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A Branch&Cut approach to recharging and refueling infrastructure planning

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A B S T R A C T

We consider the facility location problem of installing a refueling and recharging infrastructure for vehicles with a strongly limited driving range. For this purpose, a novel problem formulation is introduced that is based on an analogy to the well-known duality relationship of Max Flow and Min Cut. In order to optimally solve this problem, a decomposition-based Branch&Cut approach is developed that iteratively generates violated inequalities and so-called zero-half-cuts as specific cutting planes. A comprehensive computational study on two real-world road networks reveals that this considerable tightening of partial problems in each node enables an efficient enumeration process whereby even large scale instances are solved to optimality for the first time.

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1. Introduction

This study deals with the planning of a refueling and recharging infrastructure for vehicles with a strongly limited driving range. While such networks exist for vehicles with traditional fuel combustion engines that have a substantially larger driving range, comparable infrastructures are still lacking in most countries for electric or alternative fuel vehicles. Hence, substantial efforts are being made to reduce these deficits and to establish these vehicles in the market (Upchurch, Kuby, & Lim, 2009). Primarily, these efforts relate to the establishment of a competitive refueling and recharging infrastructure on a nationwide basis (Chung & Kwon, 2015; Lim & Kuby, 2010). From this perspective, the present study considers a specific facility location problem that pursues a maximal coverage of the expected travel demands of potential customers by opening a limited number of refueling or recharging stations (henceforth referred to as *stations*) in the network. The quality of a found network structure is measured by the attained fulfillment degree of the total demand of all customers, in what follows denoted as the *service level*. Alternatively, one may seek to develop a network structure that attains a predetermined service level with a minimum number of opened stations. The model defines the demand to be covered by OD-pairs, i.e., combinations of an origin and a destination of travel, weighted by an estimated number of corresponding customers.

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1.1. Literature review

The modeling and solving of facility location planning problems for the design of a refueling and recharging infrastructure is a vital research area in the recent literature. In order to provide a flexible measure of demand satisfaction, an increasing number of papers adopted the concept of covering OD-pairs. Among them, the Flow Capturing Location Model (FCLM) originally proposed by Hodgson (1990) defines for every demand a fixed set of covering stations, where at least one of them has to be opened to assure a feasible coverage. However, the strongly limited range of electric and alternative fuel vehicles frequently requires more than one stoppage for recharging for a considered OD-pair. Consequently, Kuby and Lim (2005) propose an extension of the model by defining for each demand a set of feasible station combinations while a chosen coverage requires the assignment of one of these combinations. Since all predefined combinations contain solely stations that are located on the shortest path from origin to destination, the flexibility of the network design is strongly limited. In order to overcome this limitation, the approach of Kim and Kuby (2012) allows predefined deviations from the respective shortest path. However, these deviations are restricted to some predetermined routes. In contrast to this, a more flexible determination of covering stations in a network design is integrated in the approaches and models of Lim and Kuby (2010), MirHassani and Ebrazi (2013), and Kim and Kuby (2013). Most recently, Yıldız, Arslan, and Karaşan (2016) propose the *Refueling Station Location Problem (RSLP)* with routing. In order to solve the problem, the authors design a Branch&Price approach that is based on an explicit enumeration of variables.

Table 1
Literature overview:

Article	Variant[a]	Coverage criterion	Method[b]
Hodgson (1990)	Max	Single station from fixed set	M, H
Kuby and Lim (2005)	Max	Fixed sets on shortest paths	M
Lim and Kuby (2010)	Max	Flexible on shortest paths	H
Kim and Kuby (2012)	Max	Fixed sets on deviation paths	M
Capar and Kuby (2012)	Max	Flexible on single path	M
Kim and Kuby (2013)	Max	Flexible on deviation paths	H
Capar et al. (2013)	Max/Min	Flexible on single path	M
MirHassani and Ebrazi (2013)	Max/Min	Flexible on shortest paths	M
Li and Huang (2014)	Min	Fixed sets on deviation paths	M, H
Yıldız et al. (2016)	Max (Min)	Flexible on deviation paths	M, B&P
This article	Max (Min)	Flexible on deviation paths	B&C

[a] Max = RSLP-Max, Min = RSLP-Min
[b] M = IP-Model, H = Heuristic, B&P = Branch&Price, B&C = Branch&Cut

A further distinction of the approaches in the literature results from the pursued objective functions, i.e., the considered *optimization variant*. *RSLP-Min* seeks to guarantee a given service level with a minimum amount of resources. On the contrary, *RSLP-Max* denotes models that apply a given set of resources in order to cover a maximally weighted set of demands. For both types of optimization variant, various model formulations, construction and improvement heuristics are proposed in the literature.

Table 1 provides a brief overview of the approaches in the literature categorized by the optimization variant, the criterion used to state coverage of demand and the applied solution methodology. Furthermore, the literature additionally proposes specific extensions to the aforementioned location models in order to strengthen their applicability. For instance, Upchurch et al. (2009) develop the capacitated flow refueling location model (CFRLM) that additionally integrates limited capacities for stations in order to limit the number of vehicles refueled at the same station. The approach of Chung and Kwon (2015) considers the fact that the installation of a network of stations is a long-term process that usually requires several periods to be completed. Hence, the model restricts the number of newly established stations per period while pursuing a maximization of possible traffic over all periods.

The literature cited thus far and the new approach presented in this contribution focus on the appropriate placement of stations in a network. However, there is also an emerging stream in the literature, in which this facility location aspect is embedded in the context of the Vehicle Routing Problem. In addition to the determination of feasible vehicle routes that cover all customers, the set of opened recharging stations is planned simultaneously - see, e.g., Schiffer and Walther (2017, 2018); Yang and Sun (2015).

1.2. Contribution and paper structure

This paper provides the following main contributions:

- We develop a new model for RSLP-Max which originates in the extension of the well-known duality relationship of Max Flow and Min Cut to the concepts of OD-covers and OD-cuts. By enabling a suitable decomposition, this modeling yields a considerably tighter problem definition: We determine the selection of stations to be opened by solving an LP-Relaxation that is based on so-called OD-cut inequalities, whereas an iteratively applied graph-algorithmic component derives necessary conditions for the coverage of demands.
- We propose a novel fast separation algorithm for the determination of violated OD-cut inequalities. It is based on shortest path computations in a transformed network and column generation.
- We design a specific separation and selection mechanism for zero-half-cuts (originally proposed by Caprara & Fischetti, 1996) that significantly reduces the remaining integrality gap of the applied LP-relaxation.
- We integrate all components into a Branch&Cut approach that, for the first time, optimally solves considerably-sized, real-world instances of RSLP-Max.
- We conduct a computational study that evaluates the performance of the new Branch&Cut procedure in direct comparison with the well-known recent approach of Yıldız et al. (2016) in selected real-world Californian and German road networks. For this purpose, the study covers a broad variety of experiments with up to five settings for the vehicle range, up to four deviation thresholds (including the case of no deviation) and up to three sets of candidate sites per network. This study underlines that the new approach overcomes previous limitations by enabling a flexible location planning of stations and attains optimal solutions even in networks of considerable size.

The remainder of the paper is structured as follows: Basic mathematical instruments for formalizing the coverage of demand by using OD-covers and the novel OD-cuts are introduced in Section 2. Section 3 applies the concept of OD-cuts in order to derive a more suitable formulation of RSLP-Max. Subsequently, Section 4 introduces the main components of the proposed Branch&Cut approach. Computational results that were obtained for two real-world road networks are presented in Section 5. Section 6 concludes the article and opens perspectives for further research.

2. Basic concepts

Each opening of a station in the considered *basic road network* $G'(P, R)$ aims at extending the coverage of existing demands Q of potential customers. The basic road network $G'(P, R)$ comprises a set of nodes P and a set of road segments R and is assumed to be a connected and directed graph with possibly asymmetric edge lengths. A demand $q \in Q$ is modeled by an ordered tuple $(o_q, d_q) \in P \times P$, commonly denoted as an *OD-pair*. It represents a set of customers that demand to travel from an origin $o_q \in P$ to a destination $d_q \in P$ in the network. The size of this customer set determines the weight w_q of demand q. The set of origins $O \subseteq P$ is given by $O := \{o_q | q \in Q\}$ and the set of destinations $D \subseteq P$ by $D := \{d_q | q \in Q\}$. Furthermore, the set $V \subseteq P$ comprises all locations that can be used to set up a station, i.e., the candidate set. Due to the limited driving range of the utilized vehicles, the possible coverage of a demand with a significant travel distance requires the installation of one or more stations along the driving tour. In order to attain a reasonable compromise between investment costs and customer inconvenience, stations enabling the coverage of an OD-pair do not have to belong to a shortest path between origin and destination, but have to be positioned such that a predetermined maximum length

Table 2
Notation.

Q	Set of demands
$\lambda \geq 1$	Deviation factor (setting $\lambda = 1$ forbids any deviation)
$G'(P, R)$	Basic (i.e., original) road network
$G(N, E)$	Transformed (i.e., condensed) network
$V \subseteq P$	Set of candidate sites to set up a station
O	Set of demand origins
D	Set of demand destinations
o_q	Origin of demand $q \in Q$
d_q	Destination of demand $q \in Q$
$\delta^{\min}/\delta^{\max}$	Min./max. distance between used stations on a trip
$\delta_o^{\max}$	Max. distance of used station from $o \in O$
$\delta_d^{\max}$	Max. distance of used station to $d \in D$
$\delta_{v,w}$	Shortest distance between v and w
δ_q	Shortest distance from o_q to d_q ($q \in Q$)
$\delta_q^{\max} := \lambda \cdot \delta_q$	Maximal feasible distance for fulfilling demand $q \in Q$
$\Delta(v), \Delta(o), \Delta(d)$	Neighbors of $v \in V, o \in O, d \in D$
$\delta_{u,w}(S)$	Shortest distance between u and w in $G(N, E)$ with respect to $S \subseteq V$
κ	Total number of stations to be opened.
$\mathcal{F}(q)$	Set of OD-covers for $q \in Q$ (see Definition 2).
$\mathcal{C}(q)$	Set of OD-cuts for $q \in Q$ (see Definition 8).

of a feasible trip for this demand is not exceeded. This threshold models the customizable *deviation tolerance* of customers, in what follows calculated by a corresponding *deviation factor* $\lambda \geq 1$. Table 2 provides an overview of the notations used.

2.1. Deviation and range restrictions

We denote by $\delta_{v,w}$ the length of a shortest path between $v \in P$ and $w \in P$ in $G'(P, R)$, whereby the abbreviation $\delta_q := \delta_{o_q,d_q}$ gives the length of the shortest path for demand $q \in Q$. Due to the deviation factor λ, the maximum length of a trip that covers a demand $q \in Q$ and therefore connects o_q with d_q amounts to $\delta_q^{\max} := \lambda \cdot \delta_q$. Moreover, we introduce the parameters $\delta^{\min}$ and $\delta^{\max}$ that restrict the *minimum* and *maximum allowed distance* between two consecutively visited stations. While $\delta^{\max}$ results from technical restrictions, $\delta^{\min}$ aims at reducing customer inconvenience as it avoids unwanted iterative stops within a short distance. While $\delta^{\min} = 0$ deactivates this restriction, setting $\delta^{\min}$ to strictly positive values does not rule out the possibility of opening two stations $v \in V$ and $w \in V$ within a distance of $0 < \delta_{v,w} < \delta^{\min}$. However, in this case, stations v and w can not be used consecutively on a feasible trip for any demand $q \in Q$. As it may happen that recharging is not possible at an origin o_q or at an destination d_q of a demand $q \in Q$, additional range thresholds $\delta_o^{\max} \leq \delta^{\max}$ and $\delta_d^{\max} \leq \delta^{\max}$ are imposed for the first and final charging stops on a trip.

In what follows, we consider only *non-trivial demands* $q \in Q$ that require at least one recharging stop due to $\delta_q > \min\{\delta_o^{\max}, \delta_d^{\max}\}$. For each $v \in V$, we introduce the set

$$\Delta(v) := \{w \in V \mid \delta^{\min} \leq \delta_{v,w} \leq \delta^{\max}\} \tag{1}$$

as the set of adjacent candidate sites. For $o \in O$ and $d \in D$, we define in the same manner

$$\Delta(o) := \{v \in V \mid \delta_{o,v} \leq \delta_o^{\max}\} \text{ and}$$
$$\Delta(d) := \{v \in V \mid \delta_{v,d} \leq \delta_d^{\max}\}. \tag{2}$$

2.2. Transformed network

Based on this notion of adjacency, we convert the network $G'(P, R)$ into the *transformed network* $G(N, E)$ that is considered in the following. This transformation is well known in the literature, see, e.g., MirHassani and Ebrazi (2013), Adler, Mirchandani, Xue, and Xia (2016), and Yıldız et al. (2016). The node set N of G is given by

$$N := V \cup O \cup D. \tag{3}$$

Please note that a node of the basic road network may be doubled or even tripled in the transformed network if it takes the roles of a candidate site, an origin and a destination at the same time. The edge set E of G is given by

$$\begin{aligned} E := &\{(v, w) \in V \times V \mid w \in \Delta(v)\} \\ &\cup\{(o, v) \in O \times V \mid v \in \Delta(o)\} \\ &\cup\{(v, d) \in V \times D \mid v \in \Delta(d)\}. \end{aligned} \tag{4}$$

Each edge $(u, w) \in E$ obtains the weight $\delta_{u,w}$ that coincides with the shortest distance between u and w in the basic road network G'. Therefore, any path in G corresponds to a walk (repetitive visits of nodes in P may occur) in G' that complies with the distance restrictions imposed by the values $\delta^{\min}$, $\delta^{\max}$, $\delta_o^{\max}$ for $o \in O$, and $\delta_d^{\max}$ for $d \in D$. In what follows, shortest path computations are mainly conducted in the transformed network $G(N, E)$. Therefore, we define:

Definition 1. For a transformed network $G(N, E)$, $S \subseteq V$, $u \in O \cup V$ and $w \in V \cup D$, we define $\delta_{u,w}(S)$ to be the length of the shortest path from u to w in $G(N, E)$ if only nodes $v \in S$ are allowed as intermediate stops. Let $\delta_{u,w}(S) := \infty$, if no such path exists. Furthermore, let $\delta_{u,u}(S) = \delta_{w,w}(S) := 0$.

Note that we have $\delta_{o,v}(S) = \delta_{o,v}$ for $v \in \Delta(o)$ and $\delta_{v,d}(S) = \delta_{v,d}$ for $v \in \Delta(d)$ for any choice of $d \in D$ and $o \in O$, respectively. This includes the case $S = \emptyset$. Moreover, it holds that $\delta_{o,v}(S) = \delta_{o,v}(S \setminus \{v\})$ and $\delta_{v,d}(S) = \delta_{v,d}(S \setminus \{v\})$ even for $v \in S$.

2.3. OD-covers

The set $S \subseteq V$ of stations to be opened unambiguously defines a solution of the considered location problem (Yıldız et al., 2016). However, in order to assess the quality of the solution, we need to identify the set of covered demands. For this purpose, the following definition now prepares a first possibility to state the coverage of a demand $q \in Q$.

Definition 2. The set $F \subseteq V$ is an **OD-cover** for demand $q \in Q$, if

$$\delta_{o_q,d_q}(F) \leq \delta_q^{\max}.$$

An OD-cover F for demand $q \in Q$ is denoted as minimal if and only if there is no $F' \subsetneq F$ that is an OD-cover for q. $\mathcal{F}(q)$ is the set of all OD-covers for q.

As we consider only non-trivial demands, we conclude that $\forall q \in Q : \emptyset \notin \mathcal{F}(q)$.

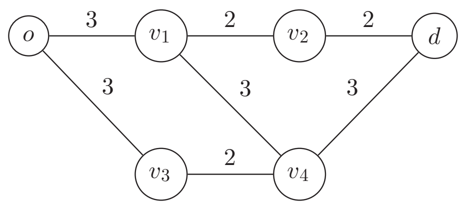

Fig. 1. Example network.

Definition 3. We denote a demand $q \in Q$ as **covered** by a set $S \subseteq V$ of stations if $S \in \mathcal{F}(q)$ holds, i.e., S is an OD-cover for $q \in Q$. We denote by

$$Q(S) := \{q \in Q \mid S \in \mathcal{F}(q)\}$$

the set of all demands covered by S.

Each combination of candidate sites which is calculated in advance in the studies of Kuby and Lim (2005) and Kim and Kuby (2012) can now be interpreted as a minimal OD-cover. However, Definition 3 also incorporates some notational flexibility as OD-covers are not necessarily minimal. The following observation investigates the structure of OD-covers:

Lemma 4. *For any non-trivial $q \in Q$ it holds that:*

$$F \in \mathcal{F}(q) \Leftrightarrow \exists v \in F : \delta_{o_q,v}(F) + \delta_{v,d_q}(F) \leq \delta_q^{\max}.$$

We denote $v \in F$ as a certificate for the OD-cover property of F.

Proof. "$\Rightarrow$": We assume that for $F \in \mathcal{F}(q)$ there is no such $v \in F$. Hence, it holds: $\forall v \in F : \delta_{o_q,v}(F) + \delta_{v,d_q}(F) > \delta_q^{\max} \geq \delta_{o_q,d_q}(F)$. But this implies $\emptyset \in \mathcal{F}(q)$ and we have a contradiction.

"$\Leftarrow$": As $v \in F$, it holds that $\delta_q^{\max} \geq \delta_{o_q,v}(F) + \delta_{v,d_q}(F) \geq \delta_{o_q,d_q}(F)$. □

The newly introduced concept of certifying nodes $v \in F$ will become an important aspect in Section 4.1 and can also be used to test whether the addition of a candidate site $v \notin F$ can make F to an OD-cover for $q \in Q$.

Lemma 5. *In a minimal OD-cover F, every $v \in F$ is a certificate for the OD-cover property of F.*

Proof. We assume that there exists a non-certifying $v \in F$. Hence, it holds that

$$\delta_{o_q,v}(F) + \delta_{v,d_q}(F) > \delta_q^{\max}.$$

As F is an OD-cover, there exists another node $v' \in F$, with $v' \neq v$ fulfilling $\delta_{o_q,v'}(F) + \delta_{v',d_q}(F) \leq \delta_q^{\max}$. Thus, we conclude that

$$\delta_{o_q,v'}(F \setminus \{v\}) + \delta_{v',d_q}(F \setminus \{v\}) \leq \delta_q^{\max}.$$

Consequently, $F \setminus \{v\}$ is an OD-cover. Since this contradicts the minimality of F, we can rule out the existence of v. □

The reverse of Lemma 5 does not hold, i.e., if we have an OD-cover F with all $v \in F$ certifying the OD-cover property of F, we cannot conclude that F is minimal.

Example 6. Consider a demand $q \in Q$ with origin o, destination d, a set of stations $V = \{v_1, v_2, v_3, v_4\}$, and vehicle ranges $\delta^{\max} = \delta_o^{\max} = \delta_d^{\max} = 3$. The transformed network with respect to q is given in Fig. 1. We assume $\delta_q = 7$ and set $\delta_q^{\max} = 8$. Then, $\{v_1, v_2\}$ as well as $\{v_3, v_4\}$ are minimal OD-covers for q. $\{v_1, v_4\}$ does not constitute an OD-cover, as the path $o - v_1 - v_4 - d$ with length 9 would not fulfill the deviation restriction. Note that set $V_{(o,d)} := \{v_1, v_2, v_3, v_4\}$ is a non-minimal OD-cover although every $v \in V_{(o,d)}$ certifies the OD-cover property of $V_{(o,d)}$.

In order to validate the OD-cover property for a station set F, Lemma 4 requires the finding of a certifying station $v \in F$. We therefore introduce for every demand $q \in Q$ the set $V_q \subseteq V$ as the set of stations that may certify the OD-cover property for q, i.e., we define

$$V_q := \{v \in V \mid \delta_{o_q,v}(V) + \delta_{v,d_q}(V) \leq \delta_q^{\max}\}. \quad (5)$$

Note that any demand $q \in Q$ with $V_q = \emptyset$ cannot be covered according to the imposed range and deviation restrictions by V nor any $S \subseteq V$. Otherwise, the (non-empty) set V_q is an OD-cover for $q \in Q$, and every OD-cover $F \in \mathcal{F}(q)$ contains an OD-cover $F' \subseteq F \cap V_q$. By sharpening a criterion originated in Yıldız et al. (2016), we define E_q as the set of all edges that are useful for the coverage of a demand $q \in Q$:

$$E_q := \{(v, w) \in E \mid \delta_{o_q,v}(V) + \delta_{v,w} + \delta_{w,d_q}(V) \leq \delta_q^{\max}\}. \quad (6)$$

The graph $G_q(V_q, E_q)$ is the transformed network of demand $q \in Q$. Note that the edges E_q of G_q are the path segments used in the Branch&Price approach of Yıldız et al. (2016).

In order to reduce the size of the sets V_q, and therefore of the transformed network G_q for all $q \in Q$, we apply the following rule:

Lemma 7. *A node $v \in V_q \subseteq V$ can be removed from V_q if at least one of the following two conditions is fulfilled:*

1. $v \in \Delta_{o_q} \wedge \{w \in V \mid (v, w) \in E_q\} \subseteq \Delta_{o_q}$
2. $v \in \Delta_{d_q} \wedge \{w \in V \mid (w, v) \in E_q\} \subseteq \Delta_{d_q}$

Proof. In both cases, given an arbitrary OD-cover $F \in \mathcal{F}(q)$ with $v \in F$, the set $F \setminus \{v\}$ still defines an OD-cover, as the first or last stop at station v is not necessary. So v can be removed from V_q. □

Note that if the maximum number of stations to be opened is limited by κ, we can neglect all demands $q \in Q$ with $\min_{F \in \mathcal{F}(q)} |F| > \kappa$ as well as all nodes $v \in V_q, q \in Q$, with $\min_{F \in \mathcal{F}(q), v \in F} |F| > \kappa$.

2.4. OD-cuts

In what follows, we prepare a new definition of demand coverage that allows for the generation of more efficient solution approaches. The basic idea stems from a dual point of view that is originated in the following Definition:

Definition 8. The set $C \subseteq V_q$ is an **OD-cut** for demand $q \in Q$, if

$$F \cap C \neq \emptyset \quad \forall F \in \mathcal{F}(q),$$

i.e., every OD-cover is intercepted by C. An OD-cut C is denoted as **minimal** (w. r. t. inclusion), if there is no set $C' \subsetneq C$, that is an OD-cut. The set of all OD-cuts for demand $q \in Q$ is denoted by $\mathcal{C}(q)$.

In order to fulfill the OD-cut property, it is obviously sufficient to have a non-empty intersection with every minimal OD-cover. The set V_q itself is an OD-cut for demand $q \in Q$ by Definition 8. Furthermore, we conclude that the sets $\Delta_{o_q} \cap V_q$ and $\Delta_{d_q} \cap V_q$ are minimal OD-cuts for $q \in Q$ after application of Lemma 7. Note that our novel definition of OD-cuts substantially generalizes the basic idea of Capar, Kuby, Leon, and Tsai (2013), which is restricted to a single path, to arbitrary deviation paths.

Example 9. In Example 6, $\{v_2, v_3\}$ is a minimal OD-cut for q as the only possible path via v_1 and v_4 is too long and, therefore, $\{v_1, v_4\}$ is not an OD-cover that has to be intersected by an OD-cut. Hence, further minimal OD-cuts are $\{v_1, v_3\}$, $\{v_1, v_4\}$ and $\{v_2, v_4\}$.

As the main result of this section, we state demand coverage by OD-cuts:

Theorem 10. *Given any set $V' \subseteq V$, demand $q \in Q$ is covered by V' if and only if $V' \cap C \neq \emptyset$ for all $C \in \mathcal{C}(q)$.*

Proof. "$\Rightarrow$": We have $V' \in \mathcal{F}(q)$. Therefore, for any $C \in \mathcal{C}(q)$ it holds $C \cap V' \neq \emptyset$ by Definition 8. For the "$\Leftarrow$" part it remains to be shown that the non-empty intersection with every OD-cut guarantees that V' is an OD-cover. We assume the opposite, i.e., we assume for $q \in Q$ that there does not exist an OD-cover $F \in \mathcal{F}(q)$ with $F \subseteq V'$. Then, the set $V_q \setminus V'$ is an OD-cut by Definition 8 and it holds $V' \cap (V_q \setminus V') = \emptyset$, which contradicts the assumed non-empty intersection of V' with every OD-cut. □

3. A new formulation for the Refueling Station Location Problem

At first, we formally define RSLP-Max on the basis of Definition 3:

Definition 11 (RSLP-Max)**.** A set of stations $S \subseteq V$ with $|S| \le \kappa$ is sought that maximizes the total weight of covered demands, i.e.,

$$\begin{aligned} \max \quad & \sum_{q \in Q(S)} w_q \\ \text{s. t.} \quad & |S| \le \kappa . \end{aligned}$$

Besides providing an IP-Model (see Section 3.1), Yıldız et al. (2016) prove that RSLP-Max is strongly $\mathcal{NP}$-hard. Different variants of RSLP-Max as RSLP-Min or a formulation with budget constraint can be found in the literature (see, e.g., Capar et al., 2013; MirHassani & Ebrazi, 2013).

3.1. The model of Yıldız et al. (2016) for RSLP-Max

Yıldız et al. (2016) provide the state of the art model for RSLP-Max. Therefore, in what follows, we restate this model by using the notation introduced in Section 2. The authors model the opening of a station $v \in V$ by a binary variable $x_v \in \{0, 1\}$, while the coverage of a demand $q \in Q$ is indicated by a binary variable $y_q \in \{0, 1\}$. The total weighted coverage to be maximized is defined by

$$\max \sum_{q \in Q} w_q \cdot y_q . \tag{7}$$

At most κ stations are allowed to be opened:

$$\sum_{v \in V} x_v \le \kappa . \tag{8}$$

In order to feasibly state the demands that are covered by the chosen set of opened stations, Yıldız et al. (2016) introduce a further set of variables $z^q_{v,w} \in \{0, 1\}$ for each edge $(v, w) \in E_q, q \in Q$. These are used to enforce the flow conditions of a feasible path leading from o_q to d_q in the transformed network if and only if $y_q = 1$ for all $q \in Q$:

$$\begin{aligned} & \sum_{v:(o_q,v) \in E_q} z^q_{o_q,v} = y_q \quad \forall q \in Q \\ & \sum_{w:(v,w) \in E_q} z^q_{v,w} = \sum_{w:(w,v) \in E_q} z^q_{w,v} \quad \forall v \in V_q, q \in Q \\ & \sum_{v:(v,d_q) \in E_q} z^q_{v,d_q} = y_q \quad \forall q \in Q \\ \sum_{(v,w) \in E_q} & \delta_{v,w} \cdot z^q_{v,w} \le \delta^{\max}_q \cdot y_q \quad \forall q \in Q \end{aligned} \tag{9}$$

Any flow on the edges also enforces an opening of the respective stations:

$$\sum_{w:(w,v) \in E_q} z^q_{w,v} \le x_v \quad \forall v \in V_q, q \in Q \tag{10}$$

Although the number of constraints and variables of the model is polynomially bounded, it may become significantly high. This results from using the sets V_q for all demands $q \in Q$ in the constraints (9) and (10) in combination with the triple indexed variables $z^q_{v,w}$. Hence, Yıldız et al. (2016) apply a Branch&Price approach that prices out the $z^q_{v,w}$ variables on demand.

Since the coverage of a demand $q \in Q$ and the opening of a station $v \in V$ are not directly linked in the definition but indirectly combined through the $z^q_{v,w}$-variables, one might be tempted to price out minimal OD-covers instead of path segments in order to get rid of the constraints given by (9). However, please note that this does not eliminate the primal degeneracy inherent in the model due to the constraint set (10).

3.2. RSLP-Max with OD-cuts

We propose the following novel mathematical definition of RSLP-Max. Herein, coverage of a demand is checked through OD-cuts (see Definition 8):

$$\max \sum_{q \in Q} w_q \cdot y_q \tag{11}$$

$$y_q \le \sum_{v \in C} x_v \quad \forall C \in \mathcal{C}(q), q \in Q. \tag{12}$$

$$\sum_{v \in V} x_v \le \kappa \tag{13}$$

$$x_v \in \{0, 1\}, v \in V \text{ and } 0 \le y_q \le 1, q \in Q \tag{14}$$

Theorem 12. *The optimization model given by (11)–(14) is an IP-Formulation that is equivalent to RSLP-Max.*

Proof. The objective function (11) and the limitation of opened stations (Restriction 13) are also part of the model of Yıldız et al. (2016). Moreover, the coverage of a demand $q \in Q$ with $y_q = 1$ is forced by the constraints (12). This results from Theorem 10 that states that the coverage of demand $q \in Q$ is established by the set of opened stations $V' = \{v \in V \mid x_v = 1\} \subseteq V$ if and only if V' has a non-empty intersection with every OD-cut $C \in \mathcal{C}(q)$. □

The new formulation of RSLP-Max contains only a variable for every candidate site $v \in V$ together with a variable for every demand $q \in Q$. As there is usually more than one OD-cut for every demand $q \in Q$, it is an extended version of the Flow Capturing Location Model (FCLM) of Hodgson (1990). The FCLM itself is a reinterpretation of the Maximal Covering Location Problem (MCLP) proposed by Church and Velle (1974). Note that the new formulation looks similar to the formulation of Capar et al. (2013). This is not surprising as OD-cuts generalize the concept of station sets proposed by Capar et al. (2013). In accordance with Church and Velle (1974), we invert the y_q-variables by introducing $\bar{y}_q := 1 - y_q$ in order to make the model more suitable for IP/LP solvers. The model then reads as follows:

$$\min \sum_{q \in Q} w_q \cdot \bar{y}_q \tag{15}$$

$$\sum_{v \in V} x_v \le \kappa \tag{16}$$

$$\sum_{v \in C} x_v + \bar{y}_q \ge 1 \quad \forall C \in \mathcal{C}(q), q \in Q. \tag{17}$$

$$x_v \in \{0, 1\}, v \in V \text{ and } \bar{y}_q \ge 0, q \in Q \tag{18}$$

Due to the non-zero right-hand side of the constraints (17) in the modified model, primal degeneracy may get reduced. Moreover, the $\bar{y}_q$-variables introduced no longer require upper bound restrictions. The modified objective function pursues the minimization of the total weight of uncovered demands. The LP-relaxation of this problem is then given by the replacement of (18) with

$$0 \le x_v \le 1, v \in V \text{ and } 0 \le \bar{y}_q \le 1, q \in Q. \tag{19}$$

Note that here the upper bounds on the demand variables are retained, as otherwise the introduction of further Integer Programming cutting planes may lead to violations of these upper bounds.

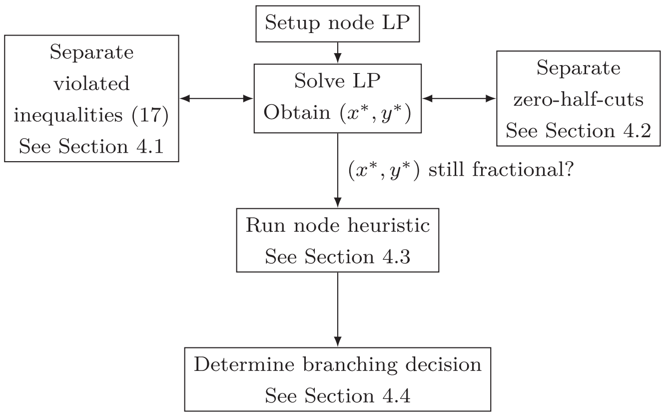

Fig. 2. The processing steps conducted by each node of the Branch&Cut approach.

4. The Branch&Cut approach

The new model for RSLP-Max possesses a moderate number of integer variables, but requires a significant number of constraints. In order to tackle such problems, Branch&Cut seems to be a suitable solution technique. It adds only violated inequalities to a row restricted model built for the LP-relaxation of the problem. The integrality of iteratively chosen decision variables is then ensured by conducting branching steps that generate the enumeration tree (see, e.g., Mitchell, 2009 for a general overview of Branch&Cut). In what follows, we introduce the problem-specific instruments required by our Branch&Cut approach (an overview of all steps conducted in each node of the enumeration process is given in Fig. 2): In Section 4.1 we describe the separation process for finding violated inequalities of type (12) or (17) for every $q \in Q$, i.e., violated inequalities that are not included in the current LP-relaxation. Due to Theorem 12, the determination of such a violated OD-cut inequality is sufficient to disprove the feasibility of a found solution of RSLP-Max. Hence, the integration of this restriction tightens the current LP-relaxation. Moreover, in order to exclude fractional solutions while keeping all feasible integer solutions, we combine inequalities of type (17) to obtain a well-known class of Integer Programming cutting planes, the so-called zero-half-cuts (Caprara & Fischetti, 1996). We describe this process in Section 4.2. However, if these cuts do not force the LP-relaxation to become integral, we apply a specific heuristic that attains an integral solution. This heuristic is illustrated in Section 4.3. Eventually, if the objective function value that is yielded by the optimal solution of the final LP-relaxation of the current node does not rule out the existence of an improved integral solution, the current node is branched. Subsequently, the enumeration is continued by choosing a new node from the priority list. We provide detailed information concerning these branching steps and the handling of the priority list in Section 4.4.

The subsequent Definition 13 categorizes stations and demands depending on the values of a considered solution of the LP-relaxation.

Definition 13. Given a fractional solution $(x^*, \bar{y}^*)$, we define the following sets of stations and demands:

1. $V_0^* := \{v \in V \mid x_v^* = 0\}$ (closed stations)
2. $V_1^* := \{v \in V \mid x_v^* = 1\}$ (opened stations)
3. $V_f^* := \{v \in V \mid 0 < x_v^* < 1\}$ (fractional stations)
4. $Q_0^* := \{q \in Q \mid \bar{y}_q^* = 0\}$ (covered demands)
5. $Q_1^* := \{q \in Q \mid \bar{y}_q^* = 1\}$ (lost demands)
6. $Q_f^* := \{q \in Q \mid 0 < \bar{y}_q^* < 1\}$ (partially covered demands)

Consequently, we denote a covered demand $q \in Q_0^*$ as *split covered*, if $V_1^* \notin \mathcal{F}(q)$.

Example 14. Consider the following setting with $Q = \{q_1, q_2\}$, $w_{q_1} = w_{q_2} = 1$ and a set of candidate sites $V = \{v_1, v_2, v_3\}$. The sets of OD-cuts for demands q_1 and q_2 are given by $\mathcal{C}(q_1) = \{\{v_1, v_2\}, \{v_1, v_3\}\}$ and $\mathcal{C}(q_2) = \{\{v_2, v_3\}\}$, respectively. In order to cover q_1, we have to open either v_1, or v_2 and v_3 simultaneously. Demand q_2 does not benefit from opening v_1. Therefore, we have to additionally open v_2 or v_3 to cover it. Thus, at least two stations are necessary to attain an objective value of 2. However, if we have only one station left for potentially covering these two demands, q_2 is split covered in order to allow a partial coverage of q_1. Specifically, by setting $x_{v_1}^* = 0$ and $x_{v_2}^* = x_{v_3}^* = 1/2$ as well as $y_{q_1} = 1/2$ and $y_{q_2} = 1$, a single open station attains an objective value of 3/2. Note that optimal solutions of the LP-relaxation for our model frequently possess combinations of split and partial coverages.

4.1. Separation of OD-cut inequalities

The finding of a violated inequality of type (12) or type (17) caused by a given solution (x^*, y^*) for a demand $q \in Q$ with $y_q^* > 0$ can be addressed by the following *violated OD-cut separation problem*:

$$y_q^* \overset{!}{>} \min \mu_q(x^*, y^*) = \sum_{v \in V_q} x_v^* \cdot s_v \tag{20}$$

$$\sum_{v \in F} s_v \geq 1 \quad \forall F \in \mathcal{F}(q) \tag{21}$$

$$s_v \in \{0, 1\} \quad \forall v \in V_q \tag{22}$$

The binary variables s_v indicate the set of stations $v \in V_q$ constituting the violated OD-cut. Constraint (21) enforces the intersection with each OD-cover $F \in \mathcal{F}(q)$ that is required by Definition 8. In order to identify a violated OD-cut, we pursue the finding of an OD-cut with minimal weight. Thus, the problem turns out to be a weighted Hitting Set Problem that is in general $\mathcal{NP}$-hard (Garey & Johnson, 1979).

While the above formulation contains a polynomial number of variables, it may possess an exponential number of constraints caused by a large number of OD-covers potentially existing for a considered demand. This problem is related to the length bounded min cut problem that is $\mathcal{NP}$-hard even in the fractional case (Baier, 2003). However, in our case, the cut is not defined on the edges E_q, but on the vertices V_q of the transformed network G_q.

In order to obtain a fast *separation algorithm*, we abstain from optimally solving the model defined by (20)–(22) in all cases since the finding of the most violated OD-cut inequality is not necessary, but rather the generation of a heuristic solution that violates Constraint (12) (or (17)) is sufficient. Alternatively, the process terminates if the existence of a violated OD-cut is disproved. In order to systematically check both possibilities, we start trying to disprove the existence of a violated OD-cut inequality (Section 4.1.1). If this is not possible, we try to construct such a violated OD-cut inequality by an assignment procedure (Section 4.1.2). However, if both attempts fail, a column generation heuristic is finally applied (Section 4.1.3). This process is sketched in Fig. 3. Furthermore, the handling of the separated inequalities in the current LP-relaxation is addressed in Section 4.1.4.

4.1.1. Disproving step

We start with a partitioning of V_q into two disjoint sets F_q^* and C_q^*. The set

$F_q^* := \{v \in V_q \mid x_v^* \geq y_q^*\}$

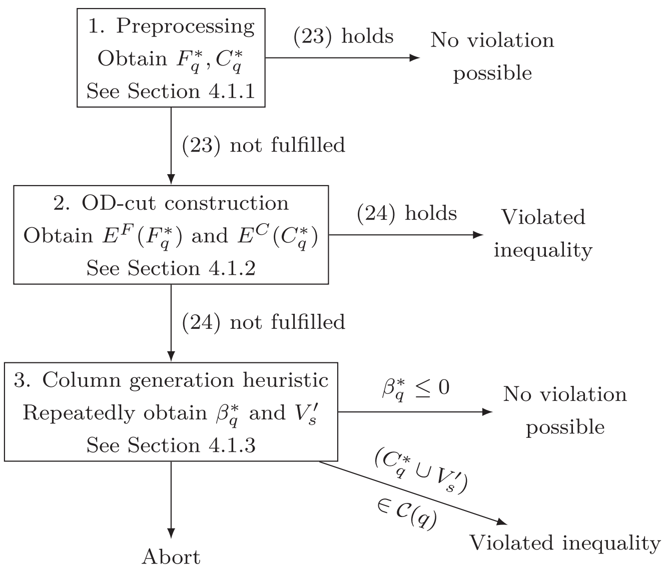

Fig. 3. Overview of the stages of the separation procedure and their possible outcomes.

is the set of all candidate stations that cannot be included in any violated OD-cut inequality. As one objective, we try to augment this set to an OD-cover, such that $F_q^* \in \mathcal{F}(q)$. Note that such an OD-cover would disprove the existence of a violated OD-cut inequality as at least one element of this OD-cover has to be in each OD-cut. Simultaneously, we try to build a violated OD-cut by constituting the set

$$C_q^* := V_q \cap V_0^*.$$

It contains all stations v that, due to $x_v^* = 0$, can be inserted into any violated OD-cut inequality. Given F_q^* and C_q^*, we introduce the set of unassigned stations through

$$N_q^* := V_q \backslash (F_q^* \cup C_q^*).$$

Hence, F_q^*, C_q^* and N_q^* build a partition of V_q. We iteratively try to extend the two sets C_q^* and F_q^* by applying the following two operations:

$$C_q^* \leftarrow C_q^* \cup \{v \in N_q^* \mid \delta_{o_q,v}(F_q^*) + \delta_{v,d_q}(F_q^*) \leq \delta_q^{\max}\}.$$

This first operation adds all stations $v \in N_q^*$ to C_q^* that, together with set F_q^*, build an OD-cover. Consequently, due to $C_q^* \cap F_q^* = \emptyset$, v has to be inserted into C_q^* in order to guarantee a non-empty intersection of C_q^* with each OD-cover. The second operation that augments the set F_q^* is given by

$$F_q^* \leftarrow F_q^* \cup \{v \in N_q^* \mid \sum_{w \in C_q^*} x_w^* + x_v^* \geq y_q^*\}.$$

This operation adds stations to set F_q^* that cannot be inserted into set C_q^* since this would abolish any possible existing violation of Constraint (12) (or (17)).

This process of extending the sets C_q^* and F_q^* is repeated until there is no station assignable or we obtain

$$F_q^* \in \mathcal{F}(q) \vee \sum_{v \in C_q^*} x_v^* \geq y_q^*. \tag{23}$$

If condition (23) is fulfilled, we disproved the existence of a violated OD-cut inequality for q.

4.1.2. OD-cut construction

Conversely, in what follows, we assume that $\sum_{v \in C_q^*} x_v^* < y_q^*$ holds and apply Algorithm 1 in order to continue the finding of a violated OD-cut inequality for demand $q \in Q$. Algorithm 1 runs in polynomial time and assigns all remaining stations $v \in N_q^*$ in an arbitrary sequence to either $E^C(C_q^*)$ or $E^F(F_q^*)$. As the aim of the algorithm is the construction of an OD-cut given by $C_q^* \cup E^C(C_q^*)$, v has to be added to $E^C(C_q^*)$ as soon as $F_q^* \cup E^F(F_q^*) \cup \{v\} \in \mathcal{F}(q)$ in order to provide a non-empty intersection with every OD-cover according to Definition 8. Otherwise, v can be safely added to $E^F(F_q^*)$, which allows us to keep the size of the resulting OD-cut as small as possible. These cognitions are formalized in Lemma 15.

Algorithm 1 OD-cut Construction

Input: N_q^*, F_q^*, C_q^* for $q \in Q$ with $F_q^* \notin \mathcal{F}(q)$
Output: Partition of N_q^* into $E^F(F_q^*)$ and $E^C(C_q^*)$,such that $C_q^* \cup E^C(C_q^*) \in \mathcal{C}(q)$ and $F_q^* \cup E^F(F_q^*) \notin \mathcal{F}(q)$.
$E^F(F_q^*) \leftarrow \emptyset$
$E^C(C_q^*) \leftarrow \emptyset$
while $N_q^* \neq \emptyset$ **do**
  Select an arbitrary $v \in N_q^*$
  $N_q^* \leftarrow N_q^* \backslash \{v\}$
  if $\delta_{o_q,v}(F_q^* \cup E^F(F_q^*)) + \delta_{v,d_q}(F_q^* \cup E^F(F_q^*)) \leq \delta_q^{\max}$ **then**
    $E^C(C_q^*) \leftarrow E^C(C_q^*) \cup \{v\}$
  else
    $E^F(F_q^*) \leftarrow E^F(F_q^*) \cup \{v\}$
  end if
end while

Lemma 15. *The set $C_q^* \cup E^C(C_q^*)$ generated by Algorithm 1 is an OD-cut for $q \in Q$.*

Proof. Assume that $C_q^* \cup E^C(C_q^*)$ is not an OD-cut for $q \in Q$. Then, the set $F' := V_q \backslash (C_q^* \cup E^C(C_q^*)) = F_q^* \cup E^F(F_q^*)$ is an OD-cover for $q \in Q$. We consider a minimal OD-cover $F'' \subseteq F'$. As $F_q^* \notin \mathcal{F}(q)$, we conclude that $F'' \cap E^F(F_q^*) \neq \emptyset$. Due to Lemma 5, we know that every station of set F'' and, therefore, of set $F'' \cap E^F(F_q^*)$ testifies the OD-cover property. Hence, this applies to all stations that were added to $E^F(F_q^*)$ during the application of Algorithm 1. We consider the lastly added station $v \in E^F(F_q^*)$. For this station, we have

$$\begin{aligned} &\delta_{o_q,v}((F_q^* \cup E^F(F_q^*)) \backslash \{v\}) + \delta_{v,d_q}((F_q^* \cup E^F(F_q^*)) \backslash \{v\}) \\ &\leq \delta_{o_q,v}(F'' \backslash \{v\}) + \delta_{v,d_q}(F'' \backslash \{v\}) \\ &= \delta_{o_q,v}(F'') + \delta_{v,d_q}(F'') \leq \delta_q^{\max}. \end{aligned}$$

Therefore, Algorithm 1 would have assigned station v to set $E^C(C_q^*)$ and not to $E^F(F_q^*)$. □

Moreover, by applying Algorithm 1 to a given set $C_q^* \subseteq V_q$, we are able to verify its OD-cut property if and only if $E^C(C_q^*) = \emptyset$:

Lemma 16.

$$C_q^* \in \mathcal{C}(q) \Leftrightarrow E^C(C_q^*) = \emptyset$$

Proof. "⇒": Consider the set $V_q \backslash C_q^*$. As $C_q^* \in \mathcal{C}(q)$, $V_q \backslash C_q^* \notin \mathcal{F}(q)$. Thus, it holds that for every $v \in V_q \backslash C_q^*$ and $E^F(F_q^*) \subseteq N_q^*$

$$\begin{aligned} &\delta_{o_q,v}(F_q^* \cup E^F(F_q^*)) + \delta_{v,d_q}(F_q^* \cup E^F(F_q^*)) \geq \\ &\quad \delta_{o_q,v}(V_q \backslash C_q^*) + \delta_{v,d_q}(V_q \backslash C_q^*) > \delta_q^{\max}. \end{aligned}$$

Therefore, Algorithm 1 assigns all stations $v \in V_q \backslash C_q^*$ to $E^F(F_q^*)$ and terminates with $E^C(C_q^*) = \emptyset$.

"⇐": Direct consequence of Lemma 15. □

In our implementation of Algorithm 1, the selection of $v \in N_q^*$ is done in non-decreasing order of $\delta_{o_q,v}(F_q^* \cup E^F(F_q^*))$ or $\delta_{v,d_q}(F_q^* \cup E^F(F_q^*))$. This enables us to embed the procedure into a slightly modified variant of Dijkstra's algorithm. As every node $v \in E^C(C_q^*)$

is necessary for providing the OD-cut property of $C_q^* \cup E^C(C_q^*)$, Algorithm 1 can also be applied for minimizing a given OD-cut $C \in \mathcal{C}(q)$ according to Definition 8. For this purpose, we set $C_q^* = \emptyset$ and $F_q^* = V_q \setminus C$, while the resulting set $E^C(\emptyset) \subseteq C$ provides a minimal OD-cut.

If for the OD-cut $C_q^* \cup E^C(C_q^*)$ returned by Algorithm 1 it holds that

$$\sum_{v \in C_q^* \cup E^C(C_q^*)} x_v^* < y_q^*, \tag{24}$$

we have found a violated OD-cut inequality and can stop the separation procedure for $q \in Q$. Otherwise, the existence of a violated OD-cut inequality cannot be ruled out and we employ a column generation heuristic as outlined in the following section:

4.1.3. Column generation heuristic

We consider the *dual version of the LP-relaxation of the violated OD-cut separation problem* (Formulas (20) to (22)) for the reduced set of OD-covers $\mathcal{F}'(q) = \{F \in \mathcal{F}(q) : F \cap C_q^* = \emptyset\}$, i.e., for the set of OD-covers that are not intercepted by C_q^*. The dual problem is defined as

$$\max \mu_q^D(x^*, y^*) := \sum_{F \in \mathcal{F}'(q)} \varphi_F \tag{25}$$

$$\sum_{F \in \mathcal{F}'(q) \wedge v \in F} \varphi_F \le x_v^* \quad \forall v \in N_q^* \quad (s_v) \tag{26}$$

$$\varphi_F \ge 0, F \in \mathcal{F}'(q). \tag{27}$$

The objective value $\mu_q^D(x^*, y^*)$ of each feasible solution of this problem is a lower bound of the optimal primal solution value $\mu_q(x^*, y^*)$. This provides us with the following upper bound value of the maximally attainable violation of any OD-cut inequality for demand $q \in Q$:

$$\beta_q^* := y_q^* - \sum_{v \in C_q^*} x_v^* - \mu_q^D(x^*, y^*) \tag{28}$$

Clearly, if we obtain $\beta_q^* \le 0$, the existence of a violated OD-cut inequality is disproved for $q \in Q$.

The initial column set $\mathcal{F}'' \subseteq \mathcal{F}'(q)$ for problem (25)–(27) is obtained during the execution of Algorithm 1, as for every $v \in E^C(C_q^*)$, it holds that $F_q^* \cup E^F(F_q^*) \cup \{v\} \in \mathcal{F}'(q)$ for $q \in Q$. If optimally solving the problem with respect to $\mathcal{F}''$ returns $\beta_q^* > 0$, we investigate the found dual solution s_v^*, which is a not necessarily feasible solution of the LP-relaxation of the *violated OD-cut separation problem* (20)–(22). We sort the set of candidate stations $V_s = \{v \in N_q^* : s_v^* > 0\}$ in sequence of non-increasing $e_v^* := s_v^*/x_v^*$-values. Note that this efficiency measure is well-defined as we initially inserted all nodes $v \in V_q$ with $x_v^* = 0$ into the set C_q^*. The subsequent OD-cut candidate $C_q^* \cup V_s'$ is generated by inserting the stations of set V_s in sequence of non-increasing e_v-values into V_s' as long as $\sum_{v \in C_q^*} x_v^* + \sum_{v \in V_s'} x_v^* < y_q^*$ holds.

Hence, if $C_q^* \cup V_s'$ is an OD-cut (checked by applying Algorithm 1), a violated OD-cut inequality is found and the heuristic terminates. Otherwise, we have $E^C(C_q^* \cup V_s') \ne \emptyset$ due to Lemma 16. Hence, for each $v \in E^C(C_q^* \cup V_s')$, the set $F_q^* \cup E^F(F_q^*) \cup \{v\}$ defines again an OD-cover and, therefore, may provide a new column with positive reduced cost. Please note that we do not perform an explicit pricing here. The column generation heuristic is stopped if no column with positive reduced cost can be constructed from $E^C(C_q^* \cup V_s')$ or if a predetermined limit of iterations in our simplex implementation with product form inverse is exceeded. Otherwise, we are able to re-optimize the problem and obtain a hopefully smaller value of β_q^* and a new dual solution $s_v^*, v \in N_q^*$, which allows us to repeat the steps from above.

4.1.4. OD-cut pool

In order to speed up the separation process of violated OD-cut inequalities, we keep all obtained OD-cuts in a pool. This pool is seeded with the OD-cuts $\Delta_{o_q} \cap V_q$ and $\Delta_{d_q} \cap V_q$ for $q \in Q$. Only if this pool provides less then $\pi^{c,\min}$ violated OD-cut inequalities, do we perform the calculations in Sections 4.1.1 to 4.1.3 for those demands $q \in Q$ with $y_q^* > 0$, which do not have a corresponding violated OD-cut inequality in the pool. The violated inequalities that are actually added to the LP-relaxation in one *separation round* are given in sequence of non-increasing value

$$\gamma^*(C) := (y_q^* - \sum_{v \in C} x_v^*) \cdot w_q / (|C| + 1) \tag{29}$$

for $C \in \mathcal{C}(q), q \in Q$ until the cut limit $\pi^{c,\max}$ is reached. This measure prefers most violated OD-cut inequalities of highly weighted demands that yield a sparse coefficient matrix. However, if less than $\pi^{c,\min}$ violated OD-cut inequalities are still found during the separation round, we continue the separation process with the possible construction of zero-half-cuts which are described in the following section:

4.2. Zero-half-cuts

In order to tighten the LP-relaxation, we combine OD-cut inequalities of type (17) together with the lower bound constraints $-x_v \le 0, v \in V$ and $-\bar{y}_q \le 0, q \in Q$. The aim is to separate so-called zero-half-cuts (Caprara & Fischetti, 1996) that constitute a subclass of the well-known Gomory-Chvatal-Cuts. It is worth mentioning that there are various further Integer Programming cutting planes besides zero-half-cuts available that may be useful to tighten the LP-relaxation (see Dey & Molinaro, 2018 for a recent survey). However, our choice of zero-half-cuts was motivated by the following facts: First, the respective algorithm turns out to be numerically stable as the current values of the solution $(x^*, \bar{y}^*)$ are only used to determine a set of relevant OD-cut inequalities. Second, we are able to obtain a considerable number of violated inequalities during one round. Third, from a theoretical point of view, the convex hull of all integral solutions can be obtained under very mild conditions by an exclusive (possibly iterative) application of zero-half-cuts (see Gentile, Ventura, & Weismantel, 2006).

Definition 17. Given a fractional solution $(x^*, \bar{y}^*)$ of the problem defined by (15)–(18), we define

$$\mathcal{C}^* := \bigcup_{q \in Q} \Big\{ -\sum_{v \in C} x_v - \bar{y}_q \le -1 \mid C \in \mathcal{C}(q) \wedge C \cap V_f^* \ne \emptyset \wedge \sum_{v \in C} x_v^* + \bar{y}_q^* = 1 \Big\},$$

as the set of all OD-cut inequalities of type (17) written in "≤-form" without slack that belong to the partially or split covered demands. Furthermore, the set of lower bound constraints without slack is given by

$$\mathcal{L}^* := \{-x_v \le 0 : v \in V_0^*\} \cup \{-\bar{y}_q \le 0 : q \in Q_0^*\}.$$

Note that for every constraint in $\mathcal{C}^*$, there is no variable representing a station $v \in V_1^*$ or a demand $q \in Q_1^*$ included. The pool of OD-cuts (see Section 4.1.4) provides a subset of $\mathcal{C}^*$, namely, OD-cut inequalities, that are now fulfilled without slack, but were usually violated in a former separation round.

In our implementation, we scan the pool of OD-cuts (see Section 4.1.4) in order to obtain a subset of $\mathcal{C}^*$, as it stores all OD-cuts whose corresponding OD-cut inequalities were added to the LP-relaxation due to a former violation.

Definition 18. We consider a set of inequalities $\mathcal{A}$ in "≤-form" of the problem defined by (15)–(18). Then, adding the aggregation $\sum(\mathcal{A})$, i.e., the sum of the inequalities in $\mathcal{A}$, leads to an equivalent problem. For $v \in V$, let $\xi_v(\mathcal{A}) \in \mathbb{Z}$ be the coefficient of x_v in

$\sum(\mathcal{A})$ and for $q \in Q$, let $\upsilon_q(\mathcal{A}) \in \mathbb{Z}$ be the coefficient of $\bar{y}_q$ in $\sum(\mathcal{A})$. By $\rho(\mathcal{A}) \in \mathbb{Z}$ we denote the right hand side of $\sum(\mathcal{A})$. Furthermore, the sets

$V(\mathcal{A}) := \{v \in V : \xi_v(\mathcal{A}) \neq 0\}$ and

$Q(\mathcal{A}) := \{q \in Q : \upsilon_q(\mathcal{A}) \neq 0\}$

are denoted as the station and demand support of $\mathcal{A}$, respectively.

Lemma 19. *Consider the aggregation $\sum(\mathcal{Z})$ for $\mathcal{Z} \subseteq \mathcal{C}^* \cup \mathcal{L}^*$ that fulfills the following conditions:*

1. $\rho(\mathcal{Z}) \equiv 1 \mod 2$
2. $\xi_v(\mathcal{Z}) \equiv 0 \mod 2 \quad \forall v \in V(\mathcal{Z})$
3. $\upsilon_q(\mathcal{Z}) \equiv 0 \mod 2 \quad \forall q \in Q(\mathcal{Z})$

Then, the inequality

$$\sum_{v \in V(\mathcal{Z})} \frac{\xi_v(\mathcal{Z})}{2} \cdot x_v + \sum_{q \in Q(\mathcal{Z})} \frac{\upsilon_q(\mathcal{Z})}{2} \cdot \bar{y}_q \leq \left\lfloor \frac{\rho(\mathcal{Z})}{2} \right\rfloor, \tag{30}$$

*is a **zero-half-cut** that is fulfilled by all feasible integral solutions and **maximally violated** (by 1/2).*

Proof. If x_v and $\bar{y}_q$ are integrals, the left-hand side becomes integral. Due to $\xi_v(\mathcal{Z}) \equiv \upsilon_q(\mathcal{Z}) \equiv 0 \mod 2$, the left-hand side stays integral after being divided by 2. Hence, the right-hand side, that is also divided by 2, stays larger or equal even after being rounded down. However, for fractional solutions, the fulfillment without slack of all considered constraints in $\mathcal{Z}$ leads to the maximum attainable violation of 1/2 due to the rounding operation on the right-hand side (for the derivation and maximum violation of zero-half-cuts see Caprara & Fischetti, 1996 and Caprara, Fischetti, & Letchford, 2000). □

Note that, due to the lacking slack in the inequalities of set $\mathcal{Z}$, the fractional solution $(x^*, \bar{y}^*)$ used for the determination of $\mathcal{C}^*$ and $\mathcal{L}^*$ violates the added inequality. Therefore, the zero-half-cut tightens the model.

Example 20 (cont. Example 14)**.** Setting $\mathcal{Z} = \mathcal{C}^* \cup \{-\bar{y}_{q_2} \leq 0\}$ for the fractional solution $x^*_{v_1} = 0, x^*_{v_2} = x^*_{v_3} = 1/2, \bar{y}_{q_1} = 1 - y_{q_1} = 1/2$, and $\bar{y}_{q_2} = 1 - y_{q_2} = 0$, we obtain for $\sum(\mathcal{Z})$:

$$-2 \cdot x_{v_1} - 2 \cdot x_{v_2} - 2 \cdot x_{v_3} - 2 \cdot \bar{y}_{q_1} - 2 \cdot \bar{y}_{q_2} \leq -3$$

The zero-half-cut according to (30) is given by

$$-x_{v_1} - x_{v_2} - x_{v_3} - \bar{y}_{q_1} - \bar{y}_{q_2} \leq -2,$$

i.e., to satisfy both demands, at least two stations have to be opened, and the fractional solution $(x^*, \bar{y}^*)$ violates the constraint by 1/2.

For our problem, the cuts are quite dense as all variables appearing in at least one constraint of $\mathcal{Z}$ possess a non-zero coefficient in the resulting zero-half-cut.

If existing, a set $\mathcal{Z} \subseteq \mathcal{C}^* \cup \mathcal{L}^*$ of constraints leading to a maximally violated zero-half-cut can be determined by the solution of an equation system over the field $GF(2)$ in polynomial time (Caprara et al., 2000). Herein, every OD-cut inequality in $\mathcal{C}^*$ is associated with a variable indicating its choice for $\mathcal{Z}$. The restrictions result from the conditions 1–3 of Lemma 19, which have only to be fulfilled for variables $x_v, v \in V^*_f$ and $\bar{y}_q, q \in Q^*_f$, respectively. Subsequently, the further variables $x_v, v \in V^*_0$ and $\bar{y}_q, q \in Q^*_0$ are addressed by including the lower bound inequalities in $\mathcal{L}^*$ in a post-processing phase. We refer to Reinelt and Wenger (2006) for more details. Note that the general version of the zero-half-cut separation problem that is not focused on maximally violated zero-half-cuts is strongly $\mathcal{NP}$-hard (Caprara & Fischetti, 1996).

If the equation system for the determination of maximally violated zero-half-cuts is solvable, we usually obtain a high number of valid zero-half-cuts due to the free variables (that correspond to certain OD-cut inequalities (17)) in the equation system. Therefore, in this case, after solving the equation system and subsequently fixing all free variables to zero, we obtain a first zero-half-cut. Subsequently, additional zero-half-cuts are generated by setting each free variable to one (i.e., inserting the corresponding OD-cut inequality (17) into the zero-half-cut) while keeping the others at zero and updating the solution of the equation system (cf. Reinelt & Wenger, 2006). The density of the resulting zero-half-cuts forces us to carefully select those cuts that should enter the current LP-relaxation, as otherwise increasing LP solution times might vitiate the positive effect of a sharper bound attained by zero-half-cuts (Andreello, Caprara, & Fischetti, 2007). For this purpose, we identify for each zero-half-cut $\mathcal{Z}$ the minimum weight of an included demand by $q(\mathcal{Z}) := \arg\min_{q \in Q(\mathcal{Z})} w_q$, while for two zero-half-cuts $\mathcal{Z}$ and $\mathcal{Z}'$ with $q(\mathcal{Z}') = q(\mathcal{Z})$, we delete the one with a larger number of non-zero coefficients (ties are broken arbitrarily). Finally, the remaining zero-half-cuts are iteratively added to the LP-relaxation in sequence of non-increasing $w_{q(\mathcal{Z})}$-values if the demands occurring in this cut do not occur in all previously added cuts. Note that this zero-half-cut filtering pursues finding a larger variety of higher weighted demands that are violated by the current fractional solution. Moreover, the focus on limiting the number of non-zero coefficients improves the numerical solvability of the resulting LP-relaxation.

Furthermore, in order to additionally enable the insertion of demands $q \in Q$ for which the current solution $(x^*, \bar{y}^*)$ provides only a single OD-cut inequality without slack (17), we explore combinations with a single domain constraint with slack (18) for all $q \in Q^*_f$. Specifically, if it holds that $\bar{y}^*_q < 0.5$, we try to generate a zero-half-cut $\mathcal{Z}$ with $\mathcal{Z} \subseteq \mathcal{C}^* \cup \mathcal{L}^* \cup \{-\bar{y}_q \leq 0\}$ that fulfills the conditions 1–3 of Lemma 19. In the opposite case, i.e., if we have $\bar{y}^*_q \geq 0.5$, a zero-half-cut $\mathcal{Z}$ is sought with $\mathcal{Z} \subseteq \mathcal{C}^* \cup \mathcal{L}^* \cup \{\bar{y}_q \leq 1\}$. Note that in this second case, the opposite signs of the coefficients remove variable $\bar{y}_q$ from the zero-half-cut. Furthermore, in both cases, due to a maximum slack of 0.5, a violation of at least 0.25 occurs for the solution $(x^*, \bar{y}^*)$ if a zero-half-cut can be generated by solving the corresponding equation system.

4.3. A primal greedy Heuristic

In order to obtain lower bounds based on a feasible integral solution of RSLP-Max, we apply a greedy heuristic denoted as Algorithm 2. Note that a solution $S \subseteq V$ is uniquely defined by the

Algorithm 2 Greedy algorithm.

Input: $V^*_1 \subset V$ as opened stations, $V' \subseteq V^*_f \cup V^*_0$ as the candidate set
Output: A solution $S \subseteq V' \cup V^*_1$
  $S \leftarrow V^*_1$
  while $|S| < \kappa$ **do**
    $w = \arg\max_{v \in V' \setminus S} \theta_v(S)$
    $S \leftarrow S \cup \{w\}$
  end while

set of opened stations, as the set of covered demands is then given by the set $Q(S)$ (see Definition 3). Algorithm 2 iteratively complements the set V^*_1 of stations opened by the fractional LP-Solution $(x^*, \bar{y}^*)$ by adding further stations out of set V' until κ stations are opened. For this purpose, stations $v \in V'$ are selected in sequence of non-increasing $\theta_v(S)$-values that depend on the current solution S. Note that we remove candidate stations from V that are already fixed to zero in the current node of the Branch&Cut tree. Such a zero-fixation may either result from a branching decision or is caused by a reduced cost fixing at the current node.

Our heuristic slightly extends the greedy approach by Lim and Kuby (2010) and Kim and Kuby (2013) who propose to define $\theta_v(S)$ for $v \in V \setminus S$ as the attained increase of the objective function value if station v is opened, i.e., added to the current set S (cf. Definitions 1 and 3):

$$\theta_v(S) := \sum_{q \in Q_v(S)} w_q, \tag{31}$$

where $Q_v(S)$ is given as

$$Q_v(S) := \{q \in Q \backslash Q(S) : \delta_{o_q,v}(S) + \delta_{v,d_q}(S) \leq \delta_q^{\max}\} \tag{32}$$

However, taking only the immediate improvement into account seems to be inappropriate in cases where more than one station is needed for covering a demand. Those demands $q \in Q$ are only considered by the given measure if the respective stations in V_q happen to have been opened in previous iterations due to their contributions to the coverage of other demands. In order to reduce this shortcoming, we extend the measure $\theta_v(S)$ by including a discount factor $\mu_{v,q}(S)$. For the current set S, $\mu_{v,q}(S)$ estimates the number of additionally needed stations (including v) for covering demand $q \in Q \backslash Q(S)$. It holds that

$$\mu_{v,q}(S) := \begin{cases} 1 & \Leftrightarrow q \in Q_v(S) \\ \mu'_{v,q}(S) > 1 & \text{otherwise.} \end{cases} \tag{33}$$

In this calculation, for current set S, $\mu'_{v,q}(S)$ is an estimate of the number of additional stations needed such that v can certify the existence of an OD-cover for q according to Definition 2. Using this estimate as a discount factor, we apply the following $\theta_v(S)$-measure:

$$\theta_v(S) := \sum_{q \in Q \backslash Q(S) \wedge v \in \{v' \in V_q | \mu_{v',q}(S) + |S| \leq \kappa\}} w_q / \mu_{v,q}(S). \tag{34}$$

4.4. Branching scheme

Since each solution is unambiguously defined by the set of opened stations (Yıldız et al., 2016), we conduct branching by fixing respective station variables x_v for $v \in V$. For a given fractional solution $(x^*, \bar{y}^*)$, a branching candidate is determined by

$$v' = \arg\max_{v \in V_f^*} \theta_v(V_1^*). \tag{35}$$

Herein, we define $\theta_v(V_1^*)$ as in Formula 34. Ties are broken arbitrarily except for the case when it holds that $\forall v \in V_f^* : \theta_v(V_1^*) = 0$. v' is then chosen from V_f^* by applying the non-cannibalizing selection criterion of Hodgson (1990). After choosing v', we create two child nodes with the additional constraints $x_{v'} = 0$ (down child) and $x_{v'} = 1$ (up child).

The choice of the next node of the Branch&Cut tree to be considered in the enumeration process is based on a combination of its bound and the *fractionality of the corresponding LP-Solution*. The fractionality of an LP-Solution $(x^*, \bar{y}^*)$ is determined by the ratio $\frac{|V_f^*|}{|V_f^*|+|V_1^*|}$. The lower bound z^{LB} at a given time results from the attained objective function value of the best feasible solution that is found during the elapsed enumeration process. We denote the largest upper bound over all nodes (i.e., currently, one of the most promising partial solutions) generated at a given time as z^{UB}, while $z^{UB}(v)$ gives the upper bound of node v. In order to continue the enumeration process, we select the node v' with lowest fractionality that fulfills $(z^{UB} - z^{UB}(v)) \leq \sigma^s \cdot (z^{UB} - z^{LB})$. The parameter $0 \leq \sigma^s \leq 1$ defines a threshold on the accepted deviation of the bound $z^{UB}(v)$ of the selected node v from the maximum bound with respect to the current optimality gap $z^{UB} - z^{LB}$.

However, instead of always selecting a new node from the tree, we continue the evaluation of the children nodes of the up child v', as long as $(z^{UB} - z^{UB}(v')) \leq \sigma^c \cdot (z^{UB} - z^{LB})$ is fulfilled. For this purpose, the parameter $0 \leq \sigma^c \leq 1$ defines a (different) threshold on the continuation of the branching at the up child. This allows us to speed up the enumeration process since the LP-relaxation is frequently easier to solve at a direct successor node.

Note that our implementation allows a multi-threaded exploration of the solution space that is based on an opportunistic selection of nodes to be branched.

5. Computational results

In what follows, we evaluate the efficiency of the proposed Branch&Cut algorithm by means of a computational study that is based on two real-world road networks. The first road network comprises motorways and national roads in Germany with $|P| = 5642$ and $|R| = 8017$. Every vertex of this network is either a motorway junction or a crossing point of two or more national roads. The set of considered demands for this network is given by a selection of the highest weighted county to county ($|O| = |D| = 430$) demands based on traffic data from the year 2010 (BMVI, 2014).

The second real-world road network is the representation of the Californian road network as given in Li, Cheng, Hadjieleftheriou, Kollios, and Teng (2005) with $|P| = 21{,}693$ and $|R| = 21{,}048$. For the demands, we use the 2010 Census Data (Bureau, 2010) in order to identify 208 urbanized areas and urban clusters used as origins and destinations. The weights of the demands are determined using the gravity model approach of Hodgson (1990). In all experiments, the parameter setting $\delta^{\min} = 0$, $\delta_o^{\max} = \delta_d^{\max} = \delta^{\max}/2$ is applied. It implements the so-called half tank assumption that models that people want to leave the origin and reach the destination of a demand with, at least, a half-full tank. In order to solely consider unrelated demands, we combine all demands q_1, $q_2 \in Q$ with $o_{q_1} = d_{q_2}$ and vice versa into a new demand q' with weight $w_{q'} = w_{q_1} + w_{q_2}$, as the basic road networks considered in the experiments are undirected and symmetric. Therefore, all listed results are based on unrelated demands.

As increasing vehicle ranges (i.e., larger values of $\delta^{\max}$) also augment the set of demands, that are considered as trivial during the solution process, we introduce the following notation:

$$Q_{\delta^{LB}}^{\delta^{UB}} := \{q \in Q \mid \delta^{LB} < \delta_q \leq \delta^{UB}\}. \tag{36}$$

The demand structure can be found in Table 3. It lists, for different range criteria with respect of $\delta^{\max}$, the corresponding number of demands $q \in Q$, whose values δ_q fulfill the given distance criteria. Additionally, the weighted market share is given in percent. As one can observe, the demand weights of the real data from the German network as well as of the gravity model for California have a strong bias towards short distance demands, which is additionally emphasized with an increasing value of $\delta^{\max}$. Please note that in the following, the set of considered demands always depends on the range setting of the corresponding instance. For the first set of tests, we employ all demands $q \in Q$ with $\delta_q > \delta^{\max}/2$, i.e., we consider the set $Q_{\delta^{\max}/2}^{\infty}$.

Our computational study extends existing analyses in the literature by considering a further dimension that determines a condensation degree of the set of stations $V \subseteq P$ in the basic road network that are being examined to be opened. We obtain these candidate sets by applying the following filtering procedure:

Definition 21. Given a distance δ_f and a parameter $0 \leq \varepsilon \leq 1$, we denote a station $v \in V$ as dominating compared to $w \in V$, whenever

1. $\delta_{v,w} \leq \delta_f$
2. $\sum_{\{q \in Q | v \in V_q\}} w_q > \sum_{\{q \in Q | w \in V_q\}} w_q$
3. $\sum_{\{q \in Q | w \in V_q \wedge v \notin V_q\}} w_q \leq \varepsilon \cdot \sum_{\{q \in Q | w \in V_q\}} w_q$

All dominated stations are excluded from V and we obtain $V' \subseteq V$ as the condensed set of stations. We determine the condensation

Table 3
Demand structure.

$\delta^{\max}$ (kilometres)	California					Germany				
	$\lvert Q^{\infty}_{\delta^{\max}/2}\rvert$	$\lvert Q^{\delta^{\max}}_{\delta^{\max}/2}\rvert$		$\lvert Q^{\infty}_{\delta^{\max}}\rvert$		$\lvert Q^{\infty}_{\delta^{\max}/2}\rvert$	$\lvert Q^{\delta^{\max}}_{\delta^{\max}/2}\rvert$		$\lvert Q^{\infty}_{\delta^{\max}}\rvert$	
75	19,490	493	(15%)	18,997	(85%)	15,290	139	(7%)	15,151	(93%)
100	19,362	855	(40%)	18,507	(60%)	15,290	993	(45%)	14,297	(55%)
125	19,124	1,183	(44%)	17,941	(56%)	15,290	2,081	(67%)	13,209	(33%)
150	18,997	1,715	(55%)	17,282	(45%)	15,151	3,171	(76%)	11,980	(24%)
175	18,747	2,191	(55%)	16,556	(45%)	14,745	4,101	(77%)	10,644	(23%)

Table 4
Instance properties with respect to different range and deviation settings.

$\delta^{\max}$	λ	V^{easy}			V^{medium}			V^{hard}		
		$\lvert V\rvert$	$\Sigma_q\lvert V_q\rvert$	Max%	$\lvert V\rvert$	$\Sigma_q\lvert V_q\rvert$	Max%	$\lvert V\rvert$	$\Sigma_q\lvert V_q\rvert$	Max%
Germany										
75 kilometres	1	727	$2.48 \cdot 10^4$	52.22	982	$6.51 \cdot 10^4$	73.07	1,318	$1.28 \cdot 10^5$	90.06
	1.05	774	$1.37 \cdot 10^5$	65.55	1,009	$2.78 \cdot 10^5$	82.50	1,338	$4.32 \cdot 10^5$	93.67
	1.1	785	$2.61 \cdot 10^5$	71.61	1,020	$4.53 \cdot 10^5$	86.78	1,375	$6.61 \cdot 10^5$	95.09
	1.2	803	$4.74 \cdot 10^5$	80.22	1,037	$7.36 \cdot 10^5$	91.16	1,393	$1.04 \cdot 10^6$	96.99
100 kilometres	1	784	$6.85 \cdot 10^4$	84.55	1,005	$1.07 \cdot 10^5$	93.53	1,331	$1.51 \cdot 10^5$	98.01
	1.05	791	$2.47 \cdot 10^5$	89.94	1,013	$3.41 \cdot 10^5$	96.25	1,348	$4.6 \cdot 10^5$	98.78
	1.1	796	$3.82 \cdot 10^5$	92.62	1,027	$5.12 \cdot 10^5$	97.42	1,378	$6.86 \cdot 10^5$	99.12
	1.2	814	$6.01 \cdot 10^5$	96.33	1,041	$7.95 \cdot 10^5$	98.27	1,392	$1.07 \cdot 10^6$	99.55
125 kilometres	1	788	$8.67 \cdot 10^4$	93.54	1,006	$1.15 \cdot 10^5$	97.69	1,332	$1.53 \cdot 10^5$	99.55
	1.05	794	$2.66 \cdot 10^5$	95.83	1,015	$3.49 \cdot 10^5$	98.94	1,348	$4.63 \cdot 10^5$	99.71
	1.1	803	$4 \cdot 10^5$	96.91	1,029	$5.2 \cdot 10^5$	99.51	1,378	$6.91 \cdot 10^5$	99.84
	1.2	815	$6.19 \cdot 10^5$	98.55	1,041	$8.03 \cdot 10^5$	99.75	1,392	$1.07 \cdot 10^6$	99.92
150 kilometres	1	789	$8.91 \cdot 10^4$	96.17	1,006	$1.16 \cdot 10^5$	99.03	1,332	$1.53 \cdot 10^5$	99.77
	1.05	794	$2.69 \cdot 10^5$	97.61	1,015	$3.5 \cdot 10^5$	99.41	1,349	$4.63 \cdot 10^5$	99.82
	1.1	803	$4.03 \cdot 10^5$	98.32	1,029	$5.22 \cdot 10^5$	99.77	1,379	$6.91 \cdot 10^5$	99.91
	1.2	816	$6.22 \cdot 10^5$	99.14	1,043	$8.04 \cdot 10^5$	99.90	1,394	$1.07 \cdot 10^6$	99.97
175 kilometres	1	789	$8.99 \cdot 10^4$	98.11	1,006	$1.15 \cdot 10^5$	99.44	1,332	$1.51 \cdot 10^5$	99.81
	1.05	794	$2.7 \cdot 10^5$	98.93	1,015	$3.49 \cdot 10^5$	99.74	1,349	$4.61 \cdot 10^5$	99.84
	1.1	804	$4.04 \cdot 10^5$	99.26	1,029	$5.2 \cdot 10^5$	99.89	1,379	$6.89 \cdot 10^5$	99.94
	1.2	816	$6.22 \cdot 10^5$	99.79	1,043	$8.02 \cdot 10^5$	99.95	1,394	$1.07 \cdot 10^6$	100.00
California										
75 kilometres	1	257	$1.31 \cdot 10^4$	29.68	377	$8.08 \cdot 10^4$	76.15	515	$2.13 \cdot 10^5$	88.84
	1.05	299	$2.36 \cdot 10^5$	56.35	427	$5.83 \cdot 10^5$	88.86	614	$9.19 \cdot 10^5$	89.99
	1.1	325	$5.31 \cdot 10^5$	79.66	476	$9.89 \cdot 10^5$	93.29	665	$1.46 \cdot 10^6$	90.33
	1.2	368	$9.6 \cdot 10^5$	94.07	497	$1.61 \cdot 10^6$	99.20	681	$2.3 \cdot 10^6$	94.47
100 kilometres	1	309	$1.14 \cdot 10^5$	90.05	387	$1.76 \cdot 10^5$	98.08	516	$2.32 \cdot 10^5$	98.73
	1.05	340	$5.77 \cdot 10^5$	93.06	443	$7.35 \cdot 10^5$	98.53	618	$9.49 \cdot 10^5$	98.89
	1.1	372	$8.79 \cdot 10^5$	98.57	482	$1.13 \cdot 10^6$	98.74	665	$1.49 \cdot 10^6$	98.97
	1.2	385	$1.33 \cdot 10^6$	99.27	497	$1.73 \cdot 10^6$	99.77	681	$2.31 \cdot 10^6$	99.11
125 kilometres	1	314	$1.33 \cdot 10^5$	91.81	390	$1.83 \cdot 10^5$	99.02	516	$2.33 \cdot 10^5$	99.82
	1.05	343	$5.98 \cdot 10^5$	93.50	442	$7.47 \cdot 10^5$	99.35	619	$9.5 \cdot 10^5$	99.95
	1.1	370	$8.99 \cdot 10^5$	98.83	482	$1.14 \cdot 10^6$	99.38	663	$1.49 \cdot 10^6$	100.00
	1.2	384	$1.35 \cdot 10^6$	99.97	496	$1.74 \cdot 10^6$	100.00	680	$2.31 \cdot 10^6$	100.00
150 kilometres	1	316	$1.4 \cdot 10^5$	93.05	390	$1.85 \cdot 10^5$	99.95	516	$2.35 \cdot 10^5$	100.00
	1.05	343	$6.04 \cdot 10^5$	94.12	444	$7.48 \cdot 10^5$	100.00	621	$9.5 \cdot 10^5$	100.00
	1.1	369	$9.05 \cdot 10^5$	99.55	482	$1.14 \cdot 10^6$	100.00	664	$1.49 \cdot 10^6$	100.00
	1.2	383	$1.36 \cdot 10^6$	100.00	496	$1.74 \cdot 10^6$	100.00	681	$2.32 \cdot 10^6$	100.00
175 kilometres	1	315	$1.43 \cdot 10^5$	99.44	390	$1.86 \cdot 10^5$	99.99	516	$2.34 \cdot 10^5$	100.00
	1.05	343	$6.06 \cdot 10^5$	99.98	444	$7.48 \cdot 10^5$	100.00	621	$9.49 \cdot 10^5$	100.00
	1.1	373	$9.06 \cdot 10^5$	100.00	486	$1.14 \cdot 10^6$	100.00	666	$1.49 \cdot 10^6$	100.00
	1.2	385	$1.36 \cdot 10^6$	100.00	497	$1.74 \cdot 10^6$	100.00	681	$2.32 \cdot 10^6$	100.00

degree by setting the parameters δ_f and ε: By being set to moderate values, the parameter δ_f ensures that a station may dominate only stations in its direct vicinity that are contributing less to the covering of demands. However, the parameter ε allows to keep those alternatives located nearby if these stations are assumed to be useful in satisfying different demands to a certain extent.

By applying a deviation factor of $\lambda = 1.0$, a vehicle range of 100 kilometres and a minimum distance of 100 kilometres for all considered demands, we create the following *three candidate sets* per network by individual degrees of condensation: The set V^{easy} results from applying $\varepsilon = 0.4$ and $\delta_f = 50$ kilometres, V^{medium} is obtained by setting $\varepsilon = 0.3$ and $\delta_f = 40$ kilometres, whereas V^{hard} possesses the parameter values $\varepsilon = 0.2$ and $\delta_f = 30$ kilometres. Since complexity is driven by network density, a more restrictive filtering reduces the resulting complexity.

Throughout the computational tests, we apply up to five vehicle ranges with $\delta^{\max} \in [75\,\text{kilometres}, 175\,\text{kilometres}]$ and up to four deviation factors $\lambda \in [1,\ 1.2]$. The characteristics of the instances with respect to their range and deviation settings as well as the choice of the candidate set are given in Table 4. The columns $|V|$ indicate the number of stations that enable the coverage of at least one demand in the considered candidate sets. The complexity impact of a larger allowed deviation λ becomes obvious by considering the value $\Sigma_{q \in Q}|V_q|$. Note that this value drives the number of constraints in the model of Yıldız et al. (2016) (defined by (9) and (10)). The maximum objective value that is attained by the opening

Table 5
Attained results for the model of Yıldız et al. (2016) on $Q^{\infty}_{\delta^{\max}/2}$ solved with Gurobi 8.1.

λ	κ	$\delta^{\max}=75$ kilometres	$\delta^{\max}=100$ kilometres	$\delta^{\max}=125$ kilometres	$\delta^{\max}=150$ kilometres	$\delta^{\max}=175$ kilometres
Germany						
1	5	3.51 seconds	6.02 seconds	6.06 seconds	5.29 seconds	6.67 seconds
	10	4.32 seconds	7.78 seconds	8.49 seconds	15.15 seconds	7.94 seconds
	20	1.96 seconds	5.66 seconds	9.93 seconds	14.55 seconds	17.91 seconds
	50	2.20 seconds	16.94 seconds	38.24 seconds	105.09 seconds	140.01 seconds
1.05	5	28.78 seconds	38.41 seconds	21.82 seconds	35.34 seconds	48.61 seconds
	10	44.13 seconds	180.48 seconds	78.49 seconds	111.21 seconds	80.42 seconds
	20	31.01 seconds	673.19 seconds	169.58 seconds	635.21 seconds	2,220.90 seconds
	50	53.82 seconds	23.19% – 23.3%	29.26% – 30.12%	34.1% – 34.41%	38.33% – 38.66%
1.1	5	275.83 seconds	225.97 seconds	48.04 seconds	84.63 seconds	202.17 seconds
	10	522.33 seconds	885.32 seconds	517.76 seconds	344.78 seconds	1,440.86 seconds
	20	87.24 seconds	8,551.11 seconds	17.76% – 18.21%	20.55% – 20.76%	23.77% – 24.18%
	50	2,757.60 seconds	25.31% – 26.93%	32.37% – 33.82%	37.85% – 38.82%	43.88% – 44.59%
California						
1	5	0.32 seconds	1.42 seconds	5.16 seconds	6.77 seconds	6.76 seconds
	10	0.79 seconds	7.31 seconds	14.33 seconds	14.61 seconds	13.12 seconds
	20	0.70 seconds	10.07 seconds	15.95 seconds	19.94 seconds	28.40 seconds
	50	0.60 seconds	11.12 seconds	24.51 seconds	32.36 seconds	51.10 seconds
1.05	5	1.37 seconds	6.13 seconds	19.21 seconds	36.00 seconds	145.48 seconds
	10	8.57 seconds	62.85 seconds	139.88 seconds	498.84 seconds	2,970.52 seconds
	20	39.87 seconds	8,828.36 seconds	5,186.80 seconds	72.01% – 72.22%	79.55% – 80.88%
	50	371.26 seconds	82.35% – 83.06%	87.36% – 87.71%	89.36% – 90.2%	0% – 147.51%
1.1	5	2.39 seconds	10.53 seconds	43.04 seconds	80.27 seconds	212.06 seconds
	10	16.99 seconds	5,699.80 seconds	758.08 seconds	1,228.05 seconds	0% – 145.43%
	20	433.84 seconds	68.92% – 70.01%	76.47% – 76.93%	0% – 154.47%	0% – 145.43%
	50	67.38% – 67.61%	0% – 125.07%	0% – 134.33%	0% – 154.47%	0% – 145.43%

of all stations is given in the "Max%"-column. Clearly, since more demands may be covered by an identical set of stations, this value does not decrease with an ascending deviation level.

All tests of the following analysis are conducted on Personal Computers operated under Linux and equipped with an Intel Core i7-5820 K CPU running with 3.3 gigahertz and 64 gigabytes working memory. The algorithms are coded in C++ and use up to six threads simultaneously. None of the running times given include any preprocessing steps that are necessary for the application of both approaches (e.g., the calculation of V_q, $\forall q \in Q$).

5.1. Results for the Model of Yıldız et al. (2016)

In order to validate the new Branch&Cut approach, we directly compare its performance with the solving of the model of Yıldız et al. (2016) that can be seen as the state of the art approach for RSLP-Max. As the model of Yıldız et al. (2016) extends the model of MirHassani and Ebrazi (2013), which does not allow any deviation (i.e. $\lambda = 1$) to the case with deviation (i.e. $\lambda > 1$), the following computational tests also provide a comparison with the model of MirHassani and Ebrazi (2013) for all instances with $\lambda = 1$. For this purpose, we implement the entire model definition (including all path segments) in Gurobi 8.1 and use the IP-Solver mode with default settings and a predetermined time limit of three hours, i.e., 10,800 seconds. Due to an unmanageable complexity for smaller condensation degrees (leading to the substantially larger station sets V^{medium} and V^{hard}), the model of Yıldız et al. (2016) is only solved with the smallest station set V^{easy} for both networks combined with three deviation settings ($\lambda \in \{1.0,\ 1.05,\ 1.1\}$) and four limits for the maximal number of stations to be opened κ. Therefore, this first test bed comprises altogether 120 instances. The attained results are presented in Table 5.

For all instances in Table 5, we give either the computation time needed to solve the instance to optimality, or the corresponding lower and upper bounds in percent of weighted total coverage of all demands in cases where the time limit of 10,800 seconds was exceeded. By analyzing the measured results in Table 5, it becomes obvious that, even with a limited set of candidate sites, a practical application of the approach of Yıldız et al. (2016) is strongly limited. For instance, by applying the larger deviation factor $\lambda := 1.1$, in some instances even the initial LP cannot be solved by the dual simplex within the given time limit. Hence, in these cases, a large upper bound is attained from the current dual solution whereas the lower bound results from the trivial solution that does not open any station at all.

5.2. Results for the Branch&Cut approach

In contrast to the application of the approach of Yıldız et al. (2016), the new Branch&Cut approach attains useful results during the predetermined time limit for all tested condensation degrees, i.e., for all resulting station sets. Hence, the second test bed is considerably extended by combining each of the three station sets with four limits for the maximum number of stations to be opened ($\kappa \in \{5,\ 10,\ 20,\ 50\}$) and four different deviation factors λ. This results in 480 instances. Analogous to the first test bed, a time limit of three hours is imposed while Gurobi 8.1 is applied for solving each LP-relaxation of the RSLP-Max formulation that is generated by the approach. Furthermore, the maximal number of violated OD-cuts added to the LP-relaxation in one separation round is limited to $\pi^{c,\max} = 200$. Zero-half-cuts are additionally separated in every node of the enumeration tree whenever a minimum number of $\pi^{c,\min} = 100$ violated OD-cut inequalities could not be generated in the current separation round. However, if the total number of zero-half-cuts and violated OD-cuts together still does not reach $\pi^{c,\min} = 100$ and the obtained solution is not integral, the search for additional violated inequalities is stopped. If the last solution is integral, feasibility has to be subsequently checked by continuing the separation process. Note that, in contrast to this, in the root node the separation process is stopped only if no violated inequality is found.

The enumeration process is performed by using up to six threads in parallel. Note that this is equivalent to the automatic setting of Gurobi applied to test the approach of Yıldız et al. (2016). During the enumeration process, the parameter setting $\sigma^s = 0.1$ and $\sigma^c = 0.5$ is applied (see Section 4.4).

Table 6
Results of the Branch&Cut approach attained for the road network of California.

$Q^{\infty}_{\delta^{max}/2}$		$\delta^{max}=7$ kilometres		$\delta^{max}=100$ kilometres		$\delta^{max}=125$ kilometres		$\delta^{max}=150$ kilometres		$\delta^{max}=175$ kilometres	
λ	κ	T(seconds)	C(%)	T(seconds)	C(%)	T(seconds)	C(%)	T(seconds)	C(%)	T(seconds)	C(%)
V^{easy}											
1	5	< 0.1†	9.37	< 0.1†	24.93	0.27	25.97	0.29†	30.03	0.42‡	30.30
	10	0.10	13.54	0.26†	39.63	0.44†	42.51	0.82	46.99	0.55†	49.69
	20	< 0.1	19.69	0.42†	56.12	0.52†	59.61	0.67	64.13	0.92	70.65
	50	< 0.1	26.37	0.55†	75.29	0.62‡	81.06	0.86	84.78	0.64‡	92.66
1.05	5	< 0.1	13.32	0.18†	25.36	0.48	27.85	0.48‡	31.79	0.81	35.95
	10	0.78	22.79	0.54†	42.07	0.77†	45.62	2.64	49.58	2.27	57.17
	20	0.66	31.08	6.21	60.69	5.22	66.19	7.49	72.08	12.89	80.12
	50	1.40	44.33	11.02	82.79	33.78	87.46	23.95	89.98	10.81	97.33
1.1	5	< 0.1	19.77	0.21†	30.33	0.64	33.17	0.47†	41.40	0.77†	41.55
	10	0.17†	34.05	0.75	47.02	1.22	51.33	1.69†	60.07	9.73	62.28
	20	0.94	49.31	31.75	69.14	2.79‡	76.56	33.45	82.92	90.49	85.90
	50	11.50	67.42	29.89	91.41	42.32	94.98	35.92	97.23	12.89	98.71
1.2	5	< 0.1†	22.82	0.22†	30.66	0.35†	38.50	0.59‡	43.44	1.32‡	49.02
	10	0.41	37.60	1.45	48.94	3.24	58.94	26.79	66.26	2.67‡	74.05
	20	0.83†	59.93	51.34	75.00	5.81	84.27	29.52	89.52	74.72	91.24
	50	1.76	84.99	2.60‡	94.82	23.97	97.87	18.34	99.06	13.30	99.56
V^{medium}											
1	5	< 0.1	19.96	0.24	26.24	0.29‡	36.38	0.41‡	39.83	0.66	36.57
	10	0.13†	32.25	0.37‡	42.49	0.62‡	49.48	0.71†	55.54	0.79	54.56
	20	0.31	46.45	0.59‡	60.89	2.51	65.21	3.44	70.25	2.73	71.86
	50	0.26‡	61.42	1.12	82.13	0.77‡	88.66	0.85‡	92.25	0.64‡	94.59
1.05	5	0.15	21.58	0.21†	28.28	0.33†	37.75	0.55‡	41.53	1.32	40.26
	10	0.65	35.14	0.60‡	44.38	1.66	53.13	1.77†	59.92	2.81‡	60.43
	20	0.56‡	51.98	15.93	64.79	1.67†	74.13	19.10	79.55	76.30	81.84
	50	6.24	71.50	16.15	88.62	18.65	94.00	22.93	96.36	35.44	97.65
1.1	5	0.21	21.72	0.26†	30.33	0.36†	37.96	0.55‡	43.44	1.69	44.65
	10	0.74	36.74	1.03	47.00	1.37†	55.88	10.47	63.43	22.75	66.96
	20	0.78†	55.60	44.08	69.14	28.50	79.74	54.48	85.06	180.14	87.40
	50	12.12	79.07	72.12	92.14	49.19	95.87	36.92	97.97	32.49	98.93
1.2	5	< 0.1†	26.09	0.26†	30.66	0.49†	38.95	0.69‡	47.08	1.48†	55.12
	10	0.40†	42.04	1.67	48.87	2.06‡	59.52	71.96	68.38	2.99†	78.10
	20	3.46‡	62.95	8.89‡	75.81	11.53	85.63	5.92	90.59	104.94	92.39
	50	29.94	88.74	24.11	95.46	128.57	98.24	24.81	99.23	75.49	99.61
V^{hard}											
1	5	0.21	17.04	0.31	26.08	0.30‡	32.19	0.39†	41.44	0.85	36.73
	10	0.49	26.88	0.49‡	42.05	0.63‡	50.01	0.69†	57.95	1.33	55.92
	20	0.47†	43.82	0.73‡	60.61	0.89†	67.14	15.83	71.30	2.83	73.35
	50	2.73	64.17	1.97	83.84	1.11‡	90.10	1.59‡	92.84	1.84	95.24
1.05	5	0.11‡	19.89	0.52	26.70	0.55	33.48	0.51†	42.87	0.83†	39.92
	10	1.58	31.27	0.70‡	44.29	1.02†	51.85	1.32‡	60.74	7.74	59.73
	20	1.01	48.83	4.12‡	66.23	3.04‡	74.76	72.70	78.95	103.59	81.78
	50	23.36	71.49	7.01	89.82	71.68	94.72	82.16	96.79	94.84	97.95
1.1	5	0.12‡	20.65	0.92	27.25	0.88	34.01	0.62†	45.60	2.38	45.03
	10	0.48‡	33.46	0.80†	46.47	1.63†	53.66	36.03	63.92	24.65	66.87
	20	2.56	51.57	244.20	69.39	42.75	77.35	377.46	83.44	409.86	87.41
	50	37.51	75.82	96.74	92.54	160.37	96.43	145.40	98.14	149.78	99.01
1.2	5	0.14†	23.77	0.33‡	30.82	0.44†	38.98	0.78†	47.76	4.20	50.61
	10	0.93	37.54	1.39†	52.13	5.54	59.67	941.14	67.57	54.11	76.79
	20	98.87	59.69	4.20†	78.56	40.49	84.66	153.65	89.92	528.16	92.76
	50	9.63	84.37	52.77	95.52	261.87	98.28	110.88	99.32	67.58	99.67

The results attained for both networks on 480 (in total) instances, which are depicted in Tables 6 and 7, clearly underline that the proposed Branch&Cut approach significantly outperforms the approach of Yıldız et al. (2016) in terms of computation times in seconds (column T(s)). This can mainly be ascribed to the modified problem definition and, as a consequence, to the insights that are gained from the introduction of OD-cuts. These improvements become particularly obvious by analyzing the results attained for the Californian network (see Table 6). Herein, the conducted decomposition defers the complex path finding task on this dense network to the faster working OD-cut separation process. As a consequence, all 240 instances are solved to optimality far below the predetermined time limit of 10,800 seconds. Not surprisingly, the optimal coverage (column C(%)) increases with ascending deviation levels λ as well as with an increasing number of placed stations κ. However, as the underlying demand set changes, the market share of covered demands can also decrease if range settings increase (see optimal values for $\delta^{max}=150$ and $\delta^{max}=175$ in the Californian network on candidate sets V^{medium} and V^{hard}). Note further that many instances can be optimally solved within the root node, which is indicated by a †, or a ‡ in the case that zero-half-cuts were separated and might have helped to find the integral solution.

The yielded results of the Branch&Cut approach for the German road network are given in Table 7. Although the computation times are higher in comparison to the measured results for the Californian network, the maximum computation time of 2035 seconds is still far below the time limit of 10,800 seconds. Note that this higher complexity mainly results from the fact that in the German road network, the sets of alternative stations are considerably

Table 7
Results of the Branch&Cut approach attained for the road network of Germany.

$Q^{\infty}_{\delta^{\max}/2}$		$\delta^{\max}=75$ kilometres		$\delta^{\max}=100$ kilometres		$\delta^{\max}=125$ kilometres		$\delta^{\max}=150$ kilometres		$\delta^{\max}=175$ kilometres	
λ	κ	T(seconds)	C(%)	T(seconds)	C(%)	T(seconds)	C(%)	T(seconds)	C(%)	T(seconds)	C(%)
V^{easy}											
1	5	8.90	1.14	0.89	2.50	0.32†	4.98	0.45†	5.44	0.39†	6.16
	10	0.40	2.59	0.85	4.93	0.57‡	8.05	0.87	8.80	0.66†	9.99
	20	0.39	5.41	0.42‡	9.69	1.02	13.29	0.96	14.43	1.69	16.15
	50	0.47	12.37	2.93	19.25	3.00	25.33	4.31	27.75	2.76	30.76
1.05	5	1.54	1.98	1.82	3.00	0.50†	5.82	0.61†	6.76	0.65‡	7.33
	10	0.88	4.03	2.41	6.02	1.15†	9.65	1.14‡	11.31	0.99†	12.91
	20	0.47	7.97	3.10	11.31	3.14	15.62	5.27	18.38	5.94	20.80
	50	1.88	16.30	12.93	23.21	19.64	29.68	11.22	34.17	18.98	38.34
1.1	5	9.59	1.98	1.95	3.79	0.61†	6.60	1.14‡	7.50	1.17‡	8.16
	10	1.95	4.27	4.16	7.25	4.27	10.70	1.56	12.46	3.13	14.26
	20	0.75‡	9.01	6.49	13.31	7.14	17.87	8.01	20.65	17.69	23.84
	50	6.40	18.29	28.83	26.34	24.72	33.21	31.14	38.23	30.75	44.13
1.2	5	4.35	2.40	1.39	4.95	0.96‡	8.11	1.31‡	8.87	2.79	10.20
	10	1.76	5.66	2.70	9.45	10.90	12.78	3.04‡	15.03	22.87	17.38
	20	7.26	10.98	14.23	16.42	12.47	21.47	10.49	25.07	24.47	29.24
	50	151.07	21.66	105.05	31.74	105.17	38.66	85.79	45.44	215.42	51.91
V^{medium}											
1	5	0.71	1.44	0.37‡	3.08	0.40†	5.01	0.44†	5.55	0.60	6.20
	10	1.48	2.89	1.48	5.55	1.14	8.24	1.11	9.33	0.64†	10.71
	20	1.22	5.88	1.45	10.34	0.80‡	13.88	1.82	15.37	2.26	16.92
	50	1.25	14.21	4.71	20.59	8.12	25.61	4.44	28.63	6.81	31.58
1.05	5	21.16	2.22	1.33	3.68	0.57†	5.82	0.64†	6.76	0.72‡	7.33
	10	2.34	4.32	3.24	6.63	1.58†	9.77	1.24‡	11.32	1.20‡	12.91
	20	3.49	8.45	3.18	12.20	3.99	15.94	4.14	18.67	7.26	21.01
	50	4.32	18.11	107.21	24.67	31.89	30.65	22.93	34.71	27.60	39.59
1.1	5	44.18	2.22	1.51	3.94	0.68†	6.60	1.10‡	7.62	1.01‡	8.24
	10	4.01	4.56	4.20	7.66	3.58	10.82	1.90	12.59	5.91	14.28
	20	3.51	9.50	5.65	14.08	6.21	18.34	12.18	20.90	37.31	23.92
	50	7.03	20.53	55.52	28.38	101.41	34.34	38.19	39.37	76.34	44.81
1.2	5	8.66	2.53	2.65	5.17	1.86	8.12	1.38‡	9.23	1.62	10.70
	10	4.01	5.80	4.46	9.70	11.55	12.95	5.15	15.48	35.50	17.73
	20	11.92	11.31	15.58	17.12	58.33	21.56	23.23	25.44	72.99	29.60
	50	127.80	23.80	230.08	33.09	264.95	39.37	156.58	46.46	180.26	53.59
V^{hard}											
1	5	4.74	1.65	1.42	2.82	0.46†	5.01	0.48†	5.55	0.88	6.20
	10	2.86	3.23	2.46	5.30	1.63	8.24	1.19	9.33	0.75†	10.71
	20	4.20	6.49	2.68	10.28	1.24‡	13.88	2.30	15.57	2.08	17.24
	50	8.25	15.22	20.50	21.27	16.14	26.11	11.13	29.07	7.87	32.10
1.05	5	3.76	2.39	8.02	3.50	0.59‡	5.95	0.58†	7.23	0.81‡	7.59
	10	4.39	4.32	4.95	6.69	1.86†	9.91	1.64‡	11.87	2.30	13.05
	20	4.95	8.68	5.51	12.34	6.66	16.05	7.17	19.29	6.97	21.83
	50	11.91	18.95	193.78	25.86	37.88	31.47	62.63	35.79	40.28	40.39
1.1	5	1.87	2.51	3.71	3.99	0.77†	6.76	0.90‡	8.06	1.17‡	8.42
	10	5.38	4.89	3.73	8.03	4.10	11.22	5.82	13.34	6.13	14.76
	20	4.09	10.08	9.28	14.41	12.23	18.55	7.68	21.98	25.71	25.57
	50	111.33	21.43	97.11	29.22	120.47	34.86	86.57	40.21	102.42	45.96
1.2	5	12.36	2.57	1.11‡	5.44	2.00	8.29	1.20‡	9.61	2.18‡	10.65
	10	10.83	5.91	4.87	10.04	14.21	13.32	8.33	15.99	43.76	17.89
	20	26.71	11.74	73.20	17.51	74.78	21.82	40.42	26.51	103.43	30.11
	50	1,474.19	24.72	2,034.87	33.80	1,008.68	39.68	301.95	47.20	688.96	53.82

larger, while the basic road network is smaller. Therefore, there is a greater computational burden on the LP-Solver that has to solve bigger LP-relaxations of the RSLP-Max formulation due to larger OD-cuts. Also the separation algorithms of Section 4.1 are increasingly challenged with a growing number of candidate stations in an OD-cut.

5.3. Long distance demands

Table 8 depicts the measured results of the Branch&Cut approach if we filter out shorter demands by imposing a minimum distance of $\delta^{\max}$. As detailed analyses reveal that the coverage of demands with a short distance is frequently possible by the opening of a single station, the greedy heuristic is able to provide useful lower bounds. This can primarily be attributed to the fact that the demands with up to 100 kilometres distance account for between 40% (California) and 45% (Germany) of all mapped demands (see Table 3). Thus, a reasonable preselection of candidates covering various short distance demands attains promising results. In contrast to this, the coverage of longer distance demands ($>\delta^{\max}$) increasingly requires the opening of, at least, two stations. Thus, filtering out shorter demands substantially rises the complexity of the instances to be solved.

As a consequence, by solely considering the demands in $Q^{\infty}_{\delta^{\max}}$, the computation times of the Branch&Cut approach are significantly increased, while a substantial variance can be observed. But, despite these deteriorations, the novel algorithm is still able to solve almost all instances to optimality within the time limit of 10,800 seconds. Since this also applies to the most complex instances with station set V^{hard}, the practicability of the proposed Branch&Cut approach is further underlined. While all 96 instances of the Californian network are optimally solved, the time limit of

Table 8
Results of the Branch&Cut approach attained on long distance demands.

$Q^{\infty}_{\delta^{max}}$		Germany				California			
λ	κ	$\delta^{max} = 100$ kilometres		$\delta^{max} = 150$ kilometres		$\delta^{max} = 100$ kilometres		$\delta^{max} = 150$ kilometres	
		T(seconds)	UB%	T(seconds)	UB%	T(seconds)	UB%	T(seconds)	UB%
V^{easy}									
1	5	0.38	3.26	10.46	2.42	0.11	17.96	0.36	22.48
	10	0.40†	5.86	24.17	4.48	0.36	31.25	0.59†	37.59
	20	1.65	9.65	12.05	8.95	0.53	50.90	0.60†	57.21
	50	2.23	19.32	11.76	20.86	0.48†	76.63	0.60‡	86.11
1.05	5	0.82	3.85	36.99	3.11	0.41	17.98	0.59†	29.47
	10	3.10	6.54	123.29	6.48	0.69‡	34.13	29.64	43.10
	20	5.33	11.61	68.61	13.56	32.80	55.19	26.91	72.52
	50	33.00	24.22	4,647.99	30.72	9.29	85.66	22.21	94.81
1.1	5	3.77	3.86	181.77	3.63	0.19†	21.93	0.64†	29.87
	10	17.75	6.72	65.97	8.62	0.68†	41.10	45.76	49.32
	20	12.79	12.42	255.35	17.68	32.67	62.12	64.48	78.95
	50	998.91	27.10	(37.62%)	38.31	47.32	90.81	200.71	97.13
1.2	5	49.61	4.23	462.77	4.90	0.22†	23.45	1.62†	30.80
	10	45.92	7.90	642.18	11.28	0.99†	45.08	42.09	61.32
	20	146.65	15.08	1,368.88	22.82	111.12	70.77	468.61	86.19
	50	636.00	32.95	(42.8%)	48.27	80.92	94.52	51.68	99.05
V^{medium}									
1	5	0.23†	3.69	20.66	2.42	0.20	17.96	0.40†	25.68
	10	24.35	6.16	20.69	4.59	0.38†	33.22	1.36	38.32
	20	5.45	10.03	18.63	8.95	1.63	52.01	0.97‡	61.73
	50	9.15	19.63	24.21	21.45	1.34	78.45	0.72‡	89.04
1.05	5	0.48†	4.28	84.23	3.13	0.36	18.96	0.82‡	28.87
	10	33.60	6.99	378.27	6.59	1.18	36.94	26.78	45.79
	20	7.60	12.22	297.97	13.93	16.78	57.86	63.64	73.69
	50	790.51	24.90	(31.29%)	31.97	44.58	87.06	91.13	95.43
1.1	5	1.08†	4.28	168.74	3.93	0.72	19.38	0.88†	30.41
	10	65.85	7.24	217.33	8.81	2.43	38.68	32.83	53.90
	20	39.71	13.20	497.59	18.29	83.48	62.92	241.75	80.76
	50	1,817.47	28.50	(36.82%)	39.83	178.91	91.80	74.47	97.74
1.2	5	19.31	4.37	1,047.11	5.06	1.03	21.06	15.36	31.04
	10	27.57	8.52	1,207.42	11.72	8.48	41.91	94.31	61.88
	20	222.88	15.97	(23.15%)	23.32	25.80	72.35	298.30	87.98
	50	(33.61%)	33.73	(42.75%)	49.77	168.80	95.05	204.36	99.19
V^{hard}									
1	5	0.32†	3.69	8.62	2.51	0.32	17.96	0.44†	26.89
	10	45.88	6.17	29.34	4.64	0.67	33.01	1.37	39.75
	20	27.91	10.04	37.42	9.24	3.44	52.16	2.37	62.57
	50	11.74	20.50	42.45	22.06	1.98	80.66	3.76	89.38
1.05	5	0.69†	4.28	95.89	3.33	0.43	19.11	0.68‡	29.80
	10	54.76	7.05	255.52	6.95	1.45	37.09	12.56	48.25
	20	35.57	12.45	111.78	14.83	20.21	58.68	73.49	73.96
	50	4,271.38	26.03	(32.12%)	32.99	60.41	88.50	136.26	95.86
1.1	5	1.43†	4.28	201.43	4.22	0.71	20.36	1.33†	30.41
	10	26.95	7.49	188.86	9.41	4.36	39.82	83.15	52.72
	20	46.14	13.81	1,537.37	18.72	38.21	64.77	625.08	80.31
	50	(29.89%)	29.90	(37.1%)	40.67	618.92	92.30	366.89	97.88
1.2	5	27.72	4.46	5,147.89	5.08	1.90	22.67	10.57	31.04
	10	104.39	8.76	4,335.57	11.97	32.85	44.31	119.19	62.30
	20	1,524.79	16.48	(24%)	24.12	72.41	75.46	1,419.20	88.42
	50	10,310.10	34.92	(41.56%)	51.04	525.10	95.99	1,794.46	99.31

10,800 seconds prevents the finding of an optimal solution or the proof of optimality in just 12 of 96 cases for the German network. In these cases, Table 8 reports in brackets the attained lower bound value.

As short distance demands may be mainly induced by commuters, their exclusion may be reasonable as it leads to practically relevant subproblems. Frequently, commuters can use an already existing charging infrastructure at their demand origin (home location) as well as at their demand destination (work location), wherefore the applied aforementioned half tank assumption may be too pessimistic. In contrast to this, a substantially larger proportion of the long distance demands might actually depend on the charging infrastructure erected by the stations to be opened. Hence, covering these demands by a sophisticated infrastructure may be the crucial part of a real-world application.

6. Conclusion and outlook

This paper contributes a new approach for the planning of a recharging and refueling infrastructure. By introducing OD-cuts, we propose a novel definition of the well-known Refueling Station Location Problem (RSLP) and generate a new corresponding Branch&Cut approach. As a consequence, the computational intractability that mainly results from a considerable increase of binary variables for instances of realistic size is significantly reduced. Specifically, by applying the proposed Branch&Cut approach, real-world instances of practical size with respect to $|V|$, $|Q|$, and κ are solved to optimality for the first time.

However, besides these promising results, more detailed analyses of instances with a dominating proportion of long distance demands also reveal that the attained competitive upper bounds cannot readily be converted into lower bounds of comparable quality. Instead, in order to find an improved incumbent solution, the enumeration process frequently has to evaluate numerous nodes before the greedy heuristic is successfully applied. Hence, future research shall be devoted to developing a more sophisticated heuristic. Note that respective state-of-the-art heuristics, which can be found in the literature (Kim & Kuby, 2013; Lim & Kuby, 2010), require a considerable number of solution evaluations. This is a prohibitive computational burden for instances with realistic values for $|V|$, $|Q|$, and κ.

Due to its significant practical relevance, the modeling of capacity restrictions at the recharging stations is another aspect that has gained increasing interest in the recent literature (see, e.g., Upchurch et al., 2009; Zhang, Kang, & Kwon, 2017). Since long waiting times for recharging would cause considerable customer inconvenience and, as a direct consequence, may even endanger the market adoption of electric cars, its modeling is essential. However, it is worth mentioning that this extension substantially changes the basic problem structure. Note that in the original RSLP model, the coverage of a demand depends only on the set of opened stations, but is not influenced by other demands. In the capacitated case, however, demands to be covered may compete for opened stations since their usage on necessary paths is limited. However, the development of algorithms that are based on sophisticated decompositions may, also in this case, be a promising field of future research.

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