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Fatigue lifetime estimation of bearing pin of console manipulator loaded with multiaxial random loading.

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Abstract

Fatigue lifetime estimation of load bearing parts of construction is a key factor in a design of technique used for long term operation. Complex loading in combination with complex geometry of the most loaded parts represents only one of many problems that engineers have to deal with. One of the most commonly used approaches how to deal with this problem is to use the technical standards. This approach, despite its indisputable advantages (clear and comprehensible steps of calculation), has several significant shortcomings (non-economical oversized design). In this paper we propose procedure of how to estimate fatigue lifetime of construction parts that are loaded with random operational loading. The result of proposed procedure is distribution function of fatigue lifetime which takes into account the complex geometry of the designed part and random character of the complex loading (multiaxial stress state) in a hotspot. This approach is then demonstrated on fatigue lifetime estimation of bearing pin of a console manipulator loaded by random operational loading. Loading process was obtained using set of strain gages. The measured loading process was then transformed into stress histories in critical hotspot using FEM. Method of processing the stress loading histories in critical hotspot into a form usable in combination with hypotheses for fatigue calculation under multiaxial stress states is presented.

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Keywords: multiaxial fatigue; variable amplitude; nonproportional loading

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1. Introduction

Estimation of fatigue life time of mechanical components working under real operational loadings (usually multiaxial variable amplitude loading) could be considered as an ultimate task in a field of fatigue life time analysis. Due to the complexity of the problem, there are many factors that have crucial influence on the final lifetime estimation. Moreover, a great deal of these problems do not have widely accepted solution and usually there are multiple ways how to solve them. One way how to tackle these problems is to use the methodology given by standards. These standards normally use some simplification to make the estimation more feasible for engineers. However, as a result of these simplifications, the methodology given by standards is normally suitable only for infinite life calculation, and when used in finite fatigue life time estimation, it usually leads to oversized components, or in some cases, it can even lead to the design error on non-conservative side. Due to these facts, enormous effort was and still is put forth to postulate methodology for finite fatigue life estimation under variable non-proportional random loading [1,2,3,4,5]

Nomenclature

| | |
|----------------------------|---|
| σ_f' | Fatigue strength coefficient |
| b_σ | Fatigue strength exponent |
| τ_f' | Shear fatigue strength coefficient |
| b_τ | Shear fatigue strength exponent |
| k_{fin} | Findley criterion parameter |
| τ_f^* | Findley shear fatigue strength coefficient |
| σ_x, σ_y | Normal stress component |
| τ_{xy} | Shear stress component |
| $\sigma_{x'}, \sigma_{y'}$ | Normal stress component in rotated coordinate system |
| $\tau_{x'y'}$ | Shear stress component in rotated coordinate system |
| M | Bending moment |
| M_x | X-component of bending moment |
| M_y | Y-component of bending moment |
| α | Angle of bending moment orientation |
| τ_{eq} | Equivalent shear stress amplitude |
| σ_a | Normal stress amplitude |
| σ_n | Normal stress acting on plane |
| σ_{max} | Maximal normal stress acting on plane |
| τ_a | Shear stress amplitude |
| τ_{am} | Shear stress amplitude with nonzero meanvalue |
| τ_m | Shear stress mean value |
| N_f | Number of cycle to failure |
| CI | Confidence interval |
| N_{fi} | Number of cycle to failure corresponding to i-th amplitude |
| n_i | Number of occurrence |
| D | Damage |
| β | angel between critical place and coordinate system of measured bending moment |

In the pages below, authors show how to deal with problems that engineers have to overcome during fatigue life time estimation under multiaxial random loadings. Step by step, analysis of real component (bearing pin of pneumatic handler) loaded with random loading is shown. Usage of multiple techniques is shown for various problems and brief comparison of results and small critical discussion is provided.

2. Obtaining of loading signal during standard operational cycle

The first step in life time estimation is the identification of critical places and the definition of loading signal in these hot spots. Strain gauges have been used for measurement of loading signals and FEM calculation has been used for identification of critical places on a bearing pin.

2.1. Loading signal measurement

Strains have been measured on the smooth part of a bearing pin using foliage strain gauges. The gauges have been placed on the pin in the place where no stress or strain concentrations exist. Localization and set up of strain gauges can be seen in fig.1. Standard foliage 120 ohm strain gauges have been used and the strains have been measured using National Instrument data acquisition system.

Since only stresses caused by bending moment M are significant on the smooth part of the bearing pin, the magnitude and orientation of the bending moment can be calculated from three measured strain signals.

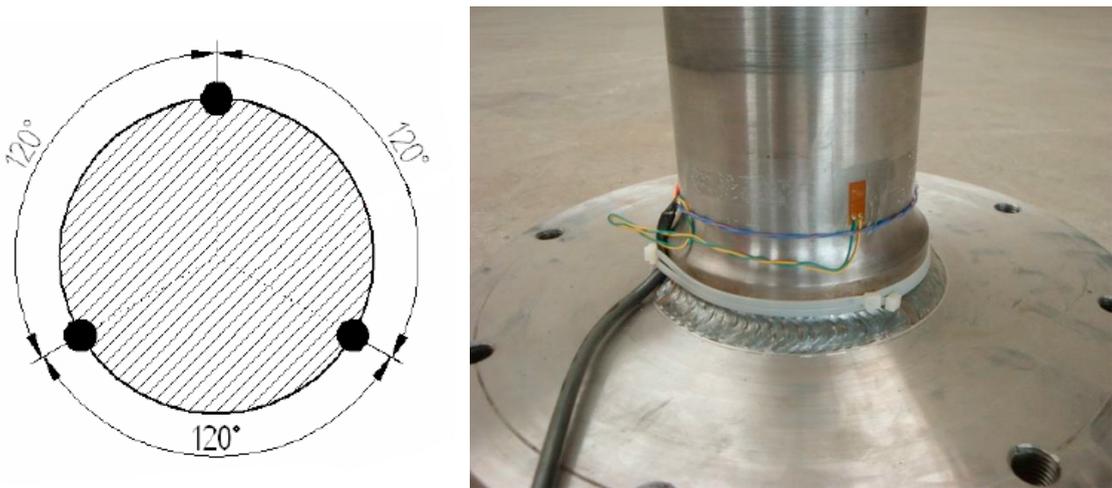


Fig. 1 Position of strain gauges on bearing pin.

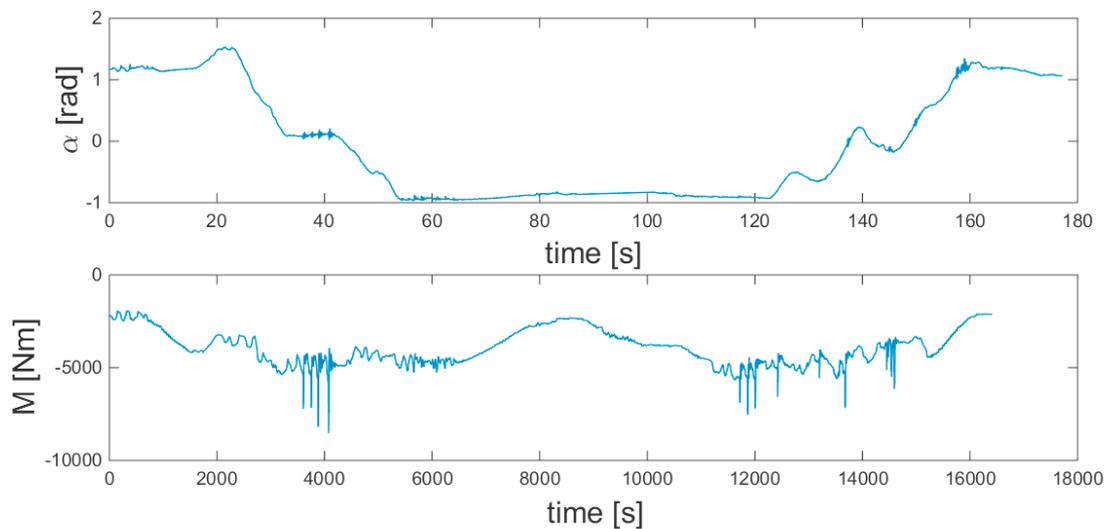


Fig. 2 time behavior of direction α and magnitude M of bending moment.

The magnitude of bending moment in a form of time function, as well as its direction, have been calculated using methodology described by Benca and Podebradsky [6], and later developed for industrial usage and long term monitoring by Chmelko at al. [7].

The final measured magnitude and direction of bending moment can be seen in fig. 2. The measured signal represents standard operational loading unit. The unit consists of the loading and the unloading of weight and moving the weight around a handler. The final fatigue life time will be represented by number of repetitions of measured loading unit.

2.2. FEM analysis of bearing pin

The FEM model was used to identify the critical location. The 3D model of bearing pin has been used for FEM calculation in ANSYS simulation software. The tetrahedral elements have been used for meshing of the model. Two independent simulations were carried on. In both of them, the model has been loaded with the same bending moment magnitude, but in one simulation, the direction of the moment was rotated by angle $\pi/2$ around central axis. The critical place in bottom rib of bearing pin was identified. Stress field in form of von Mises stress is presented for both cases in fig. 3.

The influential matrix correlating stresses on a free surface in a critical place with x and y component of bending moment was obtained from FEM simulation. The influential matrix is used to relate the x and y component of the measured bending moment to the local surface stresses $\sigma_x, \sigma_y, \tau_{xy}$ [8]. Influence matrix for critical place on bottom rib is as follows:

$$\begin{bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \end{bmatrix} = \begin{bmatrix} 1.32 & 14.22 \\ 2.14 & 22.24 \\ -1.48 & -2.37 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix} \quad (1)$$

M_x and M_y components of the bending moment can be calculated using moment magnitude and direction angle shown in fig. 2.

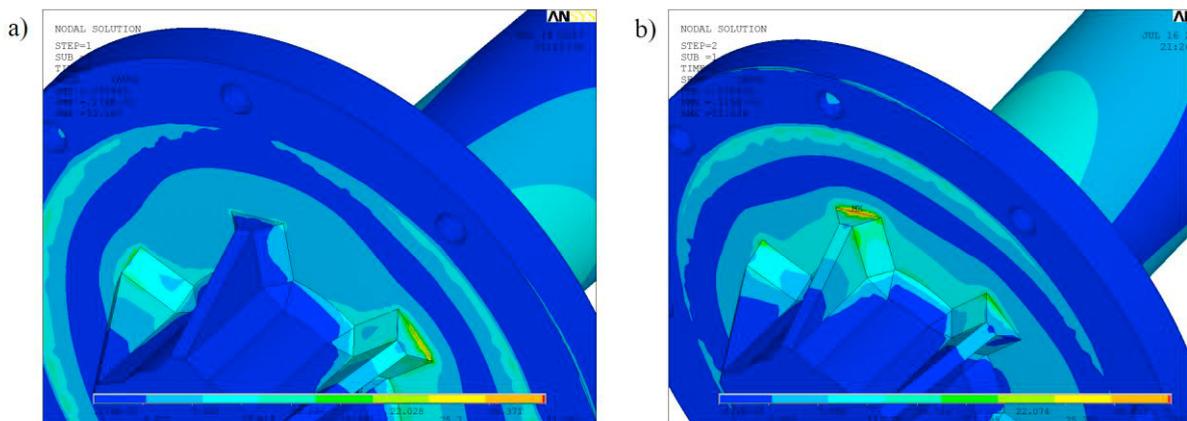


Fig. 3 (a) Bending moment in x direction; (b) bending moment in y direction.

3. Signal processing

The stress loading signal acquired by methodology described in the previous chapter needs to be modified into a form suitable for fatigue life time estimation. In the case of uniaxial fatigue analysis, this is not a problem and it can be done using standardized rainflow algorithm which extracts stress amplitudes from general loading signal into

form usable for fatigue damage simulation models. However, there is no widely accepted cycle counting methodology for fatigue life time analysis under multiaxial loading (which often is non-proportional).

Nowadays, two most common approaches are used for cycle (or halfcycle) identification and extraction from general loading signals.

The first approach identifies extractable cycles based on analysis of all stress (strain) tensor components (usually some kind of equivalent stress or strain is used). Extracted cycles can be directly used for the calculation of equivalent stress (strain) amplitudes or can be recalculated into various material planes to be used with damage models based on critical plane. The biggest advantage of this method is that the extracted cycles are directly usable in damage models based on stress or strain invariants or damage models using all stress (strain) tensor components. In this category, the most known method for cycle identification is proposed by Wang and Brown [9,10]. Their method is usable for any proportional or non-proportional multiaxial loading signals of strains or stresses. Wang-Brown's method is based on cycle counting in von Mises equivalent strain (or stress). The main problem with using Mises equivalent strain (stress) is the loss of loading signal sign, since Mises strain or stress is always positive. This can lead to the underestimating of the loading histories that have negative and positive parts. To overcome this problem, the relative Mises strain (stress) is introduced. It is calculated based on the difference between each of strain (stress) components during loading history and initial point in each count cycle. The Meggiolaro and Castro [3,4] later modified the Wang-Brown method using reduced five-dimensional (or lower dimensional) strain (or stress) sub-space. By using this ideas, the modified method is simplified and also solves some problems of the original method that can lead to the extraction of cycles with lower range of strains (or stresses) [4].

The second approach represented by method proposed by Banantine and Socie [2] is based on recalculating the loading signal to different plane in critical hot-spot which provides normal and shear signal histories in each plain. Then suitable rainflow method is applied on either shear or normal strain (stress) histories, depending on the chosen damage model. The rainflow method has to be modified to keep track of events happening in the second (non extracted) loading signal (normal or shear strain in case of extracting from shear or normal strain respectively).

Banantine-Socie counting approach has been applied on measured signal in each calculated plane. The histogram of extracted rainflow matrix for chosen plane is shown in fig. 4.

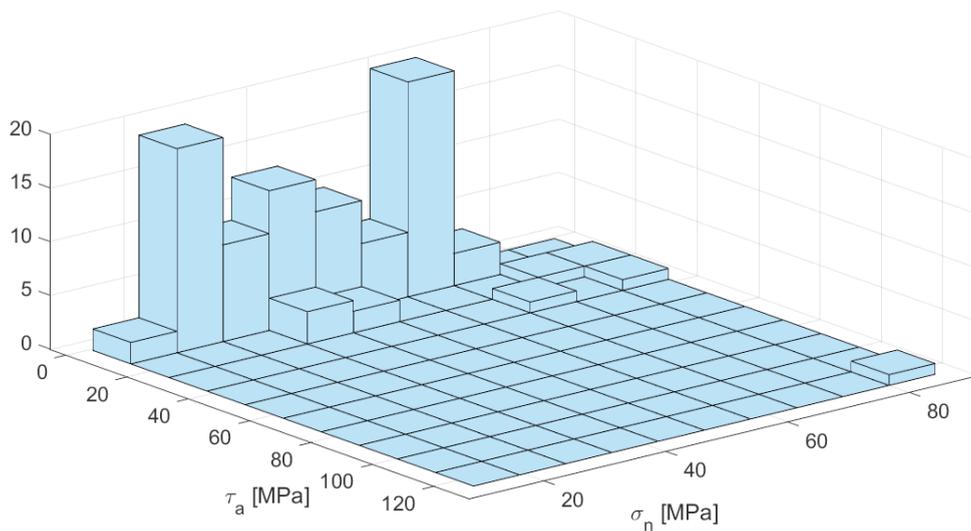


Fig. 4 Rain-flow matrix.

4. Damage model

Multiple approaches to tackle the problem of multiaxial fatigue have been proposed. Nowadays, a widely used method is so called critical plane approach. This method is based on the calculation of equivalent shear (normal) strains or stresses calculated from strain or stress components acting on particular plane. The equivalent strain (stress) then can be calculated in multiple planes. Based on the used damage model, one of the examined planes is considered as the critical one and equivalent strain (stress) in this plane is then used for fatigue life time estimation.

In this paper, two hypotheses based on critical plane approach were used for the calculation of the equivalent strain amplitude in a critical plane. The first one, the Findley damage model [11], is one of the most commonly used models in multiaxial fatigue calculations. The second model used is the model proposed by authors in [12]. Both chosen models are based on the calculation of an equivalent stress. Because bearing pin is designed to work in high cycle fatigue region, stress based approach is suitable for fatigue life time estimation, as in the high cycle fatigue region the cyclic stress is the main factor influencing fatigue damage.

Findley proposed criterion in linear form [11]. He suggested that normal stress, σ_n , on shear plane has linear influence on the alternating shear stress τ_a . The critical plane is then the plane with maximal value of equivalent shear stress τ_{eq} . In case of finite long-life fatigue estimation, the Findley criterion has the following form:

$$\tau_{eq} = (\tau_a + k_{fin} \sigma_n)_{\max} = \tau_f^* (2N_f)^b \quad (2)$$

In this formula, k_{fin} , b and τ_f^* represent the material parameters that have to be obtained experimentally. When using this criterion for variable amplitude non-proportional multiaxial loading, damage in each plane should be counted and then the plane with maximal damage during whole loading unit is the critical plane with the lowest fatigue life time.

The second criterion used for fatigue evaluation of bearing pin was the criterion proposed by Margetin and Durka [12]. It is a quadratic criterion based on material parameters obtained from Basquin equations of pure torsion and tension/compression loading. The criterion is derived from McDiarmid criterion [13] and has the following form:

$$\tau_{eq} = \sqrt{\left(\left(\frac{2\tau'_f}{\sigma'_f} \right)^2 \left(\frac{2\sigma_{\max}}{\sigma'_f} \right)^{\frac{2(b_\tau - b_\sigma)}{b_\sigma}} - 1 \right) \sigma_{\max}^2 + \tau_a^2} = \tau'_f (2N_f)^{b_\tau} \quad (3)$$

Critical plane in this criterion is the plane with the maximal shear stress amplitude. When used under variable amplitude non-proportional loading, damage in every plane should be counted and then the plane with the maximal damage during whole loading unit is identified as the critical plane. This criterion, unlike Findley criterion, considers different slopes of Basquin curves under pure torsion and pure tension/compression loading modes. On the other hand, Findley criterion has natural ability to predict fatigue life time of non-proportional loading signals more accurately due to its definition of the critical plane (the problem of Margetin-Durka criterion, as well as many other criterions that have critical plane defined based on the stress or strain range, is that the critical plane is defined inconsistently with its normal definition in the case of non-proportional variable loading).

Since loading process after extraction has shear amplitudes with mean value, the appropriate model has to be used to recalculate the stress amplitudes with mean value into amplitudes without mean value. Mean shear stress correction model (eq. 4) proposed by Wang and Miller [14] has been used.

$$\tau_a = \tau_{am} e^{\frac{|\tau_m|}{\tau'_f}} \quad (4)$$

Both of these models have to be used with damage cumulation rule for predicting finite fatigue life time. Linear damage cumulation rule, also known as Palgren-Miner rule (eq. 4), has been used.

$$D = \sum \frac{n_i}{N_{fi}} \quad (5)$$

Material parameters for fatigue calculation (taken from author's previous work [12]) are summarized in table 1. All material parameters correspond to 97,5 % confidence interval.

Table 1. Material properties

| Axial | CI (97,5%) | Torsional | CI (97,5%) | Findley crit. | - |
|------------------|------------|----------------|------------|------------------|-------|
| σ_f [MPa] | 619 | τ_f [MPa] | 536 | k_{fin} [-] | 0,131 |
| b_s [-] | -0,0531 | b_t [-] | -0,0732 | τ_f^* [MPa] | 555 |

Accumulated damages as a function of plane orientation for both criterions are shown in fig. 5. The increments of direct cosine θ and ϕ of the evaluated planes were set to 20° and 2° respectively. In this setting, the total number of failure planes evaluated is $9 \times 90 = 810$.

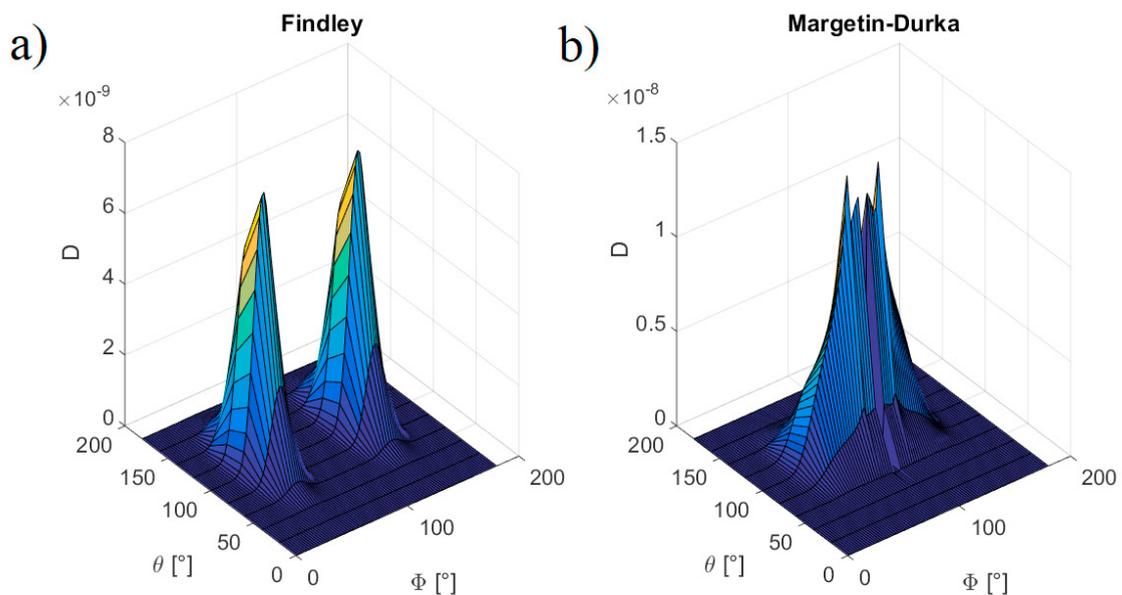


Fig. 5 Damage accumulated on plane as function of direct cosine (a) Findley criterion; (b) Margetin-Durka criterion.

5. Calculation of fatigue life time distribution function

Using calculated cumulative damage in each plane, fatigue life time can be estimated for each of the planes. Then the plane with the lowest fatigue life time (or the highest damage) is the critical plane. One of the biggest problems in fatigue life time estimation of components loaded with real operational loadings is the natural randomness of the loading signal. During multiaxial fatigue estimation, this problem tends to be even bigger, as alongside standard randomness of stress (strain) the change of phase shifting between normal and shear stresses can occur. Kliman

proposed methodology [15,16] to solve this problem by phase shifting normal and shear stress signal. Using this technique, multiple possible loading segments are created. Then calculations of fatigue life time for each segment (shifted configuration) have to be done. The set of calculated life times can then be used for creating the fatigue life time distribution function.

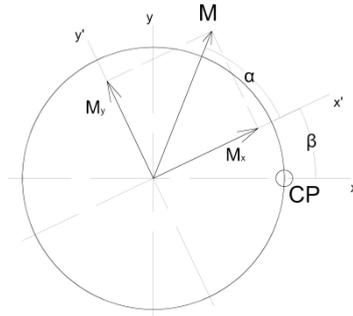


Fig. 6 Angle β between critical place (CP) and coordinate system of measured moment.

In case of bearing pin, the randomness of loading condition is caused by the uncertain position of handler operator. Due to this phenomena, the angle between the critical place of bearing pin and the coordinate system of measured loading signal (we will call it β) can change between each of the loading unit repetitions (fig.6). As the real position of the handler operator is random, fatigue life time has been calculated for multiple segments (position angle β varying between 0 and 180° with increment of 5°). The result from this calculation is set of 37 estimated fatigue life times which can be used to create fatigue life time distribution function. Fatigue life time distribution function for Findley, as well as for Margetin-Durka criterion, can be seen in figure 7.

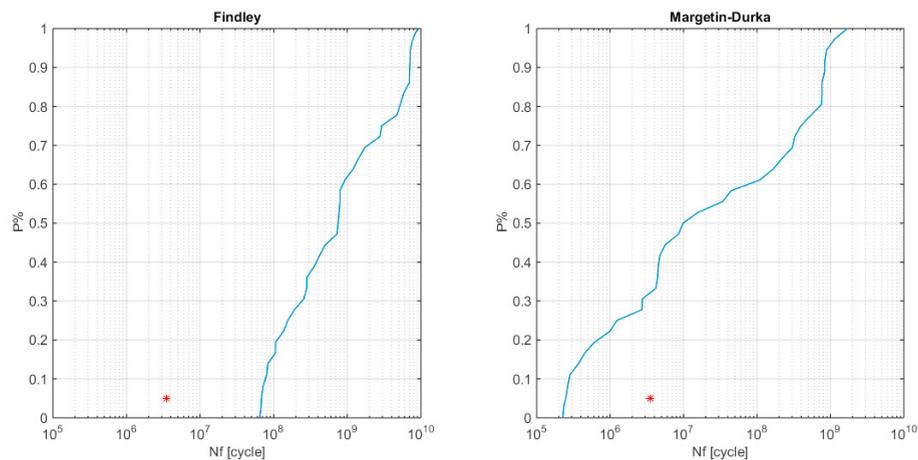


Fig. 7 Fatigue life time distribution function (a) Findley criterion; (b) Margetin-Durka criterion.

The red point represents the demanded working point (defined by the company producing handler). If the red point lies on the left side of the distribution function, the estimated life time is higher than the demanded life time. As can be seen from figure 7, the use of Margetin-Durka criterion leads to estimation of significantly higher damage and lower life time than Findley criterion does. This result is given by the fact that Margetin-Durka criterion overestimates fatigue damaging in “noncritical” planes (planes where shear τ_a is not the biggest for given loading cycle). On the other hand, Findley criterion by definition considers critical plane as the plane with the maximal damage so it is more suitable for non-proportional variable amplitude loading.

6. Conclusion

The fatigue life time estimation of bearing pin loaded with real operational loading was carried out in this paper. The result of this estimation are two fatigue life time distribution functions shown in figure 7. During life time estimation, the following major problems have to be solved:

- The appropriate counting method for loading cycle identification has to be chosen. There are two most popular approaches for this task. One is represented by Wang-Brown method which identifies loading half cycles based on relative equivalent strain (stress). The second one is represented by Bannantine-Socie method which is based on modified rainflow extraction from one stress component in chosen plane (and keeping the information of other stress components during extracted cycle). For critical plane fatigue criteria the Bannantine-Socie method is a better choice, as it extracts cycles based on events happened in particular planes.
- The multiaxial fatigue criterion for calculating equivalent stress (strain) has to be chosen correctly. Due to non-proportionality of loading signals, the critical plane changes its orientation cycle by cycle. The final damage has to be summed in multiple searched planes so the criteria that define critical plane based on maximal damage parameter work better for non-proportional loading. As can be seen in figure 7, Margetin-Durka criterion overestimates damage in planes where stress amplitude τ_a isn't maximal.
- The random nature of loading process has to be considered. This can be done by creating multiple possible load segments. Then fatigue life time for each segment can be calculated, and this set of fatigue life times can be used for creating fatigue life time distribution function.

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