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Elastic plastic approximation procedure for notched bodies subjected to thermal transient loadings

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Abstract

Components of power plants are often subjected to thermo-mechanical loading conditions. Thermal loadings alone are strain-controlled loadings, inducing locally high mechanical strains and stresses, which may result in low cycle fatigue issues. Furthermore, these cycles are mixed with numerous cycles of lower stress and strain ranges. Regarding applicable design codes such as ASME, French RCC-M or German KTA, fatigue evaluation of such components can be based on the simplified elastic plastic fatigue analysis as the standard option and alternatively on elastic plastic finite element analysis. With regard to processing of long load-time histories (e.g. within an online or offline fatigue monitoring approach), elastic plastic finite element analyses are too time-consuming and not feasible. In contrast, the simplified elastic plastic fatigue analysis is a comparatively fast method, but may yield overly conservative results (and in some rare cases underestimate elastic plastic strain ranges). This may lead to unsatisfactory results by neglecting important influences (cyclic plastic deformation behavior, load sequence and mean stress). In order to consider effects of load sequence and mean stress in fatigue evaluation, it is necessary to calculate the local stress-strain paths over the entire load-time history, using the elastic plastic deformation behavior of the material. The application of commonly used notch approximation procedures (e.g. Neuber's rule, equivalent strain energy density method) fail under thermo-mechanical loading conditions by overestimating the local stresses and strains.
As a general application e.g. for the purpose of long-term fatigue monitoring, measured or calculated temperature-time sequences have to be transferred to fatigue relevant stress and strain time sequences at critical locations. In order to support this task, a fast approximation procedure will be developed in order to overcome the shortcomings of plasticity estimation as an essential part of the fatigue analysis.

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1. Introduction

By monitoring of components in power and other technical plants, the operators should be qualified to ensure a safe long-term operation with the benefit of a more economical usage of their resources. The realistic consideration of loads and plasticity are two major factors influencing the results of the fatigue analysis. The processing of long load-time histories derived from the monitoring for the fatigue evaluation is only possible under the assumption of a linear elastic material behavior. The feasibility is assured by the proportional interdependence between the loading and the local quantities as well as the applicability of the principle of superposition in case of various load cases.

In the context of a power plant typical loading, because of a thermal loading with high temperature ranges, high elastic plastic deformations may result in addition to purely mechanical induced stresses and strains by internal pressure and external piping loads. Under the consideration of plastic deformations, the linear behavior between stress and strain as well as the principle of superposition does not exist anymore. The computing of a component under variable amplitude loading by application of a nonlinear kinematic hardening rule is very time-consuming and with respect to a huge number of load reversal points, not executable.

Under these conditions, it seems to be more efficient to perform an elastic plastic calculation just at a fatigue critical location of the component, better than to perform an elastic plastic calculation for the whole component over the entire lifetime. The simplified elastic plastic fatigue analysis, utilized by the technical codes [1-3], follows these principles by a linear elastic analysis combined with a local plastification factor (K_e-factor), however in relation to many practice-relevant examples the results show a strong tendency to too short lifetimes [4]. In contrast to this fact, the development requirements to improved K_e or direct methods becomes apparent.

The aim of a new or improved procedure must be to combine the benefits of the two possible methods, either ‘fast and conservative’ (e.g. Fast Fatigue Evaluation) or ‘time-consuming and realistic’ (cyclic elastic plastic simulation of the component) toward ‘fast and realistic’.

The established method of plasticity correction by the K_e-factor is based on fictitiously elastic calculations taking just the strain ranges into account. Load sequence and mean stresses are usually considered in the design fatigue curve or by means of specific correction factors. By the new method, it will be possible to consider load sequence effects and mean stresses individually for each cycle in a subsequent damage calculation. The aspired solution bases on an incremental procedure, which takes force-controlled as well as strain-controlled (thermal) loadings into account.

Commonly used approximation procedures for the determination of local stresses and strains in notched bodies, for example Neuber [5], Seeger-Beste [6] or the ‘Equivalent Strain Energy Density’ (ESED) method [7] require a spatially limited plastic deformation under structural mechanical loadings. In case of an unlimited spatial plastic deformation, additional terms are needed [8]. The applicability of the approximation procedures (mentioned above) for thermal loadings is not given, even for the case of spatially limited plastic deformations, the component behavior influences the local behavior by its geometry and material behavior.

Nomenclature

Variables

δ	incremental step
Δ	range (of stress or strain)
f	incremental plastification factor
F	force
K_e, K_n, K_ν	code based plastification factors
σ, ε	stress and strain
S_n	equivalent linearized stress range
S_m	design stress intensity value
S_a	effective stress amplitude for damage calculation

S_p	peak stress
S_lt	thermal induced local stress
S_tb	thermal induced bending stress
t	time
Indexes	
a	amplitude
e	fictitious elastic/pseudo elastic value
el, pl	elastic or plastic part
m, t	mechanical or thermal induced value
tot	total value (sum of elastic and plastic part)

2. Theoretical

2.1. Approximation procedures

The most commonly used approximation procedure for notches under mechanical loadings is known as Neuber's rule [5]:

$$\sigma\,\varepsilon = \sigma^\mathrm{e}\varepsilon^\mathrm{e} \tag{1}$$

The resulting product of stress and strain (elastic strain energy) for a steady state under the usage of a linear elastic stress-strain law (indicated with e for the fictitiously elastic solution also known also known as pseudo elastic) is equal to the product of stress and strain under the usage of an elastic plastic stress-strain law (elastic plastic strain energy). With the knowledge of the fictitiously elastic solution, it is possible to recalculate the elastic plastic solution by a given stress-strain curve (for example defined by three parameters with Ramberg-Osgood [9]). It was shown later by Buczynski and Glinka. [10] that Neuber's rule can also be used in a kind of incremental formulation, every change of the fictitiously elastic state (by $\delta\sigma^\mathrm{e}$ and $\delta\varepsilon^\mathrm{e}$) leads to a change in stress and strain (by $\delta\sigma$and $\delta\varepsilon$):

$$\sigma\,\delta\varepsilon + \delta\sigma\,\varepsilon + \delta\sigma\,\delta\varepsilon = \sigma^\mathrm{e}\,\delta\varepsilon^\mathrm{e} + \delta\sigma^\mathrm{e}\,\varepsilon^\mathrm{e} + \delta\sigma^\mathrm{e}\,\delta\varepsilon^\mathrm{e} \tag{2}$$

Molski and Glinka postulated an approximation [7] for stress and strain in notches, based on the 'Equivalent Strain Energy Density' (ESED, just another definition of strain energy), which can also be written in a kind of incremental formulation:

$$\sigma^\mathrm{e}\,\delta\varepsilon^\mathrm{e} + \frac{1}{2}\delta\sigma^\mathrm{e}\,\delta\varepsilon^\mathrm{e} = \int_{\varepsilon}^{\varepsilon+\delta\varepsilon} \sigma\,\mathrm{d}\varepsilon \approx \sigma\,\delta\varepsilon + \frac{1}{2}\delta\sigma\,\delta\varepsilon \tag{3}$$

Both approximation formulae are only applicable for mechanical loadings and a spatial located plastic deformation.

2.2. Simplified elastic plastic fatigue analysis

The simplified elastic plastic fatigue analysis, as part of the nuclear standard codes e.g. ASME code and the German KTA [1,3] uses factiously elastic solutions too. Based on fictitiously elastic solutions, the stress components in a cross section are getting linearized and divided into a membrane and a bending stress, the strain ranges are taken which are leading to the highest equivalent stress between two points of time in between a cycle. The resulting equivalent strain range S_n is taken to calculate the plastification factor K_e, which depends on equivalent stress range of the membrane and bending stress S_n, as well as on a material dependent value S_m. The decisive damage parameter S_a is a stress amplitude, calculated by the product of the plastification factor K_e and the half equivalent peak stress range:

$$\mathrm{S}_a = \frac{1}{2} S_{\mathrm{p}} K_{\mathrm{e}} \tag{4}$$

The stress amplitude S_{a} is proportional to the total (elastic and plastic) strain amplitude $\varepsilon_{\mathrm{a}}^{\mathrm{tot}}$ by the Young's modulus E ($\varepsilon_{\mathrm{a}}^{\mathrm{tot}}=\varepsilon_{\mathrm{a}}^{\mathrm{el}}+\varepsilon_{\mathrm{a}}^{\mathrm{pl}}=S_{\mathrm{a}}/E$)

The French RCC-M code [2] separates mechanical induced (index m) from thermal induced (index t) loadings, the calculation of the plastification factor $K_{\mathrm{e,m}}$ for the mechanical loading follows the procedure of ASME and KTA standard, the plastification factor for the thermal induced loading follows a different way and is generally considered less conservative:

$$\mathrm{S}_a = \frac{1}{2}\left(S_{\mathrm{p,m}} K_{\mathrm{e,m}} + S_{\mathrm{p,t}} K_{\mathrm{e,t}}\right) \tag{5}$$

Another variation of the simplified fatigue analysis is Adam's proposal [11], which is also adopted to the ASME code [12]. The proposal separates the loading into a mechanical and a thermal loading too, with the difference that only the thermal local induced stresses (S_{lt}) and the thermal bending induced stresses (S_{tb}) are treated separately, the thermal induced membrane stresses are treated like the mechanical induced stresses.

$$\mathrm{S}_a = \frac{1}{2}\left(K_{\mathrm{e}}(S_{\mathrm{p}} - S_{\mathrm{lt}} - S_{\mathrm{tb}}) + K_{\nu} S_{\mathrm{lt}} + K_{\nu} K_{\mathrm{n}} S_{\mathrm{tb}}\right) \tag{6}$$

The plastification factor K_{ν} for the thermal induced local and bending stress depends in contrast to French RCC-M code not on the linearized equivalent stress range, it depends on the equivalent peak stress range. K_{n} is still 1.0, when the notch is modelled sufficiently detailed. This kind of more sophisticated plasticity correction factor is actually still under discussion.

2.3. Previous study

In a previous study by the author [13] it was shown that it is possible to recalculate the elastic plastic behavior in a notch subjected to thermal and mechanical induced loadings based on the results of a fictitiously elastic solution. A notched plate under cyclic tension (mechanical loading) and a cyclic thermal transient (thermal loading) was used (s. Fig. 1). The solution was separated into an elastic plastic solution for the mechanical loading (achieved with Neuber's rule) and an elastic plastic solution for the thermal loading.

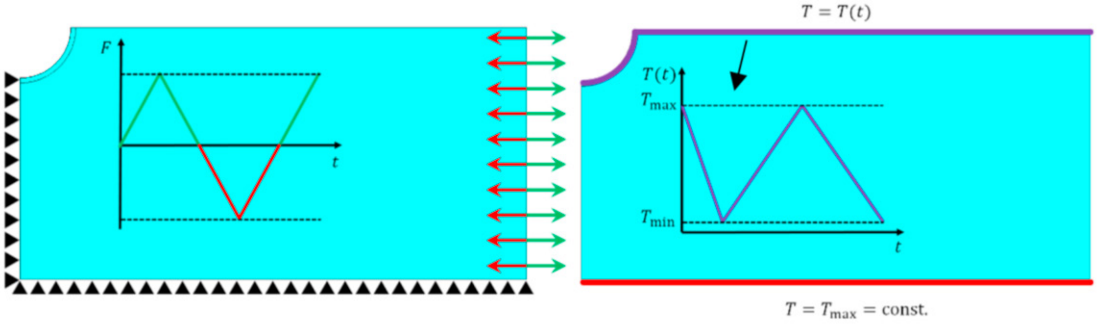

Fig. 1. Plate with mechanical loading (left) and thermal loading (right) [13]

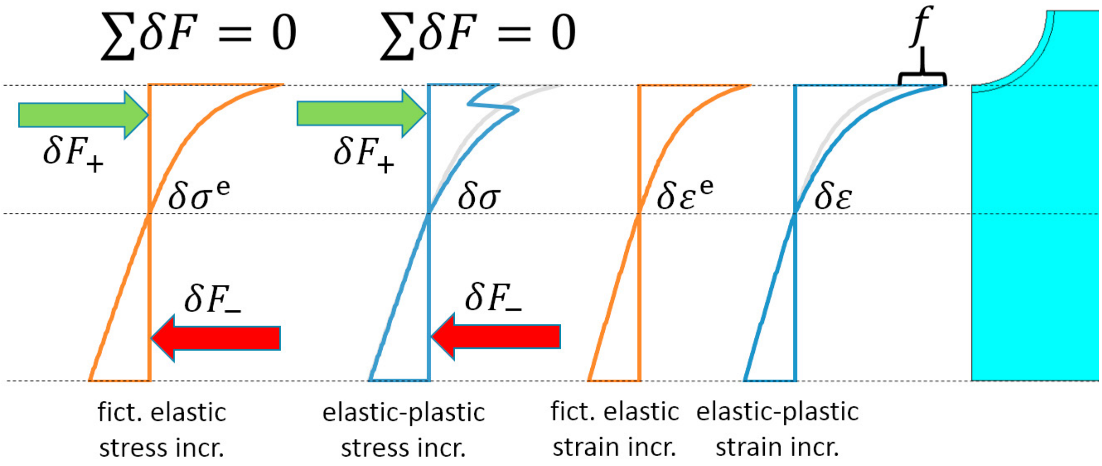

Fig. 2. Resulting stress and strain increments in the cross section for thermal induced loading, pseudo elastic stress increments in contrast to elastic-plastic stress increments (left), pseudo elastic strain increments in contrast to elastic-plastic strain increments (right)[13]

The elastic plastic solution for the thermal loading was achieved by the restauration of the inner equilibrium, by this it was necessary to consider the stresses in the whole cross section. Fig. 2 shows the resulting stress increments for a change of thermal loading, for the fictitiously elastic solution as well as for the elastic plastic solution in contrast to the fictitiously elastic one. Without any external forces, the inner forces must be in equilibrium. By this condition, the incremental changes in stress over the cross section must be in equilibrium too, for the fictitiously elastic solution as well as for the elastic plastic solution. When the material in the notch reaches the yield strength, the stress growth rate drops down to a lower level caused by the lower tangent modulus in plastic region. To fulfil the equilibrium continuously, the elastic plastic strain increments need to be higher than the fictitiously elastic strain increments (s. Fig. 2, right), the calculation to be done iteratively. The ratio between the elastic plastic strain increment and the fictitious strain increment is denoted as incremental plastification factor f. The incremental plastification factor depends on the loading, the geometry and the elastic plastic material behavior, for a comparison with the code based plastification factor K_e, a modified plastification factor K_e^{mod} can be calculated by the quotient of the elastic plastic strain range to the fictitious elastic strain range:

$$K_e^{\mathrm{mod}} = \frac{\sum \delta\varepsilon(t) f(t)}{\sum \delta\varepsilon^{\mathrm{e}}(t)} = \frac{\Delta\varepsilon}{\Delta\varepsilon^{\mathrm{e}}} \tag{7}$$

For the combination of both elastic plastic solutions in the notch (for mechanical and thermal loading) to a total solution (index tot), two incremental formulations have been used. Both incremental formulations comply with the principle of superposition in pure elastic region, but they do not follow the principle of superposition in plastic region. The first formulation is equivalent to Neuber's rule, but for a combination of two partial solutions, not the strain energies need to be summed:

$$\begin{gathered}(\sigma_{\mathrm{m}} + \sigma_{\mathrm{t}})(\delta\varepsilon_{\mathrm{m}} + \delta\varepsilon_{\mathrm{t}}) + (\delta\sigma_{\mathrm{m}} + \delta\sigma_{\mathrm{t}})(\varepsilon_{\mathrm{m}} + \varepsilon_{\mathrm{t}}) + (\delta\sigma_{\mathrm{m}} + \delta\sigma_{\mathrm{t}})(\delta\varepsilon_{\mathrm{m}} + \delta\varepsilon_{\mathrm{t}}) \\ = \sigma^{\mathrm{tot}}\delta\varepsilon^{\mathrm{tot}} + \delta\sigma^{\mathrm{tot}}\varepsilon^{\mathrm{tot}} + \delta\sigma^{\mathrm{tot}}\delta\varepsilon^{\mathrm{tot}}\end{gathered} \tag{8}$$

The second formulation is equivalent to the ESED method:

$$(\sigma_{\mathrm{m}} + \sigma_{\mathrm{t}})(\delta\varepsilon_{\mathrm{m}} + \delta\varepsilon_{\mathrm{t}}) + \frac{1}{2}(\delta\sigma_{\mathrm{m}} + \delta\sigma_{\mathrm{t}})(\delta\varepsilon_{\mathrm{m}} + \delta\varepsilon_{\mathrm{t}}) = \sigma^{\mathrm{tot}}\delta\varepsilon^{\mathrm{tot}} + \frac{1}{2}\delta\sigma^{\mathrm{tot}}\delta\varepsilon^{\mathrm{tot}} \tag{9}$$

Within this case study, the temperature range was kept constant at 300 K, the nominal stress range was kept constant too with 100 MPa. As well as the mean stress of the mechanical loading was chosen to 0, 50 and -50 MPa, an In-Phase (IP) and Out-of-Phase (OP) loading (between thermal and mechanical loading) have been examined. The resulting strain paths show a good agreement with the elastic plastic FE-results, the maximum deviation between approximated solution and FE solution is about 15% in strain ranges and about 8% in stress ranges. The approximated solutions leading to higher stress and strain ranges than FE solutions, approximated results according to Neuber's rule more than with the ESED formulation.

2.4. Fast Fatigue Evaluation

Elementary stress solutions (full stress tensor) are prepared based on linearly elastic finite element analysis (FEA) both for thermal and mechanical stresses in an initialization step. These elementary stress solutions, e.g. stress responses to the standardized elementary transients, in terms of all components of the stress tensor are saved in a data base. In the following working step these tailored solutions are scaled by application of the really occurring operational loads and the real component stress time histories are calculated. The quality of the calculated stress time histories is equal to a genuine finite element solution. The time of execution is only a fraction of the FEA processing time (typically 5 per cent). The calculation of partial usage factors is based on the usual rainflow cycle counting mechanisms and the Palmgren Miner damage accumulation rule. For more details concerning the Fast Fatigue Evaluation approach see reference [14]. The Fast Fatigue Evaluation approach as modular part of the AREVA Fatigue Concept (AFC) can be based on the standard plasticity correction approaches of the relevant design codes, can be combined with advanced K_e-factors (see [4]) and – as a perspective – with the plasticity correction method described herein.

3. New Proposal

The method to recalculate the elastic plastic behavior by the restauration of the inner equilibrium presented above requires a fictitiously elastic calculated solution for the whole cross section. Based on a linear elastic FE calculation, this requirement is still present. To link this method with the Fast Fatigue Evaluation approach, several steps have to be taken:

1. Creation of fictitious stress along a path through the cross section by the values of the Fast Fatigue Evaluation with membrane stress, bending stress, peak stress and an additional value for the stress gradient at the notched surface (for every time step and every relevant stress component), the approximately calculated stresses need to fulfil the equilibrium condition
2. Creation of temperatures along a path through the cross section by the values of the Fast Fatigue Evaluation with membrane value, bending value, peak value and an additional value for the temperature gradient at the notched surface (for every time step and every relevant strain component)
3. Calculation of fictitiously elastic strain along a path by the created stresses (for every time step), the elastic strains and resulting thermal strains (by the created temperatures) need to fulfil the kinematic constraints
4. Calculation of incremental changes in strain between the defined load steps

Now, the elastic plastic approximation procedure can be applied. The example used for the previous study allowed taking assumptions for simplification: Due to the dominant uniaxial behavior, stress and strain longitudinal to the cross section as well as the shear components have been neglected. With an increasing complexity by multiaxial stresses in the cross section or a decreasing size of the notch in comparison to the components dimension, additional constraints have to be considered. Not only the global equilibrium has to be considered, even the local equilibrium and the kinematic constraints. Within this procedure, the elastic plastic material law has to be solved iteratively for every time step as well as for every defined point in the cross section. The procedure is more computationally expensive than the simplified fatigue analysis, in contrast to a full elastic plastic finite element analysis (FEA) much less computationally expensive, but still able to solve in real time. The acceleration in computing, in contrast to an elastic plastic FEA is based on the reduction from a spatial problem to a plane problem respectively a plane problem to a one dimensional problem (which does not induce a reduction of the stress and strain tensor). As mentioned above, the exclusive consideration of the strain ranges on the load side within the simplified fatigue analysis, requires the consideration of mean stress and load sequence effects generalized on the resistance side. With the new proposal, the mean stress and load sequence effects can be considered individually for each load cycle.

Fig. 3 shows a comparison between the new proposal for fatigue evaluation and the conventional (code based) fatigue evaluation.

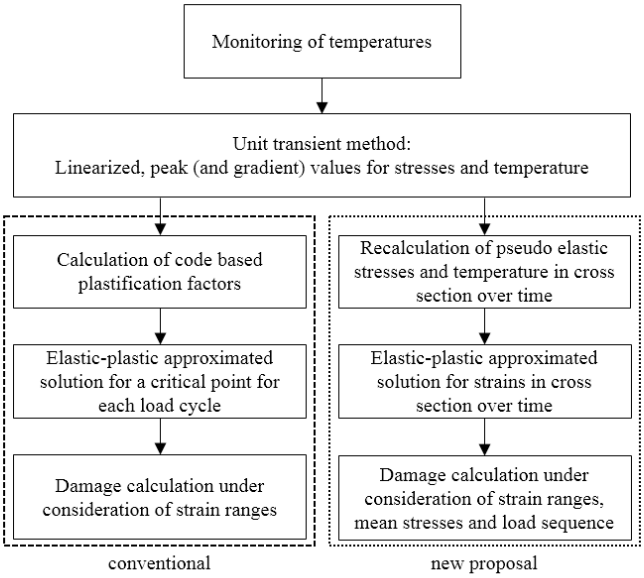

Fig. 3. New proposal in contrast to conventional (code based) proposal for the fatigue evaluation

4. Conclusion and Outlook

Load assumptions and the consideration of the cyclic elastic plastic deformation behavior of the component are two major factors of influence on the quality of a detailed fatigue analysis. The load time histories of the components can be determined in a highly realistic way as part of a fatigue monitoring concept [14]. This relieves conservatisms with respect of the loads. However, the consideration of plasticity by way of plasticity correction factors (K_e-factors) mostly induces highly conservative calculation results compared to elastic plastic analyses. The logical consequence is the development of more realistic plasticity correction procedures which constitutes the main issue of the work presented in this paper. This approach keeps the convenience and speed of executing linear elastic stress analyses. The integration of the concept in the Fast Fatigue Evaluation approach [14] combines the advantages of realistic determination of load time histories and high quality plasticity correction.

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