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The affect heuristic and stock ownership: A theoretical perspective*

Jiang Luo[1] | **Avanidhar Subrahmanyam**[2]

[1]Nanyang Business School, Nanyang Technological University, Singapore City, Singapore
[2]Anderson Graduate School of Management, University of California at Los Angeles, Los Angeles, CA, USA

Correspondence
Avanidhar Subrahmanyam, Anderson Graduate School of Management, University of California at Los Angeles, Los Angeles, CA, USA.
Email: subra@anderson.ucla.edu

Abstract
We consider asset prices and informational efficiency in a setting where owning stock confers direct utility due to an affect heuristic. Specifically, holding equity in brand name companies or those indulging in "socially desirable" activities (e.g., environmental consciousness) confers positive consumption benefits, whereas investing in "sin stocks" yields the reverse. In contrast to settings based on wealth considerations alone, expected stock prices deviate from expected fundamentals even when assets are in zero net supply. Stocks that yield high direct utility are, on average, more informationally efficient as they stimulate more entry into the market for these stocks and, consequently, more information collection. The analysis also accords with a value effect, high valuations of brand-name stocks, abnormally positive returns on "sin stocks," volume premia in the cross-section of returns, proliferation of mutual funds and ETFs, and yields untested implications. If, as psychological literature suggests, agents derive greater utility from successful companies by "basking in reflected glory," then asset prices react to public signals non-linearly, leading to booms and busts, as well as crashes and recoveries.

KEYWORDS
asset prices, behavioral preferences, informational efficiency, stock ownership

1 | INTRODUCTION

In recent years, a body of anecdotal and empirical evidence indicates that perceptions of firms' products influence market valuations and investment choices. For example, the market valuation of Tesla recently exceeded that of Ford Motors, even as Tesla's sales volume was about 1% of that of Ford.[1] Billett, Jiang, and Rego (2014) show that stocks of companies with prestigious brands have higher market/book ratios and earn lower average returns in the cross section. Keloharju, Knüpfer, and Linnainmaa (2012) show that investors prefer to trade stocks of firms whose products they use. Hong and Kacperczyk (2009) show that "sin" (tobacco, gambling) stocks earn positive abnormal (risk-adjusted) returns. Further, the number of mutual funds in the U.S. rivals or exceeds the number of publicly traded individual firms, a proliferation that seems to

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emanate from investor tastes, since the funds' aggregate performance, net of fees, underwhelms.[2] In addition, several exchange-traded funds (ETFs) in the U.S. cater to tastes.[3]

To explain the above phenomena, we consider a setting where the affect associated with the product(s) of a firm (viz. Chaudhuri & Holbrook, 2001) carries over to the decision to invest in a firm's stock. Finucane, Alhakami, Slovic, and Johnson (2000) characterize this aspect of choice as an affect *heuristic*, which is the notion that "…images, marked by positive and negative affective feelings, guide judgment and decision making" (p. 3). Thus, we propose, for example, that some users of Apple products have positive affect toward Apple stock due to a favorable inclination toward its products. This reason for holding Apple complements traditional risk-reward considerations. Similarly, another premise is that some investors who care about the environment buy shares, for instance, in Tesla Motors, again for reasons that go beyond wealth-related motives. Indeed, plenty of references in the popular press allude to the notion that investors often "fall in love" with stocks.[4] Fama and French (2007) indicate that the standard assumption that "investment assets are not also consumption goods" is "unrealistic." Further, Nagy and Obenberger (1994) find in a survey of individual investors that a primary reason for stock investment is "feelings for a firm's products and services," which is an emotional reason for attachment to a firm's stock. Keloharju *et al.* (2012), in empirically linking investors' preference for firms' stock with usage of the corresponding products, state that a "setup in which customer-investors regard stocks as consumption goods, not just as investments, seems to best explain [their] results." Beal, Goyen, and Philips (2005) argue that owning socially responsible investments leads to an increase in psychic well-being.[5] Hong and Kacperczyk (2009) mention that "there is clearly a societal norm against funding operations that promote human vice, and consequently many investors may not want themselves or others to support these companies by investing in their stocks."

What is the effect of non-wealth stock-holding motives on asset prices, incentives to acquire information, and informational efficiency? We address this question, and, in turn, derive results that accord with many stylized facts and yield several other untested implications. Our analysis considers an equilibrium with asymmetric information where stock ownership provides direct consumption benefits. In our model (based on Grossman & Stiglitz, 1980), there are four class of agents, the standard informed and uninformed traders, noise traders, and a class of utility-maximizing traders that value stock for intrinsic reasons, in addition to incorporating a traditional risk-reward tradeoff. We term the last class "*A* traders," for want of a better term. Such traders can be proxied by retail investors who are less sophisticated than neoclassical utility-maximizers, as well as professional money managers who cater to their investors' tastes. We consider multiple securities and also allow these agents to acquire information about stocks.

We characterize equilibria with information acquisition and derive cross-sectional implications for expected stock returns. We find that stocks that yield extreme utility (or disutility) tend to be, on average, the most informationally efficient and least volatile since they yield high certainty equivalents for *A* traders and thus stimulate entry and information collection by these agents. Further, expected market-to-book ratios (proxied by the ratio of the expected market price to the unconditional expected value of the stock) are higher for stocks that appeal to investors more strongly. This is because in equilibrium, the expected prices of these stocks positively deviate from expected fundamentals. Since it is these stocks that also generate lower expected returns, our model is consistent with the value effect documented by Fama and French (1992), among others. Going beyond just explaining the value effect, our approach predicts that specific proxies for direct utility from owning stock are related to market/book ratios and future returns. Thus, we argue that there will be a negative relation between such proxies (such as brand visibility or environmental responsibility) and average returns. Similarly, there will be a positive relation between a proxy for disutility (such as whether a stock is a "sin stock") and average returns. We predict that such relations will tend to be more pronounced for stocks which yield extreme direct utility and in which *A* traders face lower costs of information acquisition, for example, due to more stringent disclosure policies.[6]

Keloharju *et al.* (2012) and Frieder and Subrahmanyam (2005) show that individual investors are respectively attracted to stocks whose products they use and whose brands are familiar; in our setting these findings obtain because agents derive direct utility from holding stocks of companies with visible products and brands.[7] Also, we demonstrate that if investors derive disutility from holding "sin" stocks, such stocks earn positive abnormal returns, as in Hong and Kacperczyk (2009). Further, we predict that trading volume should be highest for stocks that yield the greatest levels of utility or disutility. We provide other untested implications that cross-sectionally relate proxies for direct utility in a stock to the type of clientele (institutional vs. retail) that holds that stock. We also show that, under reasonable conditions, mutual funds or ETFs that cater to investor tastes earn negative expected returns in equilibrium, thus rationalizing the subpar performance of mutual funds extensively documented since Jensen (1968).

We extend our model to a dynamic setting, where *A* traders make an entry decision based on past fundamentals. Psychological literature (e.g., Cialdini *et al.*, 1976), considers the penchant for individuals to associate with successful ventures and people, i.e., to "bask in reflected glory." We thus propose that *A* traders derive greater utility (disutility) from holding stock with good (bad) past fundamentals. We find that if fundamental-related signals are above a certain threshold, A traders enter[8] and prices overreact to these positive signals (and subsequently correct). Similarly, there is a crash and recovery following periods where fundamentals are below a lower threshold. We predict that such booms and busts will be higher in stocks held by relatively unsophisticated retail investors, since those agents are more likely to derive direct utility from past firm fundamentals.

Note that when agents receive direct utility from owing a stock, the expected return to these investors from holding the stock can be less than zero. Our model accords with the notion that some financial market agents lose money on average by trading stocks; see, for example, Barber and Odean (2001) or Odean (1999). This feature is common to other models of financial markets, e.g., Kyle (1985) and its various extensions, where it is standard to assume that some agents earn negative expected profits. We note, however, that our model predicts negative expected profits only in stocks where agents derive extreme utility or disutility from stock ownership; in the intermediate cases, profits can be positive. We also point out that valuing stock as a consumption good is not the same as trading on irrational (or mistaken) beliefs. In our framework, agents who derive such direct utility calculate all exogenous parameters correctly and have rational expectations, but just maximize a different objective relative to neoclassical agents who trade for traditional wealth-related reasons.

Our work is related to Barberis and Shleifer (2003) on style investing. In their setting, investors categorize assets into different styles and move money across these styles depending on past performance.[9] We complement this approach via modeling investors who value stock because of the attributes of the company or product, rather than a broad style. Dorn and Sengmueller (2009) consider the notion that investors derive entertainment from trading equities. In contrast, we examine a framework where agents derive direct utility from holding specific stocks, as opposed to deriving utility from trading stocks generically.[10] Fama and French (2007) suggest that variation in tastes for different stocks can cause agents to hold a different portfolio than the standard tangency portfolio in a mean-variance setting. Since their focus is on portfolio choice, they desist from endogenizing the effect of differing tastes on the equilibrium. We instead focus on a closed-form equilibrium to consider the informational efficiency of prices, the cross section of holdings, value effects, volume premia, and equilibrium expected profits when A traders are present in the market, and also provide untested implications.[11]

A relevant issue in our model is whether our A traders present an arbitrage opportunity. On this point, we note that our model does allow for agents who are free to arbitrage pricing discrepancies. Nevertheless, such discrepancies remain because arbitrage is risky, which bounds positions. Another issue is whether A traders form a sufficiently significant mass to affect prices in reality. In this regard, Barber, Odean, and Zhu (2009) and Kumar and Lee (2006) both provide evidence that retail investors (who are more likely to be unsophisticated A traders) do move markets. Further, there has been pressure on several public pension funds to effectively become A traders by pressures to avoid "sin" stocks and predilection toward environmentally conscious stocks,[12] and it is well-known (see., e.g., Gompers & Metrick, 2001) that large institutions do have an effect on prices.

This paper is organized as follows. Section 2 presents the basic one-security model with symmetric information. Section 3 presents the multi-asset setup with information acquisition. Section 4 endogenizes participation by A traders in the stock market. Section 5 presents a dynamic extension. Section 6 concludes. All proofs of propositions and corollaries, unless otherwise stated, appear in Appendix A.

2 | A SIMPLE MODEL

We first consider a simple one-security model with symmetric information that conveys some basic intuition, before moving on to multiple securities and allowing for information acquisition. A single stock is traded at Date 0. At Date 1, the stock pays off a liquidation dividend

$$V = \bar{V} + \theta + \epsilon.$$

$\bar{V}$ is a positive constant, which represents the expected dividend. The variables θ and ϵ represent exogenous technology shocks; ϵ is not revealed until Date 1, but θ can be observed by investors at Date 0. These variables have zero mean and are mutually independent and normally distributed. Throughout the paper, we denote the variance of any generic random variable, η, by ν_η.

A mass m of agents, where $m \in [0, 1]$ is a positive constant, have a standard exponential utility function. Specifically, for the i'th agent,

$$U(W_{i1}) = -\exp(-\gamma W_{i1}),$$

where W_{i1} is his wealth at Date 1 and γ is a positive constant representing his absolute risk aversion coefficient. We also assume there is a mass $1 - m$ of agents we call "*A* traders;" these obtain direct consumption benefits or costs from holding the stock. The *i*'th *A* trader has the utility function:

$$U_A(W_{i1}, C_i^X) = -\exp(-\gamma W_{i1} - C_i^X),$$

where C_i^X is the *extra* utility of holding stocks derived from non-wealth related considerations.

Where does direct utility or disutility of holding stock emanate from? We propose the affective bias or heuristic, wherein the benefit from something that evokes a positive feeling is perceived to be high (Finucane *et al.*, 2000) and vice versa. This implies a positive perceived benefit in brand name companies or companies committed to socially desirable causes.[13] Indeed, Schoenbachler, Gordon, and Aurand (2004) show that agents who are loyal to certain brands also tend to invest in the stock of the firm that manufactures the brand. Frieder and Subrahmanyam (2005) also show that brand perceptions influence individual stockholdings. In addition, investment in socially responsible companies is often termed ethically desirable and justifiable beyond risk/reward considerations (e.g., Sparkes, 2008). Negative affect is created by companies that run counter to societal causes such as environmental consciousness. For example, there has been a recent impetus to divest from firms that use fossil fuels, and invest in firms that develop renewable energy sources.[14] Further, some societal norms discourage investment in "sin" stocks such as those of firms which manufacture tobacco products or run casinos (Geczy, Stambaugh, & Levin, 2005). In terms of real-world proxies for trader type, generally, we expect *A* traders to be retail investors. Professional money managers are unlikely to form non-wealth-related attachments to stocks and therefore will tend to be non-*A* traders, unless they cater to their clients' tastes. When we relate our results to empirical work, we will primarily rely on the preceding arguments.

We let C_i^X take a simple linear form in X_i, the quantity of the stock held after trading is complete, i.e., $C_i^X = AX_i$. Thus, the *i*'th *A* trader's utility function can be expressed as:

$$U_A(W_{i1}, X_i) = -\exp(-\gamma W_{i1} - AX_i).$$

If *A* is positive (negative) then the agent gets extra utility (disutility) from holding a long (short) position but disutility from a short position. We allow investors to freely short-sell. Imposing short-selling constraints causes a loss of tractability in our exponential-normal framework, but promises no additional insights.[15]

The *i*'th trader is endowed with $\bar{W}_{i0}$ units of the risk-free asset. As is standard, we also assume that there is an exogenous shock that influences participation of unmodeled "noise traders" in the financial market, and, in turn, affects the supply of shares of the stock available to investors that we model. We represent this additional per capita supply by z, which is normally distributed with mean zero and is independent of all other random variables. We normalize the mean supply of the stock to zero. This is for notational simplicity, but also implies that the unconditional risk premium is zero, which allows us to cleanly identify the effect of *A* traders on security prices. Let the price and return of the risk free asset be 1.

Proposition 1. *In equilibrium, the price of the stock is given by*

$$P = \bar{V} + \alpha + \theta - \gamma \nu_\epsilon z,$$

where $\alpha = (1 - m)A/\gamma$.

As can be seen from Proposition 1, the term α arises due to *A* traders. This term can be positive or negative depending on the sign of *A*.

Write the stock return as

$$V - P = \epsilon - \alpha + \gamma \nu_\epsilon z.$$

It follows that the expected return and the return volatility can be expressed as[16]

$$E(V - P) = -\alpha, \text{ and } \mathrm{Var}(V - P) = \nu_\epsilon + (\gamma \nu_\epsilon)^2 \nu_z.$$

There are two notable observations. First, given $A > 0$, α is higher (the expected return $E(V - P)$ is lower), the greater is the mass of *A* traders. Intuitively, a greater mass of *A* traders (higher $1 - m$) enhances their impact on stock prices.

Second, the return volatility does not depend on A. We show in Sections 3 and 4.1 that with asymmetric information, these results are not necessarily true.

It is instructive to examine how A influences the equilibrium amount of stocks held by various agents. Let X_{NA} and X_A denote the demands of non-A and A traders in the stock. The proof of Proposition 1 in Appendix A yields the following expressions:

$$X_{NA}(P,\theta)=\frac{-\alpha+\gamma\nu_\epsilon z}{\gamma\nu_\epsilon},\ \text{and}\ X_A(P,\theta)=\frac{A/\gamma-\alpha+\gamma\nu_\epsilon z}{\gamma\nu_\epsilon}.$$

Taking unconditional expectations yields

$$E[X_{NA}(P,\theta)]\propto-\alpha\propto-A/\gamma,\ \text{and}\ E[X_A(P,\theta)]\propto A/\gamma-\alpha\propto A/\gamma,$$

where the $\propto$ follows from the expression of α in Proposition 1. These expressions demonstrate that the fraction A/γ represents the extra demand for the stock created by the A traders. A positive (negative) A increases (decreases) stock holdings relative to the standard utility-of-wealth setting. Note that the deviation from the standard demand of an exponential utility agent is decreasing in γ, the risk aversion coefficient. As risk aversion becomes large, the cost of deviating from the standard position rises, which shrinks the extra demand due to direct utility from owning stock. An immediate observation is that for $A>0$, non-A traders tend to short or hold less of the stock, whereas A traders tend to be heavily long in the stock (a reverse argument applies for $A<0$).

3 | MULTIPLE ASSETS AND ASYMMETRIC INFORMATION

We now extend the simple setting of the previous section to add asymmetric information and multiple securities. This allows us to obtain cross-sectional implications and to explore how incentives to collect information and thus informational efficiency are affected by A trading.

Consider a simple model Grossman and Stiglitz (1980)-type model. There are J stocks traded at Date 0. At Date 1, the j'th stock pays off a liquidation dividend

$$V_j=\bar{V}_j+\theta_j+\epsilon_j.$$

$\bar{V}_j$ is a positive constant, which represents the expected dividend. The variable ϵ_j is not revealed until Date 1, but θ_j can be observed by informed investors at Date 0. These variables have zero mean and are mutually independent and normally distributed. We assume independent stock payoffs for simplicity.[17]

As before, there is a mass m of non-A traders and a mass $1-m$ of A traders. Within the former class, a mass $m\lambda_j$ observe θ_j prior to trading, and a mass $m(1-\lambda_j)$ of uninformed agents do not. $m\in[0,1]$ is a positive constant, which represents the sum of informed and uninformed masses. $\lambda_j\in[0,1]$ is an endogenous parameter to be determined. The utility function of the i'th non-A trader is again:

$$U(W_{i1})=-\exp(-\gamma W_{i1}),$$

where W_{i1} is his wealth at Date 1 and γ is a positive constant representing his absolute risk aversion coefficient.

Further, there is a mass of $(1-m)\lambda_{Aj}$ of informed A traders and $(1-m)(1-\lambda_{Aj})$ of uninformed A traders. Again, $\lambda_{Aj}\in[0,1]$ is an endogenous parameter to be determined. The i'th A trader has the utility function:

$$U_A(W_{i1},C_i^X)=-\exp(-\gamma W_{i1}-C_i^X).$$

Again, we let C_i^X take a simple linear form in X_{ij}, the quantity of the j'th stock held after trading is complete, i.e., $C_i^X=\sum_{j=1}^J(A_jX_{ij})$. Thus, the A trader's utility function can be expressed as follows:

$$U_A(W_{i1},X_{i1},\ldots,X_{iJ})=-\exp\left[-\gamma W_{i1}-\sum_{j=1}^J(A_jX_{ij})\right].$$

The costs for the non-A and A traders to learn the realization of θ_j are denoted by c_j and c_{Aj}, respectively. The i'th trader is endowed with $\bar{W}_{i0}$ units of the risk-free asset. The noisy per capita supply in stock j is denoted by z_j, which is normally distributed with mean zero and is independent of all other random variables.

Let the price and return of the risk free asset be 1. We assume standard rational expectations, i.e., that an uninformed *A* or non-*A* trader conditions on the stock price to form his demand. We then have the following result:

Proposition 2. *In equilibrium, the price of stock* j *takes the following form:*

$$P_j = \bar{V}_j + \alpha_j + \beta_j \omega_j(\theta_j, z_j),$$

where $\omega_j(\theta_j, z_j)$ or simply $\omega_j = \theta_j - \delta_j z_j$ has a variance $\nu_{\omega_j} = \nu_{\theta_j} + \delta_j^2 \nu_{z_j}$. The parameters δ_j, α_j, *and* β_j *are positive constants.*

It is shown in Appendix A that the sign of α_j depends wholly on the sign of A_j. Thus, in the cross section, stocks in which *A* traders receive disutility from direct ownership (i.e., $A_j < 0$) will earn higher average returns relative to those with no *A* trading, and vice versa.

The equilibrium with endogenous information acquisition requires equating the expected utility from acquiring information net of acquisition cost to the expected utility from not acquiring information. This comparison needs to be done for both the *A* and non-*A* traders. The following lemma describes the ensuing equilibrium conditions.

Lemma 1.

(i) If $\phi_j \equiv \exp(2\gamma c_j) \times \frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)} - 1$ is negative (positive), then the non-*A* trader prefers to become informed by spending c_j (remain uninformed). If $\phi_j = 0$, then he is indifferent between becoming informed and remaining uninformed.

(ii) If $\phi_{Aj} \equiv \exp(2\gamma c_{Aj}) \times \frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)} - 1$ is negative (positive), then the *A* trader prefers to become informed by spending c_{Aj} (remain uninformed). If $\phi_{Aj} = 0$, then he is indifferent between becoming informed and remaining uninformed.

The full characterization of equilibria where the costs of information acquisition c_j and c_{Aj} vary arbitrarily becomes quite complex. We therefore impose a restriction on these costs. Given the premise in Section 2 that *A* traders tend to be less sophisticated than non-*A* traders, we let $c_{Aj} > c_j$, so that it is more expensive for *A* traders to acquire information than for regular traders. Define $\Delta_j \equiv \frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}[1 - \frac{\nu_{\theta_j}}{\nu_{\theta_j} + (\gamma \nu_{\epsilon_j}/m)^2 \nu_{z_j}}] + 1$ and $\Gamma_j \equiv \frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}[1 - \frac{\nu_{\theta_j}}{\nu_{\theta_j} + (\gamma \nu_{\epsilon_j})^2 \nu_{z_j}}] + 1$. The theorem below then describes the equilibrium.

Proposition 3. *When* $c_{Aj} > c_j$, *the equilibrium with endogenous information acquisition for stock* j *varies across five ranges for parameter values as follows:*

- Range 1: *If* $\exp(2\gamma c_j) \geq \Delta_j$, *then no agent is informed, i.e.,* $\lambda_j = 0$ *and* $\lambda_{Aj} = 0$.
- Range 2: *If* $\Gamma_j < \exp(2\gamma c_j) < \frac{\nu_{\theta_j}}{\nu_{\epsilon_j}} + 1$, *then* $\lambda_j \in (0, 1)$ *is interior, and* $\lambda_{Aj} = 0$.
- Range 3: *If* $\exp(2\gamma c_j) \leq \Gamma_j \leq \exp(2\gamma c_{Aj})$, *then* $\lambda_j = 1$ *and* $\lambda_{Aj} = 0$.
- Range 4: *If* $\Gamma_j < \exp(2\gamma c_{Aj}) < \Delta_j$, *then* $\lambda_j = 1$ *and* $\lambda_{Aj} \in (0, 1)$ *is interior.*
- Range 5: *If* $\exp(2\gamma c_{Aj}) \leq \Gamma_j$, *then* $\lambda_j = 1$ *and* $\lambda_{Aj} = 1$.

Depending on the relative sizes of the costs of information acquisition, one can have equilibria where neither the *A* or the non-*A* traders collect information, only one class collects information, or both classes collect information. Proposition 3 can further be explained as follows. Because $c_{Aj} > c_j$, *A* traders find it more expensive to acquire information than non-*A* traders. Thus, only when non-*A* traders acquire information do *A* traders also do so. Further, when some or all of *A* traders find it profitable to acquire information, all non-*A* traders acquire information. The five-range scheme in Proposition 3 simply describes this hierarchical structure in detail.

Using the above results, we obtain the following comparative static on α_j, the extra term in the price which is due to *A* trading.

Corollary 1. α_j *can be written as* $\alpha_j \equiv \kappa_j A_j/\gamma$, *where* $\kappa_j \in [0, 1]$.

(i) κ_j *does not depend on* A_j.

(ii) *In Ranges 1–2 and 4–5 specified in Proposition 3,* κ_j *decreases in* m.

(iii) *In Range 3 specified in Proposition 3,*

(a). *suppose* $m \geq 1/2$ *or* $\frac{\nu_{\epsilon_j}}{\nu_{\theta_j}} \geq \frac{(1-2m)^2}{8(1-m)}$. *Then,* κ_j *also decreases in* m.

(b). *Suppose instead that* $m < 1/2$ *and* $\frac{\nu_{\epsilon_j}}{\nu_{\theta_j}} < \frac{(1-2m)^2}{8(1-m)}$. *Then, there exist two positive quantities* $\underline{\nu}_{z_j}$ *and* $\bar{\nu}_{z_j}$ *such that*

(1). κ_j *increases in* m *if* $\nu_{z_j} \in (\underline{\nu}_{z_j}, \bar{\nu}_{z_j})$ *and*

(2). *decreases in* m *if* $\nu_{z_j} \notin (\underline{\nu}_{z_j}, \bar{\nu}_{z_j})$.

(iv) κ_j *decreases in* c_{Aj} *within Range 4, and does not depend on* c_{Aj} *within the other ranges.*

Note that α_j measures the unconditional deviation of stock prices from their fundamental values, and arises due to *A* traders. If $A_j = 0$, then $\alpha_j = 0$. If $m = 1$ so that the mass of *A* traders is zero, then $\kappa_j = \alpha_j = 0$.

Given that α_j arises solely due to *A* traders, it might seem intuitive that α should be higher, the greater is the mass of *A* traders (as in Section 2). This, however, is not the case. The reason is that there are two opposing effects on α_j as m increases. First, a smaller mass of *A* traders (higher m) directly reduces their impact on stock prices. Second, a smaller mass of *A* traders, which implies more non-*A* traders (who can spend less to acquire information), can increase how much information is revealed by the price. This reduces the conditional risk borne by *A* traders and causes them to trade more aggressively, which can increase the impact of *A* traders on stock prices. In Corollary 1, we provide conditions under which one or the other effects dominate. Note that when $\lambda_j = \lambda_{Aj} = 0$, as in Range 1, changing m does not change the mass of informed *A* or non-*A* traders, so the second effect does not operate. In addition, in Ranges 2 and 4, a change in m changes λ_j and λ_{Aj} in such a way that the second effect is negated. In Range 5, all agents are fully informed, so again, the second effect does not operate. This means the ambiguity in the sign of $d\kappa_j/dm$ arises only in Range 3.[18]

Part (iv) of Corollary 1 indicates that in Range 4, for a higher c_{Aj}, the sensitivity of α_j to A_j is lower (lower κ_j). There are two reasons for this. First, as *A* traders find it more expensive to acquire information (a higher c_{Aj}), there will be more uninformed *A* traders, who trade less aggressively than the informed. Second, the stock price becomes less informative, which makes the uninformed *A* traders even more conservative in their trading. Taken together, *A* trading has a smaller effect on the stock price.

The term δ_j in $\omega_j = \theta_j - \delta_j z_j$ is related to the informativeness of stock prices about θ_j. Thus, it is worth considering the behavior of δ_j across the ranges in Proposition 3. If δ_j is very small (large), then the stock price is very informative (uninformative). It is easy to show that δ_j weakly decreases in m with the comparative static holding strongly only in Range 3. The level of δ_j is related to the mass of informed traders, $m\lambda_j + (1-m)\lambda_{Aj}$, which may or may not be related to m depending on the ranges of parameter values that are specified in Proposition 3. In both Ranges 1 and 5, δ_j does not depend on m. The reason for this is that in these ranges, no trader and all traders, respectively, choose to become informed and so the masses of informed (trivially) do not depend on m. The coefficient δ_j does not depend on m in Ranges 2 and 4 either. The reason for this is that in both ranges, the mass of informed, $m\lambda_j + (1-m)\lambda_{Aj}$, is interior. For example, in Range 2, $\lambda_{Aj} = 0$ and an increase in m is accompanied by a decrease in λ_j. The mass of informed, $m\lambda_j$, does not depend on m. In Range 3, the mass of informed is m because $\lambda_j = 1$ and $\lambda_{Aj} = 0$. Therefore, δ_j decreases in m.

To further illustrate the results above, consider two extreme cases of m. In the first case, $m = 1$, so there are no *A* traders in the economy. In the second case, $m = 0$, so there are only *A* traders in the economy. As $c_j < c_{Aj}$, λ_j in the basic non-*A* economy is greater than λ_{Aj} in the only-*A* economy. Therefore, prices in the economy without *A* traders are more informative. Suppose $c_j = c_{Aj}$. Then, λ_j in the non-*A* economy equals λ_{Aj} in the only-*A* economy. Thus, prices in the two economies are equally informative.

3.1 | Cross-sectional variation in equilibrium holdings

It is instructive to examine how *A* influences the equilibrium amount of stocks held by various agents. Let X_{Ij} and X_{Uj} denote the demands of informed and uninformed non-*A* traders in the j'th stock, and let the additional subscript *A* denote the corresponding quantities for *A* traders. Further, define $\Psi_j \equiv \nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j}$. The proof of Proposition 2 in Appendix A yields the following results:

$$X_{Ij} = \frac{\theta_j - \alpha_j - \beta_j\omega_j}{\gamma\nu_{\epsilon_j}}, \quad X_{Uj} = \frac{\frac{\nu_{\theta_j}}{\nu_{\omega_j}}\omega_j - \alpha_j - \beta_j\omega_j}{\gamma\Psi_j},$$

with $X_{AIj} = [A_j/(\gamma^2 \nu_{\epsilon_j})] + X_{Ij}$ and $X_{AUj} = [A_j/(\gamma^2 \Psi_j)] + X_{Uj}$. Taking expectations, it follows that

$$E\Big[X_{Ij}(P_j, \theta_j)\Big] \propto -\alpha_j \propto -A_j/\gamma, \quad E\Big[X_{Uj}(P_j, \omega_j)\Big] \propto -\alpha_j \propto -A_j/\gamma,$$

$$E\Big[X_{AIj}(P_j, \theta_j)\Big] \propto A_j/\gamma - \alpha_j \propto A_j/\gamma, \text{ and } E\Big[X_{AUj}(P_j, \omega_j)\Big] \propto A_j/\gamma - \alpha_j \propto A_j/\gamma,$$

where the $\propto$ follows from the expression of α_j in the proof of Proposition 2 (see Equation (14) in Appendix A).

There are three observations. First, suppose that A_j is positive (for negative A_j, the ensuing discussion has a convenient reverse interpretation). We see that non-A traders tend to short or hold less of the stock, whereas A traders tend to be long in the stock. Note that if α_j is large enough, informed agents may short the stock even if their private information signal θ_j is positive. Thus, the effect of A traders causes a tendency for the direction of informed trading to oppose the direction of the informed signal if both A_j and the informed signal θ_j are of the same sign. Third, among A traders, informed traders long the stock more aggressively than uninformed traders.

3.2 | The cross section of expected prices and returns

We now discuss cross-sectional implications for prices and expected returns. It follows from Proposition 2 that the expected stock price and the return can be expressed as

$$E(P_j) = \bar{V}_j + \alpha_j, \text{ and } E(V_j - P_j) = -\alpha_j.$$

The following corollary then obtains in a straightforward manner.

Corollary 2.

(i) The average price, $E(P_j)$, varies positively with A_j in the cross section. It is higher (lower) than the fundamental level, $\bar{V}_j$, if A_j is positive (negative).
(ii) The expected return, $E(V_j - P_j)$, varies negatively with A_j in the cross-section. It is negative (positive) if A_j is positive (negative).
(iii) Under the conditions in Parts (ii), (iii) (a), and (iii) (b) (2) of Corollary 1, the sensitivities of $E(P_j)$ and $E(V_j - P_j)$ to A_j decrease in m.
(iv) The sensitivities of $E(P_j)$ and $E(V_j - P_j)$ to A_j weakly decrease in c_{Aj}.

Corollary 2 implies that in the cross section, high A_j stocks will command low expected returns and vice versa. Particularly, stocks with $A_j > 0 (< 0)$ earn negative (positive) returns. Therefore, consistent with intuition, stocks are overvalued if agents desire them as consumption goods and vice versa.

Further, this effect (i.e., high A_j stocks will command low expected returns and vice versa) tends to be more pronounced when A traders find it easy to acquire information (low c_{Aj}).

3.3 | Expected profits of A traders

We now consider whether A traders, on average, make or lose money via their trading activity. We calculate these profits gross of the costs of information acquisition costs; net of costs, of course, the expected profits would be lower. Henceforth, "expected profit" denotes the gross expected profit from trading activity alone.

Straightforward calculations yield

$$E(\Pi_{AIj}) = \frac{(A_j/\gamma - \alpha_j)(-\alpha_j) + \text{Var}(\theta_j - P_j)}{\gamma \nu_{\epsilon_j}}. \tag{1}$$

It follows from Proposition 2 that the first term above, $(A_j/\gamma - \alpha_j)(-\alpha_j)$, is negative and proportional to $(A_j/\gamma)^2$. The second term is positive. Thus, if the absolute value of A_j is sufficiently high, informed A traders earn negative expected profits in stock j. We also have that

$$E(\Pi_{AUj}) = \frac{(A_j/\gamma - \alpha_j)(-\alpha_j) + \text{Var}(\theta_j - P_j) - \text{Var}(\theta_j - \frac{\nu_{\theta_j}}{\nu_{\omega_j}}\omega_j)}{\gamma(\nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})}, \tag{2}$$

Thus, if $|A_j|$ is high, then the uninformed A traders also lose money on average.

Comparing the numerators of Equations (2) and (3) suggests that the uninformed A trader can lose more money relative to the informed. However, comparing the denominators suggests that the uninformed will lose less relative to the informed. Possessing private information tends to reduce the losses of A traders. But the fact that the uninformed traders bear more risk than the informed, and thus tend to take less aggressive positions, tends to temper their losses. So, on net, whether the uninformed A traders incur greater expected losses than the informed ones, is ambiguous, and depends on parameter values.

Taken together, however, our results indicate that if $|A_j|$ is high then A traders' overall expected profits in stock j are negative. Thus, our model is able to explain investor losses on average, viz. Barber and Odean (2001), even as these investors maximize expected utility. It may be argued that losing money on average because one obtains direct utility from owning stock is not necessarily "irrational," since stock viewed in this way is merely a consumption good such as a DVD collection, or antiques, or any of a large number of non-monetary possessions.

3.4 | Trading volume

We now examine trading volume within our model. The expected trading volume is given by the sums of the expected absolute changes in each type of agent's position via trading in the market for stock j. Note that in our setting, the initial endowment of shares is normalized to zero. We can express the total expected trading volume in stock j as

$$\begin{aligned} T_j \equiv 0.5E\Big[& m\lambda_j \times |X_{Ij}(P_j,\theta_j)| + m(1-\lambda_j) \times |X_{Uj}(P_j,\omega_j)| \\ &+ (1-m)\lambda_{Aj} \times |X_{AIj}(P_j,\theta_j)| + (1-m)(1-\lambda_{Aj}) \times |X_{AUj}(P_j,\omega_j)|\Big]. \end{aligned} \tag{3}$$

We then have the following result.

Corollary 3. *The expected trading volume,* T_j, *increases in* $|A_j|$.

Thus, stocks in which agents have a greater level of utility or disutility from ownership exhibit greater trading volume. Investment in such stocks generates greater deviation from the initial endowment (which, in our setting, is normalized to zero for convenience), generating higher trading volume.

3.5 | Empirical implications

We now consider the empirical implications of our model. To do so, it is useful to first identify what investor types are likely to be A traders in our model, and what attributes might be related to their A_j. As we pointed out earlier in Section 2, we expect A traders to be retail investors and money managers that cater to their clients' tastes. (We model the latter possibility in Section 4.2.) Further, we expect that companies that appeal to societal causes such as environmental consciousness (e.g., Tesla, and firms that develop renewable energy sources), companies with a high corporate social responsibility (CSR) score,[19] and companies with high brand appeal (e.g., Apple and Google) have a positive A_j. In contrast, companies that do not appeal to social norms, for example, "sin" stocks such as those of firms which manufacture tobacco products or run casinos (Geczy *et al.*, 2005) have a negative A_j. We now use the above observations to derive several implications. For each of these, the preceding material in parentheses indicates specific sections or results that yield the predictions.

3.5.1 | Investing clienteles

Our analysis in Section 3.1 implies that positive A_j companies will be held more aggressively by A traders. Thus, we obtain the following prediction:

Prediction 1. (Section 3.1) *Companies that appeal to societal causes such as environmental consciousness, companies with a high CSR score, and companies with high brand appeal are held more heavily by retail investors.*

This prediction accords with Schoenbachler *et al.* (2004) who show that agents who are loyal to certain brands also tend to invest in the stock of the firm that manufactures the brand, and Frieder and Subrahmanyam (2005) who demonstrate that individual investors are more heavily represented in brand name stocks.

We also obtain the following untested prediction:

Prediction 2. (Section 3.1) *Companies that do not appeal to social norms such as "sin" stocks are held relatively less heavily by retail investors.*

3.5.2 | The cross section of stock returns

Defining the expected "market-to-book" ratio to be $E(P_j)/\bar{V}_j$, Corollary 2(i) indicates that this ratio is increasing in A_j. Thus, $A_j > 0$ increases the tendency for a stock to be categorized as a "glamour" stock. Conversely, a stock becomes a "value" stock when agents derive low utility (or disutility) from holding the stock (i.e., A_j is low or negative). Corollary 2 also implies that in the cross section, high A_j stocks will command low expected returns and vice versa. We then have the following predictions:

Prediction 3. (Corollary 2) *Companies that appeal to societal causes such as environmental consciousness, companies with a high CSR score, and companies with high brand appeal have a higher market/book ratio and a lower return.*

Prediction 4. (Corollary 2) *Companies that do not appeal to social norms such as "sin" stocks have a lower market/book ratio and a higher return.*

The above predictions are consistent with the value premium (see, e.g., Fama & French, 1992). Note that in our setting, market-to-book proxies for how much utility agents receive from direct stock ownership, A_j, which distinguishes our framework from others.

Prediction 3 also accords with the market valuation of Tesla exceeding that of Ford Motors on sales volume that is 1% of that of Ford (viz. Footnote 1) because of Tesla's appeal to environmental consciousness, and the overvaluation of companies that are "in fashion," such as dot-com companies in the late 1990s (viz. Ofek & Richardson, 2003). This prediction further implies that the underperformance of IPOs documented, for example, in Loughran and Ritter (1995) is likely to be particularly pronounced for IPOs with strong brand recognition, such as Twitter and Facebook. Prediction 3 also accords with Billett *et al.* (2014), which demonstrates that brand prestige of a firm's products is associated with lower average returns in the cross section. One possible strategy for increasing brand awareness by firms is to enhance product advertising. Lou (2014) and Chemmanur and Yan (2009) present evidence that increases in expenditures on advertisements are associated with lower future returns in the cross section. Prediction 4 accords with the evidence that "sin" stocks earn positive returns, viz. Hong and Kacperczyk (2009).

It may be argued that an inverse empirical proxies for c_{Aj}, the cost of information acquisition incurred by A-traders, is the informativeness of the firm's disclosures.[20] From Corollary 2(iv), we then have the following untested prediction:

Prediction 5. (Corollary 2(iv)) *Predictions 3 and 4 will tend to be more pronounced for stocks of firms with more stringent disclosure policies.*

3.5.3 | Trading volume and the cross section of stock returns

Corollary 3 implies that the cross-sectional variation in volume will be positively linked to proxies for $|A_j|$. Thus, we obtain the following prediction:

Prediction 6. (Corollary 3) *Stocks that yield extreme values of direct utility or disutility have the highest trading volume in the cross section.*

The above proposition indicates that stocks with extremely high brand appeal, for example, will be accompanied by high values of A_j and thus command high volume.

Note that our model does not directly link expected returns to volume since expected returns depend on the signed A_j (see Corollary 2), whereas Corollary 3 involves the absolute value of A_j. However, the following prediction obtains:

Prediction 7. (Corollaries 2 and 3) *If positive* A_j *stocks dominate in the cross section, there is a negative relation between trading volume and average returns, and vice versa.*

The empirical evidence indicates that positive A_j stocks are more likely than negative A_j ones (indeed, Hong and Kacperczyk (2009) indicate that "sin" stocks form less than 1% of total market capitalization). Recognizing that positive A_j stocks prevail in the cross section, the above prediction is consistent with the negative relation between trading volume and average returns documented, for example, in Datar, Naik, and Radcliffe (1998) and Brennan, Chordia, and Subrahmanyam (1998).[21]

Baker and Stein (2004) provide an alternative explanation for the volume premium that is based on sentiment. They argue that high volume implies high sentiment, and, under short-selling constraints, extreme optimism, that is reversed out the following month. We complement their argument by predicting that stocks with the highest direct utility (e.g., those that have high brand visibility) will have the highest volume and the greatest reversals. Further, we can use the same logic as that for Prediction 5 to obtain the following untested prediction:

Prediction 8. (Corollary 2(iv) and Prediction 7) *The relation between trading volume and average returns will tend to be particularly pronounced for stocks of firms with more stringent disclosure policies.*

4 | ENDOGENOUS PARTICIPATION BY *A* TRADERS

So far the mass of A traders has been taken as exogenous. However, A traders, who are likely to be less sophisticated than non-A traders, may face cognitive limitations or direct costs of information search that preclude them from participating in the stock market. Accordingly, we now endogenize A traders' participation by imposing an entry cost in addition to costs of information acquisition. This yields results on the impact of A traders on the informational efficiency of prices, and on mutual funds or ETFs that are set up to cater to investor tastes.

4.1 | Participation, information efficiency, and return volatility

We first consider a setting where A traders directly participate in the stock market (no mutual funds are allowed). In this framework, for the A traders, there are three stages in the determination of the equilibrium: A traders first decide whether to participate in a particular stock, then they choose whether to be informed, and subsequently their trade quantities. There is a mass m of the traditional (non-A) traders, which is fixed, but the proportion of these agents that are informed (λ_j) is endogenous. There is a mass $1 - m$ of *potential* A traders. A mass m_{1j} of those traders enter in stock j, and the proportion of these that are informed (λ_{Aj}) is endogenous. m_{1j} is endogenized via an entry cost, so that a mass $1 - m - m_{1j}$ of traders does not participate. Thus, suppose that A traders need to pay a cost c_{pj} to trade stock j.

The informativeness of stock prices can be measured as $[\text{corr}(\theta_j, P_j)]^2$, and the noisiness is the inverse of the informativeness. It can be shown that if m_{1j} is exogenously given, then the noisiness (or informativeness) does not depend on A_j. However, when m_{1j} is endogenous, we have the following theorem.

Proposition 4.

(i) m_{1j} increases in $|A_j|$.
(ii) The noisiness in stock prices weakly decreases in $|A_j|$.
(iii) This relation in (ii) is stronger (more negative) when c_{Aj} is high.

The intuition behind the theorem is that an extreme A_j implies that agents derive more expected utility from trading the stock, thus increasing the mass of A traders for a given cost of entry. After entry, as these A traders endogenously choose to acquire and trade based on private information, the informational efficiency of the stock price improves. The proof of the theorem shows that this second-stage effect, wherein an increase in the total mass of A traders also leads to an increase in the mass of informed A traders and thus improves informational efficiency, is strongest under parameter values such that all entering A traders choose to gather information. This is where the comparative static in Part (ii) holds strongly. The intuition for Part (iii) of the theorem is the following. When c_{Aj} is high, m_{1j} is low. In this case, a small increase in $|A_j|$, which leads to an increase in m_{1j}, has a big impact on price informativeness. When c_{Aj} is low, m_{1j} is already very high, and a small increase in m_{1j} in response to an increase in $|A_j|$ has little impact on the amount of information conveyed by the stock price.

We next describe return volatility in our setting:

Corollary 4. *Both the conditional variance (conditional on* P_j*) and the unconditional variance of the return,* $V_j - P_j$, *weakly decrease in* $|A_j|$.

The reasoning behind the corollary is simply that an extreme A_j improves the informational efficiency of the price via an increase in the participating mass of A traders, which tends to reduce return volatility conditional on stock prices. Further, since the greater mass of A traders that accompanies the more extreme A_j is better able to absorb the noise or liquidity trades, the unconditional return volatility also decreases. This result arises from asymmetric information and contrasts with the result in Section 2 that return volatility does depend on A. Thus, overall, we find that stocks in which agents derive higher utility or disutility from trading are more informationally efficient and less volatile, even though the mean prices of these stocks deviate unconditionally from expected values.

From the above analysis, we can obtain the following prediction:

Prediction 9. (Proposition 4) *Stocks that yield extreme values of direct utility or disutility,* ceteris paribus, *have greater levels of stock price informativeness.*

This implication can be tested using proxies for direct utility and c_{Aj} discussed in Section 3.5 and measures of price informativeness developed, for example, in Chen, Goldstein, and Jiang (2006).

4.2 | Mutual funds

In 2015, there were more than 9,000 mutual funds in the USA, and their number exceeds the number of actively traded equities.[22] In this subsection, we provide a rationale for why such funds may underperform (Jensen, 1968; Carhart, 1997) but still flourish. We explore the idea that mutual funds can be established that cater to investors' tastes. These funds can earn negative expected returns in equilibrium, if the taste preference to which they cater is sufficiently strong. We focus on mutual funds, but our ideas also apply to the plethora of ETFs in the U.S. scenario (see the introduction).

Again, consider the economy specified in Section 4.1. For simplicity, let $c_{Aj} > 0 \quad \forall j$ be sufficiently high; specifically, suppose that $\exp(2\gamma c_{Aj}) \geq \frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\left[1 - \frac{\nu_{\theta_j}}{\nu_{\theta_j} + (\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}\right] + 1$. Further, let $\forall j\ c_{pj} > 0$ be sufficiently low.[23] The equilibrium in this case is derived in the proof of Proposition 4. Particularly, because $\forall j\ c_{Aj}$ is sufficiently high, the entering A traders do not acquire information and remain uninformed, i.e., $\forall j\ \lambda_{Aj} = 0$. Because $\forall j\ c_{pj}$ is sufficiently low, the mass $1 - m$ of A traders enter and trade every stock, i.e., $\forall j\ m_{1j} = 1 - m$. The equilibrium pricing function for each stock j is given by Propositions 2 and 3.

Now add a mutual fund to the economy. The key aspect of the fund is that it does not face the participation costs to invest in the stock market. We also assume for simplicity that the mutual fund does not acquire information either because it finds it expensive or because it has no skill to do so. The fund stipulates in the investment mandate to invest in the top J_A stocks with the highest $A_j > 0$, indexed by $j = 1, \ldots, J_A$. If an A trader invests through the mutual fund in the J_A stocks, then the mutual fund charges him a fee F_A, trades on his behalf, and transfers the equilibrium return to him.

Because the fund acts as a simple pass-through, it does not change the nature of the pricing equilibrium. The highest fee it charges for each A trader, who invests through it, equals $F_A = \sum_{j=1}^{J_A} c_{pj} > 0$ (for a higher fee, an A trader will prefer to invest himself, instead of using the fund). It follows from Equation (2) that for each A trader, the fund delivers a profit

$$E(\Pi_A) = \sum_{j=1}^{J_A} E(\Pi_{AUj}) = \sum_{j=1}^{K} \frac{(A_j/\gamma - \alpha_j)(-\alpha_j) + \mathrm{Var}(\theta_j - P_j) - \mathrm{Var}(\theta_j - \frac{\nu_{\theta_j}}{\nu_{\omega_j}}\omega_j)}{\gamma(\nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})},$$

where only the item $(A_j/\gamma - \alpha_j)(-\alpha_j)$ is related to A_j and is linear in $-A_j^2$.

The above analysis implies that if $\exists A_j > 0$ that is sufficiently high, then the fund delivers a negative expected profit to the A trader. It is straightforward to extend this analysis to cases where funds can be set up to avoid stocks that confer disutility, or cater to subsets of investors. Also observe that since all the fund does is transfer the equilibrium return to the investor for a fixed fee, it can also be interpreted as a financial institution setting up an ETF.

Based on our discussion in Section 3.5 of the attributes related to A_j, we obtain the following prediction:

Prediction 10. (Section 4.2) *Funds that cater to tastes such as environmental consciousness and corporate social responsibility, and avoiding "sin" stocks, tend to underperform more than other funds.*

5 | BOOMS AND BUSTS, CRASHES AND RECOVERIES

We now consider a dynamic extension of our multi-asset setting where the entry of A traders is endogenous, and they derive greater utility (disutility) from holding stock with high (low) fundamentals. Such dependence can arise from the psychological tendency to derive satisfaction from being associated with the success of entities other than oneself (Cialdini & De Nicholas, 1989).[24] The dynamic extension is in the spirit of Brown and Jennings (1989) and Grundy and McNichols (1989). For convenience, we suppress the index for stock j, and analyze a generic stock in the ensuing analysis. We also assume that the information endowments of agents are exogenous; however, as will be clear, our intuition is not critically dependent on these assumptions.

The stock is traded at Dates 0, 1, 2, and 3. At Date 3, it pays off a liquidation dividend

$$V = \bar{V} + \theta_1 + \theta_2 + \epsilon.$$

$\bar{V}$ is a positive constant, which represents the expected dividend. The variables θ_1, θ_2, and ϵ represent exogenous technology shocks, which are mutually independent and multivariate normally distributed with mean zero. For convenience, we let θ_1 and θ_2 have the same variance ν_θ.

A continuous mass m of non-A traders is present in the market at all dates. The i'th non-A trader's utility function is the standard exponential:

$$U(W_{i3}) = -\exp(-\gamma W_{i3}),$$

where W_{i3} is his final wealth. He is endowed with $\bar{W}_{i0}$ units of the risk-free asset at Date 0. These traders are informed agents, in that they observe θ_1 at Date 1 and θ_2 at Date 2.

There is also a continuous mass $1 - m$ of A traders who may choose to enter the market at Date 2. The i'th entering A trader's utility function takes the form

$$U_A(W_{i3}, C_{i3}^X) = -\exp(-\gamma W_{i3} - C_{i3}^X),$$

where C_{i3}^X, the *extra* utility beyond traditional wealth considerations, is a simple linear function of X_{i2}, the quantity of stock he has bought at Date 2 and continues to hold until the end of the game; specifically, $C_{i3}^X = A(\theta_1)X_{i2}$. Thus, the utility function of the A trader can be expressed as:

$$U_A(W_{i3}, X_{i2}) = -\exp\Big[-\gamma W_{i3} - A(\theta_1)X_{i2}\Big].$$

The trader is endowed with $\bar{W}_{i2}$ units of the risk-free asset right before he enters at Date 2. Each such trader observes θ_1 at Date 2 following his entry decision. As motivated at the beginning of this section, the direct utility obtained by the A trader by investing in the stock is related to the previous fundamental performance of the stock. We let $A(\theta_1) = a\theta_1$, where a is a positive constant.

As is standard, we also assume that at Dates 1 and 2, there are unmodeled noise traders in the market who affect the supply of shares available at each of the dates. We represent these additional per capita supplies by z_1 and z_2, which are normally distributed with mean zero and common variance ν_z, and are independent of all other random variables. We let the additional per capital supply at Date 0 be zero.

Let the price of the stock at Date j=0,1, and 2 be P_j, and the price and return of the risk free asset be 1. The A traders choose whether to enter the stock market at Date 2 conditional on the date-1 stock price P_1. After entering at Date 2, they infer an estimate of θ_2 from the date-2 stock price P_2. Define $D \equiv m_1/[\nu_\theta(1 - \nu_\theta \nu_\omega^{-1})]$, and in turn, $\alpha \equiv Da\gamma^{-1}[m\nu_\epsilon^{-1} + D]^{-1}$, $\beta \equiv [m\nu_\epsilon^{-1} + D\nu_\theta\nu_\omega^{-1}][m\nu_\epsilon^{-1} + D]^{-1}$, and $\mu \equiv [\nu_\omega^{-1} + Dm_1^{-1}(\beta - \nu_\theta\nu_\omega^{-1})^2]^{-1}$. Finally, let $\tau(m_1) \equiv \gamma\mu\beta^2 m^{-1}[1 + \alpha\{1 - Dm_1^{-1}\mu\beta(\beta - \nu_\theta\nu_\omega^{-1})\}]$. Then, the following result obtains in this setting.

Proposition 5. *The equilibrium in the dynamic setting is characterized by the following:*

- *Given that a mass* m_1 *of* A *traders enter at Date 2, stock prices are given by*

$$
\begin{aligned}
P_0 &= \bar{V},\\
P_1 &= \bar{V} + \theta_1 + [1 - \mu\beta D m_1^{-1}(\beta - \nu_\theta\nu_\omega^{-1})]\alpha\theta_1 - \gamma\mu\beta^2 m^{-1} z_1,\\
P_2 &= \bar{V} + \theta_1(1 + \alpha) + \beta\omega(\theta_2, z_2),
\end{aligned}
$$

where $\omega(\theta_2, z_2) = \theta_2 - \delta z_2$, *with* $\delta = \gamma\nu_\varepsilon/m$.
- *There exists a function* $\xi(m_1, (\theta_1 - \tau(m_1)z_1)^2)$ *such that for sufficiently high* c_p *(which excludes the corner solution in which some* A *traders always enter), if* $\xi(0, (\theta_1 - \tau(0)z_1)^2) \geq 0$*, then no* A *traders will enter so* $m_1 = 0$*. If* $\xi(1 - m, (\theta_1 - \tau(1 - m)z_1)^2) \leq 0$*, then all* A *traders will enter so* $m_1 = 1 - m$*. If* $\xi(0, (\theta_1 - \tau(0)z_1)^2) < 0 < \xi(1 - m, (\theta_1 - \tau(1 - m)z_1)^2)$*, then an interior* m_1 *is given by* $\xi(m_1, (\theta_1 - \tau(m_1)z_1)^2) = 0$*.*

The function ξ in the above proposition is complex and is explicitly defined in Equation (26) of Appendix A. An interesting observation is that there can be a price "buildup and crash," (or a "crash and recovery") which is related to $\alpha(m_1)\theta_1$. This phenomenon occurs only when there is a large price movement at Date 1. To illustrate this, consider a positive $\theta_1 > 0$. Let θ_2, z_1, z_2, and ϵ equal their mean, i.e., zero. If θ_1 is sufficiently high, then all A traders enter ($m_1 = 1 - m$), and

$$
\begin{aligned}
P_1 &= \bar{V} + \theta_1 + \left[1 - \frac{\mu(1-m)\beta(1-m)(\beta(1-m) - \nu_\theta/\nu_\omega)}{\nu_\theta(1 - \nu_\theta/\nu_\omega) + \nu_\epsilon}\right]\alpha(1-m)\theta_1,\\
P_2 &= \bar{V} + \theta_1 + \alpha(1-m)\theta_1.
\end{aligned}
$$

but $V = \bar{V} + \theta_1$. The term in P_1, $\left[1 - \frac{\mu(1-m)\beta(1-m)(\beta(1-m) - \nu_\theta/\nu_\omega)}{\nu_\theta(1 - \nu_\theta/\nu_\omega) + \nu_\epsilon}\right]\alpha(1 - m)\theta_1$, arises because non-$A$ traders foresee that all the mass $1 - m$ of A traders enter at Date 2 and cause a deviation in the expected value of P_2 from fundamentals, $\alpha(1 - m)\theta_1$. This term appears when θ_1 surpasses a threshold. This suggests a boom in stock price at Date 1.

Note that non-A traders do not trade up to the scale to incorporate the whole $\alpha(1 - m)\theta_1$ in P_1 because of a hedging concern. This hedging demand can be seen in the derivation of X_{R1}, regular traders' demand for stock at Date 1, in the proof of Proposition 5 (see Equations (37) and (38)). This hedging concern leaves a space for a price buildup from Date 1 to Date 2,

$$
P_2 - P_1 = \frac{\mu(1-m)\beta(1-m)(\beta(1-m) - \nu_\theta/\nu_\omega)}{\nu_\theta(1 - \nu_\theta/\nu_\omega) + \nu_\epsilon}\alpha(1-m)\theta_1.
$$

As the entire mass $1 - m$ of A traders enter at Date 2, they push the stock price even further away from the fundamental level on average. Of course, from Date 2 to Date 3, the stock price drops from P_2 to V. The total drop equals $\alpha(1 - m)\theta_1$.

In Panel A of Figure 1, we plot a price path with the parameter values $\bar{V} = 10$, $\gamma = 0.2$, $a = 0.25$, $\nu_\theta = 1$, $\nu_z = 8$, $\nu_\varepsilon = 1.2$, $m = 0.5$, and $c_p = 2.5$. The realizations of θ_2, z_1, z_2, and ε, are assumed to be zero, i.e., their mean. We let $\theta_1 = 3.808$, which is sufficiently high to induce all the mass $1 - m$ of A traders to enter. We can see that A traders' entry causes a stock price boom and bust, in that the stock price jumps by about 50% across Dates 0 and 2 and drops by more than 10% across Dates 2 and 3.

Panel B of Figure 1 plots another price path. We let $\theta_1 = -3.808$, which is sufficiently low to induce the entire mass of $1 - m$ of A traders to enter. In this case, the entry of A traders causes a stock price crash and recovery. We can further show that if $\theta_1 \in [-3.189, 3.189]$ (with the other parameters remaining the same as in the figure), then no A traders will enter, and there is no boom and bust, or crash and recovery.

We can obtain the following prediction:

Prediction 11. (Section 5) *Stock prices experience reversals following large price moves.*

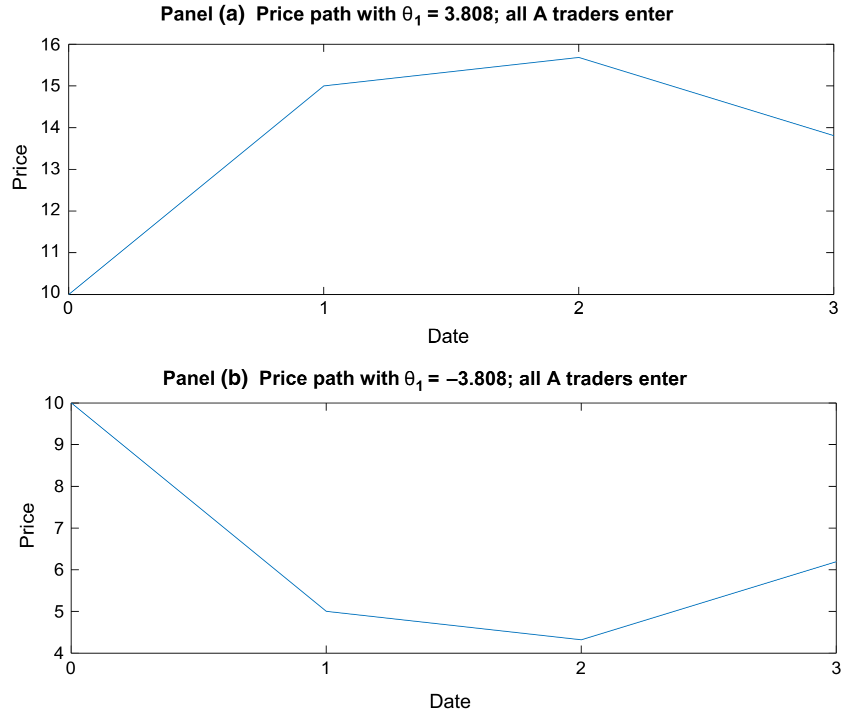

FIGURE 1 Booms and busts in asset prices [Colour figure can be viewed at wileyonlinelibrary.com]

Note: In Panel A of this figure, we plot a price path when the entire mass $1 - m$ of A traders enters by letting $\theta_1 = 3.808$. In Panel B, we assume that $\theta_1 = -3.808$, which again ensures that all A traders enter. We assume the parameter values $\bar{V} = 10$, $\gamma = 0.2$, $a = 0.25$, $\nu_\theta = 1$, $\nu_z = 8$, $\nu_\epsilon = 1.2$, $m = 0.5$, and $c_p = 2.5$. The realizations of θ_2, z_1, z_2, and ϵ, are assumed to be zero, i.e., their mean.

The crash and recovery in our setting is consistent with Cox and Peterson (1994), who document that large stock price drops are accompanied by reversals. Similarly, our analysis accords, for example, with Singleton (2013), who provides evidence of booms and busts in the oil industry. Finally, our analysis also explains phenomena like the "dot-com" boom and bust in asset prices (Brunnermeier & Nagel, 2004). The distinguishing feature of our theory is the triggering of extreme overreaction only when the fundamental-related signal crosses exogenous thresholds. Further, under the reasonable conjecture that utility-dependence on past stock fundamentals is more likely to arise for relatively unsophisticated retail investors, our analysis suggests that the phenomenon in Prediction 11 is more likely to obtain for stocks heavily held by such investors.

6 | CONCLUSION

How are asset prices and incentives to acquire information and thus informational efficiency affected when owning stock in a firm is a consumption good? To address this issue, we consider a setting where some agents (termed "A traders") derive direct utility or disutility from specific investments. Such benefits and costs, arise, for example, from brand appeal for a company's product, causes such as caring for the environment, and wanting to desist from investing in "sin" stocks. We embed agents whose holdings are affected by such consumption gain or loss in a standard model with asymmetric information.

We are able to solve for an analytic equilibrium in a framework where informed and uninformed A traders co-exist with traditional agents who maximize the utility of terminal wealth. We find that stocks that yield extreme direct utility are, on average, informationally more efficient, as they raise certainty equivalents for holding the stock. This allows for greater participation by A traders, which, in turn, tends to raise the total mass of informed traders in the market.

In our model, the expected price deviates from expected fundamentals even when assets are in zero net supply. The deviation increases in the level of utility derived from direct stock ownership. There is a value-glamour effect in the cross section; with the specific implication that stocks that yield high direct utility are glamour stocks that yield low future returns whereas stocks that yield low (or negative) direct utility are value stocks that earn high abnormal returns. This part of the analysis concurs with Billett *et al.* (2014), who demonstrate that equities of firms with brands that are perceived as prestigious have lower book/market multiples and earn lower average returns in the cross section. Our analysis further implies that the relation between direct utility and returns will tend to be particularly pronounced for firms in which A traders face lower costs of information acquisition such as those with more stringent disclosure policies. We also show that cross sectionally, stocks that yield extreme direct utility will have the highest trading volume. We provide additional implications that cross sectionally relate firms' products to the clientele that holds the firms' equities. Specifically, we propose that stocks of companies with familiar, brand-name products are more likely to be held by retail investors, and the reverse is true for "sin stocks."

We show that, under reasonable conditions, mutual funds or ETFs that cater to investors' tastes earn negative expected returns in equilibrium. A dynamic extension of our model demonstrates that booms and busts in asset prices arise if A traders receive direct utility from owning stocks with high (low) past performance, owing, for instance, to a need to associate with successful ventures (Cialdini *et al.*, 1976). The basic idea is that A traders enter if disclosures about fundamentals are above a certain threshold, which causes a non-linear reaction to signals, but only if the signals cross an exogenous threshold. Under the plausible assumption that retail investors are more likely to derive direct utility from past firm fundamentals, our analysis indicates that stock price reversals after large price moves are more likely to obtain in stocks heavily held by such investors.

Our analysis opens up new vistas for financial research. Specifically, the presence of A traders may substantially influence liquidity and volume in a non-competitive setting, unlike the competitive setting we consider, where price impacts are assumed to be zero. Further, we have not analyzed dynamic implications where agents may exit firms that cease to provide them utility (owing, for example, to a loss of brand equity via, say, a spinoff of a visible brand). Similarly, investors might enter if firms raise their brand awareness and prestige, or cease doing business in arenas that are often deemed socially undesirable, such as tobacco or casinos. Such entry and exits might have interesting implications for price moves greater than that warranted by fundamentals. These extensions are left for future research.

ENDNOTES

[1] "Tesla, on a hot streak, passes ford in investor value," *Wall Street Journal*, 4/4/2017.

[2] See, for example, Massa (1998), Fama and French (2010), and Jensen (1968).

[3] For example, the fund with ticker symbol CATH is a Catholic Values ETF, whereas EQLT represents a "Workplace Equality Fund" (see etfdb.-com). Many such examples exist.

[4] For example, see "The new tech-stock temptation," by Liam Pleven and Liz Moyer, *Wall Street Journal*, March 6, 2015; and "Do we fall in love with our investments?," by Graham Witcomb, available at https://intelligentinvestor.com.au/Do-we-fall-in-love-with-our- investments.

[5] While these authors do model psychic utility in conjunction with the traditional utility of wealth, they do not consider the market equilibrium that results in their setting. In a similar vein, Statman (2010) conjectures that "some socially responsible investors are willing to sacrifice investment profits for human rights."

[6] Our rationale for the value effect is complementary to the overconfidence-based argument in (Daniel, Hirshleifer, & Subrahmanyam, 2001), since our direct utility proxies that help explain price-scaled ratios and returns are not inherently related to overconfidence.

[7] Larkin (2013) shows how brand loyalty increases debt capacity by allowing for a steadier revenue stream and, in turn, lower cash flow uncertainty.

[8] Supporting the notion that good performance attracts A traders, Chordia, Huh, and Subrahmanyam (2007) show that net buying pressure is higher for stocks that have experienced increases in market prices (which proxy for past fundamentals).

[9] Peng and Xiong (2006) show that limited investor attention can lead to category-based learning behavior, causing asset-return comovement within sectors.

[10] A related paper, Luo and Subrahmanyam (2016), considers a setting where agents receive direct utility from trading, i.e., view the act of trading as a consumption good. In contrast, in the present paper, the utility emanates from the *signed* holding of a stock.

[11] Friedman and Heinle (2016) present a model related to ours where different agents value asset payoffs differently. Specifically, although there is no asymmetric information, some agents prefer corporations with a sense of corporate social responsibility. In this setting, there are incentives for firms to spin off divisions that appeal to different clienteles. In contrast, we consider information asymmetry, endogenize the entry of A traders, and also study dynamics of asset prices.

[12] See, for example, "CalPERS mulls ending tobacco investment ban," available at http://www.pionline.com/article/20160404/PRINT/304049985/calpers-mulls-ending-tobacco-investment-ban. Hong and Kacperczyk (2009) find that "norm-constrained" pension plans scale back investment in "sin" stocks.

[13] Our model accords with this bias if investors derive a benefit of A per share beyond the cash payout from a stock, or simply mistakenly believe that they will derive an additional nonstochastic cash flow beyond the actual payoff V. The latter construct works because of the exponential form of the utility function wherein an additional term A in the perceived payoff from the asset yields our formulation.

[14] See, for example, "World's biggest sovereign wealth fund dumps dozens of coal companies," *The Guardian*, 2/5/2015.

[15] Note that the final wealth also is a function of the position taken in risky assets. Our formulation assumes that A traders evaluate the utility of wealth and the utility of holding stock separately and multiplicatively. As we will see later, our functional form leads to an intuitively appealing expression for the optimal position taken by the agent.

[16] As is standard in exponential-normal settings, we measure expected returns via expected price changes (viz. Hong & Stein, 1999).

[17] We show in an Online Appendix that adding a linear factor structure to the payoffs does not change our main results.

[18] The analysis of Corollary 1 remains mostly the same even if A traders have to pay a very high c_{Aj} to acquire information, so they will never acquire information (i.e., $\lambda_{Aj} = 0$). In this case, there are only Ranges 1, 2, and 3 as given in Proposition 3.

[19] See, for example, Kim, Park and Wier (2012) for approaches to measuring companies' CSR scores.

[20] See, for example, Lang and Lundholm (1996) on how to measure the information content of disclosures by firms. The assumption here is that disclosure informativeness influences c_{Aj} rather than c_j, since institutions have access to management and social networks in finance (viz. Cohen, Frazzini, & Malloy, 2010; Frankel, Johnson, & Skinner, 1999) that retail investors (likely the A-traders) do not.

[21] Several studies find that the Amihud (2002) measure of liquidity, which is computed as the average ratio of volatility to volume over a month, is priced. In a recent paper, Lou and Shu (2014) show that the denominator (volume) plays the predominant role in this pricing, which accords with our result relating volume to future returns.

[22] See, for example, https://www.statista.com/topics/1441/mutual-funds/.

[23] The specific bound on c_p is based on a function ζ, defined in Equation (26) in Appendix A, which governs A traders' entry into the market. The condition is $\zeta(m_{1j}, A_j^2, c_p) \leq 0$ for $m_{1j} \in [0, 1 - m]$.

[24] In the disposition effect (Odean, 1998), agents are reluctant to sell losers, which is at odds with the notion that they eschew stocks that have lost money. However, the disposition effect only applies to stocks agents actually own, whereas we propose that agents derive utility (disutility) from *buying* stocks that have good (subpar) fundamentals.

[25] If $y \sim N(\bar{y}, 1)$, then $E(\exp(-ty^2)) = \frac{1}{\sqrt{1+2t}}\exp(-\frac{t\bar{y}^2}{1+2t})$.

[26] If $y \sim N(\bar{y}, \nu)$, then $E[|y|] = \sqrt{\nu}\left[2\phi(\frac{\bar{y}}{\sqrt{\nu}}) + \frac{\bar{y}}{\sqrt{\nu}}(1 - 2\Phi(\frac{-\bar{y}}{\sqrt{\nu}}))\right]$, where $\phi(.)$ and $\Phi(.)$ denote the p.d.f. and c.d.f. of standard normal distribution. $\frac{\partial(E[|y|])}{\partial\bar{y}} = 1 - 2\Phi(\frac{-\bar{y}}{\sqrt{\nu}})$ is positive if $\bar{y} > 0$ and negative if $\bar{y} < 0$. This implies that $E[|y|]$ increases in $|\bar{y}|$.

[27] If $Y \sim N(0, \Sigma)$, then $E\left[\exp(-\Gamma'Y - 0.5Y'\Omega Y)\right] = \frac{|\Sigma^{-1}+\Omega|^{-1/2}}{|\Sigma|^{1/2}}\exp\left[0.5\Gamma'(\Sigma^{-1} + \Omega)^{-1}\Gamma\right]$.

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SUPPORTING INFORMATION

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APPENDIX

Proof of Proposition 1. Write the i'th A trader's wealth at Date 1 as $W_{i1} = \bar{W}_{i0} + X_i(V - P)$. He needs to choose X_i to maximize

$$E\Big[U_A(W_{i1}, X_i)|\theta\Big] = E\Big[-\exp(-\gamma\bar{W}_{i0} - \gamma X_i(V-P) - AX_i)|\theta\Big]$$
$$= -\exp\Big[-\gamma\bar{W}_{i0} - \gamma X_i(A/\gamma + E(V|\theta) - P) + 0.5\gamma^2 X_i^2 \text{Var}(V|\theta)\Big].$$

The f.o.c. w.r.t. X_i implies that his demand can be expressed as follows:

$$X_A(P,\theta) = \frac{A/\gamma + E(V|\theta) - P}{\gamma \text{Var}(V|\theta)} = \frac{A/\gamma + \bar{V} + \theta - P}{\gamma \nu_\epsilon}. \tag{4}$$

The second order condition (s.o.c.) holds obviously in the above case, and all other cases below, so explicit reference to the s.o.c. will henceforth be omitted. We can similarly show that a non-A trader's demand (that with $A = 0$) can be expressed as:

$$X_{NA}(P,\theta) = \frac{E(V|\theta) - P}{\gamma \text{Var}(V|\theta)} = \frac{\bar{V} + \theta - P}{\gamma \nu_\epsilon}. \tag{5}$$

Equations (4) and (5) and the market clearing condition $z = mX_{NA}(P,\theta) + (1 - m)X_A(P,\theta)$ implies that P takes the form given in Proposition 1.

Proof of Proposition 2. In the multi-asset setup, stock payoffs take the independent-normal structure. This implies that a trader's expected (negative exponential) utility takes a multiplicative form, in which each multiplicative component represents his expected utility obtained from trading a specific stock. His optimization problem regarding a specific stock is independent of his optimization problem in the other stocks. Therefore, in the following derivation and all the other derivations for the multi-asset setup, we restrict our attention to his optimization problem in only one stock. This is for notational convenience. It is straightforward to extend the analysis to include his optimization problem for other stocks. Conjecture that the price of stock j takes the following linear form:

$$P_j = \bar{V}_j + \alpha_j + \beta_j \omega_j(\theta_j, z_j), \tag{6}$$

where $\omega_j(\theta_j, z_j)$, or simply $\omega_j = \theta_j - \delta_j z_j$, has a variance $\nu_{\omega_j} = \nu_{\theta_j} + \delta_j^2 \nu_{z_j}$. The constant parameters, δ_j, α_j, and β_j, are to be determined.

The i'th informed A trader spends c_{Aj} to observe θ_j. Write his wealth at Date 1 as $W_{i1} = \bar{W}_{i0} - c_{Aj} + X_{ij}(V_j - P_j)$. He needs to choose X_{ij} to maximize

$$E\Big[U_{AI}(W_{i1}, X_{ij})|\theta_j\Big] = E\Big[-\exp(-\gamma(\bar{W}_{i0} - c_{Aj}) - \gamma X_{ij}(V_j - P_j) - A_j X_{ij})|\theta_j\Big]$$
$$= -\exp\Big[-\gamma(\bar{W}_{i0} - c_{Aj}) - \gamma X_{ij}(A_j/\gamma + E(V_j|\theta_j) - P_j) + 0.5\gamma^2 X_{ij}^2 \text{Var}(V_j|\theta_j)\Big]. \tag{7}$$

The f.o.c. w.r.t. X_{ij} implies that his demand can be expressed as follows:

$$X_{AIj}(P_j, \theta_j) = \frac{A_j/\gamma + E(V_j|\theta_j) - P_j}{\gamma \text{Var}(V_j|\theta_j)} = \frac{A_j/\gamma + \bar{V}_j + \theta_j - P_j}{\gamma \nu_{\epsilon_j}}. \tag{8}$$

The i'th uninformed A trader does not observe θ_j, but learns ω_j from P_j in Equation (6). Write his wealth at Date 1 as $W_{i1} = \bar{W}_{i0} + X_{ij}(V_j - P_j)$. He needs to choose X_{ij} to maximize

$$E\Big[U_{AU}(W_{i1}, X_{ij})|\omega_j\Big] = E\Big[-\exp(-\gamma\bar{W}_{i0} - \gamma X_{ij}(V_j - P_j) - A_j X_{ij})|\omega_j\Big]$$
$$= -\exp\Big[-\gamma\bar{W}_{i0} - \gamma X_{ij}(A_j/\gamma + E(V_j|\omega_j) - P_j) + 0.5\gamma^2 X_{ij}^2 \text{Var}(V_j|\omega_j)\Big]. \tag{9}$$

The f.o.c. w.r.t. X_{ij} implies that his demand can be expressed as:

$$X_{AUj}(P_j, \omega_j) = \frac{A_j/\gamma + E(V_j|\omega_j) - P_j}{\gamma \text{Var}(V_j|\omega_j)} = \frac{A_j/\gamma + \bar{V}_j + (\nu_{\theta_j}/\nu_{\omega_j})\omega_j - P_j}{\gamma(\nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})}. \tag{10}$$

We can similarly show that the i'th non-A informed and uninformed trader's demand (those with $A_j = 0$) can be expressed as follows:

$$X_{Ij}(P_j, \theta_j) = \frac{E(V_j|\theta_j) - P_j}{\gamma \text{Var}(V_j|\theta_j)} = \frac{\bar{V}_j + \theta_j - P_j}{\gamma \nu_{\epsilon_j}}, \tag{11}$$

$$X_{Uj}(P_j, \omega_j) = \frac{E(V_j|\omega_j) - P_j}{\gamma \text{Var}(V_j|\omega_j)} = \frac{\bar{V}_j + (\nu_{\theta_j}/\nu_{\omega_j})\omega_j - P_j}{\gamma(\nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})}. \tag{12}$$

Define

$$N_{1j} = \frac{m\lambda_j}{\gamma \nu_{\epsilon_j}}, \quad N_{2j} = \frac{m(1 - \lambda_j)}{\gamma\left[\nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j}\right]},$$

$$N_{3j} = \frac{(1 - m)\lambda_{Aj}}{\gamma \nu_{\epsilon_j}}, \text{ and } N_{4j} = \frac{(1 - m)(1 - \lambda_{Aj})}{\gamma\left[\nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j}\right]}.$$

It follows from Equations (8), (10), (11), and (12), that the market clearing condition requires

$$\begin{aligned} z_j &= m\lambda_j \times X_{Ij}(P_j, \theta_j) + m(1 - \lambda_j) \times X_{Uj}(P_j, \omega_j) \\ &\quad + (1 - m)\lambda_{Aj} \times X_{AIj}(P_j, \theta_j) + (1 - m)(1 - \lambda_{Aj}) \times X_{AUj}(P_j, \omega_j) \\ &= N_{1j} \times (\bar{V}_j + \theta_j - P_j) + N_{2j} \times (\bar{V}_j + \frac{\nu_{\theta_j}}{\nu_{\omega_j}}\omega_j - P_j) \\ &\quad + N_{3j} \times (A_j/\gamma + \bar{V}_j + \theta_j - P_j) + N_{4j} \times (A_j/\gamma + \bar{V}_j + \frac{\nu_{\theta_j}}{\nu_{\omega_j}}\omega_j - P_j) \end{aligned} \tag{13}$$

Solving for P_j from Equation (13) verifies the conjectured price function in Equation (6). The parameters δ_j, α_j, and β_j are given by $\delta_j = [N_{1j} + N_{3j}]^{-1}$,

$$\alpha_j = \frac{N_{3j} + N_{4j}}{N_{1j} + N_{2j} + N_{3j} + N_{4j}} \times A_j/\gamma, \text{ and} \tag{14}$$

$$\beta_j = \frac{N_{1j} + N_{3j} + (N_{2j} + N_{4j})\frac{\nu_{\theta_j}}{\nu_{\omega_j}}}{N_{1j} + N_{2j} + N_{3j} + N_{4j}}, \tag{15}$$

Proof of Lemma 1. Consider an A trader, who trades stock j. If he spends c_{Aj} to acquire the information θ_j, then his expected utility is given by Equation (7). Substituting for his optimal demand from Equation (8) yields

$$\begin{aligned} E\left[U_{AI}(W_{i1}, X_{ij})|\theta_j\right] &= -\exp\left[-\gamma(\bar{W}_{i0} - c_{Aj}) - 0.5(A_j/\gamma + \bar{V}_j + \theta_j - P_j)^2/\nu_{\epsilon_j}\right] \\ &= -\exp\left[-\gamma(\bar{W}_{i0} - c_{Aj}) - 0.5\frac{\text{Var}(\theta_j|\omega_j)}{\nu_{\epsilon_j}} Y_j^2\right], \end{aligned}$$

where $Y_j \equiv \frac{A_j/\gamma + \bar{V}_j + \theta_j - P_j}{\sqrt{\text{Var}(\theta_j|\omega_j)}}$ with $Y_j|\omega_j \sim N(\frac{A_j/\gamma + \bar{V}_j + E(\theta_j|\omega_j) - P_j}{\sqrt{\text{Var}(\theta_j|\omega_j)}}, 1)$. Here P_j is a public signal which contains the same information as ω_j (from Proposition 2). It follows that

$$
\begin{aligned}
&E\left[U_{AI}(W_{i1},X_{ij})|\omega_j\right]\\
&=-E\left[\exp\left[-\gamma(\bar{W}_{i0}-c_{Aj})-0.5\frac{\mathrm{Var}(\theta_j|\omega_j)}{\nu_{\epsilon_j}}Y_j^2\right]|\omega_j\right]\\
&=-\frac{\exp(-\gamma(\bar{W}_{i0}-c_{Aj}))}{\sqrt{1+\mathrm{Var}(\theta_j|\omega_j)/\nu_{\epsilon_j}}}\exp\left[-0.5\frac{\frac{\mathrm{Var}(\theta_j|\omega_j)}{\nu_{\epsilon_j}}\left(\frac{A_j/\gamma+\bar{V}_j+E(\theta_j|\omega_j)-P_j}{\sqrt{\mathrm{Var}(\theta_j|\omega_j)}}\right)^2}{1+\mathrm{Var}(\theta_j|\omega_j)/\nu_{\epsilon_j}}\right]\\
&=-\exp(-\gamma(\bar{W}_{i0}-c_{Aj}))\times\sqrt{\frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)}}\exp\left[-0.5\frac{(A_j/\gamma+E(V_j|\omega_j)-P_j)^2}{\mathrm{Var}(V_j|\omega_j)}\right],
\end{aligned}
\tag{16}
$$

where the second equality follows from the fact in Footnote 25, and the last equality follows from the facts $\mathrm{Var}(V_j|\theta_j)=\nu_{\epsilon_j}$ and $\mathrm{Var}(V_j|\omega_j)=\nu_{\epsilon_j}+\mathrm{Var}(\theta_j|\omega_j)$.[25]

If he does not spend c_{Aj} to acquire the information θ_j, then his expected utility is given by Equation (9). Substituting for his optimal demand from Equation (10) yields

$$
E\left[U_{AU}(W_{i1},X_{ij})|\omega_j\right]=-\exp\left[-\gamma\bar{W}_{i0}-0.5\frac{(A_j/\gamma+E(V_j|\omega_j)-P_j)^2}{\mathrm{Var}(V_j|\omega_j)}\right].
\tag{17}
$$

Comparing Equations (16) and (17),

$$
E\left[U_{AI}(W_{i1},X_{ij})-U_{AU}(W_{i1},X_{ij})|\omega_j\right]=\left[\exp(\gamma c_{Aj})\sqrt{\frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)}}-1\right]E\left[U_{AU}(W_{i1},X_{ij})|\omega_j\right].
$$

It follows that the difference between an agent's ex ante utilities from becoming informed and remaining uninformed is given by

$$
E\left[U_{AI}(W_{i1},X_{ij})-U_{AU}(W_{i1},X_{ij})\right]=\left[\exp(\gamma c_{Aj})\sqrt{\frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)}}-1\right]E\left[U_{AU}(W_{i1},X_{ij})\right].
\tag{18}
$$

Denote $\phi_{Aj}\equiv\exp(2\gamma c_{Aj})\times\frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)}-1$. If ϕ_{Aj} is negative (positive), then the difference in the ex ante utility in Equation (18) is positive (negative) because $E\left[U_{AU}(W_{i1},X_{ij})\right]<0$, and the A trader prefers to become informed (remain uninformed). If $\phi_{Aj}=0$, then he is indifferent between becoming informed and remaining uninformed. We can similarly derive ϕ_j as given in Lemma 3 for a non-A trader. In his case, he needs to spend c_j to acquire information and he equivalently has $A_j=0$.

Proof of Proposition 3. From Proposition 2 and Lemma 1,

$$
\delta_j=\frac{\gamma\nu_{\epsilon_j}}{m\lambda_j+(1-m)\lambda_{Aj}},
\tag{19}
$$

$$
\phi_j=\exp(2\gamma c_j)\times\frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)}-1=\frac{\exp(2\gamma c_j)}{\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\left(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+\delta_j^2\nu_{z_j}}\right)+1}-1,
\tag{20}
$$

$$
\phi_{Aj}=\exp(2\gamma c_{Aj})\times\frac{\mathrm{Var}(V_j|\theta_j)}{\mathrm{Var}(V_j|\omega_j)}-1=\frac{\exp(2\gamma c_{Aj})}{\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\left(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+\delta_j^2\nu_{z_j}}\right)+1}-1.
\tag{21}
$$

Next, using the entry conditions, we propose that the interior value of λ_j in Range 2 is given by

$$
\frac{\exp(2\gamma c_j)}{\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\left[1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+\left(\frac{\gamma\nu_{\epsilon_j}}{m\lambda_j}\right)^2\nu_{z_j}}\right]+1}-1=0,
\tag{22}
$$

and that of $\lambda_{Aj} \in (0, 1)$ in Range 4 is given by

$$\frac{\exp(2\gamma c_{Aj})}{\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\left[1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+\left(\frac{\gamma\nu_{\epsilon_j}}{m+(1-m)\lambda_{Aj}}\right)^2\nu_{z_j}}\right]+1}-1=0. \tag{23}$$

We can use λ_j and λ_{Aj} given in Proposition 3 and in Equations (22) and (23) to compute ϕ_j and ϕ_{Aj} and verify that $0 \le \phi_j < \phi_{Aj}$ in Range 1, $0 = \phi_j < \phi_{Aj}$ in Range 2, $\phi_j \le 0 \le \phi_{Aj}$ in Range 3, $\phi_j < 0 = \phi_{Aj}$ in Range 4, and $\phi_j < \phi_{Aj} \le 0$ in Range 5. It follows from Lemma 3 that λ_j and λ_{Aj} given in Proposition 3 represent an equilibrium. In the remainder of this proof, we show that these quantities represent the unique equilibrium. We use two facts implied by Equations (19) to (21). First, $\phi_j < \phi_{Aj}$ because $c_j < c_{Aj}$. Second, ϕ_j and ϕ_{Aj} increase in λ_j and λ_{Aj}.

In Range 1, if the equilibrium is not $\lambda_j = \lambda_{Aj} = 0$ (note that $\lambda_j, \lambda_{Aj} \in [0, 1]$), $0 < \phi_j < \phi_{Aj}$ from Equations (19) to (21). Lemma 1 implies that all non-A and A traders prefer to remain uninformed, i.e., $\lambda_j = \lambda_{Aj} = 0$. This constitutes a contradiction. Thus, in equilibrium, $\lambda_j = \lambda_{Aj} = 0$.

In Range 2, if $\lambda_{Aj} > 0$, $\phi_{Aj} \le 0$ (from Lemma 1) and thus $\phi_j < 0$ (because $\phi_j < \phi_{Aj}$). Lemma 1 implies that all non-A traders prefer to become informed, i.e., $\lambda_j = 1$. $\lambda_{Aj} > 0$ and $\lambda_j = 1$ lead to $\phi_j > 0$ in Equation (20), which conflicts with $\phi_j < 0$. Thus, in equilibrium, $\lambda_{Aj} = 0$. A unique λ_j, in this case an interior λ_j, is specified by $\phi_j = 0$ because ϕ_j increases in λ_j.

In Range 3, if $\lambda_j < 1$, $\phi_j \ge 0$ (from Lemma 1) and thus $\phi_{Aj} > 0$ (because $\phi_j < \phi_{Aj}$). Lemma 1 implies that all A traders prefer to remain uninformed, i.e., $\lambda_{Aj} = 0$. $\lambda_j < 1$ and $\lambda_{Aj} = 0$ lead to $\phi_j < 0$ in Equation (20), which conflicts with $\phi_j \ge 0$. Thus, in equilibrium, $\lambda_j = 1$. Given $\lambda_j = 1$, if $\lambda_{Aj} > 0$, $\phi_{Aj} \le 0$ (from Lemma 1). $\lambda_j = 1$ and $\lambda_{Aj} > 0$ lead to $\phi_{Aj} > 0$ in Equation (21), which conflicts with $\phi_{Aj} \le 0$. Thus, in equilibrium, $\lambda_{Aj} = 0$.

In Range 4, if $\lambda_j < 1$, $\phi_j \ge 0$ (from Lemma 1) and thus $\phi_{Aj} > 0$ (because $\phi_j < \phi_{Aj}$). Lemma 1 implies that all A traders prefer to remain uninformed, i.e., $\lambda_{Aj} = 0$. $\lambda_j < 1$ and $\lambda_{Aj} = 0$ lead to $\phi_{Aj} < 0$ in Equation (21), which conflicts with $\phi_{Aj} > 0$. Thus, in equilibrium, $\lambda_j = 1$. A unique λ_{Aj}, in this case an interior λ_{Aj}, is specified by $\phi_{Aj} = 0$ because ϕ_{Aj} increases in λ_{Aj}.

In Range 5, if the equilibrium outcome is not $\lambda_j = \lambda_{Aj} = 1$, then $\phi_j < \phi_{Aj} < 0$ from Equations (19) to (21). Lemma 1 implies that all non-A and A traders prefer to become informed by spending c_j or c_{Aj}. This constitutes a contradiction. Thus, in equilibrium, $\lambda_j = \lambda_{Aj} = 1$.

In sum, the λ_j and λ_{Aj} given in Proposition 3 represent the unique equilibrium.

Proof of Corollary 1.

(i) From Proposition 2, we can write $\alpha_j = \kappa_j A_j/\gamma$, where $\kappa_j = \frac{N_{3j}+N_{4j}}{N_{1j}+N_{2j}+N_{3j}+N_{4j}} \in [0, 1]$ does not depend on A_j.

(ii) Consider Ranges 1–2 and 4–5 given in Proposition 3, we can use Propositions 2 and 3 to compute κ_j.

In Range 1, $\lambda_j = \lambda_{Aj} = 0$. It follows that $\kappa_j = 1 - m$ decreases in m.

In Range 2, $\lambda_{Aj} = 0$. It is straightforward to show that $m\lambda_j$ and $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m\lambda_j}$ do not depend on m. It follows that $\nu_{\omega_j} = \nu_{\theta_j} + \delta_j^2\nu_{z_j}$ also does not depend on m. Write $\kappa_j = \frac{\frac{1-m}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}}{\frac{m\lambda_j}{\gamma\nu_{\epsilon_j}}+\frac{1-m\lambda_j}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}}$, which obviously decreases in m.

In Range 4, $\lambda_j = 1$. Define $Q_j \equiv m + (1-m)\lambda_{Aj}$. Now, it can easily be shown that Q_j and $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m+(1-m)\lambda_{Aj}}$, do not depend on m. It follows that $\nu_{\omega_j} = \nu_{\theta_j} + \delta_j^2\nu_{z_j}$ also does not depend on m. Write $\kappa_j = \frac{\frac{Q_j-m}{\gamma\nu_{\epsilon_j}}+\frac{1-Q_j}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}}{\frac{Q_j}{\gamma\nu_{\epsilon_j}}+\frac{1-Q_j}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}}$, which obviously decreases in m.

In Range 5, $\lambda_j = \lambda_{Aj} = 1$. It follows that $\kappa_j = 1 - m$ decreases in m.

(iii) Now consider Range 3 given in Proposition 3.

In Range 3, $\lambda_j = 1$ and $\lambda_{Aj} = 0$. It follows that $\delta_j = \gamma\nu_{\epsilon_j}/m$, $\nu_{\omega_j} = \nu_{\theta_j} + \delta_j^2\nu_{z_j} = \nu_{\theta_j} + (\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}$, and $\kappa_j = \frac{1}{1+\frac{m}{1-m}\left(\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\left(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}\right)+1\right)}$. Note that

$$\frac{\partial}{\partial m}\left[\frac{m}{1-m}(\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}})+1)\right]$$
$$=\frac{1}{(1-m)^2}(\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}+1)-\frac{1}{1-m}\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}\frac{2\nu_{\theta_j}(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{(\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j})^2}$$
$$\propto \nu_{\theta_j}\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}+\nu_{\epsilon_j}-2(1-m)\nu_{\theta_j}^2\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{(\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j})^2}.$$

Denote $G(\nu_{z_j}) \equiv \nu_{\theta_j}\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}+\nu_{\epsilon_j}-2(1-m)\nu_{\theta_j}^2\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{(\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j})^2}$. For κ_j to increase (decrease) in m, it suffices that $G(\nu_{z_j})$ is negative (positive).

It is straightforward to show that

$$G(\nu_{z_j})|_{\nu_{z_j}=0}>0,\ G(\nu_{z_j})|_{\nu_{z_j}\to\infty}>0,\ \text{and}\ \frac{\mathrm{d}G(\nu_{z_j})}{\mathrm{d}\nu_{z_j}}\propto 1-2(1-m)\frac{\nu_{\theta_j}-(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}.$$

Suppose $m \geq 1/2$. Then $\frac{\mathrm{d}G(\nu_{z_j})}{\mathrm{d}\nu_{z_j}} \geq 0$ (because $1-2(1-m)\frac{\nu_{\theta_j}-(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}$ is monotonic in ν_{z_j} and is non-negative for both $\nu_{z_j}=0$ and $\nu_{z_j}\to\infty$). It follows that $G(\nu_{z_j})>0$ and therefore κ_j decreases in m.

Suppose $m < 1/2$. As ν_{z_j} increases from 0 to ∞, $\mathrm{d}G(\nu_{z_j})/\mathrm{d}\nu_{z_j}$ is first negative and then positive. $G(\nu_{z_j})$ reaches a minimum when $1-2(1-m)\frac{\nu_{\theta_j}-(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}=0$.

$$\begin{aligned}\min G(\nu_{z_j}) &= \nu_{\theta_j}\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}\left[1-2(1-m)\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}\right]+\nu_{\epsilon_j}\\
&=\nu_{\theta_j}\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}\left[2(1-m)\frac{\nu_{\theta_j}-(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}-2(1-m)\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}\right]+\nu_{\epsilon_j}\\
&=-2(1-m)\nu_{\theta_j}(\frac{(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}})^2+\nu_{\epsilon_j}\\
&=\frac{(1-2m)^2\nu_{\theta_j}}{8(1-m)}+\nu_{\epsilon_j},\end{aligned}$$

where the second and fourth equalities follow from $1-2(1-m)\frac{\nu_{\theta_j}-(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}}=0$.

If $\frac{\nu_{\epsilon_j}}{\nu_{\theta_j}} \geq \frac{(1-2m)^2}{8(1-m)}$, then $G(\nu_{z_j}) \geq \min G(\nu_{z_j}) \geq 0$ and therefore κ_j decreases in m.

If $\frac{\nu_{\epsilon_j}}{\nu_{\theta_j}} < \frac{(1-2m)^2}{8(1-m)}$, then $\min G(\nu_{z_j})<0$. As ν_{z_j} increases from 0 to ∞, note from the above that $\mathrm{d}G(\nu_{z_j})/\mathrm{d}\nu_{z_j}$ is first negative and then positive. It follows that $G(\nu_{z_j})$ is first positive (because $G(\nu_{z_j})|_{\nu_{z_j}=0}>0$), then decreases and reaches its m&inimum $\min G(\nu_{z_j})<0$, and finally increases to be positive again (because $G(\nu_{z_j})|_{\nu_{z_j}\to\infty}>0$). Therefore, there exists two positive quantities $\underline{\nu}_{z_j}$ and $\bar{\nu}_{z_j}$ such that if $\nu_{z_j}\leq\underline{\nu}_{z_j}$ or $\nu_{z_j}\geq\bar{\nu}_{z_j}$, $G(\nu_{z_j})$ is positive (and κ_j decreases in m). If $\nu_{z_j}\in(\underline{\nu}_{z_j},\bar{\nu}_{z_j})$, $G(\nu_{z_j})$ is negative (and κ_j increases in m).

(iv) An observation from Propositions 2 and 3 is that c_{Aj} does not affect the equilibrium α_j in Ranges 1–3 and 5 given in Proposition 3. Thus, we just need to focus on Range 4. Note from Equation (21) that λ_{Aj} decreases in c_{Aj}. Consequently, $Q_j \equiv m+(1-m)\lambda_{Aj}$ as defined in the above (ii) decreases in c_{Aj}. Further, $\nu_{\omega_j}=\nu_{\theta_j}+\delta_j^2\nu_{z_j}$, where $\delta_j=\frac{\gamma\nu_{\epsilon_j}}{m+(1-m)\lambda_{Aj}}=\frac{\gamma\nu_{\epsilon_j}}{Q_j}$, increases in c_{Aj}. It follows immediately that

$$\kappa_j=\frac{\frac{Q_j-m}{\gamma\nu_{\epsilon_j}}+\frac{1-Q_j}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}}{\frac{Q_j}{\gamma\nu_{\epsilon_j}}+\frac{1-Q_j}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}}=1-\frac{\frac{m}{\gamma\nu_{\epsilon_j}}}{\frac{Q_j}{\gamma\nu_{\epsilon_j}}+\frac{1-Q_j}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}},$$

decreases in c_{Aj}.

Proof of Corollary 2. It follows from Proposition 2 that $E(P_j)=\bar{V}_j+\alpha_j$ and $E(V_j-P_j)=-\alpha_j$. This corollary thus obtains directly from the theorem and Corollary 1.

Proof of Corollary 3. Consider the i'th non-A informed trader first. His demand is given in Equation (11). It follows from Propositions 2 and 3 that

$$X_{Ij}(P_j,\theta_j)=\frac{\bar{V}_j+\theta_j-P_j}{\gamma\nu_{\epsilon_j}}=\frac{\theta_j-\alpha_j-\beta_j\omega_j}{\gamma\nu_{\epsilon_j}}\sim N\left(\frac{-\alpha_j}{\gamma\nu_{\epsilon_j}},\frac{\mathrm{Var}(\theta_j-\beta_j\omega_j)}{\gamma^2\nu_{\epsilon_j}^2}\right),$$

where the mean, $-\frac{\alpha_j}{\gamma\nu_{\epsilon_j}}$, is proportional to A_j, and the variance, $\frac{\mathrm{Var}(\theta_j-\beta_j\omega_j)}{\gamma^2\nu_{\epsilon_j}^2}$, does not depend on A_j. It follows from the fact in Footnote 26 that the expectation of the absolute value of his trading, $E\left[|X_{Ij}(P_j,\ \theta_j)|\right]$, increases in $|-\frac{\alpha_j}{\gamma\nu_\epsilon}|$ and therefore in $|A_j|$.[26]

Similarly, the expectations of the absolute values of other types of traders' trading also increase in $|A_j|$. Noting from Proposition 3 that λ_j and λ_{Aj} do not depend on A_j, we conclude that the trading volume, T_j, as defined in Equation (3), increases in $|A_j|$.

Proof of Proposition 4. This proof includes four steps.

Step 1: We follow the proofs of Propositions 2 and 3 to show that there is a rational expectations equilibrium in which the price function for stock j takes the linear form specified in Propositions 2 and 3, except that $1-m$ is replaced by m_{1j}.

Specifically, the parameters, δ_j, α_j, and β_j, are given by

$$\delta_j=\frac{1}{N_{1j}+N_{3j}},\ \alpha_j=\frac{N_{3j}+N_{4j}}{N_{1j}+N_{2j}+N_{3j}+N_{4j}}\times A_j/\gamma,\ \text{and}\ \beta_j=\frac{N_{1j}+N_{3j}+(N_{2j}+N_{4j})\frac{\nu_{\theta_j}}{\nu_{\omega_j}}}{N_{1j}+N_{2j}+N_{3j}+N_{4j}},$$

where

$$N_{1j}=\frac{m\lambda_j}{\gamma\nu_{\epsilon_j}},\quad N_{2j}=\frac{m(1-\lambda_j)}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})},$$

$$N_{3j}=\frac{m_{1j}\lambda_{Aj}}{\gamma\nu_{\epsilon_j}},\ \text{and}\ N_{4j}=\frac{m_{1j}(1-\lambda_{Aj})}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j})+\nu_{\epsilon_j})}.$$

λ_j and λ_{Aj} vary across five ranges for parameter values.

Range 1: If $\exp(2\gamma c_j)\geq\nu_{\theta_j}/\nu_{\epsilon_j}+1$, then $\lambda_j=0$ and $\lambda_{Aj}=0$.

Range 2: If $\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}})+1<\exp(2\gamma c_j)<\nu_{\theta_j}/\nu_{\epsilon_j}+1$, then $\lambda_j\in(0,1)$ is given by

$$\frac{\exp(2\gamma c_j)}{\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\frac{\gamma\nu_{\epsilon_j}}{m\lambda_j})^2\nu_{z_j}})+1}-1=0,\tag{24}$$

and $\lambda_{Aj}=0$.

Range 3: If $\exp(2\gamma c_j)\leq\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\gamma\nu_{\epsilon_j}/m)^2\nu_{z_j}})+1\leq\exp(2\gamma c_{Aj})$, then $\lambda_j=1$ and $\lambda_{Aj}=0$.

Range 4: If $\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}})^2\nu_{z_j}})+1<\exp(2\gamma c_{Aj})<\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\frac{\gamma\nu_{\epsilon_j}}{m})^2\nu_{z_j}})+1$, then $\lambda_j=1$ and $\lambda_{Aj}\in(0,1)$ is given by

$$\frac{\exp(2\gamma c_{Aj})}{\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}\lambda_{Aj}})^2\nu_{z_j}})+1}-1=0.\tag{25}$$

Range 5: If $\exp(2\gamma c_{Aj})\leq\frac{\nu_{\theta_j}}{\nu_{\epsilon_j}}(1-\frac{\nu_{\theta_j}}{\nu_{\theta_j}+(\frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}})^2\nu_{z_j}})+1$, then $\lambda_j=1$ and $\lambda_{Aj}=1$.

Step 2: We consider A traders' entry decision. In this step, we focus on Ranges 1–4 specified in Step 1.

If an A trader does not enter, then his expected utility equals $-\exp(-\gamma\bar{W}_{i0})$. If he enters, he may choose to become informed by spending c_{Aj} or remain uninformed. Here, we consider only his expected utility if he chooses to remain uninformed. There is no need to consider his expected utility if he choose to become informed, either because none of them will do so (in Ranges 1–3), or because after accounting for the cost of information acquisition c_{Aj}, this expected utility equals the one he obtains from remaining uninformed (in Range 4).

It follows from Equation (17) in the proof of Lemma 1 that an uninformed A trader who chooses to enter has the following expected utility:

$$E\Big[U_{AU}(W_{i1},X_{ij})|\omega_j\Big] = -\exp\Big[-\gamma(\bar{W}_{i0}-c_{pj}) - 0.5\frac{(A_j/\gamma+E(V_j|\omega_j)-P_j)^2}{\mathrm{Var}(V_j|\omega_j)}\Big]$$
$$= -\exp\Big[-\gamma(\bar{W}_{i0}-c_{pj}) - 0.5\frac{(A_j/\gamma-\alpha_j-(\beta_j-\nu_{\theta_j}/\nu_{\omega_j})\omega_j)^2}{\mathrm{Var}(V_j|\omega_j)}\Big]$$
$$= -\exp\Big[-\gamma(\bar{W}_{i0}-c_{pj}) - 0.5\frac{\Delta_{Z_j}}{\mathrm{Var}(V_j|\omega_j)}Z_j^2\Big],$$

where $Z_j \equiv \frac{A_j/\gamma-\alpha_j-(\beta_j-\nu_{\theta_j}/\nu_{\omega_j})\omega_j}{\sqrt{\Delta_{Z_j}}} \sim N(\frac{A_j/\gamma-\alpha_j}{\sqrt{\Delta_{Z_j}}},1)$ and $\Delta_{Z_j} \equiv (\beta_j-\nu_{\theta_j}/\nu_{\omega_j})^2\nu_{\omega_j}$. It follows from the fact in Footnote 25 that

$$E\Big[U_{AU}(W_{i1},X_{ij})\Big] = -\frac{\exp(-\gamma(\bar{W}_{i0}-c_{pj}))\times\exp\Big[-0.5\frac{(A_j/\gamma-\alpha_j)^2}{\Delta_{Z_j}+\mathrm{Var}(V_j|\omega_j)}\Big]}{\sqrt{1+\Delta_{Z_j}/\mathrm{Var}(V_j|\omega_j)}}.$$

A simple comparison between the A trader's expected utilities from entering (and trading) and from not entering suggests his entering decision is based on the function

$$\zeta(m_{1j},A_j^2) \equiv \frac{\exp(\gamma c_{pj})\times\exp\Big[-0.5\frac{(A_j/\gamma-\alpha_j)^2}{\Delta_{Z_j}+\mathrm{Var}(V_j|\omega_j)}\Big]}{\sqrt{1+\Delta_{Z_j}/\mathrm{Var}(V_j|\omega_j)}} - 1. \tag{26}$$

As Step 1 suggests, $\zeta(m_{1j}, A_j^2)$ is a function of A_j^2 because $(A_j/\gamma - \alpha_j)^2$ is proportional to A_j^2. It is also a function of m_{1j} because the items, α_j and β_j (in Δ_{Z_j}), are dependent on m_{1j}. If $\zeta(m_{1j}, A_j^2) > 0$, then the A trader will not enter. If $\zeta(m_{1j}, A_j^2) < 0$, then he will enter. If $\zeta(m_{1j}, A_j^2) = 0$, then he is indifferent between entering and not entering. Therefore, the equilibrium, if $\zeta(0, A_j^2) \geq 0$, $m_{1j} = 0$; if $\zeta(1-m, A_j^2) \leq 0$, $m_{1j} = 1-m$; if $\zeta(0, A_j^2) < 0 < \zeta(1-m, A_j^2)$, an interior $m_{1j} \in (0, 1-m)$ is specified by $\zeta(m_{1j}, A_j^2) = 0$.

To show that m_{1j} is uniquely determined based on the function $\zeta(m_{1j}, A_j^2)$ (particularly, that an interior $m_{1j} \in (0, 1-m)$ is specified by $\zeta(m_{1j}, A_j^2) = 0$) and that m_{1j} increases in $|A_j|$, it suffices to show that $\zeta(m_{1j}, A_j^2)$ decreases in A_j^2 and increases in m_{1j}. Consider the expression for $\zeta(m_{1j}, A_j^2)$ in Equation (26). Account for the fact that in Ranges 1–4, the items, $m_{1j}\lambda_{Aj}$, δ_j, ν_{ω_j}, and $\mathrm{Var}(V_j|\omega_j)$, do not depend on m_{1j} (since in Ranges 1–3, $\lambda_{Aj} = 0$, so $m_{1j}\lambda_{Aj} = 0$; in Range 4, $m_{1j}\lambda_{Aj}$ is endogenously determined by Equation (25)). It is evident that $\zeta(m_{1j}, A_j^2)$ decreases in A_j^2 because in Equation (26), only the item, $(A_j/\gamma - \alpha_j)^2$, is related to and is linear in A_j^2. It is straightforward to show, after plugging in the derived β_j, that $\Delta_{Z_j} = (\beta_j - \frac{\nu_{\theta_j}}{\nu_{\omega_j}})^2\nu_{\omega_j}$ decreases in m_{1j}. Thus, the denominator in the expression of $\zeta(m_{1j}, A_j^2)$ in Equation (26) decreases in m_{1j}. We can also show, after plugging in the above derived α_j and β_j (in Δ_{Z_j}) and taking derivatives w.r.t. m_{1j}, that the numerator in the expression of $\zeta(m_{1j}, A_j^2)$ in Equation (26) increases in m_{1j}. It follows that $\zeta(m_{1j}, A_j^2)$ increases in m_{1j}. Taken together, in Ranges 1-4, m_{1j} is uniquely determined by the function $\zeta(m_{1j}, A_j^2)$ (particularly, an interior $m_{1j} \in (0, 1-m)$ is specified by $\zeta(m_{1j}, A_j^2) = 0$). Moreover, m_{1j} increases in $|A_j|$.

Step 3: We consider A traders' entering decision. In this step, we focus on Range 5 specified in Step 1.

If an A trader does not enter, then his expected utility equals $-\exp(-\gamma\bar{W}_{i0})$. If he enters, then he will spend c_{Aj} to become informed. In Range 5, $\lambda_{Aj} = 1$ and $\lambda_j = 1$. It follows from Step 1 that $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}}$, $\alpha_j = \frac{m_{1j}}{m+m_{1j}} \times A_j/\gamma$, and $\beta_j = 1$. It follows from Equation (16) in the proof of Lemma 3 that an entering informed A trader has the following expected utility:

$$E\Big[U_{AI}(W_{i1},X_{ij})|\theta_j\Big] = -\exp\Big[-\gamma(\bar{W}_{i0}-c_{pj}-c_{Aj}) - 0.5\frac{(A_j/\gamma+E(V_j|\theta_j)-P_j)^2}{\mathrm{Var}(V_j|\theta_j)}\Big]$$
$$= -\exp\Big[-\gamma(\bar{W}_{i0}-c_{pj}-c_{Aj}) - 0.5(A_j/\gamma-\alpha_j+\delta_j z_j)^2/\nu_{\epsilon_j}\Big]$$
$$= -\exp\Big[-\gamma(\bar{W}_{i0}-c_{pj}-c_{Aj}) - 0.5\frac{\delta_j^2\nu_{z_j}}{\nu_{\epsilon_j}}Z_j^2\Big],$$

where we define $Z_j \equiv \frac{A_j/\gamma - \alpha_j + \delta_j z_j}{\delta_j \sqrt{\nu_{z_j}}} \sim N(\frac{A_j/\gamma - \alpha_j}{\delta_j \sqrt{\nu_{z_j}}}, 1)$. It follows from the fact in Footnote 25 that

$$E\Big[U_{AI}(W_{i1}, X_{ij})\Big] = -\frac{\exp(-\gamma(\bar{W}_{i0} - c_{pj} - c_{Aj})) \times \exp\Big[-0.5\frac{(A_j/\gamma-\alpha_j)^2}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}}\Big]}{\sqrt{1+\delta_j^2\nu_{z_j}/\nu_{\epsilon_j}}}.$$

A simple comparison between the A trader's expected utilities from entering (and trading) and from not entering suggests his entering decision is based on the function

$$\zeta(m_{1j}, A_j^2) \equiv \frac{\exp(\gamma c_{pj})\exp(\gamma c_{Aj}) \times \exp\Big[-0.5\frac{(A_j/\gamma-\alpha_j)^2}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}}\Big]}{\sqrt{1+\delta_j^2\nu_{z_j}/\nu_{\epsilon_j}}} - 1. \tag{27}$$

As Step 1 suggests, $\zeta(m_{1j}, A_j^2)$ is a function of A_j^2 because $(A_j/\gamma - \alpha_j)^2$ is proportional to A_j^2. It is also a function of m_{1j} because α_j and δ_j can depend on m_{1j}. If $\zeta(m_{1j}, A_j^2) > 0$, the A trader won't enter. If $\zeta(m_{1j}, A_j^2) < 0$, he will enter. If $\zeta(m_{1j}, A_j^2) = 0$, he is indifferent between entering and not entering. Therefore, the equilibrium m_{1j} is given as follows. If $\zeta(0, A_j^2) \geq 0$, $m_{1j} = 0$; if $\zeta(1 - m, A_j^2) \leq 0$, $m_{1j} = 1 - m$; if $\zeta(0, A_j^2) < 0 < \zeta(1 - m, A_j^2)$, an interior $m_{1j} \in (0, 1 - m)$ is specified by $\zeta(m_{1j}, A_j^2) = 0$.

To show that m_{1j} is uniquely determined based on the function $\zeta(m_{1j}, A_j^2)$ (particularly, that an interior $m_{1j} \in (0, 1 - m)$ is specified by $\zeta(m_{1j}, A_j^2) = 0$) and that m_{1j} increases in A_j, it suffices to show that $\zeta(m_{1j}, A_j^2)$ decreases in A_j^2 and increases in m_{1j}. Consider the expression of $\zeta(m_{1j}, A_j^2)$ in Equation (27). It is evident that $\zeta(m_{1j}, A_j^2)$ decreases in A_j^2 because in Equation (27), only the item, $(A_j/\gamma - \alpha_j)^2$, is related to and is linear in A_j^2. Because $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}}$ decreases in m_{1j}, the denominator in the expression of $\zeta(m_{1j}, A_j^2)$ in Equation (27) decreases in m_{1j}. We can also show, after substituting the expressions for α_j and δ_j derived above, and taking derivatives w.r.t. m_{1j}, that the numerator in the expression of $\zeta(m_{1j}, A_j^2)$ in Equation (27) increases in m_{1j}. It follows that $\zeta(m_{1j}, A_j^2)$ increases in m_{1j}. Taken together, in Range 5, m_{1j} is uniquely determined based on the function $\zeta(m_{1j}, A_j^2)$ (particularly, an interior $m_{1j} \in (0, 1 - m)$ is specified by $\zeta(m_{1j}, A_j^2) = 0$). Moreover, m_{1j} increases in A_j.

Step 4: Let $\rho_{\theta_j} \equiv \text{corr}(\theta_j, P_j)$. We show that $1/\rho_{\theta_j}^2$ weakly decreases in $|A_j|$. Note that $1/\rho_{\theta_j}^2 = 1/\text{corr}(\theta_j, P_j)^2 = \nu_{\omega_j}/\nu_{\theta_j} = 1 + \delta_j^2\nu_{z_j}/\nu_{\theta_j}$. It suffices to show that δ_j^2 weakly decreases in $|A_j|$ or equivalently A_j^2.

In Ranges 1, 2, and 3 specified in Step 1 of this proof, $\lambda_{Aj} = 0$ and $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m\lambda_j}$. λ_j can be 0, interior (specified by Equation (24)), or unity; none of these outcomes depend on $|A_j|$. Thus, δ_j does not involve $|A_j|$ either. In Range 4, $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}\lambda_{Aj}}$ can be determined by Equation (25), which does not depend on $|A_j|$. In Range 5, $\lambda_j = \lambda_{Aj} = 1$. It follows that

$$\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m+m_{1j}},$$
$$A_j/\gamma - \alpha_j = \frac{m}{m+m_{1j}}A_j/\gamma = m\delta_j A_j/(\gamma^2\nu_{\epsilon_j}) = \sqrt{m^*}\delta_j A_j,$$

where $\sqrt{m^*} \equiv m/(\gamma^2\nu_{\epsilon_j})$ Then, we can write from Equation (27) that

$$\nu_{\epsilon_j}\exp(2\gamma c_{pj})\exp(2\gamma c_{Aj}) \times \exp(-\frac{m^*\delta_j^2A_j^2}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}}) - (\nu_{\epsilon_j} + \delta_j^2\nu_{z_j}) = 0, \tag{28}$$

which specifies δ_j^2 as a function of A_j^2 and c_{Aj}. Taking the implicit derivative of δ_j^2 w.r.t. A_j^2 yields

$$\frac{\mathrm{d}(\delta_j^2)}{\mathrm{d}(A_j^2)} = -\frac{\nu_{\epsilon_j}\exp(2\gamma c_{pj})\exp(2\gamma c_{Aj}) \times \exp(-\frac{m^*\delta_j^2A_j^2}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}})\frac{m^*\delta_j^2}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}}}{\nu_{\epsilon_j}\exp(2\gamma c_{pj})\exp(2\gamma c_{Aj}) \times \exp(-\frac{m^*\delta_j^2A_j^2}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}})\frac{m^*A_j^2\nu_{\epsilon_j}}{(\nu_{\epsilon_j}+\delta_j^2\nu_{z_j})^2} + \nu_{z_j}}$$
$$= -\frac{m^*\delta_j^2}{\frac{m^*A_j^2\nu_{\epsilon_j}}{\nu_{\epsilon_j}+\delta_j^2\nu_{z_j}} + \nu_{z_j}} < 0,$$

where the second equality follows from Equation (28) which specifies δ_j^2 as a function of A_j^2 and c_{Aj}. We can similarly show that $\frac{\mathrm{d}(\delta_j^2)}{\mathrm{d}c_{Aj}} > 0$. It follows that

$$\frac{\mathrm{d}}{\mathrm{d}c_{Aj}}\left[\frac{\mathrm{d}(\delta_j^2)}{\mathrm{d}(A_j^2)}\right] \propto -\frac{\mathrm{d}(\delta_j^2)}{\mathrm{d}c_{Aj}} < 0.$$

In sum, $1/\rho_{\theta_j}^2$ weakly decreases in $|A_j|$. This property holds strictly in Range 5, and is stronger (more negative) when c_{Aj} is high.

Proof of Corollary 4. (i) The variance of $V_j - P_j$ conditional on P_j or equivalently, ω_j, is given by

$$\mathrm{Var}(V_j - P_j|\omega_j) = \mathrm{Var}(\theta_j + \epsilon_j|\omega_j) = \nu_{\theta_j}(1 - \nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j} = \nu_{\theta_j}(1 - \rho_{\theta_j}^2) + \nu_{\epsilon_j}.$$

From Step 4 of the proof of Proposition 4, ρ_{θ_j} weakly increases in $|A_j|$. Therefore, this conditional variance weakly decreases in $|A_j|$. This monotonic property holds strictly in Range 5 specified in the proof of Proposition 4, in which $\lambda_j = \lambda_{Aj} = 1$.

(ii) Using the law of total variance, we can write the unconditional variance of $V_j - P_j$ as

$$\begin{aligned}\mathrm{Var}(V_j - P_j) &= E\left[\mathrm{Var}(V_j - P_j|\omega_j)\right] + \mathrm{Var}\left[E(V_j - P_j|\omega_j)\right] \\ &= \mathrm{Var}(V_j - P_j|\omega_j) + \mathrm{Var}\left[(\nu_{\theta_j}/\nu_{\omega_j} - \beta_j)\omega_j\right] \\ &= \mathrm{Var}(V_j - P_j|\omega_j) + \frac{(\delta_j^2\nu_{z_j})^2}{\nu_{\theta_j} + \delta_j^2\nu_{z_j}}\ell_j^2,\end{aligned}$$

where $\ell_j = \frac{N_{1j} + N_{3j}}{N_{1j} + N_{2j} + N_{3j} + N_{4j}}$, and the $N_{.j}$'s are specified in the proof of Proposition 4. In this unconditional variance, the first term, $\mathrm{Var}(V_j - P_j|\omega_j)$, weakly decreases in $|A_j|$ as shown above. We show that the second term, $\frac{(\delta_j^2\nu_{z_j})^2}{\nu_{\theta_j} + \delta_j^2\nu_{z_j}}\ell_j^2$, also decreases in $|A_j|$ as follows.

From the proof of Proposition 4, m_{1j} increases in $|A_j|$. Thus, it suffices to show that the term $\frac{(\delta_j^2\nu_{z_j})^2}{\nu_{\theta_j} + \delta_j^2\nu_{z_j}}\ell_j^2$ decreases in m_{1j}. Consider Ranges 1–4 specified in the proof of Proposition 4. It is easy to show that δ_j does not depend on m_{1j} (the logic is similar to that used in the proof of Corollary 1). We then need to show that ℓ_j decreases in m_{1j}.

In Range 1, $\ell_j = 0$. In Range 2, $\ell_j = \frac{m\lambda_j/(\gamma\nu_{\epsilon_j})}{\frac{m\lambda_j}{\gamma\nu_{\epsilon_j}} + \frac{m(1-\lambda_j) + m_{1j}}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})}}$. $\nu_{\omega_j} = \nu_{\theta_j} + \delta_j^2\nu_{z_j}$ and $m\lambda_j$ (specified by Equation (24)) do not depend on m_{1j}. Therefore, ℓ_j decreases in m_{1j}. In Range 3, similarly as in Range 2, $\ell_j = \frac{m/(\gamma\nu_{\epsilon_j})}{\frac{m}{\gamma\nu_{\epsilon_j}} + \frac{m_{1j}}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})}}$ decreases in m_{1j}. In Range 4, $\ell_j = \frac{(m + m_{1j}\lambda_{Aj})/(\gamma\nu_{\epsilon_j})}{\frac{m + m_{1j}\lambda_{Aj}}{\gamma\nu_{\epsilon_j}} + \frac{m_{1j} - m_{1j}\lambda_{Aj}}{\gamma(\nu_{\theta_j}(1-\nu_{\theta_j}/\nu_{\omega_j}) + \nu_{\epsilon_j})}}$. ν_{ω_j} and $m_{1j}\lambda_{Aj}$, which is specified by Equation (25), also do not depend on m_{1j}. Therefore, ℓ_j decreases in m_{1j}.

Finally, consider Range 5 specified in proof of Proposition 4. In this range, $\lambda_j = \lambda_{Aj} = 1$, $\beta_j = 1$, and $\delta_j = \frac{\gamma\nu_{\epsilon_j}}{m + m_{1j}}$ decreases in m_{1j}. We can compute $\mathrm{Var}(V_j - P_j) = \nu_{\epsilon_j} + \delta_j^2\nu_{z_j}$, which obviously decreases in m_{1j}.

Proof of Proposition 5. This proof includes four steps. We use backward induction.

Step 1: We derive the stock price at Date 2, P_2, in this step. Suppose that given the stock price at Date 1, P_1, a mass $m_1(P_1)$ or simply $m_1 \in [0, 1 - m]$ (which we will derive in Step 3) of A traders enter at Date 2.

Conjecture that the stock price, P_2, takes the following linear form:

$$P_2 = \bar{V} + \theta_1 + \alpha(m_1)\theta_1 + \beta(m_1)\omega(\theta_2, z_2), \tag{29}$$

where $\omega(\theta_2, z_2)$ or simply $\omega = \theta_2 - \delta z_2$ has a variance $\nu_\omega = \nu_\theta + \delta^2\nu_z$. The constant parameter, δ, and the functions of m_1, $\alpha(m_1)$ and $\beta(m_1)$, are to be determined.

Write the i'th regular non-A trader's wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(V - P_2)$. He needs to choose X_{i2} to maximize

$$\begin{aligned} E\Big[U_R(W_{i3})|P_2,\theta_1,\theta_2\Big] &= E\Big[-\exp(-\gamma W_{i2} - \gamma X_{i2}(V-P_2))|P_2,\theta_1,\theta_2\Big] \\ &= -\exp\Big[-\gamma W_{i2} - \gamma X_{i2}(E(V|\theta_1,\theta_2)-P_2) + 0.5\gamma^2 X_{i2}^2 \mathrm{Var}(V|\theta_1,\theta_2)\Big]. \end{aligned} \tag{30}$$

The f.o.c. w.r.t. X_{i2} implies that his demand can be expressed as follows:

$$X_{R2}(P_2,\theta_1,\theta_2) = \frac{E(V|\theta_1,\theta_2)-P_2}{\gamma \mathrm{Var}(V|\theta_1,\theta_2)} = \frac{\bar{V}+\theta_1+\theta_2-P_2}{\gamma \nu_\epsilon}. \tag{31}$$

The i'th entering A trader does not observe θ_2. But he learns ω from P_2 in Equation (29). Write his wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(V - P_2)$. He needs to choose X_{i2} to maximize

$$\begin{aligned} E\Big[U_A(W_{i3},X_{i2})|P_2,\theta_1,\omega\Big] &= E\Big[-\exp(-\gamma W_{i2} - \gamma X_{i2}(V-P_2) - A(\theta_1)X_{i2})|P_2,\theta_1,\omega\Big] \\ &= -\exp\Big[-\gamma W_{i2} - \gamma X_{i2}(A(\theta_1)/\gamma + E(V|\theta_1,\omega)-P_2) + 0.5\gamma^2 X_{i2}^2 \mathrm{Var}(V|\theta_1,\omega)\Big]. \end{aligned} \tag{32}$$

The f.o.c. w.r.t. X_{i2} implies that his demand can be expressed as follows:

$$X_{A2}(P_2,\theta_1,\omega) = \frac{A(\theta_1)/\gamma + E(V|\theta_1,\omega)-P_2}{\gamma \mathrm{Var}(V|\theta_1,\omega)} = \frac{A(\theta_1)/\gamma + \bar{V}+\theta_1+(\nu_\theta/\nu_\omega)\omega-P_2}{\gamma(\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon)}. \tag{33}$$

From Equations (31) and (33), the market clearing condition requires

$$\begin{aligned} z_2 &= m \times X_{R2}(P_2,\theta_1,\theta_2) + m_1 \times X_{A2}(P_2,\theta_1,\omega) \\ &= m \times \frac{\bar{V}+\theta_1+\theta_2-P_2}{\gamma\nu_\epsilon} + m_1 \times \frac{A(\theta_1)/\gamma+\bar{V}+\theta_1+(\nu_\theta/\nu_\omega)\omega-P_2}{\gamma(\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon)} \\ &= \frac{m}{\gamma\nu_\epsilon}\times\theta_2 + \frac{m_1}{\gamma(\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon)}\times(\nu_\theta/\nu_\omega)\omega + \frac{m_1}{\gamma(\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon)}\times A(\theta_1)/\gamma \\ &\quad - \Big(\frac{m}{\gamma\nu_\epsilon} + \frac{m_1}{\gamma(\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon)}\Big)(\alpha(m_1)\theta_1 + \beta(m_1)\omega), \end{aligned} \tag{34}$$

where the last equality obtains by plugging in the expression of P_2 in Equation (29). Note that $A(\theta_1) = a\theta_1$. Equation (34) implies that in the conjectured price function P_2 in Equation (29),

$$\delta = \frac{\gamma\nu_\epsilon}{m}, \quad \alpha(m_1) = \frac{\frac{m_1}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}{\frac{m}{\nu_\epsilon}+\frac{m_1}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}\times(a/\gamma), \quad \beta(m_1) = \frac{\frac{m}{\nu_\epsilon}+\frac{m_1}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}\times\frac{\nu_\theta}{\nu_\omega}}{\frac{m}{\nu_\epsilon}+\frac{m_1}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}.$$

Step 2: We derive the stock price at Date 1, P_1, in this step. Suppose that given P_1, a regular non-A trader anticipates that a mass $m_1^*(P_1)$ or simply $m_1^* \in [0, 1 - m]$ of A traders will enter at Date 2, and that the stock price P_2 will take the linear form given by Equation (29). From a notational standpoint, we do not distinguish between the actual m_1 and his anticipated m_1^* here, but bring out this distinction in Step 3.

Consider the i'th regular non-A trader's expected utility at Date 2 given by Equation (30). Substituting for his optimal demand for the stock from Equation (31) yields

$$\begin{aligned} E\Big[U_R(W_{i3})|P_2,\theta_1,\theta_2\Big] &= -\exp\Big[-\gamma W_{i2} - 0.5(\bar{V}+\theta_1+\theta_2-P_2)^2/\nu_\epsilon\Big] \\ &= -\exp\Big[-\gamma W_{i2} - 0.5(\alpha(m_1)\theta_1+\beta(m_1)\omega-\theta_2)^2/\nu_\epsilon\Big], \end{aligned} \tag{35}$$

where the last equality obtains by plugging in the expression of P_2 in Equation (29).

At Date 1, the regular non-A trader does not know ω and θ_2. Therefore, we need to take expectations of his utility in Equation (35) over ω and θ_2. We use iterative expectation as follows. First, we take expectation of his utility in Equation (35) over θ_2 conditional on ω. Consider Equation (35). $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$, where the stock price P_2 is given by Equation (29), is dependent on θ_1 and ω, but does not depend on θ_2. We use this fact below. Denote $Y \equiv \frac{\alpha(m_1)\theta_1 + \beta(m_1)\omega - \theta_2}{\sqrt{\mathrm{Var}(\theta_2|\omega)}}$ with $Y|\theta_1,\omega \sim N(\frac{\alpha(m_1)\theta_1 + (\beta(m_1) - \nu_\theta/\nu_\omega)\omega}{\sqrt{\mathrm{Var}(\theta_2|\omega)}}, 1)$. From the fact in Footnote 25 and the facts, $\mathrm{Var}(V|\theta_1,\theta_2) = \nu_\epsilon$ and $\mathrm{Var}(V|\theta_1,\omega) = \nu_\epsilon + \mathrm{Var}(\theta_2|\omega)$,

$$\begin{aligned}
E\Big[U_R(W_{i3})|P_2,\theta_1,\omega\Big] &= E_{\theta_2|\omega}\Big[-\exp\Big[-\gamma W_{i2} - 0.5(\alpha(m_1)\theta_1 + \beta(m_1)\omega - \theta_2)^2/\nu_\epsilon\Big]\Big] \\
&= -\exp(-\gamma W_{i2}) \times E\Big[\exp\Big[-0.5\frac{\mathrm{Var}(\theta_2|\omega)}{\nu_\epsilon}Y^2\Big]|\theta_1,\omega\Big] \\
&= -\frac{\exp(-\gamma W_{i2})}{\sqrt{1+\mathrm{Var}(\theta_2|\omega)/\nu_\epsilon}}\exp\Bigg[-0.5\frac{\frac{\mathrm{Var}(\theta_2|\omega)}{\nu_\epsilon}\Big(\frac{\alpha(m_1)\theta_1 + (\beta(m_1)-\nu_\theta/\nu_\omega)\omega}{\sqrt{\mathrm{Var}(\theta_2|\omega)}}\Big)^2}{1+\mathrm{Var}(\theta_2|\omega)/\nu_\epsilon}\Bigg] \\
&= -\exp\Big[-\gamma W_{i2} - 0.5\frac{(\alpha(m_1)\theta_1 + (\beta(m_1)-\nu_\theta/\nu_\omega)\omega)^2}{\mathrm{Var}(V|\theta_1,\omega)}\Big]\sqrt{\frac{\mathrm{Var}(V|\theta_1,\theta_2)}{\mathrm{Var}(V|\theta_1,\omega)}},
\end{aligned} \tag{36}$$

where $E_{\theta_2|\omega}$ indicates taking expectation over θ_2 conditional on ω. The expression in the square root above is non-stochastic and depends only on exogenous parameters. We continue to take expectation of Equation (36) over ω. In Equation (36), substitute $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$, where the stock price P_2 obtains from Equation (29). Take one more expectation over ω:

$$\begin{aligned}
E\Big[U_R(W_{i3})|P_1,\theta_1\Big] &\propto E_\omega\Big[-\exp\Big[-\gamma W_{i2} - 0.5\frac{(\alpha(m_1)\theta_1 + (\beta(m_1)-\nu_\theta/\nu_\omega)\omega)^2}{\mathrm{Var}(V|\theta_1,\omega)}\Big]\Big] \\
&= -\exp\Big[-\gamma W_{i1} - \gamma X_{i1}(\bar{V} + \theta_1 + \alpha(m_1)\theta_1 - P_1) - 0.5\alpha(m_1)^2\theta_1^2/\mathrm{Var}(V|\theta_1,\omega)\Big] \\
&\quad \times E_\omega\Big[\exp\Big[-(\gamma X_{i1}\beta(m_1) + \frac{(\beta(m_1)-\nu_\theta/\nu_\omega)\alpha(m_1)\theta_1}{\mathrm{Var}(V|\theta_1,\omega)})\omega - 0.5\frac{(\beta(m_1)-\nu_\theta/\nu_\omega)^2\omega^2}{\mathrm{Var}(V|\theta_1,\omega)}\Big]\Big] \\
&= -\exp\Big[-\gamma W_{i1} - \gamma X_{i1}(\bar{V} + \theta_1 + \alpha(m_1)\theta_1 - P_1) - 0.5\frac{\alpha(m_1)^2\theta_1^2}{\mathrm{Var}(V|\theta_1,\omega)}\Big] \\
&\quad \times \sqrt{\mu(m_1)/\nu_\omega} \times \exp\Big[0.5\mu(m_1)(\gamma X_{i1}\beta(m_1) + \frac{(\beta(m_1)-\nu_\theta/\nu_\omega)\alpha(m_1)\theta_1}{\mathrm{Var}(V|\theta_1,\omega)})^2\Big],
\end{aligned} \tag{37}$$

where $\mu(m_1) = \Big[1/\nu_\omega + \frac{(\beta(m_1)-\nu_\theta/\nu_\omega)^2}{\mathrm{Var}(V|\theta_1,\omega)}\Big]^{-1}$, and the last equality follows from the fact in Footnote 27 below.[27]

At Date 1, the regular non-A trader needs to choose X_{i1} to maximize the expected utility in Equation (37). The f.o.c. w.r.t. X_{i1} implies that his demand can be expressed as:

$$X_{R1} = \frac{\frac{\bar{V}+\theta_1+\alpha(m_1)\theta_1 - P_1}{\mu(m_1)\beta(m_1)} - \frac{(\beta(m_1)-\nu_\theta/\nu_\omega)\alpha(m_1)\theta_1}{\mathrm{Var}(V|\theta_1,\omega)}}{\gamma\beta(m_1)}. \tag{38}$$

The market clearing condition, $z_1 = m \times X_{i1}$, implies

$$P_1 = \bar{V} + \theta_1 + \Big[1 - \frac{\mu(m_1)\beta(m_1)(\beta(m_1)-\nu_\theta/\nu_\omega)}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}\Big]\alpha(m_1)\theta_1 - \frac{\gamma\mu(m_1)\beta(m_1)^2}{m}z_1. \tag{39}$$

Step 3: We consider A traders' participation decision at Date 2 in this step. We first derive the mass m_1 of A traders who will enter at Date 2. Note that this m_1 is dependent on the stock price P_1 at Date 1, and that P_1 is dependent on regular non-A traders' anticipation of m_1^* (see Step 2). Then, we will require that in equilibrium, regular non-A traders' anticipation must be correct, i.e., $m_1^* = m_1$. This gives the specification for m_1.

Right before Date 2, an A trader learns from the stock price P_1 in Equation (39), $s^* = \theta_1 - \tau(m_1^*)z_1$, where

$$\tau(m_1^*) = \frac{\gamma\mu(m_1^*)\beta(m_1^*)^2/m}{1+\left[1-\frac{\mu(m_1^*)\beta(m_1^*)(\beta(m_1^*)-\nu_\theta/\nu_\omega)}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}\right]\alpha(m_1^*)}.$$

s^* has a variance $\nu_s^* = \nu_\theta + \tau(m_1^*)^2\nu_z$. Here, we use m_1^* to indicate that P_1 is based on regular non-A traders' anticipated m_1.

We need to compare an A trader's expected utility from entering (and trading) and not entering. If an A trader does not enter, then his expected utility equals $-\exp(-\gamma\bar{W}_{i2})$. If he enters and trades, then his expected utility at Date 2 is given by Equation (32). Accounting for the participation cost, c_p, and plugging in the optimal demand for the stock from Equation (33) yields

$$\begin{aligned} E\left[U_A(W_{i3},X_{i2})|P_2,\theta_1,\omega\right] &= -\exp\left[-\gamma(\bar{W}_{i2}-c_p)-0.5\frac{(A(\theta_1)/\gamma+\bar{V}+\theta_1+(\nu_\theta/\nu_\omega)\omega-P_2)^2}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}\right] \\ &= -\exp\left[-\gamma(\bar{W}_{i2}-c_p)-0.5\frac{(A(\theta_1)/\gamma-\alpha(m_1)\theta_1-(\beta(m_1)-\nu_\theta/\nu_\omega)\omega)^2}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}\right], \end{aligned} \tag{40}$$

where the last equality obtains by substituting for P_2 from Equation (29). Here we use the actual mass m_1 of A-traders.

Prior to Date 2, the A trader knows only s^* and does not know $P2$, θ_1, and ω. Thus, we need to take expectations of Equation (40) conditional on s^*. Note that $A(\theta_1) = a\theta_1$. Denote

$$Y \equiv \frac{(a/\gamma-\alpha(m_1))\times\theta_1-(\beta(m_1)-\nu_\theta/\nu_\omega)\omega}{\sqrt{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}}$$

with $Y|s^* \sim N\left(\frac{(a/\gamma-\alpha(m_1))\nu_\theta/\nu_s\times s^*}{\sqrt{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}},1\right)$. It follows from the fact in Footnote 25 that

$$\begin{aligned} &E\left[U_A(W_{i3},X_{i2})|s^*\right] \\ &= E\left[-\exp\left[-\gamma(\bar{W}_{i2}-c_p)-0.5\frac{(A(\theta_1)/\gamma-\alpha(m_1)\theta_1-(\beta(m_1)-\nu_\theta/\nu_\omega)\omega)^2}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}\right]|s^*\right] \\ &= -\exp\left[-\gamma(\bar{W}_{i2}-c_p)\right] \\ &\quad\times E_{Y|s^*}\left[\exp(-0.5\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}Y^2)\right] \\ &= -\frac{\exp\left[-\gamma(\bar{W}_{i2}-c_p)\right]}{\sqrt{1+\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}} \\ &\quad\times\exp\left[-0.5\frac{\frac{(a/\gamma-\alpha(m_1))^2(\nu_\theta/\nu_s^*)^2\times s^{*2}}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}{1+\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}\right]. \end{aligned}$$

A simple comparison between the A trader's expected utilities from entering (and trading) and from not entering indicates that his entry decision is based on the function

$$\xi(m_1,s^{*2}) \equiv \frac{\exp\left[\gamma c_p-0.5\frac{\frac{(a/\gamma-\alpha(m_1))^2(\nu_\theta/\nu_s^*)^2\times s^{*2}}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}{1+\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}\right]}{\sqrt{1+\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\nu_\theta/\nu_s^*)+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}}-1. \tag{41}$$

If $\xi(m_1, s^{*2}) < 0$, then the A trader will enter. If $\xi(m_1, s^{*2}) > 0$, then he will not enter. If $\xi(m_1, s^{*2}) = 0$, then he is indifferent between entering and not entering.

To show that m_1 is uniquely specified by the function $\xi(m_1, s^{*2})$ (particularly, that an interior $m_1 \in (0, 1 - m)$ is uniquely determined by $\xi(m_1, s^{*2}) = 0$), it suffices to show that $\xi(m_1, s^{*2})$ decreases in s^{*2} and increases in m_1. Consider the expression for $\xi(m_1, s^{*2})$ in Equation (41). It is evident that $\xi(m_1, s^{*2})$ decreases in s^{*2}. To show that $\xi(m_1, s^{*2})$

increases in m_1, note from the proof in Step 1 that $\delta = \gamma\nu_\varepsilon/m$ and therefore $\nu_\omega = \nu_\theta + \delta^2\nu_z$ do not depend on m_1, and that

$$a/\gamma - \alpha(m_1) = \frac{m/\nu_\epsilon \times (a/\gamma)}{\frac{m}{\nu_\epsilon} + \frac{m_1}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}, \text{ and } \beta(m_1) - \nu_\theta/\nu_\omega = \frac{m/\nu_\epsilon \times (1-\nu_\theta/\nu_\omega)}{\frac{m}{\nu_\epsilon} + \frac{m_1}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}},$$

are positive and decrease in m_1. Substituting for the quantities in the right-hand sides above into Equation (41), it is straightforward to show that $\xi(m_1, s^{*2})$ increases in m_1. Taken together, m_1 is uniquely specified by the function $\xi(m_1, s^{*2})$.

In equilibrium, non-A traders' conjecture of m_1 must be correct, i.e., $m_1^* = m_1$. Then, $s^* = \theta_1 - \tau(m_1)z_1$, and its variance $\nu_s^* = \nu_\theta + \tau(m_1)^2\nu_z$. Substituting for these quantities into the expression for $\xi(m_1, s^{*2})$ in Equation (41) yields

$$\xi(m_1, (\theta_1 - \tau(m_1)z_1)^2) \equiv \frac{\exp\left[\gamma c_p - 0.5\frac{\frac{(a/\gamma-\alpha(m_1))^2(\frac{\nu_\theta}{\nu_\theta+\tau(m_1)^2\nu_z})^2\times(\theta_1-\tau(m_1)z_1)^2}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}{1+\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\frac{\nu_\theta}{\nu_\theta+\tau(m_1)^2\nu_z})+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}\right]}{\sqrt{1+\frac{(a/\gamma-\alpha(m_1))^2\nu_\theta(1-\frac{\nu_\theta}{\nu_\theta+\tau(m_1)^2\nu_z})+(\beta(m_1)-\nu_\theta/\nu_\omega)^2\nu_\omega}{\nu_\theta(1-\nu_\theta/\nu_\omega)+\nu_\epsilon}}} - 1. \tag{42}$$

Therefore, the equilibrium m_1 is specified as follows. First fix θ_1 and z_1; if $\xi(0, (\theta_1 - \tau(0)z_1)^2) \geq 0$, no A traders enter, so that $m_1 = 0$; if $\xi(1 - m, (\theta_1 - \tau(1 - m)z_1)^2) \leq 0$, all A traders enter, so that $m_1 = 1 - m$; if $\xi(0, (\theta_1 - \tau(0)z_1)^2) < 0 < \xi(1 - m, (\theta_1 - \tau(1 - m)z_1)^2)$, an interior m_1 is given by $\xi(m_1, (\theta_1 - \tau(m_1)z_1)^2) = 0$.

Step 4: This step shows that in equilibrium, $P_0 = \bar{V}$.

Consider the i'th regular non-A trader's expected utility at Date 1 given by Equation (37). Substituting for his optimal demand from Equation (38), and using the fact implied by the market clearing condition, $z_1 = m \times X_{i1}$, yields

$$\begin{aligned} E\left[U_R(W_{i3})|P_1, \theta_1\right] &\propto -\sqrt{\mu(m_1)/\nu_\omega} \times \exp\Big[-\gamma W_{i1} - 0.5\frac{\alpha(m_1)^2\theta_1^2}{\mathrm{Var}(V|\theta_1,\omega)} \\ &\quad + 0.5\mu(m_1)(\frac{\beta(m_1)-\nu_\theta/\nu_\omega}{\mathrm{Var}(V|\theta_1,\omega)})^2\alpha(m_1)^2\theta_1^2 - 0.5\mu(m_1)(\gamma X_{i1}\beta(m_1))^2\Big] \\ &= -\sqrt{\mu(m_1)/\nu_\omega} \times \exp(-\gamma W_{i1} - 0.5\kappa_\theta(m_1)\theta_1^2 - 0.5\kappa_z(m_1)z_1^2), \end{aligned} \tag{43}$$

where

$$\begin{aligned} \kappa_\theta(m_1) &= \left[\frac{1}{\mathrm{Var}(V|\theta_1,\omega)} - \mu(m_1)(\frac{\beta(m_1)-\nu_\theta/\nu_\omega}{\mathrm{Var}(V|\theta_1,\omega)})^2\right] \times \alpha(m_1)^2, \\ \kappa_z(m_1) &= \mu(m_1)(\frac{\gamma\beta(m_1)}{m})^2. \end{aligned}$$

Note that, as Step 3 demonstrates, m_1 is also determined by θ_1 and z_1.

At Date 0, the regular non-A trader does not observe θ_1 and z_1. Therefore, we need to take expectations of his utility in Equation (43) over θ_1 and z_1. Substituting $W_{i1} = \bar{W}_{i0} + X_{i0}(P_1 - P_0)$, expressing P_1 in Equation (39) as $\bar{V} + p_\theta(m_1)\theta_1 - p_z(m_1)z_1$ where

$$\begin{aligned} p_\theta(m_1) &= 1 + \left[1 - \frac{\mu(m_1)\beta(m_1)(\beta(m_1)-\nu_\theta/\nu_\omega)}{\mathrm{Var}(V|\theta_1,\omega)}\right] \times \alpha(m_1), \\ p_z(m_1) &= \gamma\mu(m_1)\beta(m_1)^2/m, \end{aligned}$$

and taking expectations over θ_1 and z_1 yields

$$\begin{aligned} E\left[U_R(W_{i3})|P_0\right] \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -\sqrt{\mu(m_1)/\nu_\omega} \times \exp\Big[-\gamma\bar{W}_{i0} - \gamma X_{i0}(\bar{V} + p_\theta(m_1)\theta_1 - p_z(m_1)z_1 - P_0) \\ - 0.5\kappa_\theta(m_1)\theta_1^2 - 0.5\kappa_z(m_1)z_1^2\Big]d\Phi(\theta_1/\sqrt{\nu_\theta})d\Phi(z_1/\sqrt{\nu_z}). \end{aligned} \tag{44}$$

The non-A trader needs to choose X_{i0} to maximize the unconditional expected utility in Equation (44). Taking the f.o.c. w.r.t. X_{i0} and imposing $X_{i0} = 0$ from the market clearing condition yields

$$0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\gamma(\bar{V} + p_\theta(m_1)\theta_1 - p_z(m_1)z_1 - P_0)$$
$$\times \exp\Big[-\gamma\bar{W}_{i0} - 0.5\kappa_\theta(m_1)\theta_1^2 - 0.5\kappa_z(m_1)z_1^2\Big]\mathrm{d}\Phi(\theta_1/\sqrt{\nu_\theta})\mathrm{d}\Phi(z_1/\sqrt{\nu_z}).$$

For $P_0 = \bar{V}$ to be the equilibrium stock price, it suffices that

$$0 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\gamma(p_\theta(m_1)\theta_1 - p_z(m_1)z_1)\times$$
$$\times \exp\Big[-\gamma\bar{W}_{i0} - 0.5\kappa_\theta(m_1)\theta_1^2 - 0.5\kappa_z(m_1)z_1^2\Big]\mathrm{d}\Phi(\theta_1/\sqrt{\nu_\theta})\mathrm{d}\Phi(z_1/\sqrt{\nu_z}). \tag{45}$$

Observe that in Equation (45), the integration occurs across the signed θ_1 and z_1. Note also that Also, m_1 is determined by θ_1 and z_1 as per Equation (42), and that θ_1 and z_1 are normally distributed with mean zero. Equation (45) then follows from the fact that the distributions of θ_1 and z_1 are symmetric about zero.