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Original Article

Phase shift, amplification, oscillation and attenuation of solitons in nonlinear optics

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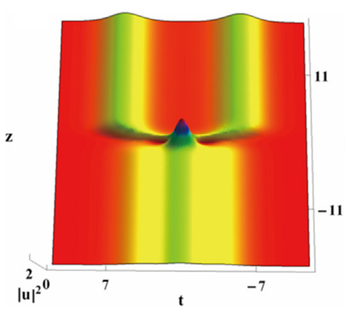
HIGHLIGHTS

- Effects of the reciprocal of group velocity on solitons were discussed.
- Energy exchange of solitons occured during the phase shift.
- Solitons in a bound state were amplified or attenuated.
- Parabolic soliton interactions were analysed to decrease the interactions.
- Parabolic solitons can be reduced to dromion-like structures.

GRAPHICAL ABSTRACT



Interactions between solitons can be controlled through adjusting the phase shift of solitons.

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ABSTRACT

In nonlinear optics, the soliton transmission in different forms can be described with the use of nonlinear Schrödinger (NLS) equations. Here, the soliton transmission is investigated by solving the NLS equation with the reciprocal of the group velocity $\beta_1(z)$, the group velocity dispersion coefficient $\beta_2(z)$ and nonlinear coefficient $\gamma(z)$. Two-soliton solutions for the NLS equation are obtained through the Hirota method. According to the solutions obtained, $\beta_1(z)$ and $\gamma(z)$ with different function forms are taken to study the characteristics of solitons. The effect of the phase shift on the soliton interaction is discussed, and the non-oscillating soliton amplification, which is transmitted in a bound state, is explored. Parabolic solitons with oscillations are analysed. Moreover, parabolic solitons can be reduced to dromion-like structures. Results indicate that the transmission of solitons can be adjusted with the group velocity dispersion and Kerr nonlinearity coefficients. The phase shift, amplification, oscillation and attenuation of solitons can also be controlled by other related parameters. This work accomplishes the theoretical study of transmission characteristics of optical solitons in spatially dependent inhomogeneous optical fibres. The conclusions of this research have theoretical guidance for the research of optical amplifier, all-optical switches and mode-locked lasers.

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Introduction

Solitons have been investigated in such fields as mathematics and physics [1–6]. They can propagate in a long distance without changes in their waveform, velocity and amplitude [7–10]. The soliton phenomena are closely related to nonlinear evolution equation models [11–17]. Solitons are also applied in particle physics, fluid mechanics, Bose-Einstein condensation and nonlinear optics [18–20].

Some researchers have studied solitons by solving the nonlinear evolution equations and analysing their soliton solutions [21–25]. As one of the classic nonlinear evolution equations, the nonlinear Schrödinger (NLS) equation can be solved to obtain soliton solution, and has been widely investigated by using different methods [26–32]. With the differential quadrature method, the dynamic problems constructed by the NLS equation have been analysed [26]. The existence and stability of the standing wave solutions for the NLS equation in n-dimensional space have been studied [27]. Using the generalized exponential rational function, a new method to solve the exact special solutions of the NLS equation has been proposed [28]. The unified method has been used to acquire optical soliton solutions of the NLS equation [29]. Moreover, the stability of full dimensional KAM tori for the NLS equation has been proved [30]. Local Cauchy theory for the NLS equation has been discussed [31], and nonlinear instability of half-solitons has been analysed [32].

However, the soliton transmission process can be simulated more accurately with the variable coefficient NLS (vcNLS) equation, when the transmission medium or boundary condition is not uniform [33–40]. For the vcNLS equation, dynamics of solitons have been explored [33], and soliton interactions have been discussed [34,35]. In addition, the breather-to-soliton transitions for the vcNLS equation have been found [36], and nonautonomous multi-peak solitons have been obtained [37].

The different transmission characteristics of solitons can be obtained by solving this vcNLS equation,

$$i\frac{\partial u}{\partial z}+i\beta_1(z)\frac{\partial u}{\partial t}+\beta_2(z)\frac{\partial^2 u}{\partial t^2}+\gamma(z)|u|^2u=0, \quad (1)$$

where $u(z,t)$ describes the temporal envelope of solitons; z and t represent the longitudinal coordinate and the time in the moving coordinate system, respectively; and $\beta_1(z)$, $\beta_2(z)$ and $\gamma(z)$ are related to the reciprocal of the group velocity, group velocity dispersion (GVD) coefficient and the nonlinearity coefficient, respectively. If $\beta_1(z)=0$, Eq. (1) will become the standard vcNLS equation.

Eq. (1) can be used to describe the transmission of solitons. The discussion of $\beta_1(z)$ and $\beta_2(z)$ is helpful to the development of dispersion management communication systems [38]. In addition, the study of $\beta_1(z)$, $\beta_2(z)$ and $\gamma(z)$ is of significance to the experimental and engineering application of mode-locked fiber lasers and nonlinear optics. The rogue wave solutions for Eq. (1) have been solved with similarity transformation [39], and the properties of oscillating solitons for Eq. (1) have been analysed [40].

However, parallel solitons, parabolic solitons and dromion-like solitons obtained by taking different functions for $\beta_1(z)$ and $\gamma(z)$ have not been reported. The effect of the phase shift on the transmission of parallel solitons will be discussed. Here, the non-oscillating parallel soliton amplification, which transmits in bound states, will be studied. Parabolic solitons with oscillations will be analysed. The method of the parabolic soliton reducing to the dromion-like structure will be proposed by adjusting the corresponding parameters.

In Section ‘Material and methods’, the two-soliton solutions will be solved through use of the Hirota method. In Section ‘Results and discussion’, different soliton transmission characteristics are analysed with the variable coefficients taking different functions. And in Section ‘Conclusions’, the conclusions will be derived.

Material and methods

Firstly, the following independent variable transformation is introduced to solve the bilinear form according to the Hirota method [40],

$$u(z,t)=\frac{g(z,t)}{f(z,t)}, \quad (2)$$

where $g(z,t)$ and $f(z,t)$ are the complex function and real function, respectively. Then the bilinear form is obtained,

$$\left[iD_z+i\beta_1(z)D_t+\beta_2(z)D_t^2\right]g\cdot f=0,$$

$$\beta_2(z)D_t^2f\cdot f-\gamma(z)gg^*=0.$$

where $*$ is the complex conjugate. Moreover, the bilinear operators D_z and D_t are defined as follows

$$D_z^mD_t^n(G\cdot F)=\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^m\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^nG(z,t)F(z',t')|_{z'=z,t'=t}.$$

According to the Hirota method, the bilinear form can be solved by the following power series expansions of $g(z,t)$ and $f(z,t)$,

$$g(z,t)=\varepsilon g_1(z,t)+\varepsilon^3g_3(z,t)+\varepsilon^5g_5(z,t)+\cdots,$$

$$f(z,t)=1+\varepsilon^2f_2(z,t)+\varepsilon^4f_4(z,t)+\varepsilon^6f_6(z,t)+\cdots,$$

Here ε is a formal expansion parameter.

In order to obtain the two-soliton solutions, it can be assumed that

$$g(z,t)=\varepsilon g_1(z,t)+\varepsilon^3g_3(z,t),$$

$$f(z,t)=1+\varepsilon^2f_2(z,t)+\varepsilon^4f_4(z,t),$$

where

$$g_1(z,t)=e^{Q_1(z,t)}+e^{Q_2(z,t)}, f_4(z,t)=n_5(z)e^{Q_1(z,t)+Q_2(z,t)+Q_1^*(z,t)+Q_2^*(z,t)},$$

$$g_3(z,t)=m_1(z)e^{Q_1(z,t)+Q_2(z,t)+Q_1^*(z,t)}+m_2(z)e^{Q_1(z,t)+Q_2(z,t)+Q_2^*(z,t)},$$

$$f_2(z,t)=n_1(z)e^{Q_1(z,t)+Q_1^*(z,t)}+n_2(z)e^{Q_2(z,t)+Q_1^*(z,t)}+n_3(z)e^{Q_1(z,t)+Q_2^*(z,t)}+n_4(z)e^{Q_2(z,t)+Q_2^*(z,t)}.$$

In addition,

$$Q_1(z,t)=k_{11}(z)+ik_{12}(z)+(w_{11}+iw_{12})t+\theta_{11}+i\theta_{12},$$

$$Q_2(z,t)=k_{21}(z)+ik_{22}(z)+(w_{21}+iw_{22})t+\theta_{21}+i\theta_{22}.$$

Finally, one can obtain the solutions,

$$m_1(z)=\frac{s_{12}\gamma(z)}{8w_{21}^2s_{11}\beta_2(z)}, m_2(z)=\frac{s_{12}\gamma(z)}{8w_{21}^2s_{11}\beta_1(z)},$$

$$\beta_2(z)=c\gamma(z), n_5(z)=\frac{\left[(w_{11}-w_{21})^2+(w_{12}-w_{22})^2\right]^2}{64w_{11}^2w_{21}^2\left[(w_{11}+w_{21})^2+(w_{12}-w_{22})^2\right]^2},$$

$$k_{11}(z)=-\int[w_{11}\beta_1(z)+2w_{11}w_{12}\beta_2(z)]dz, n_1(z)=\frac{\gamma(z)}{8w_{11}^2\beta_2(z)},$$

$$k_{21}(z)=-\int[w_{21}\beta_1(z)+2w_{21}w_{22}\beta_2(z)]dz, n_2(z)=\frac{\gamma(z)}{2s_{21}\beta_2(z)},$$

$$k_{12}(z) = -\int [w_{12}\beta_1(z) - w_{11}^2\beta_2(z) + w_{12}^2\beta_2(z)]dz, n_3(z) = \frac{\gamma(z)}{2s_{11}\beta_2(z)},$$

$$k_{22}(z) = -\int [w_{22}\beta_1(z) - w_{21}^2\beta_2(z) + w_{22}^2\beta_2(z)]dz, n_4(z) = \frac{\gamma(z)}{8w_{21}^2\beta_2(z)},$$

and

$$s_{11} = (w_{11} + iw_{12} + w_{21} - iw_{22})^2, s_{12} = (w_{11} + iw_{12} - w_{21} - iw_{22})^2,$$

$$s_{21} = (w_{11} - iw_{12} + w_{21} + iw_{22})^2, s_{22} = (w_{11} - iw_{12} - w_{21} + iw_{22})^2.$$

Results and discussion

By analysing the preceding two-soliton solutions, we find that if $c = 1$, then the GVD coefficient $\beta_2(z)$ is equal to the nonlinearity coefficient $\gamma(z)$. Therefore, the different soliton transmission characteristics can be obtained by discussing the reciprocal of the group velocity $\beta_1(z)$ and the nonlinearity coefficient $\gamma(z)$.

In order to obtain the effect of the phase shift on parallel soliton transmission, one takes $\beta_1(z)$ as the Gauss function, and $\beta_2(z) = \gamma(z) = 2\beta_1(z)$ as shown in Fig. 1. In Fig. 1(a) and (b), the phase shift of two solitons to the left influences the interaction between two solitons. In order to keep two solitons from interacting with each other after the phase shift, one can adjust the parameter w_{11}. Comparing Fig. 1(b) with Fig. 1(c) or Fig. 1(a) with Fig. 1(d), one can find that the direction of the phase shift is related to the signs of w_{12} and w_{22}. When w_{12} and w_{22} are positive, two solitons shift to the left, while two solitons move in opposite directions when they are negative. The larger the value of $|w_{12}|$ or $|w_{22}|$, the greater is the phase shift distance of the soliton. Therefore, one can adjust the value of w_{22} as shown in Fig. 2 in order to separate the two interacting solitons after the phase shift. Moreover, the amplitude of the soliton increases as $|w_{11}|$ or $|w_{21}|$ increases.

To observe whether soliton energy exchange occurs during the phase shift, we can measure the amplitude of the soliton before and after the phase shift as shown in Fig. 2. In Fig. 2(a) and (b), the amplitudes of two solitons before and after the phase shift have obvious changes. However, the amplitudes of two solitons before and after the phase shift are almost the same in Fig. 2(c) and (d). Therefore, when two solitons are simultaneously shifted to the left, the energy transfer between solitons occurs. When two solitons are in opposite phase shifts, they remain constant in energy. Moreover, it can be seen from Figs. 1(c) and 2(c) that two solitons retain shape and amplitude after the collision except for a certain phase shift, which indicates that the collision is elastic. These analyses can help eliminate the interaction between solitons and change the direc-

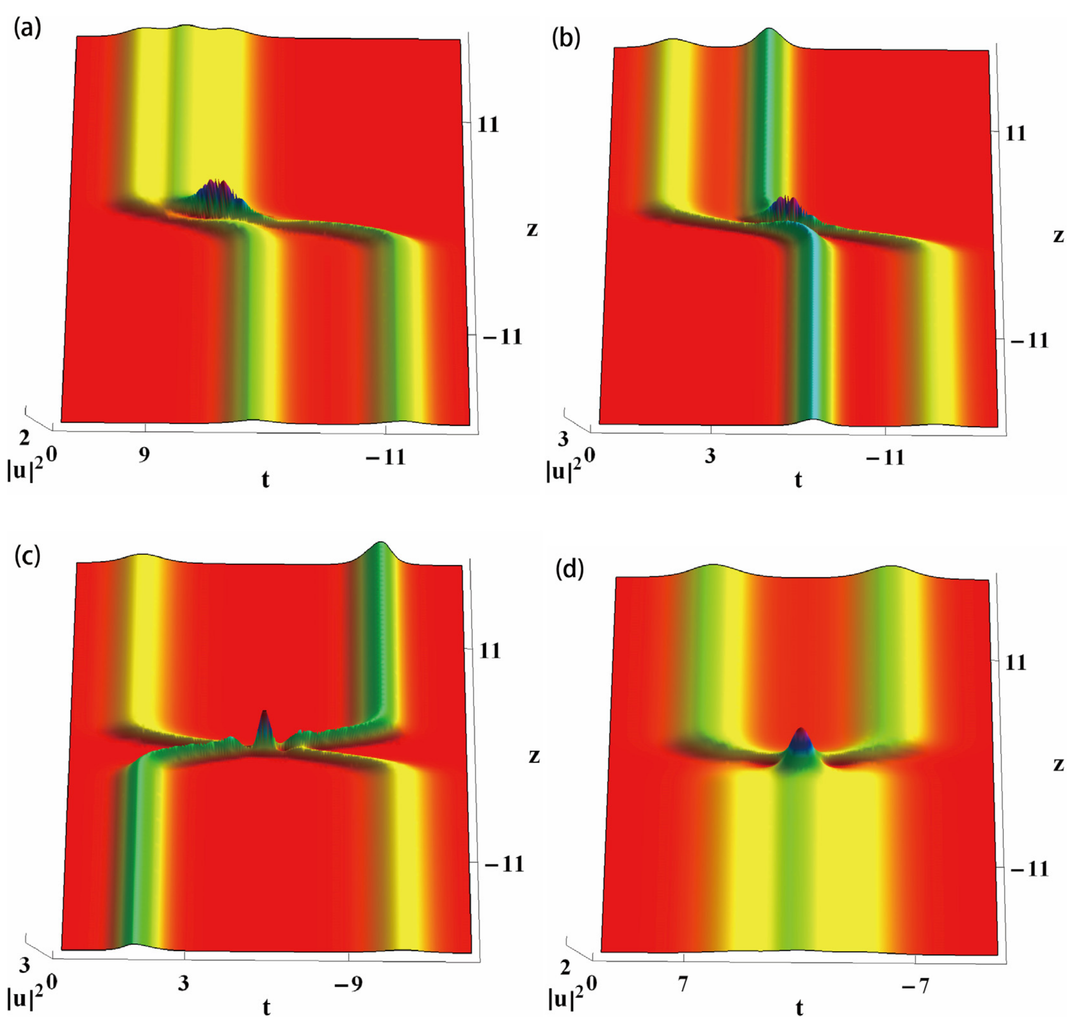

Fig. 1. The phase shift has an effect on the transmission of parallel solitons. The parameters are $\theta_{11} = -1, \theta_{12} = 2, \theta_{21} = -1, \theta_{22} = 2, \beta_1(z) = 2e^{-0.5z^2}$ and $\gamma(z) = 2\beta_1(z)$ with (a): $w_{11} = -0.5$, $w_{12} = 0.031$, $w_{21} = 0.5$ and $w_{22} = 0.69$; (b): $w_{11} = -0.78$, $w_{12} = 0.031$, $w_{21} = -0.5$ and $w_{22} = 0.69$; (c): $w_{11} = -0.78$, $w_{12} = -1.1$, $w_{21} = -0.5$ and $w_{22} = 0.69$; (d): $w_{11} = 0.5$, $w_{12} = 0.031$, $w_{21} = -0.5$ and $w_{22} = 0.53$.

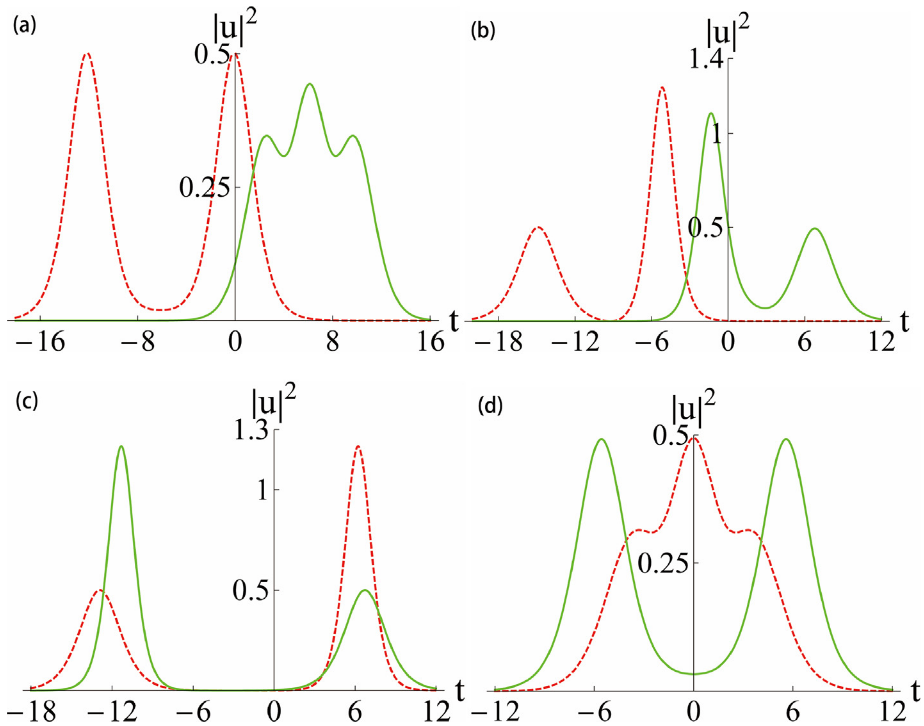

Fig. 2. Amplitude comparison between two solitons at $z = -11$ (red dotted line) and $z = 11$ (green solid line) in (a): Fig. 1(a); (b): Fig. 1(b); (c): Fig. 1(c); (d): Fig. 1(d).

tion of the soliton transmission. This feature is useful for reducing the bit error rate, improving the quality of optical communication and studying all-optical switches.

Then, the effect of the phase shift on parallel solitons with the oscillation is discussed. In order to obtain the parallel solitons with oscillations, $\beta_1(z) = 2e^{-5z^2} + \sin z$ and $\gamma(z) = 4e^{-0.5z^2}$ are taken as Fig. 3 shows. In Fig. 3(a) and (b), one can find that the amplitude and phase shift of solitons can be changed by adjusting the parameters. The amplitude of the left soliton increases as $|w_{11}|$ or $|w_{21}|$ is increased. However, the amplitude of the right soliton increases with the decreasing of $|w_{11}|$ or the increasing of $|w_{21}|$. The direction of the phase shift for two solitons is related to the sign of w_{12} and w_{22}. The phase shift distance increases as the values of $|w_{12}|$ and $|w_{22}|$ are increased. And when two solitons are in opposite phase shifts, elastic collision occurs during the phase shift. Because they have no change in amplitude and shape before and after the phase shift as shown in Fig. 4(a) and (b). From Fig. 3(c) and (d), one can see that not only are two solitons simultaneously shifted to the right, but also two solitons are shifted to the left at the same time. Further, in the process of the phase shift, the energy of two solitons is exchanged. Their amplitude changes before and after the phase shift, as shown in Fig. 4(c) and (d). These investigations can help weaken the energy exchange with the oscillatory parallel soliton during the phase shift. These studies can be helpful in the study of mode-locked fibre lasers.

The effect of the phase shift on the parallel soliton transmission and energy exchange was investigated. Next, we discuss the non-oscillating parallel soliton amplification, which transmits in bound states. In order to study the transmission, one takes $\beta_1(z) = -\frac{0.55}{1+5z}$ and $\gamma(z) = 4e^{-0.5z^2}$ as shown in Fig. 5. Near the $z = 0$ point, it is mainly the sudden changes in $\beta_1(z)$ and $\gamma(z)$ that lead to change in the interaction between two solitons. As a result, two solitons have exchanged energy.

In Fig. 5(a), the left soliton amplitude increases and the right soliton amplitude decreases near the $z = 0$ point. Energy is exchanged from the right soliton to the left. The situation in Fig. 5(a) and (d) is just the opposite. In Fig. 5(b), the amplitudes of two solitons are reduced at the same time near the $z = 0$ point, while the amplitudes of two solitons are amplified in Fig. 5(c). And in Fig. 5(a) and (d), the interaction between two solitons has also changed near the $z = 0$ point. This is because the inverse function taken by $\beta_1(z)$ is changed suddenly around the point of $z = 0$. However, before and after the point $z = 0$ in Fig. 5(b) and (c), the interaction between two solitons does not change. Moreover, the amplitude is also affected by $w_{ij}(i = 1, 2, j = 1, 2)$.

Their changes in amplitude are shown in Fig. 6. The interaction between two solitons is inelastic. The energy is exchanged, and the amplitude changes between them without any outside influence. This can provide some help in the development of optical amplifiers.

To investigate the effect of $\beta_1(z)$ and $\gamma(z)$ on other types of soliton transmissions, one considers $\beta_1(z) = -\cos(\arccos z)$ and $\gamma(z) = \sin 2z$. Then, a set of parabolic solitons with oscillations is obtained. In Fig. 7(a), two parabolic solitons maintain their original oscillation intensity due to the weak interaction. In Fig. 7(b), the interaction between two parabolic solitons causes the intensity of the inner parabolic soliton oscillations to become smooth. However, the interaction between parabolic solitons in Fig. 7(c) tends to smooth the intensity of the outer parabolic soliton oscillations. By analysing the parameters, one finds that w_{12} affects the soliton oscillation intensity of the inner side, and the inner soliton oscillates are exacerbated as $|w_{12}|$ increases. The outer soliton oscillation intensity is controlled by w_{22} and exacerbated with the increasing of $|w_{22}|$. Moreover, the amplitude of the inner soliton is mainly controlled by $|w_{11}|$, while the amplitude of the outer soliton is mainly influenced by $|w_{21}|$. The amplitude of the inner and outer solitons is increased with increasing $|w_{11}|$ and $|w_{21}|$, respectively. When $|w_{11}| \to 0$ or $|w_{21}| \to 0$, the two solitons decay into a soliton as shown in Fig. 7(d). Those studies are of great help in controlling the oscillation intensity of parabolic solitons so that it can promote theoretical and experimental research on soliton transmission and collision.

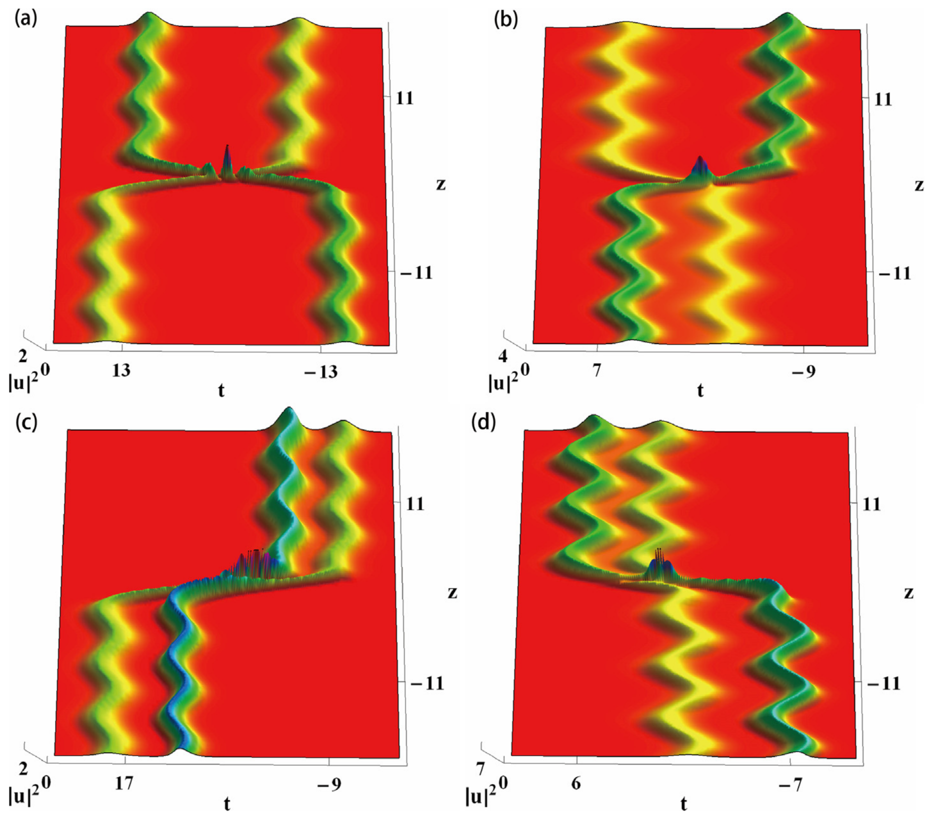

Fig. 3. Phase shift affects the parallel soliton with oscillations. The parameters are $\theta_{11} = -1$, $\theta_{12} = 2$, $\theta_{21} = -1$, $\theta_{22} = 2$, $\beta_1(z) = 2e^{-5z^2} + \sin z$ and $\gamma(z) = 4e^{-0.5z^2}$ with (a): $w_{11} = 0.47$, $w_{12} = -1.3$, $w_{21} = -0.56$ and $w_{22} = 1.2$; (b): $w_{11} = -0.84$, $w_{12} = -0.59$, $w_{21} = 0.53$ and $w_{22} = 0.22$; (c): $w_{11} = 0.5$, $w_{12} = -1.4$, $w_{21} = 0.75$ and $w_{22} = -0.84$; (d): $w_{11} = -1.3$, $w_{12} = 0.41$, $w_{21} = 0.81$and $w_{22} = 0.19$.

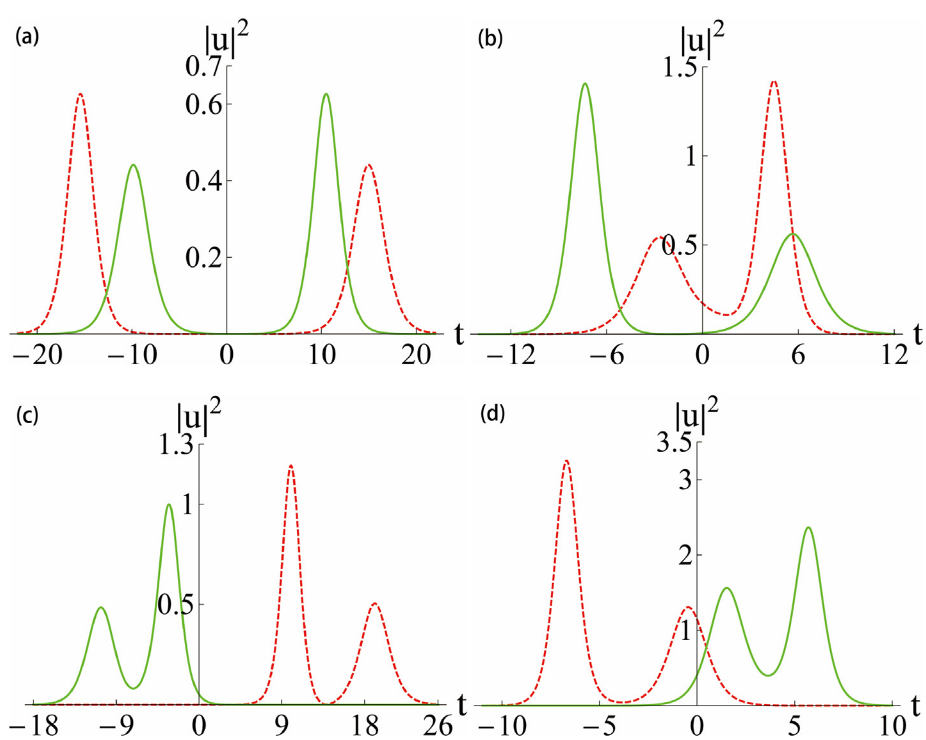

Fig. 4. Amplitude comparison between two solitons at $z = -11$ (red dotted line) and $z = 11$ (green solid line) in (a): Fig. 3(a); (b): Fig. 3(b); (c): Fig. 3(c); (d): Fig. 3(d).

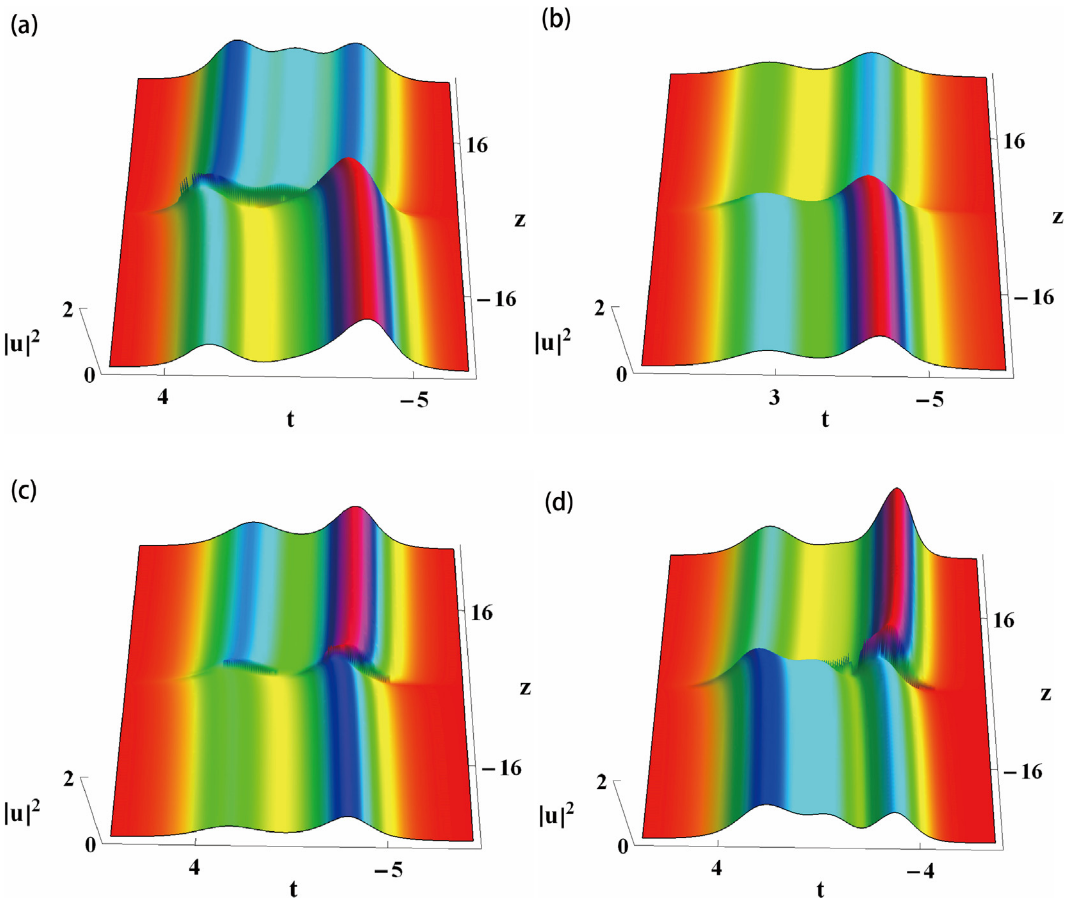

Fig. 5. The parallel soliton amplification in bound states. The parameters are $\theta_{11} = -1$, $\theta_{12} = 2$, $\theta_{21} = -1$, $\theta_{22} = 2$, $\beta_1(z) = -\frac{0.55}{1+5z}$ and $\gamma(z) = 4e^{-0.5z^2}$ with (a): $w_{11} = 0.8$, $w_{12} = 0.69$, $w_{21} = -0.78$ and $w_{22} = -0.41$; (b): $w_{11} = 0.59$, $w_{12} = -0.8$, $w_{21} = -0.38$ and $w_{22} = -0.55$; (c): $w_{11} = 0.5$, $w_{12} = 0.69$, $w_{21} = -0.94$ and $w_{22} = 0.59$; (d): $w_{11} = -0.94$, $w_{12} = 0.67$, $w_{21} = 0.63$and $w_{22} = -0.63$.

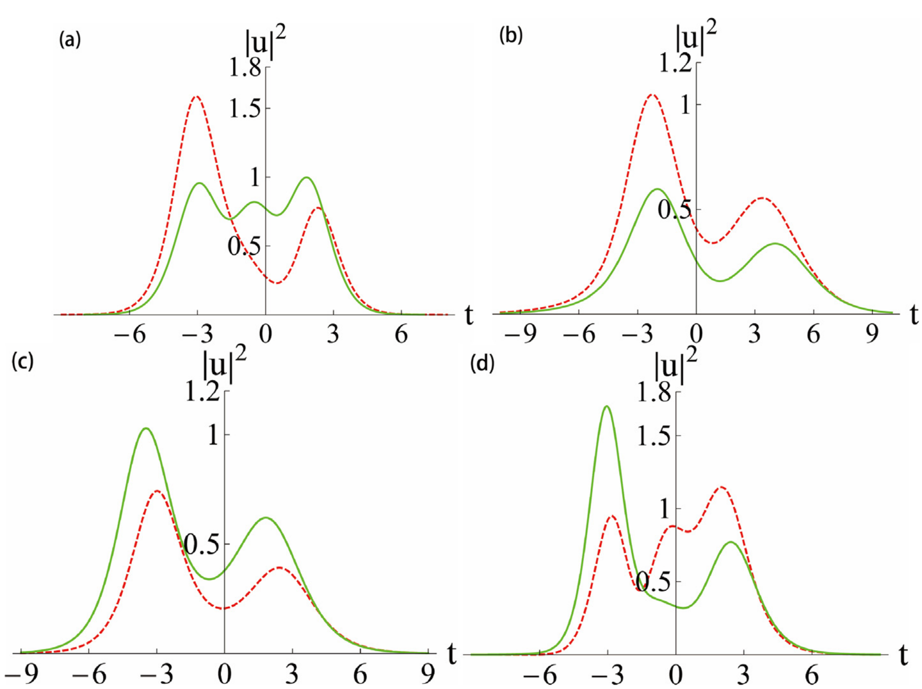

Fig. 6. Amplitude comparison between two solitons at $z = -16$ (red dotted line) and $z = 16$ (green solid line) in (a): Fig. 5(a); (b): Fig. 5(b); (c): Fig. 5(c); (d): Fig. 5(d).

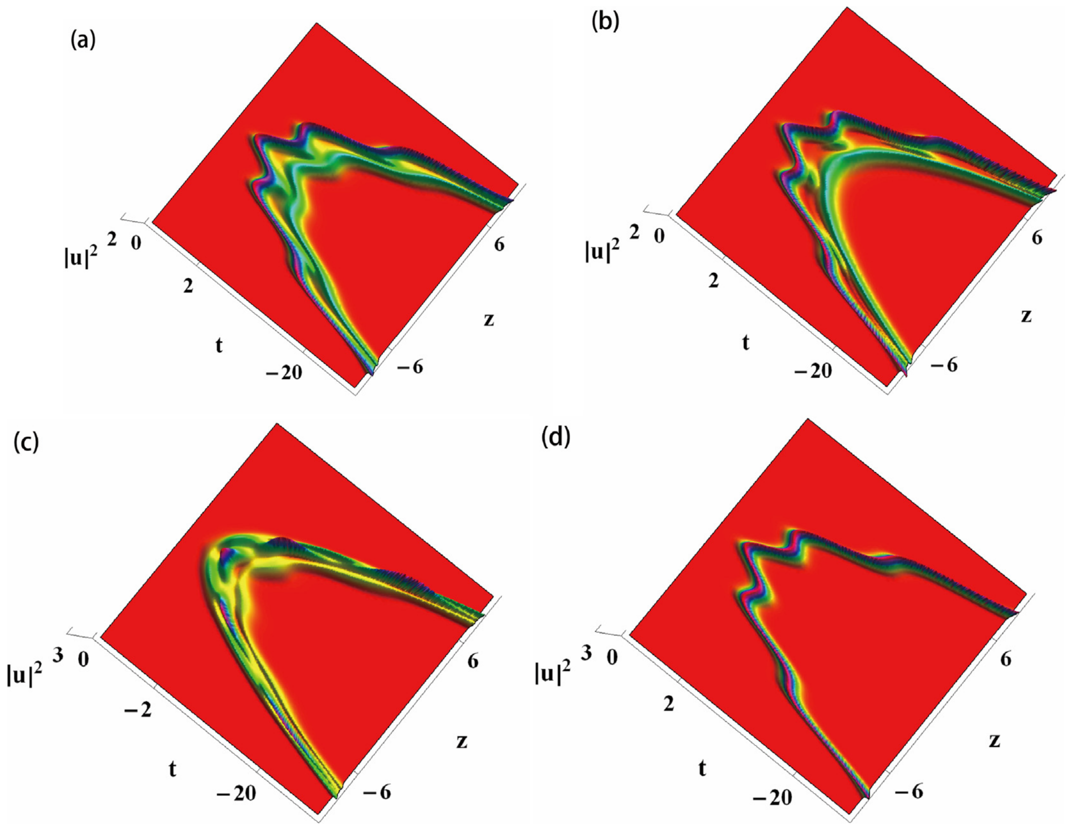

Fig. 7. The interactions between parabolic solitons with oscillations. The parameters are $\theta_{11} = -1$, $\theta_{12} = 2$, $\theta_{21} = -1$, $\theta_{22} = 2$, $\beta_1(z) = -\cos(\arccos z)$ and $\gamma(z) = \sin 2z$ with (a): $w_{11} = -0.59$, $w_{12} = -0.94$, $w_{21} = 0.83$ and $w_{22} = -1.9$; (b): $w_{11} = -0.59$, $w_{12} = 0.016$, $w_{21} = 0.83$ and $w_{22} = -1.9$; (c): $w_{11} = -0.59$, $w_{12} = -0.94$, $w_{21} = 0.83$ and $w_{22} = 0.25$; (d): $w_{11} = -0.031$, $w_{12} = -0.94$, $w_{21} = 0.83$ and $w_{22} = -1.9$.

In order to further explore the effect of $\beta_1(z)$ and $\gamma(z)$ on other types of soliton transmissions, one takes $\beta_1(z) = \arcsin z$ and $\gamma(z) = \mathrm{arc\,sinh}\, z$ as shown in Fig. 8, so that the parabolic solitons can decay into two dromion-like structures. The method of how to adjust the parameters to achieve the suitable attenuation is proposed. The attenuation of solitons is mainly influenced by w_{11}, w_{12} and w_{22}. The smaller the $|w_{11}|$ value, the faster the decay rate becomes. Only when w_{12} and w_{22} are positive can the parabolic solitons be attenuated to the dromion-like structures. Further, as w_{12} and w_{22} increase, the decay rate becomes faster. Moreover, the distance between two dromion-like structures can be adjusted by $w_{ij}(i = 1, 2, j = 1, 2)$ as shown in Fig. 8(a) and (b). Their opening directions are related to the sign of A in $\gamma(z) = \mathrm{arcsinh}Az$. These parametric analyses are of great significance in the development of Bose-Einstein condensation.

Conclusions

Two soliton solutions of Eq. (1) were obtained through use of the Hirota method. By analysing the solution, different function forms for $\beta_1(z)$ and $\gamma(z)$ were taken to obtain different types of soliton transmission. The effect of the phase shift on the transmission of smooth parallel solitons was discussed when $\beta_1(z) = 2e^{-0.5z^2}$ and

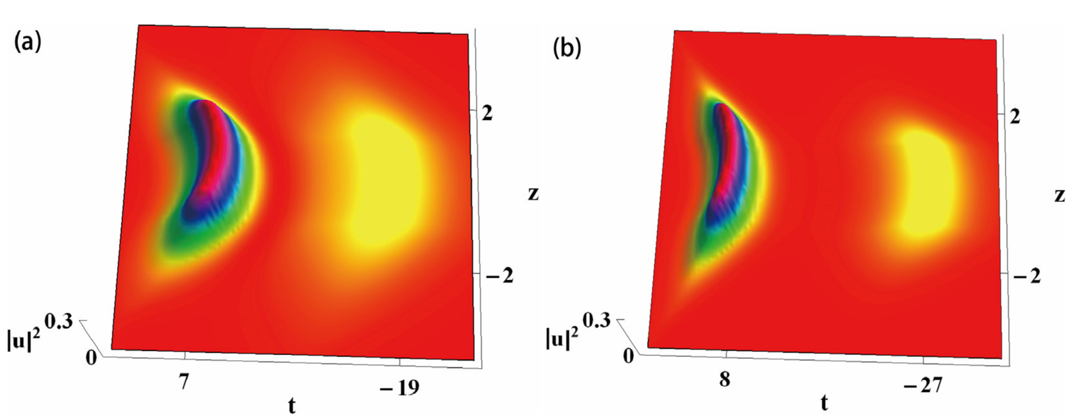

Fig. 8. The parabolic solitons decay into two dromion-like structures. The parameters are $\theta_{11} = 1$, $\theta_{12} = 1$, $\theta_{21} = 1$, $\theta_{22} = 1$, $\beta_1(z) = \arcsin z$ and $\gamma(z) = \mathrm{arc\,sinh}\, z$ with (a): $w_{11} = -0.33$, $w_{12} = 0.88$, $w_{21} = 0.14$ and $w_{22} = 0.36$; (b): $w_{11} = -0.33$, $w_{12} = 0.88$, $w_{21} = 0.14$ and $w_{22} = 0.89$.

$\gamma(z) = 2\beta_1(z)$. If one takes $\beta_1(z) = 2e^{-5z^2} + \sin z$ and $\gamma(z) = 4e^{-0.5z^2}$, the influence of the phase shift on parallel solitons was presented. In addition, the soliton amplification in a bound state has been explored when $\beta_1(z) = -\frac{0.55}{1+5z}$. When $\beta_1(z) = -\cos(\arccos z)$ and $\gamma(z) = \sin 2z$, parabolic solitons with oscillations are demonstrated. Moreover, parabolic solitons have been reduced to dromion-like structures while $\beta_1(z) = \arcsin z$ and $\gamma(z) = \text{arc}\sinh z$. Those presented results are applicable to theoretical analysis and experimental research of optical amplifier, all-optical switches and mode-locked lasers.

Conflict of interest

The authors declare they have no conflict of interest.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

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