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Analog Full Duplex Amplify-and-Forward Relay for Power Line Communication Networks

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Abstract—Signal relays in power line communication networks are usually implemented using the decode-and-forward protocol, which requires complex relays. In this paper, we propose a real time amplify-and-forward relay that is mainly based on an analog full duplex architecture. The speed of analog circuits enables the amplification of the incoming signal without introducing significant communication delay. Our system level analysis explains the optimization of the relay design and, based on a measured channel database, shows what increase in communication rate can be achieved by using the proposed solution.

Index Terms—Amplify-and-forward, relay, full duplex, power line communications.

I. INTRODUCTION

POWER line communication (PLC) networks are characterized by strong multipath propagation, by the presence of high noise and attenuation that the signals undergo. Different signal processing techniques have been proposed to cope with each of these issues [1].

To tackle the high attenuation, multi-hop communication via relay nodes has been proposed both in the context of in-home broadband communications [2] and of narrowband communications in distribution grids [3]. These papers propose different methods to enhance the relay efficiency based on the classical decode-and-forward (DF) approach. Although the throughput of the network is enhanced by the presence of one or more relays, the performance is limited by the overhead and the delay generated by the relay nodes. The amplify-andforward (AF) approach has also been proposed for PLC, but only in a time division duplexing context, therefore sharing the same limitations of the DF approach [4], [5], [6], [7].

In this letter, we exploit the full duplex (FD) technology, to propose FD AF for PLC networks. Although the FD technology has been widely investigated in the digital subscriber line (DSL) and wireless contexts, it has been studied only recently in the context of PLC [8], and the application to AF relays has, to our knowledge, not yet been discussed. Different FD AF approaches and optimal algorithms have been presented in the context of wireless networks. They have specifically addressed: reducing the self-interference [9], maximizing the relay gain [10], finding its optimal location [11], and optimally switching between HD and FD relaying [12]. All these works assume the signal processing to be performed in digital domain, where optimal solutions can be found. Furthermore, since orthogonal frequency division multiplexing (OFDM) signals are commonly deployed, the

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minimum delay between the received and forwarded signal at the digital domain relay is at least one OFDM symbol. This means that the forwarded signal acts as an interferer for the newly received OFDM symbol, thus requiring a high level of self-interference cancellation in the FD transceiver chain.

The aforementioned solutions can be adapted to the PLC context where, however, the algorithms should be tuned to cope with the linear periodic time variant (LPTV) channel behaviour (as done in [8] for interference cancellation), strong noise, high attenuation, and the jagged channel impedance. Additionaly, since PLC is a baseband communication technology, differently from wireless, it is also possible to design a wide-band analog FD AF relay using analog passive or active circuits. The advantage of such an approach is the reduced processing time, since the intrinsic delay of analog circuits in the band of interest (up to 80 MHz for broadband PLC) is typically negligible w.r.t. the sampling time of the treated signals. For this reason, the self interference at the relay does not act as an interferer but only as a gain factor for the forwarded signal. Another advantage, is the overall reduced cost of the relay, which requires only an analog circuit instead of a full power line DF modem.

The purpose of this letter is to perform a system analysis of such an analog FD AF relay and to propose, based on the properties of the PLC channel, a simple analog adaptive circuit to maximize the signal amplification at the relay, while avoiding any instability due to the positive feedback chain.

The rest of the paper is organized as follows. In Section II, we present the system model. Our proposed circuit architecture follows in Section III. Section IV is dedicated to the results and conclusions follow in Section V.

II. SYSTEM MODEL

The system model of a SISO FD AF relay is presented in Fig. 1. As discussed in the introduction, since the signal undergoes processing by only analog circuits, differently from digital solutions that would require digital signal acquisition and buffering, we can neglect the presence of signal processing delay. The relay input-output relation in frequency domain is

$$Y = \frac{AH_{SR}}{1 - A\left(H_E - \hat{H}_E\right)}X + \frac{A}{1 - A\left(H_E - \hat{H}_E\right)}N, \quad (1)$$

where Y is the signal transmitted by the relay, X is the signal generated at the source, N is the noise process, H_{SR} is the transfer function from the source to the relay, A is the so called feed-forward transfer function, which represents the signal amplification operated by the relay, and H_E is the transfer function of the echo (self-interference) created by the channel.



Fig. 1. System model of an amplify-and-forward relay.

 H_E can be to some degree directly reduced by modifying the matching impedance of the circulator employed for FD transmission (see also Fig. 2) [13]. The relay might have some further ability to reduce the self-interference, whose effect is modeled by the transfer function \hat{H}_E . \hat{H}_E is generated based on an estimate of H_E , so that the residual self-interference $\bar{H}_E = H_E - \hat{H}_E$ is minimized and possibly only the signal of interest $H_{SR}X$ is amplified. We notice from (1) that the proposed system amplifies the noise as much as the intended signal, therefore it does not modify the SNR at the relay (assuming ideal electronics with total noise figure equal to one). All the aforementioned quantities and those written later in this section are function of frequency, which is not made explicit to ease the notation.

Equation (1) represents a positive feedback system, which is stable when it satisfies the Nyquist criterion [14]. The Nyquist criterion includes a condition on the number of poles of both the numerator and denominator of (1). We now examine the presence of poles in H_{SR} and H_E that are given by the direct and echo channels. In the case of PLC, we write H_{SR} as

$$H_{SR} = \prod_{n=1}^{N} \alpha_n e^{\gamma_n} \frac{1 - \beta_n}{1 - \beta_n e^{-2\gamma_n}},$$
 (2)

where α_n , β_n and γ_n are complex functions of frequency, and in particular we have $|\beta_n| \leq 1$ (cfr. [15, Eq. 21 and 26]). On the other hand, H_E cannot be written in closed form, but it exhibits as well a series of poles in the form $1 - \beta_n e^{-2\gamma_n}$. The condition

$$1 - \beta_n e^{-2\gamma_n} = 0 \tag{3}$$

is only met when $\beta_n = 1$ and $\gamma_n = 0$, which is an impractical case in PLC, since it is verified only when the source is connected to a short circuit [15]. Moreover, the condition $\beta_n = 1$ alone is also not verified in real environments [6]. We can therefore assume that both H_{SR} and H_E have no poles and zeros that cause instability in typical real environments.

The verification of the Nyquist criterion is then left to A and \overline{H}_E , which have to be designed such that

$$P_A = N_{1-A\bar{H}_E},\tag{4}$$

where P_A is the number of poles of A with real part greater than 0, and $N_{1-A\bar{H}_E}$ is the number of anti-clockwise turns of the Nyquist curve of $A\bar{H}_E$ around the point 1. This means that a careful design of A and \bar{H}_E would allow to obtain stability even in the presence of positive feedback. However,



Fig. 2. Sketch of a realization of an analog FD AF relay (details of the circulator on top). The buffers refer to impedance decoupling stages.

 \bar{H}_E depends on the medium and therefore it might not be accurately synthesized. In this work, we assume not to have precise information about the medium (i.e., both H_E and H_{SR} are unknown). Hence, to be on a safe side the stability criterion reduces to

$$P_A = N_{1-A\bar{H}_E} = 0, (5)$$

which in other terms is met whenever

$$\left|A\bar{H}_E\right| < 1 \tag{6}$$

for any frequency. The stability criterion (6) has to be met when the FD AF relay is implemented with no-poles filters.

To summarize, when the relay cannot access precise information about H_E , or when no-poles filters are used, the stability criterion (6) is required. Conversely, when infinite impulse response (IIR) filters are used and knowledge of H_E is provided, the broader stability criterion (4) can be used.

III. PROPOSED DESIGN METHOD

The purpose of the relay design is to maximize the amplification gain A given to the incoming signal, while maintaining the system stability. This implies that also \hat{H}_E has to be tuned so that \bar{H}_E is minimized (cfr. (6)). In this section, we present some characteristics of the PLC medium that help us design A and \bar{H}_E and formulate the relay gain maximization problem.

Concerning H_E , it is known that echo mitigation can be realized in three ways (and their combinations): using digital filters, using analog filters and matching the impedance at the circulator. While the first two methods rely on the use of the filter \hat{H}_E , the third acts on the circulator impedance to directly reduce the value of H_E . In this letter, we focus only on this third method, therefore we assume $\hat{H}_E = 0$. We consider the relay to be equipped with a circulator (see Fig. 2), which enables efficient FD communications [8], [13]. Using this device, H_E becomes proportional to the input reflection coefficient ρ_{IN} defined as

$$\rho_{IN} = \frac{Z_{IN} - Z_M}{Z_{IN} + Z_M},\tag{7}$$

where Z_{IN} is the input impedance of the PLC network seen by the relay and Z_M is the matching impedance on the circulator. Therefore, the problem of minimizing H_E reduces to the problem of tuning Z_M so that $Z_M \simeq Z_{IN}$. Given the wide variations of Z_{IN} over frequency in PLC networks, a

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Fig. 3. Average experimental values of \bar{H}_E and H_{SR} .

precise broadband estimate of Z_{IN} is practically unfeasible with analog components. However, a first or second order Z_M already gives a good approximation of Z_{IN} [13]. In fact, such a Z_M ends up matching Z_C , the characteristic impedance of the cable which the relay is branched to. Such matching suppresses the near-end echo, i.e., the echo due to the impedance mismatch at the junction with the PLC network, whereas it does not suppress the far-end echo, i.e., the echo due to all the other mismatches along the network. Fig. 3a shows that a tunable first order Z_M reduces the echo more than a fixed resistive Z_M , which is the standard matching impedance proposed in the PLC literature [8]. With (6) in mind, such low values of H_E allow the relay to increase the gain provided to the forwarded signal, without reaching system instability.

Concerning A, it can be as simple as a constant multiplicative gain. However, we notice that, due to the PLC medium, both H_E and H_{SR} show on average a decreasing amplitude with frequency (see Figs. 3a and 3b). It makes sense then to apply a pre-emphasis filter before the gain stage, as often done in the AF literature [10], which uniforms the attenuation over the band of interest and helps boosting the gain.

In this work, we limit our search to a first order high pass filter. Using for example a simple RC filter, the transfer function A becomes

$$A(f) = \frac{j2\pi fRCG}{1+j2\pi fRC} = \frac{j2\pi f\tau G}{1+j2\pi f\tau},$$
(8)

where G is the gain, R is the resistance, C is the capacity, and $\tau = RC$.

In order to maximize the amplitude of the relay transfer function (first term in (1)), the filter parameters in (8) can be tuned by solving the nonlinear optimization problem:

$$\max_{G,\tau} \int_{f_1}^{f_2} \left| \frac{j 2\pi f \tau G H_{SR}}{1 + j 2\pi f \tau (1 - G \bar{H}_E(f))} \right|^2 df, \qquad (9)$$

under the constraints

$$P_X(f) \left| \frac{j2\pi f \tau G H_{SR}(f)}{1 + j2\pi f \tau (1 - G\bar{H}_E(f))} \right|^2 < P_R(f),$$

$$\tau_M > \tau > 0,$$

$$G > 0,$$
 (10)

where $f_2 - f_1$ is the frequency band of interest, $P_X(f)$ is the power spectral density (PSD) of the source signal, $P_R(f)$ is the PSD limit specified by the PLC regulations, and τ_M is the maximum time constant required by the PLC regulations to filter out the mains power. Herein, we assume $P_X(f) =$ $P_R(f) =$ -50 dBm/Hz. We remark that (10) is more restrictive than the simple stability condition (6). In fact, in order to satisfy a power limit, the system already has to be stable.

The optimization problem can be either solved entirely with analog circuits [16] or in digital domain with a twodimensional search aiming at keeping the complexity low. Since, as mentioned, the PLC channel is LPTV, this optimization problem can be either solved continuously, if analog circuits are deployed, or a set of optimal G_i and τ_i can be saved for each possible state *i* of the network. Then, a microcontroller can be used to control and select the parameters.

IV. RESULTS

The results have been obtained using a provided dataset, whose statistics have been shown to have general validity [17]. It consists of the measured transmission matrix of 1312 in-home channels, with frequencies in the band 1-80 MHz. To assess the performance of the proposed relay, we assume it to be positioned on the source-destination backbone, so that we can apply the chain rule to obtain an equivalent channel from each possible measured link. We then run the optimization algorithm presented in Section III to find the optimal tuning of the relay. We also computed the sourceto-destination achievable rate values R_R and R_D , with and without relay respectively, defined as

$$R_{R} = \int_{f_{1}}^{f_{2}} \log_{2} \left[1 + \frac{P_{X}(f) |H_{SD}(f)|^{2} |1 + H_{R}(f)|^{2}}{\sigma_{N}^{2}(f) \left(1 + |H_{R}(f)H_{RD}(f)|^{2} \right)} \right] df$$
(11)

and

$$R_D = \int_{f_1}^{f_2} \log_2 \left[1 + \frac{P_X(f) \left| H_{SD}(f) \right|^2}{\sigma_N^2(f)} \right] df, \qquad (12)$$

where we assume H_{RD} to be the transfer function from the relay to the destination and $H_{SD} = H_{SR}H_{RD}$. $\sigma_N^2(f)$ is the PSD of the noise N, which in our simulation is zero mean colored Gaussian with floor at -135 dBm/Hz, as often assumed in PLC. Finally, H_R is the optimized transfer function of the relay $H_R = A/(1 - A\bar{H}_E)$.

Concerning the optimization of the relay parameters G and τ , Fig. 4 shows that the gain \overline{G} is on average around 10 dB. On the other hand, the median value of the cutoff frequency $f_C = 1/2\pi\overline{\tau}$ is around 3 MHz, since \overline{H}_E is on average greater than H_{SR} and it has on average high values only for very low frequencies (cfr Fig. 3). However, the effect of



Fig. 4. CDF of G (left) and f_c (right).



Fig. 5. Increase of the achievable rate due to the presence of the relay.

the echo strongly limits the achievable value of $|H_R|$, whose mean value is on average just equal to 3 (CDF not shown for brevity).

Considering now the full source-relay-destination link, the presence of the relay can actually boost the achievable rate, as shown in Fig. 5. Here we plot the ratio R_R/R_D as a function of the SNR at the destination when no relay is present, defined as

$$SNR_D = \frac{\int_{f_1}^{f_2} P_X(f) \left| H_{SD}(f) \right|^2 df}{\int_{f_1}^{f_2} \sigma_N^2(f) df}.$$
 (13)

The figure shows that, when SNR_D is rather high, the relay does not significantly increase the achievable rate. On the other hand, the proposed relaying technique finds its best application in channels characterized by low values of SNR_D . This is due to the fact that channels with low attenuation perform already well so that relaying is not more beneficial, while channels that have high attenuation can get more benefit. For example, the achievable rate is on average doubled w.r.t. no relaying when $SNR_D = 10$ dB. It should also be noted that the deployment of the proposed relay is beneficial in most cases.

V. CONCLUSIONS

In this paper, we presented an analog full duplex amplifyand-forward relay for PLC. Such relay comprises a circulator with an input impedance matching circuit, an high-pass filter and an amplifier. The optimal parameters of the impedance matching filter can be set once at the time of installation, while those of the high-pass filter need to be tuned for the specific source-relay link. Numerical simulations have been run to assess the performance of the proposed relay, relying on channels coming from a measurement dataset. The results show that such relay provides a significant increase in the achievable rate w.r.t. direct transmission, when the signal-tonoise ratio at the receiver is rather low. Future research efforts include the use of more complex filters, and the application to different PLC scenarios, such as low or medium voltage distribution grids.

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