Games and Economic Behavior 113 (2019) 497–514

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Citizens or lobbies: Who controls policy?

Paolo Roberti

Department of Economics, University of Bologna, Piazza Scaravilli 2, 40126 Bologna, Italy

ARTICLE INFO

Article history:
Received 19 December 2016
Available online 31 October 2018

JEL classification:
D72
D74
D78

Keywords:
Voting
Lobbying
Citizen-candidate

ABSTRACT

This paper analyzes a model of electoral competition with uncertainty on the policy implemented by candidates. I show that this uncertainty can induce risk-averse voters to elect politicians whose policies are biased. I apply these results to a lobbying game, where candidates hold private information about their willingness to pander to lobbies once elected. I show that voters elect politicians who implement policies biased in favor of the lobby. Increasing the probability of non-pandering candidates can increase the effect of lobbying. The model thus demonstrates that uncertainty on the influence of special interests can lead to large effects of lobbying on policy.

Citizens are often uncertain about the policies candidates will implement once elected (Baker et al., 2016). Indeed, it is common for politicians to deviate from their electoral platform. This work shows how this uncertainty affects electoral outcomes and policies when voters are risk averse. In particular, voters select candidates who carry out policies which can be very different, in expected terms, from the ones adopted when there is no uncertainty.

These results can be better understood by looking at an important source of uncertainty over policy: the influence of lobbies on elected politicians. Large parts of electoral campaigns are devoted to either showing that a candidate cannot be trusted due to the controlling influence of special interests, or on the contrary, that a candidate can be trusted because she will not pander to lobbies. For example, during the 2016 US Presidential campaign the New York Times Editorial Board warned Hillary Clinton that her level of trustworthiness among voters was weak, when dealing with the Wall Street lobby.[1] The debate at hand hinges on an inherent uncertainty faced by voters. While candidates often promise that their concerns are driven solely by policy, it is difficult to predict how they will react to offers made by special interest groups once in office. Indeed, citizens often express disappointment in their elected representatives' decisions precisely on issues affected by lobbying. For example, a recent poll by CBS News/New York Times documented that 59% of Americans felt angry and disappointed by the results of a Senate vote which struck down a bipartisan measure for expanding background checks on gun owners, a topic that was subject to extensive lobbying by the National Rifle Association. Republican (86%), Democratic (95%) and Independent (83%) voters all favored this policy.[2] Moreover, multiple polls conducted by Gallup show that, when American voters are asked whether they think their congressional representatives focus on the needs of special interest groups or the needs of their constituents, half of the respondents answer special interests.[3] This range of responses suggests

E-mail address: proberti4242@gmail.com.

1 See http://www.nytimes.com/2016/02/26/opinion/mrs-clinton-show-voters-those-transcripts.html.

2 More information is available at http://www.cbsnews.com/news/poll-59-of-americans-unhappy-about-senate-gun-vote/.

3 More information available at http://www.gallup.com/poll/1600/congress-public.aspx.

https://doi.org/10.1016/j.geb.2018.10.009
0899-8256/

that elected politicians vary in their behavior with special interest groups or that some voters are not well informed about the linkages between their representatives and lobbies.

Theories that investigate lobbying under uncertainty typically assess cases of special interest groups acting before the elections by providing candidates with campaign contributions. Much of that work focuses on the informational content that lobbying can provide to voters because special interest groups are assumed to be more informed than voters with regards to candidates' skills.[4]

This paper takes a new approach, analyzing instead a model of lobbying in which rational voters are uncertain about the willingness of a candidate to trade policy for favors from lobbies, if elected. This theory allows for an examination of how uncertainty on the responsiveness to lobbying – which, in reality, is widespread – shapes the impact of special interest groups on policy. I outline a model of lobbying with risk-averse citizens whose preferences are distributed on a unidimensional policy space. Voters cast their vote and elect a politician from the pool of citizens who receives, in turn, a contribution proposal from a lobby, in exchange for a policy more in line with the lobby's preferences. There is uncertainty with regard to two types of citizens: *saints* and *merchants*. Saints do not value contributions from lobbies if elected. Therefore they do not pander to the interests of lobbies and simply implement their preferred policy. Merchants, instead, positively value these donations and, if elected, implement a policy which is a compromise between their preferred policy and the lobby's preferred policy.[5]

The main finding is that lobbying is effective: a lack of information on the responsiveness of politicians to lobbying induces risk-averse voters to elect candidates whose policies are biased in favor of the lobby. Studies on lobbying have generally either found that lobbying elected representatives influences policy, assuming that voters are not rational (Grossman and Helpman, 1994), or that rational voters in equilibrium are able to neutralize the effect of special interest groups (Besley and Coate, 2001). In the latter case, voters anticipate the distortion of lobbying on policy and strategically elect politicians with offsetting preferences who, after being lobbied, carry out policies which are equal to those implemented in a model without lobbying. In the model proposed in this paper, voters instead understand that lobbying distorts policies if the candidate is a merchant. They are, however, unable to tell if a candidate is a merchant or a saint and thus base their vote on an expected policy. The theory shows that the closer the politician's preferred policy is to the lobby's preferred policy, the lower the uncertainty regarding the policy she implements. For example, uncertainty concerning the relationship between a pro-gun Republican and the National Rifle Association does not overly worry voters because the behavior of this politician does not change significantly if she panders to the lobby or not. On the contrary, uncertainty with regard to the relationship between a Democrat and the NRA matters more to voters, because it implies a larger variation in the politician's implemented policy. Candidates with larger variance of implemented policies are less appealing to risk-averse voters who in equilibrium elect a candidate whose expected policy is closer to the lobby's preferred policy, because this implies a lower uncertainty.

Moreover, if the responsiveness of merchants to lobbying decreases, i.e. they value less the contribution from the special interest group, the effect of lobbying is reduced: merchants implement policies which are closer to the policy implemented by saints, reducing the uncertainty on the equilibrium policy. More interestingly, the probability of saint types in the population of citizens has a non-monotonic effect on the influence of lobbying on policy. If there are only merchants and the probability of saint types increases, the effect of lobbying on policy grows. Indeed, if there are only merchant types, citizens completely offset lobbying, because they correctly anticipate the implemented policy by the elected politician. Increasing the probability of saints creates uncertainty on the behavior of politicians and thus on implemented policies. Citizens in equilibrium elect a candidate whose expected policy is biased in favor of the lobby because it reduces the uncertainty on policies. If instead there is already a large uncertainty on the pool of candidates, because the probabilities of merchants and saints are balanced, increasing the probability of saints reduces the effect of lobbying on policy.

This study builds on political economy literature investigating the effectiveness of lobbying on policy, dating back to Grossman and Helpman (1994) and Besley and Coate (2001). The main contributions of the present paper to this strand of literature are the introduction of a simple way of creating uncertainty on the potential influence of lobbies and the investigation of its consequences in terms of policy and candidates' selection.[6] The inability of lobbies to influence policy, the equilibrium result proposed by Besley and Coate (2001), suggests that a richer model is needed to explain the effect of lobbies on policy. As previously discussed, the uncertainty over responsiveness to lobbying has been shown to be a relevant issue in voters' perception. Moreover, adding this specific feature to the model by Besley and Coate (2001) can shed light on the effectiveness of the policy levers put in place to address lobby influence. The regulation of lobbying activity has usually

[4] Prat (2002a, 2002b) and Coate (2004) show, for example, that lobbying reduces voters' welfare, in that either the distortion from lobbying is higher than the informational benefit to voters, or voters anticipate the distortion and some consequently switch to the candidate who does not sell out to special interest groups, thwarting the informational benefit of contributions. Ashworth (2006) shows that if there is an incumbency advantage and lobbies provide campaign contributions, the introduction of public financing of electoral campaigns can have ambiguous effects on voters' welfare. Lima and Moreira (2014) analyze a game where lobbies have private information about the competitiveness of their productive sector. In Sobbrio (2011), lobbies influence voters' decision through biased media.

[5] This dichotomy resembles the distinction used by Callander (2008) between policy motivated and office motivated candidates, the difference being that merchants are motivated by a specific perk of office – the lobbying contribution – which is a function of the implemented policy.

[6] In Felli and Merlo's (2006, 2007) rational voter models lobbying affects policy, meaning that there are no equilibria where a very extreme policy is implemented. Their analysis provides an intuition of why politicians would collude with antithetical lobbies, but leaves open the question of how and when citizens can neutralize lobbying.

had the objective of increasing transparency in the relation between lobbies and politicians. The results of this work confirm the value of these policies because increasing information on the responsiveness of politicians to lobbying gives voters the ability to properly counteract undue influence, even when politicians are discovered to cater to lobbies. Indeed, such policy has been shown to be a promising step forward in increasing public trust in government (OECD, 2014). Another relevant contribution in this field is the work by Acemoglu et al. (2013) which shows that incumbent politicians may choose policies left of the median, in order to signal to voters that they will not cater to the interests of right-wing elites, if reelected.[7] The present study differs from their work mainly because there is heterogeneity in candidates' bliss points and politicians cannot signal their integrity before (re)election. While, as shown in the equilibrium analysis, when candidates are heterogeneous in their bliss points, voters choose candidates whose expected policy is biased in the direction of the lobby, signals by candidates as analyzed by Acemoglu et al. (2013) can push in the opposite direction. In the concluding remarks I discuss promising areas of future research, where both features are present.

This paper also speaks to the literature on citizen-candidates pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997), particularly citizen-candidate models under uncertainty and the work of Grosser and Palfrey (2014), who show that private information on ideal points induces substantial political polarization. The theoretical framework in this paper complements this literature by focusing on uncertainty about the trade-off between a politician's utility from policy and another source of utility, namely contributions from lobbies.

Moreover, by showing that the effect of lobbying on policy is only present when there is uncertainty on the behavior of elected politicians, this theory can help explain the mixed results observed in the empirical literature on the influence of special interest groups.[8]

Finally, this study documents a new channel through which information on candidates conveyed, for instance by the media, can – removing the ex-ante uncertainty on the pool of merchants and saints – increase the power of voters over policy decisions. However, media can be captured by ruling elites.[9] In this respect media can perform the lobby's role, exchanging favors for policy. For example, Chakraborty and Ghosh (2016) analyze a model where candidates pander to ideologically biased media to obtain favorable endorsements.

1. The model

A society is composed by a continuum of citizens represented by set N. Each citizen has a preferred policy in the policy space $I = [0, 1]$ and cares about the implemented policy; citizen i's utility function is

$$U(q, y, i) = u(q, q^i) + \rho^i y,$$

where $q \in I$ is the implemented policy and q^i is citizen i's bliss point. Following Besley and Coate (2001) and Felli and Merlo (2006), I assume that function $u(q, q^i)$ is strictly concave in q, single-peaked and symmetric around q^i. In other words, voters are risk averse with respect to policies. The utility is linear in money, which is denoted by y. The marginal utility of money for citizen i is denoted by ρ^i. This component of the citizens' utility function will be useful when lobbying is introduced. Let f denote the density function that describes the distribution of citizens' bliss points on the policy space I and $q^M \in I$ the bliss point of the median citizen.

As in Felli and Merlo (2006), I assume the concave function $u(q, q^i)$ takes the following form:

$$u(q, q^i) = -\left(q - q^i\right)^2.$$

Using quadratic utility serves two purposes:

1. As shown by Duggan (2014), it ensures the existence of a Condorcet winner when citizens vote over lotteries.[10]
2. It guarantees closed form solutions.

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[7] Similarly, Smart and Sturm (2013) analyze the usefulness of term limits in lowering the reelection incentives of politicians, who would otherwise choose welfare decreasing policies to signal that they are not biased.

[8] Extensive empirical work has endeavored to assess the effect of lobby donations on policy; see for example Wright (1990), Stratmann (2002), Bronars and Lott (1997). Ansolabehere et al. (2003) survey this literature and conclude that lobby contributions do not seem to affect policy. Stratmann (2005) finds just the opposite, through a meta-analysis of the papers surveyed by Ansolabehere et al. (2003). Recent work focusing on revolving door phenomenons, such as that of Blanes i Vidal et al. (2012) and Luechinger and Moser (2014), indirectly find large effects of lobbying on political outcomes.

[9] See the theoretical contribution by Besley and Prat (2006) and the empirical analysis by McMillan and Zoido (2004). Interestingly, there is also evidence of an opposite media bias: Ansolabehere et al. (2005) show that media reports about lobbying contributions to politicians are inflated. For a review of the literature on the political economy of mass media, see Prat and Stromberg (2013).

[10] Duggan (2014) shows the existence of a Condorcet winner when voting over lotteries, if voter i's utility has a representation of the form $u^i(q) = \alpha^i v(q) - c(q) + \beta^i$, $v, c : I \to \mathbb{R}$, which is satisfied by quadratic utilities. See also the seminal works by Zeckhauser (1969) and Shepsle (1970) for a discussion on the existence of Condorcet winners when lotteries are admissible. Banks and Duggan (2006) provide an example with concave utilities that are a decreasing function of $|q - q^i|$, where there is no Condorcet winner when voting over lotteries.

Uncertainty over implemented policies
I assume that if elected, citizen i either implements her bliss point, q^i, with probability p, or a distortion of policy, $Q(q^i)$, with probability $1-p$. I also assume that $Q(\cdot)$ weakly increases with q^i and is twice differentiable in q^i. Thus the bliss point and the distortion of policy co-move. The distortion $Q(\cdot)$ can be considered a deviation from the candidates' bliss point, taking place after elections. There can be different rationales behind this deviation: lobbying can be one example, mistakes by politicians another one.

Voting
The winner of the election is selected according to a Condorcet method where all citizens run as candidates. Condorcet methods elect the candidate who would win a two-candidate election against each of the other candidates using a plurality vote. Moreover, preferences are single-peaked, therefore the Condorcet winner is the candidate preferred by the median voter. If there is more than one Condorcet winner, ties are broken fairly.

Timing
The timing of the game is as follows:

1. Voters observe the bliss points of candidates and select the winner according to a Condorcet method,
2. Nature draws the policy implemented by the winner,
3. The politician implements the policy drawn by Nature.

All proofs are in Appendix A.

2. The effect of uncertainty over policy: equilibrium policy

I find the subgame perfect Nash equilibrium (SPNE) of the game by backward induction. If there is no uncertainty over the implemented policy of candidates, the median voter theorem predicts that the bliss point of the median voter is implemented. When there is uncertainty regarding the policies candidates will implement, policies can be influenced in two different ways. Uncertainty can influence the *ex-post* policy (i.e. the policy implemented after Nature draws the policy of the elected politician) or the policy expected by voters before Nature moves. Influence over the implemented policy once Nature moves is unavoidable (and therefore uninteresting to show) because voters cannot predict Nature's draw: with some probability the *ex-post* policy will be different from q^M. I thus employ an *ex-ante* perspective, focusing on the policy that voters expect from the elected politician before Nature draws it. Let me define the expected policy of candidate P: $\bar{q}^P := pq^P + (1-p)Q(q^P)$, and the variance of policy: $\text{Var}^P := p(q^P)^2 + (1-p)(Q(q^P))^2 - (\bar{q}^P)^2$.

Definition 1. An equilibrium expected policy $\bar{q}^P$ is biased if it is different from the median citizen's preferred policy q^M.

Each voter i has the following expected utility from the election of candidate P:

$$\mathbb{E}[u(q^{*P_t}, q^i)] = -(q^i - \bar{q}^P)^2 - \text{Var}^P. \tag{1}$$

The winner of the election is the citizen whose bliss point maximizes the expected utility of the median voter:

$$\max_{q^P \in [0,1]} -(q^M - \bar{q}^P)^2 - \text{Var}^P. \tag{2}$$

Let me define by $q^{\min}$ the bliss point q^P which minimizes Var^P. If $q^{\min} \leq q^M$, the Condorcet winner cannot have a bliss point in $[0, q^{\min}[$, because electing a candidate with bliss point $q^{\min}$ implies a lower variance of policy and an expected policy closer to q^M, than all candidates with bliss point in $[0, q^{\min}[$. If $q^{\min} \leq q^M$, I denote by *restricted support* the interval $[q^{\min}, 1]$. Similarly, if $q^{\min} \geq q^M$, I denote by *restricted support* the interval $[0, q^{\min}]$. Let me define by R the candidate whose expected implemented policy is equal to the median voter's bliss point: $\bar{q}^R = pq^R + (1-p)Q(q^R) = q^M$. The following proposition states conditions on the distortion of policy $Q(\cdot)$ for the existence and uniqueness of a solution to the first order condition in problem (2).

Proposition 1. *If the variance weakly increases with bliss point q^P in the restricted support, the solution to the first order condition in problem (2) exists when either there is q^P such that $Q(q^P) = q^M$ and $\frac{\partial Q}{\partial q^P}(1) \neq 0$ or if $\frac{\partial Q}{\partial q^P}(1) = 0$. The solution is unique if $Q(\cdot)$ is weakly concave in q^P. If the variance weakly decreases with bliss point q^P in the restricted support, the solution exists when either there is q^P such that $Q(q^P) = q^M$ and $\frac{\partial Q}{\partial q^P}(0) \neq 0$ or if $\frac{\partial Q}{\partial q^P}(0) = 0$. The solution is unique if $Q(\cdot)$ is weakly convex in q^P.*

The conditions given in Proposition 1 are satisfied for different distortions of policy. Two examples are $Q^P(q^P) = \frac{1}{a}q^P, 1 < a < 1/q^M$ and $Q^P(q^P) = 0$. In the first case the distortion of policy is a convex combination of the politician's bliss point and policy 0, in the second one the distortion of policy is independent of q^P and is always equal to policy 0.[11] The following Proposition shows how the bias of policy is affected by uncertainty on the implemented policy. I assume that the distortion of policy $Q(\cdot)$ satisfies the conditions stated in Proposition 1.

Proposition 2. *If the variance of policy Var^P is a weakly monotonic function of the bliss point of candidate P in the restricted support and is strictly monotonic for $q^P = q^R$, the elected politician's expected policy is different from the bliss point of the median voter and is biased in the direction where the variance of policy is decreasing.*

Given that voters are risk averse, they also consider the variance of the implemented policy when they choose the politician. If, in expected terms, a politician implements a policy that is very close to the median voter's bliss point, but highly uncertain, the median voter can prefer a politician who implements a policy which in expectation is farther from q^M, but with lower variance. In order to relate the monotonicity of variance with the lotteries over policies introduced in this model, I compute the marginal effect of q^P on variance Var^P[12]:

$$\frac{\partial \mathrm{Var}^P}{\partial q^P} = 2p(1-p)(q^P - Q(q^P))\left(1 - \frac{\partial Q(q^P)}{\partial q^P}\right). \tag{3}$$

If the distortion of policy is on the left of the candidate's bliss point, $Q(q^P) < q^P$, and moves right by less than one when the candidate's bliss point moves right by one unit, $\frac{\partial Q(q^P)}{\partial q^P} < 1$, the variance of the implemented policy increases as the politician's bliss point moves right. Therefore voters elect a candidate who implements a policy which is leftward biased, in order to compensate for a higher variance.

Proposition 3 *(Comparative statics and voter welfare). If there exists a bliss point q^P such that its distortion is equal to the median voter's bliss point, $Q(q^P) = q^M$, and the conditions stated in Propositions 1 and 2 are satisfied, the following holds:*
(i) Median voter welfare is non-monotonic with respect to probability p: at low (high) values of p, median voter welfare decreases (increases) with p;
(ii) The bias $|\bar{q}^P - q^M|$ induced by uncertainty is non-monotonic with respect to the probability p: at low (high) values of p, the bias increases (decreases) with p.

The policy bias is related to the uncertainty of policy. If p is low, there is little uncertainty on the implemented policy because it is very likely that the politician will implement the distortion of policy. Therefore, if p increases, uncertainty will increase inducing voters to select a candidate with a larger policy bias. Median voter welfare decreases because variance increases. Conversely, if there is large uncertainty and p increases, uncertainty on policy decreases.

Note that a violation of the conditions of Proposition 2 does not imply that there is no effect of uncertainty on policy. Those conditions are sufficient to identify the direction of the policy bias. If the variance of policy is non-monotonic with respect to q^P, there can be a bias on policy because the maximand of expression (2) can be different from q^R, but the direction of the bias would be unclear.

3. Uncertainty over policy: lobbying

In this section I analyze a particular reason, discussed in the introduction, why there can be uncertainty over policy: the activity of lobbies. Let me add another player to the game, the lobby, with the following utility function:

$$V(q, y) = u(q, q^L) + y,$$

where q^L is the lobby's bliss point and q is the implemented policy. The utility of the lobby is linear in money y. The lobby is a non-elected political agent who can influence policy through a monetary contribution to the elected politician.[13] For simplicity I assume that there is only one lobby, although the model can easily be extended to multiple lobbies that compete for influence, as discussed in the concluding remarks. I also assume that the special interest group lobbies for a policy on the left of the policy space: $q^L < q^M$.[14]

[11] The proof of Proposition 1 in Appendix A shows that these distortions of policy satisfy the conditions stated in the Proposition.

[12] The expression for the derivative $\frac{\partial \mathrm{Var}^P}{\partial q^P}$ is computed in the proof of Proposition 2.

[13] Thus, as in Besley and Coate (2001) and Felli and Merlo (2006), I abstract from another source of influence wielded by special interest groups, namely campaign contributions. As mentioned, considerable attention has been devoted in the literature to the issue of uncertainty and lobbying through campaign contributions. See Groll and Ellis (2014) for a model of lobbying where the effect of special interest groups is mediated by commercial lobbying firms.

[14] This model can thus be applied to an election where a powerful special interest group lobbies for a policy opposed by the majority of citizens.

Uncertainty about citizens' preferences
I assume that there are two types of citizens, defined by their responsiveness to a lobby's contribution, $\rho^t, t \in T := \{m, s\}$. For simplicity I assume $\rho^m = \rho > 0$ and $\rho^s = 0$, but all results extend to $\rho^m > \rho^s > 0$. Merchants (type m) care about policy but they also consider lobby contributions, while saints (type s) are solely policy motivated.[15]

Saints are thus uncompromising citizens who do not trade policy for the favors of special interest groups. The probability of citizen i of being a saint is denoted by p. Responsiveness to lobbying is the only private information citizens have. All citizens' bliss points are common knowledge. They can thus be interpreted as the long-standing positions of citizens, known by voters. The responsiveness to lobbying is instead private information, in that voters observe the pandering of candidates to lobbies only once they are elected to office.

Lobbying
After being elected, politician P and the lobby bargain over policy q to be implemented and over a monetary transfer R that the lobby gives to the politician. Policy q and transfer R are the result of a Nash bargaining in which the politician has bargaining power k:

$$\max_{q \in I, R \geq 0} \left[u(q, q^P) + \rho^t R - u(q^P, q^P)\right]^k \left[u(q, q^L) - R - u(q^P, q^L)\right]^{1-k}, \tag{4}$$

such that

$$\begin{cases} u(q, q^P) + \rho^t R - u(q^P, q^P) \geq 0, \\ u(q, q^L) - R - u(q^P, q^L) \geq 0. \end{cases}$$

The status quo utility of the politician is $u(q^P, q^P)$ because with no lobbying the politician would implement her bliss point q^P, while the status quo utility of the lobby is $u(q^P, q^L)$. The utility levels $u(q, q^P) + \rho^t R - u(q^P, q^P)$ and $u(q, q^L) - R - u(q^P, q^L)$ are respectively the politician's and the lobby's gains from the bargaining. The lobbying process defined here includes some of the most relevant cases studied in the lobbying literature. Indeed, in Besley and Coate (2001), the lobbying process is built so that the lobby has all the bargaining power, which corresponds to $k = 0$ in this model. In Felli and Merlo (2006), the politician receives the entire surplus of the negotiation, which matches the case $k = 1$. Parameter k serves the purpose of showing that the model's results do not depend on specific assumptions on politician's bargaining power over the lobby.

Summarizing, the timing of the game is as follows:

1. voters observe the candidates' bliss points and select the winner according to a Condorcet method,
2. the elected politician reveals her type,
3. the politician and the lobby engage in a bargaining over policy q and lobby's contribution R,
4. the politician implements policy q and the lobby makes the monetary transfer R to the politician.

In the next section I analyze the equilibrium of this game and the conditions under which uncertainty on the influence of lobbying affects the equilibrium policy. In Besley and Coate's (2001) rational voter model all citizens are merchants, and lobbying is ineffective because voters are able to anticipate that special interest groups will distort policy. Thus when voting, they do not consider the candidate's bliss point in their utility functions, but rather the policy resulting from lobbying. Hence, in equilibrium voters are able to counteract lobbies by electing a candidate with offsetting policy preferences. The equilibrium policies of a model with lobbying are the same of a model without lobbying.

This result is retained as a special case within the framework analyzed in this paper. If there is no lobbying in the model (e.g. all citizens are saints), the median voter is elected and the equilibrium policy is q^M, the median voter's bliss point. If, as in Besley and Coate, there are only merchants, the elected candidate is the citizen who implements q^M. The elected candidate is not the median citizen but a candidate with a bliss point larger than q^M. In this case, lobbying does not affect policy.

I anticipate this basic result in order to define the influence of lobbying.

Definition 2. Lobbying is effective in equilibrium, if the equilibrium expected policy $\bar{q}^P$ is different from the median citizen's preferred policy q^M: $\bar{q}^P \neq q^M$. If $\bar{q}^P < q^M$ the equilibrium expected policy is biased in favor of the lobby, while if $\bar{q}^P > q^M$ the equilibrium expected policy is biased against the lobby.

[15] Lobby contributions could be considered a bribe, which would imply that merchants are corruptible citizens while saints are honest citizens. However, lobbies commonly offer favors to elected politicians that are permitted by law. Take for example, the revolving door phenomenon, where politicians are hired by the private industries that they regulate, reflecting the existence of a lawful intertemporal exchange of policy for money. An example which falls in a gray legal area is the release of privileged information by private firms that influence financial investments of politicians, see Eggers and Hainmueller (2014).

4. Results

I find the subgame perfect Nash equilibrium (SPNE) of the game by backward induction. When useful for the intuition, I keep in the notation a general utility from policy u. I first show the outcome of the bargaining game between the elected politician P and the lobby.

Proposition 4 *(Implemented policy). The implemented policy is a convex combination of the bliss point of the politician, q^P, and the bliss point of the lobby, q^L:*

$$q^{*P_t} = \frac{q^P + \rho^t q^L}{1+\rho^t}.$$

The lobby's contribution is the following:

$$\tilde{R} = k\big[u(q^{*P_t}, q^L) - u(q^P, q^L)\big] + (1-k)\frac{1}{\rho^t}\big[u(q^P, q^P) - u(q^{*P_t}, q^P)\big],$$

if $\rho^t \neq 0$. The contribution $\tilde{R}$ is zero, if $\rho^t = 0$.

If the elected politician is a saint, her ρ^t is equal to zero and she implements her bliss point. If the elected politician is a merchant, she caters to the lobby and the implemented policy is between her bliss point and the lobby's one: $q^L < q^{*P_t} < q^P$. Moreover, as in Besley and Coate (2001) and Felli and Merlo (2006), the implemented policy has the realistic feature of depending on the bliss points of the lobby and the politician. Indeed, having a moderate or an extremist special interest group, which lobbies the politician, has an effect on the implemented policy. Similarly, having a moderate or an extremist politician matters for policy. As assumed in the general model, the distortion of policy which in the model of lobbying is the policy implemented by the merchant, co-moves with the politician's bliss point. However, the implemented policy does not depend on the bargaining power k. Indeed, the solution of problem (4) with respect to q, is equivalent to the maximization of expression $u(q, q^P) + \rho^t u(q, q^L)$. The negotiation maximizes the joint surplus of the two players, where the utility from the lobby's policy is weighted by ρ^t, given that the lobby's contribution is the means by which the lobby transmits its preferences to the politician. In the negotiation, both players are willing to compromise on the policy. For this reason, the farther q^P is from q^L, the larger the distortion of lobbying on policy: $q^P - q^{*P_t} = \frac{\rho^t}{1+\rho^t}\left(q^P - q^L\right)$. The same result holds for the lobby: the farther q^P is from q^L, the greater the distance of q^L from the policy: $q^{*P_t} - q^L = \frac{1}{1+\rho^t}\left(q^P - q^L\right)$.

The equilibrium contribution distributes the surplus of the bargaining to the two players. If the politician has no bargaining power, $k = 0$, all the surplus goes to the lobby. Thus the lobby compensates the politician for the loss incurred in the implementation of q^{*P_t}, with respect to q^P, but the politician does not gain anything from the negotiation. If $k = 1$, the lobby transfers all its gain from the implementation of q^{*P_t} to the politician. This is clear from the politician's indirect utility:

$$k\frac{(\rho^t)^2}{1+\rho^t}(q^P - q^L)^2,$$

which, differently from the implemented policy, depends positively on the politician's bargaining power k. If $\rho^t = 0$, the indirect utility is zero, because the politician would implement her bliss point q^P. The indirect utility is computed in Appendix A.

At the voting stage the Condorcet winner is selected. Here citizens try to anticipate the distortion created by the lobby. They do not in fact value the bliss point q^P of the politician in their utility function, but rather the implemented policy q^{*P_t}, internalizing the distortion performed by the special interest group. The lobbying subgame showed that candidates with different responsiveness to the lobby's contribution implement different policies. Therefore, the uncertainty concerning the candidates' responsiveness to the lobby's contribution creates uncertainty concerning their implemented policies. The expected policy can be expressed as follows: $\bar{q}^P = \frac{1+\rho p}{1+\rho}q^P + \frac{\rho(1-p)}{1+\rho}q^L$. Expression (3) implies that in this game, the variance of the implemented policy by candidate P increases with the candidate's bliss point because $q^P > Q(q^P) = q^{*P_m}$ for all $q^P > q^L$ and $\frac{\partial Q}{\partial q^P} = \frac{1}{1+\rho} < 1$. In particular, the variance depends positively on the distance between the candidate's bliss point and the lobby's preferred policy:

$$\mathrm{Var}^P = v(q^P - q^L)^2,$$

where quantity v is defined as follows: $v := \frac{\rho^2 p(1-p)}{(1+\rho)^2}$. The formula of variance is computed in Appendix A. If q^P is closer to q^L, the merchant's implemented policy becomes closer to her bliss point q^P, which in turn is the policy implemented by the saint, hence the variance associated to P decreases. The closer the politician is to the lobby, the lower the uncertainty as to how she behaves.

If there is no uncertainty (e.g. all candidates are merchants), the Condorcet winner would be the candidate who implements the median voter's bliss point. Such a candidate would have a bliss point q^P larger than q^M: $\frac{q^P+\rho q^L}{1+\rho}=q^M$. Note that if the responsiveness to lobbying ρ is sufficiently large, there is no candidate P whose preferences for policy offset lobbying: $\frac{1+\rho q^L}{1+\rho}>q^M$. Thus lobbying would affect policies no matter what voters do. I exclude this possibility in order to focus the analysis on cases where voters are potentially able to offset lobbying, even if there are only merchants in the pool of candidates. Thus I assume the following upper bound on ρ:

$$\rho \leq \frac{1-q^M}{q^M-q^L}. \tag{5}$$

When there is uncertainty, the Condorcet winner is not the candidate whose expected policy is the median voter's bliss point: $\bar{q}^P=q^M$. Indeed, the median voter can increase her utility by electing a candidate whose expected policy is lower than q^M because, by having a bliss point closer to q^L, such candidate exhibits a lower variance of implemented policies.

Proposition 5 *(Voting equilibrium). The equilibrium expected policy is*

$$\bar{q}^P=\frac{(1+\rho p)^2 q^M+\rho^2 p(1-p)q^L}{(1+\rho p)^2+\rho^2 p(1-p)}.$$

Lobbying is effective because the equilibrium policy is biased in favor of the lobby: $\bar{q}^P<q^M$.

Thus, lobbying creates uncertainty on the policy implemented by politicians. This in turn influences the choice of risk-averse voters who elect a candidate whose expected policy is biased in favor of the lobby. Clearly, this result only holds if voters are risk-averse. If voters are risk-neutral, lobbying does not affect the implemented policy.

Let me show, with a numerical example, the location of the equilibrium expected policy in the policy space. Let me consider $q^M=1/2, q^L=0, \rho=2, p=1/4$. This choice of parameters implies $\bar{q}^P=3/4\cdot 1/2=3/8$, which is lower than 1/2, the bliss point of the median voter. Note that, changing only p in the set of parameters, from 1/4 to 1/16, reduces the policy bias, as $\bar{q}^P$ gets closer to q^M: $\bar{q}^P=27/64>3/8=24/64$. For these values of the parameters, reducing the probability of having a saint type in the pool of candidates, decreases the effect of lobbying on policy.

The following proposition states the conditions under which this result holds.

Proposition 6 *(Comparative statics). The following holds:*
(i) If the merchants' responsiveness to lobbying ρ decreases, the policy bias $|q^M-\bar{q}^P|$ shrinks.
(ii) The probability that a citizen is a saint has a non-monotonic effect on the policy bias: if $p\leq\bar{p}:=\frac{1}{2+\rho}$, then increasing the probability p of saint types amplifies the bias, if $p>\bar{p}$, then increasing p reduces the bias.
(iii) Median voter welfare and the policy bias have opposite comparative statics with respect to ρ and p.

Fig. 1 illustrates the relationship between the probability p of saint types and the policy bias $|q^M-\bar{q}^P|$, with the same choice of parameters as in the previous numerical example: $q^M=1/2, q^L=0, \rho=2$.

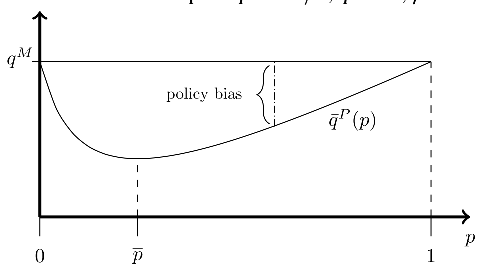

Fig. 1. Increasing the probability p of saint politicians has a non-monotonic effect on the policy bias $|q^M-\bar{q}^P|$, reaching a maximum at $p=\bar{p}$.

Decreasing the responsiveness to lobbying reduces the policy bias because merchants are more similar to saints in their utility and implemented policies. The probability p has a non-monotonic effect. On one hand, when all citizens are merchants, if the probability of having citizens who are saints becomes positive, the uncertainty on the implemented policy is amplified. Thus voters react to the larger uncertainty choosing a candidate with an expected policy closer to the lobby. On the other hand, when there is a high probability that a citizen is a saint, a further increase of p reduces the uncertainty over types, and, consequently, on implemented policies. Therefore, voters choose a candidate whose expected policy is closer to the median voter's bliss point.[16]

Comparative statics regarding the responsiveness to lobbying ρ are useful to understand the effect of lobbying on median voter welfare. Indeed, median voter welfare decreases with ρ. Given that, if $\rho = 0$, there is no lobbying in the model, this implies that the presence of a lobby reduces median voter welfare. The same reasoning cannot be applied to utilitarian welfare, $\int_{[0,1]} \mathbb{E}[u(q^{*P_t}, q^i)] f dq^i$. With quadratic utility, the comparative statics on utilitarian welfare are equal to comparative statics on the mean voter welfare, where the mean voter is the voter with a bliss point equal to the mean of the distribution f of bliss points. Therefore, utilitarian welfare does not necessarily follow the same comparative statics of the median voter welfare. One could think that, if q^M and the mean of distribution f are far apart and the mean is close to the lobby's bliss point, the presence of lobbying unequivocally increases utilitarian welfare. This is not the case because increasing ρ increases variance, which reduces mean voter welfare. Thus the effect of lobbying on utilitarian welfare, if q^M and the mean of distribution f are not close, is unclear.[17]

The presence of uncertainty at the voting stage implies that all players of the game would be better off if there was no election and the median voter could bargain directly with the lobby, with the status quo utilities of the two players given by the expected utility they receive in equilibrium in the present game.[18] This reasoning shows that, in the presence of risk-averse players and uncertainty on policy-relevant features of the preferences of politicians, indirect democracy can lead to inefficiencies.

Propositions 5 and 6 are the equivalent of Propositions 2 and 3, applied to the case of lobbying. In particular, assumption (5) ensures that Proposition 3 is valid in the model of lobbying.

5. Concluding remarks

This work studies the effect of uncertainty over policy on voting behavior, applying the results to an electoral model of lobbying, in which there is uncertainty with regard to the responsiveness of citizens to the lobby's contribution. The model demonstrates that uncertainty on the influence of special interests can lead to large effects of lobbying on policy. Moreover, the comparative statics on the probability of non-pandering candidates contribute to the public debate on the supposed need for politicians who do not "sell-out" to special interest groups, suggesting that this may not reduce the power of lobbies over policy.[19] These results are robust to natural extensions and different assumptions on the behavior of players, as the following discussion shows.

1) The model studies the effect of a unique lobby on policy. This can be a reasonable assumption if there is a special interest group that dominates the lobbying market. Still, in many situations multiple special interest groups with opposing preferences intervene in the lobbying process. The model can be easily extended to the case of multiple lobbies. Let me assume that there are multiple lobbies with bliss points $q^l, l \in L$. Lobby l has the following utility:

$$V(q, y, l) = \eta^l u(q, q^l) + y,$$

where $\eta^l \geq 0$ is the relative intensity of preference for policy of lobby l. Parameter η^l creates additional heterogeneity among lobbies: lobbies with larger η have a larger willingness to contribute in order to move the implemented policy towards their bliss point. After being elected, politician P and all lobbies bargain over policy q to be implemented, and monetary transfer R^l that lobby l gives to the politician.

$$\max_{q \in I, R^l \geq 0, l \in L} \left[u(q, q^P) + \rho^t \sum_{l \in L} R^l - u(q^P, q^P) \right] \prod_{l \in L} \left[\eta^l u(q, q^l) - R^l - \eta^l u(q^P, q^l) \right].$$

such that

$$\begin{cases} u(q, q^P) + \rho^t \sum_{l \in L} R^l - u(q^P, q^P) \geq 0, \\ \eta^l u(q, q^l) - R^l - \eta^l u(q^P, q^l) \geq 0, \quad l \in L. \end{cases}$$

For simplicity I assume that all participants to the bargaining process have the same bargaining power.

[16] When p is low, the expected policy, and therefore the policy implemented by merchant, is close to q^M. When p is close to 1, again the expected policy, and therefore the policy implemented by the saint, is close to q^M. Therefore, the median voter has higher utility when the merchant type of P is elected, if $p < 1/(2 + \rho)$. Otherwise she is better off with the saint type of P. A proof is available in Appendix A.

[17] If the mean is different from q^M, there are three components of the comparative statics of ρ on utilitarian welfare: the increase in variance induced by ρ, the effect of ρ on the distance between the mean and the expected policy $\bar{q}^P$, the effect of ρ on the utilitarian welfare, given by its effect on the bliss point q^P of the elected politician. The first component is negative, the second one is positive if the mean is lower than $\bar{q}^P$. The third component is positive if the mean is lower than q^M and $\rho < 1/\sqrt{p}$. A detailed analysis is present in Appendix A.

[18] Indeed the median voter and the lobby receive respectively in equilibrium $u(\bar{q}^P, q^M) - \text{Var}^P$ and $u(\bar{q}^P, q^L) - \text{Var}^P$. In order to show that there is room for bargaining, it is sufficient to show that the present utilities are not on the Pareto frontier: for example, if the players agree on implementing $q = \bar{q}^P$, they increase their utility from the status quo, because they implement the same policy they would implement in the status quo, without the negative effect of uncertainty.

[19] See http://www.nytimes.com/2013/10/08/opinion/politicians-for-sale.html.

The bargaining process is a game with transferable utility, therefore by Myerson (2013, p. 385), policy q maximizes:

$$u(q,q^P)+\rho^P\sum_{l\in L}\eta^l u(q,q^l).$$

Note that this maximization shows the equivalence of such game with a menu-auction model of lobbying, where all lobbies offer truthful contributions, in the sense of Bernheim and Whinston (1986). The implemented policy is a convex combination of the politician's bliss point and the bliss point of all lobbies, where each bliss point is weighted by the willingness η to contribute: $q^{*P}=\frac{q^P+\rho^P\sum_{l\in L}\eta^l q^l}{1+\rho^P\sum_{l\in L}\eta^l}$. Lobbies with a larger η have a larger influence on policy. The model analyzed in this paper includes this case, if q^L is defined as follows: $q^L:=\sum_{l\in L}\eta^l q^l$. Clearly, the results of this model carry interesting implications only if $q^L\neq q^M$, which in the case of multiple lobbies means that lobbies do not perfectly compensate each other.[20]

2) A second extension relates to the number of types that are considered in this work. While the main model analyzes two types, the theory can be extended to any finite number or to a continuum of types. Let me first analyze a finite number of types. Let me define $\Psi:=\{\rho_1,...,\rho_n\}, \rho_1=0<\rho_2<...<\rho_n$, the finite support of a probability mass function $g(\rho_l)\geq 0$ of types, where $\sum_{l=1}^n g(\rho_l)=1$. Let me denote the expected policy by $\bar{q}^P=\sum_l q^{*P}g(\rho_l)$, where q^{*P} depends on ρ_l as follows: $q^{*P}=q^P/(1+\rho_l)+\rho_l q^L/(1+\rho_l)$. Let me denote the variance of the implemented policy as follows: $\text{Var}^P=\sum_l(q^{*P}-\bar{q}^P)^2 g(\rho_l)$. Let me now define expectation and variance for a continuum of types. Let me define $[0,\rho]$ the support of a probability density function $g(\rho^P)$ of types, where g is a non-negative Lebesgue-integrable function such that $\int_{[0,\rho]}g(\rho^P)d\rho^P=1$. Let me denote the expected policy by $\bar{q}^P=\int_{[0,\rho]}q^{*P}g(\rho^P)d\rho^P$, where q^{*P} depends on ρ^P as follows: $q^{*P}=q^P/(1+\rho^P)+\rho^P q^L/(1+\rho^P)$. Let me denote the variance of the implemented policy as follows: $\text{Var}^P=\int_{[0,\rho]}(q^{*P}-\bar{q}^P)^2 g(\rho^P)d\rho^P$. Considering $\bar{q}^P$ and Var^P as just defined, Proposition 5 follows from the same reasoning applied in the main analysis because the expected utility of risk averse voters with quadratic utility can be decomposed as shown in equation (1), independently of the underlying probability structure. While the main result can be generalized without loss, the Bernoulli distribution considered in the main analysis gives a clearer intuition of the comparative statics with respect to relevant parameters, such as ρ and p.

3) While an efficient bargaining between the politician and the lobby implies that variance decreases, as distance $|q^P-q^L|$ decreases, there are potentially other mechanisms that can push in the opposite direction. Consider a variation of the model in which the distance $|q^P-q^L|$ increases the demand of precision of signals that voters receive about a candidate's type. Such demand of information would dampen the positive relationship between $|q^P-q^L|$ and variance Var^P, implying that in equilibrium there could be a lower (or no) effect of lobbying on policy. Note that as long as the median voter is pivotal in the election outcome, he will demand information in order to increase the probability that a policy close to q^M is implemented, which is unrelated to the distance $|q^P-q^L|$. A reason why the demand of information can be positively related to $|q^P-q^L|$ is the presence of partisan voters or party members with the same bliss point of candidates, who suffer a disutility from the deviation of candidate P from her bliss point: $q^P-q^{*P_t}=\frac{\rho^t}{1+\rho^t}\left(q^P-q^L\right)$. Similarly, if the probability of being a saint increases with distance $|q^P-q^L|$, possibly because of a prior positive selection of saints depending on their distance to the lobby, there would be a non-monotonic effect of $|q^P-q^L|$ on the variance of policy, because of the non-monotonic effect of p on the variance. Depending on where the variance peaks and thus on the marginal effect of $|q^P-q^L|$ on p, this additional mechanism could increase or decrease the effect of lobbying on policy. These two mechanisms suggest that the results presented in this work are a first step in showing that uncertainty on policy induced by different types of candidates can play a role in the effectiveness of lobbying and that this effect can be mitigated or exacerbated by other channels.

4) Finally, the lobbying model presented in this work is equivalent to a model where candidates are merchants and the lobby is active with probability $1-p$, in which case the bargaining takes place, and with the complementary probability the lobby is inactive, and the politician implements her bliss point. The main result and comparative statics are the same. It is worth noticing that in general lobbies are long-term actors in politics,[21] while there is a large turnover of politicians, suggesting that uncertainty is more likely to be related to candidates' behavior, than to lobby's activity.

A promising area of future research might be to extend this model by giving candidates the possibility of sending signals to voters before elections, in order to communicate that they will not pander to lobbies. One signal could be electoral campaign promises of no-pandering to lobbies. Unfortunately, there is an incentive for all candidates to make such promises

[20] This extension considers the set of lobbies as exogenous, differently from Felli and Merlo (2006), who investigate how politicians endogenously select the set of lobbies to negotiate with. An endogenous choice of lobbies would have implications for this model, because electing different politicians would imply having different lobbies involved in the negotiation, and a non-monotonic effect of q^P on Var^P, which makes the direction of the bias unclear, as underlined at the end of Section 2.

[21] A few examples: the National Rifle Association was founded in 1871; the American Bankers Association, very active in lobbying on financial trade regulation, was founded in 1875; Boeing, which, according to the Center for Responsive Politics (2017), spends a substantial amount of money in political donations, was founded in 1916.

and lofty speeches quickly turn to cheap talk. There are other, more costly, signals that candidates can use to show voters that they will not cater to lobbies. For example, some US Presidential candidates have explicitly declined money from donors related to special interest groups. During his 2012 electoral campaign, Obama refused money from “Washington lobbyists or corporate interests”.[22]

Acknowledgments

I am grateful to Antonio Nicolò, Andrea Mattozzi, Arnaud Dellis, Sandro Brusco, Jörgen Weibull, Tim Besley, Stefano Bolatto, Giuseppe Pignataro, Elena Lucchese, Salvatore Piccolo, Giorgio Gulino and all the participants of the Fourth Workshop on Institutions, Individual Behaviour and Economic Outcomes in Alghero, Italy, the 2014 Political Economy Workshop in Milan, Italy, the 2014 ASSET conference, and the seminar in Boğaziçi University, Istanbul, Turkey, for helpful comments and suggestions.

Appendix A

Proof of Propositions 1 and 2. The Condorcet winner is the citizen P whose bliss point q^P maximizes:

$$-(q^M-\bar{q}^P)^2-\mathrm{Var}^P,$$

with q^P in the restricted support. The first order condition is:

$$2(q^M-\bar{q}^P)\frac{\partial\bar{q}^P}{\partial q^P}=\frac{\partial\mathrm{Var}^P}{\partial q^P}. \tag{6}$$

The sign of the left hand side depends on the sign of expression $q^M-\bar{q}^P$, because $\frac{\partial\bar{q}^P}{\partial q^P}$ is positive: $\frac{\partial\bar{q}^P}{\partial q^P}=p+(1-p)\frac{\partial Q}{\partial q^P}>0$. If $\frac{\partial\mathrm{Var}^P}{\partial q^P}$ is weakly positive and positive for $q^P=q^R$, the first order condition is satisfied for a value of q^P such that $\bar{q}^P<q^M$. The opposite is true if $\frac{\partial\mathrm{Var}^P}{\partial q^P}$ is weakly negative and negative for $q^P=q^R$. Now I investigate the existence and uniqueness of a solution to the first order condition. I compute the derivative of the variance, $p(q^P)^2+(1-p)(Q(q^P))^2-(\bar{q}^P)^2$, with respect to q^P:

$$\begin{aligned}\frac{\partial\mathrm{Var}^P}{\partial q^P}&=2pq^P+2(1-p)Q(q^P)\frac{\partial Q}{\partial q^P}-2\bar{q}^P\left(p+(1-p)\frac{\partial Q}{\partial q^P}\right)=2p(q^P-\bar{q}^P)+2(1-p)\frac{\partial Q}{\partial q^P}\left(Q(q^P)-\bar{q}^P\right)=\\&2p(q^P-pq^P-(1-p)Q(q^P))+2(1-p)\frac{\partial Q}{\partial q^P}\left(Q(q^P)-pq^P-(1-p)Q(q^P)\right)=\\&2p(1-p)\left[(q^P-Q(q^P))+\frac{\partial Q}{\partial q^P}(Q(q^P)-q^P)\right]=2p(1-p)(q^P-Q(q^P))\left(1-\frac{\partial Q(q^P)}{\partial q^P}\right).\end{aligned}$$

Inserting this expression in the first order condition, I obtain the following:

$$\begin{aligned}&\left(q^M-pq^P-(1-p)Q(q^P)\right)\left(p+(1-p)\frac{\partial Q}{\partial q^P}\right)=p(1-p)(q^P-Q(q^P))\left(1-\frac{\partial Q(q^P)}{\partial q^P}\right)\\&q^M\left(p+(1-p)\frac{\partial Q}{\partial q^P}\right)=pq^P\left[p+(1-p)\frac{\partial Q(q^P)}{\partial q^P}+(1-p)\left(1-\frac{\partial Q(q^P)}{\partial q^P}\right)\right]+\\&(1-p)Q(q^P)\left[\left(p+(1-p)\frac{\partial Q}{\partial q^P}\right)-p\left(1-\frac{\partial Q(q^P)}{\partial q^P}\right)\right]q^M=\frac{pq^P+(1-p)Q(q^P))\frac{\partial Q}{\partial q^P}}{p+(1-p))\frac{\partial Q}{\partial q^P}}\end{aligned}$$

If $Q(q^P)\le q^P$ for every q^P in the restricted support, and $\frac{\partial Q(q^P)}{\partial q^P}\le 1$, it must be that, for $q^P=q^{\min}$, the right hand side is lower than or equal to $q^{\min}\le q^M$. If, when $q^P=1$, the following inequality is satisfied: $\frac{p+(1-p)Q(1))\frac{\partial Q}{\partial q^P}(1)}{p+(1-p))\frac{\partial Q}{\partial q^P}(1)}\ge q^M$, by the intermediate value theorem, there is a solution to the first order condition. Necessary and sufficient conditions are that either $\frac{\partial Q}{\partial q^P}(1)\ne 0$ and there exists q^P such that $Q(q^P)=q^M$, or $\frac{\partial Q}{\partial q^P}(1)=0$. In the first case, $Q(1)$ must be larger than or equal to $Q(q^P)=q^M$, and the convex combination between $Q(1)$ and 1 is larger than or equal to q^M, for every $p\in[0,1]$. In the second case, the right hand side is equal to 1, if $q^P=1$. If $Q(q^P)\ge q^P$ for every q^P in the restricted support, and

[22] See http://abcnews.go.com/blogs/politics/2011/10/obama-campaign-tops-one-million-donors/.

$\frac{\partial Q(q^P)}{\partial q^P} \leq 1$, a similar reasoning applies: necessary and sufficient conditions are that either $\frac{\partial Q}{\partial q^P}(0) \neq 0$ and there exists q^P such that $Q(q^P) = q^M$, or $\frac{\partial Q}{\partial q^P}(0) = 0$.

In order to study the uniqueness of the solution, I compute the second derivative:

$$-2\left(\frac{\partial \bar{q}^P}{\partial q^P}\right)^2 + 2(q^M - \bar{q}^P)(1-p)\frac{\partial^2 Q}{\partial (q^P)^2} - 2p(1-p)\left(1 - \frac{\partial Q}{\partial q^P}\right)^2 + 2p(1-p)(q^P - Q(q^P))\frac{\partial^2 Q}{\partial (q^P)^2}.$$

Let me consider the case in which $\frac{\partial \mathrm{Var}^P}{\partial q^P}$ is weakly positive and positive for $q^P = q^R$. All addenda in the second derivative are negative, if $Q(\cdot)$ is weakly concave in q^P, $q^P \geq Q(q^P)$, for every q^P in the restricted support and $\bar{q}^P < q^M$. By equation (6), there can only be a solution for q^P, if $\bar{q}^P < q^M$. If $\bar{q}^P > q^M$, the second order derivative can change sign, but there cannot be another solution to equation (6). Therefore the first order condition identifies a unique maximum. A similar reasoning applies to the case in which $\frac{\partial \mathrm{Var}^P}{\partial q^P}$ is weakly negative and negative for $q^P = q^R$, with the difference that $Q(\cdot)$ has to be weakly convex in q^P and $q^P \leq Q(q^P)$, for every q^P in the restricted support.

Finally, let me show that the distortions of policy $Q^P(q^P) = \frac{1}{a}q^P, a > 1$ and $Q^P(q^P) = 0$ satisfy Proposition 1. When $Q^P(q^P) = \frac{1}{a}q^P, a > 1$, the variance increases when q^P moves right: $\frac{\partial \mathrm{Var}^P}{\partial q^P} = 2p(1-p)(1-1/a)^2 q^P > 0$. $\frac{1}{a}q^P = q^M$ implies $q^P = aq^M$, which exists in the policy space $[0, 1]$ if $aq^M < 1$. Moreover $\frac{\partial Q}{\partial q^P}(1) = \frac{1}{a} \neq 0$. Finally $Q(\cdot)$ is linear in q^P: the conditions stated in Proposition 1 are satisfied. When $Q^P(q^P) = 0$ Proposition 1 is satisfied, because the variance increases when q^P moves right, $\frac{\partial \mathrm{Var}^P}{\partial q^P} = 2p(1-p)q^P > 0$, moreover $\frac{\partial Q}{\partial q^P}(1) = 0$, and $Q(\cdot)$ is constant in q^P. □

Proof of Proposition 3. I will first prove point (i), and subsequently point (ii).

If there exists q^P such that $Q(q^P) = q^M$, the equilibrium expected policy $\bar{q}^P$ is equal to q^M for $p = 0$ and $p = 1$, because variance is zero. For these two values of p, there is no uncertainty and the indirect utility of the median voter reaches its largest possible value, 0. By Proposition 2, if $\frac{\partial \mathrm{Var}^P}{\partial q^P}$ is weakly positive and positive for $q^P = q^R$, for $p \in]0, 1[$, the expected policy is on the left of the median voter's bliss point. Hence median voter welfare $-|\bar{q}^P - q^M| - \mathrm{Var}^P$ is negative. For the intermediate value theorem, there must exist a value of $p \in]0, 1[$, not necessarily unique, such that the derivative of median voter welfare with respect to p is zero. Let me define the lowest and largest values of p, such that the derivative of median voter welfare with respect to p is zero, respectively as $\bar{p}_1$ and $\bar{p}_2$. For $p < \bar{p}_1$, median voter welfare decreases with p, for $p > \bar{p}_1$, median voter welfare increases with p. This proves point (i).

The same reasoning can be applied to prove point (ii) because, as proved in the previous paragraph, for $p = 0$ and $p = 1$, the bias $|\bar{q}^P - q^M|$ is 0, while for $p \in]0, 1[$, the expected policy is on the left of the median voter's bliss point. □

A.1. Equilibrium of the lobbying subgame

Proof of Proposition 4. Let me solve problem (4) for $\rho^t = 0$. In this case the politician's surplus $u(q, q^P) - u(q^P, q^P)$ is negative for all $q \neq q^P$, and it is equal to 0 if $q = q^P$, because q^P maximizes function $u(q, q^P)$ with respect to q. Therefore, the only admissible solution to the Nash bargaining is $q = q^P$. At the same time, if $\rho^t = 0$, the objective function of the Nash bargaining is a decreasing function of R, thus the equilibrium transfer $\tilde{R}$ is equal to $= 0$. Let me now prove that the Nash bargaining problem has a unique maximum, under the conditions that both surpluses are larger than zero. I then solve the system of the first order conditions for q and R and prove that the surpluses are both positive, computed in this solution. First, I prove that the logarithmic transformation of the function in the maximization problem is concave. I compute the Hessian matrix and prove that both second order derivatives are negative and the determinant of the Hessian matrix is positive. I define $S_P := u(q, q^P) + \rho^t R - u(q^P, q^P)$ the surplus of the politician, and $S_L := u(q, q^L) - R - u(q^P, q^L)$ the surplus of the lobby. Moreover I denote by u_{jP} the j-th derivative of function $u(q, q^P)$ with respect to q. Similarly I denote by u_{jL} the j-th derivative of function $u(q, q^L)$ with respect to q. The first order derivative of the logarithmic transformation with respect to q is the following:

$$k\frac{u_{1P}}{S_P} + (1-k)\frac{u_{1L}}{S_L}.$$

The first order derivative of the logarithmic transformation with respect to R is the following:

$$k\frac{\rho^t}{S_P} - \frac{1-k}{S_L}.$$

The second order derivative of the logarithmic transformation with respect to q is the following:

$$k\frac{S_P u_{2P} - u_{1P}^2}{S_P^2} + (1-k)\frac{S_L u_{2L} - u_{1L}^2}{S_L^2}. \tag{7}$$

u_{2P} and u_{2L} are negative, thus quantity (7) is negative. The second order derivative of the logarithmic transformation with respect to R is the following:

$$-\frac{k(\rho^t)^2}{S_P^2}-\frac{1-k}{S_L^2},$$

which is negative as well. The cross derivative is the following:

$$-k\rho^t\frac{u_{1P}}{S_P^2}+(1-k)\frac{u_{1L}}{S_L^2}.$$

The determinant is the following:

$$\begin{aligned}
&-k^2(\rho^t)^2\frac{S_Pu_{2P}-u_{1P}^2}{S_P^4}-(1-k)k\frac{S_Pu_{2P}-u_{1P}^2+(\rho^t)^2S_Lu_{2L}-(\rho^t)^2u_{1L}^2}{S_L^2S_P^2}-\\
&(1-k)^2\frac{S_Lu_{2L}-u_{1L}^2}{S_L^4}-\frac{k^2(\rho^t)^2u_{1P}^2}{S_P^4}-\frac{(1-k)^2u_{1L}^2}{S_L^4}+\frac{2k(1-k)\rho^tu_{1P}u_{1L}}{S_L^2S_P^2}=\\
&-k^2(\rho^t)^2\frac{u_{2P}}{S_P^3}-(1-k)k\frac{S_Pu_{2P}-u_{1P}^2+(\rho^t)^2S_Lu_{2L}-(\rho^t)^2u_{1L}^2}{S_L^2S_P^2}-\\
&(1-k)^2\frac{u_{2L}}{S_L^3}+\frac{2k(1-k)\rho^tu_{1P}u_{1L}}{S_L^2S_P^2}.
\end{aligned}$$

All addenda in the last expression are positive, hence the determinant is positive. Let me solve problem (4) for $\rho^t\neq 0$. The transfer R is computed, solving the maximization (4), under the condition $R\geq 0$ and $q\in I$. I take the logarithmic transformation of the function in the maximization problem (4) and deriving with respect to R the following first order condition is found: $\frac{k\rho^t}{u(q,q^P)+\rho^tR-u(q^P,q^P)}-\frac{1-k}{u(q,q^L)-R-u(q^P,q^L)}=0$, which brings the following solution: $\tilde{R}(q)=k[u(q,q^L)-u(q^P,q^L)]+(1-k)\frac{1}{\rho^t}\left[u(q^P,q^P)-u(q,q^P)\right]$. Deriving the same logarithmic transformation with respect to q the following first order condition is found: $-2k[u(q,q^L)-R-u(q^P,q^L)](q-q^P)-2(1-k)[u(q,q^P)+\rho^tR-u(q^P,q^P)](q-q^L)=0$. Substituting $R=\tilde{R}$ in expressions $[u(q,q^L)-R-u(q^P,q^L)]$ and $[u(q,q^P)+\rho^tR-u(q^P,q^P)]$ I obtain respectively $(1-k)[1/\rho^tu(q,q^P)+u(q,q^L)\ -1/\rho^tu(q^P,q^P)-u(q^P,q^L)]$ and $k\rho^t[1/\rho^tu(q,q^P)+u(q,q^L)-1/\rho^tu(q^P,q^P)-u(q^P,q^L)]$, which can be substituted in the first order condition for q:

$$-2k(1-k)\left[1/\rho^tu(q,q^P)+u(q,q^L)-1/\rho^tu(q^P,q^P)-u(q^P,q^L)][q-q^P+\rho^t(q-q^L)\right]=0.$$

Either expression $E:=[1/\rho^tu(q,q^P)+u(q,q^L)-1/\rho^tu(q^P,q^P)-u(q^P,q^L)]$ or $q-q^P+\rho^t(q-q^L)$ are zero. If $E=0$, given that $(1-k)E$ and $k\rho^tE$ are the surpluses respectively of the lobby and the politician, both assume value 0. If $q-q^P+\rho^t(q-q^L)=0$, the solution would be $q^{*P_t}=(q^P+\rho^tq^L)/(1+\rho^t)$. If I substitute q^{*P_t} in E, it becomes $\rho^t/(1+\rho^t)(q^P-q^L)^2$, which is positive, implying that both surpluses are positive. Hence solving for $E=0$ cannot bring to a maximizer. Therefore q^{*P_t} and $\tilde{R}(q^{*P_t})$ are respectively the policy and the transfer that solve the Nash bargaining problem. □

The computation of politician P's indirect utility

The indirect utility of politician P is computed as follows:

$$\begin{aligned}
U(q^{*P},\tilde{R},P)&=u(q^{*P},q^P)+\rho^tk\left[u(q^{*P_t},q^L)-u(q^P,q^L)\right]-(1-k)u(q^{*P},q^P)=\\
k\left\{u(q^{*P},q^P)+\rho^t\left[u(q^{*P_t},q^L)-u(q^P,q^L)\right]\right\}&=k\left\{-\left(\frac{(1+\rho^t)q^P-q^P-\rho^tq^L}{1+\rho^t}\right)^2+\right.\\
\left.\rho^t\left[-\left(\frac{(1+\rho^t)q^L-q^P-\rho^tq^L}{1+\rho^t}\right)^2+(q^P-q^L)^2\right]\right\}&=(-\rho^t-1+(1+\rho^t)^2)\frac{k\rho^t}{(1+\rho^t)^2}\left(q^P-q^L\right)^2=\\
&\frac{k\left(\rho^t\right)^2}{1+\rho^t}\left(q^P-q^L\right)^2.
\end{aligned}$$

A.2. Equilibrium of the voting subgame

The computation of Var^P
The variance of implemented policies can be computed as follows:

$$\mathbb{E}(q^{*P})^2 = (1-p)\left(\frac{q^P+\rho q^L}{1+\rho}\right)^2 + p(q^P)^2,$$

$$\mathbb{E}^2(q^{*P}) = \left((1-p)\frac{q^P+\rho q^L}{1+\rho} + pq^P\right)^2,$$

$$\begin{aligned}\mathrm{Var}^P = \mathbb{E}(q^{*P})^2 - \mathbb{E}^2(q^{*P}) =\\ p(1-p)\left(\frac{q^P+\rho q^L}{1+\rho}\right)^2 + p(1-p)(q^P)^2 - 2p(1-p)q^P\frac{q^P+\rho q^L}{1+\rho} =\\ \frac{p(1-p)}{(1+\rho)^2}\Big\{(q^P)^2+\rho^2(q^L)^2+2\rho q^P q^L+(1+\rho)^2(q^P)^2\\ -2(1+\rho)(q^P)^2 - 2(1+\rho)\rho q^P q^L\Big\} = \frac{\rho^2 p(1-p)}{(1+\rho)^2}\left(q^P-q^L\right)^2.\end{aligned}$$

Proof of Proposition 5. In the voting subgame the Condorcet winner is the candidate with a bliss point that solves the following problem:

$$\max_{q^P\in[0,1]} -(q^M-\bar{q}^P)^2 - v(q^P-q^L)^2,$$

where $\bar{q}^P = \frac{1+\rho p}{1+\rho}q^P + \frac{\rho(1-p)}{1+\rho}q^L$. The first order condition is as follows:

$$+\left(q^M-\bar{q}^P\right)\left(\frac{1+\rho p}{1+\rho}\right) - v\left(q^P-q^L\right) = 0,$$

$$q^P = \frac{\frac{1+\rho p}{1+\rho}q^M + \left(v-\rho(1-p)\frac{1+\rho p}{(1+\rho)^2}\right)q^L}{\left(\frac{1+\rho p}{1+\rho}\right)^2 + v} = \frac{(1+\rho p)(1+\rho)q^M - \rho(1-p)q^L}{(1+\rho p)^2+\rho^2 p(1-p)}. \tag{8}$$

In order to avoid a corner solution, q^P should be in the following interval: $q^P \in [0,1]$. The difference $q^P - q^M$ is positive, thus q^P is larger than or equal to 0:

$$\begin{aligned}q^P - q^M \geq 0 \Leftrightarrow\\ (1+\rho p)(1+\rho)q^M - \rho(1-p)q^L - (1+\rho p)^2 q^M - \rho^2 p(1-p)q^M \geq 0 \Leftrightarrow\\ (1+\rho p)(1+\rho-1-\rho p)q^M - \rho(1-p)q^L - \rho^2 p(1-p)q^M \geq 0 \Leftrightarrow\\ \rho(1-p)(1+\rho p-\rho p)q^M - \rho(1-p)q^L \geq 0 \Leftrightarrow\\ \rho(1-p)(q^M-q^L) \geq 0.\end{aligned}$$

Moreover q^P is lower than or equal to 1:

$$\begin{aligned}q^P \leq 1 \Leftrightarrow\\ (1+\rho p)(1+\rho)q^M - \rho(1-p)q^L \leq (1+\rho p)^2 + \rho^2 p(1-p) \Leftrightarrow\\ (1+\rho)q^M + (1+\rho)\rho p q^M - \rho q^L + \rho p q^L \leq 1 + 2\rho p + \rho^2 p \Leftrightarrow\\ \rho p \geq \frac{(1+\rho)q^M - 1 - \rho q^L}{(1+\rho)(1-q^M)+1-q^L}.\end{aligned}$$

The denominator is positive. The numerator is instead non positive, because $\rho \leq \frac{1-q^M}{q^M-q^L}$, as assumed by Inequality (5). Therefore the last inequality is satisfied.

The equilibrium expected policy is computed, substituting in $\bar{q}^P$ the solution (8) for the Condorcet winner q^P:

$$\bar{q}^P = \frac{\left(\frac{1+\rho p}{1+\rho}\right)^2 q^M + \left(\frac{1+\rho p}{1+\rho} v - \frac{\rho(1-p)}{1+\rho}\left(\frac{1+\rho p}{1+\rho}\right)^2\right) q^L}{\left(\frac{1+\rho p}{1+\rho}\right)^2 + v}$$
$$+ \frac{\frac{\rho(1-p)}{1+\rho}\left(\frac{1+\rho p}{1+\rho}\right)^2 q^L + \frac{\rho(1-p)}{1+\rho} v q^L}{\left(\frac{1+\rho p}{1+\rho}\right)^2 + v} =$$
$$\frac{(1+\rho p)^2 q^M + \rho^2 p(1-p) q^L}{(1+\rho p)^2 + \rho^2 p(1-p)}.$$

The equilibrium expected policy is lower than q^M, because it is a convex combination of q^L and q^M, therefore it lies in between the two bliss points. □

Proof of Proposition 6. The sign of the derivative of the expected policy with respect to the responsiveness to lobbying of merchant types is computed as follows:

$$\frac{\partial \bar{q}^P}{\partial \rho} < 0 \Leftrightarrow$$
$$\left((1+\rho p)^2 + p(1-p)\rho^2\right)\left(2(1+\rho p)pq^M + 2\rho p(1-p)q^L\right) -$$
$$\left((1+\rho p)^2 q^M + p(1-p)\rho^2 q^L\right)(2(1+\rho p)p + 2\rho p(1-p)) < 0 \Leftrightarrow$$
$$(q^M - q^L)\left(\rho^2 p^2(1-p)(1+\rho p) - \rho p(1-p)(1+\rho p)^2\right) < 0 \Leftrightarrow$$
$$-(q^M - q^L)\rho p(1-p)(1+\rho p) < 0.$$

The last inequality is satisfied. Hence the derivative of $\bar{q}^P$ with respect to ρ is negative. If ρ increases, the bias $q^M - \bar{q}^P$ increases.

The sign of the derivative of the expected policy with respect to the probability of saint types is computed as follows:

$$\frac{\partial \bar{q}^P}{\partial p} \geq 0 \Leftrightarrow$$
$$\left(2(1+\rho p)\rho q^M + \rho^2(1-2p)q^L\right)\left((1+\rho p)^2 + \rho^2 p(1-p)\right) -$$
$$\left((1+\rho p)^2 q^M + \rho^2 p(1-p)q^L\right)\left(2\rho(1+\rho p) + \rho^2(1-2p)\right) \geq 0 \Leftrightarrow$$
$$(q^M - q^L)\rho^2(1+\rho p)\left(2\rho p(1-p) - (1+\rho p)(1-2p)\right) \geq 0 \Leftrightarrow p \geq \frac{1}{2+\rho}.$$

Thus, the effect p on $\bar{q}^P$ is non-monotonic. If $p < \bar{p} = \frac{1}{2+\rho}$, increasing p increases the bias $q^M - \bar{q}^P$. If $p \geq \bar{p}$, increasing p reduces the bias.

Here I compute the effect of ρ and p on the median voter's welfare. By the envelope theorem, when deriving the median voter's indirect utility with respect to the parameters of the model, I do not take into account the effect of the parameters on the Condorcet winner's bliss point q^P:

$$\frac{\partial}{\partial \rho}\mathbb{E}(u(q^{*P}, q^M)) = 2(q^M - \bar{q}^P)\frac{\partial \bar{q}^P}{\partial \rho} - \frac{\partial v}{\partial \rho}(q^P - q^L)^2 =$$
$$\frac{2(q^M - \bar{q}^P)}{(1+\rho)^2}\left((pq^P + (1-p)q^L)(1+\rho) - (1+\rho p)q^P - \rho(1-p)q^L\right) -$$
$$\frac{(q^P - q^L)^2}{(1+\rho)^4}\left(2\rho p(1-p)(1+\rho)^2 - 2\rho^2(1+\rho)p(1-p)\right) =$$
$$-\frac{2(q^M - \bar{q}^P)}{(1+\rho)^2}(1-p)\left(q^P - q^L\right) - \frac{(q^P - q^L)^2}{(1+\rho)^4}2\rho(1+\rho)p(1-p) < 0$$

Thus, voter welfare decreases with ρ.

$$\frac{\partial}{\partial p}\mathbb{E}(u(q^{*P}, q^M)) =$$
$$2(q^M - \bar{q}^P)\frac{\rho}{1+\rho}(q^P - q^L) - \frac{\rho^2}{(1+\rho)^2}(1-2p)(q^P - q^L)^2 =$$
$$\frac{\rho}{1+\rho}(q^P - q^L)\left(2(q^M - \bar{q}^P) - \frac{\rho}{1+\rho}(1-2p)(q^P - q^L)\right).$$

I compute the following expressions:

$$q^M - \bar{q}^P = \frac{\rho^2 p(1-p)(q^M - q^L)}{(1+\rho p)^2 + \rho^2 p(1-p)},$$
$$q^P - q^L = \frac{(1+\rho p)(1+\rho)q^M - (1+\rho p)(\rho(1-p) + 1 + \rho p)q^L}{(1+\rho p)^2 + \rho^2 p(1-p)} =$$
$$\frac{(1+\rho p)(1+\rho)(q^M - q^L)}{(1+\rho p)^2 + \rho^2 p(1-p)}.$$

Substituting these two expressions in the derivative of the median voter's welfare with respect to p, I obtain:

$$\frac{\rho^2(q^P - q^L)(q^M - q^L)}{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))}\left(2\rho p(1-p) - (1+\rho p)(1-2p)\right).$$
$$\frac{\partial}{\partial p}\mathbb{E}(u(q^{*P}, q^M)) \geq 0 \Leftrightarrow$$
$$2\rho p(1-p) - (1+\rho p)(1-2p) \geq 0 \Leftrightarrow$$
$$\rho p + 2p - 1 \geq 0 \Leftrightarrow$$
$$p \geq \frac{1}{2+\rho}.$$

Thus, median voter's welfare and the policy bias have opposite comparative statics. $\square$

Is the median voter better off with the saint or the merchant?

The median voter has higher utility from the election of the saint type of P, with respect to the merchant type, if

$$-\left(q^M - q^P\right)^2 > -\left(q^M - \frac{q^P + \rho q^L}{1+\rho}\right)^2 \Leftrightarrow$$
$$q^P - q^M < q^M - \frac{q^P + \rho q^L}{1+\rho} \tag{9}$$

$q^P - q^M$ has been computed in the proof of Proposition 5:

$$q^P - q^M = \frac{(1-p)\rho\left(q^M - q^L\right)}{(1+\rho p)^2 + \rho^2 p(1-p)}.$$

$q^M - \frac{q^P + \rho q^L}{1+\rho}$ is computed as follows:

$$q^M - \frac{q^P + \rho q^L}{1+\rho} =$$
$$\frac{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))q^M - (1+\rho p)(1+\rho)q^M}{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))} +$$
$$\frac{\rho(1-p)q^L - \rho((1+\rho p)^2 + \rho^2 p(1-p))q^L}{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))} =$$
$$\frac{(1+\rho)((1+\rho p)\rho p + \rho^2 p(1-p))q^M + \rho((1-p)(1-\rho^2 p) - (1+\rho p)^2)q^L}{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))} =$$
$$\frac{\rho p(1+\rho)((1+\rho p) + \rho(1-p))q^M + \rho((1-p)(1-\rho^2 p) - (1+\rho p)^2)q^L}{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))} =$$
$$\frac{\rho(1+\rho)^2 p q^M - \rho(1+\rho)^2 p q^L}{(1+\rho)((1+\rho p)^2 + \rho^2 p(1-p))} =$$

$$\frac{\rho(1+\rho)p(q^M-q^L)}{(1+\rho p)^2+\rho^2 p(1-p)}.$$

Solving inequality (9), substituting these two expressions, I obtain:

$$\rho(1-p) < \rho(1+\rho)p \Leftrightarrow p > \frac{1}{2+\rho}. \qquad \square$$

Utilitarian welfare analysis

With quadratic utility, the comparative statics on utilitarian welfare are equal to comparative statics on the mean voter welfare. Therefore, to compute the effect of ρ on the utilitarian welfare I take advantage of the computation of the derivative of median voter welfare with respect to ρ in the proof of Proposition 6, where q^M is substituted with $\int_{[0,1]} q^i f dq^i := \bar{q}$. Moreover I add the effect of ρ on mean voter welfare given by its effect on q^P, because the envelope theorem does not hold:

$$\frac{\partial}{\partial \rho}\int\limits_{[0,1]} \mathbb{E}\left(u(q^{*P},q^i)\right) f dq^i =$$
$$-\frac{2(\bar{q}-\bar{q}^P)}{(1+\rho)^2}(1-p)\left(q^P-q^L\right)-\frac{(q^P-q^L)^2}{(1+\rho)^4}2\rho(1+\rho)p(1-p)+$$
$$\left(2\left(\bar{q}-\bar{q}^P\right)\left(\frac{1+\rho p}{1+\rho}\right)-2v\left(q^P-q^L\right)\right)\frac{\partial q^P}{\partial \rho}$$

The first addendum is positive if $\bar{q} < \bar{q}^P$. The second addendum is negative. The first factor in the third addendum, if $\bar{q} = q^M$, is the derivative of median voter welfare with respect to q^P, which is zero because of the first order condition. Given that it increases with $\bar{q}$, it must be negative for $\bar{q} < q^M$, and positive otherwise. The sign of the derivative of q^P with respect to ρ is given by the following computation:

$$\frac{\partial q^P}{\partial \rho} = \frac{(1+2\rho p+\rho^2 p)(pq^M+q^M+2\rho p q^M-q^L+pq^L)}{((1+\rho p)^2+\rho^2 p(1-p))^2}$$
$$-\frac{(q^M+\rho p q^M+\rho q^M+\rho^2 p q^M-\rho q^L+\rho p q^L)(2p+2\rho p)}{((1+\rho p)^2+\rho^2 p(1-p))^2} =$$
$$\frac{(q^M-q^L)(1-p)(1-\rho^2 p)}{((1+\rho p)^2+\rho^2 p(1-p))^2}.$$

Therefore $\frac{\partial q^P}{\partial \rho}$ is positive if $\rho \le 1/\sqrt{p}$.

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