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Highlights

- Investigated original equipment manufacturer's pricing under secondary market
- Effects of presence of secondary market on decisions and consumer welfare
- Examined how the dynamic pricing and competition affect the optimal decisions

Effects of a secondary market on original equipment manufactures' pricing, trade-in remanufacturing, and entry decisions

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Abstract: As environmental regulations are becoming increasingly strict, firms are setting up remanufacturing systems and using trade-in programmes that take back used products to stimulate demand. Meanwhile, they are starting to sell remanufactured products to secondary markets to avoid the problem of cannibalization. In this study, we establish a two-period model in which a monopolistic original equipment manufacturer (OEM) offers a trade-in programme to improve sales and collect used products. At the same time, the OEM can elect to remanufacture these used products and resell them to a secondary market. The results for the static pricing case show that the two primary driving factors, customers' maximum willingness to pay into the secondary market and production cost, produce different outcomes. Depending on the relationship between these two key factors, seven outcomes exist. Specifically, although all used products are collected and the secondary market is available, the OEM may not remanufacture or may partially remanufacture. We study the above problem using a dynamic pricing case in which the product price during the second period is different from that in the first period. We find that the OEM prefers to offer a menu such that all rather than just some holders participate in the trade-in programme. Furthermore, in the dynamic pricing case, all rather than some of these used products are remanufactured, in contrast

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with the static pricing case. However, the layout of the OEM's trade-in and remanufacturing policies under the static pricing case is similar to that under the dynamic pricing case. We further extend our study to include a competitive situation and find that the results for the core model can essentially be reproduced under competition.

Keywords: Supply chain management; trade-in remanufacturing; pricing; secondary market; competition

1 Introduction

Generally, a consumer can obtain a price discount when buying a new product and returning his/her used product through a trade-in programme. A successful trade-in programme is currently regarded as an important way to increase customer demand and recapture used products (Xiao, 2017; Yin, Li, & Tang, 2015). According to a report by the International Data Corporation, global shipments of smartphones reached 1.47 billion units in 2016, a considerable number of which were for replacement. The high demand for replacements (and the subsequent high number of shipments) is due to the product's short lifecycle and frequent technology upgrades. Meanwhile, an increasing number of enterprises have begun to vigorously implement trade-in services. For example, Apple has a reuse and recycling programme in China that encourages regular Chinese consumers to purchase again. Apple's competitors, such as Huawei, Xiaomi, and Samsung, also offer trade-in rebates to customers willing to return their used products. China's automotive industry has implemented a similar trade-in programme. In 2016, ten governmental departments, including the Ministries of Finance and Commerce, issued measures to oversee auto replacement policies. These expanded the scope of subsidiary objects and increased the number of subsidies compared with 2009. They also aimed to accelerate the upgrading of older cars and boost domestic demand. Nowadays, trade-in programmes are a popular approach to expanding product sales and stimulating market demand (Desai, Purohit, & Zhou, 2016; Yin & Tang, 2014).

Although trade-in programmes provide benefits for enterprises, the operations involved encounter many challenges. A fundamental problem with implementing a trade-in programme is how to deal with the used products properly. It is rarely cost-effective for firms to directly disassemble used products for their salvage value. However, the cannibalization that occurs between new and remanufactured products must be resolved when remanufacturing is chosen. Companies adopt various strategies when facing these concerns. For example, to prevent cannibalization between new and remanufactured products, Apple has implemented a disassembly and recycling strategy for used products collected through its reuse and recycling programme (Apple, 2017 Environmental Responsibility Report). Other companies remanufacture their used products and resell them together with their new products (Abbey & Blackburn, 2015; Atasu, Sarvary, & Wassenhove, 2008; Debo, Toktay, & Van Wassenhove, 2005; Ferrer & Swaminathan, 2010). However, these two alternatives only partially solve the problems associated with remanufacturing and reselling. New solutions to these problems are urgently needed.

Entering a separate secondary market (e.g., an overseas secondary market) appears to be one way to achieve a suitable balance in the situation described above. Fortunately, with the rapid development of globalization, it has become increasingly easy for firms to enter such markets. For example, Toyota sells and leases its used/remanufactured automobiles to an international secondary market (Lacourbe, 2016). Other automobile companies, such as BMW, Ford, and Mercedes-Benz, often collect their used products through trade-in programmes and then sell them to a secondary market after refurbishing and remanufacturing them (see https://www.bmwusa.com/; https://www.ford.com/; https://www.mercedes-benz.com/en/). However, OEMs may face competition in the secondary market. For example, different automobile companies commonly sell their remanufactured vehicles in the same secondary market. A similar phenomenon is found in other industries. For example, in the smartphone industry, Aihuishou, an independent online recycling platform, takes back used smartphones and computers and resells them in the secondary market after remanufacturing them (see http://shan.aihuishou.com/). Gazelle collects old iPhones from consumers at low prices and sells the remanufactured iPhones to the Asian and African markets (Aflaki & Mazahir, 2015). Therefore, it is necessary to investigate how the secondary market affects firms' trade-in policies, especially in competitive situations.

In the secondary market, some potential links between the demand for remanufactured products, trade-in programmes and remanufacturing exist. Clearly, the demand for remanufactured products is affected by the supply of remanufactured products. However, the supply is constrained by the volume of used products collected through trade-in programmes. Additionally, the volume of used products relies on the demand for new products and trade-in rebates. It is therefore essential for firms to clarify the relationship between the demand for remanufactured products, the trade-in programme, and the remanufacturing operation. In particular, firms should seek to understand the mechanisms of interaction between these factors.

Although many studies investigate the role of trade-in programmes in improving demand (Desai et al., 2016; Xiao, 2017; Yin et al., 2015), few explore trade-in programmes from the perspective of reverse logistics management. They do not consider the problems with remanufacturing and remarketing used products collected through trade-in programmes. Instead, they assume that the firms acquire only the residual value of the used products (Li & Xu, 2015; Miao, Fu, Xia, & Wang, 2017; Ray, Boyaci, & Aras, 2005). To fill this gap, we aim to address these problems explicitly and provide firms with theoretical guidelines for practical operations.

Therefore, this study aims to answer the following questions.

(1) What is the OEM's optimal trade-in and remanufacturing strategy? There is a secondary market and the OEM can either choose to do nothing and obtain the residual value of the used products or remanufacture and sell the remanufactured products to the secondary market.

(2) If the OEM adopts a dynamic pricing strategy, what should it do to optimize operational decisions and achieve maximum profits? What is the effect of the dynamic pricing strategy on the OEM's trade-in and remanufacturing policies?

(3) Given that there are potential competitors in the secondary market (i.e., competitive case), should the OEM still undertake remanufacturing and sell the remanufactured products to the secondary market? What are the effects of competition?

In this study, we consider a two-period model in which a monopolistic OEM sells new products in the first period and introduces a trade-in programme in the second period. The OEM has two strategies to deal with the used products collected through the trade-in programme. The first is a single-market strategy in which the OEM does not undertake remanufacturing but retains the residual value of the used products. The second is a dual-market strategy in which the OEM either remanufactures and sells the remanufactured products to the secondary market or retains the used products' residual value. Under this framework, we study the OEM's profit-maximizing problem under both the single- and dual-market strategies. First, we explore these questions under static and dynamic pricing models to emphasize the robustness of the results. Under the static pricing framework, we also consider a competitive case in which a potential competitor sells substitutable products in the secondary market.

The main **contributions** of this study are as follows. To best of our knowledge, we are the first to jointly consider the optimization of trade-in programmes, pricing strategies and the resale of used products. Although some studies examine the problems of trade-in programmes and pricing (Miao et al., 2017; Ray et al., 2005; Xiao, 2017), they do not consider the problem of reselling used products collected through trade-in programmes. This study finds that an OEM may not remanufacture or partially remanufacture even if all of its used products have been collected and the secondary market is available.

Previous studies of trade-in programmes, pricing strategies and remanufacturing assume that the price of a remanufactured product is exogenous (Yin & Tang, 2014; Zhang & Zhang, 2018; Zhu & Wang, 2018). However, the price of remanufactured products is endogenous, which is an important consideration for this study. In this situation, more interesting strategies are feasible and we can provide a layout of the optimal strategies on a two-dimensional plane. In addition, we can consider a setting in which competition exists in the secondary market. The literature on dual-market strategies generally only looks at monopolistic environments (Aflaki & Mazahir, 2015; Alev, Agrawal, & Atasu, 2016). We find that under some conditions, an OEM may be more likely to enter the secondary market.

The rest of this study is organized as follows. Section 2 provides a literature review and explains our contributions in related fields. Section 3 describes our research problem and model. Section 4 discusses the analysis. Section 5 presents a dynamic pricing model. Section 6 extends our research to include a competitive case. Section 7 presents the discussion. Section 8 briefly summarizes our research and indicates future directions.

2 Related literature

This study lies at the intersection of research on closed-loop supply chains and research on marketing. The research closest to our work addresses the following areas: (1) reverse logistics and closed-loop supply chains; (2) closed-loop supply chains and trade-in programmes; and (3) remanufacturing and secondary markets.

Reverse logistics and closed-loop supply chains have been widely examined for several decades. Many such studies explore the reverse channel choice (Choi, Li, & Xu, 2013; Chuang, Wang, & Zhao, 2014; Wu & Zhou, 2017) and the strategic management of new and remanufactured products (Abbey & Blackburn, 2015; Debo et al., 2005; Ferrer & Swaminathan, 2010). More details on these topics are found in Souza (2013) and Govindan, Soleimani, and Kannan (2015). In this study, we also focus on the management of new and remanufactured products. However, unlike previous works, we do not focus on managing the problem of cannibalization. Instead, we concentrate on how firms can avoid the problem of cannibalization by entering a secondary market. The other distinction between this study and previous research lies in the way in which used products are taken back. In this study, we focus on a trade-in programme that jointly affects the performance of used product collection and the sale of new products. This approach is more interesting and novel.

Studies of closed-loop supply chains and trade-in programmes typically investigate the role of trade-ins in stimulating demand, promoting recycling, and enhancing corporate profits (Kim et al. 2011; Li & Xu, 2015; Yin et al., 2015). For example, Ray et al. (2005) analyse a firm's durable goods pricing strategies, namely a uniform pricing model, an age-independent differentiation pricing model and an age-dependent differentiation pricing model. Rao, Narasimhan, and John (2009) investigate a firm's pricing and trade-in decisions under the coexistence of old and new products and find that trade-ins can reduce the inefficiency caused by the "lemon market."[2] Chen (2015) and Chen and Hsu (2015) investigate firms' optimal trade-in strategies when recovery cost and customer strategic behaviour are considered. Zhu, Wang, Chen, and Chen (2016) examine the effects of trade-ins and discover that they can improve a firm's competitive advantage in a horizontal competitive situation. Agrawal, Ferguson, and Souza (2016) study an OEM's use of a trade-in programme to reduce competitive stress from a third-party remanufacturer. Zhang and Zhang (2018) examine the effects of trade-in programmes on profits and environmental performance when consumers are strategic. Xiao (2017) considers the manufacturing, remanufacturing and trade-in programme decisions of companies when the primary market is saturated. Miao et al. (2017) explore the effects of a trade-in programme on a manufacturer's reverse channel choice and environmental performance. The results show that the manufacturer and supply chain systems benefit from the trade-in strategy implemented by the manufacturer. However, environmental performance is superior when the retailer carries out a trade-in programme. Liu, Zhai, and Chen (2018a) investigate the optimal pricing strategy under a trade-in programme in which consumers' strategic behaviour is considered. Cao, Xu, Bian, and Sun (2018) explore the optimal trade-in policies of a business-to-consumer platform. These studies explore the trade-in programme's role from a closed-loop supply chain perspective. Most assume that used products bring constant value to firms and seldom consider the problems with remanufacturing and remarketing the used products.

Unlike the above-referenced studies, we construct a theoretical model that includes the remanufacturing and remarketing of used products. A distinctive characteristic of our model is its internal constraint on the quantity of the remanufactured products and the volume of the used products collected through the trade-in programme. In addition, the price of remanufactured products, assumed to be

[2] For the definition of "lemon market," please see Akerlof (1978).

exogenous in previous works, is endogenous in this study.

This study also relates to research on remanufacturing and secondary markets. Robotis, Bhattacharya, and Wassenhove (2005) consider a supply chain system consisting of two suppliers and one retailer, in which the retailer determines the remanufacturing investment and the procurement decisions in the secondary market. Angelus (2011) explores an inventory control problem within multiple periods in which the enterprise can sell its overstock in the secondary market. Huang, Gu, Ching, and Siu (2014) analyse the inventory control and coordination problems of a dominant manufacturer in a competitive supply chain system in which retailers can resell returned products in the second-hand market. Their results show that a second-hand market helps to increase the number of wholesale products but intensifies conflict among retailers. Lacourbe (2016) studies the effects of an international secondary market on leasing and exporting strategies for durable goods. Aflaki and Mazahir (2015) study a monopolistic company's manufacturing and remanufacturing decisions under two strategies: single-market and dual-market. Xiong, Zhao, Xiong, and Li (2016) study the effects of a manufacturer's upgrade strategy on the decision to enter the secondary market, and provide the optimal entry conditions. Alev et al. (2016) verify the effects of extended producer responsibility on the secondary market intervention strategy of a durable goods manufacturer that recycles used products from the secondary market.

Of the studies noted above, Aflaki and Mazahir (2015) is the closest to the current study. However, unlike the OEM in Aflaki and Mazahir's study, the OEM in this study not only makes trade-in and remanufacturing policies but also considers the internal constraint between the quantity of remanufactured goods and the used products collected through the trade-in programme. The study by Xiong et al. (2016) is also similar to this study. Like them, we investigate a firm's decision to enter a secondary market. However, this study focuses on exploring the effects of the trade-in programme rather than the upgrade strategy on the OEM's entry decision. Further, we consider a competitive case in which the OEM determines whether to enter a secondary market and the appropriate strategy to use. Table 1 presents a comparison of this study with the above-mentioned related studies.

Table 1. Comparison of this study with related studies

Literature	Trade-in programme	Secondary market	Periods	Endogenous price of RP
Ray et al. (2005)	✓	×	1	×
Rao et al. (2009)	✓	✓	2	×
Yin et al. (2014)	✓	×	2	×

Chen (2015)	✓	×	1	×
Aflaki et al. (2015)	×	✓	1	✓
Chen et al. (2015)	✓	×	1	×
Agrawal et al. (2015)	✓	×	2	✓
Zhang et al. (2015)	✓	×	2	×
Miao et al. (2016)	✓	×	2	×
Xiao (2016)	✓	✓	1	×
Xiong et al. (2016)	×	✓	2	✓
Zhu et al. (2016)	✓	×	2	×
Desai et al. (2016)	✓	×	1	×
Miao et al. (2017)	✓	×	2	×
This study	✓	✓	**2**	✓

Note: RP: remanufactured products.

3 Model development

In this study, we consider a monopolistic OEM that sells new products with a two-period lifecycle in a primary market and collects used products through a trade-in programme. These collected products can be remanufactured and sold to a separate secondary market. The related notation and assumptions are summarized in Table 2. *The proofs for all propositions are provided in Appendix A of the supplementary materials and all tables that include the optimal decisions are provided in the appendix.*

3.1 Consumer choice in the primary market

We consider a two-period model that captures customers' purchasing choices for products with a two-period life cycle, based on their willingness to pay. Specifically, customers have to decide whether to buy new products in period 1. In period 2, those who bought new products in period 1 can elect to keep their used products or replace them through a trade-in programme, in which case they are eligible for a trade-in rebate. The setting considered in this study is similar to the settings used by Agrawal et al. (2016), Atasu et al. (2008) and Miao et al. (2017). Suppose that the customers are heterogeneous in their willingness to pay θ for a new product in the primary market, which is uniformly distributed on $[0,1]$. In the first period, given the retail price p_{1n}, the customer surplus from purchasing the new product is $\theta - p_{1n}$. Obviously, customers are willing to purchase the new product if and only if $\theta - p_{1n} \geq 0$. Therefore, the total demand for new products in the first period can be given by

$$q_{1n} = 1 - p_{1n}. \quad (1)$$

Table 2. Summary of related notation and assumptions

Symbol	Definition
θ	Consumers' willingness to pay in the primary market
v	Consumers' willingness to pay in the secondary market
n	New products
r	Remanufactured products
t	Trade-in
C	Potential competitor
M	OEM
p_{1n}	Retail price for new products in the first period
p_{2n}	Retail price for new products in the second period; $p_{1n} \geq p_{2n}$
p_n	Retail price for new products under the static pricing strategy; $p_{1n} = p_{2n} = p_n$
p_t	Trade-in rebate
p_r	Retail price for remanufactured products
p_C	Retail price for competitive products
q_{1n}	Sales of new products in the first period
q_{2n}	Sales of new products in the second period
q_t	Volume of used products collected through the trade-in programme
q_r	Sales of remanufactured products
q_C	Sales of competitive products
s	Salvage value of used products, and satisfies $c_n > s$.
ϕ	Durability of new products, and satisfies $1 > \phi > s$.
δ_i	Difference between the remanufactured product and the competitive product, where $i = H, L$, corresponding to the cases in which the potential competitor sells high- and low-quality products respectively
c_n	Production cost for new products
c_r	Remanufacturing cost; satisfies $c_r < c_n$.
c_i	Production cost for competitive products
F	Upfront investment for entering the secondary market
π_C^j	Profit of the competitor under the case j, and $j = DL, DH$, which respectively denotes the dual market with a low-quality competitor and the dual market with a high-quality competitor.
π_M^k	Profit of the OEM under the case k, and $k = SM, DM, SD, DD, DL, DH$, which respectively represent the single-market strategy (*SM*), dual-market strategy (*DM*), single market with dynamic pricing strategy (*SD*), dual market with dynamic pricing strategy (*DD*), dual market with a low-quality competitor strategy (*DL*), and dual market with a high-quality competitor strategy (*DH*).

Assumptions	
A1	Consumers' willingness to pay θ in the primary market is uniformly distributed over $[0,1]$.
A2	Consumers' willingness to pay v in the secondary market is uniformly distributed over $[0, b]$, and $b \leq 1$.
A3	θ and v are independently distributed over the corresponding range.
A4	For tractability, we normalize the total market size of the primary market to 1. This assumption has been widely used in relevant studies, such as Rao et al. (2009), Atasu et al. (2008), Atasu and Souza (2013) and Miao et al. (2017).

Before the second period, customers in the primary market are divided into two segments, i.e., holders who bought new products in the first period and non-holders who did not buy new products in the first period. In the second period, holders can elect to participate in trade-in programmes and return used products or hold on to their used products; non-holders can elect to purchase a new product or not make a purchase. Suppose that the products depreciate with use: the rate of depreciation is $(1 - \phi)$ and satisfies $0 \leq \phi < 1$. To some extent, ϕ implies the product's level of durability. If a holder chooses to participate in the trade-in programme, the surplus he obtains is $\theta - p_{2n} + p_t$ and $\phi\theta$ otherwise. Obviously, a holder will be willing to participate in the trade-in programme if $\theta - p_{2n} + p_t \geq \phi\theta$. If a non-holder elects to purchase a new product, the surplus he obtains is $\theta - p_{2n}$ and zero otherwise. Similarly, a non-holder will be willing to purchase a new product if and only if $\theta - p_{2n} \geq 0$. Therefore, the total demand for new products in the second period can be expressed by

$$q_{2n} = 1 - \frac{p_{2n} - p_t}{1 - \phi} + p_{1n} - p_{2n}, \tag{2}$$

where $1 - \frac{p_{2n} - p_t}{1 - \phi}$ represents the quantity of holders who participate in trade-in programmes and $p_{1n} - p_{2n}$ represents the quantity of non-holders who are willing to buy a new product in the second period. In addition, the constraint $p_{1n} \geq p_{2n}$ should hold to ensure that the demand of non-holders in the second period is non-negative.

3.2 Consumer choice in the secondary market

We assume that in the secondary market, the customer's willingness to pay for a unit remanufactured product is v, which is uniformly distributed on $[0, b]$ and $b \leq 1$. This assumption aligns with Aflaki and Mazahir (2015) and Lacourbe (2016). Given the remanufactured product price p_r, consumers that are willing to pay v can obtain a $v - p_r$ surplus by buying a unit remanufactured product. Thus, consumers will be willing to purchase a remanufactured product if $v - p_r \geq 0$, and demand for remanufactured products can be expressed by $q_r = 1 - p_r/b$.

Suppose that a competitive firm sells competitive products in the secondary market. The customer's willingness to pay for a unit competitive product is δv, where the parameter δ captures the differentiation between the remanufactured product and the competitive product. In particular, if $\delta > 1$ (or $\delta < 1$), the competitive products are of a higher (or lower) quality than the remanufactured ones. To some extent, this parameter reflects the degree of competition between the remanufactured and competitive products. When $\delta \to 1$, the degree of competition reaches its highest point. Given the competitive products' price p_C, the net surplus from customer purchases of the unit competitive product is $\delta v - p_C$.

We use δ_L to denote the scenario in which the competitor sells low-quality products. At the moment, we have $\delta_L < 1$, and the demand functions for the competitive products and remanufactured products are respectively given by

$$q_C = \frac{p_r - p_C}{(1 - \delta_L)b} - \frac{p_C}{b\delta_L} \text{ and } q_r = 1 - \frac{p_r - p_C}{(1 - \delta_L)b}. \tag{3.1}$$

Similarly, δ_H is used to denote the case in which the competitor sells a high-quality product, and $\delta_H > 1$. Thus, the demands for the competitive and remanufactured products are respectively given by

$$q_C = 1 - \frac{p_C - p_r}{(\delta_H - 1)b} \text{ and } q_r = \frac{p_C - p_r}{(\delta_H - 1)b} - \frac{p_r}{b}. \quad (3.2)$$

3.3 OEM's market strategy

Suppose that the OEM has two strategies: a single-market strategy and a dual-market strategy. Under the single-market strategy, the OEM sells new products to customers in both periods but implements a trade-in programme in the second period. As there is no secondary market, the OEM only needs to determine the retail prices and trade-in rebates. In this case, the OEM obtains the salvage value of the used products collected by the trade-in programme. The situation, however, is different under the dual-market strategy.

Under the dual-market strategy, the OEM sells new products in the primary market in both periods and implements a trade-in programme in the second period. As there is a secondary market, the OEM can choose to remanufacture all, some, or none of the used products, and then sell them in the secondary market during the second period. Note that the quantity of remanufactured products will never be greater than that of the used products collected through the trade-in programme. In addition, the quantity of the used products collected through the trade-in programme will never be bigger than the sales of new products sold in the first period. In other words, two potential constraints exist in this situation, which are expressed mathematically by $q_r \leq q_t$ and $q_t \leq q_{1n}$. The structure of the OEM's market strategy is shown in Figure 1.

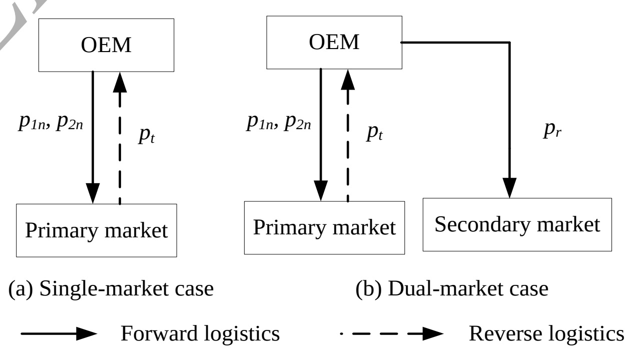

Figure 1. Two market strategies with a trade-in programme

The sequences of events under the single- and dual-market strategies are shown in Figure 2.

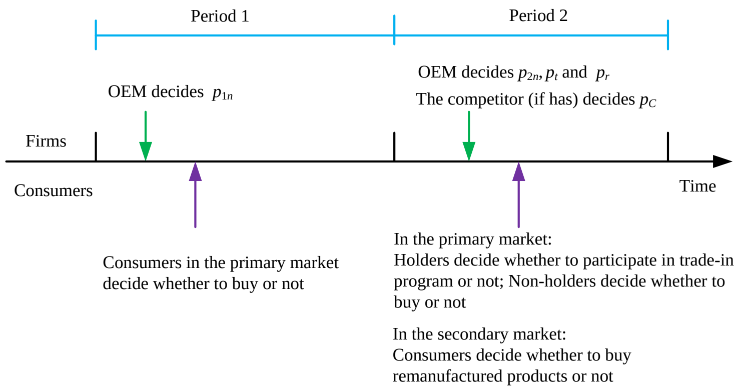

Figure 2. Sequence of events under the single- and dual-market strategies

4 Dual market with static pricing strategy

In this section, we assume that the OEM has a static pricing policy (or static pricing strategy), i.e. $p_{1n} = p_{2n} = p_n$. First, we examine the OEM's optimal decisions under the single-market strategy, which serve as a benchmark. Then we explore the OEM's optimal decisions under the dual-market strategy. Finally, we analyse the effects of the secondary market on the OEM's optimal decisions and consumer welfare.

4.1 Benchmark case: Single market with static pricing strategy

Under the single-market strategy, the OEM sells its products in the primary market during the first period and implements a trade-in programme in the second period. As there is no secondary market in this case, the OEM receives only the salvage value of the used products. As $p_{1n} = p_{2n} = p_n$, based on equations (1) and (2), the profit-maximizing problem of the OEM can be expressed by

$$\max_{(p_n,p_t)} \pi_M^{SM} = (p_n - c_n)(1 - p_n) + (p_n - c_n - p_t)\left(1 - \frac{p_n - p_t}{1-\phi}\right) + s\left(1 - \frac{p_n - p_t}{1-\phi}\right),$$

$$\text{s.t.}\ 1 - p_n \geq 1 - \frac{p_n - p_t}{1-\phi} \geq 0. \tag{4}$$

The first term represents the revenue received from selling new products in the first period, and the second term corresponds to the revenue received from implementing the trade-in programme. The third term refers to the total salvage value of the used products. The constraints ensure that the quantity of the used products collected through trade-ins is non-negative and is less than the sale of new products in the first period.

Through equation (4), we can obtain the following results.

Lemma 1. *Under the single-market strategy, the OEM's optimal trade-in and remanufacturing policies are as follows:*

(1) If $c_n > 1 + s - \phi$, *the OEM should offer a menu of retail prices and trade-in rebates such that no holders participate in a trade-in programme, i.e., no trade-in and remanufacturing (NTNR).*

(2) If $1 + s - \phi \geq c_n \geq s/\phi$, *the OEM should offer a menu of retail prices and trade-in rebates such that partial holders participate in the trade-in programme, i.e., partial trade-in and no remanufacturing (PTNR).*

(3) If $s/\phi > c_n$, *the OEM should offer a menu of retail prices and trade-in rebates such that all holders participate in the trade-in programme, i.e., all trade-in and no remanufacturing (ATNR).*

The optimal retail prices and trade-in rebates are summarized in Table 3.

Lemma 1 depicts the OEM's optimal trade-in scheme and remanufacturing policies under the single-market strategy. As there is no secondary market, the OEM receives only the salvage value of the used products, and does not need to consider remanufacturing decisions. However, the OEM must take into account the performance of the trade-in programme. From Lemma 1, relying on the range of production costs, there are three types of trade-in scheme (namely *NTNR*, *PTNR*, and *ATNR*), in line with Miao et al. (2017). From Figure 3, we can observe that under the single-market strategy, the optimal trade-in rebate and retail price increase with the production cost, but the volume of holders participating in the trade-in programme decreases. As the increase in production cost leads to an increase in the retail price, demand drops in the first period. Furthermore, the trade-in price (i.e., retail price minus trade-in rebate) increases. This makes holders unwilling to participate in the trade-in programme. Thus, the number of holders participating in the trade-in programme falls.

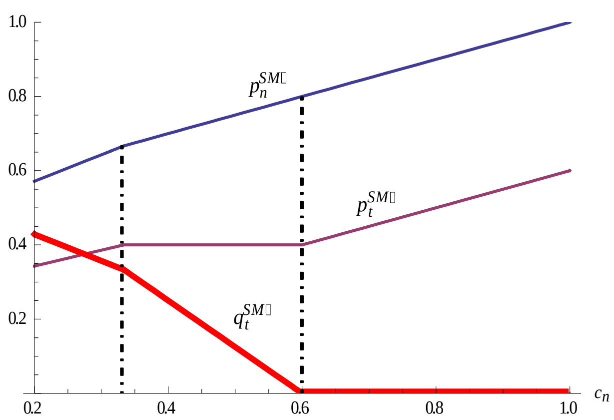

Figure 3. p_n^{SM*}, p_t^{SM*}, and q_t^{SM*} vs. production cost c_n. ($s = 0.2$, $c_r = 0.1$ and $\phi = 0.6$)

4.2 Dual market with static pricing strategy

Under the dual-market strategy, the OEM sells new products in the first period and launches a trade-in programme in the primary market during the second period. Subsequently, the used products collected can be remanufactured and sold to the secondary market or disassembled to obtain their salvage value. Similarly, the profit-maximizing problem of OEM can be expressed by

$$\max_{(p_n,p_t,p_r)} \pi_M^{DM} = (p_n - c_n)(1-p_n) + (p_n - c_n - p_t)\left(1 - \frac{p_n-p_t}{1-\phi}\right) + s(1 - \frac{p_n-p_t}{1-\phi} - \left(1 - \frac{p_r}{b}\right)) + \left(1 - \frac{p_r}{b}\right)(p_r - c_r) - F,$$

$$\text{s.t.}\, 1 - p_n \geq 1 - \frac{p_n-p_t}{1-\phi} \geq 0 \text{ and } 1 - \frac{p_n-p_t}{1-\phi} \geq 1 - \frac{p_r}{b} \geq 0. \quad (5)$$

The first term represents the revenue earned from selling new products in the first period, and the second term corresponds to the revenue earned from the trade-in programme. The third term refers to the salvage value of the used products collected through trade-in but not remanufactured. The fourth term refers to the revenue earned from remanufacturing and reselling used products. The last term is the upfront investment cost of entering the secondary market. The first constraint ensures that the quantity of the used products collected through the trade-in programme is non-negative and is less than the sale of new products in the first period. The second constraint ensures that the quantity of remanufactured products is non-negative and smaller than that of the used products.

Proposition 1 sets forth the optimal strategies of the OEM under the dual-market strategy.

Proposition 1. *Under the dual-market strategy, the OEM's optimal trade-in and remanufacturing policies are as follows:*

(1) If $b \leq s + c_r$, the OEM's optimal policies are identical to those under the single-market strategy; in other words, no used products are remanufactured even though the secondary market is available.

(2) If $s + c_r < b$, then

(i) when $c_n > 1 + b - \phi - c_r$, the OEM should offer a menu such that no holders participate in a trade-in programme, i.e., NTNR;

(ii) when $\Delta \geq c_n \geq (bs + (s + c_r)(1-\phi))/b$, the OEM should offer a menu such that only partial holders participate in the trade-in programme, but all of the used products are remanufactured and sold to the secondary market, i.e., partial trade-in but all remanufacturing (PTAR), where $\Delta = c_r/(b-\phi)$ $if\, b \leq \phi + c_r$ or otherwise $\Delta = 1 + b - \phi - c_r$;

(iii) when $c_n \geq \max\{c_r/(b-\phi), (bs+(s+c_r)(2-\phi))/2b\}$ *and* $b \geq \phi + c_r$*, the OEM should offer a menu such that all holders participate in the trade-in programme and all of the used products are remanufactured and sold to the secondary market, i.e., all trade-in and remanufacturing (ATAR);*

(iv) when $(bs+(s+c_r)(1-\phi))/b > c_n \geq s/\phi$*, the OEM should offer a menu such that only partial holders participate in the trade-in programme, but only some of the used products are remanufactured and sold to the secondary market, i.e., partial trade-in and remanufacturing (PTPR);*

(v) when $\min\{(bs+(s+c_r)(2-\phi))/2b, s/\phi\} > c_n$*, the OEM should offer a menu such that all holders participate in the trade-in programme, but only some of the used products are remanufactured and sold to the secondary market, i.e., all trade-in and partial remanufacturing (ATPR).*

The optimal prices and trade-in rebates are summarized in Table 4.

Proposition 1 shows that the production cost and customers' maximum willingness to pay in the secondary market under the dual-market strategy are key to the OEM's trade-in and remanufacturing decisions. Unlike Miao et al. (2017), we find that when a secondary market is available, used products will encounter different processes based on the relationship between production cost and customers' maximum willingness to pay in the secondary market.

To be specific, when the production cost is small, all used products are collected through the trade-in programme. As the cost of collecting used products through the trade-in programme is low, the incremental revenue from the trade-in programme is greater than the corresponding incremental cost. However, with an increase in customers' maximum willingness to pay, then none, some or all of these used products will be remanufactured and sold to the secondary market. The main driver is customers who come to the secondary market and are willing to pay a higher price for remanufactured products. Therefore, the OEM is incentivized to remanufacture more used products and sell them to the secondary market.

We find that when the production cost increases, the OEM wants to offer a menu such that only some or none of the holders participate in the trade-in programme. As high production costs lead to an increase in the cost of collecting used products, the OEM's incentive is reduced. However, a notable observation is that the result is significantly different when the customers' maximum willingness to pay is (relatively) high. The main driver is that the incremental revenue gained from remanufacturing is greater than the incremental cost of collection and remanufacturing in this case. This not only compensates for the loss of collection, but also results in a positive margin. Therefore, the OEM prefers to offer a menu that attracts all (some) of the holders to participate in the trade-in programme in this case (i.e., *ATAR*). A numerical example, where the related parameters are $\phi = 0.4$, $s = 0.2$, and $c_r = 0.1$, is provided to achieve a better understanding of Proposition 1. From Figure 4, it can be seen that the OEM can choose different trade-in and remanufacturing policies according to the relationship between the production cost and customers' maximum willingness to pay in the secondary market.

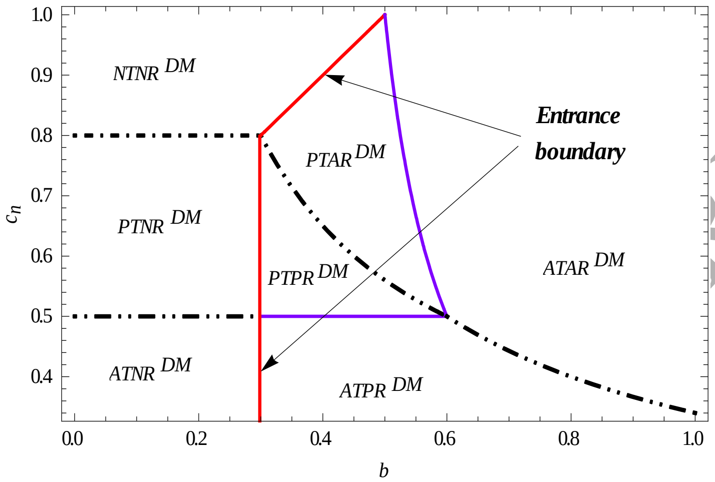

Figure 4. Layout of the optimal trade-in and remanufacturing policies on the $b - c_n$ plane

Based on Proposition 1, it is easy to obtain the following corollary.

Corollary 1. *Without considering the upfront investment F required to enter the secondary market, the dual-market strategy is (weakly) better than the single-market strategy; that is, the dual-market strategy is a (weakly) dominant strategy.*

Obviously, under the dual-market strategy, the secondary market becomes an alternative option for the OEM. Without considering the upfront investment, the OEM always wants to enter the secondary market if it can obtain higher revenue from remanufacturing and selling remanufactured products than disassembling them. In this case, Corollary 1 is a natural consequence; that is to say, the dual-market strategy is never worse than the single-market strategy.

4.3 Effects of the presence of a secondary market

The previous sections show that the secondary market has a significant effect on the OEM's trade-in and remanufacturing policies. However, the effect of the secondary market on the OEM's optimal decisions remains unclear. To this end, we provide a comparison of the dual-market strategy and single-market strategy.

Based on Lemma 1 and Proposition 1, comparing the optimal retail price and trade-in price (i.e., retail price minus trade-in rebate), we have the following proposition.

Proposition 2. *The retail price of new products under the dual-market strategy is smaller than or equal to that under the single-market strategy; i.e.,*$p_n^{DM*} \le p_n^{SM*}$*. In addition, holders pay a lower trade-in price under the dual-market strategy than they do under the single-market strategy; i.e.,*$p_n^{SM*} - p_t^{SM*} \ge p_n^{DM*} - p_t^{DM*}$.

Proposition 2 suggests that the secondary market not only helps to decrease the retail price but also reduces the trade-in price. With an increase in customers' maximum willingness to pay, remanufacturing will become more profitable than the salvage value of the used products. This encourages the OEM to reduce the retail price, increase the demand during the first period, and reduce the trade-in price. More holders will participate in the trade-in programme and return their used products. Obviously, under the dual-market strategy, customers can also benefit from purchasing new products because they pay a lower retail price.

This proposition shows how the secondary market affects the trade-in price. The trade-in price is lower under the dual-market strategy, so holders can obtain a higher surplus from participating in the trade-in programme than they can under the single-market strategy. The driver is that when a secondary market exists, the OEM prefers to charge a low retail price and provide a high trade-in rebate. This attracts more holders, who purchased new products in the first period, to participate in the trade-in programme in the second period.

We now focus on exploring the effects of the secondary market on consumer surplus in the primary market. The consumer surplus can be expressed by

$$CS = \int_{p_n}^{1} (\theta - p_n)\, d\theta + \int_{\frac{p_n - p_t}{1-\phi}}^{1} [(1-\phi)\theta - p_n + p_t]\, d\theta, \tag{3}$$

where the first term represents the surplus from purchasing new products in the first period and the second term is the surplus from participating in the trade-in programme in the second period. Based on Proposition 2, we can easily derive the comparison result for consumer surplus.

Proposition 3. *The presence of the secondary market can help increase consumer welfare in the primary market, i.e.,* $CS^{DM} \ge CS^{SM}$*, and "=" is used if* $s + c_r > b$ *or* $c_n > 1 + b - \phi - c_r$*, and* $s + c_r > b$.

Proposition 3 suggests that the consumer surplus under the dual-market strategy is identical to that under the single-market strategy when the OEM does not enter the secondary market; i.e., the customers' maximum willingness to pay in the secondary market is low and the production cost is high. However, when the OEM enters the secondary market, the consumer surplus under the dual-market strategy is more than it is under the single-market strategy. As such, customers' maximum willingness to pay in the secondary market is high and the production cost is (relatively) low. This proposition implies that customers are more likely to benefit from the presence of the secondary market.

5 Dual market with dynamic pricing strategy

In this section, we aim to explore the optimal decisions of the OEM when it adopts a dynamic pricing strategy in which the retail prices of new products in both periods are different. Firstly we analyse the optimal decision of the OEM under a single market with dynamic pricing strategy, which provides a baseline for our further analysis.

5.1 Benchmark case: Single market with dynamic pricing strategy

If a dynamic pricing strategy is adopted under the single-market situation, then the OEM charges a different retail price in a different period. Based on equations (1) and (2), the profit maximization problem can be given by

$$\max_{(p_{1n},p_{2n},p_t)} \pi_M^{SD} = (p_{1n}-c_n)(1-p_{1n}) + (p_{2n}-c_n-p_t)\left(1-\frac{p_{2n}-p_t}{1-\phi}\right) + (p_{2n}-c_n)(p_{1n}-p_{2n}) + s\left(1-\frac{p_{2n}-p_t}{1-\phi}\right),$$

$$\text{s.t.}\ 1-p_{1n} \geq 1-\frac{p_{2n}-p_t}{1-\phi} \geq 0. \tag{6}$$

Lemma 2. *Under the single market with dynamic pricing strategy, the optimal pricing and trade-in decisions of the OEM are: (1) if* $c_n > 1+s-\phi$*, the OEM should follow an NTNR strategy; (2) if* $1+s-\phi \geq c_n \geq (1+3s-\phi)/(1+2\phi)$*, the OEM should follow a PTNR strategy; and (3) if* $(1+3s-\phi)/(1+2\phi) > c_n$*, the OEM should follow an ATNR strategy. The optimal retail price and trade-in rebate are summarized in Table 5.*

From Lemma 2, we learn that the OEM's optimal trade-in and remanufacturing policies under the single market with the dynamic strategy are essentially identical to those under the single-market strategy. However, some differences still exist between them. Compared with the single market with static pricing strategy, under the single market with dynamic pricing strategy, the OEM tends to charge a higher retail price in the first period and a lower retail price and trade-in rebate in the second period, i.e., $p_{1n}^{SD*} > p_n^{SM*}$, $p_{2n}^{SD*} < p_n^{SM*}$ and $p_t^{SD*} < p_t^{SM*}$. In addition, the OEM is more likely to offer a menu such that all holders participate in the trade-in programme and return their used products, because the threshold in this situation is higher than it is under the single-market strategy, i.e., $(1+3s-\phi)/(1+2\phi) > s/\phi$.

5.2 Dual market with dynamic pricing strategy

When the OEM adopts a dynamic pricing strategy in the dual-market situation, the OEM similarly has a profit maximization problem, which is expressed by

$$\max_{(p_{1n},p_{2n},p_t,p_r)} \pi_M^{DD} = (p_{1n}-c_n)(1-p_{1n}) + (p_{2n}-c_n-p_t)\left(1-\frac{p_{2n}-p_t}{1-\phi}\right) + (p_{2n}-c_n)(p_{1n}-p_{2n}) + s\left(1-\frac{p_{2n}-p_t}{1-\phi}-\left(1-\frac{p_r}{b}\right)\right) + \left(1-\frac{p_r}{b}\right)(p_r-c_r) - F,$$

$$\text{s.t.}\ 1-p_{1n} \geq 1-\frac{p_{2n}-p_t}{1-\phi} \geq 0 \text{ and } 1-\frac{p_{2n}-p_t}{1-\phi} \geq \left(1-\frac{p_r}{b}\right) \geq 0. \quad (7)$$

Based on the optimization theory, the optimal decisions of the OEM can be obtained as follows.

Proposition 4. *Under the dual market with dynamic pricing strategy, the OEM's optimal strategies are as follows:*

(1) If $b \leq s + c_r$, *the OEM's optimal strategies are identical to those under the single market with dynamic pricing strategy, i.e., no used products are remanufactured and sold even if the secondary market is available.*

(2) If $s + c_r < b$,

(i) the OEM should offer a menu such that no holders participate in the trade-in programme when $c_n > 1 + b - \phi - c_r$, *i.e., NTNR;*

(ii) the OEM should offer a menu such that only partial holders participate in the trade-in programme, but all of the used products are remanufactured and resold when $1 + b - \phi - c_r \geq c_n \geq \max\{\frac{1+b-\phi-3c_r}{1-2b+2\phi}, \frac{s(1+b-\phi)+(1-\phi)c_r}{b}\}$, *i.e., PTAR;*

(iii) the OEM should offer a menu such that all holders participate in the trade-in programme, and all of the used products are remanufactured and resold when $\frac{1+b-\phi-3c_r}{1-2b+2\phi} > c_n \geq \frac{7(s+c_r)+4(bs-\phi(s+c_r))-b}{6b}$, *i.e., ATAR;*

(iv) the OEM should offer a menu such that only partial holders participate in the trade-in

programme, and some of the used products are remanufactured and resold when $\frac{s(1+b-\phi)+(1-\phi)c_r}{b} > c_n \geq \frac{1+3s-\phi}{1+2\phi}$, *i.e., PTPR;*

(v) the OEM should offer a menu such that all holders participate in the trade-in programme, but some of the used products are remanufactured and resold when $\min\{\frac{7(s+c_r)+4(bs-\phi(s+c_r))-b}{6b}, \frac{1+3s-\phi}{1+2\phi}\} > c_n$, *i.e., ATPR.*

The optimal pricing decisions are summarized in Table 6.

Proposition 4 illustrates the OEM's optimal pricing and trade-in strategies under the dual market with dynamic pricing strategy. It is clear that under the dual market with dynamic pricing strategy, the structure of the OEM's optimal trade-in policy is similar to that under the dual-market strategy. However, under the dual market with dynamic pricing strategy, the OEM is more likely to offer a menu such that all used products are collected through a trade-in programme. The OEM can adjust the retail price (by decreasing the retail price of new products) in the second period to motivate more holders to participate in the trade-in programme and return their used products. In addition, the entry barriers in these two strategies are identical. A dynamic pricing strategy is used to increase the volume of holders participating in the trade-in programme. Furthermore, the entrance boundary is affected by the marginal cost of remanufacturing (i.e.,$s + c_r$) and by the features of the secondary market (e.g., customers' maximum willingness to pay). To better understand this result, Figure 5 is provided, where $\phi = 0.4$, $s = 0.2$, and $c_r = 0.1$. As shown in Figure 5, the OEM can choose different policies according to the relationship between the production cost and customers' maximum willingness to pay in the secondary market.

Corollary 2. *The OEM will enter the secondary market if the incremental profit is greater than the upfront investment, i.e.,* $\pi_M^{DD} - \pi_M^{SD} > F$; *in particular, the dual market with dynamic pricing strategy is (weakly) better than the single market with dynamic pricing strategy if* $F \equiv 0$.

Similar to Corollary 1, under the dual market with dynamic pricing strategy, the secondary market becomes an alternative option for the OEM. Compared with the single market with dynamic pricing, if the OEM can acquire more profit by entering the secondary market, then the dual market with dynamic pricing strategy is the better choice, and vice versa. In particular, when the upfront investment equals zero, Corollary 2 is a natural consequence; i.e., the dual-market strategy is never worse than the single market with dynamic pricing strategy.

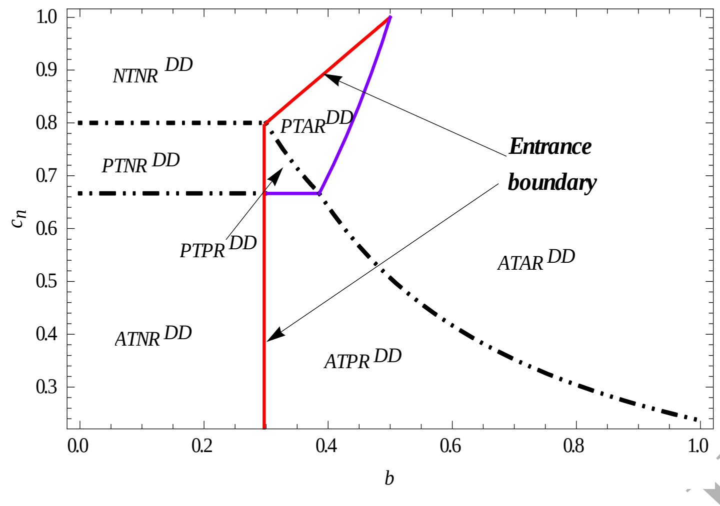

Figure 5. Layout of the optimal trade-in and remanufacturing policies on the $b - c_n$ plane

5.3 Effects of a secondary market and dynamic pricing

Firstly, in this section, we explore the effects of the secondary market under a dynamic pricing model to prove the robustness of our results under the static pricing situation. From Lemma 2 and Proposition 4, we draw the following conclusions.

Proposition 5. *The retail prices of new products under the dual market with dynamic pricing strategy are smaller than or equal to those under the single market with dynamic pricing strategy; i.e.,* $p_{1n}^{DD*} \leq p_{1n}^{SD*}$ *and* $p_{2n}^{DD*} \leq p_{2n}^{SD*}$. *Holders pay a lower price for a new product than under the single market with dynamic pricing strategy; i.e.,* $p_{2n}^{SD*} - p_t^{SD*} \geq p_{2n}^{DD*} - p_t^{DD*}$. *In addition, the presence of the secondary market helps increase CS in the primary market; i.e.,* $CS^{DD} \geq CS^{SD}$.

As presented in Proposition 5, the OEM tends to charge lower retail prices for new products within two periods under the dynamic pricing model. In addition, holders pay a lower trade-in price for a new product if they participate in the trade-in programme. Therefore, customers in the primary market can significantly benefit from the presence of the secondary market. These findings show that the results derived from the static pricing model are still tenable in the dynamic pricing situation.

Next, we investigate how the OEM's pricing strategy (i.e., dynamic pricing) affects its strategy decisions. Figure 6 is provided to further this goal.

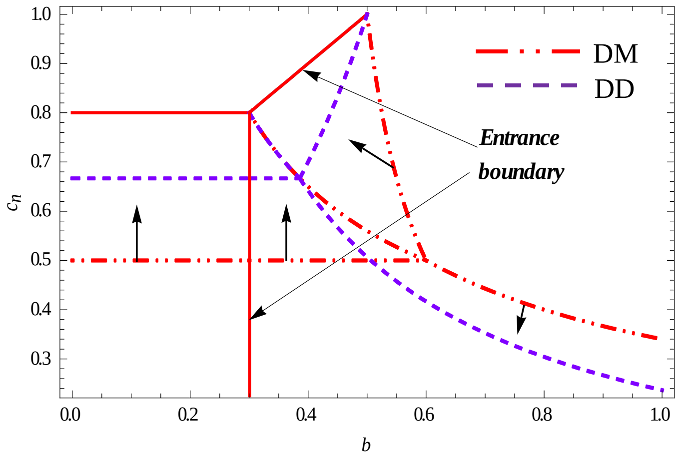

Figure 6. Effects of dynamic pricing on the OEM's strategies ($c_r = 0.1, s = 0.2$ and $\phi = 0.4$).

From Figure 6, we find that in a dual market with dynamic pricing, compared with the dual market with static pricing strategy, the OEM is more likely to offer a menu such that all holders participate in the trade-in and return their used products (even when the conditions under which no holders participate in the trade-in remain constant). Furthermore, we learn that the OEM is more likely to remanufacture all used products and resell them to the secondary market. In other words, the possible fields in which all used products are remanufactured and resold to the secondary market become bigger than those under the dual market with static pricing strategy. The main driver is that the demand for new products in the first period falls when the retail price in the first period increases (i.e., $p_{1n}^{DD*} > p_n^{DM*}$). The possibility of collecting all used products increases because the retail price in the second period decreases, i.e., $p_{2n}^{DD*} < p_n^{DM*}$.

6 Competitive situations

In this section, we consider a competitive situation in which a competitor selling substitutable products aims to enter the secondary market. This phenomenon is very common in the automobile industry. For example, BMW, Ford and Mercedes-Benz recycle their used products through trade-in programmes in developed countries and then sell them in developing regions (e.g., Southeast Asia and Africa) after refurbishing and remanufacturing them. Here, we focus on investigating whether the OEM should enter the secondary market when facing potential competition. If the OEM enters the secondary market, which trade-in and remanufacturing policy should it adopt? To this end, we consider two scenarios in which the OEM faces a competitor that may sell low- or high-quality products in the secondary market. For tractability, we assume that the OEM adopts a static pricing strategy and that the OEM and the competitor plan to enter the secondary market simultaneously. A Nash game occurs between them. In addition, we consider the following two additional cases: (1) The OEM and the potential competitor aim to enter the secondary market simultaneously, but a Stackelberg game occurs between them in which the OEM is the leader and the potential competitor is the follower; and (2) the competitor is already operating in the market, and the OEM determines whether it should enter the secondary market. *We provide the results and proofs in Appendix B of the supplementary materials*.

6.1 Dual market with a low-quality competitor

In this case, we assume that a potential competitor that sells low-quality substitutable products also aims to enter the secondary market during the second period. The OEM sells new products in the first period and launches a trade-in programme in the second period. Meanwhile, the OEM can choose to remanufacture used products and sell them to the secondary market in the second period. Based on equation (3.1), we can see that the profit maximization problems of the OEM and the competitor are, respectively:

$$\max_{(p_n,p_t,p_r)} \pi_M^{DL} = (p_n - c_n)(1 - p_n) + (p_n - c_n - p_t)\left(1 - \frac{p_n - p_t}{1-\phi}\right) + s\left(1 - \frac{p_n - p_t}{1-\phi} - \left(1 - \frac{p_r - p_C}{(1-\delta_L)b}\right)\right) +$$

$$(p_r - c_r)\left(1 - \frac{p_r - p_C}{(1-\delta_L)b}\right) - F,$$

$$\text{s.t.}\ 1 - \frac{p_n - p_t}{1-\phi} \geq 1 - \frac{p_r - p_C}{(1-\delta_L)b} \geq 0 \ \text{ and } \ 1 - p_n \geq 1 - \frac{p_n - p_t}{1-\phi} \geq 0. \tag{8}$$

$$\max_{p_2} \pi_C^{DL} = \left(\frac{p_r - p_C}{(1-\delta_L)b} - \frac{p_C}{b\delta_L}\right)(p_C - c_L). \tag{9}$$

Similarly, using the optimization theory, the result below can be obtained.

Proposition 6. *Under the dual market with a low-quality competitor case, if* $(2-\delta_L)(s + c_r) > c_L > (2-\delta_L)(c_r(4-\delta_L) - \phi\delta_L)/(4-\delta_L)$, *the optimal trade-in and remanufacturing policies of the OEM are as follows:*

(1) If $b \leq ((2-\delta_L)(s+c_r) - c_L)/2(1-\delta_L)$, *the OEM's optimal strategies are identical to those under the single market strategy.*

(2) If $((2-\delta_L)(s+c_r) - c_L)/2(1-\delta_L) < b$, *then the OEM's optimal strategy is*

(i) the NTNR strategy, such that no holders participate in the trade-in programme when $c_n > \frac{(2-\delta_L)(1+b-\phi-c_r)-(b\delta_L - c_L)}{2-\delta_L}$;

(ii) the PTAR strategy, such that only partial holders participate in the trade-in programme, but all of

the used products are remanufactured and resold when $\frac{(2-\delta_L)(1+b-\phi-c_r)-(b\delta_L-c_L)}{2-\delta_L} \geq c_n \geq \max\{\frac{b(1-\delta_L)\delta_L+2c_L-2(2-\delta_L)c_r}{2(2-\delta_L)\phi-b(4-\delta_L)(1-\delta_L)}, \frac{2(1-\phi)[(2-\delta_L)(b(1-\delta_L)+s+c_r)-c_L]}{b(4-\delta_L)(1-\delta_L)} - 1 + s + \phi\}$*;*

(iii) the ATAR strategy, such that all holders participate in the trade-in programme, and all of these used products are remanufactured and resold when $\frac{b(1-\delta_L)\delta_L+2c_L-2(2-\delta_L)c_r}{2(2-\delta_L)\phi-b(4-\delta_L)(1-\delta_L)} > c_n \geq \frac{(2-\phi)[(2-\delta_L)(b(1-\delta_L)+s+c_r)-c_L]}{b(4-\delta_L)(1-\delta_L)} - \frac{2-s-\phi}{2}$*;*

(iv) the PTPR strategy, such that only partial holders participate in the trade-in programme, and some of these used products are remanufactured and resold when $\frac{2(1-\phi)[(2-\delta_L)(b(1-\delta_L)+s+c_r)-c_L]}{b(4-\delta_L)(1-\delta_L)} - 1 + s + \phi > c_n \geq \frac{s}{\phi}$*;*

(v) the ATPR strategy, such that all holders participate in the trade-in programme, but some of the used products are remanufactured and resold when $\min\{\frac{(2-\phi)[(2-\delta_L)(b(1-\delta_L)+s+c_r)-c_L]}{b(4-\delta_L)(1-\delta_L)} - \frac{2-s-\phi}{2}, \frac{s}{\phi}\} > c_n$*.*

The optimal decisions made under the different scenarios are summarized in Table 7.

To achieve a better understanding of Proposition 6, we provide a numerical example where $\phi = 0.32$, $\delta_L = 0.8$, $c_L = 0.12$, $s = 0.2$, and $c_r = 0.1$. Proposition 6 shows that the OEM's trade-in and remanufacturing policies rely significantly on the relationship between the production cost and customers' maximum willingness to pay in the secondary market. In addition, in the dual market with a lower-quality competitor, the OEM is less likely to enter the secondary market when the entry barrier increases with competition. It is clear that, when potential competition exists in the secondary market, the OEM's decision is affected by the profitability of the competitor's products, in addition to its own production cost and customers' maximum willingness to pay in the secondary market.

Specifically, when customers' maximum willingness to pay in the secondary market is high, the OEM will offer a menu such that all holders are willing to participate in the trade-in and return their used products. Furthermore, all (or some) of these used products are remanufactured and resold to the secondary market according to their production cost level. Interestingly, with the change in the production cost, the OEM can choose the optimal trade-in and remanufacturing policy from a strategy portfolio consisting of *NTNR*, *PTAR*, *ATAR*, and *ATPR* when the customers' maximum willingness to pay is relatively high or *NTNR*, *PTAR*, *PTPR*, and *ATPR* when it is relatively low. However, if the customers' maximum willingness to pay in the secondary market is very low, the OEM will never remanufacture and then enter the secondary market. The optimal strategies in this case are identical to those under the single-market strategy.

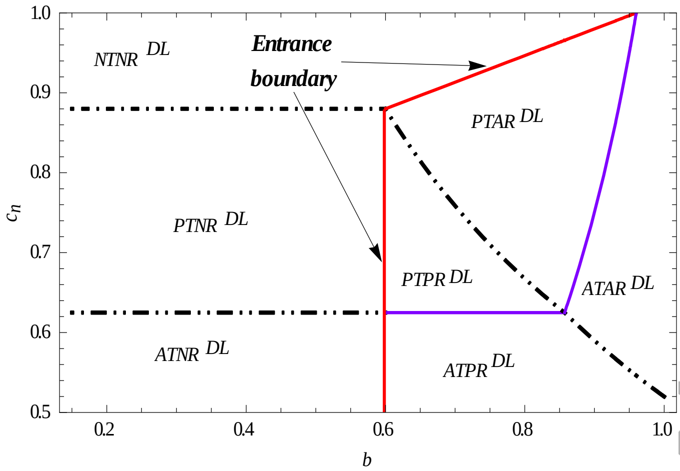

Figure 7. Layout of the optimal trade-in and remanufacturing strategies on the $b - c_n$ plane

Corollary 3. *When* $1 + s - \phi \geq c_n$, *when the difference between the remanufactured and competitive products decreases, i.e.,* δ_L *increases, the OEM is less likely to remanufacture and enter the secondary market when* $c_L < s + c_r$, *and not if* $c_L > s + c_r$.

Corollary 3 demonstrates that the competitor's production cost plays an important role in driving the OEM's optimal decisions. When the difference between remanufactured and competitive products diminishes, i.e., the competition between remanufactured and substitutable products increases, the competitor's quality advantage decreases. The OEM is more likely to remanufacture and enter the secondary market when the competitor's production cost is high. The entry barrier also decreases. However, when the competitor's production cost is low, the OEM is less likely to remanufacture and enter the secondary market if the differences between the remanufactured and competitive products increase. Although the competitor is the underdog, so to speak, in product quality, it has a cost advantage when its production cost is low. Moreover, the entry barrier increases.

6.2 Dual market with a high-quality competitor

When a potential competitor that sells high-quality substitutable products aims to enter the secondary market during the second period, based on equation (3.2), we can see that the profit maximization problems of the OEM and the competitor respectively are as follows:

$$\max_{(p_n,p_t,p_r)} \pi_M^{DH} = (p_n - c_n)(1 - p_n) + (p_n - c_n - p_t)\left(1 - \frac{p_n - p_t}{1-\phi}\right) + s\left(1 - \frac{p_n - p_t}{1-\phi} - \left(\frac{p_C - p_r}{(\delta_H - 1)b} - \right.\right.$$

$$\left.\left.\frac{p_r}{b}\right)\right) + (p_r - c_r)\left(\frac{p_C - p_r}{(\delta_H - 1)b} - \frac{p_r}{b}\right) - F,$$

$$\text{s.t.}\, 1 - \frac{p_n - p_t}{1-\phi} \geq \frac{p_C - p_r}{(\delta_H - 1)b} - \frac{p_r}{b} \geq 0 \text{ and } 1 - p_n \geq 1 - \frac{p_n - p_t}{1-\phi} \geq 0. \quad (10)$$

$$\max_{p_2} \pi_C^{DH} = \left(1 - \frac{p_C - p_r}{(\delta_H - 1)b}\right)(p_C - c_H). \quad (11)$$

Similarly, the following result can be obtained using the optimization theory.

Proposition 7. *Under the dual market with a high-quality competitor case, if* $2s > \phi$ *and* $c_H \leq ((2\delta_H - 1)(\phi(2\delta_H - 1) + c_r(4\delta_H - 1)))/(4\delta_H - 1)$*, the optimal trade-in and remanufacturing policies of the OEM are as follows:*

(1) If $b \leq [(2\delta_H - 1)(s + c_r) - c_H]/(\delta_H - 1)$*, the OEM's optimal strategies are identical to those in the single market strategy.*

(2) If $b \geq [(2\delta_H - 1)(s + c_r) - c_H]/(\delta_H - 1)$*, the OEM's optimal strategy is*

(i) the NTNR strategy, such that no holders participate in the trade-in programme when $c_n > \frac{b(\delta_H-1)+(2\delta_H-1)(1-\phi-c_r)+c_H}{2\delta_H-1}$*;*

(ii) the PTAR strategy, such that only partial holders participate in the trade-in programme, but all of the used products are remanufactured and resold when $\frac{b(\delta_H-1)+(2\delta_H-1)(1-\phi-c_r)+c_H}{2\delta_H-1} \geq c_n \geq \max\{\frac{(2\delta_H-1)(b(\delta_H-1)+2\delta_H c_r)-2\delta_H c_H}{b(\delta_H-1)(4\delta_H-1)-2(2\delta_H-1)\delta_H\phi}, \frac{2s(2\delta_H-1)(1-\phi)+b(\delta_H-1)(s(4\delta_H-1)+(2\delta_H-1)(1-\phi))+2\delta_H(1-\phi)((2\delta_H-1)c_r-c_H)}{b(\delta_H-1)(4\delta_H-1)}\}$*;*

(iii) the ATAR strategy, such that all holders participate in the trade-in programme and all of the used products are remanufactured and resold when $\frac{(2\delta_H-1)(b(\delta_H-1)+2\delta_H c_r)-2\delta_H c_H}{b(\delta_H-1)(4\delta_H-1)-2(2\delta_H-1)\delta_H\phi} > c_n \geq \frac{2s\delta_H(2\delta_H-1)(2-\phi)+b(\delta_H-1)(s(4\delta_H-1)+(2\delta_H-1)(2-\phi))+2\delta_H(2-\phi)((2\delta_H-1)c_r-c_H)}{2b(\delta_H-1)(4\delta_H-1)}$*;*

(iv) the PTPR strategy, such that only partial holders participate in the trade-in programme and some of the used products are remanufactured and resold when $\frac{2s(2\delta_H-1)(1-\phi)+b(\delta_H-1)(s(4\delta_H-1)+(2\delta_H-1)(1-\phi))+2\delta_H(1-\phi)((2\delta_H-1)c_r-c_H)}{b(\delta_H-1)(4\delta_H-1)} > c_n \geq s/\phi$*;*

(v) the ATPR strategy, such that all holders participate in the trade-in programme, but some of the used products are remanufactured and resold when $\min\{\frac{2s\delta_H(2\delta_H-1)(2-\phi)+b(\delta_H-1)(s(4\delta_H-1)+(2\delta_H-1)(2-\phi))+2\delta_H(2-\phi)((2\delta_H-1)c_r-c_H)}{2b(\delta_H-1)(4\delta_H-1)}, s/\phi\} > c_n$.

The optimal decisions under the different scenarios are summarized in Table 8.

From Proposition 7, we can observe that the structure of the OEM's optimal policies is similar to that in the dual market with a low-quality competitor. However, the threshold values are different from those in the dual market with a low-quality competitor. Specifically, when the customers' maximum willingness to pay remains the same, the OEM will never choose the *ATAR* strategy, regardless of the production cost. The reason is that the OEM faces a disadvantage because remanufactured products are less popular than the competitive products. In this case, it is detrimental to the OEM to remanufacture all of its used products.

However, we can also see that the strategy selected by the OEM ranges from *NTNR* to *PTAR*, *ATAR*, and *ATPR* when customers' maximum willingness to pay in the secondary market is relatively low, or from *NTNR* to *PTAR*, *PTPR*, and *ATPR* when customers' maximum willingness to pay in the secondary market is relatively high, with a change in production cost. Finally, when customers' maximum willingness to pay in the secondary market is low, the optimal strategy of the OEM may change from *NTNR* to *PTNR* and *ATNR* with the decrease in production costs. This is similar to the single-market strategy. To achieve a better understanding of Proposition 7, we provide a numerical example (i.e., Figure 8) where $c_r = 0.1$, $s = 0.2$, $\phi = 0.32$, $c_H = 0.24$, and $\delta_H = 1.2$. In Figure 8, the parameter b changes from 0 to 3, which helps us better understand the result even if b is less than one.

Based on Proposition 7, the following corollary is obtained:

Corollary 4. *When* $1 + s - \phi \geq c_n$, *when the difference between the remanufactured and competitive products decreases, i.e.,* δ_H *decreases, the OEM wishes to undertake remanufacturing and enter the secondary market when* $c_H > s + c_r$, *and not if* $c_H < s + c_r$.

Corollary 4 shows that the competition between remanufactured and substitutable products increases with a decrease in the difference between remanufactured and competitive products. When the competitor's production cost is high, the OEM prefers to undertake remanufacturing and enter the secondary market because it acquires a cost advantage. However, when the competitor's production cost is low, the OEM prefers not to do so because the competitor has the cost advantage.

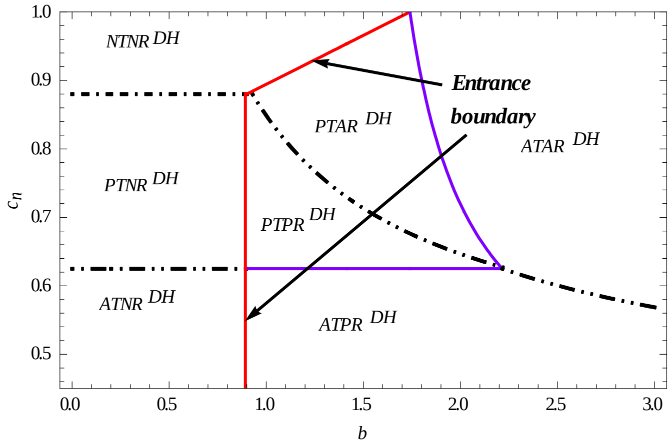

Figure 8. Layout of the optimal trade-in and remanufacturing strategies on the $b - c_n$ plane

6.3 Effects of the presence of competitors

In this section, we explore the influence of competition on the OEM's trade-in and remanufacturing policies. Figure 9 is provided to help further this goal, where the parameters are $c_r = 0.1$, $s = 0.2$, $\phi = 0.32$, $c_L = 0.12$, $\delta_L = 0.8$ $c_H = 0.24$, and $\delta_H = 1.2$.

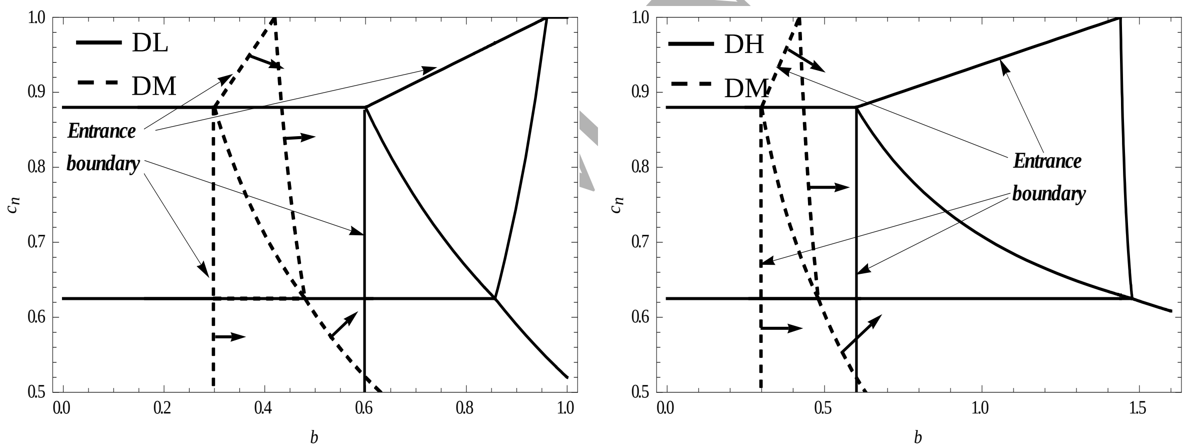

Figure 9. Effects of a competitor on the OEM's strategy

Figure 9 shows that compared with the dual-market strategy, the presence of a competitor, regardless of its type (i.e., low-quality or high quality), makes the OEM less likely to enter the secondary market. In particular, with a potential competitor in the secondary market, the OEM becomes less likely to undertake remanufacturing and enter the secondary market even if the secondary market is available. Furthermore, the OEM is much more likely to refrain from remanufacturing all used products if the competition comes from a high-quality competitor. In other words, the possible field in which the OEM remanufactures all used products narrows significantly compared with the low-quality competitor case. Obviously, the OEM is at a distinct disadvantage when the competitor has the competitive advantage in product quality. In such a case, remanufacturing all used products is more likely to adversely affect the OEM.

7 Discussion

We summarize the results based on our prior analysis and attempt to provide some managerial implications for the firm's operations using the proposed model.

Managerial Implications 1: *The implications of having a secondary market include the following.*

(1) Compared with the single-market strategy, regardless of the pricing model, the presence of a secondary market can help increase the OEM's profit without considering upfront investment.

(2) Compared with the single-market strategy, regardless of the pricing model, the presence of a secondary market leads to a decrease in the retail price and trade-in price and an increase in consumer welfare.

(3) Compared with the static pricing model, in the dynamic pricing case, the OEM should collect all (rather than some) used products through a trade-in programme and remanufacture and resell all (rather than some) of them to the secondary market.

Under the dual-market strategy, the secondary market serves as an alternative option, and firms want to remanufacture and sell the remanufactured products to the secondary market if customers' willingness to pay is higher than the sum of the remanufacturing cost and salvage value. Thus, when a firm is motivated to enter the secondary market, it is more likely to charge a low retail price and offer a low trade-in price (i.e., retail price minus trade-in rebate) to attract more holders to its trade-in programme and encourage holders to return their used products. Obviously, without considering the upfront investment required to enter the secondary market, undertaking remanufacturing and entering the secondary market are beneficial not only to the OEM, but also to customers of the primary and secondary markets. In other words, all parties (except for the competitor) can benefit from the presence of a secondary market. However, if it is not possible to avoid upfront investment, undertaking remanufacturing and entering the secondary market may adversely affect the OEM, even though it is good for its customers.

Managerial Implications 2: *In terms of trade-in and remanufacturing policy, we find the following implications:*

(1) The OEM can choose one of seven types of trade-in and remanufacturing strategy according to the relationship between customers' maximum willingness to pay in the secondary market and the production cost, regardless of whether a potential competitor exists and what type of competitor it is.

(2) The higher the customers' maximum willingness to pay in the secondary market and the lower the

production cost of the OEM, the more used products the OEM should collect and remanufacture.

Clearly, the production cost affects the cost of recycling used products. Generally, information about production cost is kept confidential by the OEM. Therefore, accurately acquiring such information can help the OEM control its competitive advantage and the supply of remanufactured products. Alternatively, the customers' maximum willingness to pay in the secondary market impacts the demand for remanufactured products, and acquiring this information can help the OEM gauge this demand effectively. Therefore, it is necessary for the OEM to acquire accurate information about these two elements. Taking Apple's iPhone as an example, although the production cost is relatively high, many firms, including Apple itself, are willing to take back used iPhones and resell them to the secondary market. The main driver is the customers' maximum willingness to pay for second-hand iPhones, which is extremely high. In other words, there is sufficient demand for refurbished iPhones. Hence, Apple has enough incentive to implement a trade-in programme and resell its used phones to the secondary market (Apple online store, 2017). Other remanufacturers also take back Apple's old products.

Managerial Implications 3: *In terms of the effects of competition in the secondary market, we find the following implications:*

(1) Compared with the monopolistic case, OEMs are less likely to undertake remanufacturing and enter the secondary market in competitive situations.

(2) In a competitive situation in which competition increases, the OEM is less likely to enter the secondary market if the competitor's production cost is low, but is more likely to enter the secondary market if the competitor's production cost is high.

When there is a potential competitor in the secondary market, customers' maximum willingness to pay in that market affects the demand for remanufactured products and the competitor's products. Furthermore, the competitor's production cost indirectly determines the relative competitive advantages of the OEM and the competitor. All of these factors play important roles in the OEM's choice of its trade-in and remanufacturing policies and entry strategy. Realistically, it is necessary to do some market research to collect information on the market demand and the competitor's production cost. This helps firms not only to gauge customer demand, but also to master their own competitive advantages and disadvantages.

8 Conclusions

In this study, we consider a two-period model in which a monopolistic OEM sells new products in the first period and offers a trade-in programme in the second period. These products, collected from the trade-in programme, can be remanufactured and sold in the secondary market. We obtain the OEM's optimal pricing, trade-in and remanufacturing decisions, and provide the possible conditions under which the OEM will choose one of seven strategies. Customers' maximum willingness to pay in the secondary market and the production cost play an important role in driving the OEM's optimal trade-in and remanufacturing policy. We provide the layouts of the OEM's optimal trade-in and remanufacturing policy on a two-dimensional plane under dual market conditions with static or dynamic pricing strategies. In addition, we extend our research to include the case in which a potential competitor exists in the secondary market, and we discuss the OEM's optimal pricing, trade-in, and remanufacturing decisions in addition to the layouts of the OEM's optimal strategies in the two-dimensional space.

We find that the competitor's production cost and the difference between the remanufactured and substitutable products jointly affect the OEM's remanufacturing and entry decisions. In particular, the entry barrier decreases with a decrease in the difference between the remanufactured and competitive products when the competitor's production cost is high. Otherwise, the entry barrier increases with a decrease in the difference between remanufactured and competitive products. We also consider the situations in which (1) the OEM serves as a Stackelberg leader and the competitor as a follower; and (2) the competitor is already operating in the secondary market, and the OEM considers whether to enter it. We find that the results derived from the monopolistic and competitive settings can be successfully reproduced.

In this study, we attempt to provide a foundation for future research on the joint operations of firms with trade-ins and remanufacturing. In future, we will investigate trade-in and remanufacturing decisions when considering firms' upgrade strategy. Several studies show that upgrade strategies have a vital effect on the profits and entry decisions of third-party remanufacturers (Xiong et al., 2016). In addition, horizontal/vertical competitive cases in the primary market are an interesting field to study. Zhu et al. (2016), for example, report that a trade-in programme helped to improve the competitive advantages of a firm under a horizontally competitive case. Finally, more attention should be paid to customer behaviour (e.g., strategic behaviour, mental accounting and loss aversion) in the future (Liu, Zhai, & Chen, 2018b; Okada, 2001).

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Table 3. Optimal retail price and trade-in rebate in the single market strategy

Strategies	p_n^{SM*}	p_t^{SM*}
$NTNR^{SM}$	$\frac{1+c_n}{2}$	$\frac{2\phi+c_n-1}{2}$
$PTNR^{SM}$	$\frac{1+c_n}{2}$	$\frac{s+\phi}{2}$
$ATNR^{SM}$	$\frac{2-s-\phi+2c_n}{2(2-\phi)}$	$\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)}$

Table 4. Optimal decisions in the dual-market strategy

Strategies	p_n^{DM*}	p_t^{DM*}	p_r^{DM*}
$NTNR^{DM}$	$\frac{1+c_n}{2}$	$\frac{2\phi+c_n-1}{2}$	b
$PTNR^{DM}$	$\frac{1+c_n}{2}$	$\frac{s+\phi}{2}$	b
$ATNR^{DM}$	$\frac{2-s-\phi+2c_n}{2(2-\phi)}$	$\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)}$	b
$PTAR^{DM}$	$\frac{1+c_n}{2}$	$\frac{(1+b-\phi)\phi+bc_n-(1-\phi)c_r}{2(1+b-\phi)}$	$\frac{b(1+b-\phi+c_n+c_r)}{2(1+b-\phi)}$
$PTPR^{DM}$	$\frac{1+c_n}{2}$	$\frac{s+\phi}{2}$	$\frac{b+s+c_r}{2}$
$ATAR^{DM}$	$\frac{2+b-\phi+2c_n+c_r}{4+2b-2\phi}$	$\frac{\phi(2+b-\phi+2c_n+c_r)}{2(2+b-\phi)}$	$\frac{b(2+b-\phi+2c_n+c_r)}{2(2+b-\phi)}$
$ATPR^{DM}$	$\frac{2-s-\phi+2c_n}{2(2-\phi)}$	$\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)}$	$\frac{b+s+c_r}{2}$

Table 5. Optimal decisions in the single market with dynamic pricing strategy

Strategies	p_{1n}^{SD*}	p_{2n}^{SD*}	p_t^{SD*}
$NTNR^{SD}$	$\frac{2+c_n}{3}$	$\frac{1+2c_n}{3}$	$\frac{3\phi+2c_n-2}{3}$
$PTNR^{SD}$	$\frac{2+c_n}{3}$	$\frac{1+2c_n}{3}$	$\frac{3s+3\phi+c_n-1}{6}$
$ATNR^{SD}$	$\frac{2(2-s-\phi)+3c_n}{7-4\phi}$	$\frac{2-s-\phi+(5-2\phi)c_n}{7-4\phi}$	$\frac{(2-s-\phi)(1-2\phi)+(2+\phi)c_n}{7-4\phi}$

Table 6. Optimal decisions in the dual market with dynamic pricing strategy

Strategies	p_{1n}^{DD*}	p_{2n}^{DD*}	p_t^{DD*}	p_r^{DD*}
$NTNR^{DD}$	$\frac{2+c_n}{3}$	$\frac{1+2c_n}{3}$	$\frac{3\phi+2c_n-2}{3}$	b
$PTNR^{DD}$	$\frac{2+c_n}{3}$	$\frac{1+2c_n}{3}$	$\frac{3s+3\phi+c_n-1}{6}$	b
$ATNR^{DD}$	$\frac{2(2-s-\phi)+3c_n}{7-4\phi}$	$\frac{2-s-\phi+(5-2\phi)c_n}{7-4\phi}$	$\frac{(2-s-\phi)(2\phi-1)+(2+\phi)c_n}{7-4\phi}$	b
$PTAR^{DD}$	$\frac{2+c_n}{3}$	$\frac{1+2c_n}{3}$	$\frac{(1+b-\phi)(3\phi-1)+(1+4b-\phi)c_n-3(1-\phi}{6(1+b-\phi)}$	$\frac{b(1+b-\phi+c_n+c_r)}{2(1+b-\phi)}$
$PTPR^{DD}$	$\frac{2+c_n}{3}$	$\frac{1+2c_n}{3}$	$\frac{3s+3\phi+c_n-1}{6}$	$\frac{b+s+c_r}{2}$
$ATAR^{DD}$	$\frac{3c_n+2(2+b-\phi+c_r}{7+4b-4\phi}$	$\frac{2+b-\phi+(5+2b-2\phi)c_n+c}{7+4b-4\phi}$	$\frac{(2+2b+\phi)c_n+(-1+2\phi)(2+b-\phi+c_r)}{7+4b-4\phi}$	$\frac{b(3c_n+2(2+b-\phi+c_r)}{7+4b-4\phi}$
$ATPR^{DD}$	$\frac{2(2-s-\phi)+3c_n}{7-4\phi}$	$\frac{2-s-\phi+(5-2\phi)c_n}{7-4\phi}$	$\frac{(2-s-\phi)(2\phi-1)+(2+\phi)c_n}{7-4\phi}$	$\frac{b+s+c_r}{2}$

Table 7. Optimal decisions in the dual market with a low-quality competitor case

Strategies	Decisions
$NTNR^{DL}$	$p_n^{DL*}=\frac{1+c_n}{2},\ p_t^{DL*}=\frac{2\phi+c_n-1}{2},\ p_r^{DL*}=\frac{2b-2b\delta_L+c_L}{2-\delta_L},\ p_2^{DL*}=\frac{b(1-\delta_L)\delta_L+c_L}{2-\delta_L}$
$PTNR^{DL}$	$p_n^{DL*}=\frac{1+c_n}{2},\ p_t^{DL*}=\frac{s+\phi}{2},\ p_r^{DL*}=\frac{2b-2b\delta_L+c_L}{2-\delta_L},\ p_2^{DL*}=\frac{b(1-\delta_L)\delta_L+c_L}{2-\delta_L}$
$ATNR^{DL}$	$p_n^{DL*}=\frac{2-s-\phi+2c_n}{2(2-\phi)},\ p_t^{DL*}=\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)},\ p_r^{DL*}=\frac{2b-2b\delta_L+c_L}{2-\delta_L},\ p_2^{DL*}=\frac{b(1-\delta_L)\delta_L+c_L}{2-\delta_L}$
$PTAR^{DL}$	$p_n^{DL*}=\frac{1+c_n}{2}$, $p_t^{DL*}=\frac{b(1-\delta_L)\delta_L+2(1+b(1-\delta_L))(2-\delta_L)\phi-2(2-\delta_L)\phi^2+2(1-\phi)c_L+b(4-\delta_L)(1-\delta_L)c_n-2(2-\delta_L)(1-\phi)c_r}{2(b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi))}$, $p_r^{DL*}=\frac{(2+b-b\delta_L-2\phi)c_L+2b(1-\delta_L)(1+b(1-\delta_L)-\phi+c_n+c_r)}{b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi)}$, $p_2^{DL*}=\frac{2(1+b(1-\delta_L)-\phi)c_L+b\delta_L(1-\delta_L)(1+b(1-\delta_L)-\phi+c_n+c_r)}{b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi)}$
$PTPR^{DL}$	$p_n^{DL*}=\frac{1+c_n}{2},\ p_t^{DL*}=\frac{s+\phi}{2},\ p_r^{DL*}=\frac{c_L+2(b+s-b\delta_L+c_r)}{4-\delta_L},\ p_2^{DL*}=\frac{2c_L+\delta_L(b+s-b\delta_L+c_r)}{4-\delta_L}$
$ATAR^{DL}$	$p_n^{DL*}=\frac{(2-\delta_L)(2+b(1-\delta_L)-\phi+2c_n+c_r)-c_L}{b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi)}$, $p_t^{DL*}=\frac{\phi((2-\delta_L)(2+b(1-\delta_L)-\phi+2c_n+c_r)-c_L)}{b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi)}$, $p_r^{DL*}=\frac{(4+b-b\delta_L-2\phi)c_L+2b(1-\delta_L)(2+b(1-\delta_L)-\phi+2c_n+c_r)}{b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi)}$, $p_2^{DL*}=\frac{2(2+b(1-\delta_L)-\phi)c_L+b\delta_L(1-\delta_L)(2+b(1-\delta_L)-\phi+2c_n+c_r)}{b(4-\delta_L)(1-\delta_L)+2(2-\delta_L)(2-\phi)}$
$ATPR^{DL}$	$p_n^{DL*}=\frac{2-s-\phi+2c_n}{2(2-\phi)},\ p_t^{DL*}=\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)},\ p_r^{DL*}=\frac{c_L+2(b+s-b\delta_L+c_r)}{4-\delta_L}$, $p_2^{DL*}=\frac{2c_L+\delta_L(b+s-b\delta_L+c_r)}{4-\delta_L}$

Table 8. Optimal decisions in the dual market with a high-quality competitor case

Strategies	Decisions
$NTNR^{DH}$	$p_n^{DH*}=\frac{1+c_n}{2},\ p_t^{DH*}=\frac{2\phi+c_n-1}{2},\ p_r^{DH*}=\frac{b(\delta_H-1)+c_H}{2\delta_H-1},\ p_2^{DH*}=\frac{\delta_H(b(\delta_H-1)+c_H)}{2\delta_H-1}$
$PTNR^{DH}$	$p_n^{DH*}=\frac{1+c_n}{2},\ p_t^{DH*}=\frac{s+\phi}{2},\ p_r^{DH*}=\frac{b(\delta_H-1)+c_H}{2\delta_H-1},\ p_2^{DH*}=\frac{\delta_H(b(\delta_H-1)+c_H)}{2\delta_H-1}$
$ATNR^{DH}$	$p_n^{DH*}=\frac{2-s-\phi+2c_n}{2(2-\phi)},\ p_t^{DH*}=\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)},\ p_r^{DH*}=\frac{b(\delta_H-1)+c_H}{2\delta_H-1},\ p_2^{DH*}=\frac{\delta_H(b(\delta_H-1)+c_H)}{2\delta_H-1}$
$PTAR^{DH}$	$p_n^{DH*}=\frac{1+c_n}{2}$, $p_t^{DH*}=\frac{2\delta_H\phi(2\delta_H-1)(1-\phi)+b(\delta_H-1)(1-2\phi-\delta_H(2-6\phi))+2\delta_H(1-\phi)c_H+b(\delta_H-1)(4\delta_H-1)c_n-2\delta_H(2\delta_H-1)(1-\phi)c_r}{2(b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(1-\phi))}$, $p_r^{DH*}=\frac{(b(\delta_H-1)+2\delta_H(1-\phi))c_H+b(\delta_H-1)(b(\delta_H-1)+2\delta_H(c_n+c_r))}{b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(1-\phi)}$, $p_2^{DH*}=\frac{\delta_H(2(b(\delta_H-1)+\delta_H-\delta_H\phi)c_H+b(\delta_H-1)(2b(\delta_H-1)+2\delta_H-1+\phi-2\delta_H\phi+c_n+c_r))}{b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(1-\phi)}$
$PTPR^{DH}$	$p_n^{DH*}=\frac{1+c_n}{2},\ p_t^{DH*}=\frac{s+\phi}{2},\ p_r^{DH*}=\frac{b(\delta_H-1)+2s\delta_H+c_H+2\delta c_r}{4\delta_H-1},\ p_2^{DH*}=\frac{\delta_H(s+2b(\delta_H-1)+2c_H+c_r)}{4\delta_H-1}$
$ATAR^{DH}$	$p_n^{DH*}=\frac{b(\delta_H-1)(3\delta_H-1)+\delta_H(2\delta_H-1)(2-\phi)-\delta_H c_H+\delta_H(2\delta_H-1)(2c_n+c_r)}{b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(2-\phi)}$, $p_t^{DH*}=\frac{\phi(b(3\delta_H-1)(\delta_H-1)+\delta_H(2\delta_H-1)(2-\phi)-\delta_H c_H+\delta_H(2\delta_H-1)(2c_n+c_r))}{b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(2-\phi)}$, $p_r^{DH*}=\frac{(\delta_H(\delta_H-1)+2\delta_H(2-\phi))c_H+b(\delta_H-1)(b(\delta_H-1)+4\delta_H c_n+2\delta_H c_r)}{b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(2-\phi)}$, $p_2^{DH*}=\frac{\delta_H(2(b(\delta_H-1)+\delta_H(2-\phi))c_H+b(\delta_H-1)(2\delta_H(\delta_H-1)-2+4\delta_H+\phi-2\delta_H\phi+2c_n+c_r))}{b(\delta_H-1)(4\delta_H-1)+2\delta_H(2\delta_H-1)(2-\phi)}$
$ATPR^{DH}$	$p_n^{DH*}=\frac{2-s-\phi+2c_n}{2(2-\phi)},\ p_t^{DH*}=\frac{\phi(2-s-\phi+2c_n)}{2(2-\phi)},\ p_r^{DH*}=\frac{b(\delta_H-1)+2s\delta_H+c_H+2\delta_H c_r}{4\delta_H-1},\ p_2^{DH*}=\frac{\delta_H(s+2b(\delta_H-1)+2c_H+c_r)}{4\delta_H-1}$