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Coordination through cooperative advertising in a two-period consumer electronics supply chain

Yi He[a], Hang Wang[b], Qiang Guo[c,*], Qingyun Xu[d]

[a] School of Management, Hainan University, Haikou, Hainan, PR China
[b] School of Management, Nanchang University, Nanchang, Jiangxi, PR China
[c] College of Tourism, Hainan University, Haikou, Hainan, PR China
[d] School of Economics and Management, Harbin Institute of Technology, Harbin, Heilongjiang, PR China

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ABSTRACT

Firms in the consumer electronics industry frequently launch new styles of their products, which leads to a "two-period" phenomenon in their product sales. Only a few published articles have considered two-period models in cooperative advertising. This paper investigates co-op advertising strategies in a two-period supply chain consisting of a single manufacturer and a single retailer. Utilizing game theory, we consider two different scenarios: a decentralized scenario with a cooperative advertising program and an integrated scenario. Aside from these scenarios, we propose a supply chain contract to coordinate this supply chain system. This paper has the following conclusions: (i) the manufacturer usually does not provide the same advertising subsidy strategy for the two generation products in the same period; (ii) the manufacturer may provide a low subsidy rate to the retailer if the advertising long-term effect is strong; and (iii) we demonstrate that the two-way subsidy contract can achieve a perfect supply chain coordination if a transfer payment exists.

1. Introduction

As consumer demands change rapidly and science and technology continuously develop, the competition among firms is becoming fiercer, particularly in the consumer electronics (e.g., mobile phones, personal computer) industry. Consumer electronics have been constantly upgraded. As Samsung Executive Deputy President Ravinder Zutshi said, "No product has a life cycle of more than 12 months." Most mobile phone producers update their products within a year. For instance, Apple, Inc. launched the iPhone 4 in June 2010, and from then on, Apple, Inc. consecutively launched the iPhone 4S, iPhone 5, iPhone 5S, iPhone 6, iPhone 6S, iPhone 7, iPhone 8, and iPhone XS in the subsequent eight years. Apple, Inc. usually releases a new style in September each year. As the new product comes to market, previous products are gradually phased out of the market.

Firms in the consumer electronics industry frequently launch new styles of their products, which leads to a "two-period" phenomenon in product sales. The first period is a product's normal selling time, which is from the time a new product is launched into the market to the period that a newer-generation product appears in the market. The second period is a product's last salvage time, which is from the time that a newer-generation product appears in the market to the time this product exits the market. For instance, Apple, Inc. launched the iPhone 8 in September 2017, and the iPhone XS was launched in September 2018. For the iPhone 8, the first period is from September 2017 to September 2018, and the second period is from September 2018 to the present. In the two periods of product sales, different marketing tools are applied. For instance, Apple, Inc. usually does not do discounts during the product's regular selling season, while it tends to adopt price promotion when a new generation product is about to be released (Farfan, 2017).

Cooperative advertising is an effective marketing tool for product sales, brand promotion, and market exploitation. It is a cooperative mechanism where the manufacturer grants a subsidy to the retailer to motivate it to put more money into advertising, which leads to additional sales of the product to the retailer as well as the manufacturer. The supply chain members' profits will be improved by adopting a cooperative advertising program.

Cooperative advertising programs are extensively practiced. Dant and Berger (1996) stated that cooperative advertising programs finance 25%–40% of retailers' local advertisements. The expenditure on cooperative advertising in the U.S. was approximately $15 billion in 2000 (Nagler, 2006) and $50 billion in 2010 (Yan, 2010). Intel Inside is a typical and successful cooperative advertising program where Intel

* Corresponding author. College of Tourism, Hainan University, No. 58, Renmin Road, Haikou City, Hainan, 570228, China.
E-mail address: 13648669169@163.com (Q. Guo).

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shares 40% of advertising costs (30% in China) for computer manufacturers who combine Intel's image or logo in the advertisement. Researchers have also paid significant attention to cooperative advertising. There are two main types of cooperative advertising models: one is the static model (Berger, 1972; Huang and Li, 2001; Li et al., 2002; Yue et al., 2006; Xie and Neyret, 2009; Xie and Wei, 2009; Yang et al., 2013; He et al., 2014; Gou et al., 2014b; Karray and Amin, 2015; Yan et al., 2016; Karray et al., 2017; Martín-Herrán and Sigué, 2017; Ahmadi-Javid and Hoseinpour, 2018; Zhou et al., 2018), and the other is the dynamic model (Sethi, 1983; Chintagunta and Jain, 1992; Jørgensen et al., 2000, 2001; Karray and Zaccour, 2005; Nair and Narasimhan, 2006; He et al., 2009, 2013; Gou et al., 2014a; Zhang et al., 2013; Chutani and Sethi, 2018; Guo and Ma, 2018; De Giovanni et al., 2019).

Almost all the research in co-op advertising has concentrated on a one-period model. Few publications have focused on the two-period model (He et al., 2014; Karray et al., 2017; Martín-Herrán and Sigué, 2017). In contrast to the abovementioned studies, this paper considers the competition between two generation products and the long-term advertising effect and then utilizes the two-period models to investigate co-op advertising strategies of supply chain members and proposes a supply chain contract to coordinate this supply chain. Our study seeks to provide solutions to the following problems:

(1) What are the optimal advertising efforts that the retailer will adopt during the two different periods? In the second period, will the retailer advertise for the first-generation product? Under what situation will the retailer advertise for the two generations of products?
(2) What are the influences of the long-term advertising effect on channel members' decisions?
(3) What subsidy strategy will the manufacturer adopt? Will the manufacturer increase the subsidy rates for the two generations of products at the same time?
(4) How should this supply chain system be coordinated?

The rest of our study is structured as follows. Section 2 reviews the literature. Section 3 describes the basic model. The equilibrium solutions for supply chain members in the different scenarios are discussed in Sections 4 and 5, respectively. Section 6 proposes a supply chain contract to coordinate the supply chain system. Conclusions are presented in Section 7. All the proofs of the results are in the Appendix.

2. Literature review

This paper is related to several streams of cooperative advertising studies, each of which is reviewed below.

A group of studies discussed the cooperative advertising strategies in the one-period supply chain, and the related cooperative advertising models can be divided into the static model and the dynamic model. Considering a static framework, the first mathematical discussion on cooperative advertising was published by Berger (1972), who proposed that profits may be increased significantly by quantitative analysis compared with simplistic fifty–fifty cost sharing. Huang and Li (2001) extended the cooperative advertising model by dividing advertising into local advertising and national advertising. This research discussed three cooperative advertising models: two noncooperative games and one cooperative game, wherein the latter maximizes system profits. Li et al. (2002) performed further analysis on the manufacturer-dominated relationship in which the manufacturer optimizes profits without reducing the retailer's profit. Some papers extended the advertising models to include decisions on pricing. Yue et al. (2006) studied the coordination of cooperative advertisement in a two-level supply chain in which the manufacturer provides price deductions to customers. Karray and Zaccour (2006) introduced the competition of the retailer's store brand into the cooperative advertising. Xie and Neyret (2009) categorized co-op advertising and pricing strategies into four classic types (i.e., Nash, Stackelberg retailer, Stackelberg manufacturer, and one cooperative game) in a two-member marketing channel. Xie and Wei (2009) addressed a coordination mechanism that relies on both wholesale price and manufacturer's participation rate in a two-member distribution channel. Yang et al. (2013) studied the effects of the retailer's fairness in a supply chain. Gou et al. (2014a, 2014b) considered the threshold effect in cooperative advertising and explored the influences of advertising threshold on channel members' decisions. Karray and Amin (2015) studied the influences of co-op advertising in a supply chain with competing retailers considering both advertising and pricing as decision variables. Zhao et al. (2016) showed that the manufacturer can benefit from sharing part of a retailer's advertising expenditure when the price elasticity is larger than a certain value. Yan et al. (2016) explored the cooperative advertising strategies in the dual-channel distributions under the environment of demand uncertainty. Zhou et al. (2018) studied the optimal cooperative advertising strategies in a two-echelon supply chain with risk-averse agents; the results showed that the profits of both agents are improved when the leader is more risk averse than the follower in the decentralized scenario.

Many dynamic cooperative advertising models were based on the advertising goodwill model of Nerlove and Arrow (1962) in the one-period supply chain. Chintagunta and Jain (1992) built a dynamic model for determining the optimal marketing efforts for one manufacturer and one retailer in a two-member marketing channel. Jørgensen et al. (2000) considered four scenarios utilizing different games and showed that supporting both short- and long-term advertising provides more profit to both channel members than from other scenarios. Assuming that marginal returns to goodwill are decreasing, Jørgensen et al. (2001) investigated dynamic advertising and promotion strategies in two noncooperative games and showed that whether or not the goodwill stock has a decreasing marginal effect on sales, both channel members receive large payoffs. Karray and Zaccour (2005) studied co-op advertising under the circumstance that the retailer sold both its own products and the manufacturer's products. He et al. (2013) explored the effects of competitive intensity on the player's profit in a supply chain with competing manufacturers and a single retailer. Zhang et al. (2013) considered the influences of consumer's reference price on the co-op advertising decisions of all channel members. Gou et al. (2014a, 2014b) focused on the horizontal cooperative program and evaluated three cooperative scenarios. De Giovanni et al. (2019) studied the influences of the effects of cooperative advertising programs on the channel members' inventory management and pricing decisions: the results showed that the cooperative program could improve all channel members' profits in only a few cases. Sethi (1983) proposed another classical dynamic model. This research did not consider advertising goodwill but found a relationship between advertising efforts and market share. Following Sethi (1983), He et al. (2009) discussed the changes in a retailer's advertising efforts under the decentralized system and vertically integrated system. Chutani and Sethi (2018) investigated the optimal cooperative advertising strategies in a “multiple manufacturers and multiple retailers” supply chain, and analyzed the impacts of competition on the channel members' optimal decisions.

Cooperative advertising research studies focusing on the two-period supply chain have been limited. He et al. (2014) utilized a two-period model to investigate the co-op advertising in a fashion and textiles supply chain, and provided a supply chain contract to achieve supply chain coordination. Considering the pricing and advertising strategies, Martín-Herrán and Sigué (2017) investigated the manufacturer and retailer's cooperative advertising strategies over a two-period planning horizon in three advertising arrangements. Considering the competitive market, Karray et al. (2017) also utilized two-stage game theoretic models to study two competing manufacturers' cooperative advertising strategies. In contrast to the abovementioned literature, this paper considers the competition between two generation products and explores the potential coordinating power of the two-way subsidy contract in this two-period supply chain.

3. Basic model

In this study, we consider a manufacturer-retailer supply chain. The selling season is divided into two periods, making it different from previous cooperative advertising models. The firm is assumed to sell a style of Product 1 (first-generation product) in the first period, and the firm will launch a new style of Product 2 (second-generation product) at the beginning of the second period and sell Product 1 simultaneously. Assume the two generations of products' design levels of the manufacturer are x_1 and x_2, respectively. The retailer advertises the products. The advertising efforts on the product $j(j = 1,2)$ during the period $i(i = 1,2)$ are a_{ij}. Therefore, a_{11} denotes the advertising efforts on the first-generation product during the first period, and these advertising efforts are mainly focused on brand promotion, while the retail promotion is commonly used during the second period, which is denoted by a_{21}. The demand for the product j during period i is D_{ij}, and the marginal profits of manufacturer and retailer are ρ_{mij} and ρ_{rij}, respectively.

As in many other papers (Karray and Zaccour, 2007; He et al., 2014; Jørgensen and Zaccour, 2014; Yan et al., 2014), we express the demand function of the first-generation product in the first period as:

$$D_{11} = \alpha_{11} + \lambda x_1 + \beta a_{11}, \tag{1}$$

where $\alpha_{11} > 0$ is the base market size of the first-generation product in the first period. Parameters λ and β are both positive constants that denote the influence of product design levels and advertising efforts on demand, respectively. In our model, we assume the product price is an exogenous variable due to the following reasons: (i) Many firms adopt a fixed price strategy when they launch the different generation products, such as Apple, Inc. launching the iPhone 6S (16G) at a price of ¥5288 in China, which is the same price as that of the iPhone 6 (16G). (ii) This assumption is also found in the literature related to co-op advertising (Chintagunta and Jain, 1992; Huang and Li, 2001; Zhang et al., 2013; Gou et al., 2014a, 2014b).

Competition exists between Product 1 and Product 2. The new style of a product attracts the attention of consumers due to its more powerful functions or more elegant appearance, which leads to the sales decrease of the product from the previous generation. Meanwhile, the advertising of Product 2 also has a passive effect on the demand for Product 1. In the second period, Product 1 obviously becomes an obsolete product when the next-generation Product 2 is launched. Therefore, the design levels of Product 1 do not affect demand in the second period. Additionally, the retailer's advertising efforts during the first period also have a long-term effect on the second period demand (Martín-Herrán and Sigué, 2017; Karray et al., 2017). Then, the demand function of Product 1 in the second period is:

$$D_{21} = \alpha_{21} + \gamma_1 a_{11} + \theta\beta a_{21} - \mu\lambda x_2 - \delta\beta a_{22}, (0 < \delta < 1, \delta < \theta, 0 < \mu < 1) \tag{2}$$

where $\alpha_{21} > 0$ is the base market size of the first-generation product in the second period. The parameters μ and δ are both larger than 0 and smaller than 1. Items $\mu\lambda$ and $\delta\beta$ denote the influence of Product 2's design levels and advertising efforts on the demand for Product 1, respectively. Moreover, $\theta\beta$ denotes the effect of Product 1's advertising efforts on its own demand. The condition $\delta < \theta$ implies that the product's own advertising efforts has a greater effect on the demand than that of other products. Item $\gamma_1 a_{11}$ represents the long-term effect on Product 1's second period demand. If the retailer's first period advertising efforts are contributed to the firm's brand image, this item takes a positive value, while if the advertising efforts hurt the firm's brand image, this item takes a negative value (Jørgensen et al., 2003; Martín-Herrán and Sigué, 2017; Karray et al., 2017). In our study, the retailer's advertising efforts during the first period contributed to improving the manufacturer's brand image (goodwill); therefore, these advertising efforts have a positive long-term effect.

In the second period, Product 1's advertising efforts have a passive influence on the demand for Product 2. In addition, Product 1's advertising efforts during the first period contributed to improving the manufacturer's brand image (goodwill); therefore, these advertising efforts also positively affect the new Product 2's demand. Similar to the assumption of Nair and Narasimhan (2006), this paper assumes that both competitive effects are the same. Therefore, the demand function of the new Product 2 in the second period is as follows:

$$D_{22} = \alpha_{22} + \gamma_2 a_{11} + \beta a_{22} + \lambda x_2 - \delta\beta a_{21}, \tag{3}$$

According to Gavious and Lowengart (2012), we assume the design cost function of the manufacturer's design levels is as follows:

$$C(x_j) = x_j^2, j \in \{1,2\}. \tag{4}$$

Similar to previous studies, such as that of Zhang et al. (2013), the advertising cost function follows this quadratic form:

$$C(a_{ij}) = \frac{1}{2}a_{ij}^2. \tag{5}$$

The manufacturer's marginal profit of product j, $(j = 1,2)$ in period i, $(i = 1,2)$ is ρ_{mij}, and the retailer's marginal profit of generation j in period i is ρ_{rij}. Therefore, the profit functions of both channel members are:

$$\pi_m = \rho_{m11}D_{11} + \rho_{m21}D_{21} + \rho_{m22}D_{22} - x_1^2 - x_2^2, \tag{6}$$

and

$$\pi_r = \rho_{r11}D_{11} + \rho_{r21}D_{21} + \rho_{r22}D_{22} - \frac{1}{2}(a_{11}^2 + a_{21}^2 + a_{22}^2), \tag{7}$$

In the following sections, we will compare and analyze the optimal decisions of supply chain members in two different scenarios, i.e., (i) a decentralized scenario within a cooperative advertising program; and (ii) an integrated scenario.

4. Decentralized scenario within a cooperative advertising program

In this section, we consider a cooperative advertising program between the channel members, that is, the manufacturer grants a subsidy to the retailer to motivate its advertising spending. Consider the subsidy rates ϕ_{ij} $(0 \le \phi_{ij} \le 1)$ and let the superscript $(\cdot^D)$ denote the optimal outcome for this scenario; then, the channel members' profit functions are changed into:

$$\pi_m^D = \rho_{m11}D_{11} + \rho_{m21}D_{21} + \rho_{m22}D_{22} - x_1^2 - x_2^2 - \frac{1}{2}\phi_{11}a_{11}^2 - \frac{1}{2}\phi_{21}a_{21}^2 - \frac{1}{2}\phi_{22}a_{22}^2, \tag{8}$$

$$\pi_r^D = \rho_{r11}D_{11} + \rho_{r21}D_{21} + \rho_{r22}D_{22} - \frac{1}{2}(1-\phi_{11})a_{11}^2 - \frac{1}{2}(1-\phi_{21})a_{21}^2 - \frac{1}{2}(1-\phi_{22})a_{22}^2. \tag{9}$$

Decision sequences of the channel members are constructed as follows: (i) The manufacturer first determines the design levels of Product 1 and grants the subsidy rate ϕ_{11} at the beginning of the first period. (ii) The retailer then decides on the advertising efforts of Product 1. (iii) At the beginning of the second period, the manufacturer first determines the design levels of Product 2 and provides the subsidy rates ϕ_{21} and ϕ_{22}. (iv) The retailer then decides on the advertising efforts of the two generations of products.

We solve the equilibrium solutions by conducting backward induction. In the decentralized scenario within a cooperative advertising program, the optimal design levels of the manufacturer's two generations of products are:

$$x_1^D = \frac{1}{2}\lambda\rho_{m11}, \quad (10)$$

$$x_2^D = \begin{cases} \frac{1}{2}\lambda(\rho_{m22} - \mu\rho_{m21}) & if\ (\rho_{m22} - \mu\rho_{m21}) > 0, \\ 0 & else. \end{cases} \quad (11)$$

and the advertising efforts of the retailer in the two different periods are:

$$a_{11}^D = \frac{\beta\rho_{r11} + \gamma_1\rho_{r21} + \gamma_2\rho_{r22}}{1 - \phi_{11}}, \quad (12)$$

$$a_{21}^D = \begin{cases} \frac{\beta(\theta\rho_{r21} - \delta\rho_{r22})}{1 - \phi_{21}} & if\ (\theta\rho_{r21} - \delta\rho_{r22}) > 0, \\ 0 & else. \end{cases} \quad (13)$$

$$a_{22}^D = \begin{cases} \frac{\beta(\rho_{r22} - \delta\rho_{r21})}{1 - \phi_{22}} & if\ (\rho_{r22} - \delta\rho_{r21}) > 0, \\ 0 & else. \end{cases} \quad (14)$$

According to Eqs. (10)–(14), we obtain two important conclusions related to the products' design levels and advertising efforts.

Proposition 1. *(i) The product's design levels are directly influenced by the manufacturer's marginal profit. (ii) The subsidy rates positively influence only the retailer's advertising efforts. (iii) The advertising efforts are directly affected by the retailer's marginal profit. (iv) The retailer may not always advertise for the two generation products.*

From Proposition 1 (i), the following implications can be obtained. First, the manufacturer's marginal profit, ρ_{m11} and ρ_{m22}, positively affects the design levels of the first- and second-generation products, respectively (i.e., $dx_1^D/d\rho_{m11} \geq 0$ and $dx_2^D/d\rho_{m22} \geq 0$). Additionally, the marginal profit of the manufacturer's first-generation product is independent of the design levels of the second-generation product (i.e., $dx_2^D/d\rho_{m11} = 0$). Second, the marginal profit of the manufacturer's first-generation product in period 2 has a negative impact on the design levels of the second-generation product (i.e., $dx_2^D/d\rho_{m21} \leq 0$). As a result, when the first-generation product still has strong profitability in the second period, the manufacturer may reduce the investment in the design levels of the second-generation product. In practice, Xiaomi, a famous mobile phone brand in China, released the Xiaomi 4 in July 2014. This mobile phone has been overwhelmingly accepted by the majority of consumers. During the "double eleven" (the biggest online shopping festival in China) in 2014, Xiaomi 4 was the sales champion of the three major e-commerce platforms (Taobao, JD.com, and Suning). However, Xiaomi did not release a new generation product during the subsequent year as usual, but turned its attention to another series of products. Not until February 2016 was the new generation of the same series, Xiaomi 4S, released[1]. Perhaps the flowing proposition can explain why Xiaomi postponed the release of its new products.

Proposition 1(ii) can be interpreted from two aspects. For one thing, the subsidy rates ϕ_{11}, ϕ_{21}, and ϕ_{22} positively influence the retailer's advertising efforts a_{11}, a_{21}, and a_{22}, respectively. Therefore, to increase product sales, the manufacturer can stimulate retailers to increase their advertising efforts by offering them a higher subsidy. Moreover, the subsidy rates have no influence on the manufacturer's product design levels. The manufacturer will not reduce the investment in product design even if it provides high subsidy rates to the retailer.

Proposition 1(iii) implies that the retailer's marginal profit ρ_{r11} positively affects the advertising efforts of a retailer's first-generation product in the first period (i.e., $\partial a_{11}^D/\partial\rho_{r11} \geq 0$). Moreover, due to the long-term advertising effect, the marginal profits ρ_{r21} and ρ_{r22} also have positive effects on the retailer's advertising efforts in the first period (i.e., $\partial a_{11}^D/\partial\rho_{r21} \geq 0$ and $\partial a_{11}^D/\partial\rho_{r22} \geq 0$). This is intuitive. In industry practice, many enterprises will invest a large amount of advertising when releasing the new generation product because it represents the image of the next series of products. In addition, the marginal profit of the retailer in the second period coming from the first-generation (second-generation) product positively (negatively) affects the advertising efforts of the first-generation product and negatively (positively) affects that of the second-generation product (i.e., $\partial a_{21}^D/\partial\rho_{r21} \geq 0$, $\partial a_{21}^D/\partial\rho_{r22} \leq 0$, $\partial a_{22}^D/\partial\rho_{r21} \leq 0$, and $\partial a_{22}^D/\partial\rho_{r22} \geq 0$). This is not difficult to understand. Products in the same series often compete with each other. For example, when Apple, Inc. launched iPhone XS in September 2018, previous generations of products, such as iPhone 6, iPhone 7 and iPhone 8, were still strong competitors for iPhone XS, particularly when the price of old products decreased.

Proposition 1(iv) can be interpreted as follows. (a) From the constraint condition $\rho_{r21} > (\delta/\theta)\rho_{r22}$ of Eq. (13), we observe that when the ratio between the retailer's marginal profits ρ_{r21} and ρ_{r22} reaches a certain value δ/θ, the retailer will be motivated to spend on advertising for the first-generation product in the second period. (b) From Eq. (14), if the condition $\rho_{r22} > \delta\rho_{r21}$ holds, the retailer must spend on advertising for the second-generation product. (c) Given $0 \leq \delta < \theta$ and $0 \leq \delta < 1$, when the condition $\delta\rho_{r21} < \rho_{r22} < (\theta/\delta)\rho_{r21}$ (or $(\delta/\theta)\rho_{r22} < \rho_{r21} < \rho_{r22}/\delta$) holds, the retailer will be willing to spend on advertising for the two generations of products. Combined with (a), (b) and (c), we have that the retailer may not always advertise for the two generation products.

Then, the equilibrium solutions (such as the product design levels and advertising efforts) are substituted into Eq. (8). The optimal value of subsidy rates from the first-order derivative of π_m^D with respect to ϕ_{ij} is determined. The manufacturer's optimal subsidy rates are:

$$\phi_{11}^D = \begin{cases} \frac{\beta(2\rho_{m11} - \rho_{r11}) + \gamma_1(2\rho_{m21} - \rho_{r21}) + \gamma_2(2\rho_{m22} - \rho_{r22})}{\beta(2\rho_{m11} + \rho_{r11}) + \gamma_1(2\rho_{m21} + \rho_{r21}) + \gamma_2(2\rho_{m22} + \rho_{r22})} \\ if\quad \beta(2\rho_{m11} - \rho_{r11}) + \gamma_1(2\rho_{m21} - \rho_{r21}) + \gamma_2(2\rho_{m22} - \rho_{r22}) > 0, \\ 0 \quad else. \end{cases} \quad (15)$$

$$\phi_{21}^D = \begin{cases} \frac{2(\theta\rho_{m21} - \delta\rho_{m22}) - (\theta\rho_{r21} - \delta\rho_{r22})}{2(\theta\rho_{m21} - \delta\rho_{m22}) + (\theta\rho_{r21} - \delta\rho_{r22})} & if \quad \begin{pmatrix} (\theta\rho_{m21} - \delta\rho_{m22}) > \\ 0.5(\theta\rho_{r21} - \delta\rho_{r22}) > 0 \end{pmatrix}, \\ 0 \quad else. \end{cases} \quad (16)$$

$$\phi_{22}^D = \begin{cases} \frac{2(\rho_{m22} - \delta\rho_{m21}) - (\rho_{r22} - \delta\rho_{r21})}{2(\rho_{m22} - \delta\rho_{m21}) + (\rho_{r22} - \delta\rho_{r21})} & if \quad \begin{pmatrix} (\rho_{m22} - \delta\rho_{m21}) > \\ 0.5(\rho_{r22} - \delta\rho_{r21}) > 0 \end{pmatrix}, \\ 0 \quad else. \end{cases} \quad (17)$$

From Eqs. (15)–(17), four important results regarding subsidy rates proposed in Proposition 2 are obtained.

Proposition 2.

(i) *In the first period, the marginal profit ρ_{m11} has a positive influence on the subsidy rate, while the marginal profit ρ_{r11} (the long-term effect γ_i) negatively affects the subsidy rate.*
(ii) *In the second period, if the manufacturer increases the subsidy rate of one generation of product, then it will decrease that of the other generation of product accordingly.*
(iii) *If the ratio between the manufacturer's marginal profits $\rho_{m21}(\rho_{m22})$ and $\rho_{m22}(\rho_{m21})$ increases, then the manufacturer will improve the subsidy rate of the first (second) - generation product in the second period. If the ratio between the retailer's marginal profits $\rho_{r21}(\rho_{r22})$ and $\rho_{r22}(\rho_{r21})$ increases, then the manufacturer will reduce the subsidy rate of the first (second) -generation product in the second period.*

Proposition 2(i) provides some significant results. First, when the manufacturer has a large marginal profit, it will offer a high subsidy rate to stimulate the retailer' spending on advertisement to make a high profit. In contrast, if the retailer' marginal profit is high, it will have a strong incentive to spend more on advertising, although the manufacturer does not offer an advertising allowance. In addition, if the long-term advertising effect is strong, i.e., the advertisement in period 1 has a positive effect on the demand for products in period 2, according to

[1] https://www.mi.com/about/history/.

Proposition 1 the retailer will spend more on the advertising in the first period; consequently, the manufacturer will reduce the advertising subsidy for the retailer in the first period.

Proposition 2(ii) is true because $\partial\phi_{22}^D/\partial\rho_{r22} < 0$, $\partial\phi_{21}^D/\partial\rho_{r22} > 0$, $\partial\phi_{22}^D/\partial\rho_{r21} > 0$, and $\partial\phi_{21}^D/\partial\rho_{r21} < 0$. When the retailer's marginal profit of the second-generation product ρ_{r22} increases, the manufacturer will reduce the subsidy rate of the second-generation product and increase that of the first-generation product in the second period. Similarly, when the manufacturer increases the subsidy rate of the second-generation product, it will reduce the subsidy rate for the first-generation product correspondingly. This conclusion implies that the manufacturer usually does not provide the same advertising subsidy strategy for both products in one period. In other words, the manufacturer usually does not simultaneously increase or decrease the subsidy rate for two generations of products.

Proposition 2(iii) is valid. Given that parameters θ and δ are fixed, the item $(\theta\rho_{m21} - \delta\rho_{m22})$ increases as the ratio ρ_{m21}/ρ_{m22} increases, which leads to an increase in subsidy rate ϕ_{21}^D. In contrast, when the ratio ρ_{r21}/ρ_{r22} increases, the subsidy rate ϕ_{21}^D will decrease. This conclusion is consistent with Proposition 2(i). For the manufacturer, it will offer higher advertising subsidy for the product with higher profitability. This is not contrary to common sense. However, the manufacturer will reduce the advertising subsidy for the product that brings higher profit to retailers. This can be explained as follows. When the retailer can obtain a higher profit from selling the product, it will actively invest more in advertising. At this moment, the advertising efforts may have reached the manufacturer's expectations. Therefore, the manufacturer will reduce the advertising subsidy for the product.

Substituting Eqs. (10)–(17) into Eqs. (8) and (9), we obtain the channel members' profits in the first case as follows: π_m^D and π_r^D.

5. Integrated scenario

In this section, the manufacturer is vertically integrated by the retailer as a single firm to establish a benchmark for supply chain coordination. Let the superscript $(\cdot^I)$ denote the optimal outcome for this scenario. Then, the integrated system's profit function is:

$$\begin{aligned}\pi^I = \pi_m + \pi_s &= (\rho_{m11} + \rho_{r11})(\alpha_{11} + \lambda x_1 + \beta a_{11}) - (a_{11}^2 + a_{21}^2 + a_{22}^2)/2\\ &+ (\rho_{m21} + \rho_{r21})(\alpha_{21} + \gamma_1 a_{11} + \theta\beta a_{21} - \mu\lambda x_2 - \delta\beta a_{22})\\ &+ (\rho_{m22} + \rho_{r22})(\alpha_{22} + \gamma_2 a_{11} + \beta a_{22} + \lambda x_2 - \delta\beta a_{21}) - x_1^2 - x_2^2.\end{aligned} \quad (18)$$

Adopting standard backward induction, we obtain the equilibrium solutions as follows:

$$x_1^I = \frac{1}{2}\lambda(\rho_{r11} + \rho_{m11}), \quad (19)$$

$$x_2^I = \begin{cases}\frac{1}{2}\lambda(\rho_{r22} - \mu\rho_{r21} + \rho_{m22} - \mu\rho_{m21}) & if\ (\rho_{r22} - \mu\rho_{r21} + \rho_{m22} - \mu\rho_{m21} > 0),\\ 0 & else.\end{cases} \quad (20)$$

$$a_{11}^I = \beta(\rho_{m11} + \rho_{r11}) + \gamma_1(\rho_{m21} + \rho_{r21}) + \gamma_2(\rho_{m22} + \rho_{r22}), \quad (21)$$

$$a_{21}^I = \begin{cases}\beta(\theta\rho_{r21} - \delta\rho_{r22} + \theta\rho_{m21} - \delta\rho_{m22}) & if\ (\theta\rho_{r21} - \delta\rho_{r22} + \theta\rho_{m21} - \delta\rho_{m22} > 0),\\ 0 & else.\end{cases} \quad (22)$$

$$a_{22}^I = \begin{cases}\beta(\rho_{r22} - \delta\rho_{r21} + \rho_{m22} - \delta\rho_{m21}) & if\ (\rho_{r22} - \delta\rho_{r21} + \rho_{m22} - \delta\rho_{m21} > 0),\\ 0 & else.\end{cases} \quad (23)$$

Together with the results of decentralized scenario, the optimal product design levels and advertising efforts in the integrated system are regarded as larger than those in the decentralized scenario. In addition, the whole supply chain profit is also improved, and the supply chain surplus π^s can be given by Eq. (24):

$$\pi^s = \pi^I - \pi^D = \pi^I - \pi_m^D - \pi_r^D. \quad (24)$$

To the best of our knowledge, it is indeed difficult for two independent firms to determine advertising strategies as a single firm for legal or other reasons. Channel members need to design a feasible supply chain contract to make both channel members choose the optimal decisions of the supply chain system in the decentralized scenario. Therefore, this study designs a supply chain contract to coordinate the two-period consumer electronics supply chain.

6. Supply chain coordination

Our study introduces a two-way subsidy contract to coordinate the two-period consumer electronics supply chain. Not only does the manufacturer share a part of the advertising expenditure with subsidy rates ϕ_{ij}, but the retailer also offers the subsidy rate ψ_j to the manufacturer's product design costs. Downstream firms typically participate in the upstream firms' product research to improve product quality. Some studies have found that the manufacturer was disposed to grant a subsidy to its supplier's quality improvement cost (Chao et al., 2009; He et al., 2016). The downstream firm (a buyer)'s involvement in quality improvement can significantly affect all channel members' profits (Zhu et al., 2007).

Let the superscript $(\cdot^C)$ denote the optimal outcome for this scenario; thus, the profit function of manufacturer is changed to:

$$\begin{aligned}\pi_m^C = &\rho_{m11}D_{11} + \rho_{m21}D_{21} + \rho_{m22}D_{22} - (1 - \psi_1)x_1^2 - (1 - \psi_2)x_2^2 - \frac{1}{2}\phi_{11}a_{11}^2\\ &- \frac{1}{2}\phi_{21}a_{21}^2 - \frac{1}{2}\phi_{22}a_{22}^2,\end{aligned} \quad (25)$$

and retailer's profit function is:

$$\begin{aligned}\pi_r^C = &\rho_{r11}D_{11} + \rho_{r21}D_{21} + \rho_{r22}D_{22} - \frac{1}{2}(1 - \phi_{11})a_{11}^2 - \frac{1}{2}(1 - \phi_{21})a_{21}^2\\ &- \frac{1}{2}(1 - \phi_{22})a_{22}^2 - \psi_1 x_1^2 - \psi_2 x_2^2.\end{aligned} \quad (26)$$

From Eqs. (25) and (26), we obtain the optimal product design levels and advertising efforts through backward induction. The two-generation products' design levels are:

$$x_1^C = \frac{\lambda\rho_{m11}}{2(1 - \psi_1)}, \quad (27)$$

$$x_2^C = \begin{cases}\frac{\lambda(\rho_{m22} - \mu\rho_{m21})}{2(1 - \psi_2)} & if\ (\rho_{m22} - \mu\rho_{m21}) > 0,\\ 0 & else.\end{cases} \quad (28)$$

and the optimal advertising efforts are:

$$a_{11}^C = \frac{\beta\rho_{r11} + \gamma_1\rho_{r21} + \gamma_2\rho_{r22}}{1 - \phi_{11}}, \quad (29)$$

$$a_{21}^C = \begin{cases}\frac{\beta(\theta\rho_{r21} - \delta\rho_{r22})}{1 - \phi_{21}} & if\ (\theta\rho_{r21} - \delta\rho_{r22}) > 0,\\ 0 & else.\end{cases} \quad (30)$$

$$a_{22}^C = \begin{cases}\frac{\beta(\rho_{r22} - \delta\rho_{r21})}{(1 - \phi_{22})} & if\ (\rho_{r22} - \delta\rho_{r21}) > 0,\\ 0 & else.\end{cases} \quad (31)$$

The expressions of product design levels imply that the more the retailer provides subsidy rates to the manufacturer, the larger the positive influence on the manufacturer's product design levels. The expressions of advertising efforts imply that the more the manufacturer shares advertising expenditure, the larger the retailer's advertising spending.

We let $x_1^C = x_1^I$, $x_2^C = x_2^I$, $a_{11}^C = a_{11}^I$, $a_{21}^C = a_{21}^I$, and $a_{22}^C = a_{22}^I$, and solve the five equations. The exact values of ϕ_{ij} and ψ_j can be obtained, and the supply chain is coordinated. The whole supply chain profit is maximal when the supply chain member makes decisions unilaterally. The optimal solutions of the subsidy rates in the supply chain contract are:

$$\psi_1^C = \frac{\rho_{r11}}{\rho_{r11} + \rho_{m11}}, \tag{32}$$

$$\psi_2^C = \begin{cases} \frac{\rho_{r22} - \mu\rho_{r21}}{\rho_{r22} - \mu\rho_{r21} + \rho_{m22} - \mu\rho_{m21}} & if\,(\rho_{r22} - \mu\rho_{r21} > 0,\ \rho_{m22} - \mu\rho_{m21} > 0), \\ 0 & else. \end{cases} \tag{33}$$

$$\phi_{11}^C = \frac{\beta\rho_{m11} + \gamma_1\rho_{m21} + \gamma_2\rho_{m22}}{\beta(\rho_{m11} + \rho_{r11}) + \gamma_1(\rho_{m21} + \rho_{r21}) + \gamma_2(\rho_{m22} + \rho_{r22})}, \tag{34}$$

$$\phi_{21}^C = \begin{cases} \frac{\theta\rho_{m21} - \delta\rho_{m22}}{\theta\rho_{r21} - \delta\rho_{r22} + \theta\rho_{m21} - \delta\rho_{m22}} & if\,(\theta\rho_{r21} - \delta\rho_{r22} > 0,\ \theta\rho_{m21} - \delta\rho_{m22} > 0), \\ 0 & else. \end{cases} \tag{35}$$

$$\phi_{22}^C = \begin{cases} \frac{\rho_{m22} - \delta\rho_{m21}}{\rho_{r22} - \delta\rho_{r21} + \rho_{m22} - \delta\rho_{m21}} & if\,(\rho_{r22} - \delta\rho_{r21} > 0,\ \rho_{m22} - \delta\rho_{m21} > 0), \\ 0 & else. \end{cases} \tag{36}$$

From Eqs. (32)–(36), we obtain three important results in Proposition 4.

Proposition 3.

(i) *In the two-way subsidy contract, the manufacturer will grant a higher subsidy rate to the retailer.*
(ii) *In the first period, the retailer (manufacturer) will grant a subsidy rate to the manufacturer (retailer) as long as it has a positive marginal profit.*
(iii) *In the first period, as the retailer's marginal profit increases, the retailer will offer a larger subsidy to the manufacturer. In contrast, if the manufacturer's marginal profit is high, then the retailer will offer a low subsidy rate.*
(iv) *In the second period, if the retailer's marginal profit of the first-generation product is high, then it will offer a low subsidy rate to the manufacturer on the second-generation product design.*

Proposition 3 presents several important results. First, comparing Eqs. (34)–(36) with Eqs. (15)–(17), we obtain $\phi_{ij}^C > \phi_{ij}^D$, which indicates that the manufacturer might contribute more advertising costs if it asks the retailer to share the design costs. Therefore, the advertising efforts in the two-way subsidy contract are also larger than that in the decentralized scenario. Second, from Eqs. (32) and (34), it is easy to find that the manufacturer and the retailer are willing to offer a subsidy in the first period as long as they can obtain a profit. Third, the implication of Proposition 3(iii) and (iv) is consistent with that of Proposition 2. Therefore, it will not be repeated here. Moreover, in this supply chain contract, if the subsidy rates ϕ_{ij} and ψ_i take the values that are given by Eqs. (32)–(36), then the product design levels and advertising efforts are the same with those in the integrated system. As a result, the whole supply chain profit in the two-way subsidy contract will equal that in the integrated scenario.

It is interesting and meaningful that the two-way subsidy contract can achieve supply chain coordination. Namely, when the manufacturer offers an advertising subsidy for the retailer, and the retailer provides a subsidy for the manufacturer's product design, the whole supply chain profit will reach the ideal level. Therefore, how to guarantee that the retailer participates in this contract will be a crucial issue. In practice, it is not uncommon for upstream and downstream enterprises to cooperate with each other. Leng et al. (2016) found that the retailer played an important role in the manufacturer's product quality assurance.

Although the whole supply chain profit is improved in the two-way subsidy contract, we also need to examine the effects of this contract on the retailer's (or manufacturer's) profit. Is it always better for the retailer to join the two-way subsidy contract? Note that it is difficult to compare the supply chain members' profits from the formula. Therefore, we use the numerical analysis to illustrate the effects of the two-way subsidy contract on the profits for each channel member and the whole supply chain. The following parameters are fixed with the following values: $\alpha_{11} = 400$, $\alpha_{21} = 200$, $\alpha_{22} = 400$, $\lambda = 5$, $\beta = 3$, $\mu = 0.3$, $\theta = 0.8$, $\gamma_1 = 0.1$, $\gamma_2 = 0.1$, and $\delta = 0.3$. The values of the other parameters are adjusted.

In Fig. 1, we let $\rho_{m22} = 5$, $\rho_{r22} = 4$, $\rho_{m21} = 2$, $\rho_{r21} = 2$, and $\rho_{m11} + \rho_{r11} = 10$; define the value of the x-axis $\rho_{r11}/(\rho_{m11} + \rho_{r11})$; and change it from 0 to 1. In Fig. 2, we let $\rho_{m11} = 5$, $\rho_{r11} = 4$, $\rho_{m21} = 2$, $\rho_{r21} = 2$, and $\rho_{m22} + \rho_{r22} = 10$; define the value of the x-axis $\rho_{r22}/(\rho_{m22} + \rho_{r22})$; and change it from 0 to 1. In Fig. 3, we let $\rho_{m11} = 5$, $\rho_{r11} = 4$, $\rho_{m22} = 5$, $\rho_{r22} = 4$, and $\rho_{m21} + \rho_{r21} = 6$; define the value of the x-axis $\rho_{r21}/(\rho_{m21} + \rho_{r21})$; and change it from 0 to 1. The relationships between the marginal profit ratio and the profit gap are presented in the following figures.

Figs. 1 and 2 imply that:

(i) The manufacturer probably gains more profit when it joins the supply chain contract rather than when the manufacturer unilaterally decides the subsidy rates. When each channel member has an approximately equal marginal profit, the manufacturer is more disposed to join the supply chain contract. In this condition, the manufacturer can obtain a larger profit. If the retailer's marginal profit ratio is high, its profit gap is much larger than that of the manufacturer. In this case, the manufacturer's attitude toward the retailer can determine whether the manufacturer will join the supply chain contract.
(ii) If the retailer's marginal profit ratio remains at a relatively low level, then its profit gap is negative. Thus, the retailer will not join the supply chain contract due to the loss in profit. The condition for the manufacturer is the reverse. The manufacturer could offer a transfer payment to the retailer to encourage it to join the supply chain contract.
(iii) The trend of the whole supply chain's profit gap is similar to that of the retailer. If the retailer has a high marginal profit ratio, then the whole supply chain's profit in the supply chain contract is higher.

Fig. 3 shows that the higher the marginal profit ratio $\rho_{r21}/(\rho_{m21} + \rho_{r21})$ is, the lower the retailer's profit in the supply chain contract will be. Therefore, the retailer is willing to join the supply chain contract when its old product's marginal profit ratio is at a low

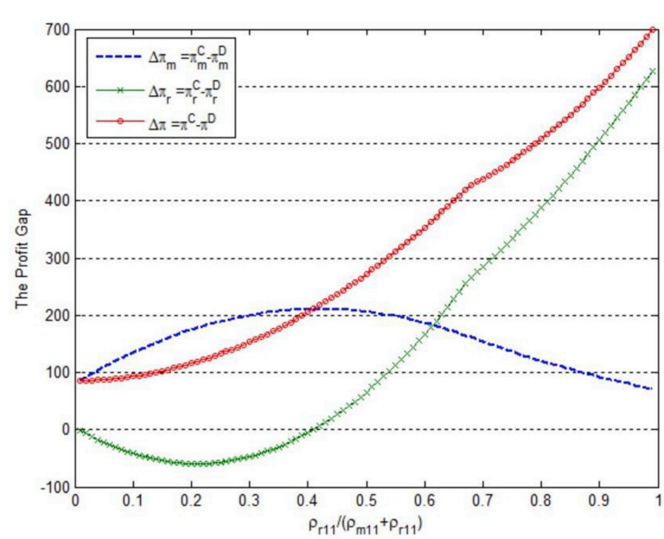

Fig. 1. The relationship between the profit gap and the marginal profit ratio.

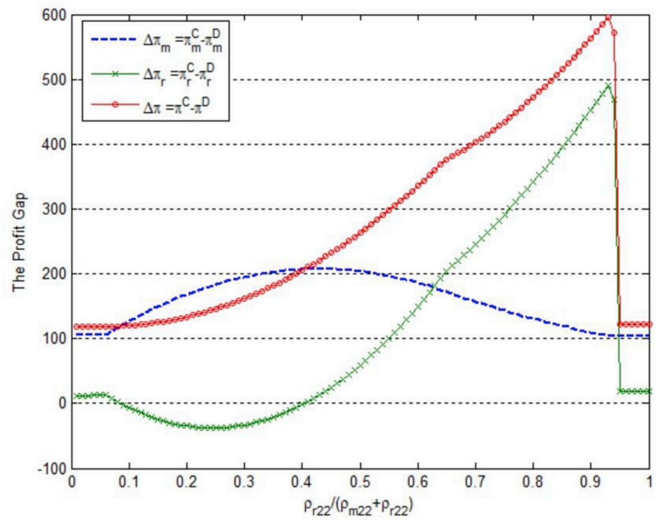

Fig. 2. The relationship between the profit gap and the marginal profit ratio.

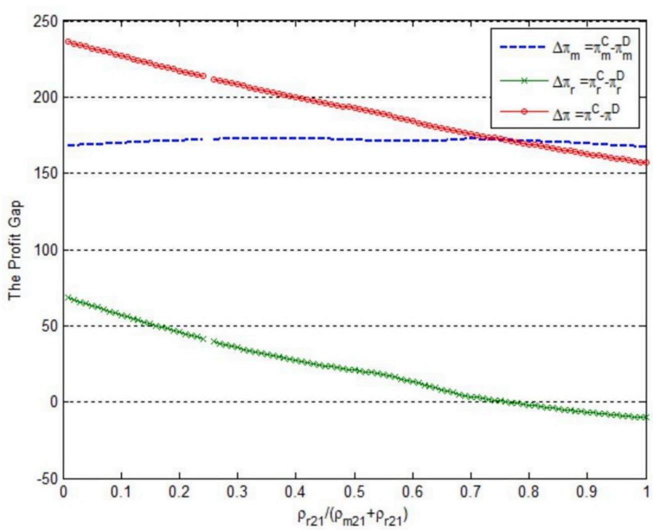

Fig. 3. The relationship between the profit gap and the marginal profit ratio.

level. The results of Figs. 1 and 2 reveal that if the new product's marginal profit for the retailer is at a high level, then the retailer is more willing to join the supply chain contract. Meanwhile, the reverse condition will occur if the old product's marginal profit for the retailer is high, and the whole supply chain's profit will decrease with this condition.

According to the numerical analysis, the retailer's profit may suffer in this contract if it has a low marginal profit. In this situation, we may need introduce a transfer payment to ensure all channel members' profits are improved, i.e., a fixed amount (T) transferred from the manufacturer to the retailer (or vice versa, if T is negative) to ensure a fair distribution of the supply chain surplus π^s from Eq. (24).

Then, the profit functions of all channel members are

$$\pi_m^* = \pi_m^C - T, \tag{37}$$

$$\pi_r^* = \pi_r^C + T. \tag{38}$$

In the current study, we assume that the supply chain members have the same risk attitude. Therefore, the supply chain members equally allocate the supply chain surplus π^s; this assumption is found in the literature (Nash, 1950; Chiang et al., 2003; Leng and Zhu, 2009). Therefore, if the manufacturer provides a transfer payment to the retailer, then let $\pi_m^* = \pi_m^D + 0.5\pi^s$ (or $\pi_r^* = \pi_r^D + 0.5\pi^s$), and we thus obtain the transfer payment T as follows:

$$T = \pi_m^C - 0.5(\pi^I + \pi_m^D - \pi_r^D), \quad or \quad T = 0.5(\pi^I + \pi_r^D - \pi_m^D) - \pi_r^C. \tag{39}$$

To achieve the perfect supply chain coordination, a two-way subsidy contract can be set up with parameters of ϕ_{ij}, ψ_j and T given by Eqs. (32)–(36) and (39), respectively.

7. Conclusion

Firms in the consumer electronics industry frequently launch new styles of their products, which leads to a "two-period" phenomenon of product sales. Related firms need to determine the appropriate marketing tools to respond to the change. Almost all of the research in co-op advertising has concentrated on a one-period supply chain. This paper looks at a setting where two generations of products are involved in the market and investigates the equilibrium product design levels and co-op advertising strategies of supply chain members in a two-period consumer electronics supply chain.

This paper obtains the following conclusions. First, if the manufacturer's marginal profit of the first-generation product is high in the second period, the manufacturer will not spend money on the second-generation product's design levels. Second, in the second period, the retailer not only advertises for the second-generation product but is also motivated to spend on advertising for the first-generation product if the ratio between ρ_{r21} and ρ_{r22} reaches a certain value δ/θ. Third, the manufacturer usually does not provide the same advertising subsidy strategy for the two generation products in the same period. When the manufacturer increases the subsidy rate of the second-generation product' advertising costs, it will decrease that of the first-generation product. Fourth, the manufacturer may provide a low subsidy rate to the retailer if the advertising long-term effect is strong. Finally, we demonstrate that the two-way subsidy contract can achieve supply chain coordination, and it can always bring an extra benefit for all channel members if a transfer payment exists.

Future research can focus on three aspects. First, the two-period supply chain system can be expanded to include multiple manufacturers or multiple retailers. A discussion on the competition between multiple manufacturers and retailers is worthy of study. Second, the product price is not a decision variable in our model. Yue et al. (2006) considered price discount, which led to different and interesting results. Finally, this model considers only local advertising efforts. In future research, we can consider both local and global advertising efforts.

Acknowledgment

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Appendix

Proofs of the equilibrium solutions are as follows:

In the decentralized scenario, the retailer's profit function is:

$$\pi_r = \rho_{r11}D_{11} + \rho_{r21}D_{21} + \rho_{r22}D_{22} - \frac{1}{2}(a_{11}^2 + a_{21}^2 + a_{22}^2) - \frac{1}{2}(1 - \phi_{11})a_{11}^2 - \frac{1}{2}(1 - \phi_{21})a_{21}^2 - \frac{1}{2}(1 - \phi_{22}a_{22}^2). \tag{A.1}$$

and the manufacturer's profit function is:

$$\pi_m = \rho_{m11}D_{11} + \rho_{m21}D_{21} + \rho_{m22}D_{22} - x_1^2 - x_2^2 - \frac{1}{2}\phi_{11}a_{11}^2 - \frac{1}{2}\phi_{21}a_{21}^2 - \frac{1}{2}\phi_{22}a_{22}^2, \tag{A.2}$$

The first-order conditions are:

$$\begin{cases} \partial\pi_r/\partial a_{21} = \theta\beta\rho_{r21} - \delta\beta\rho_{r22} - (1-\phi_{21})a_{21} \\ \partial\pi_r/\partial a_{22} = -\delta\beta\rho_{r21} + \beta\rho_{r22} - (1-\phi_{22})a_{22}. \\ \partial\pi_m/\partial x_2 = -\mu\lambda\rho_{m21} + \lambda\rho_{m22} - 2x_2 \end{cases} \tag{A.3}$$

The second-order conditions are:

$$\begin{cases} \partial^2\pi_r/\partial a_{21}^2 = -(1-\phi_{21}) \le 0 \\ \partial^2\pi_r/\partial a_{22}^2 = -(1-\phi_{22}) \le 0. \\ \partial^2\pi_m/\partial x_2^2 = -2 < 0 \end{cases} \tag{A.4}$$

Therefore, the retailer's (manufacturer's) profit function is concave in a_{21}, $a_{22}(x_2)$. Letting the first-order conditions be 0, we can obtain:

$$a_{21}^D = \begin{cases} \frac{\beta(\theta\rho_{r21} - \delta\rho_{r22})}{1-\phi_{21}} & \textit{if } (\theta\rho_{r21} - \delta\rho_{r22}) > 0, \\ 0 & \textit{else}. \end{cases} \tag{A.5}$$

$$a_{22}^D = \begin{cases} \frac{\beta(\rho_{r22} - \delta\rho_{r21})}{1-\phi_{22}} & \textit{if } (\rho_{r22} - \delta\rho_{r21}) > 0, \\ 0 & \textit{else}. \end{cases} \tag{A.6}$$

and

$$x_2^D = \begin{cases} \frac{1}{2}\lambda(\rho_{m22} - \mu\rho_{m21}) & \textit{if } (\rho_{m22} - \mu\rho_{m21}) > 0, \\ 0 & \textit{else}. \end{cases} \tag{A.7}$$

Substituting Eqs. (A.5), (A.6), and (A.7) into Eq. (A.2) and solving the manufacturer's profit function for optimal subsidy rates ϕ_{21} and ϕ_{22}, we obtain:

$$\max_{\phi_{21}} \pi_m(\phi_{21}), \tag{A.8}$$

and

$$\max_{\phi_{22}} \pi_m(\phi_{22}). \tag{A.9}$$

The first-order conditions are:

$$\frac{\partial\pi_m}{\partial\phi_{21}} = -\frac{\theta\beta^2\rho_{m21}(\delta\rho_{r22} - \theta\rho_{r21})}{(-1+\phi_{21})^2} + \frac{\delta\beta^2\rho_{m22}(\delta\rho_{r22} - \theta\rho_{r21})}{(-1+\phi_{21})^2} - \frac{\beta^2(\delta\rho_{r22} - \theta\rho_{r21})^2}{2(-1+\phi_{21})^2} + \frac{\beta^2\phi_{21}(\delta\rho_{r22} - \theta\rho_{r21})^2}{(-1+\phi_{21})^3}, \tag{A.10}$$

and

$$\frac{\partial\pi_m}{\partial\phi_{22}} = \frac{\delta\beta^2\rho_{m21}(\delta\rho_{r21} - \rho_{r22})}{(-1+\phi_{22})^2} - \frac{\beta^2\rho_{m22}(\delta\rho_{r21} - \rho_{r22})}{(-1+\phi_{22})^2} - \frac{\beta^2(\delta\rho_{r21} - \rho_{r22})^2}{2(-1+\phi_{22})^2} + \frac{\beta^2\phi_{22}(\delta\rho_{r21} - \rho_{r22})^2}{(-1+\phi_{22})^3}. \tag{A.11}$$

The second-order conditions are:

$$\frac{\partial^2\pi_m}{\partial\phi_{21}^2} = \frac{\beta^2(\theta\rho_{r21} - \delta\rho_{r22})}{(-1+\phi_{21})^4}[(\delta\rho_{m22} - \theta\rho_{m21})(2\phi_{21} - 2) + (\delta\rho_{r22} - \theta\rho_{r21})(\phi_{21} + 2)], \tag{A.12}$$

and

$$\frac{\partial^2\pi_m}{\partial\phi_{22}^2} = \frac{\beta^2(\rho_{r22} - \delta\rho_{r21})}{(-1+\phi_{21})^4}[(\delta\rho_{m21} - \rho_{m22})(2\phi_{22} - 2) + (\delta\rho_{r21} - \rho_{r22})(\phi_{22} + 2)]. \tag{A.13}$$

The manufacturer's profit function is strictly concave in ϕ_{21} if the condition $\phi_{21} < 2(\theta\rho_{r21} - \delta\rho_{r22} + \delta\rho_{m22} - \theta\rho_{m21})/[2(\delta\rho_{m22} - \theta\rho_{m21}) - (\theta\rho_{r21} - \delta\rho_{r22})]$ holds. Letting the first-order condition be 0, we can obtain:

$$\phi_{21}^D = \begin{cases} \frac{2(\theta\rho_{m21} - \delta\rho_{m22}) - (\theta\rho_{r21} - \delta\rho_{r22})}{2(\theta\rho_{m21} - \delta\rho_{m22}) + (\theta\rho_{r21} - \delta\rho_{r22})} & \textit{if } \begin{pmatrix} (\theta\rho_{m21} - \delta\rho_{m22}) > \\ 0.5(\theta\rho_{r21} - \delta\rho_{r22}) > 0 \end{pmatrix}, \\ 0 & \textit{else}. \end{cases} \tag{A.14}$$

If the condition $(\theta\rho_{m21} - \delta\rho_{m22}) > 0.5(\theta\rho_{r21} - \delta\rho_{r22}) > 0$ is satisfied, we can find that $\phi_{21}^D < 2(\theta\rho_{r21} - \delta\rho_{r22} + \delta\rho_{m22} - \theta\rho_{m21})/[2(\delta\rho_{m22} - \theta\rho_{m21}) - (\theta\rho_{r21} - \delta\rho_{r22})]$.

The manufacturer's profit function is strictly concave in ϕ_{22} if the condition $\phi_{22} > 2(\rho_{m22} - \delta\rho_{m21} + \delta\rho_{r21} - \rho_{r22})/[2(\rho_{m22} - \delta\rho_{m21}) - (\delta\rho_{r21} - \rho_{r22})]$ holds. Then, letting the first-order condition to 0, we can obtain:

$$\phi_{22}^{D} = \begin{cases} \frac{2(\rho_{m22} - \delta\rho_{m21}) - (\rho_{r22} - \delta\rho_{r21})}{2(\rho_{m22} - \delta\rho_{m21}) + (\rho_{r22} - \delta\rho_{r21})} & if \quad \begin{pmatrix} (\rho_{m22} - \delta\rho_{m21}) > \\ 0.5(\rho_{r22} - \delta\rho_{r21}) > 0 \end{pmatrix}, \\ 0 \quad else. \end{cases} \tag{A.15}$$

If the condition $(\rho_{m22} - \delta\rho_{m21}) > 0.5(\rho_{r22} - \delta\rho_{r21}) > 0$ is satisfied, we can find that $\phi_{22}^{D} > 2(\rho_{m22} - \delta\rho_{m21} + \delta\rho_{r21} - \rho_{r22})/[2(\rho_{m22} - \delta\rho_{m21}) - (\delta\rho_{r21} - \rho_{r22})]$. We can also find this approach in previous studies (Ghosh and Shah, 2015).

Substituting Eqs. (A.5)–(A.7), (A.14), and (A.15) into Eq. (A.1), the first-order condition is:

$$\begin{cases} \partial\pi_r/\partial a_{11} = \beta\rho_{r11} + \gamma_1\rho_{r21} + \gamma_2\rho_{r22} - (1 - \phi_{11})a_{11} \\ \partial\pi_m/\partial x_1 = \lambda\rho_{m11} - 2x_1 \end{cases}. \tag{A.16}$$

The second-order condition is:

$$\begin{cases} \partial^2\pi_r/\partial a_{11}^2 = -(1 - \phi_{11}) \le 0 \\ \partial^2\pi_m/\partial x_1^2 = -2 < 0 \end{cases}. \tag{A.17}$$

Therefore, the retailer's (manufacturer's) profit function is concave in $a_{11}(x_1)$. Letting the first-order conditions be 0, we can obtain:

$$a_{11}^{D} = \frac{\beta\rho_{r11} + \gamma_1\rho_{r21} + \gamma_2\rho_{r22}}{1 - \phi_{11}}, \tag{A.18}$$

and

$$x_1^{D} = \frac{1}{2}\lambda\rho_{m11}. \tag{A.19}$$

Other equations observed a similar process.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jretconser.2019.05.010.

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