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Licensing to a competitor and strategic royalty choice in a dynamic duopoly

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ABSTRACT

Network effects encourage original equipment manufacturers (OEMs) to expand their market size by changing their relationships with third-party manufacturers who provide compatible products from competition to coopetition. Moreover, network effects render consumer valuation inherently dynamic. Consumer perceptions of the sales quantity are continuously updated, causing the impact of the network effect to change dynamically over time. To this end, we examine technology licensing and price competition in a dynamic duopoly including an OEM and a third-party manufacturer, considering that consumer utility increases with the dynamic and evolving impact of network effects. However, because of limited product technology, the lower compatibility of third-party products reduces network effects. Thus, the third-party manufacturer licenses technology from the OEM. Because the OEM can strategically choose a static or dynamic royalty under technology licensing in a dynamic pricing game, we derive the firms' subgame-perfect Nash equilibrium decisions and profits and analyze the effects of licensing mechanisms and market factors on firms' instantaneous and steady-state equilibrium decisions and profits. Technology licensing enhances firms' profits when firms' and consumers' dynamic behaviors are more significant by exerting stronger network effects. A dynamic royalty is more effective for mitigating price competition intensity and for helping firms maintain higher sales margins. A static royalty induces a lower royalty chosen by the OEM and firms lower prices, which increases the impact of network effects and thus is generally more advantageous for the firms and for social welfare.

1. Introduction

In many markets, increases in the market sizes of certain products enhance communication among consumers and lead to a greater variety of complementary products offered, improving consumer demand for the products. These products are said to generate network effects (network externalities) that enhance the utility of a product for a consumer due to an increasing number of consumers who purchase similar products (de Palma, Leruth, & Regibeau, 1999). Many studies (e.g., Alexandrov, 2015; Chen, Doraszelski, & Harrington, 2009; Clements, 2004; Grajek, 2010; Katz & Shapiro, 1985; 1994; Malueg & Schwartz, 2006; Rasch, 2017; Wang, Chen, & Xie, 2010) have noted that product compatibility is a fundamental issue that influences network effects. For example, Grajek (2010) indicated that low compatibility reduces network effects based on empirical evidence from mobile markets. Based on empirical data from 45 product categories, Wang et al. (2010) verified that lower product compatibility has a negative impact on network effects. Moreover, network effects are important factors for OEMs (original equipment manufacturers) because they encourage the entrance of third-party manufacturers that sell substitutable and compatible products. Due to the limitations of the available technology, the strength of network effects on third-party products, as perceived by consumers, is often positively related to the product technology achieved by third-party manufacturers. This phenomenon is common in the consumer electronics and software industries. Using the lenses of digital reflex cameras as an example, third-party manufacturers (e.g., Sigma, Tamron, and Tokina) provide lenses that are compatible with those provided by an OEM (e.g., Canon, Fujifilm, Nikon, and Sony), and they compete in the market such that the availability of the third-party lenses improves consumer utility and subsequently increases the market demand of the OEM products. However, third-party lenses may be incompatible with OEM cameras without the use of the OEM's technology; for example, Sigma confessed that some of its lenses were not fully compatible with certain Nikon cameras (Thurston, 2013) and with Canon cameras (Burgett, 2016; Sigma Co., Ltd., 2016). Consequently, lower compatibility of third-party manufacturers reduces the network effects, which is

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not always beneficial to OEMs because the sales of OEM-mounted lenses can decrease. Under this circumstance, an OEM might choose to license its technology to third-party manufacturers to induce a greater network effect; for example, Sony either licensed its specifications to third-party manufacturers (Sony Co., Ltd., 2011) or decided not to license them to maintain its competitive advantage, and Canon did not license its mounted lens system to any third-party lens manufacturers (Canon U.S.A., Inc., 2015). In this study, we investigate technology licensing between an OEM and a third-party manufacturer with respect to the associated changes in the network effects and market characteristics.

Because the decisions of firms can influence network effects and because consumer perceptions of sales quantity are not stationary but are continuously updated based on their past experiences such that the impact of network effects evolves over time, customers become aware of the dynamic nature of these effects. In such a dynamic environment, OEMs can dynamically determine their licensing royalties. Under dynamic licensing mechanisms, “fluctuating royalties” that change during licensing periods with regard to variations in sales or the market environment are commonly applied (Attorney, 2011; Cunningham, 2002), for example, in the petroleum industry (Ciarlone, 2018; Kulander, 2016) and music industry (Lawrence, 2006). However, many firms may prefer to institute fixed licensing royalties rather than adjusting the royalties over time due to issues related to their profitability, possible adjustments in costs, and the willingness of licensees to accept variations in royalties. Therefore, it is crucial for firms to decide whether to adopt dynamic or static royalties in a dynamic environment (e.g., Ciarlone, 2018; Kulander, 2016). Moreover, the choice of the royalty scheme subsequently affects firms’ pricing decisions, which has a direct impact on the intensity of price competition and the network effects. Thus, it is important to examine firm preferences regarding technology licensing and the use of dynamic and static royalties. To the best of our knowledge, this issue has not been discussed in the literature.

We contribute to the research on technology licensing by presenting a differential game consisting of an OEM and a third-party manufacturer. Specifically, we demonstrate that firms compete on prices over time and that market demand is affected by dynamic network effects because consumers perceive that the network effects are continuously changing with respect to firms’ pricing decisions over time. In previous works, royalties are assumed to be either dynamic in dynamic models or static in static models. However, in this study, we analyze the impact of both dynamic and static royalties using two licensing models. We focus on firms’ SPNE (subgame-perfect Nash equilibrium) pricing decisions and licensing royalties and characterize firms’ strategic attitudes regarding technology licensing in the steady state. Specifically, we conduct analyses to obtain insights regarding (1) the time paths of the SPNE results, (2) steady-state equilibrium behaviors, (3) the performance of the different technology licensing options, and (4) firms’ preferences for royalty types.

A brief summary of the findings is as follows. (1) Regarding firms’ SPNE pricing and royalties, we find that the static royalty is less than the dynamic royalty, inducing firms to choose lower but stable prices over time and leading to greater network effects. Because technology licensing amplifies network effects, firms engage in less aggressive price competition by increasing their sales prices, especially when a dynamic royalty is used. (2) Network effects have different impacts on firms’ steady-state behaviors. When the network effects are stronger, the OEM will choose a lower royalty but a higher sales price. However, stronger network effects cause third-party manufacturers to choose a lower sales price, either in the absence of technology licensing or in the presence of technology licensing with a static royalty. The reverse behavior is true for a third-party manufacturer under technology licensing with a dynamic royalty. (3) Technology licensing is more effective for improving a firm’s profits, especially when the dynamic behaviors of firms and consumers are more significant and when the network effects have a more positive impact on the demand of the licensee. However, when the network effects are insignificant, third-party manufacturer will not accept technology licensing. Moreover, when technology licensing is ineffective for improving the impact of the network effects on the sales of the third-party products, the OEM will choose not to license its technology. (4) Licensing with a dynamic royalty is more effective in mitigating price competition; therefore, firms maintain higher sales margins. However, licensing with a static royalty induces the OEM to choose a lower royalty and firms lower prices, leading to an increase in network effects, which are generally more advantageous for the firms. When the network effects are weak, the impact of technology licensing on mitigating price competition is trivial, and technology licensing with a dynamic royalty is more advantageous for the OEM. (5) When consumers are more sensitive to network effects, technology licensing is more advantageous to social welfare. Moreover, social planners prefer firms to implement technology licensing with a static royalty; in contrast, technology licensing with a dynamic royalty generally causes higher unit royalty and sales prices, harming consumer surplus; thus, it is less preferred by the social planner.

The remainder of this paper is organized as follows. Section 2 surveys the related literature while comparing previous studies with our work. In Section 3, we develop the demands of firms from consumer utility, incorporating the network effects and building the dynamics of the network effects. Then, we derive firms’ SPNE decisions in the model without technology licensing. In Section 4, we formulate two licensing models: the first considers the static royalty mechanism, and the second considers the dynamic royalty mechanism. In Section 5, we analyze firms’ instantaneous and steady-state equilibrium behavior and then investigate the performances of licensing models in terms of firms’ profits. The final section concludes the study with a brief summary and suggests potential future research directions.

2. Literature review

Many studies have stressed the importance of network effects as determinants of firms’ competitive behaviors (e.g., Chen et al., 2009; Church & Gandal, 1992; Ho & Mallick, 2010; Kim, 2002; Malueg & Schwartz, 2006; Prasad, Venkatesh, & Mahajan, 2010; Shi, Chiang, & Rhee, 2006; Stremersch, Tellis, Franses, & Binken, 2007; Sun, Xie, & Cao, 2004). Church and Gandal (1992) proposed a theoretical model to investigate the impact of network effects on the choices of consumers and software firms with regard to which hardware network to join between two competitive but incompatible technologies. They showed that software firms prefer to support technology by adopting a standardization strategy at equilibrium such that all consumers will be on the same network, and the benefit of the network effects is maximized. Sun et al. (2004) discussed network effects in a static model considering an innovating firm’s choice of product strategies: single-product monopoly, technology licensing, product-line extension, and a combination of both licensing and product-line extension. Consumers perceive the network effects of a non-innovating firm’s products to be weaker than or equal to those of the innovating firm’s product; moreover, the strength of network effects on the non-innovating firm’s demand is associated with its quality level. These authors found that when network effects are significant, technology licensing is superior to product-line extension because of a larger market, which is a result of technology licensing; moreover, paid licensing is more beneficial for the innovator than free licensing, especially when licensing with a lump-sum royalty. Shi et al. (2006) focused on the cellular phone industry to study the influences of WNP (wireless number portability) policies on firms' profitability and consumer switching behaviors and verified their theoretical results with empirical evidence. They found that WNP policies intensify price competition and stimulate firms to generate greater network externality by providing a greater discount for on-net calls, leading to increases in consumer surpluses. However, they also indicated that the static model may lead firms to pursue a short-term gain by sacrificing a long-term benefit; therefore, it is worthwhile to extend the model to a dynamic setting. Prasad et al. (2010) examined the network effects caused by the bundling strategies of a monopolist offering two products and noted that the levels of asymmetry of the network effects and managerial costs between the two products have a significant impact on the firm's bundling strategy. Specifically, a traditional mixed bundling strategy is the dominant strategy. However, when both products have low managerial costs or larger network effects, a pure bundling strategy is more profitable, whereas when only one product has network effects, either the pure components or mixed bundling strategy is more profitable. Zhang, Wang, Qing, and Hong (2016b) analyzed the online group-pricing mechanism under the demands of both group and individual buying and found that consumers benefit from the network effects of group buying. They showed that a firm prefers a group-buying strategy under strong network effects, except when the inconvenience of developing a group-buying strategy outweighs the positive network effects.

Product technology is a critical factor that affects the intensity of network effects; therefore, it has received considerable attention in the literature. Malueg and Schwartz (2006) examined the asymmetric competition among several small firms whose products are fully compatible and a large firm that determines whether to supply a product compatible with the small firms' products. In their study, the large firm prefers incompatibility with the small firms' products when it has a larger market share. However, this is not the case when the number of small firms increases because competition among small firms causes them to engage in aggressive behavior, which results in a superior network that is more attractive to consumers than the large firm's network. Chen et al. (2009) further investigated technology compatibility using a discrete dynamic model to address the long-term market structure in the presence of network effects and numerically derived firms' MPE (Markov perfect equilibrium) decisions by assuming that consumers are myopic. Their work showed that when firms' market information is asymmetric to a certain degree, a larger firm will pursue a dominance strategy by making its product incompatible; however, this result is suppressed by firms' dynamic pricing. These scholars concluded that in the long run, compatibility is a stable choice for firms. Rasch (2017) analyzed the issues of compatibility and collusion in a duopoly with network effects and showed that when full collusion can be sustained, incompatibility is preferred by firms; when it cannot be sustained, compatibility is advantageous for firms. The aforementioned studies assumed that technology can be unilaterally developed by firms. However, practical evidence (Burgett, 2016; Sony Co., Ltd., 2011; Thurston, 2013) has revealed that some third-party firms are unable to improve technology compatibility because of the limited availability of technology; therefore, technology licensing is a predominant strategy in this situation. We contribute to the literature by incorporating the technology licensing and network effects associated with compatibility in a dynamic model such that firms' instantaneous and long-run equilibrium behavior regarding decisions and choices of technology licensing can be explored.

The literature on technology licensing and royalty schemes is vast. We review studies of technology licensing under competitive environments. Kulatilaka and Lin (2006) studied a static duopoly in which an entrant chooses between developing its own technology or licensing technology from an existing firm that determines its investment timing based on the licensing agreements. These scholars found that the existing firm can use technology licensing to dissuade the entrant from developing its own technology. Bagchi and Mukherjee (2014) analyzed the licensing contracts, i.e., per-unit royalty and fixed-fee licensing, used by an innovator that competes with multiple licensees on either price or quantity. They noted that royalty-based licensing is likely to be more profitable for the innovator and consumers than fixed-fee licensing, especially under price competition. Zhang, Shang, and Yildirim (2016a) considered technology licensing in a duopoly in which an innovator offers a fixed-fee, royalty or two-part tariff licensing scheme to a competitor in product differentiation and technology spillover. They proposed that the two-part tariff licensing is prevalent when the effect of the technology spillover is small. Hong, Govindan, Xu, and Du (2017) examined technology licensing in a closed-loop supply chain including a manufacturer and a remanufacturer that compete for quantity and collection, considered technology licensing from the manufacturer to the remanufacturer under royalty-based and fixed-fee licensing schemes, and found that fixed-fee licensing was superior to royalty-based licensing for the manufacturer. However, these studies discussed technology licensing in static settings, while research in this field regarding dynamic settings has been rather scant. For example, Fershtman and Markovich (2010) investigated dynamic R&D races with patent protection and licensing in a duopoly under a discrete setting and numerically characterized the MPE of the firms. Our work essentially differs from that of Fershtman and Markovich (2010) by establishing a continuous (differentiable) dynamic model and considering the dynamics of network effects. Denicoló and Zanchettin (2012) developed a differential dynamic model that includes two competing firms that determine their prices and R&D efforts, considered the dynamics of the quality gap and the stock of active patents, and derived the firms' closed-loop equilibrium. They found that firms may suffer from the prisoners' dilemma; therefore, they have an incentive to cooperate for a better outcome at equilibrium. By comparison, in this study, we not only consider the different dynamics and licensing issues but also focus on different equilibrium concepts. Specifically, we analyze the SPNE, ensuring that "if one player decides according to the feedback rule, then it is optimal for the others to do so as well" (Cellini & Lambertini, 2004). However, this statement is generally not true for closed-loop equilibrium. This study contributes to the literature by considering both dynamic and static royalty policies and by considering dynamic network effects in the field of technology licensing. Buratto and Zaccour (2009) examined advertising interactions between a licensor and a licensee under differential games by considering different dynamic states but assumed the unit royalty to be exogenous. Fan, Jun, and Wolfstetter (2018) determined the dynamic licensing mechanism in a two-stage process in a Cournot oligopoly under a discrete dynamic model. However, neither of these two studies (Buratto & Zaccour, 2009; Fan et al., 2018) considered competition between the licensee and licensor, product compatibility, and network effects.

In the economics and operational management fields, dynamic competition has attracted increasing attention (e.g., Chen, Dong, Rong, & Yang, 2018; Zhang, Chiang, & Liang, 2014). In the following, we focus on research regarding differential games (Cellini & Lambertini, 2004; 2007; Krishnamoorthy, Prasad, & Sethi, 2010; Liu, Zhang, Zhang, & Liu, 2017; Ouardighi & Pasin, 2006; Wang & Li, 2012; Xie & Sirbu, 1995). Xie and Sirbu (1995) examined a differential pricing game between an incumbent and an entrant and considered network effects on firms' demands associated with product compatibility. Specifically, when firms' products are compatible, the network effect is associated with the sum of the individual installed base. However, Xie and Sirbu (1995) focused their analysis on open-loop equilibrium, which is simple but weakly time consistent and not subgame perfect. In this study, we follow Xie and Sirbu (1995) by considering the network effect to be associated with the sum of the individual installed base because the firms' products are compatible. Conversely, we believe that technology licensing can be adopted for higher compatibility, leading to greater network effects. Cellini and Lambertini (2004) investigated a differential oligopoly with price dynamics and characterized different types of equilibria: open-loop equilibria, closed-loop memoryless equilibria, and feedback equilibria. Subsequently, Cellini and Lambertini (2007) continued their research by incorporating product differentiation and focusing on open-loop and closed-loop memoryless equilibria. Krishnamoorthy et al. (2010) examined dynamic duopolistic competition over prices and advertising and considered the dynamics depending on the advertising decisions of both firms. More recently, Liu et al. (2017) analyzed the dynamic interaction between a manufacturer and a supplier that are either far-sighted or myopic and discovered that far-sighted behavior is the dominant strategy for both the manufacturer and the supplier. However, a prisoner's dilemma exists, inducing each firm to act myopically for a better outcome. Wu (2018) examined a differential duopolistic game with technology licensing and discovered that technology licensing is always beneficial for the innovating firm but is not always beneficial for the non-innovating firm. When low product differentiation exists and a non-innovating firm has extensive capabilities to utilize licensed technology, the firm will be stimulated to accept a licensing agreement. Our study differs considerably from the dynamic model of Wu (2018), which did not consider network effects but considered licensing royalties to be exogenous and analyzed different dynamic states due to changes in innovation that were perceived by consumers.

Few studies have discussed firms' strategic choices between dynamic and static operational decisions in dynamic environments (Cachon & Feldman, 2010; Gayon & Dallery, 2007; Zhang, Lei, Zhang, & Song, 2017). For example, Cachon and Feldman (2010) investigated the dynamic and static pricing strategies of a monopolistic firm in the presence of strategic consumers and found that static pricing is more beneficial to the firm in most cases, especially when consumers are more strategic. Zhang et al. (2017) explored dynamic and static pricing policies in a dynamic model consisting of a manufacturer and an online retailer and assumed that the goodwill associated with the manufacturer's advertising decisions evolves over time. Based on a quantitative analysis, Zhang et al. (2017) found that a case in which the manufacturer adopts a static wholesale price and the retailer adopts a dynamic price is optimal for the retailer and the supply chain but is less profitable for the manufacturer than a casein which both firms choose dynamic prices. Nevertheless, dynamic and static royalty schemes are plausible in practice but have not been investigated. Hence, we consider two licensing models with dynamic and static royalties and compare them with a basic model without technology licensing to characterize firms' preferences for technology and royalty licensing choices.

To highlight the contributions of this study, Table 1 summarizes the issues and model settings in this study and the relevant research. Table 1 shows that this study is related to four streams of research: network effects, technology licensing, competition, and dynamic models. To the best of our knowledge, dynamic and static royalties under the continuous evolution of network effects have not been investigated in previous works. Hence, we not only bring dynamics associated with network effects into the field of technology licensing but also extend the works on licensing schemes by considering both dynamic and static royalties. In this manner, we derive the subgame perfect Nash equilibria (SPNEs) of the firms and provide insights regarding network effects on firms' preferences for technology licensing, firms' preferences for the types for licensing royalties, and the effects of market- and dynamic-related factors on firms' instantaneous and steady-state SPNE decisions and profits.

3. The Model

We consider a model of a duopolistic supply chain consisting of a brand manufacturer and a third-party manufacturer. Both firms sell substitutable yet compatible products. Because the brand manufacturer (denoted by A) develops the product specification and possesses the technology, its product has full compatibility. However, due to limitations of the available technology, the third-party manufacturer (denoted by B) provides a low-priced but partially compatible product. In this market, price competition and network effects exist; i.e., consumers are sensitive to both prices and network effects. The network effects lead consumers' utilities for both firms' products to increase in the total sales quantity. Thus, the existence of the third-party manufacturer is not always harmful to the brand manufacturer because the third-party products can satisfy the demand of low-budget consumers, which will expand the total sales quantity and improve the network effects. The firms compete over prices for market shares; however, they can choose to cooperate to enhance network effects, and thus co-opetition may emerge between the brand and third-party manufacturers. The firms can choose to cooperate through a licensing mechanism with which the brand manufacturer transfers its technology to the third-party manufacturers using an endogenous per-unit royalty. In this way, the third-party products possess technology for greater compatibility, and the network effects on consumers' utilities for purchasing third-party products can be improved.

Under technology licensing, the network effects are intensified, allowing firms to obtain higher sales margins. However, the brand manufacturer may lose its technological advantage, and the third-party manufacturer must bear a higher cost because of the licensing royalty. Hence, we investigate firms' strategic licensing behaviors by examining three models: Model N, in which technology licensing is absent; Model LD, in which technology licensing is implemented with a dynamic royalty; and Model LS, in which technology licensing is adopted with a static royalty. To capture the dynamic nature of the perceptions of consumers, we consider that consumers do not immediately perceive the actual sales quantity. As a result, the adjustment process of consumers to the sales quantity takes time. This phenomenon leads to the dynamic impact of network effects, which change depending on the firms' current decisions. These characteristics lead to a dynamic game in the supply chain. We first formulate the firms' dynamic interactions with the network effects and then investigate the dynamic characteristics of the SPNE and the stationary equilibrium with regard to the firms' decisions, profits, and licensing choices. Throughout the paper, we use the subscripts $i \in \{A, B\}$ to denote the firm and the superscripts $j \in \{N, LD, LS\}$ to denote the model.

3.1. Consumer product choice and dynamic state

Consumers make their purchase choices based on the utilities of both firms' products. The parameter θ denotes the willingness of a consumer to pay, and we capture consumer heterogeneity by assuming that θ is uniformly distributed between $-M$ and 1 (i.e., $f(\theta) \sim \text{Uniform}[-M, 1]$). Following Katz and Shapiro (1985), Sun et al. (2004) and Malueg and Schwartz (2006), we assume M to be sufficiently large to indicate that all consumers enter the market, such that the derivation can be simplified without considering corner solutions. Consumers have different valuations of firms' products; in other words, the willingness of a consumer to pay for the third-party product is a fraction, ρ ($\rho < 1$), of his/her willingness to pay for the brand product. Moreover, ρ can represent the consumer's evaluation of the third-party product; as ρ approaches

Table 1
Brief summary of issues and model settings in the literature.

Author(s) (Year)	Issues: Net. Effe.	Comp.	Lic.	Compb.	Dyn. vs. St.	Decision Variables	Model Type
This study	✓	✓	✓	✓	✓	Price & Royalty	Dynamic
Xie and Sirbu (1995)	✓	✓		✓		Price	Dynamic
Sun et al. (2004)	✓	✓	✓			Quantity	Static
Malueg and Schwartz (2006)	✓	✓	✓	✓		Quantity	Static
Chen et al. (2009)	✓	✓	✓	✓		Price & Compb.	Dynamic[a]
Bagchi and Mukherjee (2014)		✓	✓			Price or Quantity	Static
Zhang et al. (2016a)		✓	✓			Quantity & Royalty	Static
Cellini and Lambertini (2004, 2007)		✓				Quantity	Dynamic
Fershtman and Markovich (2010)		✓	✓			R&D	Dynamic[a]
Denicoló and Zanchettin (2012)		✓	✓			Quantity	Dynamic
Fan et al. (2018)		✓	✓			Quantity & Royalty	Dynamic[a]
Liu et al. (2017)						Price & Quantity	Dynamic
Wu (2018)		✓	✓			Price & Innovation	Dynamic
Cachon and Feldman (2010)		✓			✓	Price	Dynamic
Zhang et al. (2017)					✓	Price & Advertising	Dynamic

Net. Effe.: Network effects; Comp: Competition; Lic: Licensing; Compb: Compatibility; Dyn vs. St: Dynamic vs. static decisions. Superscript "[a]" refers to a discrete dynamic model.

one, consumers report an identical willingness to pay for the third-party and brand products. We assume that a type-θ consumer at time t receives utility $U_A(t) = \theta + \alpha W(t) - p_A(t)$ from the product of firm A, where $W(t)$ denotes the network effect at time t and α indicates consumer sensitivity to the network effects. A higher value for α indicates that consumers are more sensitive to the network effect; thus, the network effect is more significant. The utility of the product of firm B depends on the presence of technology licensing, i.e., $U_B(t) = \rho\theta + [I\kappa + \bar{I}\eta] \cdot \alpha W(t) - p_B(t)$, where I is an indicator function that takes the value of one when licensing is present and zero otherwise, and $\bar{I} \equiv 1 - I$. Note that we focus on the positive network effects (i.e., the consumer valuations increase with the total sales quantity). Based on empirical evidence from the home video industry (Shankar & Bayus, 2003) and theoretical studies by Sun et al. (2004) and Fainmesser and Galeotti (2016), we believe that firms face asymmetric network effects on their demand. Moreover, following previous theoretical research (Alexandrov, 2015; Chen et al., 2009; Clements, 2004; Malueg & Schwartz, 2006), we suggest that the levels of network effects on utility functions will decrease because of a lower degree of compatibility. Specifically, η ($\eta > 0$) is a discount factor indicating that consumers perceive the network effects of firm B's product to be weaker than or equal to those of firm A's product, as formulated in Sun et al. (2004), and κ is the degree of technology transfer to firm B; therefore, consumers perceive the network effect of firm B's product at the κ level. To avoid trivial cases, we assume that the effects of the willingness to pay are stronger than the network effects on consumer utilities (i.e., $\alpha < \rho$, $\kappa > \eta$ and $\kappa > 1/3$), indicating that consumers perceive sufficiently strong network effects when licensing is implemented.

Let Θ_A and Θ_B represent the sets of consumer types who buy the products of firms A and B, respectively. Then, $\Theta_A = \{\theta : U_A(t) \geq \max\{U_B(t), 0\}\}$ and $\Theta_B = \{\theta : U_B(t) \geq \max\{U_A(t), 0\}\}$. The demand quantities of the products at time t can be calculated as follows:

$$q_A(t) = \int_{\theta\in\Theta_A} f(\theta)\mathrm{d}\theta = \frac{p_A(t) - p_B(t) + \alpha\eta W(t)\bar{I} + \alpha W(t)(I\kappa - 1) + \rho - 1}{\rho - 1}, \text{ and}$$

$$q_B(t) = \int_{\theta\in\Theta_B} f(\theta)\mathrm{d}\theta = \frac{-\rho p_A(t) + p_B(t) + \alpha W(t)\left(-\eta\bar{I} - I\kappa + \rho\right)}{(\rho - 1)\rho}.$$

The total market demand (i.e., $q_A(t) + q_B(t)$) is nonstationary because it depends not only on the firms' pricing decisions but also on the impact of network effects, which evolve over time. To focus on duopolistic competition, i.e., $q_A(t), q_B(t) \geq 0$, the indifference point of θ between $U_A(t)$ and $U_B(t)$ must fall in $\Theta_A \cup \Theta_B$. Thus, we assume that the following condition holds throughout the paper:

$$\frac{p_A(t) - p_B(t) - (1-\rho)}{[1 - (I\kappa + \bar{I}\eta)]W(t)} \leq \alpha \leq \frac{\rho p_A(t) - p_B(t)}{[\rho - (I\kappa + \bar{I}\eta)]W(t)}.$$

Note that the exact values of the above condition can be computed by taking the equilibrium results into the above inequality and then solving for α.

Consistent with Xie and Sirbu (1995), we consider that the consumer utilities of both firms' products are associated with the sum of the firms' demand quantities because the products are compatible. Technology licensing can increase the compatibility of third-party products, improving the significance of network effects. Moreover, the network effects evolve; that is, consumers do not immediately perceive the actual sales quantity, and the market undergoes an adjustment process, which takes time. Thus, network effects are dependent on the firm price and consumers' past perceptions of network effects. In alignment with Sun et al. (2004), the impact of network effects is contingent on the total sales quantity, and we can simplify the model as follows: $\hat{W}(t) = q_A(t) + q_B(t) = [p_B(t) - \rho]/[\bar{I}\alpha\eta + I\alpha\kappa - \rho]$. Thus, the relationship between network effects and the firms' decisions is obtained. For $\hat{W}(t) > 0$, we assume $\rho - \alpha\kappa > 0$; therefore, $\alpha\kappa \leq \rho < 1$. The form of $\hat{W}(t)$ reveals that the network effect is associated with firm B's price, indicating that the existence of third-party products can cause the network effect to increase because the third-party products satisfy the demands of low-budget consumers. We use s ($0 < s < 1$) to represent the adjustment speed with which the network effect converges on its level in the utility function. Based on previous works (e.g., Cellini & Lambertini, 2004; 2007; Fershtman & Kamien, 1987; Piga, 2000; Xin & Sun, 2018) regarding the dynamics of prices and other dynamic states (e.g., advertising and water rights), we assume that the adjustment process of the network effect is governed by the following differential equation:

$$\dot{W} \equiv \frac{\mathrm{d}W(t)}{\mathrm{d}t} = s\left(\hat{W}(t) - W(t)\right). \tag{1}$$

The adjustment process in (1) reveals that as s decreases, the current impact of the network effect is less related to the firm's decision; therefore, the speed of convergence with $W(t)$ increases. As a result, when s approaches 0, the evolution of the network effects is independent not only of the firm's decision but also of the current network effects, indicating that consumers are myopic; thus, the evolution of the impact of network effects is insensitive to the firm price.

3.2. Model without technology licensing (Model N)

In Model N, the instantaneous profit function of firm i is $\pi_i^N(t) = q_i(t) \times (p_i(t) - c)$. To avoid trivial results, we consider that the firms set their prices in a specific range by assuming that $c < \rho$. Therefore, both firms determine their prices by maximizing their discounted stream of profits in Model N, which is formulated as $\max_{p_i(t)} \Pi_i^N = \int_0^\infty e^{-\delta t}\pi_i^N(t)\,dt$ subject to (1) and the condition $W(0) = W_0$, where δ denotes the intertemporal discount rate. A higher discount rate indicates that the firm has a myopic view of current profits.

Next, we focus on the feedback Nash equilibrium (Cellini & Lambertini, 2004; 2007; Piga, 2000), which dictates the following characteristics: the feedback Nash equilibria of the firms are time consistent and subgame perfect at all times, and if one player chooses a feedback rule, then it is optimal for the others (Piga, 2000). Note that hereafter, for ease of understanding, we omit the denotation of time and mark the steady-state equilibrium results with the superscript "$*$". Because the dynamic state involves firm B's price, we adopt Bellman's value function approach. For Model N, the feedback Nash equilibria must satisfy the following Hamilton-Bellman-Jacobi (HBJ) equation:

$$\delta V^N(W) = \max_{p_i}\left\{\pi_B^N + \frac{\mathrm{d}\,\mathrm{V}^{\mathrm{N}}(\mathrm{W})}{\mathrm{d}\,\mathrm{W}} \times \dot{W}\right\}, \tag{2}$$

where $V^N(W)$ is the value function of firm B in Model N on state W. Solving the first-order conditions $\mathrm{d}\,\mathrm{V}_{\mathrm{A}}^{\mathrm{N}}(\mathrm{W})/\mathrm{d}\,\mathrm{p}_{\mathrm{A}} = 0$ and $\mathrm{d}\,\mathrm{V}_{\mathrm{B}}^{\mathrm{N}}(\mathrm{W})/\mathrm{d}\,\mathrm{p}_{\mathrm{B}} = 0$ yields the firms' time-consistent SPNEs, which are as follows:

$$p_A^N = \frac{-3c + \Omega^N + 2(\rho - 1) + \alpha W(\eta + \rho - 2)}{\rho - 4}, \quad \text{and} \tag{3}$$

$$p_B^N = \frac{-c(\rho + 2) + 2\Omega^N + (\rho - 1)\rho + \alpha W(\eta(\rho - 2) + \rho)}{\rho - 4}, \tag{4}$$

where $\Omega^N = [(\rho - 1)\rho s V^{N'}(W)]/(\alpha\eta - \rho)$.

In Proposition 1, we derive the differential equation of the value function of firm B and the steady-state equilibrium decisions and states. For clarity, we summarize the proofs in the supplement.

Proposition 1. *At feedback Nash equilibrium, the steady state of the impact of the network effects W^{N*} and firm i's price p_i^{N*} in Model N are given by:*

$$W^{N^*} = \frac{(c(\rho+2) - 3\rho)(\alpha\eta - \rho) - 2(\rho - 1)\rho s \beta_2^N}{(\alpha\eta - \rho)(\alpha(2\eta + \rho) + (\rho - 4)\rho) + 2(\rho - 1)\rho s \beta_1^N},$$
$$p_A^{N^*} = -\frac{(\rho - 1)\rho s\left(\beta_2^N + W^{N^*}\beta_1^N\right) + (\alpha\eta - \rho)(-3c + 2(\rho - 1) + \alpha W^{N^*}(\eta + \rho - 2))}{(\rho - 4)(\rho - \alpha\eta)},$$
$$p_B^{N^*} = \frac{(\alpha\eta - \rho)(c(\rho + 2) + \rho - \rho(\rho + \alpha(\eta + 1)W^{N^*}) + 2\alpha\eta W^{N^*}) - 2(\rho - 1)\rho s\left(\beta_2^N + W^{N^*}\beta_1^N\right)}{(\rho - 4)(\rho - \alpha\eta)},$$

where β_1^N and β_2^N are detailed in the proof.

The instantaneous function of the impact of the network effects can be derived as follows:

$$W^N(t) = W^{N^*} + (W_0 - W^{N^*})e^{\frac{st\left(\alpha^2\eta(2\eta+\rho)+\alpha\rho(\eta(\rho-6)-\rho)+2(\rho-1)\rho s\beta_B^N-(\rho-4)\rho^2\right)}{(\rho-4)(\rho-\alpha\eta)^2}},$$

where $W_0 \equiv W(0)$. Substituting $W^N(t)$ from (5) into p_i^N, $q_i(t)$ and $\pi_i^N(t)$ yields firm i's instantaneous price, demand, and profit at the feedback SPNE.

In Corollary 1, we characterize the firms' steady-state equilibrium in Model N with respect to consumer sensitivity to the network effects α and the discount factor η of the network effects for firm B's product under a very slow adjustment speed of W.

Corollary 1. *As $s \to 0$, (i) W^{N*} increases in α and η; (ii) p_A^{N*} and p_B^{N*} increase in α; and (iii) p_A^{N*} decreases in η but p_B^{N*} increases in η.*

Corollary 1 shows that if the adjustment speed of the impact of the network effects is very slow, then the impact of network effects will increase with both consumer sensitivity to the network effects and the discount factor of the network effects for firm B's product; thus, consumers perceive higher network effects when the network effect is more significant for both products. Moreover, both firms increase their prices with respect to the strength of the network effects (α), implying that the strength of the network effects can mitigate the intensity of price competition such that the firms can obtain higher sales margins. An increase in η demonstrates that the discrepancy between the network effects of the firms' products is smaller. In cases with a smaller size, the competitiveness of firm B is more significant; therefore, firm B will increase the sales price for a greater sales margin. However, firm A will behave more aggressively by lowering its sales price. In addition, technology licensing can amplify the network effects such that the intensity of price competition is reduced; therefore, the firms' sales margins are improved. However, technology licensing also reduces the discrepancy between the network effects of the firms' products, which harms firm A's competitive advantage. Hence, technology licensing is not always accepted by the firms and is worthy of further investigation.

4. Models with technology licensing

When technology licensing is present, the instantaneous profit functions of firm i are as follows:

$$\bar{\pi}_A(t) = q_A(t) \times (p_A(t) - c) + \gamma\,\kappa\, q_B(t) \quad \text{and}$$
$$\bar{\pi}_B(t) = q_B(t) \times (p_B(t) - c - \gamma\kappa), \tag{5}$$

where γ is the per-unit royalty. The upfront fee $F(\kappa)$ is a lump-sum charge, and the royalty γ is a unit charge for a κ-degree of technology transfer. The firms' pricing problem in Model $j \in \{LD, LS\}$ is $\max_{p_i(t)} \Pi_i^j = \int_0^\infty e^{-\delta t}\bar{\pi}_i^j(t)\,dt$, and the HBJ equation of firm i is as follows:

$$\delta V_i^j(W) = \max_{p_i}\left\{\bar{\pi}_i + \frac{\mathrm{d}\,\mathrm{V}_{\mathrm{i}}^{\mathrm{L}}(\mathrm{W})}{\mathrm{d}\,\mathrm{W}} \times \dot{W}\right\}. \tag{6}$$

4.1. Equilibrium in the licensing model with a dynamic royalty (Model LD)

In Model LD, firm A determines the licensing royalty dynamically (i.e., the licensor (firm A) first chooses a time-consistent royalty and then the firms determine their time-consistent prices). Through backward induction, we first solve for the firms' pricing decisions, and then we solve for the licensing royalty. The firms' prices at the feedback SPNE must satisfy the HBJ equation in

(6); thus, solving $\mathrm{d}V_A^L/\mathrm{d}p_A = 0$ and $\mathrm{d}V_B^L/\mathrm{d}p_B = 0$ yields the time-consistent SPNEs of the firms as follows:

$$p_A^{LD} = \frac{(\alpha\kappa - \rho)\left(-3\gamma\kappa - 3c + 2\rho + \alpha W\left(\kappa - \hat{\rho}_2\right) - 2\right) - \rho\hat{\rho}_1\, sV_B^{L'}(W)}{\hat{\rho}_4(\rho - \alpha\kappa)}, \tag{7}$$

$$p_B^{LD} = -\frac{-\frac{2\hat{\rho}_1\rho s V_B^{L'}(W)}{\alpha\kappa-\rho} - \rho(\gamma\kappa + c - \alpha(\kappa+1)W + 1) - 2(c + \kappa(\gamma + \alpha W)) + \rho^2}{\hat{\rho}_4}. \tag{8}$$

The royalty γ is chosen by firm *A* to maximize its instantaneous profit. We solve the problem $\max_\gamma V_A^{LD}|_{p_i=p_i^{LD}}$ for the optimal time-consistent γ as follows:

Corollary 2 shows the parametric effects of α and κ on the firms' steady-state equilibria in Model *LD* under a very slow adjustment speed of *W*.

Corollary 2. *As $s \to 0$, (i) W^{LD*} increases in α and κ, (ii) p_A^{LD*} and p_B^{LD*} increase in α, (iii) p_A^{LD*} and p_B^{LD*} increase in κ, and (iv) γ^{LD*} increases in α but decreases in κ.*

From Corollary 2, we find that the trends of W^{LD*} and p_i^{LD*} are similar to those of W^{N*} and p_i^{N*}, as shown in Corollary 1. Intuitively, when consumers are more sensitive to the network effects, the impact of the network effects is greater; moreover, the network effects can decrease the intensity of price competition such that the firms can obtain higher sales margins by increasing their

$$\gamma^{LD} = \frac{\rho(\rho+2)\hat{\rho}_4\, sV_A^{LD'}(W) - 8\rho\hat{\rho}_1\, sV_B^{LD'}(W) + (\alpha\kappa - \rho)\left(-c\left(\rho^2+8\right) + \rho(\rho+8) + \alpha W\left(8\kappa + \rho^2\right)\right)}{2\kappa(\rho+8)(\alpha\kappa - \rho)}. \tag{9}$$

Following the approach adopted in Model *N*, we show the steady state of the feedback SPNE decisions in Model *LD* in Proposition 2. The proof of Proposition 2 is similar to that of Proposition 1.

Proposition 2. *At feedback Nash equilibrium, the steady state of the impact of network effects W^{LD*} and licensing royalty γ^{LD*} in Model L are given by:*

$$W^{LD*} = \frac{\rho(\rho+2)^2\, s\beta_{2A}^{LD} + 4\rho\hat{\rho}_1\, s\beta_{2B}^{LD} + \left(c(\rho+2)^2 - \rho(\rho+8)\right)(\alpha\kappa - \rho)}{(\alpha\kappa - \rho)(4\alpha\kappa + \alpha\rho(\rho+4) - 2\rho(\rho+8)) - s\rho(\rho+2)^2\beta_A^{LD} - 4\rho\hat{\rho}_1\, s\beta_{1B}^{LD}},$$

$$\gamma^{LD*} = \frac{1}{2\kappa(\rho+8)(\alpha\kappa - \rho)}\Big\{\rho(\rho+2)\hat{\rho}_4\, s\left(\beta_{2A}^{LD} + W^{LD*}\beta_{1A}^{LD}\right) - 8\rho\hat{\rho}_1\, s\left(\beta_{2B}^{LD} + W^{LD*}\beta_{1B}^{LD}\right) + (\alpha\kappa - \rho)\left(-c\left(\rho^2+8\right) + \rho(\rho+8) + \alpha W^{LD*}\left(8\kappa + \rho^2\right)\right)\Big\}.$$

The steady state of the firms' prices p_i^{LD*} is as follows:

$$p_A^{LD*} = \frac{1}{2(\rho+8)\hat{\rho}_4(\alpha\kappa - \rho)}\Big\{3\rho(\rho+2)\hat{\rho}_4\, s\left(\beta_{2A}^{LD} + W^{LD*}\beta_{1A}^{LD}\right) - 2\hat{\rho}_4\rho\hat{\rho}_1\, s\left(\beta_{2B}^{LD} + W^{LD*}\beta_{1B}^{LD}\right) + (\alpha\kappa - \rho)\left(3c\hat{\rho}_4(\rho+2) + \hat{\rho}_4(\rho+8) + \alpha W^{LD*}\left(2\kappa\hat{\rho}_4 + 3\rho^2\right)\right) + 2\alpha(\rho+8)\hat{\rho}_2 W^{LD*}(\alpha\kappa - \rho)\Big\},$$

$$p_B^{LD*} = \frac{1}{2(\rho+8)(\rho - \alpha\kappa)}\Big\{\rho s\left(-(\rho+2)^2\left(\beta_{2A}^{LD} + W^{LD*}\beta_{1A}^{LD}\right) - 4\hat{\rho}_1\left(\beta_{2B}^{LD} + W^{LD*}\beta_{1B}^{LD}\right)\right) - (\alpha\kappa - \rho)\left(c(\rho+2)^2 + \rho(\rho+8) + \alpha W^{LD*}(2\kappa(\rho+6) - \rho(\rho+4))\right)\Big\}.$$

From Proposition 2, we can obtain the instantaneous function of the impact of network effects, which is given by:

$$W^{LD}(t) = W^{LD*} + (W_0 - W^{LD*}) \times e^{\frac{st\left(-(\alpha\kappa-\rho)(4\alpha\kappa+\alpha\rho(\rho+4)-2\rho(\rho+8))+\rho(\rho+2)^2\, s\beta_{1A}^{LD}-4(\rho-1)\rho s\beta_{1B}^{LD}\right)}{2(\rho+8)(\rho-\alpha\kappa)^2}}.$$

Moreover, firm *i*'s instantaneous price, demand, and profit at

sales prices. However, under technology licensing, an increase in κ induces both firms to increase their sales prices; therefore, price competition is weakened. This result differs from that of Model *N*. The result implies that although an increase in the degree of technology transfer reduces firm *A*'s competitive advantage, technology licensing leads firm *A* to alter its aggressive behavior in competition to pursue a higher sales margin by increasing its sales price. Under technology licensing with a dynamic royalty, the network effects are effective for mitigating the intensity of price competition even when the network effects benefit the sales of the licensee's product. Regarding the network effects on the dynamic royalty, we find that when consumer sensitivity to network effects α is more significant, the licensor (i.e., firm *A*) will charge a higher royalty; thus, when higher consumer sensitivity to the network effects induces the licensee to engage in technology licensing, the licensor will charge a higher royalty. However, when the network effects have a greater impact on the licensee's product because of an increasing κ, the licensor will reduce the royalties, indicating that the licensor would prefer to decrease the royalties to increase technology licensing.

4.2. Equilibrium in the licensing model with a static royalty (Model LS)

In Model *LS*, firm *A* charges a fixed unit royalty. Thus, firm *A* chooses its licensing royalty during the first period; then, the firms determine their time-consistent prices based on the licensing royalty. Through backward induction, we first solve the firms' pricing decisions and then discuss the static royalty. In Model *LS*, the firms' pricing problem is identical to that in Model *LD*; thus, the firms' SPNE prices p_A^{LS} and p_B^{LS} are identical to those in (7) and (8), respectively.

Next, we substitute the firms' time-consistent SPNE p_i^{LS} into the HBJ equation in (6) and conjecture that the value function is quadratic: $V_B^L(W) = \frac{\beta_{1B}^{LS}}{2}W^2 + \beta_{2B}^{LS}W + \beta_{3B}^{LS}$ (i.e., $V_B^{L'}(W) = \beta_{1B}^{LS}W + \beta_{2B}^{LS}$). Solving the HBJ equation yields the solutions of β_{1B}^{LS} and β_{2B}^{LS}, as follows:

$$\beta_{1B}^{LS} = -\frac{(\alpha\kappa - \rho)\left(\hat{\rho}_4\left(-\sqrt{\zeta_1^{LS}} + \hat{\rho}_4(\rho(\delta + 2s) - \alpha\delta\kappa) + 2\alpha\kappa\rho s\right) - 8\alpha s(2\kappa + \rho)\right)}{16\rho\hat{\rho}_1 s^2},$$

$$\beta_{2B}^{LS} = -\frac{(\alpha\kappa - \rho)\left(\rho s\beta_{1B}^{LS}\left(\rho^2(\gamma\kappa + c + 4) + 8(\gamma\kappa + c) - \rho^3 - 12\rho\right) + 2\alpha\left(\rho - \kappa\hat{\rho}_2\right)(\alpha\kappa - \rho)(2\gamma\kappa + 2c - \rho)\right)}{\rho\left(8\rho\hat{\rho}_1 s^2\beta_{1B}^{LS} + (\alpha\kappa - \rho)\left(\hat{\rho}_4^2(\rho(\delta + s) - \alpha\delta\kappa) - \alpha s\left(\kappa\left(8 - \rho\hat{\rho}_4\right) + 4\rho\right)\right)\right)},$$

the feedback SPNE can be obtained by substituting $W^{LD}(t)$ from (10) into p_i^{LD}, $q_i(t)$ and $\pi_i^{LD}(t)$.

where

$$\zeta_1^{LS} = (\rho - \alpha\kappa)(\delta + 2s)\left(\hat{\rho}_4^2(-\alpha\delta\kappa + \delta\rho + 2\rho s) - 2\alpha\rho s(\kappa\rho + 8)\right).$$

The time path $\dot{W}^{LS}$ can be obtained by inserting p_i^{LS}, $V_B^L(W)$, and $V_B^{L'}(W)$ into (1); then, solving $\dot{W}^{LS}=0$ yields the steady state of W, denoted by W^{LS*}. The corresponding instantaneous function of the impact of network effects can be obtained as follows:

$$W^{LS}(t)=W^{LS^*}+(W_0-W^{LS^*})\times e^{\frac{st\left(\alpha^2\kappa(2\kappa+\rho)+\alpha\rho(\kappa(\rho-6)-\rho)+2(\rho-1)\rho s\beta_{1B}^{LS}-(\rho-4)\rho^2\right)}{(\rho-4)(\rho-\alpha\kappa)^2}}, \quad (10)$$

where

$$W^{LS^*}=\frac{(\alpha\kappa-\rho)(\gamma\kappa(\rho+2)+c(\rho+2)-3\rho)-2(\rho-1)\rho s\beta_{2B}^{LS}}{(\alpha\kappa-\rho)(\alpha(2\kappa+\rho)+(\rho-4)\rho)+2(\rho-1)\rho s\beta_{1B}^{LS}}.$$

Finally, we solve for the static unit royalty γ^{LS*}. Firm A's optimization problem is to maximize its cumulative profit, which includes the firms' SPNE prices p_i^{LS} in (7) and (8) and the instantaneous function $W^{LS}(t)$ in (10), i.e.,

$$\gamma^{LS^*}=\arg\max_{\gamma>0}\int_0^\infty\left\{\bar{\pi}_A(t)|W(t)=W^{LS}(t),\, p_i(t)=p_i^{LS}(t)\right\}\mathrm{d}\,t. \quad (11)$$

Once γ^{LS*} is obtained, then the firms' SPNE and steady-state prices, demands, and profits are revealed.

In Corollary 3, we characterize the network effects on the steady-state equilibrium results in Model *LS* regarding the conditions of γ^{LS*} under a very slow adjustment speed of W.

Corollary 3. *As $s\to 0$, (i) W^{LS*} increases in α when $\gamma^{LS^*}<\gamma_1\equiv[3\rho-c(\rho+2)]/[\kappa(\rho+2)]$ and increases in κ when $\gamma^{LS^*}<\gamma_2\equiv[2\alpha(c(\rho+2)-3\rho)]/[\rho(\rho+2)(\alpha+\rho-4)]$, (ii) $p_A^{LS^*}$ and $p_B^{LS^*}$ increase in α when $\gamma^{LS*}<\gamma_1$, and (iii) $p_A^{LS^*}$ increases in κ when*

$$\gamma^{LS^*}>\gamma_3\equiv\frac{\alpha(\alpha-\rho)(c(\rho+2)-3\rho)}{2\alpha^2\kappa^2+\rho^2(\alpha(\alpha+2\kappa-6)+12)+(\alpha-4)\alpha(2\kappa+1)\rho+(\alpha-3)\rho^3},$$

and $p_B^{LS^*}$ always increases in κ.

Corollary 3 shows that when the static royalty γ^{LS*} is smaller than γ_1 and γ_2, consumer sensitivity to network effects can increase its impact; subsequently, the firms' prices increase as the network effects become stronger. These results indicate that when the static royalty is lower than the conditions, the network effects are similar to those in Model *LD*; however, when γ^{LS*} exceeds the conditions associated with γ_1 and γ_2, consumer sensitivity to the network effects will reduce the impact of network effects. This result indicates that when the royalty is at a sufficiently high level, the firms are more aggressive and engage in fiercer price competition. In this case, consumer sensitivity to network effects will amplify the aggressive behavior of the firms, which, however, is harmful to the impact of network effects. When the value of γ^{LS*} is higher than that of γ_3, then the network effects between the licensor's and the licensee's products become more similar and the sales prices increase; however, when the value of γ^{LS*} is less than that of γ_3, firm A will behave in the opposite manner. As a result, we note that the choice of the static royalty in Model *LS* can lead the firms to behave differently than they do in Model *LD*.

We further explain Corollary 3 through a numerical example with the following settings: $s=0.2,\ \delta=0.8,\ \rho=0.7,\ c=0.1,\ \alpha=0.5,\ \kappa=0.7,\ \eta=0.5,\ F=0$, and $W_0=2$. (1) In this example, $\gamma^{LS^*}=0.184,\ \gamma_1=0.846,\ \gamma_2=0.340$, and $\gamma_3=0.100$. This result indicates that the impact of network effects and both firms' prices will increase in α and κ, demonstrating that the network effects mitigate the intensity of price competition and increase the network effect. (2) However, when α in the base setting decreases to 0.3, we find that $\gamma^{LS^*}=0.235,\ \gamma_1=0.968,\ \gamma_2=0.194$, and $\gamma_3=0.086$. The impact of the network effects decreases in κ, indicating that when the impact of the network effects on the licensee's product is similar to that on the licensor's product, the impact of the network effects decreases; thus, the licensor is likely to reduce the degree of technology licensing. (3) When ρ in the base setting decreases to 0.6, $\gamma^{LS^*}=0.094,\ \gamma_1=0.846,\ \gamma_2=0.340$, and $\gamma_3=0.100$. The licensor (firm A) decreases its sales price in κ, revealing that the impact of the network effect on the licensor's product is more significant and decreases the licensor's competitive advantage; therefore, the licensor will be more aggressive in price competition by decreasing its sales price. (4) Through these numerical examples, we find that γ_1 is at a sufficiently high value; therefore, in most cases $\gamma<\gamma_1$ holds true, indicating that consumer sensitivity to the network effects (α) is likely to increase the impact of the network effects and mitigate price competition.

5. Analysis of the equilibrium results

In this section, we discuss the characteristics of equilibrium decisions, profits, and social welfare to illustrate the firms' behaviors with respect to changes in time, network effects and consumer attributes. We perform numerical experiments by assigning different values to each parameter and consider a representative parametric settings as base examples.

5.1. Trajectories of the equilibrium results

Based on prior studies (Atasu, Sarvary, & Van Wassenhove, 2008; Hong et al., 2017; Piga, 2000), the base parametric settings are as follows: $c=0.1,\ s=0.2,\ \alpha=0.4,\ \delta=0.8,\ \eta=0.5,\ \kappa=0.7,\ \rho=0.8,\ W_0=1.5$. Fig. 1 (a) plots the trends of the impact of network effects at SPNE and indicates that the effects in Models *LD* and *LS* are smaller than those in Model *N* because the models with technology licensing mitigate the intensity of price competition, thereby inducing the firms to choose the higher sales prices, as shown in Fig. 1 (c) and (d). Subsequently, the higher sales prices cause the network effects to decrease. In Fig. 1 (b), we find that the licensor will choose a dynamic royalty higher than the static royalty, and the difference between the royalties increases over time. Because of the lower royalty, Model *LS* leads the firms to choose lower sales prices than those in Model *LD*. In addition, the static royalty will cause the firms' sales prices to be insensitive to time, especially the licensee's sales price. These results reveal that a static royalty chosen to maximize long-term profits can stabilize the firms' dynamic pricing decisions over time and is lower than the dynamic royalty, which increases the total sales quantity but decreases the sales margins of the firms.

According to Fig. 2 (a), Models *LD* and *LS* lead the licensor's profit to increase over time more than in Model *N*, indicating that technology licensing is beneficial to the licensor. Fig. 2 (b) shows that firm B's profit is initially greater in Model *N* than in the other models but decreases over time because the network effects stimulate consumers to purchase firm A's products. However, because technology licensing increases the network effects on firm B's products, firm B's profits in Models *LD* and *LS* increase over time and eventually become higher than those in Model *N*, indicating that technology licensing is profitable to the licensee in the long run. Moreover, we find that in this example, initially, a static royalty is more advantageous to the licensor; however, the increasing rate of the licensor's profit under the dynamic royalty is more significant than that under the static royalty. In the long run, the dynamic royalty is more beneficial to the licensor than the static royalty. Nevertheless, in this example, the static royalty is always more advantageous to the licensee than the dynamic royalty. Hence, the licensor and the licensee have different preferences regarding the settings of licensing royalties. For a better understanding of the firms' preferences regarding the licensing models, we

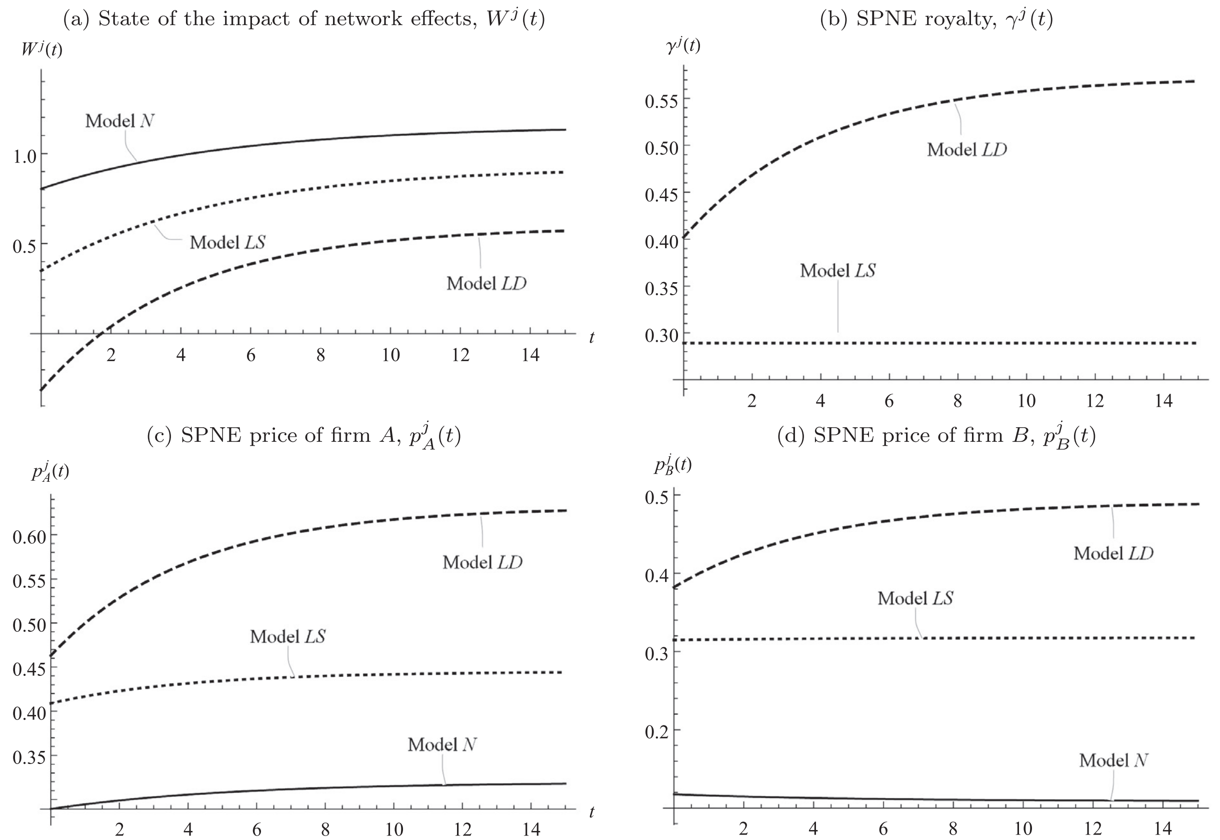

Fig. 1. Trajectories of the equilibrium results.

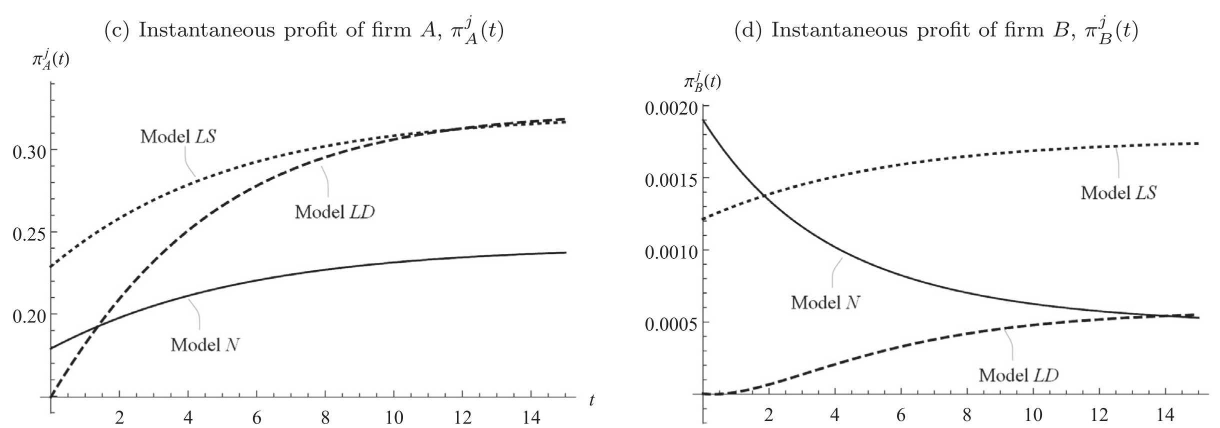

Fig. 2. Trajectories of the instantaneous profits.

conduct sensitivity analyses and compare the firms' profits at their steady-state equilibrium.

5.2. Analysis of the steady-state decisions and impact of network effects

To ensure that all the assumptions hold and the results are representative, we consider the following base settings for the equilibrium analyses under steady states: $c = 0.1$, $s = 0.2$, $\alpha = 0.5$, $\delta = 0.2$, $\eta = 0.4$, $\kappa = 0.7$, $\rho = 0.6$. According to the sensitivity analysis, the consumer valuation of firm *B*'s product (ρ) and the consumer sensitivity to the network effects (α) have significant impacts on the performances of each licensing option; thus, we focus on analysis of the steady-state equilibrium of these two parameters. Fig. 3 shows that an increase in ρ increases the competitiveness of firm *B*; therefore, it induces firm *A* to decrease the sales price, leading to more intense price competition in Model *N*. As a result, the network effects in Model *N* increase in ρ due to the low prices. However, in the presence of technology licensing, firm *A* will increase the unit royalty in ρ; therefore, firm *B* will change its behavior by increasing the sales price in ρ, which will mitigate the intensity of price competition and cause the network effects to decrease.

Fig. 4 shows that when the network effects become stronger, firm *A* becomes more competitive; therefore, firm *A*'s prices in all models will eventually increase. When firm *A*'s competitiveness increases in α, firm *B* is compelled to decrease its price to stimulate demand, and firm *A* will decrease its royalty because firm *B*'s demand shrinks. However, firm *B*'s price decreases in Models *N* and *LS* but increases in Model *LD* because Model *LD* leads firm *A* to

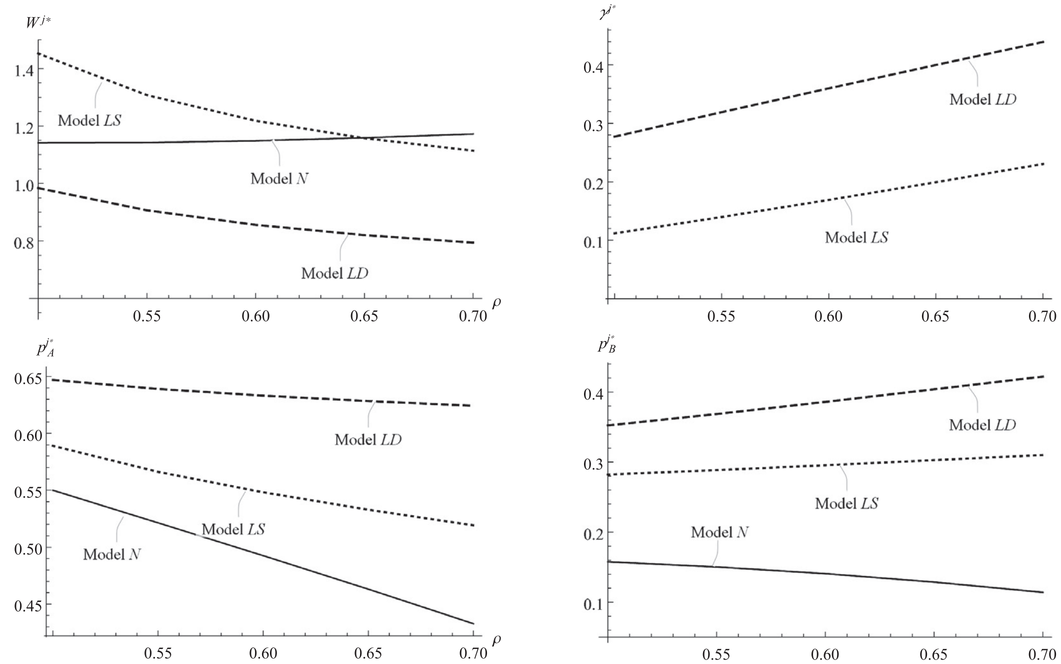

Fig. 3. Steady-state equilibrium analysis of the effect of consumer valuation of firm B's product (ρ) on the equilibrium results.

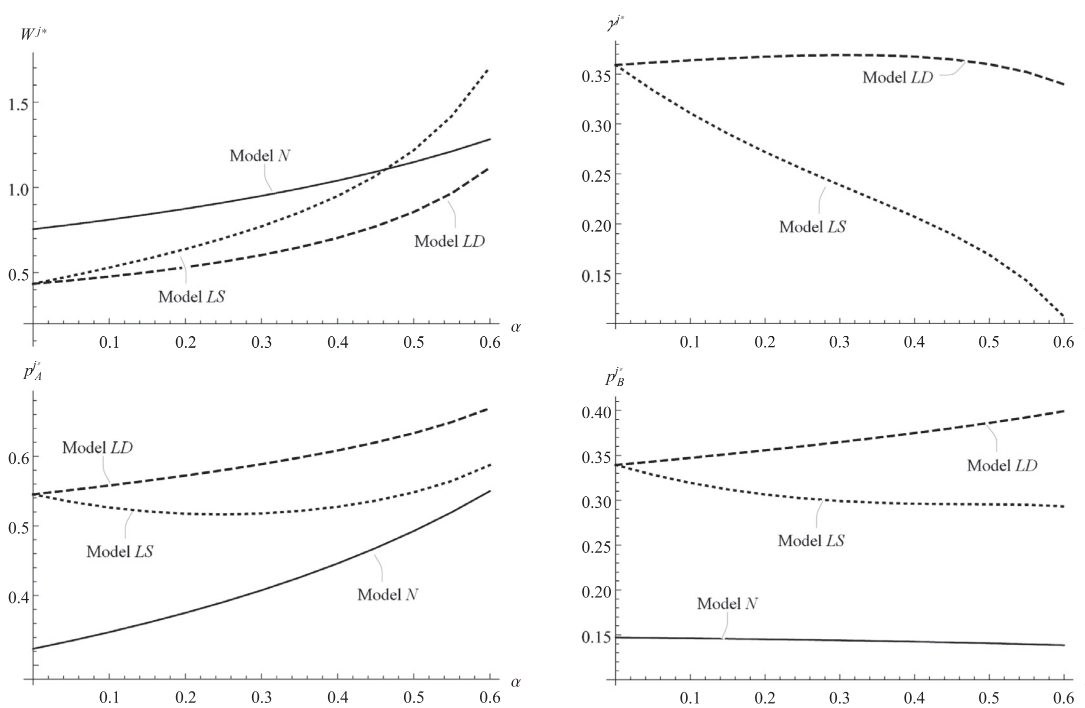

Fig. 4. Steady-state equilibrium analysis of the effect of consumer sensitivity to the network effects (α) on the equilibrium results.

choose a sufficiently high royalty; therefore, firm B sets a higher price in α. This increase in α has a direct effect on the network effect and causes it to increase. However, since p_B^{LD*} increases in α, the increasing rate of W^{LD*} is less significant than that of W^{LS*}. As a result, as the network effects become stronger, Model LS becomes more effective for increasing the total sales quantity.

By comparing Figs. 3 and 4, we first find that a dynamic royalty always causes the firms to set higher prices than those for a static royalty, indicating that the dynamic royalty is more effective in mitigating the intensity of price competition. Second, in the absence of technology licensing, the firms set the lowest prices, but the total sales quantity is not always larger than that in the licensing models, especially the model with the static royalty. Third, the dynamic royalty is higher than the static royalty, and it is significant when the network effects are stronger. Finally, in the absence of network effects (i.e., $\alpha = 0$), the licensing models perform identically at equilibrium, indicating that the choice of the royalty type is trivial to the licensor when the network effects are absent. This result verifies that the network effects are a key driver affecting the licensor's choice of the royalty types.

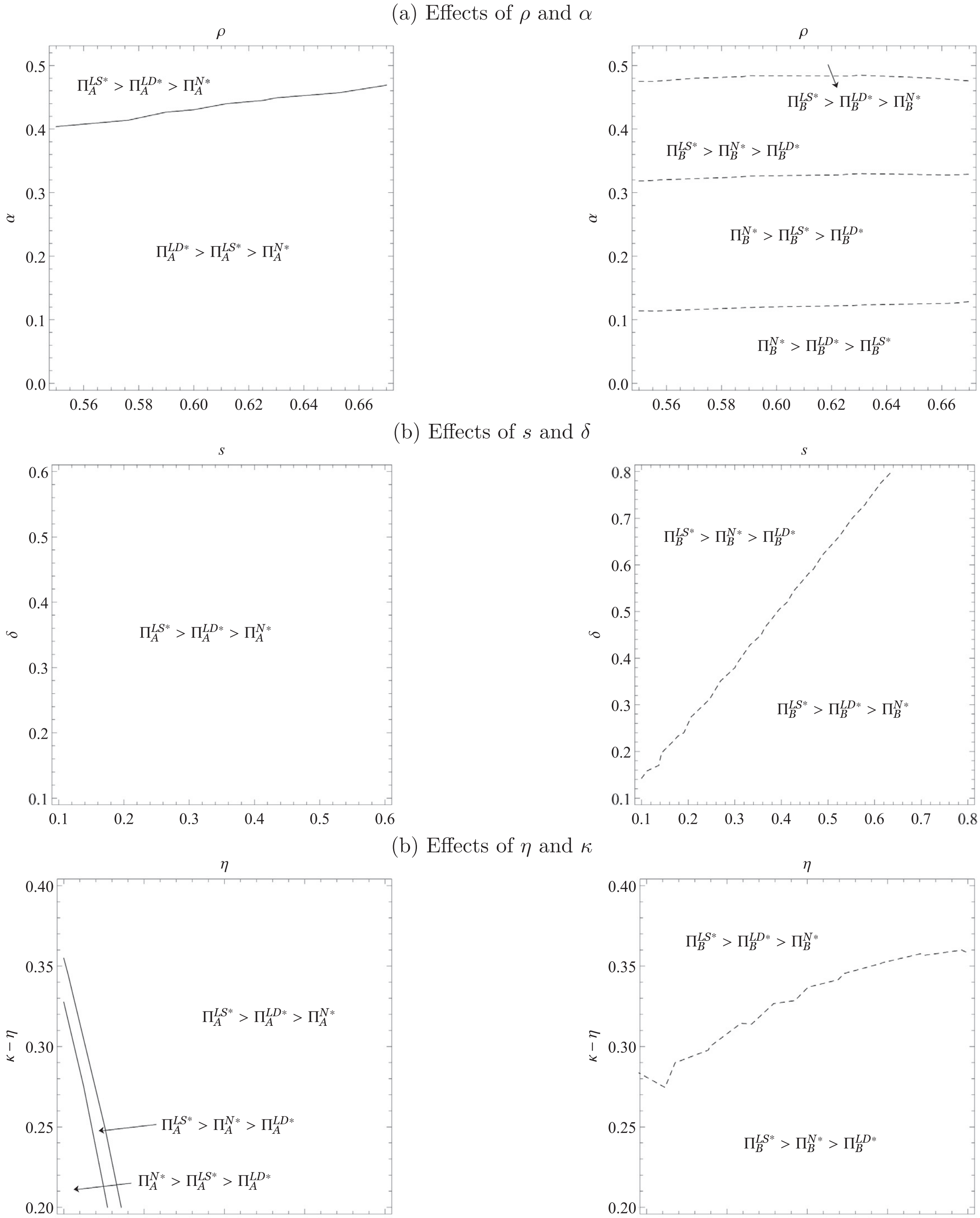

Fig. 5. Comparison of the firms' profits for all models.

5.3. Comparison of the firms' profits among the models

Fig. 5 shows the comparative results of the firms' profits for all three models. Fig. 5(a) shows that consumer sensitivity to network effects (α) and the valuation of firm *B*'s product (ρ) by consumers have different effects on firm *A*'s profits. Specifically, when α increases, the static royalty is preferable for firm *A*; however, when ρ increases, the converse is true. Moreover, α has more influence on firm *A*'s profit than ρ. Thus, the licensor's choice of licensing models mainly depends on consumer sensitivity to the network effects; specifically, an increase in consumer sensitivity to network effects causes the licensor to change its preference from the dynamic royalty (Model *LD*) to the static royalty (Model *LS*). Firm *B*'s preference for the models is more sensitive to α than to ρ. When the network effects are weak, the licensee does not consider technology licensing to be desirable. However, an increase in consumer sensitivity to the network effects will stimulate the licensee to adopt technology licensing, especially the licensing model with the static royalty. Therefore, when the network effects are stronger, both firms prefer technology licensing with the static royalty; otherwise, the licensor prefers to adopt the dynamic royalty, but the licensee is less likely to accept technology licensing.

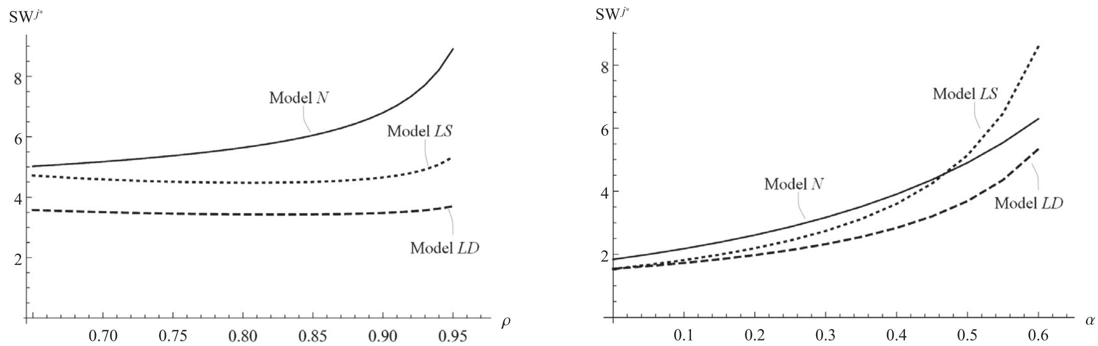

Fig. 6. Steady-state equilibrium analysis of the parametric effects on social welfare.

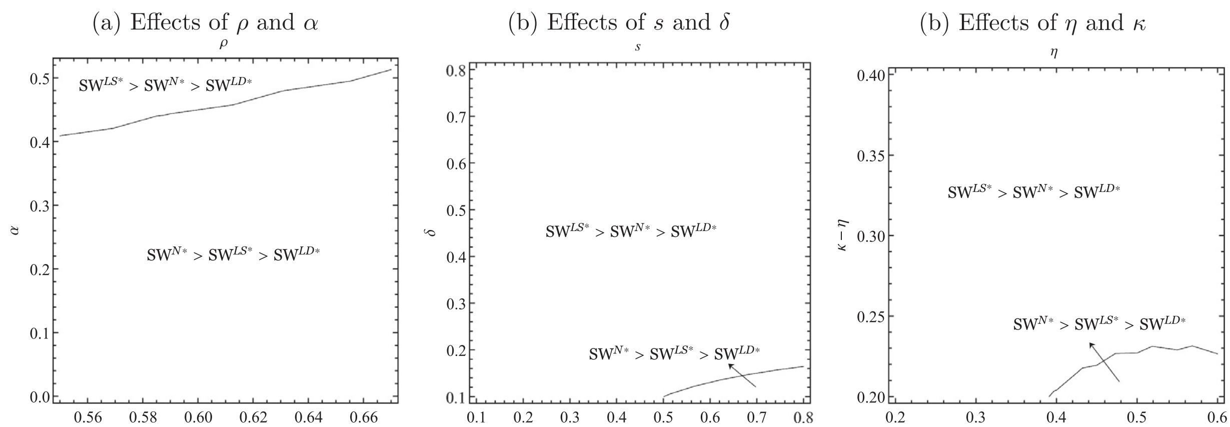

Fig. 7. Comparison of social welfare among all models.

Fig. 5(b) reveals that consumers' and firms' dynamic behaviors have a smaller impact on firm *A*'s preference for the models and do not alter the superiority of Model *LS* for firm *B*. However, when consumers and firms are less myopic (s increases, and δ decreases), technology licensing with the dynamic royalty is more profitable for firm *B* than the model without technology licensing. In addition, this result indicates that the dynamic nature of the network effects will induce firm *B* to adopt licensing with the dynamic royalty, which will be more profitable for firm *B* than the scenario without licensing.

Fig. 5(c) indicates that when the network effects on firm *B*'s demand weaken (η is lower) or when technology licensing is less effective for firm *B* to improve the network effects on its demand (the value of $\kappa - \eta$ is lower), technology licensing is less profitable for firm *A* because it is ineffective at weakening the intensity of price competition and decreases the competitiveness of firm *A*. In this case, firm *A* will not license its technology to firm *B*. To clarify, stronger network effects on firm *B*'s demand (a higher η) cause firm *B* to choose not to adopt technology licensing because its ability to improve the network effects is meaningless for firm *B*, and the demand already has strong network effects. Intuitively, the use of more effective technology for licensing to improve the network effects will increase the likelihood that firm *B* will accept the technology licensing. Additionally, for all of the parametric settings, the static royalty is more profitable for the licensee than the dynamic royalty.

5.4. Analysis of social welfare

We now consider the parametric effects on social welfare and the preference for the models by a social planner under the steady state. Following Chang, Hwang, and Peng (2013); Li and Ji (2010); Örsdemir, Kemahlioglu-Ziya and Parlaktuerk (2014); Zhao, Chen, Hong, and Liu (2014), we define social welfare as $SW^{j^*} = \Pi_A^{j^*} + \Pi_B^{j^*} + CS^{j^*}$, where CS^j ($j \in N, LD, LS$) denotes consumer surplus:

$$CS^{j^*} = \int_0^{\infty} e^{-\delta t}\left(cs(t)^j|_{p_i^j=p_i^{j^*},\gamma^j=\gamma^{j^*}}\right) dt,$$

$$cs(t)^j = \int_{\theta\in\Theta_A} U_A(t)\, f(\theta)\mathrm{d}\theta + \int_{\theta\in\Theta_B} \mathrm{U_B(t)\, f(\theta)d}\theta.$$

Fig. 6 reveals that SW^{N*} significantly increases in ρ, but the increases of SW^{LD*} and SW^{LS*} in α are more significant than that of SW^{N*}. Thus, when consumers give firm *B*'s products higher valuation, technology licensing between the firms is less preferred by the social planner. This result is obtained because when ρ increases, the models with technology licensing cause firms' sales prices to be higher than Model *N*, thus harming consumer surplus. However, when consumers are more sensitive to network effects, technology licensing is more advantageous to social welfare. This result is obtained because consumer sensitivity to network effects leads technology licensing to be more effective in improving market demand and thus benefits producer surplus (i.e., the sum of firms' profits).

Fig. 7 shows that in some cases, technology licensing conflicts with improving social welfare. Specifically, Fig. 7(a) indicates that when consumer sensitivity to network effects is high or consumer valuation of firm *B*'s products is low, technology licensing with a static royalty leads to higher social welfare than other models; thus, it is more preferred by social planners. Fig. 7(b) reveals that when consumers' and firms' dynamic behaviors are significant (s is high, and δ is low), technology licensing is less preferred by social planners. Fig. 7(b) shows that consumers' and firms' dynamic behaviors lead technology licensing to be more beneficial to the firms' profits and harm consumer surplus, thus reducing social welfare. Fig. 7(c) shows that when network effects on firm *B*'s

Table 2
Comparisons between the models.

	No technology licensing	Technology licensing Dynamic royalty	Static royalty
Royalty decision	–	High	Low
Intensity of price competition	High	Lowest	Lower
Preferred scenarios for OEM (licensor)	Network effects on the licensee's demand are insignificant	Network effects are wea	Preferred in the remaining scenarios
Preferred scenarios for 3rd-party manuf. (licensee)	Network effects are weak	Subdominant strategy when members are less myopic or licensing leads to strong network effects	Preferred in the remaining scenarios
Preferred scenarios for social planner (social welfare)	Network effects are weak, members are less myopic, and licensing improves less network effects	Not preferred by social planner	Preferred in the remaining scenarios

demand (η) are already high or when technology licensing is less effective for firm B to improve the network effects on its demand (the value of $\kappa - \eta$ is low), the increase in producer surplus cannot compensate for the decrease in consumer surplus; thus, social planners prefer firms to maintain their competition without technology cooperation. Overall, we find that social planners prefer firms to implement technology licensing with a static royalty; however, technology licensing with a dynamic royalty generally causes higher unit royalty and sales prices, harming firm B's profits and consumer surplus. Thus, it is less preferred by the social planner. As a result, when the preferences of the firms and the social planner differ, the social planner is recommended to employ some mechanisms, such as subsidy schemes for firms, to affect firms' choices of technology licensing or equilibrium decisions for higher social welfare.

6. Summary

In this paper, we examine technology licensing with network effects in the context of a dynamic competitive duopoly including an OEM and a third-party manufacturer. We develop dynamic models to compare the impact of technology licensing with both dynamic and static royalties and to account for the dynamics of the impact of network effects. To determine the characteristics of time-consistent and subgame-perfect equilibria, we derive the firms' feedback Nash equilibria. In addition, we identify the firms' instantaneous and steady-state equilibrium behavior and their preferences for different types of licensing royalties with respect to the different impacts of the parameters. Our results generate several insights, listed as follows.

- *Influences on price competition*. Network effects are generally effective for mitigating the intensity of pricing competition. This is true when technology licensing is adopted with a dynamic royalty. However, when technology licensing is implemented with a sufficiently high static royalty, the dynamic adjustment of the royalty is absent; therefore, the licensee seeks to increase and realize a quick return by lowering its prices under a high royalty. The network effects amplify this phenomenon, causing the firms to engage in aggressive price competition.
- *Impact of technology licensing*. When the dynamic behaviors of firms and consumers are more significant, technology licensing is more effective for enhancing the firms' profits, especially when technology licensing can significantly improve the strength of the network effects on the licensor's demand. In the extreme case when the dynamic behaviors of firms and consumers are significant, technology licensing may harm consumer surplus, which is not advantageous to social welfare.
- *Impact of network effects*. When the network effects are stronger, the OEM will institute a lower royalty but a higher sales price. However, the stronger network effects lead the third-party manufacturer to institute a lower sales price in both the absence of technology licensing and the presence of technology licensing with a static royalty; therefore, the total sales quantity increases. Conversely, the reverse behavior is true for the third-party manufacturer under technology licensing with a dynamic royalty. Moreover, under the stronger network effects, the social planner will encourage firms to cooperate with technology licensing.
- *Comparisons of the licensing models*. Compared with the impact of a dynamic royalty, a static royalty will lead to a lower royalty and lower prices but a larger market because of the stronger network effects. Because the static royalty increases the total sales quantity, in most cases, it is more beneficial for the firms. However, when the network effects are weak, the dynamic royalty is more profitable for the firms. When the network effects are sufficiently weak, the third-party manufacturer will not accept technology licensing. When the network effects on the demand of the third-party products are significant and when technology licensing is ineffective for improving the network effects for the third-party manufacturer, technology licensing is not beneficial for the OEM. In addition, because technology licensing with a dynamic royalty leads to a higher unit royalty and higher sales prices, harming the surplus of the licensee and consumers, technology licensing with a static royalty is commonly preferred by social planners. The main findings from the comparisons of the models are summarized in Table 2 for better understanding.

Possible extensions of our study are suggested to obtain a better understanding of the dynamic interactions that occur among firms when a technology licensing strategy is used. First, we consider that the firms provide substitutable products. It would be insightful to consider a situation in which the products are complementary because two-sided network effects (Armstrong, 2006; Rochet & Tirole, 2003) would emerge between the firms. Second, another possible direction is to divide consumers into two different types: one that is sensitive to network effects and one that is not. However, this direction would cause the demand functions to become complex. Third, it would be interesting to expand our model to examine issues of mechanism design, e.g., the design of subsidy schemes for firms or incentive policies for consumers, from the perspective of a social planner. It would be worthwhile to further investigate how to coordinate firms' licensing choices while improving social welfare.

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Supplementary material

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