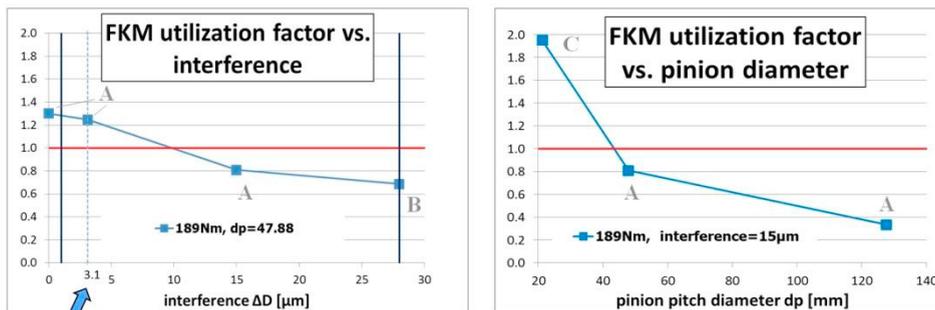


Figure 7 : different kinds of stress cycles depending on interference and pinion pitch diameter



2.5% interference probability

Figure 8 : influence of interference and pinion pitch diameter on computed connection strength

Fig.8 shows the influence of interference and pinion pitch diameter on the utilization ratio computed according to FKM-Guideline [4,8], for the load given by a 13.5kW motor (i.e.189Nm).

The lower the interference, the larger the torque transmitted through the key. The corresponding decrease of mean stress doesn't make up for the larger stress amplitude coming from key pressure. Conversely, for the maximal interference (0.028mm), the key doesn't see any torque, since interference fit alone supports the whole torque.

5. Experimental study

5.1. Static measurements

Static tests have been performed with a torque applied by a compression force at a definite distance from the shaft axis (Fig.9). They show an ultimate strength around 10 times the computed fatigue load limit, with little dependence on the torque lever arm. In both cases, a strong plastic yielding occurs in the hub wall near the critical area.

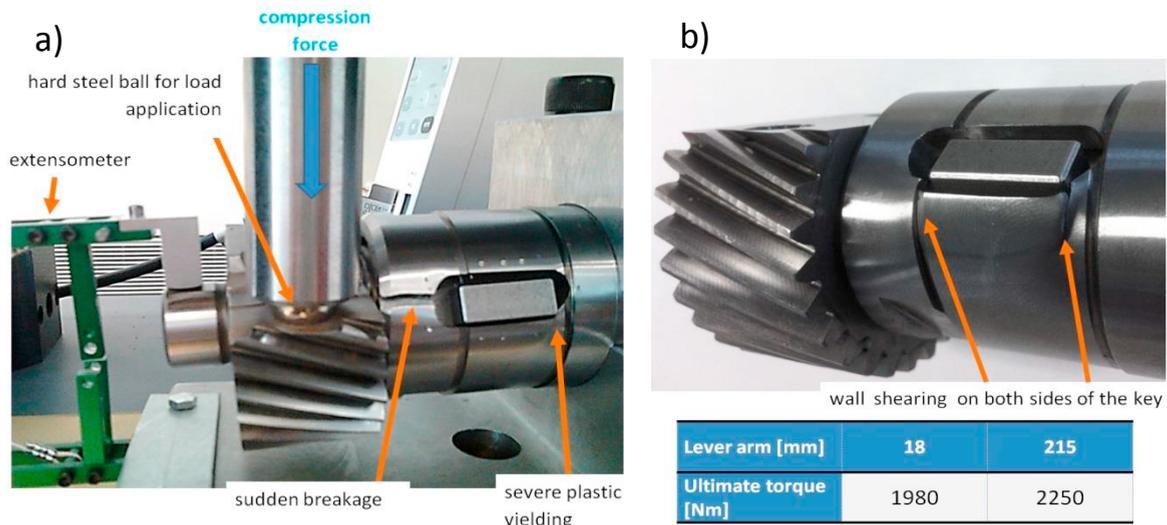


Figure 9 : static test : a) dominant radial force (lever arm 18mm) - b) dominant torque (lever arm 215mm)

5.2. Fatigue testing solutions

In order to get a representative load on the connection, a fatigue test device must address a double challenge.

Firstly, to reproduce the case A of Fig.7, it is bound to recreate the sub-synchronous combination of torque and rotative bending.

Secondly, since the key pressure is stabilized only after several revolutions, it must ensure this stabilization : if not for every load cycle, at least at the beginning of the test.

Together, these both requirements would lead to a quite slow cycle frequency, and to a complex loading device.

The suggested solution, which seems the best trade-off between simplicity and representativeness, relies on two main ideas :

- replacing the true rotative bending by a static pulsating bending, which combines easily with a torque, by means of a cantilever for example.
- forcing the stabilization by a heating coil around the hollow shaft, which cancels out the interference during the first loading (see §6.2 below).

6. Dimensioning procedure

6.1. Minimal pinion diameter

One should not overlook that the strength of the hollow shaft connection must be related to the other system components. In the present case, any torque transmission by the connection means a proportionate meshing force on the pinion teeth and a radial reaction on the motor ball bearing. It is no use designing a strong connection, if the

associated pinion or bearing doesn't reach the same strength. Moreover, the size of these neighbouring elements also dictates the dimensions of the connection.

Torque limit on pinion teeth are directly dependent on the pinion diameter. An empirical relationship is given in [11] for industrial gears, which relates the allowable torque for a pinion C_{pinion} to the cubic power of the pitch diameter d_p .

$$C_{pinion} \leq \frac{1}{2} \cdot K \cdot \frac{b}{d_p} \cdot \frac{i}{i+1} \cdot d_p^3$$

with : C_{pinion} : torque [Nmm] K : constant [MPa] b : gear width d_p : pitch diameter i : reduction ratio

This formula turns out to be quite well verified for the pinions of interest (module between 1.25 and 3 mm) with $K=4.4$, $b/d_p=1.1$, $i \leq 8$:

$$C_{pinion} \leq 2.151[MPa] \cdot d_p^3$$

This relationship between torque and pinion diameter is important for our problem, since the pinion diameter determines the radial force on the connection for a given torque. It makes therefore possible to establish a connection strength limit with the smallest possible pinion diameter, thus depending only on the torque.

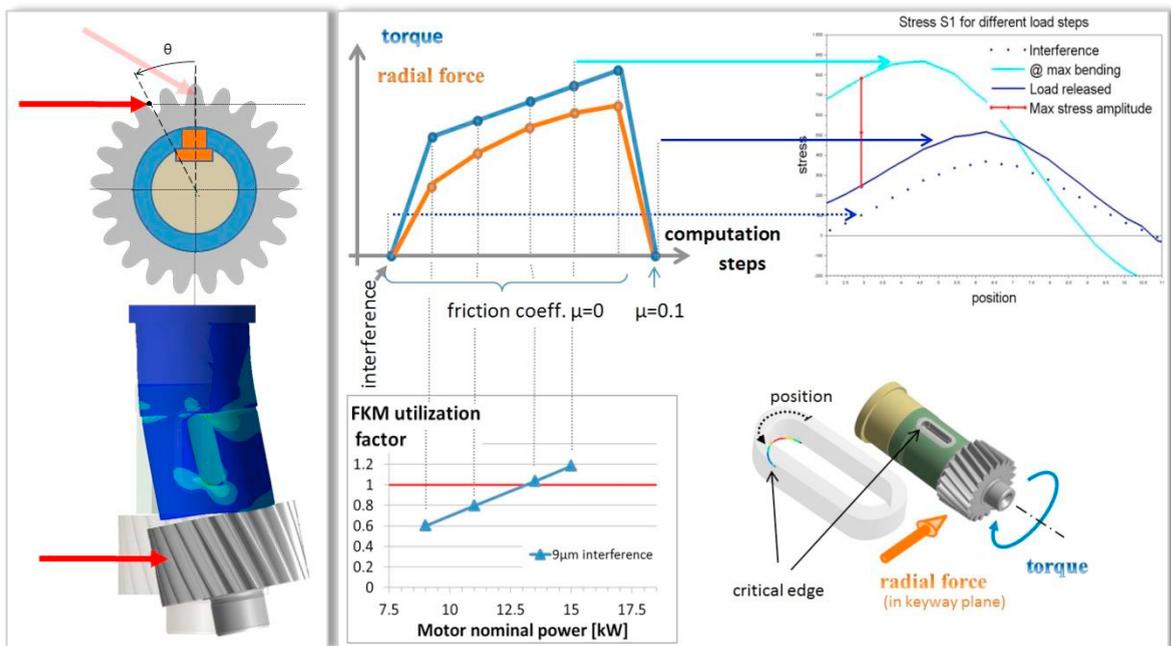


Figure 10 : a) meshing angle for maximum bending stress ; b) procedure for computing the maximum acceptable load on the connection

6.2. Assumptions

The presented dimensioning procedure relies on the following assumptions :

- the first one regards the failure mode : it is assumed that no contact fatigue occurs in the connection. This is supported by the fact that no contact fatigue failure has ever been reported on this specific kind of connection, and by the relatively thin wall of the hollow shaft.
- fatigue calculation is performed for infinite life : i.e. reference fatigue stress is the same for stress variation A on one hand, and B, C or D on the other hand (see Fig.7).
- the stabilized stress state under load is approximated by the stress state without friction.

- stress state after load release is independent of the maximal load applied before; it has been computed with a friction coefficient of 0.1, which is conservative ([12] uses a value of 0.15). Fig.11b points out the significant influence of the friction coefficient on the computed strength.
- since the system is non-linear, linear load superposition is not valid. However, as shown in Fig.8, the bending stress combined with key pressure and interference stress still determines the “worst” angular position as the angle where the bending load causes the maximum stress in the critical area. This maximum bending stress occurs for the radial force in keyway plane (Fig.10a).

6.3. Computation scheme

The computation procedure is sketched in Fig.10b. for case A, where the meshing force of Fig. 10a has been replaced by a radial force in the plane of the keyway (applied on pinion axis) and an equivalent torque. The torque is raised progressively, the radial force increasing at the same time according to the torque-pinion diameter dependency presented above (i.e. radial force \sim torque^(2/3)). Stress amplitude is then evaluated between each loaded state and the released state to get an utilization factor for different load levels.

To consider also case B, C or D, the same load sequence must be repeated with the radial force in opposite direction. The decisive stress amplitude in these cases is given by the two load steps with opposite radial force.

In the example considered here (with friction coefficient 0.1), the connection could withstand an infinite number of starts with a 13kW motor (11kW being the next lower standard power for electric motors).

Fig.11a illustrates the good accordance between full and simplified calculation for a 13.5kW motor load.

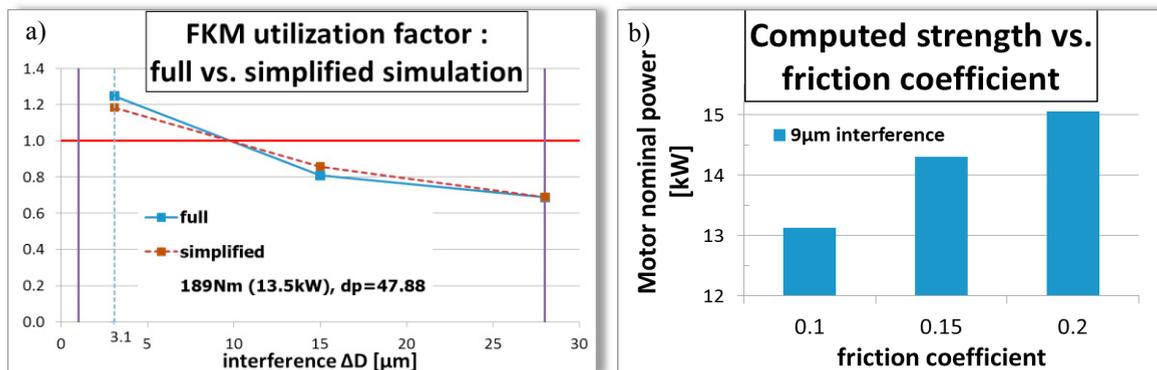


Figure 11 : a) comparison full vs. simplified simulation b) Influence of friction coefficient

7. Conclusion

This article has highlighted on an industrial example the challenge of estimating the strength of a hollow shaft-pinion connection, in the case of a specific design not covered by existing standards. Under its simplicity, such a mechanism displays a complex mechanical behaviour, due to variable load combination and to friction in the interference fit. Thanks to the simplified computation procedure laid out above, which is based on a few sensible approximations, the strength of the connection and the parameter sensitivity can be reasonably estimated (regarding especially the inevitable scatter of friction coefficient and interference).

The following elements might be suggested for further work : symmetry errors on key and keyways may lead to an additional static stress in the hollow shaft, which has been neglected so far. Neither has torque inversion been investigated. A similar fatigue assessment would admittedly be possible ; but stress surges due to shocks at reversal would not be considered. It seems therefore safer to extrapolate the results from repeated loading with the specific inversion factor given in [2]. In any case, representative fatigue test results will be obviously most valuable to assess the computed strength experimentally.

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