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Highlight

- A model for joint replacement and inventory control is considered with multi-customers.
- The replacement cost is assumed to be decreasing for End-of-Life impacts.
- Heuristic algorithms for replacement and inventory control are proposed.
- Management insights are investigated to guide similar systems.

Replacement and Inventory Control for a Multi-Customer Product Service System with Decreasing Replacement Costs

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Abstract

In a Use-Oriented Product Service System, the customers pay for a particular service without owning the product, and the profitability of the service provider (usually also the owner of the product) is determined by the product availability and how replacement and inventory control are implemented. With the advances in modern sensor and wireless communication technologies, service providers can monitor the health status of each product in use and then conduct condition-based maintenance accordingly. Meanwhile, the waste of the remaining life of replaced products should also be considered in the system's operation due to the increasing concerns about environmental impact and lean production. To improve the profitability of a Use-Oriented Product Service System, we formulate a discrete-time Markov Decision Process that maximizes the long-term revenue per period. To overcome the computational challenge of this problem, we propose a sequential heuristic solution incorporating a heuristic replacement policy along with a heuristic inventory control approach to solve the integrated model. The heuristic replacement policy is derived from the optimal control policy for the subsystem of a single customer. The inventory control heuristic determines the target inventory level according to a one-period look-ahead myopic optimization policy. The performance of the proposed solution and some useful management insights are investigated in a numerical study. In addition, sensitivity analyses by varying the replacement costs, holding cost, unit service revenue and deterioration rates are also conducted.

Keywords

OR in service industries; Production Service System; replacement policies; inventory control

1. Introduction

A Product Service System (PSS) is a novel business model relying on the value flow of function or service delivered by the service provider rather than the individual products themselves (Yang, Moore, Pu, & Wong, 2009; Reim, Parida, & Örtqvist, 2015). In Use-Oriented PSS or Product-as-a-Service, the ownership of the tangible product is retained by the service provider who collects revenue from customers according to the length and quality of the intangible service provided (Gaiardelli, Resta, Martinez, Pinto, & Albores, 2014). The typical patterns of Use-Oriented PSS are product leasing, renting, sharing, or pooling. Unexpected failures and associated downtime will negatively affect the productivity of customers and the future development of the service provider. On the other hand, if the frequency of replacement is too high, the profitability of the service provider decreases because the operation costs rise without necessarily adding revenue. In practice, the replacement policy is a function of the availability of spare parts. If the service provider needs to maintain the inventory by cooperating with some upstream companies, the system becomes an integrated system of Use-Oriented PSS and a traditional supply chain, where the innovation of the new business model endows the service provider with a dominant position in the competitive market. However, a Use-Oriented PSS involving both a replacement policy and an inventory control policy with the objective of maximizing the expected net revenue has not been well studied in the existing literature.

Another aspect that needs to be addressed in a Use-Oriented PSS is the End-of-Life cost. Corvellec & Stål (2017) showed that the use of PSSs did not reduce the waste of material and energy in the fashion industry. Thus, the cost of waste in a Use-Oriented PSS cannot be ignored due to the sharing nature of the business model. After the service provider implements the replacement action, the remaining amount of product determines the material wasted. Besides the waste of material and energy in the production process, the End-of-Life cost is significant if the product is hazardous to the environment and has a high disposal cost, such as batteries and some medical devices. While the replacement costs are often assumed to constant or increasing in the existing literature, no one takes into consideration how much life/material is left before replacement. In our research, we assume the replacement cost is a decreasing function of the health status of the in-use product. Before designing the proper method of recovery, an operation plan that considers End-of-Life costs benefits the sustainability of the business model.

Existing literature on maintenance policies for large equipment or products considers replacement as an option to enhance the reliability (Dekker, Wildeman, & Van der Duyn Schouten, 1997; Pan & Thomas, 2010; Ahmad & Kamaruddin, 2012). In the area of maintenance decision making, the use of condition-based maintenance relies on the continuous or periodic monitoring of an in-use product (Grall, Bérenguer, & Dieulle, 2002; Chen & Wu, 2007; and Alaswad & Xiang, 2017). Some researchers have extended the new maintenance paradigm by monitoring multiple in-use products without inventory considerations (Tian & Liao, 2011; Hong, Zhou, Zhang, & Ye, 2014; Shafiee, Finkelstein, & Bérenguer, 2015; Keizer, Teunter, & Veldman, 2016; and Verbert, Schutter, & Babuška, 2017). Some other papers related to joint maintenance decisions and inventory control for a condition-based decision framework are summarized in Section 2. The disadvantage of a condition-based maintenance framework is that it needs to collect data on time and accurate information on product performance which is measured by an embedded sensor. Interestingly, this kind of embedded sensor is also required by the Use-Oriented PSS because the service provider maintains the ownership of the product. For example, in bicycle-sharing systems, companies often monitor and control their bicycles using advanced information technologies. Whenever a customer drops off a bicycle, the system will immediately receive the associated service information about the customer's journey. Joo & Oh (2013) provided additional insights into the monitoring of a bicycle sharing system. Without a doubt, condition-based maintenance or replacement policies are viable in this type of Use-Oriented PSS. This paper utilizes the information from a product embedded sensor to build a dynamic decision-making model, i.e., a Markov Decision Process (MDP), for a product experiencing Markovian deterioration. We utilize the deterioration of the in-use product as a measure of "health status" to differentiate it from the states in the MDP.

For the joint decision on replacement and inventory control for a single customer, existing research has shown that an optimal policy has a monotonic structure (Kawai, 1983). Our research also indicates that the optimal policy for the replacement problem for a single customer is monotonic under certain circumstances, where the replacement action is employed when the product health status reaches a particular threshold. However, it is generally not optimal to apply a single-customer policy to a system of multiple customers (Cho & Parlar, 1991). For replacement problems, a separate single-customer policy ignores the dependent influence of multiple customers where the replacement policy should consider the entire state of the system. For example, in an inventory system, a separate single-customer policy would order a spare by treating it as the reservation for one specific customer, when in fact spares are shared by all the customers. Thus, a coordination model for multiple customers is necessary. Unfortunately, MDP models often suffer from a well-known limitation, called the "the curse of dimensionality." For a practically sized problem, it is often intractable to obtain the exact solution of an MDP. Recently, Approximate Dynamic Programming was proposed to solve practical-sized MDPs, where the value of the expected cost-to-go was approximated in various ways (Powell, 2016). Although Approximate Dynamic Programming can handle practical sized problems, it is also difficult to implement as the problem size grows. In this paper, we propose two heuristic algorithms for this challenge. While heuristic algorithms cannot guarantee global optimality, they are capable of providing satisfactory solutions (sometimes near optimal) in a reasonable amount of time.

The main contributions of this paper are threefold. First, we build a model of joint condition-based maintenance and inventory optimization that features multiple customers, ordering lead time of spares, and decreasing replacement costs. Second, some key optimal structures of replacement policy are identified for the single customer's counterpart. We propose an alternative way to calculating the marginal replacement benefit, which prevents the calculation of iterative algorithms. Third, a heuristic solution is proposed to tackle the studied model.

The remainder of this paper is organized as follows. The relevant literature on joint optimization of replacement and inventory control models is reviewed in Section 2. Our model is formulated in Section 3. In Section 4, a heuristic approach is provided to solve the joint decision-making problem. A numerical study is conducted in Section 5 that evaluates the performance of the model and heuristic solution approach. In Section 6, sensitivity analyses are conducted. Finally, the characteristics of the proposed heuristic solution and some management insights are summarized in Section 7.

2. Literature Review

A comprehensive summary of the existing literature on joint optimization of maintenance and spare parts inventory control is reviewed by Keizer, Flapper, & Teunter (2017), Van Horenbeek, Scarf, Cavalcante, & Pintelon (2013), and Kennedy, Patterson, & Fredendall (2002). In Keizer, Flapper, & Teunter (2017), they analyzed these models through structural dependence, stochastic dependence, and resource dependence. From their point of view, the condition-based maintenance model with limited spares is a kind of resource dependence, where the in-use products are connected by sharing spares. Elwany & Gebraeel (2008a) proposed a sensor data-driven model for a single-unit replacement and inventory model where the decision is based on the estimated remaining life distribution of the component. Wang, Chu, & Mao (2008a) proposed a similar single-unit model along with a threshold-based heuristic to minimize the long-term operational cost. Rausch & Liao (2010) studied a bi-objective model to minimize the inventory and expected operational cost. Their method utilized simulation optimization to determine the base-stock level and replacement threshold. Similar models can be found in Louit, Pascual, Banjevic, & Jardine (2011) and Wang, Zhao, Cheng, & Yang (2015). Recently, Z. Wang, Hu, Wang, Kong, & Zhang (2015) extended their model to account for a stochastic lead time. Different from the above papers, Icten, Shechter, Maillart, & Nagarajan (2013) considered the objective of maximizing the expected in-use time of a single-unit system with a limited number of replacements. Their paper does not address the associated ordering decision for spares.

To the best of our knowledge, there are nine papers consider the integration of condition-based maintenance and spare part inventory optimization for multiple-unit systems. These papers are summarized in Table 1, and the differences with this paper are highlighted. We only found two papers that considered a system of non-identical units. Other papers only consider the system of multiple identical units which is a special case of the more general non-identical units. However, the objective of the model in Li & Ryan (2011) is to minimize the inventory cost when the replacement-related cost is not considered. In their paper, a penalty cost is applied when products are insufficient. Meanwhile, a simultaneous failure condition is not allowed due to the use of a First-Come-First-Serve rule. Keizer, Teunter, & Veldman (2017) developed an MDP model for a multiple-unit system and solved it by using the Value Iteration algorithm, which is computationally intensive. Nguyen et al. (2017) studied the joint maintenance and inventory strategy for complex systems consisted of multiple non-identical components. Their considered the objective of total operation costs including the inspecting of inner components. Only our paper considers the objective of maximizing the net revenue, while other papers assumed that income is fixed and only consider the objective of minimizing the sum of some set of costs. A (s, S) policy checks the inventory level periodically and reorders replenishment if dropping below a certain point, which is well studied in inventory problems with uncertainties (Qin, Sun, & Lim, 2017). Most of the previous work examined the (s, S) inventory policy in the context of maintenance and spares, such as Xie & Wang (2008), Wang, Chu, & Mao (2008b), Wang, Chu, & Mao (2009), Van Horenbeek, Scarf, Cavalcante, & Pintelon (2013), and Zhang et al. (2017). Although, Keizer, Teunter, & Veldman (2017) showed that the (s, S) inventory is not optimal even when cooperating with the optimal replacement decision. Cai, Yin, Zhang, & Chen (2017) proposed an appointment policy which reserves a spare to a specific unit when the deterioration reaches a particular threshold. In their method, the inventory level would be larger than the determined level under some conditions. However, they utilized the (s, S) inventory policy when the appointed spares are wiped out. Our paper proposes a state-dependent heuristic for inventory control, which improves the existing methods of inventory control in the field of joint condition-based maintenance and spare part inventory optimization.

Table 1. Existing papers for multiple-unit systems

	Consider non-identical units	Deterioration mode	objective	Solution method
Xie &Wang (2008)		Weibull distribution	Minimize the operation cost	Genetic Algorithm
Wang, Chu, & Mao (2008b)		Markov Process	Minimize the operation cost	Monte Carlo
Wang, Chu, & Mao (2009))		Markov Process	Minimize the operation cost	Genetic Algorithm
Li & Ryan (2011)	√	Wiener Process	Minimize the inventory cost	Heuristic
Van Horenbeek, Scarf, Cavalcante, & Pintelon (2013)		Age-based	Minimize the operation cost	Monte Carlo
Keizer, Teunter, & Veldman (2017)	√	Poisson Process	Minimize the operation cost	Value Iteration
Cai, Yin, Zhang, & Chen (2017)		Wiener Process	Minimize the operation cost	Genetic Algorithm and the Monte Carlo
Zhang et al. (2017)		Exponential or gamma process	Minimize the operation cost	Genetic Algorithm
Nguyen et al. (2017)	√	Age-based	Minimize the operation cost	Monte Carlo
This paper	√	Poisson Process	Maximize the net revenue	Heuristic

3. Model Formulation

3.1 Notation

The notation for our model is summarized below in Table 2.

Table 2. Model notation

i	index of customers, $i = 1, 2 \dots \dots N$
h_i	health status of product at customer i, $h_i = 1, 2 \dots \dots H$.
k	inventory level of spare product
a_i	binary variable indicating if the product at customer i is replaced
p	variable representing the purchase amount of new spare products
$P_{i_{h_i, h_i'}}$	transition probability from health status h_i to $h_i{}'$ for customer i
γ_i	unit service revenue per unit of service at customer i
$Rev_i(h_i, a_i)$	expected revenue from customer i at health status h_i for one-period of service
c_i	penalty cost when the product at customer i is failed ($h_i = H$)
$f(h)$	replacement cost at health status h
o	unit purchase cost of a spare product
h_c	unit holding cost of a spare product per period

3.2 The description of model

In our model, we consider a discrete-time system consisting of N customers who enjoy the service of a homogeneous and interchangeable product. We name the products which are kept by the service provider for future replacement as "spare products". A service provider replaces the product at each customer from their limited spare product inventory k. The product service at each customer has a health status h_i which can be no larger than H. When $h_i = 1$, the product is as good as new, and when $h_i = H$ the product is at the end of its life. At the beginning of every period, the service provider observes the health status of the product at every customer h_i and decides on replacement a_i and ordering p. The newly purchased spare products are assumed to be as good as new and arrive at the end of this period. The replacement operation is supposed to take place immediately. During this period, revenue is collected according to the level of satisfied service demand for each customer. The service demands of customers are assumed to be independent but may not be identical in distribution. For simplification, the deterioration increments are assumed to be linear to the satisfied service demand. Thus, the expected revenue, $Rev_i(h_i)$, is determined by the unit service revenue per unit of service γ_i and health status h_i. Because the decision is made before the realization of demand, the revenue is determined by the expected value. Afterward, the demand is satisfied with the in-use product. The in-use products continue to deteriorate with each increment of health status.

Four kinds of costs are considered in our model. The penalty cost, denoted as c_i, is incurred if the product at customer i is at the end of its life. This cost compensates customers for the failure of the service. The replacement cost, denoted as $f(h)$, decreases as a function of the health status when the replacement happens. Because the replacement decision is implemented by the unique service provider, $f(h)$ is identical for all customers. The ordering decision of spare product creates a purchase cost $o \cdot p$ where o is the unit purchase cost. The remaining spare products after the replacement are assumed to incur a holding cost h_c.

3.3 Markov Decision Process Model

The MDP model, first developed by Bellman (1957), is first proposed as an approach because it is useful for a broad range of problems where the payoff is partly based on a dynamic environment and partly based on decision maker's behaviors. The essential elements of an MDP model are the states, actions, transition probability, and the reward function, all of which are explained in detail below.

State space. The state consists of the system health status and the inventory level, denoted as $\pi(h_1, h_2 \dots h_n, k)$. Due to the boundary requirement of MDP, we assume the maximum inventory level is $\bar{U}$. Thus, $h_i \in [1, H]$ and $k \in [0, \bar{U}]$. The size of state space is $H^N \cdot (\bar{U} + 1)$.

Action space. The action space in each period consists of the replacement actions for each customer and the ordering decision, denoted as $\bar{a}(a_1, a_2 \dots a_n, p)$. For $i = 1, 2 \dots \dots N$,

$$a_i = \begin{cases} 1 \text{ if the product at customer } i \text{ is replaced} \\ 0 \qquad \text{otherwise} \end{cases}.$$

There are two constraints for $\bar{a}(a_1, a_2 \dots a_n, p)$ if implemented at state π. First, the total replacements cannot exceed the existing spare products, namely

$$\sum_{i=1}^{N} a_i \leq k. \tag{1}$$

Second, the spare products in the next period cannot exceed the limitation,

$$k - \sum_{i=1}^{N} a_i + p \leq \bar{U}. \tag{2}$$

Transition probability. For any two states $\pi(h_1, h_2 \dots h_n, k)$ and $\pi'(h'_1, h'_2 \dots h'_n, k')$, the transition probability through action $\bar{a}$ is

$$P(\pi'|\pi,\bar{a}) = \begin{cases} \prod_{i=1}^{N} P_i(h_i'|h_i,a_i) \text{ if } k' = k - \sum_{i=1}^{N} a_i + p \\ 0 \quad \text{otherwise} \end{cases}, \tag{3}$$

where

$$P_i(h_i'|h_i,a_i) = \begin{cases} P_{i_{h_i,h_i'}} \text{ if } a_i = 0 \\ P_{i_{1,h_i'}} \text{ if } a_i = 1 \end{cases},$$

in which $P_{i_{h_i,h_i'}}$ in the transition matrix represents the health status of the product at customer i. The deterioration only happens in one direction. Thus $P_{i_{h_i,h_i'}} = 0$ if $h_i' < h_i$ for $\forall i$ and h_i.

Reward function. The objective of the model is to maximize the net revenue which is defined as the sum of revenues minus the total costs, which include penalty costs, replacement costs, purchase costs, and holding costs. The reward function at state π with action $\bar{a}$ is the net revenue in this period without consideration of future states, which is expressed as

$$reward(\pi,\bar{a}) = \sum_{i=1}^{N} Rev_i(h_i,a_i) - \sum_{i=1}^{N} c_i \cdot 1_{h_i=H} - \sum_{i=1}^{N} f(h_i) \cdot a_i - o \cdot p - h_c \cdot \left(k - \sum_{i=1}^{N} a_i\right), \tag{4}$$

where $1_{h_i=H}$ is the indicator function which is equal to 1 if $h_i = H$ is true and 0 otherwise. The expected revenue from customer i is

$$Rev_i(h_i,a_i) = \sum_{j=h^\wedge_i+1}^{H} P_{i_{h^\wedge_i,j}} \cdot \gamma_i \cdot (j - h^\wedge_i), \tag{5}$$

where $h^\wedge_i = \begin{cases} 1 & \text{if } a_i = 1 \\ h_i & \text{otherwise} \end{cases}$ and represents the health status of customer i after replacement.

Using Bellman's Equation (Bellman, 1954), the expected net revenue for state π is given by

$$V(\pi) = \max_{\bar{a}} reward(\pi,\bar{a}) + \sum_{\pi' \in \Pi} P(\pi'|\pi,\bar{a}) \cdot V(\pi'). \tag{6}$$

Similar to the discussion by Keizer, Teunter, & Veldman (2017), this model satisfies the Weak Unichain Assumption (Tijms, 1994) and the maximum long-term average net revenue can be obtained by using the Value Iteration algorithm. However, the experiments show that this model is very intractable when $N \geq 5$. When $N = 4$ and $H = 6$, the calculation time for an optimal control policy is less than 2 minutes. However, the calculation time surges to over 1 hour when N reaches 5. For practically sized problems, the optimal policy cannot be found in a reasonable amount of time. The next section introduces two heuristic algorithms as a way to solve this model.

4. Heuristic Solution Algorithm

We develop a heuristic-based policy using the sequential principle "replacement-first order-later." Therefore, the heuristic replacement policy is dependent solely on the current state. Before handling the situation with multiple customers, the model of replacement strategy for a single customer without new purchase is analyzed to determine the structure of the policy.

4.1 Replacement strategy for a single customer without new purchase

Following the notation developed in Section 3, the spare products are limited to a certain number and are assumed to be nonincreasing during each transition. The maximum expected reward criterion is utilized for maximizing total net revenue. Customer i represents this single customer. The state of this model is (h_i, k) which represents the health status and spare product inventory. We utilize $v(h_i, k)$ to represent the expected total net revenue without a new purchase which is different than $V(\pi)$. The mathematical expression is depicted as follows:

$$v(h_i,k) = \begin{cases} 0 & \text{if } h_i = H \text{ and } k = 0 \\ max \begin{cases} \text{"}Replace\text{"}: R(h_i,k) = Rev_i(h_i,1) - f(h_i) - c_i \cdot 1_{h_i=H} - h_c \cdot (k-1) + \sum_{h_i'=1}^{H} P_{i_{1,h_i'}} \cdot v(h_i', k-1) \\ \text{"}Keep\text{"}: K(h_i,k) = Rev_i(h_i,0) - c_i \cdot 1_{h_i=H} - h_c \cdot k + \sum_{h_i'=1}^{H} P_{i_{h_i,h_i'}} \cdot v(h_i', k) \end{cases} & \text{otherwise} \end{cases} \tag{7}$$

where $R(h_i, 0) = -\infty$ for $\forall h_i$.

State $(H, 0)$ is a special case for this model. When the product at customer i is passed its threshold and there is no more spare product, $v(H, 0)$ is defined as 0 to avoid infinitely negative net revenue. In other cases, two actions are compared to maximize the total net revenue, $R(h_i, k)$ and $K(h_i, k)$. When the spare product inventory is zero ($k = 0$), the action must be to choose ($R(h_i, 0) = -\infty$ for $\forall h_i$). The transition possibilities are the same as the model in Section 3.

First, we make the following assumption for simplifying the structure of $P_{i_{h_i,h_i'}}$.

Assumption 1. There exists a series of $P_{i_{\Delta=h}}$ $(h = 0,1 \dots H-1)$ *where* $P_{i_{h_i,h_i'}} = \begin{cases} P_{\Delta=h_i'-h_i} \text{ if } h_i' \neq H \\ \sum_{t=H-h_i}^{H-1} P_{\Delta=t} \text{ if } h_i' = H \end{cases}$ *for any customer* i.

In this assumption, $\sum_{h=0,1\dots H-1} P_{i_{\Delta=h}} = 1$ for guaranteeing $\sum_{h_i' \in [1,H]} P_{i_{h_i,h_i'}} = 1$ for any h_i and $h_{i'}$. Let us illustrate this with an example. Say a model with five states of health status ($H = 5$). $P_{i_{\Delta=h}}$ is {0.2, 0.2, 0.2, 0.2 0.2}. We can get the transition matrix like

$$P_{i_{h_i, h_i'}} = \begin{Bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & 0.2 & 0.2 & 0.2 & 0.4 \\ 0 & 0 & 0.2 & 0.2 & 0.6 \\ 0 & 0 & 0 & 0.2 & 0.8 \\ 0 & 0 & 0 & 0 & 1 \end{Bmatrix}.$$

Assumption 1 implies that the deterioration of a product is independent of the health status of the in-use product. When the deterioration exceeds or equals the remaining life of the product, the health status reaches H. In the experiment, the deterioration rate is assumed to follow a bounded Poisson Process where the maximum deterioration is $H - 1$. This assumption is reasonable and shared with other research in the literature (Keizer, Teunter, & Veldman, 2017). In the remainder of this paper, based on Assumption 1, the following lemma can be obtained.

Lemma 1. *Under Assumption 1,* $Rev_i(h, 0)$ *is nonincreasing with h.*

Proof. See Appendix A. ■

We can show that $Rev_i(h_i, 1)$ is constant in h_i according to the definition of $Rev_i(h_i, a_i)$. Here we propose Theorem 1 to show the existence of a monotonic structure for an optimal policy under certain conditions.

Theorem 1. *Under Assumption 1 and the following condition, for a fixed* $k > 0$, *there exists a threshold* ${h_i}^*$ *such that it is optimal to replace the product when* $h_i > {h_i}^*$, *and it is optimal to keep the product in use when* $h_i \leq {h_i}^*$.

(i) $f(h_i)$ *is non-concave (i.e. linear or convex) in* h_i.

(ii) $f(h_i)$ *is decreasing in* h_i.

Proof. See Appendix A. ■

Theorem 1 shows that there is a threshold of health status where the replacement is preferred under certain conditions. In the numeral study, we assume the replacement cost $f(h_i)$ is a linear decreasing function in h_i, which satisfies these conditions. Theorem 1 only consider the optimal policy for a fixed inventory level. Then Theorem 2 is proposed to determine the structure of optimal policy for different k.

Theorem 2. *Under the same condition as Theorem 1,* h_i^* *is nonincreasing in k.*

Proof. See Appendix A. ■

Based on Theorem 2, the optimal replacement policy is a two-direction Control Limit Policy similar to that defined in Icten, Shechter, Maillart, & Nagarajan (2013). The replacement is preferred when the inventory level is high. This phenomenon can be explained by the fact that a high inventory level stimulates the replacement action and reduces the holding cost. In the contrast, a low inventory level restrains the replacement action and reduces the penalty cost. In the next section, we propose a rapid method to calculate the marginal benefit for the replacement action along with a heuristic replacement policy.

4.2 Heuristic replacement policy for multiple customers

The heuristic replacement policy is based on the replacement benefit for a single customer. Define $B(h_i,k) = R(h_i,k) - K(h_i,k)$. According to Theorem 1 and Theorem 2, $B(h_i,k)$ is nondecreasing in h_i and nonincreasing in k. The replacement decision for single-customer model is preferred when $B(h_i,k) > 0$. The expected total net revenue for multiple customers without new purchase is not decomposable structurally. However, the marginal benefit of replacing the product for customer i is approximated as $B(h_i,k)$. Here we propose Theorem 3 to provide a method for calculating $B(h_i,k)$ without the requirement for any iteration algorithm.

Theorem 3. *Under the same condition as Theorem 1, for any* $k > 1$, $B(h_i,k) = \delta(h_i) + h_c \cdot k$ *if* $B(h_i,k) \geq 0$ *, where* $\delta(h_i) = P_{i_{h_i,H}} \cdot c_i - \sum_{j=h_i}^{H} P_{i_{h_i,j}} \cdot \big(f(h_i) - f(j)\big) - Rev_i(h_i,0)$ *. And* $B(h_i,k) \leq \delta(h_i) + h_c \cdot k$ *if* $B(h_i,k) < 0$.

Proof. See Appendix A. ■

According to the definition of Theorem 3, $\delta(H) = c_i$, due to that $P_{i_{H,H}} = 1$, $P_{i_{H,j(j \neq H)}} = 0$, and $Rev_i(H,0) = 0$. Thus, the replacement is preferred in health status H when $c_i + h_c > 0$. For $h_i < H$, $\delta(h_i) + h_c \cdot k$ provides an upper bound for $B(h_i,k)$. When $\delta(h_i) + h_c \cdot k < 0$, $B(h_i,k)$

is also less than 0, namely the marginal benefit for the replacement action is negative. In our heuristic replacement policy, the replacement would not be considered if $B(h_i, k) < 0$. Therefore, the heuristic replacement policy for multiple customers is composed based on $\delta(h_i) + h_c \cdot k$ instead of the exact value of $B(h_i, k)$. This method bypasses the complexity associated with separate optimization. The process for our heuristic replacement policy for multiple customers is depicted in pseudocode in Algorithm 1.

Algorithm 1. Heuristic replacement policy for multiple customers

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Input: $\pi(h_1, h_2 \dots h_n, k)$, $\delta(h_i)$, h_c

Output: $\bar{a}(a_1, a_2 \dots a_n, 0)$

Step 0: Place i in the nonincreasing order of $\delta(h_i)$. $k_{cur} = k$. $\bar{a} = (0,0 \dots .0)$

Step 1: If $k_{cur} = 0$ or $\sum_{i=1}^{n} a_i = N$, return $\bar{a}$.

Step 2: For each i starting for $i = 1$, if $a_i = 0$ and $\delta(h_i) + h_c \cdot k_{cur} > 0$, $i^* = i$ and go to Step 3. If no such i exists, return $\bar{a}$.

Step 3: $a_{i^*} = 1$, and $k_{cur} --$. Go to Step 1.

In Algorithm 1, the termination condition occurs when no spare product leaves or every customer product has been replaced. We utilize k_{cur} to record the level of available spare product. In each iteration, the algorithm finds a i which is not replaced and maximizes $\delta(h_i) + h_c \cdot k_{cur}$ as i^*. If $B(h_{i^*}, k_{cur})$ is assumed to be positive by $\delta(h_i) + h_c \cdot k_{cur}$, the algorithm decides to replace it and record it in the heuristic replacement policy. Otherwise, the algorithm is terminated because $B(h_i, k_{cur})$ are negative for every product.

Even though the heuristic replacement policy does utilize the separate results for optimal control, it is not a separate policy. The prior decision of replacement reduces the level of available spare product, which impacts the subsequent decision by increasing the threshold for replacement. In the long-term, the service provider needs to gain the maximum profit from every spare product. The performance of Algorithm 1 is evaluated in a numerical study.

4.3 Heuristic inventory control for multiple customers

Other researchers have reported that the (s, S) policy is far from optimal for the problem of spare parts inventory control with replacement (Keizer, Teunter, & Veldman, 2017). The (s, S) policy is developed to handle the demand for a stationary distribution where the realization of demand is independent of the state of the system. In the replacement problem, the replacement demand varies with the system state. Thus, the static policy for the inventory level is no longer suitable for the problem with multiple customers.

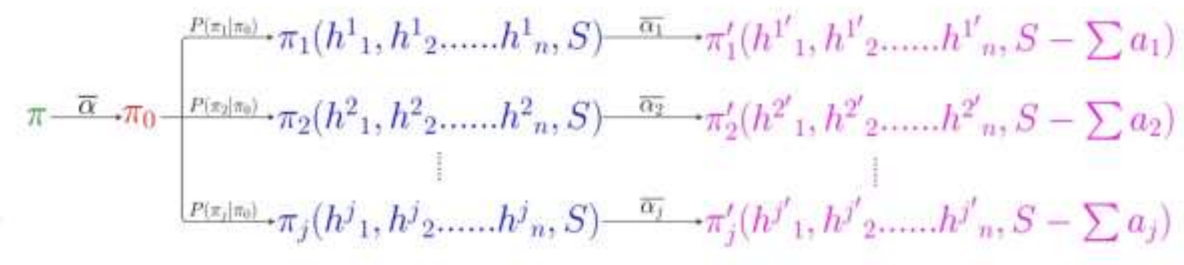

Figure 1. The transitions of states in heuristic inventory control

In this paper, we propose a one-period look-ahead Heuristic inventory control policy whose inventory level is dependent on the system's state. First, define $\pi_0(h^0{}_1, h^0{}_2 \dots\dots h^0{}_n, S)$ as the post decision state for the original state π and action $\bar{\alpha}$. Let $h^0{}_i = \begin{cases} 1, \text{ if } a_i = 1 \\ h_i, \text{ otherwise} \end{cases}$. Then, $\pi_j(h^j{}_1, h^j{}_2 \dots\dots h^j{}_n, S)$ is the system state after one-period of deterioration from π_0. $h^j{}_i \geq h^0{}_i$ for $\forall i, j$. The transition probability from π^0 to π_j is $P(\pi_j|\pi_0) = \prod_{i=1}^{N} P_{i_{h^j{}_i, h^0{}_i}}$. Then, the new replacement decision $\bar{\alpha}_j(a^j{}_1, a^j{}_2 \dots\dots a^j{}_n, 0)$ is found by Algorithm 1 based on π_j. The new post decision state $\pi'_j(h^{j'}{}_1, h^{j'}{}_2 \dots\dots h^{j'}{}_n, S - \sum_{i=1}^{n} a^j{}_i)$ is defined as the state after decision $\bar{\alpha}_j$ from π_j. $h^{j'}{}_i = \begin{cases} 1, \text{ if } a^1{}_i = 1 \\ h^j{}_i, \text{ otherwise} \end{cases}$ for $\forall i, j$. The transitions between states are depicted in Figure 1. The unsatisfied replacement demand is backlogged. Thus, the cost of new purchase is omitted in the decision for inventory control.

The future cost-to-go of state π, $\tilde{V}(\pi)$ is approximate by

$$\tilde{V}(\pi) \approx \sum_{\pi_j \in \Pi} F\left(\pi_j(h^j{}_1, h^j{}_2 \dots\dots h^j{}_n, S)\right) \cdot P(\pi_j|\pi_0), \tag{8}$$

where

$$F(\pi_j) = \sum_{i=1}^{N} Rev_i(h^{j'}{}_i, 0) - \sum_{i=1}^{N} c_i \cdot P_{i_{h^{j'}{}_i, H}} - \sum_{i=1}^{N} f(h^j{}_i) \cdot a^j{}_i - h_c \cdot \left(S - \sum_{i=1}^{N} a^j{}_i\right).$$

In particular, $F(\pi_j)$ is the myopic profitability for state π_j after the replacement action. The optimal inventory level S^* is determined by maximizing $\tilde{V}(\pi)$, namely

$$S^* = \underset{S}{argmax} \sum_{\pi_j \in \Pi} F\left(\pi_j(h^j{}_1, h^j{}_2 \ldots \ldots h^j{}_n, S)\right) \cdot P\left(\pi_j \middle| \pi_0\right). \tag{9}$$

According to (9), S^* is highly dependent on the health status of the products, where the higher level of spare products only occurs when the in-use products are about to fail. The performance of the heuristic replacement policy and the heuristic inventory control policy will be evaluated in the next section by comparing it with a global optimization policy and some existing benchmark policies.

5. Numerical Study

In this section, we compose two basic instances, one with identical customers and another with non-identical customers, to illustrate the performance of the heuristic replacement policy and heuristic inventory control policy.

5.1 Basic instances and global optimization policy

Let's begin by considering a system composed of 4 customers with independent deterioration rates for their products. In the identical instance case, the deteriorations all follow a Poisson Process with deterioration rates λ={1, 1, 1, 1}. $P_{i_{h_i, h_i'}} \sim Poisson(\lambda_i)$ if $h_i' \neq H$, and $P_{i_{h_i,H}} = 1 - \sum_{j=h_i}^{H-1} P_{i_{h_i, h_i'}}$. In the non-identical instance, the deteriorations follow Poisson distributions with different rates λ={1, 1, 1.5, 1.5}. The upper bound of health status is 6, and the upper bound of the inventory level is 4. Therefore, there are a total of $6^4 \times 5$ (i.e., 6480) states and $2^4 \times 5$ (i.e., 80) possible actions at most for each state. The parameters for the basic instances are summarized in Table 3. Except for λ, other differences between the identical and the non-identical instances are the unit service revenue and the penalty cost. The unit service revenues for the 2nd and 4th customers are two less than those of the 1st and 3rd customers, respectively. The expectations of revenue are both 20 per period. This setting imitates the situation where some customers are less sensitive to unavailability and want to pay less for service, where the penalty costs are also cut in half for balancing the interests of the service provider. The system would replace the products at the customers with high penalty to achieve higher profitability.

Table 3. The parameter of basic instances

Parameter	Description	Values in the identical instance	Values In the non-identical instance
N	the number of customers	4	4
H	the upper bound of health status levels	6	6
$\bar{U}$	the upper bound of inventory level	4	4
λ	the deterioration rates of health status	{1, 1, 1, 1}	{1, 1, 1.5, 1.5}
γ_i	unit service revenue per unit of service at customer	{5, 5, 5, 5}	{5, 3, 5, 3}
c_i	penalty costs when the product at customer i is failed	{20, 20, 20, 20}	{20, 10, 20, 10}
$f(h)$	replacement cost at health status h	{6, 5, 4, 3, 2, 1}	{6, 5, 4, 3, 2, 1}
o	unit purchase cost for a spare product	5	5
h_c	unit holding cost for a spare product	0.5	0.5

The Value Iteration algorithm is applied to find the global optimization control policy which is denoted as GO. The stopping criterion was set as $\varepsilon = 0.00005$ meaning that the gap is less than 0.005%. During the iteration, the non-optimal actions are eliminated by the methods defined by Puterman (1994).

5.2 (s, S) policies and separate optimization policy

The (s, S) policies are considered as the benchmark policies. Since we ignore fixed ordering cost, the structure of (s, S) is $(S-1, S)$. The inventory level S is selected through the iteration for 0 to $\bar{U}$. Two kinds of (s, S) policies are considered. First, the (s, S) policy is coordinated with the optimal replacement policy, denoted as $O + (s, S)$. The procedure for this policy is similar to the global optimization control policy, where the decision of p is not searched in each iteration. In the other benchmark policy, the (s, S) policy is coordinated with the heuristic replacement policy, denoted as $H + (s, S)$. The control policy is directly obtained from Algorithm 1 and the (s, S) policy. The $H + (s, S)$ policy is evaluated using the Value Iteration algorithm with a fixed policy.

The separate optimization policy, denoted as SO, is also utilized as a benchmark policy. The SO policy is a monotonic policy for each customer, where the replacement and ordering are applied when the health status reaches some determined thresholds. In the identical instance, the SO policy would order one spare product when the health status is 3 and replace the in-use product when the health status is 4. In the situation of multiple customers, the conflict between two customers when spare products are not enough to replace them both is handled by replacing the in-use product with greater deterioration (i.e., higher h_i) and larger deterioration rate. The SO policy is also evaluated using the

Value Iteration algorithm with a fixed policy.

The heuristic replacement policy is coordinated with the optimal inventory control policy, denoted as $H+O$. The handling procedure is similar to the $O+(s,S)$ policy. The heuristic inventory control has to be applied with the heuristic replacement policy in designation, thus the heuristic solution consists of the heuristic replacement policy and the heuristic inventory control is evaluated using the Value Iteration algorithm with fixed policy, denoted as HS.

5.3 *Gaps of heuristic policies and benchmark policies*

The Long-term Average Net Revenues ($LANR$) for the two instances are calculated by using the above methods. The *Gap*s are the criterion that evaluates the performance of the heuristic policies and benchmark policies in LANR, and are given by

$$Gap(*) = \frac{LANR(GO) - LANR(*)}{|LANR(*)|} \cdot 100\%, \tag{10}$$

where "$*$" can be any policy mentioned above. All the experiments are conducted on a Mono 2.6.1 in C# on a computer with a dual Intel Xeon 2.93 GHz X5670 CPU with 24 GB of memory. For the basic instances, the optimal control policies are obtained after 12 and 13 iterations respectively. The *LANR* for different policies and their *Gap*s are presented in Table 4.

In the identical and non-identical instances, the best S is 2. In the identical instance, the Value Iteration algorithm with heuristic replacement policy finds the optimal control policy in 20.1 seconds. In contrast, the Value Iteration algorithm with the GO policy finds the optimal control policy in 86.0 seconds. The HS finds near optimal (*Gap*=0.16%) control policy in 10.9 seconds. The heuristic inventory is not optimal in only 340 states (5.2% in total states). The HS even performs better than the $O+(s,S)$ policy whose calculation time is 24.5 seconds. Compared with the (s,S) policy, the heuristic inventory control policy is more suitable to coordinate with the heuristic replacement policy, where the *Gap* is improved by 0.83%. The SO policy performs worst among all the policies.

In the non-identical instance, the heuristic replacement policy finds the near optimal (*Gap*=0.08%) control policy in 21.1 seconds. The heuristic replacement policy is non-optimal in 371 states (5.7%). The other results are similar with the identical instance. Overall, the HS performs better in the identical instance case.

Table 4. The *LANR*s and *Gap*s of the heuristic policies and the benchmark policies

	The identical instance	The non-identical instance

GO	$LANR$	8.2936	5.9380
$H+O$	$LANR$	8.2936	5.9330
	Gap	0%	0.08%
HS	$LANR$	8.2801	5.9090
	Gap	0.16%	0.49%
$O+(s,S)$	$LANR$	8.2127	5.8897
	Gap	0.99%	0.82%
$H+(s,S)$	$LANR$	8.2125	5.8641
	Gap	0.99%	1.26%
SO	$LANR$	5.3297	3.7194
	Gap	55.61%	59.64%

To illustrate the overall performance of the HS, we show 20 instances with ten identical instances and ten non-identical instances with random parameters (exact numbers and distributions are provided in Appendix B). The average *Gap*s are depicted in Table 5. We find that the performance of the HS is close to the $O+(s,S)$. Meanwhile, the HS outperforms the SO on average.

Table 5. The overall performance of the heuristic solution with random parameters

	$Gap(HS)$	$Gap(O+(s,S))$	$Gap(SO)$
Identical instances	3.27%	2.94%	62.04%
Non-identical instances	5.66%	6.07%	232.83%

5.4 Cost analysis

In this section, the corresponding costs for replacement and inventory are analyzed to explore the individual performance of the various policies. The results of identical and non-identical instances are presented in Figure 2a and 2b respectively. The main difference between the SO policy and other policies is the penalty cost. It is caused by the fact that the SO policy orders a spare product and keeps it in inventory only when the health status for an in-use product reaches a threshold. Consequently, the SO policy does not order enough spare products to avoid stock-outs. The total revenue for the SO policy is impaired by the high possibility of service failure along with no spare product to replace failed products. The same phenomenon can be found in the non-identical instance. The $O+(s,S)$ policy and the $H+(s,S)$ policy are also inclined to order less spare product, which is similar to the SO policy. Notice that the S level in (s,S) is just 2, more spare products would be ordered for the condition with higher S levels though the performance is worse than current result. Compared with the optimal inventory control, the heuristic inventory control suggests ordering more spare products, which increases the holding cost. The heuristic replacement policy gets an optimal or near optimal result in the experiments, thus we cannot distinguish between the heuristic replacement policy and the optimal replacement policy. Another interesting finding is that the replacement costs among all the policies are very close and change proportionately with the ordering cost. This result is partly because of the high-performance of heuristic replacement policy and partly because of the short lead time where the delivery is very timely. Overall, the HS is not as "smart" as the optimal policy, where the average inventory level is higher by about 20 percent. The spare products in the HS are not put into use immediately as they are in the optimal control policy.

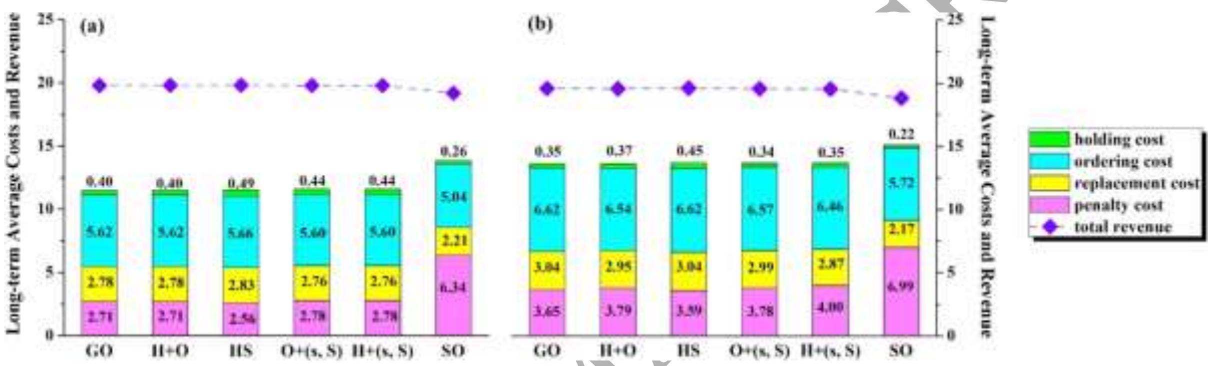

Figure 2. The long-term average costs and revenues of different policies
(a: the identical instance; b: the non-identical instance)

Another area of interest for cost analysis is based on the difference between customers. In the non-identical instance, four customers are identified as four categories in deterioration rate and unit service revenue: low-use-high-value, low-use-low-value, high-use-high-value, and high-use-low-value. Apparently, the category of high-use-high-value can bring the most revenue. We explore how the HS may deviate. Five criteria are selected to depict the detail action of the HS: average health status when replacement happens, denoted as ASR; average times for replacement per period, denoted as AR; the replacement cost, denoted as $f(h)$; the penalty cost, denoted as C; and the total revenue, denoted as Rev. All the criteria are calculated in each customer in the setting of the non-identical instance. The $O+(s,S)$ policy is also involved as a benchmark. The *Gap*s of these criteria are calculated by using the equation similar to (10). The results are illustrated in Figure 3.

In Figure 3, one can see that the $O+(s,S)$ policy does not change preference for a particular customer. The average health status when replacement happens in the $O+(s,S)$ policy is delayed a little. This result is because of insufficient ordering which will be explained in the sensitivity analysis.

However, the HS reduces the average times for replacement only for the second customer, the low-use-low-value category. For other customers, the HS increases the average times for replacement as expected. Thus, these criteria have two directions in the top part of Figure 3. This phenomenon shows that the HS over-reacts a little in the non-identical instance, where the preference to the high-value customer impairs the service level of the low-value customer. Meanwhile, the $O + (s, S)$ policy always keeps two spare products in inventory to lessen the possibility of abandoning the customer of small value. In the next section, the sensitivity analysis enables us to test the robustness of HS by varying different parameters.

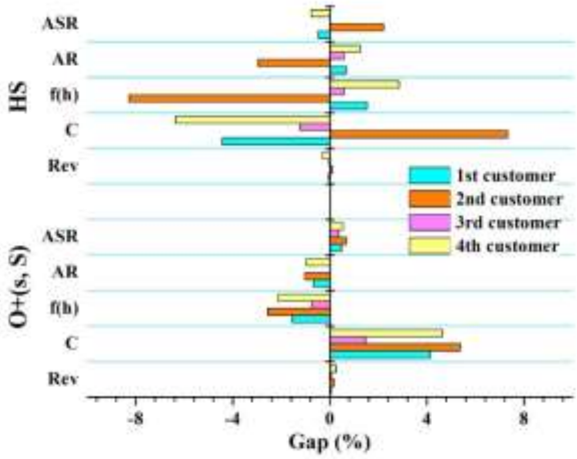

Figure 3. The Gaps of criteria for every customer in the non-identical instance

6. Sensitivity Analyses

We consider the conditions of varying unit holding cost, replacement costs, unit service revenue, and deterioration rate. Other parameters are kept the same as the basic instances. $Gap(H + O)$ is utilized to represent the performance of the heuristic replacement policy, denoted as $Gap1$. Because the heuristic inventory control policy has to coordinate with the heuristic replacement policy, $Gap(HS) - Gap(H + O)$, denoted as $Gap2$, is utilized to represent the performance of the heuristic inventory control. For the similarity between identical and non-identical instance, this sensitivity analysis focuses on the identical instance.

6.1 Influence of unit holding costs

In this experiment, we analyze the impact of h_c from 0 to 2. For a small value of h_c, the spare products are not perishable or are easy to store, where the system prefers to hold more spare products.

Alternatively, if the spare products are perishable for a very high value of h_c, such as storage batteries and radiopharmaceuticals, the system would prefer to hold fewer due to the cost. The result is depicted in Figure 4. The *Gap* of heuristic replacement policy is apparent when $h_c = 0$, while the SO and $O + (s, S)$ policies can find the optimal control. However, in the extreme experiment of $h_c = 0.05$ (1/100 of unit purchase cost), *Gap1* is relatively small (0.61%). Notice that the S level is the maximum inventory level, the $O + (s, S)$ policy would order spare products as often as possible. The SO policy also maximizes the level of spare products no matter whether replacement occurs or not. Due to that the maximum inventory level is equal to N, the replacement is not constrained by the availability of spare product. This result illustrates that the coordination effect for the system does not exist when $h_c = 0$, where we can separate the system into N subsystems for a single customer to solve efficiently. When h_c becomes larger, the coordination effect becomes significant. The performance for the SO and $O + (s, S)$ policies become worse as h_c increases, especially for the SO policy. This result illustrates that neither of these two policies can handle the coordination effect well. The *Gap* of the HS is relatively low in the setting of positive h_c by managing the coordination effect of the system.

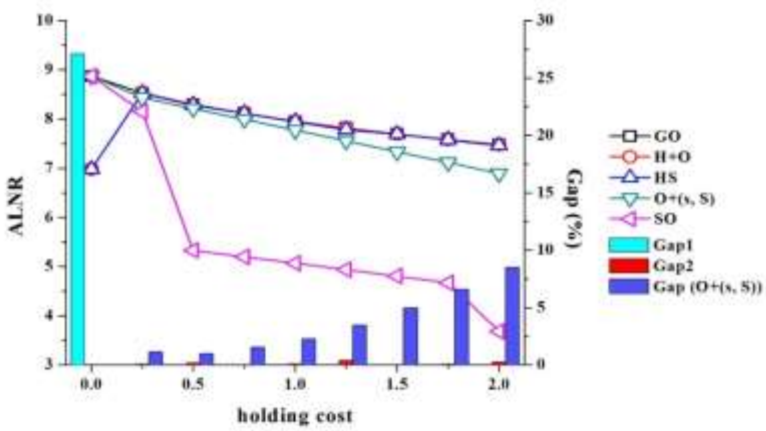

Figure 4. Average long-term net revenue and Gaps for different policies under the influence of varying unit holding cost

6.2 *Influence of replacement costs*

In this experiment, the replacement cost is varied according to multiples, Δ. The replacement cost represents the importance of the End-of-Life cost for the product. As Δ is increased, the impact of waste for the product becomes more significant to the system. The results are shown in Figure 5, where the maximum *Gap* for the HS is 7.54% when $\Delta = 1.5$. The $O + (s, S)$ policy becomes worse as Δ increases. This result can be explained by the fact that the (s, S) policy does not consider the End-of-Life cost, while our heuristic inventory control does. The maximum *Gap* for the $O + (s, S)$ policy is 5.27% when $\Delta = 3$. To illustrate the reasons for the *Gap*s when $\Delta = 1.5$ and $\Delta = 2$, the penalty costs, replacement costs, and holding costs are each analyzed in Figure 6. The total revenue and the purchase cost are always very adjacent for these two policies during this experiment. When $\Delta = 1.5$, the resulting *Gap* is driven by the holding cost, namely the HS has to keep more spare products in inventory than the optimal control. When $\Delta = 2$ or 2.5, the *Gap* comes from the higher penalty cost in the HS. However, the replacement cost for the HS is less than the optimal control. This phenomenon shows that the HS has postponed the timing for replacement while the optimal control has not. When $\Delta = 3$, the HS is almost optimal again. However, the HS always outperforms the SO policy during the experiment. In conclusion, the HS is more sensitive to the End-of-Life cost of the product than the optimal control. This weakness brings a periodic gap when the HS delays the replacement earlier than the optimal control.

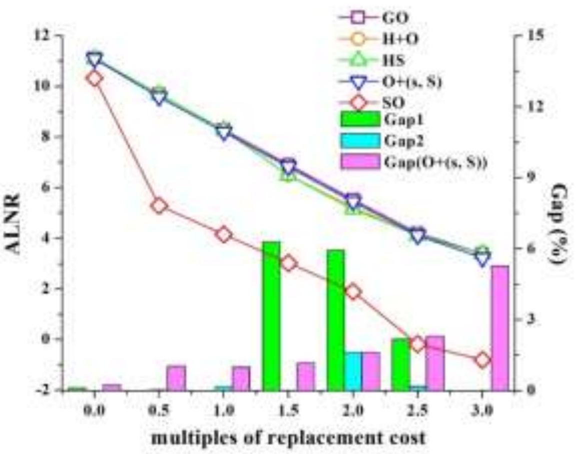

Figure 5. The average long-term net revenues and Gaps of different policies under the influence of varying replacement costs

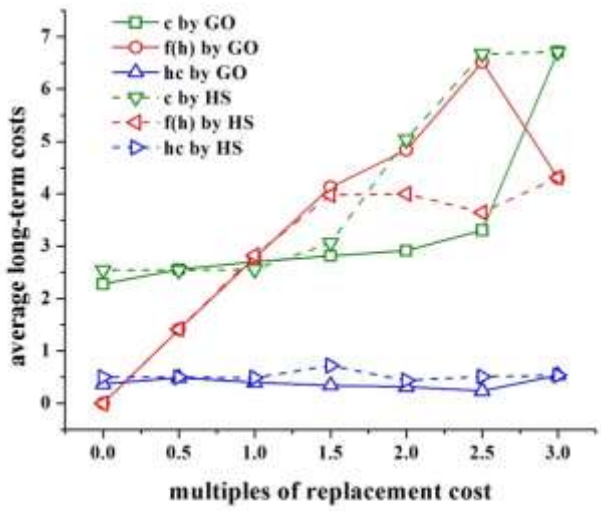

Figure 6. The cost analysis of the optimal control and heuristic solution under the influence of varying replacement cost

6.3 Influence of unit service revenue

In this experiment, the unit service revenue is varied from 1 to 20. Because the heuristic replacement policy is established based on the criterion of maximizing revenue, this experiment does not consider the extreme condition of no unit service revenue. In Figure 7, the ratio between the penalty cost and the unit service revenue is the x-axis. The *Gap* for the HS is not apparent when the ratio is between 4 and 10, namely the unit service revenue is between 5 and 2. When the unit service revenue is 20 or 10, the average health status when replacement happens is the retained for the optimal control policy. However, the HS suggests postponing the timing of replacement to health status 5. Notice that the maximum health status is 6, the HS tries to replace at the last opportunity before the in-use product is at the end of its life. When the unit service revenue is 1, the HS suggests doing the replacement earlier, while the optimal control does not. This phenomenon shows that the HS is also sensitive to the influence of the unit service revenue, which is similar to the condition in Section 6.2. The maximum *Gap* is 12.86% when the unit service revenue is 1.

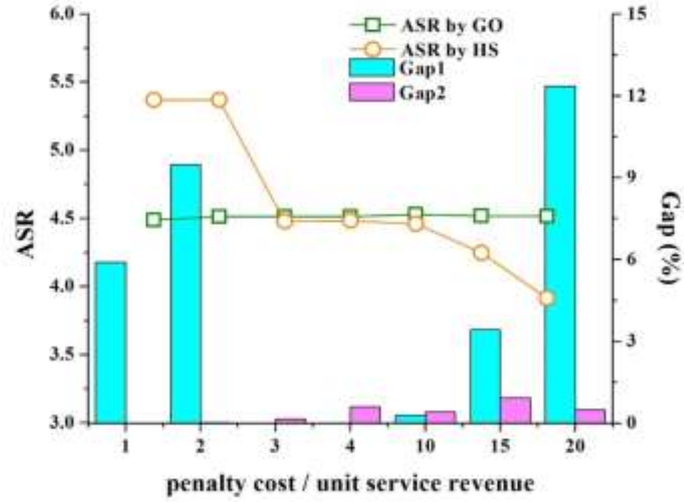

Figure 7. The Gap and average health status when replacement happens under the influence of varying unit service revenue

6.4 Influence of deterioration rate

The deterioration rate represents the expected life of the in-use product. When the deterioration rate is low, the in-use products are assumed to last longer in the hands of the customers, for example, long-term leases. In contrast, the deterioration rate is high when in-use products are expected to have rapid exchange and flow between customer and service provider, such as the condition of product sharing in the fashion industry. In this experiment, we vary the expected life for in-use products from 2 to 10 periods. The deterioration rate is defined as one fifth of the expected life of the in-use product. The results are depicted in Figure 8. When the expected life of an in-use product is 2, the SO policy is optimal as the condition for $hc = 0$. Under this condition, the coordination effort for the system can also be ignored because the exchanging of product is too frequent and the system always needs to keep as many spare products as possible. Otherwise, the HS always outperforms the SO policy when the coordination effort is considerable. When the expected life of the in-use product is between 3 and 5, the HS is optimal. When the expected life of an in-use product is larger than 6, the HS performs worse than the $O + (s, S)$ policy. The biggest *Gap* is 11.37% when the expected life of the in-use product is 9. The *Gap* for the HS comes predominately from the heuristic replacement policy, which postpones the timing of replacement for the small possibility of product/service failure.

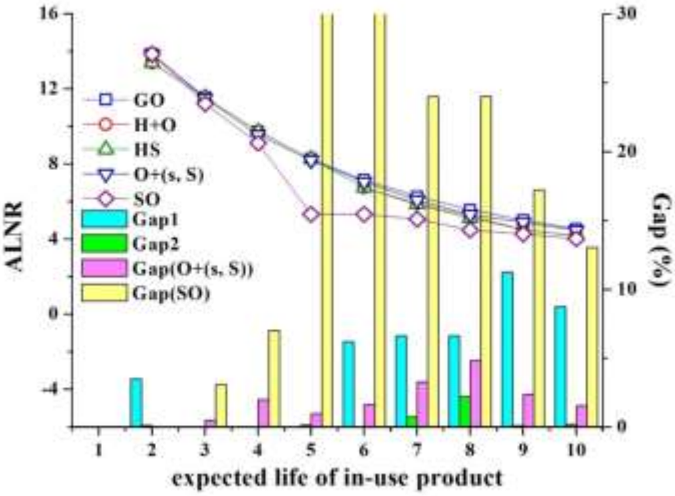

Figure 8. The average long-term net revenues and Gaps for different policies under the influence of varying deterioration rates

However, the deterioration rate is also determined by the maximum health status of product, H. For example, the deterioration rate is 5/6 when the expected life of in-use product is 6 in the original experiment. If we define $H = 7$, the deterioration rate becomes 1. Meanwhile, the unit service revenue and replacement cost should change accordingly. This relation provides a novel solution for the system when the deterioration rate is less than 1. In this experiment, the *ALNR* is 7.7202 and the *Gap* for the HS is 0.01% when $H = 7$. By comparison, the *ALNR* for the HS is 7.0473 and the *Gap* is 6.21% when $H = 6$. The HS benefits from the increment of H by 9.54%, although the calculation time also increases as H increases due to the increase in state space. In this experiment, the average calculation time for each state is 2.3 milliseconds when $H = 7$ and 1.2 milliseconds when $H = 6$.

7. Conclusions

This paper considers a dynamic model for a Use-Oriented PSS which jointly optimizes replacement and inventory decisions. In our modeling effort, we simulate the condition of decreasing replacement costs and multiple customers, which better represents the characteristics of a novel business model for Use-Oriented PSS. We proposed a sequential heuristic solution which consists of a heuristic replacement policy and a heuristic inventory control policy to solve the integrated model. The heuristic replacement policy is based on the optimization of expected total reward in separate sub-systems. The structure of marginal replacement benefit is analyzed for bypassing the calculation of iterative algorithms. The one-period look-ahead myopic optimization applies to the heuristic inventory control policy based on an approximate function of future cost-to-go. In 20 instances with random parameters, the heuristic solution finds the near-optimal results, where the overall average gaps from the Global Optimization are 3.27% and 5.66%. As a result of sensitivity analysis, the performance of the heuristic solution is satisfactory in most cases, where the largest gap from the Global Optimization is 12.86%. When compared with other benchmark policies, the performance of the heuristic solution is close to the optimal replacement policy with the best (s, S) inventory policy. Here the characteristics of the heuristic solution are summarized. First, the gap mainly comes from the heuristic replacement policy which is more sensitive to the parameters than the optimal control policy heuristic. For varying replacement cost, unit service revenue, and deterioration rate, the heuristic replacement policy advances or delays the timing of replacement. Second, the heuristic solution over reacts for the non-identical instances. For some low-value customers, who have small deterioration rates and low unit service revenue, the heuristic solution does not satisfy their service sufficiently. Third, the heuristic solution performs better when the deterioration rate is larger than 1. However, the deterioration rate can be adjusted by setting the maximum health status and period interval in the model.

Some management insights are provided based on the sensitivity analysis. First, the coordination effect for this model does not always exist. The coordination effect is related to the availability of spare products. If the spare products are always available where the inventory level is larger than the number of customers at any possible state, this system can be divided into N sub-systems for a single customer. In contrast, the coordination effect has to be considered if the spare products are limited. This paper identifies two conditions where the separate optimization is also optimal: the holding cost is close to zero compared with other parameters; the deterioration rate is very high that the system needs to keep as many spare products as possible. The evidence can be found in Section 6.1 and Section 6.4. However, the separate optimization policy and the (s, S) policy do not handle the coordination effect well. Second, the decreasing slope of the replacement cost hurts the profitability. If the End-of-life cost is significant in the system, the replacement cost decreases sharply. Meanwhile, we can find that the coordination effect is also significant along with the End-of-life cost, which is presented in Section 6.2. However, the models in this area have not considered the influence of the

End-of-life cost yet. This feature is not well illustrated in the previous literature. Third, the ratio between the penalty cost and unit service revenue needs to be considered carefully. If this ratio is too small, the heuristic solution suggests advancing the timing of replacement compared with the optimal control policy. If the ratio is too high, the heuristic solution suggests delaying the timing. Under the both conditions, the heuristic solution has about a 10% gap as shown in Section 6.3.

Future work in this area should focus on the improvement of the heuristic solutions. The sensitivity of the replacement policy needs to be reduced for more profitability. The criterion of myopic profitability may be considered as a component of a hybrid approach for improving the short-term performance of the replacement decision. While the heuristic inventory control performs well in the experiment, the problem of calculation complexity exists. For the situation where the delivery needs multiple periods of lead time, the distribution for the future state is very hard to calculate. Thus, some simulation-based approximation methods may need to be utilized to improve the practical value of our heuristic solution. The approximation bias of the heuristic inventory control is also likely to affecting the performance of our solution, which leads to a more precise approximate function.

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Appendix A.

Proof of Lemma 1. $Rev_i(h_i, 0) = \sum_{j=h_i+1}^{H} P_{i_{h_i,j}} \cdot \gamma_i \cdot (j - h_i)$.

According to Assumption 1, For any $h_i < H$,

$$Rev_i(h_i, 0) = \sum_{t=1}^{H-h_i-1} P_{\Delta=t} \cdot \gamma_i \cdot t + \sum_{t=H-h_i}^{H-1} P_{\Delta=t} \cdot \gamma_i \cdot (H - h_i), \tag{A1}$$

and

$$Rev_i(h_i + 1,0) = \sum_{t=1}^{H-h_i-1} P_{\Delta=t} \cdot \gamma_i \cdot t + \sum_{t=H-h_i}^{H-1} P_{\Delta=t} \cdot \gamma_i \cdot (H - h_i - 1). \tag{A2}$$

Thus

$$Rev_i(h_i + 1,0) - Rev_i(h_i, 0) = -\sum_{t=H-h_i}^{H-1} P_{\Delta=t} \cdot \gamma_i \leq 0. \tag{A3}$$

Proof of Theorem 1. Except $R(h_i, k) > K(h_i, k)$ for all h_i, there exists $h_i{}'(h_i{}' < H)$ which satisfies: $R(h_i{}', k) \leq K(h_i{}', k)$ and $R(h_i{}'', k) > K(h_i{}'', k)$ for all $h_i{}'' > h_i{}'$.

Define $Rev_i(h_i{}', 0) - h_c \cdot k$ as $r(h_i{}', k, 0)$ and $Rev_i(h_i{}', 1) - h_c \cdot (k - 1)$ as $r(h_i{}', k, 1)$. Such that

$$\begin{aligned} K(h_i{}', k) &= r(h_i{}', k, 0) + P_{i_{h_i{}',h_i{}'}} \cdot K(h_i{}', k) + \sum_{j=h_i{}'+1}^{H} P_{i_{h_i{}',j}} \cdot R(j, k) \\ &= \frac{r(h_i{}', k, 0) + \sum_{j=h_i{}'+1}^{H} P_{i_{h_i{}',j}} \cdot R(j, k)}{1 - P_{i_{h_i{}',h_i{}'}}}. \end{aligned} \tag{A4}$$

Notice that

$$R(j, k) = r(j, k, 1) - f(j) - c_i \cdot 1_{j=H} + \sum_{j'=1}^{H} P_{i_{1,\, j'}} \cdot v(j', k - 1), \tag{A5}$$

where $r(j, k, 1)$ is constant in j. Thus, we can find that $R(j, k) - R(h_i{}', k) = f(h_i{}') - f(j)$. (A6)

From (A6), subtracting $R(h_i{}', k)$ from both sides of (A4) yields

$$K(h_i{}', k) - R(h_i{}', k) = \frac{r(h_i{}', k, 0) + \sum_{j=h_i{}'+1}^{H} P_{i_{h_i{}', j}} \cdot \left(R(j, k) - R(h_i{}', k)\right)}{1 - P_{i_{h_i{}', h_i{}'}}}$$

$$= \frac{r(h_i{}', k, 0) + \sum_{j=h_i{}'+1}^{H} P_{i_{h_i{}', j}} \cdot \left(f(h_i{}') - f(j)\right) - P_{i_{h_i{}', H}} \cdot c_i}{1 - P_{i_{h_i{}', h_i{}'}}}, \qquad \text{(A7)}$$

where $1 - P_{i_{h_i{}', h_i{}'}} = \sum_{j=h_i{}'+1}^{H} P_{i_{h_i{}', j}}$.

According to Assumption 1, we can define $P_{\Delta \geq i} = \sum_{t=i}^{H-1} P_{i_{\Delta = t}}$. Therefore, $1 - P_{i_{h_i{}', h_i{}'}} = P_{\Delta \geq 1}$, $\sum_{j=h_i{}'+1}^{H} P_{i_{h_i{}', j}} = P_{\Delta \geq i}$, and $P_{i_{h_i{}', H}} = P_{\Delta \geq H - h_i{}'}$.

Replace $f(h_i{}') - f(j)$ with $\left(f(h_i{}') - f(h_i{}' + 1)\right) + \left(f(h_i{}' + 1) - f(h_i{}' + 2)\right) \ldots + \left(f(j-1) - f(j)\right)$ in (A7). Thus,

$$K(h_i{}', k) - R(h_i{}', k) =$$

$$\frac{r(h_i{}', k, 0) + P_{\Delta \geq 1} \cdot \left(f(h_i{}') - f(h_i{}' + 1)\right) + P_{\Delta \geq 2} \cdot \left(f(h_i{}' + 1) - f(h_i{}' + 2)\right) \ldots + P_{\Delta \geq H - h_i{}'} \cdot \left(f(H-1) - f(H)\right) - P_{\Delta \geq H - h_i{}'} \cdot c_i}{P_{\Delta \geq 1}} = ①. \qquad \text{(A8)}$$

We can replace $h_i{}'$ with $h_i{}' - 1$ in the both sides of (A4), and

$$K(h_i{}' - 1, k) = r(h_i{}' - 1, k, 0) + P_{i_{h_i{}'-1, h_i{}'-1}} \cdot v(h_i{}' - 1, k) + \sum_{j=h_i{}'}^{H} P_{i_{h_i{}'-1, j}} \cdot R(j, k). \qquad \text{(A9)}$$

Because $v(h_i{}' - 1, k) = max\{K(h_i{}' - 1, k), R(h_i{}' - 1, k)\}$, we have $v(h_i{}' - 1, k) \geq K(h_i{}' - 1, k)$.

Thus, $K(h_i{}' - 1, k) \geq \dfrac{r(h_i{}' - 1, k, 0) + \sum_{j=h_i{}'}^{H} P_{i_{h_i{}'-1, j}} \cdot R(j, k)}{1 - P_{i_{h_i{}'-1, h_i{}'-1}}}$. (A10)

Through the similar process from (A4) to (A8), the following result can be derived from (A10)

$$K(h_i{}' - 1, k) - R(h_i{}' - 1, k)$$

$$\geq \frac{r(h_i{}' - 1, k, 0) + P_{\Delta \geq 1} \cdot \left(f(h_i{}' - 1) - f(h_i{}')\right) + P_{\Delta \geq 2} \cdot \left(f(h_i{}') - f(h_i{}' + 1)\right) \ldots + P_{\Delta \geq H - h_i{}' + 1} \cdot \left(f(H-1) - f(H)\right) - P_{\Delta \geq H - h_i{}' + 1} \cdot c_i}{P_{\Delta \geq 1}} = ②. \qquad \text{(A11)}$$

We compare ① and ② in each component. According to Lemma 1, we have $r(h_i{}' - 1, k, 0) \geq r(h_i{}', k, 0)$; from condition (i) and (ii), we have $f(j) - f(j+1) \geq f(j+1) - f(j+2)$

for all $j \in [1, H-2]$; moreover, we have $P_{\Delta \geq H - h_i' + 1} \leq P_{\Delta \geq H - h_i'}$ through Assumption 1. As a result, ② ≥ ①. Namely $K(h_i' - 1, k) - R(h_i' - 1, k) \geq$ ② ≥ ① $= K(h_i', k) - R(h_i', k)$.

Besides we know that the optimal policy at (h_i', k) is "*Keep*" (as shown in (7)), namely $K(h_i', k) - R(h_i', k) \geq 0$, therefore $K(h_i' - 1, k) - R(h_i' - 1, k) \geq$ ② ≥ ① $= K(h_i', k) - R(h_i', k) \geq 0$. The optimal policy at $(h_i' - 1, k)$ is also "*Keep*".

Similarly, $K(h_i, k) \geq R(h_i, k)$ for all $h_i < h_i' - 1$.

Proof of Theorem 2. We prove by showing that $K(h_i, k) \geq R(h_i, k)$ holds if $K(h_i, k+1) \geq R(h_i, k+1)$ for any state.

If $k = 0$, $K(h_i', k) \geq R(h_i', k)$ always holds. For $k \geq 1$, we assume h_i' is the threshold of $k+1$ defined in Theorem 1. (A7) still holds, namely

$$K(h_i', k+1) - R(h_i', k+1) = \frac{r(h_i', k+1, 0) + \sum_{j=h_i'+1}^{H} P_{i_{h_i', j}} \cdot \left(f(h_i') - f(j)\right) - P_{i_{h_i', H}} \cdot c_i}{1 - P_{i_{h_i', h_i'}}} \geq 0.$$

For state (j, k), we define the action is same with state $(j, k+1)$. Due to that these actions may be non-optimal,

$$K(h_i', k) - R(h_i', k) \geq \frac{r(h_i', k, 0) + \sum_{j=h_i'+1}^{H} P_{i_{h_i', j}} \cdot \left(f(h_i') - f(j)\right) - P_{i_{h_i', H}} \cdot c_i}{1 - P_{i_{h_i', h_i'}}}. \tag{A12}$$

Because $r(h_i', k+1, 0) = Rev_i(h_i', 1) - h_c \cdot k$ and $r(h_i', k+1, 0)$ is nonincreasing in k, we have

$$\frac{r(h_i', k, 0) + \sum_{j=h_i'+1}^{H} P_{i_{h_i', j}} \cdot \left(f(h_i') - f(j)\right) - P_{i_{h_i', H}} \cdot c_i}{1 - P_{i_{h_i', h_i'}}} \geq \frac{r(h_i', k+1, 0) + \sum_{j=h_i'+1}^{H} P_{i_{h_i', j}} \cdot \left(f(h_i') - f(j)\right) - P_{i_{h_i', H}} \cdot c_i}{1 - P_{i_{h_i', h_i'}}}$$

$$\geq K(h_i', k+1) - R(h_i', k+1) \geq 0. \tag{A13}$$

Thus, $K(h_i', k) - R(h_i', k) \geq 0$.

For any h_i and k, $K(h_i, k) - R(h_i, k) \geq 0$, if $K(h_i, k+1) - R(h_i, k+1) \geq 0$.

Proof of Theorem 3. If $B(h_i, k) \geq 0$ (i.e. "*Replace*" is preferred), we have

$$K(h_i,k) = Rev_i(h_i,0) - h_c \cdot k - c_i \cdot 1_{h_i=H} + \sum_{j=h_i}^{H} P_{i_{h_i,j}} \cdot R(j,k). \tag{A14}$$

Thus, $B(h_i,k) = R(h_i,k) - K(h_i,k)$

$$= Rev_i(1,0) - h_c \cdot (k-1) - f(h_i) + \sum_{j=1}^{H} P_{i_{1,j}} \cdot v(j,k-1) - Rev_i(h_i,0) + h_c \cdot k - \sum_{j=h_i}^{H} P_{i_{h_i,j}} \cdot R(j,k). \tag{A15}$$

Notice that

$$R(j,k) = Rev_i(1,0) - h_c \cdot (k-1) - c_i \cdot 1_{j=H} - f(j) + \sum_{j=1}^{H} P_{i_{1,j}} \cdot v(j,k-1). \tag{A16}$$

Therefore,

$$B(h_i,k) = P_{i_{h_i,H}} \cdot c_i - \sum_{j=h_i}^{H} P_{i_{h_i,j}} \cdot \big(f(h_i) - f(j)\big) - Rev_i(h_i,0) + h_c \cdot k. \tag{A17}$$

Namely

$$B(h_i,k) = \delta(h_i) + h_c \cdot k, \tag{A18}$$

where

$$\delta(h_i) = P_{i_{h_i,H}} \cdot c_i - \sum_{j=h_i}^{H} P_{i_{h_i,j}} \cdot \big(f(h_i) - f(j)\big) - Rev_i(h_i,0).$$

If $B(h_i,k) < 0$,

$$K(h_i,k) \le Rev_i(h_i,0) - h_c \cdot k - c_i \cdot 1_{h_i=H} + \sum_{j=h_i}^{H} P_{i_{h_i,j}} \cdot R(j,k). \tag{A19}$$

By following the similar steps from (A14) to (A18), we have $B(h_i,k) \le \delta(h_i) + h_c \cdot k$.

Appendix B. Random Selected Instances

Table B1. The values of parameters in the overall performance test

Identical instances														
	λ_1	λ_2	λ_3	λ_4	c_1	c_2	c_3	c_4	γ_1	γ_2	γ_3	γ_4	$f(h)^*$	hc	o
1	0.99	0.99	0.99	0.99	24.71	24.71	24.71	24.71	3.39	3.39	3.39	3.39	0.35	0.78	6.25
2	1.00	1.00	1.00	1.00	17.23	17.23	17.23	17.23	6.10	6.10	6.10	6.10	1.32	0.19	4.13
3	1.46	1.46	1.46	1.46	22.04	22.04	22.04	22.04	6.11	6.11	6.11	6.11	1.49	0.44	4.97
4	1.02	1.02	1.02	1.02	13.59	13.59	13.59	13.59	5.52	5.52	5.52	5.52	1.01	0.33	6.93
5	0.77	0.77	0.77	0.77	27.81	27.81	27.81	27.81	7.10	7.10	7.10	7.10	1.01	0.48	6.69
6	0.75	0.75	0.75	0.75	13.44	13.44	13.44	13.44	6.54	6.54	6.54	6.54	1.46	0.46	5.02

7	0.73	0.73	0.73	0.73	28.59	28.59	28.59	28.59	6.10	6.10	6.10	6.10	1.47	0.96	3.21
8	0.58	0.58	0.58	0.58	16.94	16.94	16.94	16.94	5.81	5.81	5.81	5.81	0.92	0.78	3.95
9	1.17	1.17	1.17	1.17	14.44	14.44	14.44	14.44	2.89	2.89	2.89	2.89	1.28	0.67	6.79
10	0.76	0.76	0.76	0.76	18.37	18.37	18.37	18.37	5.21	5.21	5.21	5.21	1.93	0.66	6.46
Non-identical instances															
	λ_1	λ_2	λ_3	λ_4	c_1	c_2	c_3	c_4	γ_1	γ_2	γ_3	γ_4	$f(h)^*$	hc	o
1	1.30	1.20	0.73	1.14	12.89	26.00	14.87	26.51	6.65	7.47	5.06	3.91	0.34	0.39	4.50
2	0.74	0.67	1.07	1.36	22.57	24.48	15.36	29.97	5.26	7.10	5.05	6.07	1.22	0.52	3.06
3	0.51	0.84	1.25	1.04	16.12	17.26	19.75	27.37	3.41	5.90	3.55	7.08	0.45	0.09	3.46
4	0.63	0.97	0.62	1.47	18.67	10.87	26.30	13.17	2.74	3.67	5.40	7.05	0.96	0.94	4.43
5	0.77	0.93	1.25	1.12	24.30	25.66	25.89	17.68	6.36	7.32	4.02	2.56	1.41	0.41	4.91
6	1.29	0.58	1.50	1.47	17.93	16.43	13.00	29.50	6.92	6.07	5.67	5.49	1.13	0.72	2.86
7	0.76	0.54	0.72	0.65	11.22	21.05	14.10	12.00	6.89	7.12	3.12	6.02	0.40	0.68	7.18
8	1.39	1.17	0.65	0.94	18.57	25.07	11.94	21.03	6.15	4.24	4.47	4.12	0.09	0.66	7.24
9	0.81	1.31	1.41	0.66	16.95	19.21	24.44	28.21	6.26	4.98	3.87	7.13	0.38	0.11	4.77
10	1.05	0.64	1.39	1.05	15.15	12.87	20.28	13.75	4.91	3.73	7.34	5.92	0.46	0.29	7.24

$f(h)^*$ is the multiple times of original setting of replacement costs, $f(h)^* \cdot [6, 5, 4, 3, 2, 1]$.

Table B2. The distributions of parameters in the overall performance test ($U[a, b]$ is uniform distribution with lower bound *a* and upper bound *b*)

λ_i	$U[0.5, 1.5]$
c_i	$U[10, 30]$
γ_i	$U[2.5, 7.5]$
$f(h)^*$	$U[0, 2]$
hc	$U[0, 1]$
o	$U[2.5, 7.5]$

References

Ahmad, R., & Kamaruddin, S. (2012). An overview of time-based and condition-based maintenance in industrial application. *Computers & Industrial Engineering*, *63*(1), 135-149.

Alaswad, S., & Xiang, Y. (2017). A review on condition-based maintenance optimization models for stochastically deteriorating system. *Reliability Engineering & System Safety*, 157, 54-63.

Bellman, R. (1957). A Markovian decision process: DTIC Document.

Cai, J., Yin, Y., Zhang, L., & Chen, X. (2017). Joint Optimization of Preventive Maintenance and Spare Parts Inventory with Appointment Policy. *Mathematical Problems in Engineering*, *2017*.

Chen, A., & Wu, G. (2007). Real-time health prognosis and dynamic preventive maintenance policy for equipment under aging Markovian deterioration. *International Journal of Production Research*, *45*(15), 3351-3379.

Cho, D. I., & Parlar, M. (1991). A survey of maintenance models for multi-unit systems. *European Journal of Operational Research, 51*(1), 1-23.

Corvellec, H., & Stål, H. I. (2017). Evidencing the waste effect of Product-Service Systems (PSSs). *Journal of Cleaner Production*.

Dekker, R., Wildeman, R. E., & Van der Duyn Schouten, F. A. (1997). A review of multi-component maintenance models with economic dependence. *Mathematical methods of operations research, 45*(3), 411-435.

Elwany, A. H., & Gebraeel, N. Z. (2008). Sensor-driven prognostic models for equipment replacement and spare parts inventory. *IIE Transactions, 40*(7), 629-639.

Gaiardelli, P., Resta, B., Martinez, V., Pinto, R., & Albores, P. (2014). A classification model for product-service offerings. *Journal of Cleaner Production, 66*, 507-519.

Hong, H., Zhou, W., Zhang, S., & Ye, W. (2014). Optimal condition-based maintenance decisions for systems with dependent stochastic degradation of components. *Reliability Engineering & System Safety, 121*, 276-288.

Icten, Z. G., Shechter, S. M., Maillart, L. M., & Nagarajan, M. (2013). Optimal management of a limited number of replacements under Markovian deterioration. *IIE Transactions, 45*(2), 206-214.

Joo, S., & Oh, C. (2013). A novel method to monitor bicycling environments. *Transportation research part A: policy and practice, 54*, 1-13.

Kawai, H. (1983). An optimal ordering and replacement policy of a markovian degradation system under complete observation. *Journal of Operational Research Society of Japan, 26*, 279-290.

Keizer, M. C. O., Teunter, R. H., & Veldman, J. (2016). Clustering condition-based maintenance for systems with redundancy and economic dependencies. *European Journal of Operational Research, 251*(2), 531-540.

Keizer, M. C. O., Teunter, R. H., & Veldman, J. (2017). Joint condition-based maintenance and inventory optimization for systems with multiple components. *European Journal of Operational Research, 257*(1), 209-222.

Keizer, M. C. O., Flapper, S. D. P., & Teunter, R. H. (2017). Condition-based maintenance policies for systems with multiple dependent components: a review. *European Journal of Operational Research*, 261(2).

Kennedy, W., Patterson, J. W., & Fredendall, L. D. (2002). An overview of recent literature on spare parts inventories. *International Journal of Production Economics, 76*(2), 201-215.

Li, R., & Ryan, J. K. (2011). A Bayesian Inventory Model Using Real-Time Condition Monitoring Information. *Production and Operations Management, 20*(5), 754-771.

Liao, H., Elsayed, E. A., & Chan, L.-Y. (2006). Maintenance of continuously monitored degrading systems. *European Journal of Operational Research, 175*(2), 821-835.

Louit, D., Pascual, R., Banjevic, D., & Jardine, A. K. (2011). Condition-based spares ordering for critical components. *Mechanical Systems and Signal Processing, 25*(5), 1837-1848.

Nguyen, K. A., Do, P., & Grall, A. (2017). Joint predictive maintenance and inventory strategy for multi-component systems using birnbaum’s structural importance. *Reliability Engineering & System Safety*, 168.

Pan, Y., & Thomas, M. U. (2010). Repair and replacement decisions for warranted products under Markov deterioration. *IEEE transactions on reliability, 59*(2), 368-373.

Powell, W. B. (2016). Perspectives of approximate dynamic programming. *Annals of Operations Research,* *241*(1-2), 319-356.

Puterman, M. L. (1994). Markov Decision Processes: Discrete Stochastic Dynamic Programming.

Qiu, R., Sun, M., & Yun, F. L. (2017). Optimizing (s, S) policies for multi-period inventory models with demand distribution uncertainty: robust dynamic programming approaches. European Journal of Operational Research, 261(3).

Rausch, M., & Liao, H. (2010). Joint production and spare part inventory control strategy driven by condition based maintenance. *IEEE transactions on reliability, 59*(3), 507-516.

Reim, W., Parida, V., & Örtqvist, D. (2015). Product–Service Systems (PSS) business models and tactics–a systematic literature review. *Journal of Cleaner Production, 97*, 61-75.

Shafiee, M., Finkelstein, M., & Bérenguer, C. (2015). An opportunistic condition-based maintenance policy for offshore wind turbine blades subjected to degradation and environmental shocks. *Reliability Engineering & System Safety, 142*, 463-471.

Tian, Z., & Liao, H. (2011). Condition based maintenance optimization for multi-component systems using proportional hazards model. *Reliability Engineering & System Safety, 96*(5), 581-589.

Tijms, H. C. (1994). *Stochastic models: an algorithmic approach* (Vol. 303): John Wiley & Sons Inc.

Van Horenbeek, A., Buré, J., Cattrysse, D., Pintelon, L., & Vansteenwegen, P. (2013). Joint maintenance and inventory optimization systems: A review. *International Journal of Production Economics, 143*(2), 499-508.

Van Horenbeek, A., Scarf, P. A., Cavalcante, C. A., & Pintelon, L. (2013). The effect of maintenance quality on spare parts inventory for a fleet of assets. *IEEE transactions on reliability, 62*(3), 596-607.

Verbert, K., Schutter, B. D., & Babuška, R. (2017). Timely condition-based maintenance planning for multi-component systems. *Reliability Engineering & System Safety*, 159, 310-321.

Wang, L., Chu, J., & Mao, W. (2008a). A condition-based order-replacement policy for a single-unit system. *Applied mathematical modelling, 32*(11), 2274-2289.

Wang, L., Chu, J., & Mao, W. (2008b). An optimum condition-based replacement and spare provisioning policy based on Markov chains. *Journal of Quality in Maintenance Engineering, 14*(4), 387-401.

Wang, L., Chu, J., & Mao, W. (2009). A condition-based replacement and spare provisioning policy for deteriorating systems with uncertain deterioration to failure. *European Journal of Operational Research, 194*(1), 184-205.

Wang, Y., Zhao, J., Cheng, Z., & Yang, Z. (2015). Integrated decision on spare parts ordering and equipment maintenance under condition based maintenance strategy. *Eksploatacja i Niezawodność, 17*.

Wang, Z., Hu, C., Wang, W., Kong, X., & Zhang, W. (2015). A prognostics-based spare part ordering and system replacement policy for a deteriorating system subjected to a random lead time. *International Journal of Production Research, 53*(15), 4511-4527.

Xie, J., & Wang, H. (2008). Joint optimization of condition-based preventive maintenance and spare ordering policy. Paper presented at the Wireless Communications, Networking and Mobile Computing, 2008. WiCOM'08. 4th International Conference on.

Yang, X., Moore, P., Pu, J.-S., & Wong, C.-B. (2009). A practical methodology for realizing product service systems for consumer products. *Computers & Industrial Engineering, 56*(1), 224-235.

Zhang, X., & Zeng, J. (2017). Joint optimization of condition-based opportunistic maintenance and spare parts provisioning policy in multiunit systems. *European Journal of Operational Research*, 262(2).