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Decision Support

Profitability of horizontal mergers in the presence of price stickiness☆

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ABSTRACT

This paper investigates the profitability of horizontal mergers with price dynamics through the differential game approach wherein both the open and closed-loop equilibria are considered. It is shown that the incentive to merge is determined by how fast the market price adapts to the equilibrium level. When prices adjust with a very sticky mechanism, mergers emerge with a small number of insiders, even if firms play open-loop strategies, and total output reduction after a merger is not significant, even in mergers with a large number of insiders. In the case of instantaneous price adjustment, it can be shown that the relationship between the possibility of a merger and market concentration depends on the type of strategy firms play. These findings have important implications for antitrust authorities since: (a) price stickiness creates market conditions that facilitate merger practice, and (b) changes in output may not be a good benchmark for merger assessment in the case of price stickiness.

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1. Introduction

When quantity-setting firms with symmetric cost functions compete in a homogenous product market, a horizontal merger is modelled as an exogenous change in market structure. In such a setting, these mergers reduce the number of competitors in the industry. Accordingly, firms' market price and market power increase. Although non-participant firms benefit from increased market power, merger profitability is not guaranteed. In the case of linear demand and cost functions, the resulting anti-competitive forces benefit outsiders. Only when their market shares are quite high (at least 80% i.e. almost a monopoly) merging firms will favour the opportunity to merge (Gaudet & Salant, 1992; 1991; Salant, Switzer, & Reynolds, 1983). This threshold will be reduced to 50%, again a considerable market share, provided that the merged entity is not restricted to remain a Cournot player and can become a Stackelberg leader after the merger (Levin, 1990). By considering general demand functions, Cheung (1992) shows that at least half of the industry should merge in order for a merger to be profitable.

Assuming an asymmetric organization rather than considering an industry comprising entirely of identical firms, Daughety (1990) argues that in industries where almost less than one-third of firms are leaders, mergers will be profitable if they are leader-generating. The efficiency argument was first advocated by Perry and Porter (1985) who showed mergers are profitable provided that firms can benefit from some economies of scale. Then, Farrell and Shapiro (1990) discussed the issue extensively, and found that a horizontal merger which does not generate synergies will raise the price, which makes the merger profitable only when the merging firm's market share is large enough. However, the incentive to merge always exists once price is employed as a strategic variable rather than a quantity. Deneckere and Davidson (1985) demonstrate that in a differentiated product industry, mergers of any size will be beneficial if firms are engaged in a price-setting game.

The present study aims at finding out whether prices that are sticky and do not respond to market signals can be a cause of mergers. There is a lot of evidence on the frequency of changes in price, which suggests there is a significant degree of price stickiness. Price stickiness varies in different countries and different industries. For instance, in the US, on average, price-spell, the length of time that a product's price remains the same, lasts about 3.7 months according to Klenow and Kryvtsov (2008) while in Germany and Italy it lasts about 10 and 11.1 months, respectively (Hoffmann & Kurz-Kim, 2006) and (Fabiani, Gattulli, Veronese, & Sabbatini, 2010). However, the exclusion of temporary price changes (for example sales) raises this average considerably: in the US it rises to 11 months Nakamura and Steinsson (2008). Gorodnichenko and Weber (2016) made a comparison across various sectors of the US economy to show how sticky prices are in different industries. They report the average time between price

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changes of 3.7, 8.6, 5.2, 5, 7.6 and 11.8 months for agriculture, manufacturing, utilities, trade, finance and service industries, respectively. In UK manufacturing, Domberger (1979, 1980) shows that in some sectors such as engineer's small tools and instruments and electrical machinery, price stickiness is high whereas in mineral oil refining and paper and board it is low. In industries with price stickiness features, firms may have an incentive to change their productions to gain more profits for the reason that the actual market price does not instantaneously adjust to that indicated by the demand function. This study conducts an investigation into the long-term consequences of horizontal mergers in oligopolistic Cournot competition in the presence of price stickiness.[1] Here, price can be considered the state variable of a dynamic system.

In a dynamic game, the information set available to players should be specified when they make their decisions and choose their control variables at every instant. If they can observe the initial state of the world, but not the dynamics of the state, they will have to select the control actions as a function of time only; this solution concept is known as open-loop. However, if they observe and consider the evolution of the state, the solution concept will be different. Under the closed-loop memoryless decision rule, the players' current actions are conditioned on current and initial time in addition to current values of state variables (Basar & Olsder, 1982). With a feedback strategy (closed-loop perfect state information rule), control actions are chosen as a function of time and the entire path of state variables from initial to present time.

The open-loop strategy has an interesting feature that the corresponding equilibrium is much easier to compute than the memoryless closed-loop and feedback equilibria (Pineau, Rasata, & Zaccour, 2011). On the other hand, this strategy is not subgame perfect and, conceptually, is less attractive than the other two. This is due to the fact that the open-loop solution does not consider strategic interactions among the players through the evolution of state variables over time nor the associated adjustment in controls. However, when players cannot observe the world dynamics, or are forced to commit to their respective plans initially decided or when the planning horizon is short, this solution concept is realistic and more appropriate. Instead, in cases where players can observe the current state of the world and then make their decisions, closed-loop memoryless and feedback solutions are used under which competition is more intense and true interactions among players take place over time (Brekke, Cellini, Siciliani, & Straume, 2010).[2]

In a static homogeneous good model where there are no efficiency effects, if firms choose quantities (firms' decisions are strategic substitutes) a merger is not profitable unless it accounts for more than 80% of the total number of firms (Salant et al. (1983)'s results). Using an oligopolistic differential game model with sticky prices in the specific case of instantaneous price adjustment, Dockner and Gaunersdorfer (2001) showed that, contrary to the static game, if firms use feedback strategies, mergers are always profitable because quantities are far higher than the static Cournot model quantities and the corresponding equilibrium price is close to marginal cost, i.e. the Bertrand equilibrium outcome (where firms' decisions are strategic compliments).[3] Benchekroun (2003) demonstrates that when firms use open-loop strategies, a merger is profitable only if the market share of a merging firm is significantly high, which puts more emphasis on the role of closed-loop strategies to create an incentive to merge; referred to as the "closed-loop effect".[4] Esfahani and Lambertini (2012) used the same model for open-loop strategies to show that mergers between two out of three firms are profitable provided that demand is convex.[5]

Although, Dockner and Gaunersdorfer (2001) and Benchekroun (2003) have introduced price stickiness in their study to make the model dynamic, they did not consider the role of price stickiness on horizontal mergers' profitability; in fact, once the speed of price adjustment goes to infinity, price stickiness will disappear. In this paper, without introducing any specific assumption on the degree of price stickiness, the bearings of price dynamics are studied as a motive for mergers.[6] To this end, use has been made of a differential game approach with sticky price dynamics introduced by Simaan and Takayama (1978) and its extension by Fershtman and Kamien (1987) and Cellini and Lambertini (2004, 2007). Both the open-loop and the closed-loop memoryless equilibria have been considered to investigate how the speed of price adjustment can affect horizontal mergers' profitability. The results of this study show that when the price adjusts with a very sticky mechanism, privately profitable mergers emerge with a small number of insiders even if firms play open-loop strategies contrary to what Benchekroun (2003) has found for the limit case; which emphasises the "price stickiness effect" that creates an incentive to merge. Firms have incentives to increase their productions and gain more profits on the grounds that the price is sticky. However, in the long-run, the current price will adjust to the Cournot equilibrium and, hence, firms' increased productions will cause the long-term price to get closer to the competitive price, resulting in a significant reduction in firms' long-term profits. In this aggressive environment, the incentive to merge increases in order to decrease the number of competitors and recover what firms are losing slightly. Then, the impact of price stickiness on total output reduction after mergers is considered showing that even in mergers with a large number of insiders, output reductions are not significant in the event that price is very sticky. In addition, it is revealed that the relationship between the possibility of merger and market concentration depends on the nature of the competition.

The remainder of the paper is organized as follows; the model layout is presented in Section 2, the open-/closed-loop equilibria before and after the merger are illustrated in Sections 3 and 4, respectively. Section 5 assesses incentives towards mergers, and Section 6 concludes the paper.

2. The model

Consider a dynamic oligopoly market where n symmetric firms, at any $t \in [0, \infty)$, produce quantities $q_i(t) \geq 0$, $i \in \{1, 2, \ldots, n\}$, of homogeneous goods with concave technologies described by the cost functions

$$C_i(t) = cq_i(t) + \frac{1}{2}q_i^2(t), \qquad c > 0. \tag{1}$$

In each period, the product price, $\hat{p}(t)$, is determined by the inverse demand function

$$\hat{p}(t) = a - \sum_{i=1}^{n} q_i(t), \tag{2}$$

but since the price is sticky, the current market price does not instantaneously adjust to that given by the demand function,

[1] The focus is on the steady-state profit evaluation.

[2] For more discussion, see Bertinelli, Camacho, and Zou (2014); Wu (2018) and Dockner, Jorgensen, Van Long, and Sorger (2000).

[3] Fershtman and Kamien (1987, pp. 1159), have looked into the features of the feedback equilibrium in the limit when the speed of price adjustment tends to infinity and have demonstrated that in such circumstances, the feedback equilibrium coincides with the Bertrand equilibrium of the static game.

[4] Competition is stronger among firms when they use feedback strategies because they have the possibility to influence their competitors through the state variable.

[5] For more discussion about sticky price models see Colombo and Labrecciosa (2018) and Xin and Sun (2018).

[6] Since the concentration is on the merger incentives that are generated by price dynamics, scale economies has been ruled out by assumption.

meaning that $\hat{p}(t)$ will differ from the current price level, $p(t)$, and it will move according to the following equation

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s\left\{\hat{p}(t) - p(t)\right\}, \tag{3}$$

where $s \in (0, \infty)$ is a constant which determines the speed of price adjustment; the lower s is, the higher the degree of price stickiness will be. When s tends to infinity, the price is not sticky and the actual market price is equal to that given by the demand function.

Firm i's instantaneous profit function is

$$\pi_i(t) = q_i(t)\left[p(t) - c - \frac{1}{2}q_i(t)\right].$$

Therefore, its maximization problem is

$$\max_{q_i(t)} J_i = \int_0^\infty e^{-\rho t} q_i(t)\left[p(t) - c - \frac{1}{2}q_i(t)\right]dt, \tag{4}$$

subject to (3), $p(0) = p_0$ and $p(t) \geq 0$ for all $t \in [0, \infty)$. Factor $e^{-\rho t}$ discounts future gains, and the discount rate ρ is assumed to be constant and equal across firms.

The differential game is solved using both the open-loop information structure, where firms choose their production plans at the initial date and adhere to them for the entire time horizon, and the closed-loop memoryless information structure where the firms' quantity choices at any time depend on the initial and current levels of the state variable (here, price).

3. The pre-merger equilibrium

3.1. The open-loop equilibrium

In open-loop strategies, players choose a path of action $q_i(t)$ to which they commit themselves to the outset of the game. The Nash equilibrium in such strategies is an n-tuple of paths such that each player's path is the best response to its rivals' paths and results in the following steady state:

Lemma 1. *At the open-loop Nash equilibrium of the pre-merger game, the steady state equilibrium price and each firm's quantity are given by*

$$p^{OL} = a - nq^{OL}; \qquad q^{OL} = \frac{(a-c)(\rho+s)}{s+(n+1)(\rho+s)}, \tag{5}$$

where superscript OL indicates the open-loop equilibrium. The corresponding long-term profits are

$$\pi^{OL} = \frac{(a-c)^2(\rho+s)(\rho+3s)}{2[s+(n+1)(\rho+s)]^2}. \tag{6}$$

Proof. *See Appendix A.1.* □

In the limit case $(s \to \infty)$ where the price adjusts instantaneously, a firm's long-term profits are given by

$$\pi_\infty^{OL} = \frac{3(a-c)^2}{2(n+2)^2}, \tag{7}$$

which indeed corresponds to the equilibrium of a one-shot static Cournot game.

3.2. Closed-loop equilibrium

Under the closed-loop memoryless information structure, firms do not pre-commit to any path and consider, at every instant, the effects of the current level of state variables on controls at that time. The outcome of the pre-merger closed-loop memoryless game is summarized as follows:

Lemma 2. *At the closed-loop Nash equilibrium of the pre-merger game, the steady state equilibrium price and each firm's quantity are given by*

$$p^{CL} = a - nq^{CL}; \qquad q^{CL} = \frac{(a-c)(\rho+ns)}{s+(n+1)(\rho+ns)}, \tag{8}$$

where superscript CL denotes the closed-loop memoryless equilibrium. The corresponding long-term profits are

$$\pi^{CL} = \frac{(a-c)^2(\rho+ns)(\rho+(n+2)s)}{2[s+(n+1)(\rho+ns)]^2}. \tag{9}$$

Proof. *See Appendix A.2.* □

In the limit case of instantaneous price adjustment, a firm's long-term profits are given by

$$\pi_\infty^{CL} = \frac{(a-c)^2(n+2)n}{2(n(n+1)+1)^2}, \tag{10}$$

which is lower than one in (7) and implies more aggressive competition under the closed-loop information.

For later reference, let us also note that in the static game where cost and demand functions are specified by (1) and (2), in turn, when firms play *à la* Cournot and *à la* Bertrand equilibrium prices are, respectively,

$$p^{CN} = \frac{2a+nc}{n+2}, \tag{11}$$

and

$$p^{BN} = \frac{a+nc}{n+1}, \tag{12}$$

where *CN* and *BN* represent for Cournot-Nash and Bertrand-Nash equilibria, respectively.

4. The merger equilibrium

In this section, the horizontal merger of m firms $(1 < m < n)$ where they act jointly to maximize their discounted stream of profits, is considered.[7] Since $n - m$ firms stay outside the merger, the differential game becomes

$$\max_{\bar{q}_i} J^M = \int_0^\infty e^{-\rho t}\left[(p(t)-c)\sum_{i=1}^m \bar{q}_i(t) - \frac{1}{2}\sum_{i=1}^m \bar{q}_i^2(t)\right]dt, \quad i = 1, \ldots, m \tag{13}$$

$$\max_{q_j(t)} J_j = \int_0^\infty e^{-\rho t} q_j(t)\left[p(t) - c - \frac{1}{2}q_j(t)\right]dt, \qquad j = m+1, \ldots, n \tag{14}$$

subject to

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s\left\{a - \sum_{i=1}^m \bar{q}_i(t) - \sum_{j=m+1}^n q_j(t) - p(t)\right\}, \tag{15}$$

and the initial conditions $p(0) = p_0$ and $p(t) \geq 0$.

$\bar{q}_i(t) \geq 0$, $i \in \{1, 2, \ldots, m\}$ and $q_j(t) \geq 0$, $j \in \{m+1, \ldots, n\}$ denote, in turn, the output level of an insider and an outsider. Since firms are symmetric, the sequence of insiders and outsiders can be presented as $i = 1, \ldots, m$ and $j = m+1, \ldots, n$, respectively. J^M and

[7] Given the convex cost function, each plant's marginal cost is an increasing function of the quantity produced; thus, it is optimal to use all the plants by the merged entity.

J_j represent the problem of the merging firm and outsiders, respectively.

According to (13)–(15), the Hamiltonian functions of the merging firm and outsiders are

$$H^M(t) = e^{-\rho t}\left\{(p(t)-c)\sum_{i=1}^{m}\bar{q}_i(t) - \frac{1}{2}\sum_{i=1}^{m}\bar{q}_i^2(t) + \bar{\lambda}_i(t)s\left[a - \sum_{i=1}^{m}\bar{q}_i(t) - \sum_{j=m+1}^{n}q_j(t) - p(t)\right]\right\}, \tag{16}$$

$$H_j(t) = e^{-\rho t}\left\{q_j(t)\left[p(t) - c - \frac{1}{2}q_j(t)\right] + \lambda_j(t)s\left[a - \sum_{i=1}^{m}\bar{q}_i(t) - \sum_{j=m+1}^{n}q_j(t) - p(t)\right]\right\}, \tag{17}$$

where $\lambda_j(t) = \mu_j(t)e^{\rho t}$ and $\bar{\lambda}_i(t) = \bar{\mu}_i(t)e^{\rho t}$ and, $\mu_j(t)$ and $\bar{\mu}_i(t)$ are co-state variables associated with $p(t)$.

4.1. Open-loop equilibrium

Now, the post-merger Nash equilibrium under open-loop strategies is derived in this part. The outcome is summarized by the following proposition:

Proposition 1. *At the open-loop Nash equilibrium, the steady state levels of price and quantity of the merging firm and an outsider are*

$$p^{OL}_{post} = a - q^{OL}_M - (n-m)q^{OL}_O, \tag{18}$$

$$q^{OL}_M = Am(\rho + 2s), \qquad q^{OL}_O = A((m+1)s + \rho), \tag{19}$$

where A is a positive function of parameters a, c, n, m, s and ρ. Subscripts M and O show the equilibrium level of a variable for the merging firm and an outsider, and subscript "post" refers to the equilibrium price after the merger. Hence, the long-term equilibrium profits are

$$\pi^{OL}_M = \frac{A^2 m(\rho+2s)^2((2m+1)s+\rho)}{2(\rho+s)}, \tag{20}$$

$$\pi^{OL}_O = \frac{A^2(\rho+3s)((m+1)s+\rho)^2}{2(\rho+s)}. \tag{21}$$

Proof. See Appendix B. □

4.2. Closed-loop equilibrium

The outcome of the game between the merged entity and outsiders using closed-loop memoryless strategies is:

Proposition 2. *At the closed-loop Nash equilibrium, the steady state levels of price and quantity of the merging firm and an outsider are*

$$p^{CL}_{post} = a - q^{CL}_M - (n-m)q^{CL}_O, \tag{22}$$

$$q^{CL}_M = Bm(\rho + (n-m+1)s)\left(\rho + \left(m^2 - m + n + 1\right)s\right), \tag{23}$$

$$q^{CL}_O = B(\rho + (n + m(m-1))s)(\rho + s(n+1)), \tag{24}$$

where

$$B = (a-c)/\left[(n+1)\rho^2 + \left(n\left(m^2 - m + 2n + 3\right) + 2\right)\rho s + \left((n+1)\left(m^2 n - mn + n^2 + n + 1\right) - m^4 + m^3\right)s^2\right]$$

which yield the long-term equilibrium profits as follows

$$\pi^{CL}_M = \frac{1}{2}B^2 m(\rho + (n-m+1)s)(\rho + (n+m+1)s) \times (\rho + (m^2 - m + n + 1)s)^2, \tag{25}$$

$$\pi^{CL}_O = \frac{1}{2}B^2(\rho + s(n+1))^2\left(\rho + (m^2 - m + n)s\right) \times \left(\rho + (m^2 - m + n + 2)s\right). \tag{26}$$

Proof. Taking the first-order conditions w.r.t. $\bar{q}_i(t)$ and $q_j(t)$ and using (16) and (17), in turn,

$$\frac{\partial H^M(t)}{\partial \bar{q}_i(t)} = p(t) - c - \bar{q}_i(t) - \bar{\lambda}_i(t)s = 0, \tag{27}$$

$$\frac{\partial H_j(t)}{\partial q_j(t)} = p(t) - c - q_j(t) - \lambda_j(t)s = 0, \tag{28}$$

which yield the optimal closed-loop output for the insiders and outsiders respectively as follows

$$\bar{q}^{CL}_i(t) = \begin{cases}\left(p(t) - c - \bar{\lambda}_i(t)s\right) & \text{if } p(t) > c + \bar{\lambda}_i(t)s,\\ 0 & \text{otherwise,}\end{cases} \tag{29}$$

$$q^{CL}_j(t) = \begin{cases}\left(p(t) - c - \lambda_j(t)s\right) & \text{if } p(t) > c + \lambda_j(t)s,\\ 0 & \text{otherwise.}\end{cases} \tag{30}$$

The adjoint equations for the optimum are

$$-\frac{\partial H^M(t)}{\partial p(t)} - \sum_{j=m+1}^{n}\frac{\partial H^M(t)}{\partial q_j(t)}\frac{\partial q^{CL}_j(t)}{\partial p(t)} = \frac{\partial \bar{\lambda}_i(t)}{\partial t} - \rho\bar{\lambda}_i(t), \tag{31}$$

$$-\frac{\partial H_j(t)}{\partial p(t)} - \sum_{\substack{k=m+1,\\ k\neq j}}^{n}\frac{\partial H_j(t)}{\partial q_k(t)}\frac{\partial q^{CL}_k(t)}{\partial p(t)} - m\sum_{i=1}^{m}\frac{\partial H_j(t)}{\partial \bar{q}_i(t)}\frac{\partial \bar{q}^{CL}_i(t)}{\partial p(t)} = \frac{\partial \lambda_i(t)}{\partial t} - \rho\lambda_i(t). \tag{32}$$

The following transversality conditions must also hold

$$\lim_{t\to\infty}\bar{\mu}_i(t).p(t) = 0; \qquad \lim_{t\to\infty}\mu_j(t).p(t) = 0.$$

From (29) and (30) it is shown that the three strategic responses to the state variable are equal

$$\frac{\partial q^{CL}_j(t)}{\partial p(t)} = \frac{\partial q^{CL}_k(t)}{\partial p(t)} = \frac{\partial \bar{q}^{CL}_i(t)}{\partial p(t)} = 1. \tag{33}$$

Hence, using these reactions and inducing the symmetry assumption, Eqs.(31) and (32) can be rewritten as

$$\frac{\partial \bar{\lambda}(t)}{\partial t} = -m\bar{q}(t) + [\rho + (n-m+1)s]\bar{\lambda}(t), \tag{34}$$

$$\frac{\partial \lambda(t)}{\partial t} = -q(t) + [\rho + (n + m(m-1))s]\lambda(t). \tag{35}$$

Differentiating (29) and (30) w.r.t. time and using (34) and (35)

$$\frac{d\bar{q}^{CL}_i(t)}{dt} = \frac{dp(t)}{dt} - \left[-m\bar{q}(t) + [(m-n+1)s + \rho]\bar{\lambda}(t)\right]s, \tag{36}$$

$$\frac{dq^{CL}_j(t)}{dt} = \frac{dp(t)}{dt} - \left[-q(t) + \left[\left(-m^2 - n + m + 2\right)s + \rho\right]\lambda(t)\right]s. \tag{37}$$

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Using (15) and substituting for $\bar{\lambda}(t)$ and $\lambda(t)$ from (27) and (28) in (36) and (37), the following is obtained

$$\frac{d\bar{q}_i^{CL}(t)}{dt} = sa + [(n-m-1)s - \rho]c + [\rho - (n-m)s]p(t) \\ -s(n-m)q(t) + [(n-m-1)s - \rho]\bar{q}(t),$$

$$\frac{dq_j^{CL}(t)}{dt} = sa - c\Big[\rho - \big(m^2+n-m-2\big)s\Big] - sm\bar{q}(t) \\ +\Big[\rho - \big(m^2+n-m-1\big)s\Big]p(t) + \Big[\big(m^2-1\big)s - \rho\Big]q(t).$$

$d\bar{q}_i^{CL}(t)/dt = 0$, $dq_j^{CL}(t)/dt = 0$ and $dp(t)/dt = 0$, yield the steady state of the system and the equilibrium point is a saddle with (22)–(24). □

The difference between the closed- and open-loop solutions is due to the terms $\partial\bar{q}_i/\partial p$ and $\partial q/\partial p$ in equations (31) and (32) which are set equal to zero in the open-loop case.[8] In other words, in the closed-loop solution, the information regarding the dependency of other firms' supply policy on current market price is taken into account through the adjoint equation. These additional terms, which are not considered by definition in the open-loop solution, imply strategic interactions among firms. Furthermore, the adjoint equation of an insider (31) is different from that of an outsider (32). There is no strategic interaction among insiders, however, (32) shows that besides strategic interaction between an outsider and any of the insiders, there are strategic interactions among outsiders. Therefore, keeping the symmetry assumption, the two groups are necessarily asymmetric because while there is no strategic interaction among insiders, the rest of the market behave like dynamic Cournot competitors. These asymmetries between the two groups are not only with respect to first-order conditions and controls; in particular, they are with respect to co-state amounts. By construction, the list of co-state values entails that the shadow price attached by any outsider will be systematically different from that attached to price dynamics by an insider. First order conditions (27) and (28) can be rewritten as $\bar{\lambda}(t) = p(t) - c - \bar{q}(t)/s$ and $\lambda(t) = p(t) - c - q(t)/s$. Then, taking into account the fact that the output level of an outsider is greater than that of a single insider, the following result is obtained

Corollary 1. *An insider's shadow price is greater than that of an outsider's* $\big(\bar{\lambda}(t) > \lambda(t)\big)$.

This indicates that, on account of alterations in the state equation, the proportional change of the merging firm's profit is more than that of an outsider's.

5. The incentive to merge

Finding post-merger equilibrium, it is possible to investigate the profitability of a horizontal merger with price dynamics in a Cournot competition; to do that, the firms' profits before and after the merger should be compared. Any decrease in the number of strategic players is to the benefit of firms outside the merger, whereas for insiders, the incentive to merge exists if the difference between the merging firm's post-merger profits and the sum of individual insiders' profits prior to the merger is positive. In other words, in an n-firm industry, firms find it profitable to merge if, and only if, the merger profitability conditions $\pi_M^{OL} - m\pi^{OL} > 0$ and $\pi_M^{CL} - m\pi^{CL} > 0$ hold for the open and closed-loop equilibria, respectively.

[8] In the open-loop solution, the adjoint equations for an insider and an outsider, respectively, are as follows $-\frac{\partial H^M(t)}{\partial p(t)} = \frac{\partial\bar{\lambda}_i(t)}{\partial t} - \rho\bar{\lambda}_i(t)$; $\quad -\frac{\partial H_j(t)}{\partial p(t)} = \frac{\partial\lambda_i(t)}{\partial t} - \rho\lambda_i(t)$

First, the profitability of horizontal mergers for instantaneous price adjustment is investigated to examine the closed-loop effects on merger profitability and then evaluation is done for general speed of price adjustment to perceive the role of price stickiness in stimulating merger incentives. Finally, the impact of each effect on total output changes is distinguished.

5.1. Closed-loop effect

Consider the case wherein, on account of infinity of price adjustment speed, the price is not sticky. The comparison of the firms' profits suggests that:

Proposition 3. *In the case of instantaneous price adjustment, the required proportion of insiders to make the merger profitable is significantly lower when firms employ closed-loop (memoryless) strategies as compared to open-loop ones.*

Proof. Using (6), (20), (9) and (25), the merger profitability for the open and closed-loop strategies are, respectively

$$\Pi^{OL} \equiv \pi_M^{OL} - m\pi^{OL} = \frac{1}{2}(a-c)^2 F^{OL}(n,m,s,\rho)G^{OL}(n,m,s,\rho), \tag{38}$$

$$\Pi^{CL} \equiv \pi_M^{CL} - m\pi^{CL} = \frac{1}{2}(a-c)^2 F^{CL}(n,m,s,\rho)G^{CL}(n,m,s,\rho), \tag{39}$$

where functions $F(n,m,s,\rho) \in \mathbb{R}^+$ and $G(n,m,s,\rho) \in \mathbb{R}$ are introduced in Appendix C. Given instantaneous price adjustment, profitability conditions are

$$\Pi_\infty^{OL} \equiv \lim_{s\to+\infty} \Pi^{OL} \gtreqless 0 \Leftrightarrow V^{OL}(n,k) \gtreqless 0,$$

$$\Pi_\infty^{CL} \equiv \lim_{s\to+\infty} \Pi^{CL} \gtreqless 0 \Leftrightarrow V^{CL}(n,k) \gtreqless 0,$$

with

$$V^{OL}(n,k) = 3n^3k(1-k)^2 - n^2\big(15k^2 - 6k - 1\big) + 4(n+1),$$

and

$$\begin{aligned} V^{CL}(n,k) = {} & n^7k^6(n+2)(nk-1) - n^4k^4(nk-1) \\ & \times \big(n^4 + 4n^3 + n^2 - 2n - 1\big) \\ & - n^3k^3\big(5n^4 + 10n^3 + 8n^2 + 2n - 1\big) \\ & + n^2k^2(n+1)^2\big(3n^2 + 2n + 1\big) \\ & + nk(n+1)^2\big(n^4 - 2n^3 - 3n^2 - 4n - 1\big) \\ & + \big(n^3 + 2n^2 + 2n + 1\big)^2, \end{aligned}$$

where $k = m/n$. Solving $V^{OL}(n,k) = 0$ and $V^{CL}(n,k) = 0$, the required proportions of insiders to make the merger profitable for the open-loop, k^{OL}, and closed-loop, k^{CL}, equilibria are obtained.

Differentiating $V(n,k) = 0$ w.r.t. k,

$$\frac{dV}{dn} + \frac{dV}{dk}\frac{dk}{dn} = 0 \Rightarrow \frac{dk}{dn} = -\frac{dV}{dn}\Big/\frac{dV}{dk}.$$

Since $\frac{dV}{dk}$ is positive for both strategies, $\frac{dV^{OL}}{dn}\Big|_{k=k^{OL}} < 0$ and $\frac{dV^{CL}}{dn}\Big|_{k=k^{CL}} > 0$, therefore, $\frac{dk^{OL}}{dn} > 0$ and $\frac{dk^{CL}}{dn} < 0$. Moreover, at $n = 3$, $k^{OL} > k^{CL}$, which concludes the proof. □

Fig. 1 graphically illustrates the corresponding results in the $(m/n,\ n)$ space. The two curves are the loci of points where $\Pi_\infty^{OL}(n,m) = 0$ and $\Pi_\infty^{CL}(n,m) = 0$. Points above each curve represent the m/n proportion that makes the merger profitable for the

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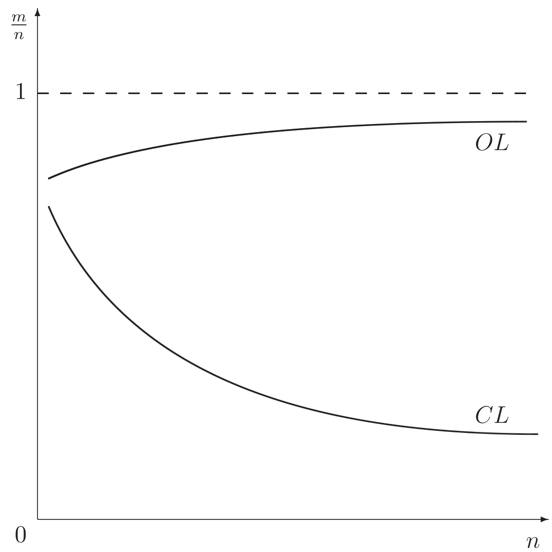

Fig. 1. The minimum proportion of firms to be merged to make a profitable merger when price adjusts immediately.

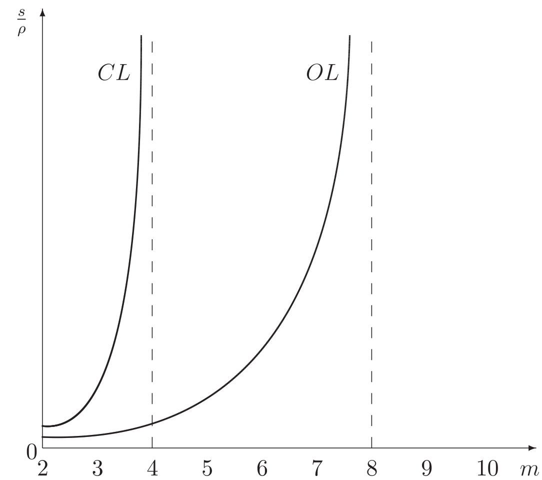

Fig. 2. Merger profitability in a 10−firm industry.

corresponding strategy. In the region below the curves, the number of insiders is not enough to mitigate intense competition sufficiently so that the merger becomes profitable. This proportion, for any n, is lower in the closed-loop equilibrium as shown plainly in Fig. 1.

The relative number of firms required for the merger to be profitable is a decreasing function of firms' population in the industry when firms play closed-loop, in contrast to when they play open-loop. Thus, in comparison, it is much easier for firms to conduct a merger in the closed-loop equilibrium. This difference is due to the fact that "open-loop" and "closed-loop" refer to two different information structures; in the closed-loop information structure, strategic interactions are explicitly incorporated in co-state equations, but in the open-loop they are not.

Our finding in the open-loop equilibrium is similar to that of Salant et al. (1983)'s because, as proved by Fershtman and Kamien (1987), when the adjustment speed tends to infinity, the static Cournot equilibrium is the limit of the open-loop Nash equilibrium; therefore, it may be argued that when market concentration is low, coordination among firms is quite a demanding task. Hence, according to Salant et al. (1983), regulators need not concern themselves with blocking mergers. However, under the closed-loop strategies, players react to "state" variable changes in each period and, thus, competition is more intense and any decrease in the number of interacting firms could affect the degree of competition and firms' profits substantially. Therefore, it could be argued that the more competitive the market is, the easier the merger to accomplish, meaning that considerably fewer firms are required to make the merger profitable.

Benchekroun (2003) and Dockner and Gaunersdorfer (2001) have shown that under the feedback rule, any merger is profitable regardless of the number of merging firms. It has to be noted that current and initial state levels are considered under the closed-loop memoryless solution while the entire past history of state is taken into account at each instant under the feedback solution. Under such conditions, competition is much tougher and gets closer to perfect competition. Therefore, any small changes in market structure can have significant effects on the price and, hence, on the profits of the merging firm. Accordingly, the larger the relevant information set is, the higher the possibility of merger between firms will be.

5.2. Price-stickiness effect

Now, the profitability of the horizontal merger will be dealt with in the presence of price stickiness. When price is sticky, firms would rather produce the correct Cournot equilibrium level of output, but this is not possible because it takes time for current prices to adjust to the dynamic Cournot equilibrium. The stickier the price is, the lesser the effects of the firms' production changes on the current price variations will be; firms will have the incentive to increase their productions and gain more profits from the price stickiness feature. However, in the long run, the current price will adjust to the Cournot equilibrium and, hence, the firms' increased production will cause the long-term price to get closer to the competitive price, causing the firms' long-term profit to reduce significantly. Such conditions provoke mergers, because in this aggressive environment, there is motivation to decrease the number of competitors through mergers in order to make a slight correction in output setting mistakes and recover what they are losing. Therefore, the stickier the price is, the higher the incentive towards a merger. This discussion amounts to the following proposition:

Proposition 4. *The incentive to merge decreases with price adjustment speed.*

Proof. Considering (38) and (39), the price stickiness effect on the merger incentive is described by

$$\frac{d\Pi}{ds} = \frac{1}{2}(a-c)^2\left(\frac{dF}{ds}G + F\frac{dG}{ds}\right), \tag{40}$$

for both *OL* and *CL*. Given the fact that $F \in \mathbb{R}^+$, and evaluating (40) at the border of merger profitability, i.e. $G(n, m, s, \rho) = 0$, $\frac{d\Pi}{ds}$ has the same sign as $\frac{dG}{ds}$ which is negative.[9] Hence, the merger profitability decreases with the speed of price adjustment. □

Fig. 2 shows merger profitability in a 10-firm industry in the (m, s) space for a given level of discount rate (ρ).[10] Curves *OL* and *CL* indicate the points at which firms are indifferent towards merging and not merging under the open and closed-loop equilibria, respectively. Mergers are profitable on the right hand side of the curves. It can be seen that even small mergers are motivated by the resulting reduction in competitivity when the price is very sticky.

[9] F and G are introduced in Appendix C.

[10] Since $n = 10$, the profitability of the merger in this setting depends only on the amounts of m and s/ρ.

The merger can be profitable even for a small number of insiders provided that the price adjustment speed is low enough, meaning merger incentives are higher when this speed is slower. Irrespective of the information structure, if the price adjusts very slowly, the equilibrium price is very close to the perfectly competitive one (in the limit, if $s \to 0$, it collapses onto the competitive price $(a+nc)/(n+1)$, as in (12)). In games where firms are Bertrand competitors in homogeneous goods, the mergers' profitability is driven by an increase in market price generated by the reduction in the population of firms, which benefits both insiders and outsiders alike.

Fig. 2 shows again that in the open-loop equilibrium, when the adjustment speed tends to infinity, the merger must involve at least 80 percent of firms to become profitable. However, in the closed-loop Nash equilibrium, as this figure clearly displays, a merger of four firms in a 10-firm industry is always profitable due to the closed-loop rule properties explained earlier.

5.3. Effects on outputs

Now, the impact of price stickiness and the closed-loop effect on total output is considered as changes in total output are important for antitrust authorities. According to Horizontal Merger Guidelines, in markets for homogeneous products, agencies regard the possibility of significant post-merger output reduction that can drive up market prices. By considering relative changes in total supply due to the merger, ΔQ, it is going to be examined whether changes in output could be a good benchmark for merger assessment in markets with homogeneous products when prices are sticky. This relative change for the open and closed-loop strategies respectively are

$$\Delta Q^{OL} = \left(q_M^{OL} + (n-m)q_O^{OL} - nq^{OL}\right)/nq^{OL}, \tag{41}$$

$$\Delta Q^{CL} = \left(q_M^{CL} + (n-m)q_O^{CL} - nq^{CL}\right)/nq^{CL}. \tag{42}$$

where q indicates the output of a firm before the merger. After the merger, q_M and q_O represent supplies of the merging entity and an outsider, respectively. Substituting from (5) and (19) into (41) and from (8), (23) and (24) into (42), the followings are obtained

$$\Delta Q^{OL} = -I(n, m, s/\rho), \tag{43}$$

$$\Delta Q^{CL} = -J(n, m, s/\rho), \tag{44}$$

where I and J are real-valued positive functions of parameters. Therefore, under both strategies, ΔQ is always negative, which means that total output reduces after the merger. However, the amount of reduction depends on the degree of price stickiness and the type of strategy firms play.

Proposition 5. *The absolute value of the total output change:*
(i) increases with the speed of price adjustment in the open-loop,
(ii) is lower in the closed-loop compared to the open-loop.

Proof. Differentiating (43) with respect to s/ρ,

$$\frac{d\left(\Delta Q^{OL}\right)}{d(s/\rho)} = -\frac{m(m-1)\left[(n-m)(m-1)\gamma^2 + (n+1)\left(4\gamma^2 + 4\gamma + 1\right)\right]}{nH^2}, \tag{45}$$

where $\gamma = s/\rho$, and $H(n, m, \gamma)$ is a real function of parameters. Clearly, $d(\Delta Q^{OL})/d(s/\rho)$ is negative which means that for a given discount rate, ρ, ΔQ^{OL} decreases with s. Since ΔQ^{OL} has a negative value, this concludes (i). Using (43) and (44) for any $s \in (0, \infty)$, we find that $\Delta Q^{CL} - \Delta Q^{OL} > 0$ which concludes (ii). □

A geometric illustration of the results is presented in Fig. 3a and b. In a 10-firm industry, Fig. 3a demonstrates the closed-loop effect and Fig. 3b represents the impact of price stickiness on industry output in the space of $(m, \Delta Q)$ and $(s/\rho, \Delta Q)$, respectively.[11] In Fig. 3a, price adjusts instantaneously and the relative changes in output are depicted for a different number of merging firms from 2 to 8. As shown, changes in total output increase with the number of insiders to the merger. This difference is greater for open-loop strategies. Clearly, as the concentration is higher after the merger, it is easier for antitrust authorities to verify. Although, a merger with a few insiders does not change output dramatically when firms play closed-loop strategies, it is already known that antitrust authorities do not care about mergers with small market shares. Consequently, absent price stickiness, it does not complicate the diagnosis for antitrust even if firms play closed-loop strategies.

Fig. 3b, for a given amount of interest rate, represents output changes after the merger based on the degree of price stickiness when 80 percent of the firms participate in the merger; taking into account a merger with a large number of insiders, would certainly help to better clarify what happens when price is sticky. This figure clearly shows that differences in total output are hard to distinguish when prices are very sticky even in cases where a large merger is performed under the open-loop strategy. Therefore, while private merger incentives increase with price stickiness, antitrust authorities will receive a weak signal from changes in output. This could generate new industry conditions which raise anticompetitive effects, and could then be detrimental to social welfare as firms are more likely to attain a collusive outcome.

6. Conclusions

This paper investigates the profitability of horizontal mergers within dynamic oligopolistic industries featuring price stickiness. In view of the fact that the focus is on price dynamics-generated incentives to merge, any efficiency effects are assumed away. When there is no cost saving, any decrease in the number of firms is socially harmful for the reason that the decrease in producer surplus does not compensate for the decline in consumer surplus. Hence, regulators must watch out for mergers motivated by the reduction of competition, when prices are sticky. In markets for homogeneous products, agencies evaluate significant output suppressions for the likely competitive effects of the merger. It turns out that when the degree of price stickiness is high, the amount of total output after a merger is not so different from that of before the merger. Thus, antitrust authorities may not carry out investigations even on big mergers with a large number of insiders when the price is very sticky, for instance in the service sector and some manufacturing industries, since they receive weak signals from output changes. In this case, permitted mergers are likely to adversely affect the competitive process, resulting in reduced social welfare.

The relationship between market concentration and price stickiness is ambiguous in the related literature. Some articles suggest that this relationship is positive, whereas others imply that it is negative. In a well-received study, Bedrossian and Moschos (1988) show that price will be stickier the higher the degree of concentration is. On the other hand, the present study's results show that price stickiness motivates mergers and, as a result, increases concentration. Therefore, according to Bedrossian and Moschos (1988) this increased concentration will reduce the speed of price adjustment, which, in turn, creates more incentive to merge and further increases concentration. This intensifies the importance of merger incentives in the presence of price stickiness.

[11] ΔQ is only a function of n, m and s/ρ.

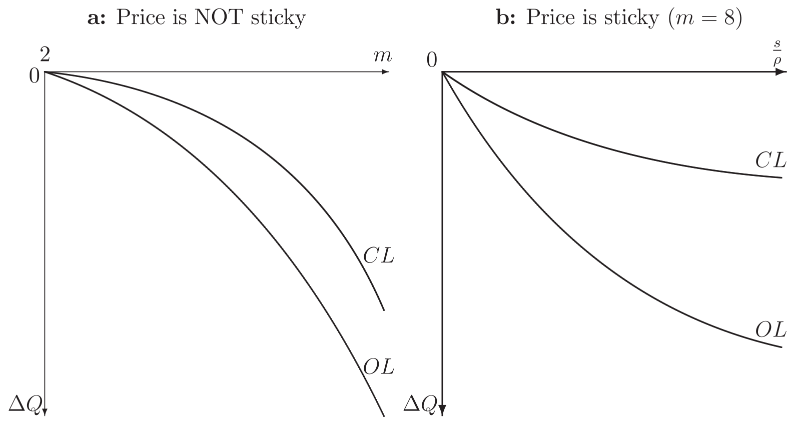

Fig. 3. Total output changes after merger in a 10−firm industry.

When price adjustment speed is high, the results of this study are in line with those of Benchekroun (2003) and Dockner and Gaunersdorfer (2001). That is, when firms play open-loop, mergers are profitable only when they result in the market structure approaching almost a monopoly (similar to the results of Salant et al. (1983) in the static competition). Whereas when firms play closed-loop, the required number of insiders to make a merger profitable is considerably lower than in open-loop. In fact, this study shows that the relative number of firms required for the merger to be profitable has two divergent trends under open and closed-loop rules, which implies that the less concentrated the market is, the easier the merger is to accomplish when firms play closed-loop in contrast to when they play open-loop. Since pushing competition has a contradictory outcome under each rule, it is worthwhile for policy makers and antitrust authorities to consider the nature of competition in the industry.

Appendix A

A1. Proof of Lemma 1:

According to Cellini and Lambertini (2004), the Hamiltonian function of problem (4) is

$$H_i(t) = e^{-\rho t}\left\{q_i(t)\left[p(t) - c - \frac{1}{2}q_i(t)\right] + \lambda_i(t)s\left[a - \sum_{i=1}^{n} q_i(t) - p(t)\right]\right\},$$

where $\lambda_i(t) = \mu_i(t)e^{\rho t}$ and $\mu_i(t)$ is the co-state variable associated with the state Eq. (3). Considering the first-order condition for firm i, we have

$$q_i(t) = \begin{cases} (p(t) - c - \lambda_i(t)s) & \text{if } p(t) > c + \lambda_i(t)s, \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

The adjoint equation for the optimum and the transversality condition are

$$-\frac{\partial H_i(t)}{\partial p(t)} = -q_i(t) + \lambda_i(t)s = \frac{\partial \lambda_i(t)}{\partial t} - \rho\lambda_i(t), \tag{47}$$

$$\lim_{t\to\infty} \mu_i(t).p(t) = 0. \tag{48}$$

Differentiating (46) and using (47) and (3) and inducing symmetry among firms, we obtain

$$\frac{dq(t)}{dt} \equiv \dot{q}(t) = as + (s+\rho)c - (2s+\rho)p(t) + [s(2-n)+\rho]q(t).$$

The equation $\dot{q}(t) = 0$ together with $\dot{p}(t) = 0$, characterizes the steady state equilibrium is (5) and (6).

A2. Proof of Lemma 2:

In the closed-loop memoryless the first-order and transversality conditions are the same as (46) and (48); but the adjoint equation is

$$-\frac{\partial H_i(t)}{\partial p(t)} - \sum_{j\neq i} \frac{\partial H_i(t)}{\partial q_j(t)} \frac{\partial q_j^{CL}(t)}{\partial p(t)} = \frac{\partial \lambda_i(t)}{\partial t} - \rho\lambda_i(t). \tag{49}$$

Now, we assume that $\frac{\partial H_i}{\partial q_j} = -\lambda_j s$ and $\frac{\partial q_j^{CL}}{\partial p} = 1$. Thus, simplifying (49), we obtain

$$\frac{\partial \lambda_i(t)}{\partial t} = -q_i(t) + \sum_i \lambda_i(t)s + \rho\lambda_i(t). \tag{50}$$

Differentiating (46) w.r.t. time, using (50) and (3) and assuming symmetry, we have

$$\frac{dq(t)}{dt} \equiv \dot{q}(t) = as + (ns+\rho)c - [(n+1)s+\rho]p(t) + (s+\rho)q(t). \tag{51}$$

The steady state equilibrium is driven from solving the system $\dot{q}(t) = 0$ and $\dot{p}(t) = 0$, which results to the equilibrium characterizes in (8) and (9).

Appendix B

Proof of proposition 1:

Taking the first-order conditions w.r.t. $\bar{q}_i(t)$ and $q_j(t)$ and using (16) and (17), in turn, we will have

$$\frac{\partial H^M(t)}{\partial \bar{q}_i(t)} = p(t) - c - \bar{q}_i(t) - \bar{\lambda}_i(t)s = 0, \tag{52}$$

$$\frac{\partial H_j(t)}{\partial q_j(t)} = p(t) - c - q_j(t) - \lambda_j(t)s = 0, \tag{53}$$

which yield the optimal output for, respectively, the insiders and outsiders as follows

$$\bar{q}_i(t) = \begin{cases} p(t) - c - \bar{\lambda}_i(t)s & \text{if } p(t) > c + \bar{\lambda}_i(t)s, \\ 0 & \text{otherwise,} \end{cases} \tag{54}$$

$$q_j(t) = \begin{cases} p(t) - c - \lambda_j(t)s & \text{if } p(t) > c + \lambda_j(t)s, \\ 0 & \text{otherwise.} \end{cases} \tag{55}$$

The adjoint equations for the optimum are

$$-\frac{\partial H^M(t)}{\partial p(t)} = -\sum_{i=1}^{m}\bar{q}_i(t) + \bar{\lambda}_i(t)s = \frac{\partial\bar{\lambda}_i(t)}{\partial t} - \rho\bar{\lambda}_i(t), \tag{56}$$

$$-\frac{\partial H_j(t)}{\partial p(t)} = -q_j(t) + \lambda_j(t)s = \frac{\partial\lambda_j(t)}{\partial t} - \rho\lambda_j(t). \tag{57}$$

The following transversality conditions must also hold

$$\lim_{t\to\infty}\bar{\mu}_i(t).p(t) = 0; \qquad \lim_{t\to\infty}\mu_j(t).p(t) = 0.$$

Differentiating (54) and (55), using (56) and (57) and invoking symmetry among the group of insiders and outsiders, we will find

$$\frac{d\bar{q}(t)}{dt} = \frac{dp(t)}{dt} - \left[(\rho+s)\bar{\lambda}(t) - m\bar{q}(t)\right]s, \tag{58}$$

$$\frac{dq(t)}{dt} = \frac{dp(t)}{dt} - [(\rho+s)\lambda(t) - q(t)]s. \tag{59}$$

Using (15), (54) and (55), we can rewrite (58) and (59) as follows

$$\frac{d\bar{q}(t)}{dt} = sa + c(\rho+s) - (\rho+2s)p(t) + (\rho+s)\bar{q}(t) - s(n-m)q(t),$$

$$\frac{dq(t)}{dt} = sa + c(\rho+s) - (\rho+2s)p(t) - sm\bar{q}(t) - [(n-m-2)s - \rho]q(t).$$

$d\bar{q}(t)/dt = 0$, $dq(t)/dt = 0$ and $dp(t)/dt = 0$, yield the steady state of the system and the equilibrium point is a saddle with (18) and (19), where

$$A = \frac{(a-c)(\rho+s)}{(\rho+s)^2 + ms[(\rho+s)(n-m+1)+s] + (\rho+s)[\rho n + s(n+m+1)]}.$$

Appendix C

In Section 5, $F^{OL}(n, m, s, \rho)$, $G^{OL}(n, m, s, \rho)$, $F^{CL}(n, m, s, \rho)$ and $G^{CL}(n, m, s, \rho)$ are given by, respectively,

$$F^{OL} = \frac{m\gamma(m-1)(\gamma+1)}{2((n+1)(\gamma+1)+\gamma)^2\left[(m-1)\gamma^2 - \gamma(\gamma+1)\left(m^2-2m-3\right) + n(\gamma+1)(\gamma(m+1)+1)+1\right]^2},$$

$$\begin{aligned}G^{OL} = &-m^3\gamma(\gamma+1)^2(3\gamma+1) + m^2\gamma(\gamma+1)(3\gamma+1)\\ &\times((2n+3)(\gamma+1)+2\gamma) - m(3\gamma+1)\left(2n\left(\gamma^3-2\gamma-1\right)\right.\\ &\left.+n^2\gamma(\gamma+1)^2 - 6\gamma^2 - 7\gamma - 2\right)\\ &-\gamma(\gamma+1)((n+1)(\gamma+1)+\gamma)^2,\end{aligned}$$

$$F^{CL} = \frac{m\gamma(m-1)}{2\left(\left(n^2+n+1\right)\gamma+n+1\right)^2\left[\left(m^3-m^4+1\right)\gamma^2 + n^2\gamma^2(m(m-1)+2) + n(\gamma+1)(\gamma(m(m-1)+2)+1) + 2\gamma\left(n^2+1\right) + n^3\gamma^2 + 1\right]^2},$$

$$\begin{aligned}G^{CL} = &-m^6\gamma^3(n\gamma+1)((n+2)\gamma+1)(m-1)\\ &+m^5\gamma^2\left(n^4\gamma^3 + 4n^3\gamma^2(\gamma+1) - \gamma(\gamma+1)^2\right.\\ &\left.-2n\left(\gamma^3-2\gamma-1\right) + n^2\gamma\left(\gamma^2+8\gamma+5\right)\right)\\ &-m^4\gamma^2\left(2n + \gamma(n+1)(5n-1) + 2\gamma^2\left(2n^3+4n^2-1\right)\right.\\ &\left.+\gamma^3\left(n\left(n^3+4n^2+n-2\right)-1\right)\right) + m^3\gamma(\gamma(n+1)+1)\\ &\left(\gamma\left(8n^2+13n+7\right) + 4n + 3 + \gamma^2\left(4n^3+15n^2+12n+3\right)\right.\\ &\left.+\gamma^3\left(5n^3+5n^2+3n-1\right)\right) - (\gamma n+\gamma+1)^2\\ &\left[\gamma\left(\left(n^2+n+1\right)\gamma+n+1\right)^2 + m\left(\left(n^2+n+1\right)\gamma+n+1\right)\right.\\ &\left(\gamma^2\left(n^2-3n-1\right) - \gamma(n+3) - 2\right) + m^2\gamma\left(n^2\gamma(3\gamma+2)\right.\\ &\left.\left.+(2n+1)(\gamma+1)^2\right)\right],\end{aligned}$$

where $\gamma = s/\rho$.

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