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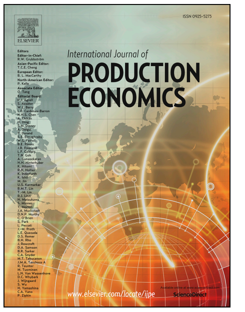

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Factory encroachment and channel selection in an outsourced supply chain

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Abstract: Business practices have demonstrated that a contract manufacturer (CM) can introduce an own-label product and thus compete with its original equipment manufacturer (OEM), i.e., factory encroachment, which has not been obtained much attention in literature. Considering a three-level outsourced supply chain consisting of a CM, an OEM, and a retailer, this paper analyzes the impact of factory encroachment on players' gains. We show that factory encroachment could implement Pareto improvement, i.e., all supply-chain players' gains increase under encroachment. We also demonstrate that factory encroachment always offers more surplus to the entire supply chain and the consumer. In addition, the most preferred channel for the supply-chain players, the entire supply-chain system, and the consumer are investigated. We find that an encroachment strategy could be simultaneously favored by all involved parties, provided there is no integration between the OEM and the retailer. However, if the OEM and the retailer act as a single entity, only the no-encroachment strategy could be favored by all parties simultaneously.

Key words: Outsourcing; encroachment; offline and online; multi-channel strategy; channel selection; game theory

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1. Introduction

As outsourcing becomes more persuasive in business practices, many contract manufacturers

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(CMs) start to encroach on original equipment manufacturers' (OEMs) territories and produce and sell its own-label products that compete with its customer OEMs' brand. Recently, for example, a project titled 'China Quality Manufacturing (CQM)', which is developed by the giant electronic retailer TAOBAO, aims to push tens thousands of Chinese CMs (factories) onto online platforms for selling its own-label products (Alibaba, 2016). Thus, in the following years, the involved factories (CMs) will serve both offline customer firms (OEMs) and online consumers. It indicates that CMs' encroachment could emerge as an important business model in O2O outsourcing practices.

CMs' managers may face two strategic decisions with regard to producing and selling own-label products. The first one is the strategic decision on introducing own-label products; and the other one is which market a CM should enter. Two markets that a CM can encroach on are obvious: the wholesale market vs. the retail market. For example, some CMs involved in the CQM project sell their products through an online wholesale market (see http://www.1688.com); while the other enterprises establish web shops on a direct-to-consumer platform (see http://q.taobao.com). These practices imply that the CMs should simultaneously satisfy the offline demand incurred by its customer firms (OEMs) and the online demand incurred by consumer or online retailers. Thus, traditional CMs with single offline channel are involved in strategic decisions of multi-channel operations. To the best of our knowledge, however, the extant literature has a theoretical gap in the decisions of factory encroachment. Moreover, the effects of a CM's encroachment on downstream firms are also ignored in the literature. Observing the managerial practices and the theoretic gaps, we propose the following research questions: *Do supply-chain players benefit or suffer from factory encroachment? How does factory encroachment affect channel selections of supply-chain players?*

To address these questions, this paper investigates a three-level outsourced supply chain consisting of a CM (supplier), an OEM (manufacturer), and a retailer. Under the outsourcing mode, the OEM designs a national brand (NB) and outsources the manufacturing to the CM. Moreover, the OEM delegates the selling to the retailer. Many giant manufacturers in practice employ or partially employ this kind of supply-chain structure, e.g., Nike, Addidas, Calvin Klein, HUGO BOSS, Apple, Xiaomi, etc.

According to the aforementioned practical cases, we consider the CM with the capability of developing a factory brand (FB) product for encroaching either on the wholesale market or on the retail market. Therefore, three common encroachment settings are investigated. The first one is Scenario ED, in which the CM opens a direct-to-consumer channel for selling the FB product. The second one is Scenario EC, which indicates that the CM produces and sells the FB product to the retailer that also sells the NB product. The last one is Scenario EE, in which the CM develops an exclusive retailer for selling the NB product. Hence, in Scenario ED, the CM encroaches on the retail market; while in Scenarios EC and EE, the CM encroaches on the wholesale market.

Note that managers in industries believe that whether factories introduce its own-label products is mainly determined by the demand of the OEMs' branded products. For example,

> Twigg (2016) stated that "In recent years, China's manufacturing sector has taken a hit due to a slowdown in global demand and the migration to cheaper sourcing centers across South East Asia. With a highly skilled workforce and not enough work, some factory owners that supply major luxury companies have decided that the solution now lies in creating brands of their own". [1]

We thus mainly model the effects of the base demands of the two products on the supply-chain players' incentives of factory encroachment. Moreover, we consider both the NB and the FB products are imperfect substitutes, which is a common assumption in the related literature (Arya et al., 2007; Cai, 2010; Chen et al., 2017; Ha et al., 2016; Yoon, 2016).

We summarize our major findings as follows.

- Given no integration between the OEM and the retailer, we show that the CM can gain more from encroachment, regardless of the encroachment setting. Moreover, both the OEM and the retailer could benefit from the CM's encroachment in Scenarios ED and EE. In Scenario EC, we show the retailer is always better off under encroachment; while the OEM is always hurt by the CM's encroachment. Therefore, Pareto gains could be obtained in Scenarios ED and EE; while cannot be

[1] In fashion industry, one may consider that a supplier CM manufacturing products and competing with its customer OEM could be uncommon. Practical cases show that it may because the factory can obtain more gains from encroachment, especially when its customer firms' orders diminish. Esquel Group Inc., for example, a CM for Calvin Klein, Hugo Boss, etc., began its own label 'PYE' since 2000 (see finance.china.com.cn for details: http://finance.china.com.cn/roll/20160720/3820150.shtml).

achieved in Scenario EC. We also show that Scenario EE could be favored by all supply-chain players simultaneously, as well as the entire supply-chain system and the consumer.

- Given vertical integration between the OEM and the retailer (labeled as the integrated OEM), we demonstrate that the CM is always better off under the encroachment, regardless of the encroachment setting. The integrated OEM can benefit from the CM's encroachment in Scenario EC while is always hurt in Scenarios ED and EE. Thus, Pareto gains can only be obtained in Scenario EC; while cannot be achieved in Scenarios ED and EE. There is no encroachment setting favored by the two supply-chain players simultaneously.
- We also demonstrate that the CM's encroachment can offer more gains for the entire supply-chain system and the consumer, regardless of the encroachment setting. Moreover, Scenarios ED, EE, and NE could be the dominant strategies for the whole supply-chain system and the consumer.

The reminder of this paper is organized as follows. Section 2 briefly reviews the related literatures, and Section 3 details our models. Given no-integration between the CM and the retailer, Sections 4 analyzes the effects of the CM's encroachment on supply-chain players' gains and the channel selection decisions. Section 5 makes similar analyses, providing the OEM and the retailer act as a single entity. Concluding remarks and future directions are presented in Section 6.

2. Literature review

This paper relates to the studies of dual-channel supply chains, which have drawn widespread attention in the literature. The main stream of the literature on this topic primarily studied whether and when manufacturer's encroachment could benefit for its retailer (Arya et al., 2007; Boyaci, 2005; Chiang et al., 2003; Li et al., 2015a; Li et al., 2013, 2015b; Tsay and Agrawal, 2004; Yao et al., 2009; Yoon, 2016). These literatures considered both supply-players set a unique decision variable: price, quantity, or inventory. Realizing non-price features could play an important role in consumer's channel selection, other literatures studied decisions of price and quality (Chen et al., 2016), price and service (Chen et al., 2008; Dumrongsiri et al., 2008; Mukhopadhyay et al., 2008), price and lead time (Hua et al., 2010), price and advertisement (Chen, 2015; Yan et al., 2006), quantity and quality (Ha et al., 2016). Still other literatures investigated pricing strategies (Bernstein et al., 2009; Cai et al., 2009; Cattani et al., 2006; Huang and Swaminathan, 2009), channel selection and (or) channel coordination in a dual-channel supply chain (Boyaci, 2005; Cai, 2010; Chen et al., 2012a). Most of these literatures considered a manufacturer-retailer channel in which the manufacturer introduces a direct online channel and investigated the impact of manufacturer's encroachment on supply-chain players' profits. However, none of these articles considered the case of a CM's encroachment in a three-level outsourced supply chain. Moreover, the CM's encroachment strategies were also ignored. Cai (2010) analyzed three different encroachment settings in a manufacturer-retailer channel, which is similar to this paper. However, our works show that several results established in Cai (2010) cannot be extended into the case of the CM's encroachment.

Because we investigate the factory encroachment problem in an outsourced supply chain, this paper relates to the stream of the literature on strategic outsourcing. Most of the literature on this topic studied the make-or-buy problem from the perspective of cost accounting methods, which is beyond the scope of this paper. We refer to Balakrishnan and Cheng (2005) for a comprehensive review. Several extant literatures analyzed players' strategic interactions in an outsourced supply chain under the environment of competition (e.g., Arya et al. (2008), Arya et al. (2013), Benjaafar et al. (2007), Chen et al. (2015), Fang and Shou (2015), Feng and Lu (2012), Jin et al. (2014), Kaya and Ozer (2009), Kaya (2011), Ma & Mallik (2016); Tang and Kouvelis (2011), Wu and Zhang (2014), Xiao et al. (2014), etc.). However, none of these researches considered the competition between an OEM and its CM.

This paper also relates to the literature of three-level supply chains. Observing supply chains in reality always contain multiple echelons, many literatures investigated a variety of coordination mechanisms that can coordinate a three-level supply chains. Examples include Ding and Chen (2008), Jaber and Goyal (2008), Jaber et al. (2010), Moussawi-Haidar et al. (2014), Munson and Rosenblatt (2001), Lee (2001), etc. In addition, other literatures analyzed some traditional operations problems in a three-level supply chain, e.g., the location-inventory problem (Tancrez et al., 2012), the network design problem (Park et al., 2010), the information-sharing problem (Sosic, 2010), the cost-sharing problem (Leng and Parlar, 2009). However, none of these literatures considered the supplier (contract manufacturer) can produce a similar product and compete with its customer firm.

Furthermore, this paper closely relates to the following papers. Lim and Tan (2010) investigated OEM's make, buy, and make-and-buy decisions, provided its CM has opportunity to be as a direct competitor. Chen et al. (2012b) considered a CM-OEM supply chain, in which the CM produces products for the OEM and an external small OEM. The main research question that the authors investigated is whether the incumbent OEM retains procurement from the CM. Wang et al. (2013) investigated strategic interactions in a CM-OEM supply chain, in which the CM acts as both upstream partner and downstream competitor to the OEM. Niu et al. (2015) considered a similar supply chain structure and investigated how pricing structure affects the equilibrium outcomes. They mainly focused on the quantity and pricing leadership. Our works differ with these papers in: (i) we consider a CM can encroach on an OEM's wholesale or a retailer's retail markets through three specified channels in a three-level outsourced supply chain; (ii) we highlight channel preference of each supply-chain player, as well as the whole supply-chain system and the consumer.

3. Model

We consider a three-level outsourced supply chain consisting of a CM, an OEM, and a retailer. The OEM outsources the manufacturing of a national brand (NB) product to the CM, and then sells it to the retailer, who will resell the product to the consumer market. In addition, we consider the CM may have an opportunity to introduce a FB product to sell it directly to a retailer or consumer. The former implies that the CM encroaches on the OEM's wholesale market; while the latter means that the CM encroaches on the territory of the retailer.

Let Q_n and Q_f denote the demands of the NB and the FB products, respectively. Sales price of the NB product is denoted as p_n and the FB product's price is represented by p_f. To characterize each product's demand function, we employ a utility function of a representative consumer introduced by Ingene and Parry (2004), which has been applied extensively in the field of marketing and operations management (Cai, 2010; Chen et al., 2016; Häckner, 2000; Hsiao and Chen, 2013; Liu et al., 2014; Singh and Vives, 1984; Symeonidis, 2003; Wu et al., 2015). The representative consumer utility (U) is given as

$$U = \alpha_n Q_n + \alpha_f Q_f - \frac{Q_n^2 + Q_f^2}{2} - kQ_nQ_f - p_nQ_n - p_fQ_f, \tag{1}$$

where $\alpha_n > 0$ and $\alpha_f > 0$ denote the NB and the FB products' base demand, respectively; parameter k $(0 \le k < 1)$ measures product substitution. When k approaches one, both products become perfect substitutes; while the demand for each product become independent when $k = 0$.

If the CM does not introduce an own-label product, i.e., $Q_f = 0$, then maximizing U with respect to Q_n yields

$$Q_n^N = \alpha_n - p_n^N \text{ and } Q_f^N = 0. \tag{2}$$

Where the superscript N denotes the case of no-encroachment. If the CM introduces a FB product, then maximizing U with respect to Q_n and Q_f yields

$$Q_n^E = \frac{\alpha_n - p_n^E - k\left(\alpha_f - p_f^E\right)}{1-k^2} \text{ and } Q_f^E = \frac{\alpha_f - p_f^E - k\left(\alpha_n - p_n^E\right)}{1-k^2}. \tag{3}$$

Where the superscript E denotes the encroachment setting.

We consider four channel scenarios, as Figure 1 shows. 1(a) is the no-encroachment scenario; while 1(b), (c), and (d) present the encroachment scenarios. 1(b) shows that the CM introduces a direct channel for selling the FB product, i.e., Scenario ED; 1(c) shows that the CM sells the FB product through a common retailer with the OEM, i.e., Scenario EC; 1(d) shows that the CM develops an exclusive retailer (retailer E) to sell the FB product, i.e., Scenario EE.

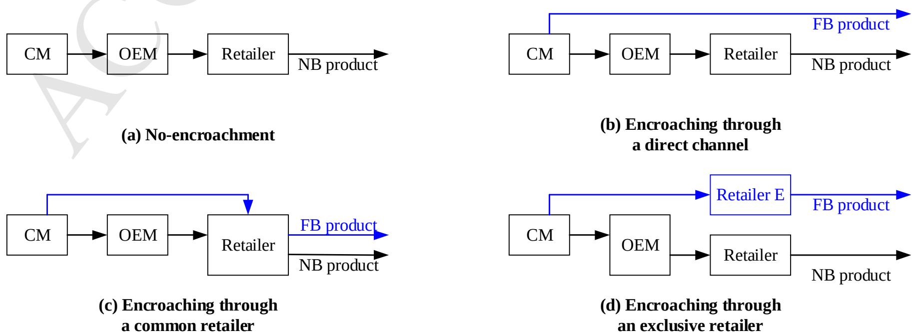

Figure 1. Channel scenarios

The product-related costs are normalized to be zero so that we can focus on the effects of the market bases. [2] Given Scenario NE, the sales price of the NB product p_n^N can be reformulated as $p_n^N = w_n^N + m_n^N + r_n^N$. Where, w_n^N is the wholesale price charged by the CM; m_n^N denotes the markup set by the OEM; and r_n^N represents the markup set by the retailer. [3] Thus, if the CM does not encroach on the market, the profits of supply-chain players are

$$\pi_C^{NE} = w_n^{NE} Q_n^N,\ \pi_O^{NE} = m_n^{NE} Q_n^N, \text{ and } \pi_R^{NE} = r_n^{NE} Q_n^N. \tag{4}$$

Where the subscripts C, O, and R represent the CM, the OEM, and the retailer, respectively. For Scenario ED, we present the profits as:

$$\pi_C^{ED} = w_n^{ED} Q_n^E + p_f^{ED} Q_f^E,\ \pi_O^{ED} = m_n^{ED} Q_n^E, \text{ and } \pi_R^{ED} = r_n^{ED} Q_n^E. \tag{5}$$

Where the superscript ED denotes the case of Scenario ED. For Scenario EC, the profits are given as

$$\pi_C^{EC} = w_n^{EC} Q_n^E + w_f^{EC} Q_f^E,\ \pi_O^{EC} = m_n^{EC} Q_n^E, \text{ and } \pi_R^{EC} = r_n^{EC} Q_n^E + \left(p_f^{EC} - w_f^{EC}\right) Q_f^E. \tag{6}$$

Where, the superscript EC denotes the case of Scenario EC; w_f^{EC} is the wholesale price of the FB product charged by the CM. For Scenario EE, the profits are

$$\pi_C^{EE} = w_n^{EE} Q_n^E + w_f^{EE} Q_f^E,\ \pi_O^{EE} = m_n^{EE} Q_n^E,\ \pi_R^{EE} = r_n^{EE} Q_n^E, \text{ and } \pi_{RE}^{EE} = \left(p_f^{EE} - w_f^{EE}\right) Q_f^E. \tag{7}$$

Where the superscript EE denotes the case of Scenario EE; the subscript RE means the exclusive retailer.

For the supply chain of the NB product, we consider the OEM act as the Stackelberg leader; while the CM and the retailer are the followers. In the outsourcing mode, it is natural for allowing the OEM to move first and the timing setting is commonly observed in many related literatures, as well as many business practices (see Wang et al. 2013 and the references therein). Moreover, the encroachment of the FB product is assumed to be set after the configuration of the NB product, which is entitled as sequential encroachment and drawn

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[2] Note that we can integrate the product-related costs (e.g., the production cost, the selling cost) into the model analysis by introducing the indices of the maximal product profitability. Denote the product-related costs of the NB and the FB products as c_n and c_f, respectively. Thus, the following analysis can be extended to the case of nonzero product-related cost by using the maximal profitability of product i $(i = n, f)$ $\alpha_i - c_i$ to substitute the parameter α_i.

[3] Similar setting of the NB product can be applied to the three encroachment scenarios.

much attention in the literature (Arya et al., 2007; Ha et al., 2016; Li et al., 2013; Yoon, 2016). Under the timing setting, we solve the game by the backward induction approach and aim to find the subgame perfect Nash equilibrium (SPNE).

4. Analysis

In this section, we will analyze the equilibrium outcomes of the above cases. Moreover, the effects of the CM's encroachment on the supply-chain players' profits and the consumer surplus are also investigated.

4.1. Scenario NE

Scenario NE implies that the supply chain only provides the NB product and the CM does not introduce an own-label product, as Figure 1(a) shows. For the no-encroachment case, the decision sequence is formulated as: (i) the OEM sets the markup m_n^{NE}; (ii) the CM and the retailer simultaneously set the wholesale price w_n^{NE} and the markup r_n^{NE}; (iii) the consumer demand is satisfied by the NB product at the given price. Given the decision sequence, we calculate the equilibrium outcomes (as Lemma 1 shows) as follows.

Lemma 1. *For the no-encroachment setting, the equilibrium solutions are*

$$m_n^{NE} = \frac{\alpha_n}{2},\ r_n^{NE} = \frac{\alpha_n}{6},\ w_n^{NE} = \frac{\alpha_n}{6}. \tag{8}$$

All proofs are presented in Appendix A. Substituting Eq. (8) into Eqs. (1)-(4), we can establish all other equilibrium outcomes of Scenario NE. See Appendix B for details.

4.2. Scenario ED

Given Scenario ED, the sequence of events is formulated as: (i) The OEM sets the markup m_n^{ED}; (ii) the CM and the retailer simultaneously set the wholesale price w_n^{ED} and the markup r_n^{ED}; (iii) The CM sets the sales price p_f^{ED}. (iv) the consumer demands of both products are satisfied. Under the timing setting, we have the following.

Lemma 2. *For the ED setting, the equilibrium solutions are*

$$m_n^{ED}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_1] \\ \frac{\alpha_n-k\alpha_f}{2} & \text{if } A\in(A_1,A_2) \\ m_n^{NE} & \text{if } A\in[A_2,+\infty)\end{cases},\quad r_n^{ED}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_1] \\ \frac{\alpha_n-k\alpha_f}{2(3-k^2)} & \text{if } A\in(A_1,A_2) \\ r_n^{NE} & \text{if } A\in[A_2,+\infty)\end{cases}, \tag{9}$$

$$w_n^{ED}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_1] \\ \frac{(2-k^2)\alpha_n-k(4-k^2)\alpha_f}{4(3-k^2)} & \text{if } A\in(A_1,A_2) \\ w_n^{NE} & \text{if } A\in[A_2,+\infty)\end{cases},\quad p_f^{ED}=\begin{cases}\frac{\alpha_f}{2} & \text{if } A\in(0,A_2) \\ \text{N/A} & \text{if } A\in[A_2,+\infty)\end{cases} \tag{10}$$

Where, $A\equiv\frac{\alpha_n}{\alpha_f}$, $A_1=k$, *and* $A_2=\frac{6-6k^2+k^4}{k(2-k^2)}$.

Note that parameter A can be explained as the base demand ratio of the two products, which also can measure the difference between the market bases of the NB product and the FB product. Threshold values A_1 and A_2 are derived by setting $Q_n^{ED}>0$ and $Q_f^{ED}>0$, respectively. Using equilibrium solutions showed in Eqs. (9) and (10) can yield all other equilibrium outcomes, as Appendix B shows.

Lemma 2 implies that the CM will encroach on the retail market if and only if $A<A_2$ is established. Otherwise, the relatively large demand base of the NB product enables the FB product not to derive a positive demand quantity. Thus, under this circumstance, Scenario ED will degenerate to the NE problem. However, if the FB product's base demand has a sufficient advantage over that of the NB product, i.e., $A<A_1$, the NB product will withdraw from the market after the encroachment of the FB product. We thus determine that if the two brands' respective base demands are sufficiently different such that $A\in(0,A_1]$ or $A\in[A_2,+\infty)$, the supply chain could only provide a single product to the consumer. Moreover, we can show $\frac{dA_1}{dk}>0$ and $\frac{dA_2}{dk}<0$, which imply that the more substitute the two products become, the smaller the region (A_1,A_2) is, the more (less) possible the market is served by a single product (two branded products). Similar results of the impact of parameter k on the width of the interval (A_1,A_2) can also be established in the other three encroachment scenarios.

Proposition 1. *For Scenario ED, given* $A\in(A_1,A_2)$, *we have*

(i) $\pi_C^{ED}>\pi_C^{NE}$;

(ii) when $0<k<0.860$, *if* $A_1<A<T_1$, *then* $\pi_O^{ED}<\pi_O^{NE}$; *otherwise if* $T_1<A<A_2$, *then* $\pi_O^{ED}>\pi_O^{NE}$; *while when* $0.860<k<1$, $\pi_O^{ED}<\pi_O^{NE}$;

(iii) when $0<k<0.948$, *if* $A_1<A<T_2$, *then* $\pi_R^{ED}<\pi_R^{NE}$; *otherwise if* $T_2<A<A_2$, *then* $\pi_R^{ED}>\pi_R^{NE}$; *while when* $0.948<k<1$, $\pi_R^{ED}<\pi_R^{NE}$.

Where, T_1 *and* T_2 *are defined as*

$$T_1 \equiv \frac{6-3k^2+\sqrt{6(6-11k^2+6k^4-k^6)}}{k(5-2k^2)},\ T_2 \equiv \frac{18-9k^2+3(3-k^2)\sqrt{2(2-3k^2+k^4)}}{k(21-14k^2+2k^4)}. \tag{11}$$

Given Scenarios NE and ED, Proposition 1 reports the preferences of the supply-chain players when the market is served by two competitive products, i.e., $A \in (A_1, A_2)$. [4] From Proposition 1, we see that the CM is always better off under Scenario NE than under Scenario NE. After the encroachment, the CM does not uniquely rely on the NB product's distribution channel. Her own brand product also has demand base and yields profit. As a result, she will raise the wholesale price of the NB product. See

$$k = A_1 < A < A_2 = \frac{6-6k^2+k^4}{k(2-k^2)} \implies w_n^{ED} - w_n^{NE} = \frac{k(3(4-k^2)\alpha_f - k\alpha_n)}{12(3-k^2)} > 0.$$

Moreover, encroachment could also increase the demand of the NB product (Notice from the difference of Q_n^{ED} and Q_n^{NE}). Thus, besides selling the FB product, the CM also can garner more profit from the increment of the wholesale price of the NB product. Therefore, she is always better off under encroachment. The result partially contradicts the case of the dual-channel setting in which a manufacturer (supplier) introduces a direct channel. Arya et al. (2007), Cai (2010), and Chiang et al. (2003) proposed that a manufacturer could decrease the wholesale price after opening a direct channel. In a three-echelon supply chain, however, we find that the CM will increase the wholesale price after introducing her own-brand product and selling through a direct channel.

Because of the competition effect induced by the FB product, the OEM and the retailer will decrease its markups to secure the NB product's demand (Notice from the values of $m_n^{ED} - m_n^{NE}$ and $r_n^{ED} - r_n^{NE}$). Hence, the unit sales price will decrease under the encroachment, which enables the NB product to generate more demand. Indeed, if the base demand ratio is sufficiently large such that $\frac{3(2-k^2)}{k(5-2k^2)} < A < A_2$, then $Q_n^{ED} > Q_n^{NE}$; otherwise if $A_1 < A < \frac{3(2-k^2)}{k(5-2k^2)}$, then $Q_n^{ED} < Q_n^{NE}$. Hence, when the NB product owns a relatively large demand base, the OEM and the retailer could be better off under the CM's encroachment.

[4] We omit the results of the cases of the market with a single product. One can easily verify that when $A \in (0, A_1]$, the CM and the consumer obtain more from Scenario ED than from Scenario NE; while the OEM and the retailer are hurt by the CM's encroachment; while when $A \in [A_2, +\infty)$, all supply-chain players and the consumer are indifferent between the Scenario ED and the Scenario NE.

However, when both two products become more substitute (k becomes larger), the CM and the OEM will be hurt by the CM's encroachment regardless of how large the demand base they have. Nevertheless, the equilibrium profit of the CM is increasing in substitute parameter k when $A \in (A_1, A_2)$, i.e., $\frac{\partial \pi_C^{ED}}{\partial k} > 0$ for $A \in (A_1, A_2)$. Thus, the channel conflict could be arisen when both products become more substitute. For example, many Chinese factories are producing own-label products that bear a strong resemblance with the products they make for global luxury companies, many of which struggles against the factory brands (Twigg, 2016). The result implies that developing a sufficiently different product could be better for the CM's encroachment.

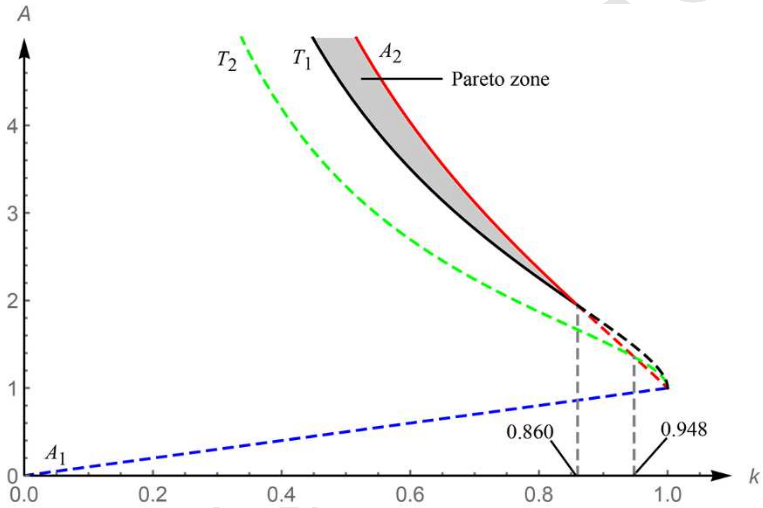


Figure 2. Pareto zone of Scenario ED

Proposition 1 also indicates that all three channel members could simultaneously benefit from the CM's encroachment, which means that Pareto improvement due to the introduction of the FB product could be realized. Note that $A_1 < T_2 < T_1 < A_2$ for $0 \leq k < 0.860$. We thus derive the Pareto zone, see the shaded region in Figure 2.

From $T_2 < T_1$ (see Figure 2), we determine that the retailer is more inclined to benefit from the CM's encroachment than the OEM because the OEM should decrease more markup for securing the NB product's demand than the retailer does. Formally, from Lemmas 1 and 2, we have $m_n^{NE} - m_n^{ED} = \frac{k\alpha_f}{2} > \frac{k(3\alpha_f - k\alpha_n)}{6(3-k^2)} = r_n^{NE} - r_n^{ED}$. As a result, the OEM requires larger demand base to keep the profit increase under Scenario ED. Moreover, we can infer that the OEM will suffer more than the retailer when they are hurt by the CM's encroachment. Figure

3 illustrates the result. We vary α_n for changing the parameter A; and the default values of parameters α_f and k are setting to $\alpha_f = 100$, $k = 0.8$. These results demonstrate that owning the Stackelberg leadership could incur more profits for the OEM under the CM's encroachment.

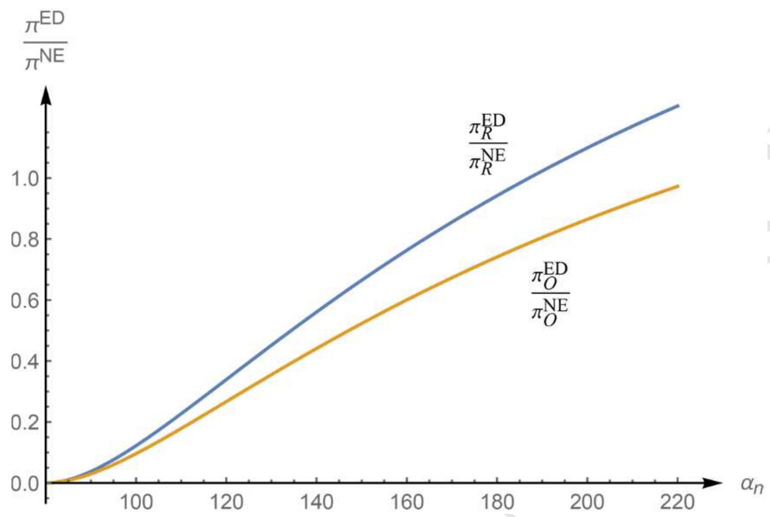


Figure 3 Profit differences of the OEM and the retailer

4.3. Scenario EC

In Scenario EC, the retailer who sells the OEM's product also distributes the CM's brand. It may be uncommonly seen that a contract manufacturer encroaches the market through its OEM's retailer. However, it is a real case in business practices. For example, a customer can either purchase an iphone or a Sharp A1 from JD.com, which simultaneously sells cellphones of the Apple and its CM Sharp. Given Scenario EC, the sequence of events is formulated as: (i) The OEM sets the markup m_n^{EC}; (ii) the CM and the retailer simultaneously set the wholesale price w_n^{EC} and the markup r_n^{EC}; (iii) The CM sets the wholesale price w_f^{EC}; (iv) the retailer determines the sales price p_f^{EC}; (v) the consumer demands of both products are satisfied. Under the timing setting, we have the following:

Lemma 3. *For the EC setting, the equilibrium outcomes are*

$$m_n^{EC} = \begin{cases} \text{N/A} & \text{if } A \in (0, A_3] \\ \frac{1}{2}\left(\alpha_n - \frac{3k\alpha_f}{4-k^2}\right) & \text{if } A \in (A_3, A_4) \\ m_n^{NE} & \text{if } A \in [A_4, +\infty) \end{cases}, \quad r_n^{EC} = \begin{cases} \text{N/A} & \text{if } A \in (0, A_3] \\ \frac{1}{6}\left(\alpha_n + \frac{3k\alpha_f}{4-k^2}\right) & \text{if } A \in (A_3, A_4) \\ r_n^{NE} & \text{if } A \in [A_4, +\infty) \end{cases},$$

$$w_n^{EC}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_3]\\ \frac{1}{6}\left(\alpha_n+\frac{3k\alpha_f}{4-k^2}\right) & \text{if } A\in(A_3,A_4)\\ w_n^{NE} & \text{if } A\in[A_4,+\infty)\end{cases},\quad w_f^{EC}=\begin{cases}\frac{\alpha_f}{2} & \text{if } A\in(0,A_3]\\ \frac{(8-3k^2)\alpha_f}{4(4-k^2)}-\frac{k\alpha_n}{12} & \text{if } A\in(A_3,A_4)\\ \text{N/A} & \text{if } A\in[A_4,+\infty)\end{cases},$$

$$p_f^{EC}=\begin{cases}\frac{3\alpha_f}{4} & \text{if } A\in(0,A_3]\\ \frac{(24-7k^2)\alpha_f}{8(4-k^2)}-\frac{k\alpha_n}{24} & \text{if } A\in(A_3,A_4)\\ \text{N/A} & \text{if } A\in[A_4,+\infty)\end{cases}. \tag{17}$$

Where $A_3=\frac{3k}{4-k^2}$ *and* $A_4=\frac{8-5k^2}{k(4-k^2)}$.

All other equilibrium results are included in Appendix B. Comparing supply-chain players' profits under Scenario EC with that of Scenario NE, we have the following:

Proposition 2. *For Scenario EC, given* $A\in(A_3,A_4)$, *we have* $\pi_C^{EC}>\pi_C^{NE}$, $\pi_O^{EC}<\pi_O^{NE}$, *and* $\pi_R^{EC}>\pi_R^{NE}$.

Comparing with the no-encroachment case, Proposition 2 reveals that the retailer will be better off under Scenario EC. However, the OEM cannot benefit from the CM's encroachment. Because the FB product also contributes to the retailer's gains, the OEM will deeply decrease the markup for arming against the CM-brand product, or else the retailer has no incentives to decrease the NB product's markup. Consequently, the OEM's gains will decrease due to the competition with the FB product. Moreover, the CM and the consumer will derive more surplus under Scenario EC than under the no-encroachment scenario.

4.4. Scenario EE

Given Scenario EE, the sequence of events is formulated as: (i) The OEM sets the markup m_n^{EE}; (ii) the CM and the retailer simultaneously set the wholesale price w_n^{EE} and the markup r_n^{EE}; (iii) The CM sets the wholesale price w_f^{EE}; (iv) the exclusive retailer (retailer E, see Figure 1) determines the sales price p_f^{EE}; (v) the consumer demands of both products are satisfied. Under the timing setting, we have the following:

Lemma 4. *For the EE setting, the equilibrium outcomes are*

$$m_n^{EE}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_5]\\ \frac{1}{2}\left(\alpha_n-\frac{k\alpha_f}{2-k^2}\right) & \text{if } A\in(A_5,A_6)\\ m_n^{NE} & \text{if } A\in[A_6,+\infty)\end{cases},\quad r_n^{EE}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_5]\\ \frac{(2-k^2)\alpha_n-k\alpha_f}{12-8k^2} & \text{if } A\in(A_5,A_6)\\ r_n^{NE} & \text{if } A\in[A_6,+\infty)\end{cases},$$

$$w_n^{EE}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_5]\\ \frac{(8-10k^2+3k^4)\alpha_n+k(8-5k^2)\alpha_f}{8(6-7k^2+2k^4)} & \text{if } A\in(A_5,A_6)\\ w_n^{NE} & \text{if } A\in[A_6,+\infty)\end{cases}; \tag{21}$$

$$w_f^{EE}=\begin{cases}\frac{\alpha_f}{2} & \text{if } A\in(0,A_6)\\ \text{N/A} & \text{if } A\in[A_6,+\infty)\end{cases},$$

$$p_f^{EE}=\begin{cases}\frac{3\alpha_f}{4} & \text{if } A\in(0,A_5]\\ \frac{(72-92k^2+29k^4)\alpha_f-k(8-10k^2+3k^4)\alpha_n}{16(6-7k^2+2k^4)} & \text{if } A\in(A_5,A_6)\\ \text{N/A} & \text{if } A\in[A_6,+\infty)\end{cases}. \tag{22}$$

Where $A_5=\frac{k}{2-k^2}$ *and* $A_6=\frac{24-36k^2+13k^4}{k(8-10k^2+3k^4)}$.

One can find all other equilibrium outcomes of Scenario EE in Appendix B. Comparing with the equilibrium profits of players of the no-encroachment case yields the following results.

Proposition 3. *For Scenario EE, given* $A\in(A_5,A_6)$*, we have:*

(i) $\pi_C^{EE}>\pi_C^{NE}$*;*

(ii) when $0<k<0.922$*, if* $A_5<A<T_3$*, then* $\pi_O^{EE}<\pi_O^{NE}$*; otherwise if* $T_3<A<A_6$*, then* $\pi_O^{EE}>\pi_O^{NE}$*; while when* $0.922<k<1$*,* $\pi_O^{EE}<\pi_O^{NE}$*;*

(iii) when $0<k<0.973$*, if* $A_5<A<T_4$*, then* $\pi_R^{EE}<\pi_R^{NE}$*; otherwise if* $T_4<A<A_6$*, then* $\pi_R^{EE}>\pi_R^{NE}$*; while when* $0.973<k<1$*,* $\pi_R^{EE}<\pi_R^{NE}$*.*

Where T_3 *and* T_4 *are defined as*

$$T_3=\frac{24-30k^2+9k^4+2\sqrt{6(24-70k^2+75k^4-35k^6+6k^8)}}{20k-24k^3+7k^5},$$

$$T_4=\frac{72-90k^2+27k^4+12(3-2k^2)\sqrt{(4-7k^2+3k^4)}}{84k-112k^3+37k^5}. \tag{23}$$

When the CM decides to introduce the FB product through an exclusive retailer, both the OEM and the retailer could be better off under the encroachment. However, if the FB product shows strong substitute with the NB product, the downstream firms are always hurt by the CM's encroachment, regardless of how large market base the NB product owns. Note that the CM always obtains more from Scenario EE than from Scenario NE. Hence, the Pareto zone could also be observed under Scenario EE, see the shaded region in Figure 4. The result is inconsistent with Cai (2010). In a two-echelon supply chain, the author demonstrated that a manufacturer favors to introduce a new retail channel; which definitely hurt the incumbent retailer. Therefore, the manufacturer's encroachment (through a new retail channel) cannot be a win-win strategy for a dual-channel supply chain. However, we show that all supply-chain players' profits could be improved when the CM introduces its own brand and sells through an exclusive retail channel.

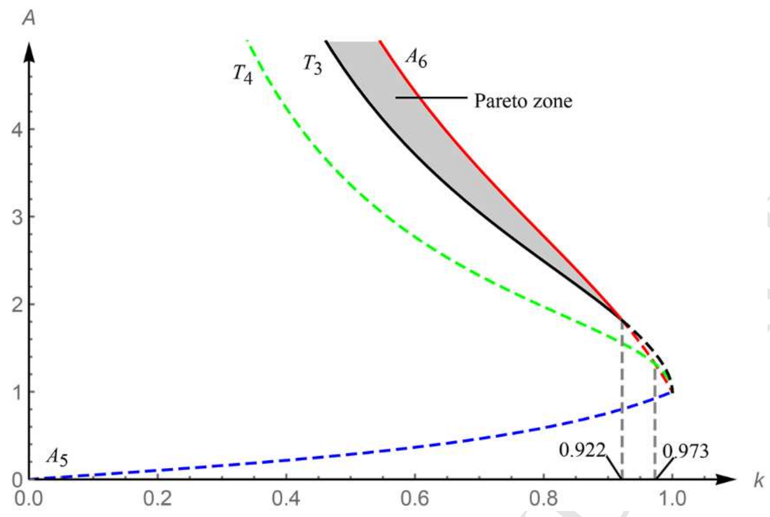


Figure 4. Pareto zone of Scenario EE

Note that the whole supply chain profit equals to $\pi_C + \pi_O + \pi_R$ and the consumer surplus could be derived by substituting the equilibrium outcomes of each scenario into Eq. (1). [5] Thus, we have the following result.

Proposition 4. *Given $A \in (A_1, A_4)$, the CM's encroachment always increases the whole supply chain profit and the consumer surplus, regardless of the encroaching manner.*

Note that the assumption $A \in (A_1, A_4)$ implies that both products can obtain nonnegative demands under each encroachment scenario. The competition incurred by the introduction of the FB product could bring the NB product's price reductions, which in turn increases the demand quantity of the NB product. Together with the direct contributions of introducing a new brand product, the whole supply chain will benefit from CM's encroachment. Moreover, the improvement of the consumer surplus is realized from the competition effect.

Combing Propositions 1-4 reveal the following insights with regard to the CM's encroachment:

[5] For Scenario EE, we also calculate the whole supply chain profit as $\pi_C^{EE} + \pi_O^{EE} + \pi_R^{EE}$ and the retailer that exclusively sells the FB product is not included in the supply chain. The assumption enables the paper to focus the effect of the encroachment on the outsourced supply chain selling the NB product. If retailer E is included in the supply chain system, we additionally require a reservation value for characterizing the profit difference between the no-encroachment case and Scenario EE.

- As long as the CM does not introduce the FB product through the retailer that also sells the NB product, encroachment could be a win-win strategy, i.e., a strategy that improves each player's profit, especially when the market base of the NB product is sufficiently pronounced and both two products are not too strong substitutes.
- In the outsourced supply chain, the two downstream firms could be hurt by the CM's encroachment. Moreover, the Stackelberg leadership enables the OEM to endure more losses than the retailer under encroachment. As a result, the retailer is more inclined to favor the CM's encroachment than the OEM does.
- The CM's encroachment can contribute to improve the whole supply chain performance, as well as the consumer surplus.

4.5. Channel selection

So far we have investigated three encroachment scenarios and analyzed the effect of the CM's encroachment on supply-players' profits, the entire supply-chain's profit and the consumer surplus. We now begin the analysis of channel preferences of the supply-chain players.

Define $\Omega \equiv \{(k, A) | 0 \leq k < 1, 0 < A < +\infty\}$ as the universal set. Thus, we have the following proposition.

Proposition 5. *Given* $(k, A) \in \Omega$, *we have:*

(i) For the CM, if $(k, A) \in R_1$, *then Scenario ED is the dominant strategy; if* $(k, A) \in R_2$, *then Scenario EE is the dominant strategy; otherwise, Scenario NE is the dominant strategy.*

(ii) For the OEM, if $(k, A) \in R_3$, *then Scenario ED is the dominant strategy; if* $(k, A) \in R_4$, *then Scenario EE is the dominant strategy; otherwise, Scenario NE is the dominant strategy.*

(iii) For the retailer, if $(k, A) \in R_5$, *then Scenario EC is the dominant strategy; if* $(k, A) \in R_6$, *then Scenario EE is the dominant strategy; otherwise, Scenario NE is the dominant strategy.*

Where,

$R_1 \equiv \{(k, A) | 0 \leq k < 1, 0 < A < A_2\}$, $R_2 \equiv \{(k, A) | 0 \leq k < 1, A_2 < A < A_6\}$;

$R_3 \equiv \{(k, A)|0 \le k < 0.860, T_1 < A < A_2\}$,

$R_4 \equiv \{(k, A)|0 \le k < 0.733, A_2 < A < A_6\} \cup \{(k, A)|0.733 \le k < 0.922, T_3 < A < A_6\}$;

$R_5 \equiv \{(k, A)|0 \le k < 1, A_3 < A < A_4\}$,

$R_6 = \{(k, A)|0 \le k < 0.649, A_4 < A < A_6\} \cup \{(k, A)|0.649 \le k < 0.973, T_4 < A < A_6\}$.

For the CM, Proposition 5 says that only if the base demand ratio A is neither too large nor too small, then Scenario ED outperforms all other three strategies; while when A is larger than the threshold value A_2, the FB product cannot obtain the positive demand, provided the CM encroaches on the retail market through a direct channel or the common retailer. As a result, Scenario EE could be the dominant strategy for the CM. However, if A is larger than the threshold value A_6, all three encroachment strategies will be degenerated to Scenario NE (see Table A1 in Appendix A for details). Hence, the CM is indifferent among the four strategies. We illustrate the CM's channel preferences in Figure 5(a). [6]

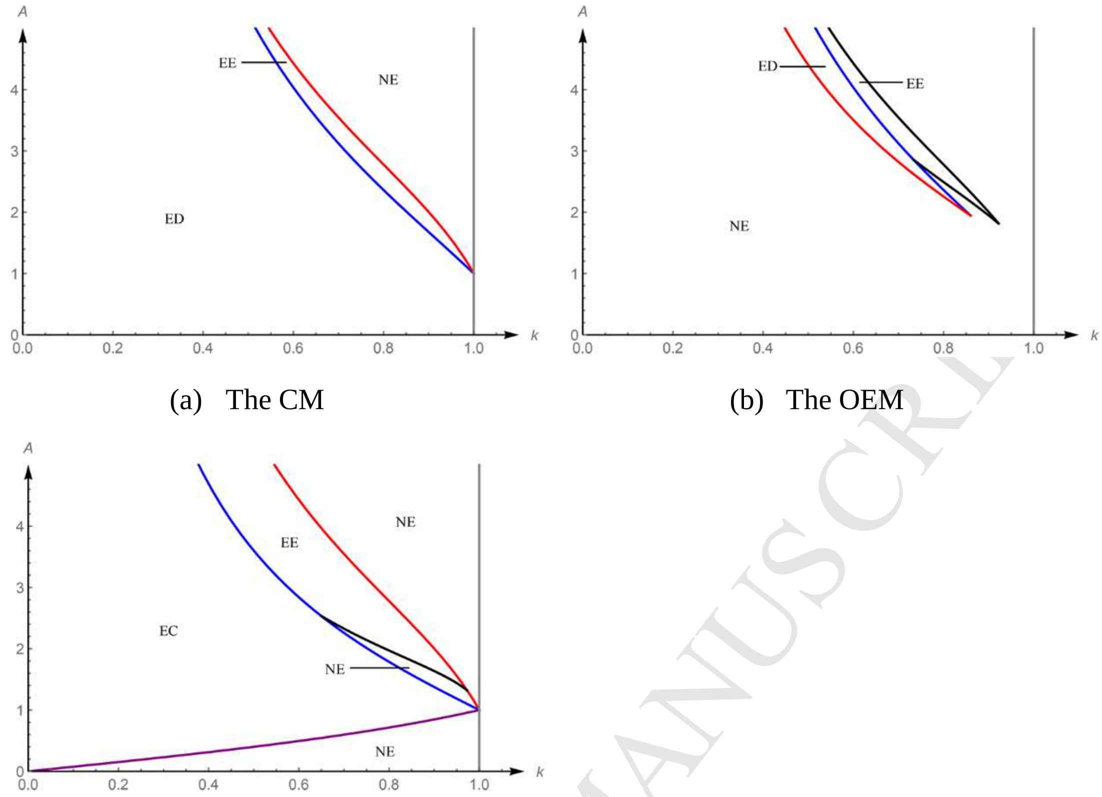
For the OEM, the encroachment strategies cannot dominate the no-encroachment strategy, except when the NB product owns a sufficiently large demand base. If the base demand ratio A becomes larger, Scenarios ED or EE could dominate the other three strategies. However, if both products are highly substitutable, the OEM will never favor the encroachment strategies. See Figure 5(b) for the illustration.

The channel preferences of the OEM also imply several insights with regard to OEM's strategic sourcing decisions. First, owning a relatively large demand base could enable the OEM to source from an external CM that has an own-label produce, especially when the CM sells its own-label product through a direct channel or an exclusive retailer; Second, an OEM will never outsource the manufacturing to a CM that produces an own-label product with strong substitutability.

The retailer may welcome the CM's encroachment under Scenario EC if the FB product could derive positive demand, because the FB product can also offer profit. However, if the base demand ratio A is sufficiently large, Scenario EE or Scenario NE enables the retailer to obtain more profits. Specifically, given $A \in (A_4, A_6)$, if A is smaller than the threshold value T_4, Scenario NE offers the most profit for the retailer; otherwise, Scenario EE

[6] In Figure 5(a), for simplicity, we denote Scenario NE as the dominant strategy for the CM when $A \in (A_6, +\infty)$. It because all the three encroachment strategies are degenerated to Scenario NE at this circumstance. Figures 5(b) and (c) can be similarly explained.

outperforms all other three strategies. See Figure 5(c) for illustration.



(a) The CM

(b) The OEM

(c) The retailer

Figure 5. The dominant strategies of supply-chain players

We also investigate the effects of channel selection on the whole supply chain profit and the consumer surplus. The most preferred channel structures of the supply chain system and the consumer are summarized in Proposition 6.

Proposition 6. *For the supply-chain system and the consumer, if* $(k, A) \in R_1$*, then Scenario ED is the dominant strategy; if* $(k, A) \in R_2$*, then Scenario EE is the dominant strategy; otherwise, Scenario NE is the dominant strategy.*

Combining the above results, we know that only Scenario EE could realize the Pareto improvement for the supply chain including the consumer. The result counters the dual-channel case studied by Cai (2010). Considering a dual channel supply chain, the author showed that developing a new retail channel could not dominate the dual channel strategy, as well as the single retail channel strategy. However, we demonstrate that developing an exclusively retailer for selling the FB product could be the most preferred encroachment strategy for all supply-chain players. We further show that Scenario EE also can improve the whole supply chain's profit and the consumer surplus. Formally, we propose Corollary 1.

Corollary 1. *Given* $(k, A) \in R_4$, *Scenario EE simultaneously improves the profits of the CM, the OEM, the (incumbent) retailer, the whole supply-chain system, and the consumer surplus.*

For the exclusive retailer, if her reservation value is lower than π_{RE}^{EE}, then Scenario EE also can improve her profit. Combining with the result showed in Corollary 1, we know that Scenario EE is the unique encroachment strategy that could benefit each party, the supply-chain system, and the consumer.

Note that the real case 'CQM' shows that many CMs encroach markets through web-based stores (see the Introduction section). Under this circumstance, the CMs should combine the offline business and the online business to operate multiple channels after encroachment. Thus, given the decentralization between the OEM and the retailer, the above results also imply the following insights with regard to CM's offline and online integration.

- Both the OEM and the retailer could welcome the CM's offline and online integration, which will increase the CM's gains.
- When the CM combine the offline OEM's market and the online wholesale market, all supply-chain players' profits could be simultaneously improved, as well as the whole supply-chain system's profit and the consumer surplus.

5. Integrated downstream firms

Many branded manufacturers, especially those in textiles, leather goods, sporting goods, and luxury goods, seek vertical integrations for pursuing competition advantages and profits (Hauptkorn et al., 2005). In practice, an OEM could integrate its retailers to distribute her own branded product. For example, Zara integrates the American Apparel (a giant retailer) for selling the branded products through its own retail channels (Lin et al., 2014). In this section, we thus investigate the impact of the CM's encroachment when the OEM and the retailer integrate as a single entity.

We also consider three encroachment scenarios studied in Section 4 and aim to analyze the effects of the CM's encroachment on supply-chain players' performances, as well as the channel selection of each member of the supply chain. Similar to the decentralized supply chain, we consider the integrated downstream firms (labeled as the integrated OEM, hereafter) play as the Stackelberg leader; while the CM is the follower. Given the downstream firms' integration, the supply chain dynamics are similar to the manufacturer's encroachment studied by Arya et al. (2007), Ha et al. (2016), Li et al. (2013) and Yoon (2016). However, this paper considers that the CM encroaches on the market by introducing a new brand product; while the extant literature assumed that a single product is distributed through either a retail channel or a direct channel. Moreover, many of these extant literatures established a manufacturer-Stackelberg; whereas we consider the downstream firms act as the Stackelberg leader, which shows more consistencies in the issues of outsourced supply chains (Chen et al., 2015; Chen et al., 2012b).

Employing the backward induction approach, we show the equilibrium outcomes of each case in Appendix C. Using the equilibrium outcomes, we derive the following:

Proposition 7. *Comparing with the case of no-encroachment,*

(i) given $A \in \left(k, \frac{2-k^2}{k}\right)$*, Scenario ED increases the CM's and the whole supply chain's profits, as well as the consumer surplus; while decreases the integrated OEM's profit;*

(ii) given $A \in \left(k, \frac{1}{k}\right)$*, Scenario EC increases the CM's, the integrated OEM's and the whole supply chain's profits, as well as the consumer surplus;*

(iii) given $A \in \left(\frac{k}{2-k^2}, \frac{4-3k^2}{k(2-k^2)}\right)$*, Scenario EE increases the CM's and the whole supply chain's profits, and the consumer surplus; while decreases the integrated OEM's profit.*

Under a similar supply-chain structure setting, Arya et al. (2007) and Cai (2010) employed manufacturer (supplier) Stackelberg models to show that a manufacturer could improve its retailer's profit by encroaching on the retail market through a direct channel. However, given both products can obtain positive demand quantities, Proposition 7 shows that the integrated OEM's profit cannot be improved when the CM encroaches on the market either through a direct channel or through an exclusive retailer. However, the CM's performance is improved under each encroachment case. This is caused by the increased competition due to the introduction of the FB product, which enables the CM to increase the wholesale price and the OEM to decrease the markup. Therefore, Pareto improvement cannot be achieved under Scenarios ED or EE. Nevertheless, if the CM encroaches on the market through the integrated OEM, then both the CM and the OEM are better off under the encroachment. One may think Scenario EC cannot be implemented in practice because an integrated OEM would not sell its CM's own brand product. Thus, the CM's encroachment could hurt the integrated OEM, and the Pareto improvement could not be realized under the downstream firms' integration. In addition, each setting of the CM's encroachment can improve the wholesale supply chain's gains and the consumer welfare.

We also show the channel preferences of the supply-chain players, the whole supply-chain system, and the consumer, as Proposition 8 shows.

Proposition 8. *Given* $(k, A) \in \Omega$, *we have:*

(i) For the CM, if $(k, A) \in \hat{R}_1$, *then Scenario ED is the dominant strategy; if* $(k, A) \in \hat{R}_2$, *then Scenario EE is the dominant strategy; otherwise, Scenario NE is the dominant strategy.*

(ii) For the integrated OEM, if $(k, A) \in \hat{R}_3$, *then Scenario EC is the dominant strategy; otherwise, then Scenario NE is the dominant strategy;*

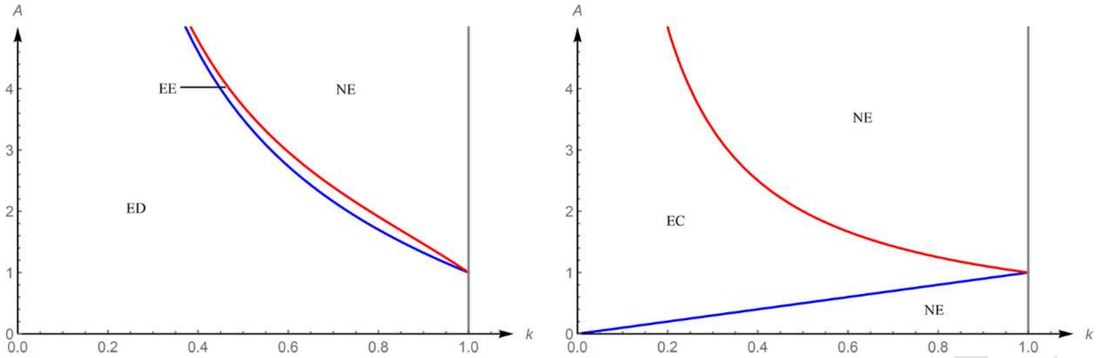
(iii) For the whole supply-chain system and the consumer, if $(k, A) \in \hat{R}_1$, *then Scenario ED is the dominant strategy; if* $(k, A) \in \hat{R}_2$, *then Scenario EE is the dominant strategy; otherwise, Scenario NE is the dominant strategy.*

Where,

$\hat{R}_1 \equiv \left\{(k, A) \middle| 0 \le k < 1, 0 < A < \frac{2-k^2}{k}\right\}$, $\hat{R}_2 \equiv \left\{(k, A) \middle| 0 \le k < 1, \frac{2-k^2}{k} < A < \frac{4-3k^2}{k(2-k^2)}\right\}$;

$\hat{R}_3 \equiv \left\{(k, A) \middle| 0 \le k < 1, k < A < \frac{1}{k}\right\}$.

When the OEM and the retailer act as a single entity, the CM could prefer to encroach on the market through a direct channel or through an exclusive retailer; whereas the integrated OEM could favor encroachment only under Scenario EC (See Figure 6 for the illustration). Consequently, none of the encroachment scenarios could be simultaneously favored by the CM and the integrated OEM. It indicates that the CM's encroachment could hardly be implemented when the two downstream firms are integrated as a single firm. Under this circumstance, adopting some useful contracts, for example the revenue-sharing contract (Cai, 2010) or the rebate contract (Taylor and Xiao, 2009), could be helpful for incentivizing the integrated OEM to adopt the CM's encroachment, which then can improve the whole supply chain's profit and the consumer surplus.



(a) The CM (b) The integrated OEM

Figure 6. The dominant strategies of the players of the supply chain with integrated OEM

The above results also present several managerial insights towards OEM's strategic sourcing decisions. Given the integration with its retailer, we see that an OEM may source from a CM with own-label product when she owns a relatively large demand base and the CM uses its retailer to sell the product; if not, an OEM will never source from a CM who also provides an own-brand product.

Consider the CM encroaches on markets through online shops, as the CQM project showed, Proposition 6 indicates that the integrated OEM will not benefit from its CM's offline and online combination. However, the CM has incentives for participating in multi-channel operations, regardless of which market it encroaches on.

6. Concluding remarks

We consider an outsourced supply chain consisting of a contract manufacturer (CM), an original equipment manufacturer (OEM) and a retailer. The OEM outsources the production of a national brand (NB) product to the CM, and the retailer is in charge of selling the product. Considering the CM can introduce a factory brand (FB) product, this paper investigates the effects of the factory encroachment on the downstream firms' (the OEM and the retailer) profits, the entire supply-chain profit and the consumer surplus under different supply chain structure setting. Namely, CM can distribute its own product either through a direct-to-consumer channel (Scenario ED), the incumbent retail-channel (Scenario EC), or through an exclusive retail-channel (Scenario EE). Given the OEM and the retailer act as self-profit maximizers, we employ an OEM Stackelberg game model to demonstrate that

Pareto improvement could be realized in Scenarios ED or EE, i.e., the CM's encroachment could improve all supply-chain players' profits. Nevertheless, Pareto improvement cannot be implemented under Scenario EC. We further consider the OEM and the retailer act as a single entity, which is titled as downstream firms' integration, and show that only Scenario EC could bring Pareto gains for supply-chain players. Additionally, we analyze the channel selections of the supply-chain players, the entire supply-chain, and the consumer with and without downstream firms' integration. It shows that Scenario EE could simultaneously improve each supply-chain player's profit for the case of no-integration; while for the case of integration, none of encroachment scenarios could be the dominant strategy for all supply-chain players.

To focus on the effects of the base demand difference, we assume that both the NB product and the FB product are equally cost-effective and the related costs are normalized to zero. One may anticipate that the NB product has cost advantages over the FB product from the perspectives of the manufacturing and (or) the selling, as several articles suggested (Arya et al., 2007; Chen etal., 2016; Ha et al., 2016; Yoon, 2016). This is indeed a case because the OEM may possess expertise on product development and channel management than the CM does. To illustrate the cost disadvantage of the FB product, we can denote that the FB product has a positive cost while the NB product's is zero. One can verify that our findings can be easily extended to the case with asymmetric costs by using a similar approach (see the first footnote in Section 3). The main insights still hold if we consider the cost asymmetry. To guarantee demand of its own brand product, the CM owning an own-label brand but with cost disadvantage will increase the wholesale price of the NB product greatly, especially when the NB product owns a relatively small demand base. As a result, the downstream firm(s) will decrease more markup for securing the NB product's market share. Therefore, the CM can benefit from the encroachment. Nevertheless, whether the downstream firm(s) could be better off under the CM's encroachment may be dependent on the cost asymmetry, base demand ratio, and the substitution degree.

Extension could be made on the leadership of the outsourced supply chain. Though business practices always demonstrate that the OEM has more power in the outsourced supply chain (e.g., Apple, Nike, Xiaomi, etc.), other literatures considered alternative supply-chain power structures, e.g., the CM-Stackelberg game (Chen et al., 2015; Choi and Fredj, 2013;

Kaya and Ozer, 2009; Wang et al., 2013), the CM-OEM bargain game (Feng and Lu, 2012, 2013), etc. It is believed that channel leaderships could incur sales and wholesale prices' variations (Choi, 1991; Choi, 1996; Jeuland and Shugan, 1983) while not essentially change the impacts of the supplier's encroachment, which is demonstrated in dual-channel supply chains (Arya et al., 2007; Yoon, 2016). We thus note that our major findings of the OEM-Stackelberg model can be extended to the alternative cases in which the CM is more powerful.

Future work could consider more general demand patterns and examine the impact of the CM's encroachment. Moreover, the CM may not observe the FB product's base demand before she makes encroachment decisions. Thus, integrating the demand uncertainty could be worth studying. Finally, designing some efficient schemes for coordinating the outsourced supply chain with the NB and the FB products could be helpful for mitigating channel conflicts due to the CM's encroachment.

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Appendix A. Proofs

Proof of Lemma 1. By the backward induction, we first formulate the retailer's and the CM's problems as

$$\begin{cases}\max_{w_n^{NE}} \pi_C^{NE} \\ \max_{r_n^{NE}} \pi_R^{NE}\end{cases} \Rightarrow \begin{cases}\frac{\partial \pi_C^{NE}}{\partial w_n^{NE}} = 0 \\ \frac{\partial \pi_R^{NE}}{\partial r_n^{NE}} = 0\end{cases} \Rightarrow \begin{cases} w_n^{NE}(m_n^{NE}) = \frac{\alpha_n - m_n^{NE}}{3} \\ r_n^{NE}(m_n^{NE}) = \frac{\alpha_n - m_n^{NE}}{3}\end{cases}. \tag{A1}$$

Thus, the OEM's problem can be formulated as

$$\max_{m_n^{NE}} \pi_O^{NE} = m_n^{NE} Q_n^N = m_n^{NE}\left(\alpha_n - (w_n^{NE} + m_n^{NE} + r_n^{NE})\right), \tag{A2}$$

$$\text{s.t. } w_n^{NE}(m_n^{NE}) = \frac{\alpha_n - m_n^{NE}}{3},\ r_n^{NE}(m_n^{NE}) = \frac{\alpha_n - m_n^{NE}}{3} \tag{A3}$$

Substituting Eqs. (A3) into (A2) and differentiating π_O^{NE} w.r.t. m_n^{NE} twice, we have $\frac{\partial^2 \pi_O^{NE}}{\partial (m_n^{NE})^2} = -\frac{2}{3}$. Thus, using the first order condition (FOC), we can derive the equilibrium m_n^{NE}, as showed in Eq. (7). Then, substituting the equilibrium m_n^{NE} into Eq. (A3), we have the other two equilibrium outcomes. ■

Proof of Lemma 2. Given the profit π_C^{ED} showed in Eq. (5), we have $\frac{\partial^2 \pi_C^{ED}}{\partial \left(p_f^{ED}\right)^2} = -\frac{2}{1-k^2}$. Because $0 \le k < 1$, thus π_C^{ED} is strictly concave in p_f^{ED}. Using the FOC yields

$$p_f^{ED}(w_n^{ED}, m_n^{ED}, r_n^{ED}) = \frac{2kw_n^{ED} + \alpha_f + k(m_n^{ED} + r_n^{ED} - \alpha_n)}{2}. \tag{A4}$$

Substituting Eq. (A4) into π_C^{ED} and π_R^{ED}, we have $\frac{\partial^2 \pi_C^{ED}}{\partial (w_n^{ED})^2} = -2$ and $\frac{\partial^2 \pi_R^{ED}}{\partial (r_n^{ED})^2} = -\frac{2-k^2}{1-k^2}$. Thus, we derive the following by using the FOCs.

$$w_n^{ED}(m_n^{ED}) = \frac{(2-k^2)(\alpha_n - m_n^{ED}) + k\alpha_f}{2(3-k^2)},\ r_n^{ED}(m_n^{ED}) = \frac{\alpha_n - m_n^{ED} - k\alpha_f}{3-k^2}. \tag{A5}$$

Using Eq. (A5), we have $\frac{\partial^2 \pi_O^{ED}}{\partial (m_n^{ER})^2} = -\frac{2-k^2}{3-4k^2+k^4}$. Setting $\frac{\partial \pi_O^{ED}}{\partial m_n^{ER}} = 0$ yields

$$m_n^{ED} = \frac{\alpha_n - k\alpha_f}{2}. \tag{A6}$$

Substituting Eqs. (A6) into (A5) yields the equilibrium w_n^{ED} and r_n^{ED}. Moreover, the equilibrium p_f^{ED} can be derived by substituting Eq. (A6) and the equilibrium w_n^{ED} and r_n^{ED} into Eq. (A4). Substituting the equilibrium w_n^{ED}, r_n^{ED} and p_f^{ED} into Eq. (3) yields

$$Q_n^{ED} = \frac{(2-k^2)(\alpha_n - k\alpha_f)}{4(3-4k^2+k^4)},\ Q_f^{ED} = \frac{(6-6k^2+k^4)\alpha_f - k(2-k^2)\alpha_n}{4(3-4k^2+k^4)}. \tag{A7}$$

To ensure the existence of the interior point solutions, we need $Q_n^{ED} > 0$ and $Q_f^{ED} > 0$. Thus, we have $A = \frac{\alpha_n}{\alpha_f} > k = A_1$ and $A = \frac{\alpha_n}{\alpha_f} < \frac{6-6k^2+k^4}{k(2-k^2)} = A_2$. If $A < A_1$, then $Q_n^{ED} \leq 0$, which means that the NB product cannot obtain the positive demand after the encroachment of the FB product. Therefore, the CM become the monopoly supplier that only provides her own brand product to the market. The demand of the FB product is $Q_f^{ED} = \alpha_f - p_f^{ED}$ and the profit of the CM is $\pi_C^{ED} = p_f^{ED} Q_f^{ED}$. Using the FOC, we have $p_f^{ED} = \frac{\alpha_f}{2}$. Moreover, if $A > A_2$, then the FB product cannot obtain the positive demand. Therefore, Scenario ED will be degenerate to the case of no-encroachment, which means that the solutions are identical with the solutions of Scenario NE. ■

Proof of Proposition 1. Subtracting π_C^{NE} from π_C^{ED} yields

$$\pi_C^{ED} - \pi_C^{NE} = \frac{L_1(A,k)\alpha_f^2}{144(3-k^2)^2(1-k^2)}. \tag{A8}$$

Where, $L_1(A,k) = k^2(24 - 19k^2 + 4k^4)A^2 - 18k(2-k^2)^2 A + 9(36 - 56k^2 + 24k^4 - 3k^6)$. Given $A \in (A_1, A_2)$ and $0 \leq k < 1$, we have

$\frac{\partial^2 L_1(A,k)}{\partial A^2} = 48k^2 - 38k^4 + 8k^6 > 0$, $\left.\frac{\partial L_1(A,k)}{\partial A}\right|_{A=A_1} = 8k(1-k^2)(3-k^2)^2 < 0$,

and $\left.\frac{\partial L_1(A,k)}{\partial A}\right|_{A=A_2} = 4k\left(19 - 28k^2 + 13k^4 - 2k^6 - \frac{2}{2-k^2}\right) > 0$.

Thus, using the FOC yields the minimal value of function $L_1(A,k)$. The solution is characterized as

$$\frac{\partial L_1(A,k)}{\partial A} = 0 \implies A^o = \frac{9(2-k^2)^2}{k(24-19k^2+4k^4)}.$$

Substituting A^o into $L_1(A,k)$ yields

$$L_1(A,k)|_{A=A^o} = \frac{36(3-k^2)^2(20-35k^2+18k^4-3k^6)}{24-19k^2+4k^4} > 0 \text{ for } 0 \leq k < 1.$$

Hence, $L_1(A,k) > 0$ when $A \in (A_1, A_2)$ and $0 \leq k < 1$, which implies that $\pi_C^{ED} > \pi_C^{NE}$ is established. Therefore, Part (i) of Proposition 1 holds. For the OEM's profit, subtracting

π_O^{NE} from π_O^{ED} yields

$$\pi_O^{ED} - \pi_O^{NE} = \frac{k\alpha_f^2 L_2(A,k)}{24(3-4k^2+k^2)}. \tag{A9}$$

Where, $L_2(A,k) = 3k(2-k^2) - 6(2-k^2)A + k(5-2k^2)A^2$. Given $A \in (A_1, A_2)$ and $0 \le k < 1$, we have $\frac{\partial^2 L_2(A,k)}{\partial A^2} = 2k(5-2k^2) > 0$. Note that $L_2(A,k)|_{A=A_1} = -2k(3-4k^2+k^2) < 0$; while if $0 \le k < 0.860$, then $L_2(A,k)|_{A=A_2} = \frac{2(1-k^2)(3-k^2)(6-12k^2+6k^4-k^6)}{k(2-k^2)^2} > 0$; otherwise if $0.860 \le k < 1$, then $L_2(A,k)|_{A=A_2} < 0$. Thus, for $0 \le k < 0.860$, solving the equation $L_2(A,k) = 0$ yields two roots:

$T_1 = \frac{6-3k^2+\sqrt{6(6-11k^2+6k^4-k^6)}}{k(5-2k^2)}$, $T_1' = \frac{6-3k^2-\sqrt{6(6-11k^2+6k^4-k^6)}}{k(5-2k^2)}$.

T_1' is omitted because $T_1' < A_1$. Therefore, when $0 \le k < 0.860$, if $A_1 < A < T_1$, then $L_2(A,k) < 0$, which implies that $\pi_O^{ED} < \pi_O^{NE}$; otherwise, if $T_1 < A < A_2$, then $L_2(A,k) > 0$, which implies that $\pi_O^{ED} > \pi_O^{NE}$. However, if $0.860 \le k < 1$, $L_2(A,k) < 0$ for $A \in (A_1, A_2)$, which indicates that $\pi_O^{ED} < \pi_O^{NE}$. We can use the similar approach to prove the π_R^{NE} and π_R^{ED}. Therefore, Parts 2 and 3 of Proposition 1 hold. ■

Proof of Lemma 3. Given the profit π_R^{EC} showed in Eq. (6), we have $\frac{\partial^2 \pi_R^{EC}}{\partial \left(p_f^{EC}\right)^2} = -\frac{2}{1-k^2}$. Using the FOC yields

$$p_f^{EC}\left(w_n^{EC}, m_n^{EC}, r_n^{EC}, w_f^{EC}\right) = \frac{kw_n^{EC} + w_f^{EC} + \alpha_f + k\left(m_n^{EC} + 2r_n^{EC} - \alpha_n\right)}{2}. \tag{A10}$$

Substituting Eq. (A10) into π_C^{EC}, we have $\frac{\partial^2 \pi_C^{EC}}{\partial \left(w_f^{EC}\right)^2} = -\frac{1}{1-k^2}$. Setting $\frac{\partial \pi_C^{EC}}{\partial w_f^{EC}} = 0$ yields

$$w_f^{EC}(w_n^{EC}, m_n^{EC}, r_n^{EC}) = \frac{2kw_n^{EC} + km_n^{EC} + \alpha_f - k\alpha_n}{2}. \tag{A11}$$

Given Eqs. (A11) and (A10), we have $\frac{\partial^2 \pi_C^{EC}}{\partial \left(w_n^{EC}\right)^2} = -2$ and $\frac{\partial^2 \pi_R^{EC}}{\partial \left(r_n^{EC}\right)^2} = -2$. Thus, we derive the following by using the FOCs.

$$w_n^{EC}(m_n^{EC}) = \frac{\alpha_n - m_n^{EC}}{3},\ r_n^{EC}(m_n^{EC}) = \frac{\alpha_n - m_n^{EC}}{3}. \tag{A12}$$

Given Eqs. (A12), (A11), and (A10), we have $\frac{\partial^2 \pi_O^{EC}}{\partial \left(m_n^{EC}\right)^2} = -\frac{4-k^2}{6(1-k^2)} < 0$. Setting $\frac{\partial \pi_O^{EC}}{\partial m_n^{EC}} = 0$ yields

$$m_n^{EC} = \frac{1}{2}\left(\alpha_n - \frac{3k\alpha_f}{4-k^2}\right). \tag{A13}$$

Substituting Eqs. (A13) into (A12) yields the equilibrium w_n^{EC} and r_n^{EC}. Substituting the results into Eq. (A11) yields the equilibrium w_f^{EC}. Moreover, the equilibrium p_f^{EC} can be established by substituting the equilibrium w_n^{EC}, r_n^{EC} and w_f^{EC} into Eq. (A10). From these outcomes, we have:

$$Q_n^{EC} = \frac{(4-k^2)\alpha_n - 3k\alpha_f}{24(1-k^2)},\ Q_f^{EC} = \frac{(8-5k^2)\alpha_f - k(4-k^2)\alpha_n}{8(4-5k^2+k^4)}. \tag{A14}$$

To ensure the existence of the interior point solutions, we need $Q_n^{EC} > 0$ and $Q_f^{EC} > 0$. Thus, we have $A > \frac{3k}{4-k^2} = A_3$ and $A < \frac{8-5k^2}{k(4-k^2)} = A_4$. If $A < A_3$, the CM becomes the monopoly supplier that only provides her own brand product to the market through the incumbent retailer. The demand of the FB product is $Q_f^{EC} = \alpha_f - p_f^{EC}$ and the profits of the retailer and the CM are $\pi_R^{EC} = (p_f^{EC} - w_f^{EC})Q_f^{EC}$ and $\pi_C^{EC} = w_f^{EC}Q_f^{EC}$. Using the FOC, we have $w_f^{EC} = \frac{\alpha_f}{2}$ and $p_f^{EC} = \frac{3\alpha_f}{4}$. Moreover, if $A > A_4$, then the FB product cannot obtain the positive demand. Therefore, Scenario EC will be degenerate to the case of no-encroachment, which means that the solutions are identical to the solutions of Scenario NE. ■

Proof of Proposition 2. Subtracting π_C^{NE} from π_C^{EC} yields

$$\pi_C^{EC} - \pi_C^{NE} = \frac{\alpha_f^2 L_3(A,k)}{96(4-k^2)^2(1-k^2)}. \tag{A15}$$

Where, $L_3(A,k) = 3(64 - 72k^2 + 17k^4) - 2Ak(64 - 44k^2 + 7k^4) + 3k^2(4-k^2)^2A^2$. Thus, we can employ the method used in the Proof of Proposition 1 to demonstrate $L_3(A,k)$ for $A \in (A_1, A_2)$ and $0 \le k < 1$, which implied that $\pi_C^{EC} > \pi_C^{NE}$ is established. Furthermore, we can show the relation $\pi_R^{EC} > \pi_R^{NE}$ by using the similar approach. For the relation of π_O^{NE} and π_O^{EC}, we have

$$\pi_O^{EC} - \pi_O^{NE} = \frac{k\alpha_f^2 L_4(A,k)}{16(4-5k^2+k^4)}. \tag{A16}$$

Where, $L_4(A,k) = 3k - 2A(4-k^2) + k(4-k^2)A^2$. Employing the method used in the Proof of Proposition 1, we determine that if $A < \bar{T}_2$ or $A > \bar{T}_2'$, then $L_4(A,k) > 0$, i.e., $\pi_O^{EC} > \pi_O^{NE}$; otherwise if $\bar{T}_2 < A < \bar{T}_2'$, then $L_4(A,k) < 0$, i.e., $\pi_O^{EC} < \pi_O^{NE}$. Parameters $\bar{T}_2$ and $\bar{T}_2'$ are the two solutions of the equation $L_4(A,k) = 0$, which are showed as:

$$\bar{T}_2 = \frac{4-k^2-2\sqrt{4-5k^2+k^4}}{4k-k^3},\ \bar{T}_2' = \frac{4-k^2+2\sqrt{4-5k^2+k^4}}{4k-k^3}.$$

Note that $\bar{T}_2 < A_3 < A_4 < \bar{T}_2'$ for $0 \le k < 1$. Therefore, $\pi_O^{EC} < \pi_O^{NE}$ is established. ■

Proof of Lemma 4. Given the profit π_C^{ED} showed in Eq. (7), we have $\frac{\partial^2 \pi_{RE}^{EE}}{\partial \left(p_f^{EE}\right)^2} = -\frac{2}{1-k^2}$. Because $0 \le k < 1$, thus π_R^{EE} is strictly concave in p_f^{EE}. Using the FOC yields

$$p_f^{EE}\left(w_n^{EE}, m_n^{EE}, r_n^{EE}, w_f^{EE}\right) = \frac{k w_n^{EE} + w_f^{EE} + \alpha_f + k\left(m_n^{EE} + r_n^{EE} - \alpha_n\right)}{2}. \tag{A17}$$

Substituting Eq. (A17) into π_C^{EE}, we have $\frac{\partial^2 \pi_C^{EE}}{\partial \left(w_f^{EE}\right)^2} = -\frac{1}{1-k^2}$. Setting $\frac{\partial \pi_C^{EE}}{\partial w_f^{EE}} = 0$ yields

$$w_f^{EE}(w_n^{EE}, m_n^{EE}, r_n^{EE}) = \frac{2k w_n^{EE} + \alpha_f + k\left(m_n^{EE} + r_n^{EE} - \alpha_n\right)}{2}. \tag{A18}$$

Given Eqs. (A18) and (A17), we have $\frac{\partial^2 \pi_C^{EE}}{\partial (w_n^{EE})^2} = -2$ and $\frac{\partial^2 \pi_R^{EE}}{\partial (r_n^{EE})^2} = -\frac{4-3k^2}{2(1-k^2)}$. Thus, we derive the following by using the FOCs.

$$w_n^{EE}(m_n^{EE}) = \frac{(4-3k^2)\alpha_n - (4-3k^2) m_n^{EE} + k\alpha_f}{12-8k^2}, \; r_n^{EE}(m_n^{EE}) = \frac{(2-k^2)\alpha_n - (2-k^2) m_n^{EE} - k\alpha_f}{6-4k^2}. \tag{A19}$$

Given Eqs. (A19), (A18), and (A17), we have $\frac{\partial^2 \pi_O^{EE}}{\partial (m_n^{EE})^2} = -\frac{8-10k^2+3k^4}{12-20k^2+8k^4} < 0$. Setting $\frac{\partial \pi_O^{EE}}{\partial m_n^{EE}} = 0$ yields

$$m_n^{EE} = \frac{1}{2}\left(\alpha_n - \frac{k\alpha_f}{2-k^2}\right). \tag{A20}$$

Substituting Eqs. (A20) into (A19) yields the equilibrium w_n^{EC} and r_n^{EE}. Substituting the results into Eq. (A18) yields the equilibrium w_f^{Ee}. Moreover, the equilibrium p_f^{EE} can be established by substituting the equilibrium w_n^{EE}, r_n^{EE} and w_f^{EE} into Eq. (A17). From these outcomes, we have:

$$Q_n^{EE} = \frac{(4-3k^2)\left((2-k^2)\alpha_n - k\alpha_f\right)}{16(3-5k^2+2k^4)}, \; Q_f^{EE} = \frac{(24-36k^2+13k^4)\alpha_f - k(8-10k^2+3k^4)\alpha_n}{16(6-13k^2+9k^4-2k^6)}.$$

To ensure the existence of the interior point solutions, we need $Q_n^{EE} > 0$ and $Q_f^{EE} > 0$. Thus, we have $A > \frac{k}{2-k^2} = A_5$ and $A < \frac{(24-36k^2+13k^4)}{k(8-10k^2+3k^4)} = A_6$. If $A < A_5$, the CM become the monopoly supplier that only provides her own brand product to the market through an exclusive retailer (retailer E). The demand of the FB product is $Q_f^{EE} = \alpha_f - p_f^{EE}$ and the profits of the retailer and the CM are $\pi_{RE}^{EE} = \left(p_f^{EE} - w_f^{EE}\right) Q_f^{EE}$ and $\pi_C^{EE} = w_f^{EE} Q_f^{EE}$. Using the FOC, we have $w_f^{EE} = \frac{\alpha_f}{2}$ and $p_f^{EE} = \frac{3\alpha_f}{4}$. Moreover, if $A > A_5$, then the FB product cannot obtain the positive demand. Therefore, Scenario EE will be degenerate to the case of no-encroachment, which means that the solutions are identical with the solutions of Scenario

NE. ■

Proof of Proposition 3. Using the similar approach showed in the proof of Proposition 2, we can verify the relation $\pi_C^{EE} > \pi_C^{NE}$. For the relation of π_O^{NE} and π_O^{EE}, we have

$$\pi_O^{EE} - \pi_O^{NE} = \frac{k\alpha_f^2 L_5(A,k)}{96(2-k^2)(3-5k^2+2k^4)}.$$

Where, $L_5(A,k) = 3k(4-3k^2) - 6(8-10k^2+3k^4)A + k(20-24k^2+7k^4)A^2$. Employing the method used in the proof of Proposition 1, we determine that if $A < T_3'$ or $A > T_3$, then $L_5(A,k) > 0$, i.e., $\pi_O^{EE} > \pi_O^{NE}$; otherwise if $T_3' < A < T_3$, then $L_5(A,k) < 0$, i.e., $\pi_O^{EE} < \pi_O^{NE}$. Parameters T_3 and T_3' are the two solutions of the equation $L_5(A,k) = 0$, which are showed as:

$$T_3' = \frac{24-30k^2+9k^4-2\sqrt{6(24-70k^2+75k^4-35k^6+6k^8)}}{20k-24k^3+7k^5},$$

$$T_3 = \frac{24-30k^2+9k^4+2\sqrt{6(24-70k^2+75k^4-35k^6+6k^8)}}{20k-24k^3+7k^5}.$$

Note that $T_3' < A_5$ for $0 \le k < 1$. Thus, T_3' is omitted. Moreover, when $0.922 \le k < 1$, $A_6 < T_3$, which implies that $L_5(A,k) < 0$ is established. Thus, $\pi_O^{EE} < \pi_O^{NE}$. While when $0 \le k < 0.922$, if $A_5 < A < T_1$, then $L_5(A,k) < 0$, i.e., $\pi_O^{EE} < \pi_O^{NE}$; otherwise if $T_1 < A < A_6$, then $L_5(A,k) > 0$, i.e., $\pi_O^{EE} > \pi_O^{NE}$. Therefore, Part (ii) follows. Furthermore, we can show Part (iii) by using the similar approach. ■

Proof of Proposition 4. Note that $A_5 < A_3 < A_1 < A_4 < A_2 < A_6$ for $0 \le k < 1$. We thus determine that both products can obtain nonnegative demands in each encroachment scenario, provided $A \in (A_1, A_4)$. Thus, using the equilibrium outcomes showed in Appendix B, we can show the relations $U^{ED} > U^{NE}$, $U^{EC} > U^{NE}$, and $U^{EE} > U^{NE}$. Similarly, we can prove the relations of $\pi_C^{ED} + \pi_O^{ED} + \pi_R^{ED} > \pi_C^{NE} + \pi_O^{NE} + \pi_R^{NE}$, $\pi_C^{EC} + \pi_O^{EC} + \pi_R^{EC} > \pi_C^{NE} + \pi_O^{NE} + \pi_R^{NE}$, and $\pi_C^{EE} + \pi_O^{EE} + \pi_R^{EE} > \pi_C^{NE} + \pi_O^{NE} + \pi_R^{NE}$. ■

Lemma A1. Given $0 < A < +\infty$, the equilibrium profits are summarized in Tables A1-A3.

Table A1. The equilibrium profit of the CM when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < A_5$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_f^2}{4}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{8}$
$A_5 < A < A_3$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_f^2}{4}$	$\frac{\alpha_f^2}{8}$	$\frac{\left((288-656k^2+488k^4-199k^6)\alpha_f^2-2k(4-3k^2)^2(4-k^2)\alpha_n\alpha_f+(8-10k^2+3k^4)^2\alpha_n^2\right)}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_3 < A < A_1$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_f^2}{4}$	$\frac{\left(9(64-72k^2+17k^4)\alpha_f^2-6k(64-44k^2+7k^4)\alpha_n\alpha_f+(4-k^2)^2(8+k^2)\alpha_n^2\right)}{288(4-k^2)^2(1-k^2)}$	$\frac{\left((288-656k^2+488k^4-199k^6)\alpha_f^2-2k(4-3k^2)^2(4-k^2)\alpha_n\alpha_f+(8-10k^2+3k^4)^2\alpha_n^2\right)}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_1 < A < A_4$	$\frac{\alpha_n^2}{36}$	$\frac{\left((36-56k^2+24k^4-3k^6)\alpha_f^2+2k(2-k^2)^2\alpha_n\alpha_f-(2-k^2)^2\alpha_n^2\right)}{16(1-k^2)(3-k^2)^2}$	$\frac{\left(9(64-72k^2+17k^4)\alpha_f^2-6k(64-44k^2+7k^4)\alpha_n\alpha_f+(4-k^2)^2(8+k^2)\alpha_n^2\right)}{288(4-k^2)^2(1-k^2)}$	$\frac{\left((288-656k^2+488k^4-199k^6)\alpha_f^2-2k(4-3k^2)^2(4-k^2)\alpha_n\alpha_f+(8-10k^2+3k^4)^2\alpha_n^2\right)}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_4 < A < A_2$	$\frac{\alpha_n^2}{36}$	$\frac{\left((36-56k^2+24k^4-3k^6)\alpha_f^2+2k(2-k^2)^2\alpha_n\alpha_f-(2-k^2)^2\alpha_n^2\right)}{16(1-k^2)(3-k^2)^2}$	$\frac{\alpha_n^2}{36}$	$\frac{\left((288-656k^2+488k^4-199k^6)\alpha_f^2-2k(4-3k^2)^2(4-k^2)\alpha_n\alpha_f+(8-10k^2+3k^4)^2\alpha_n^2\right)}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_2 < A < A_6$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\left((288-656k^2+488k^4-199k^6)\alpha_f^2-2k(4-3k^2)^2(4-k^2)\alpha_n\alpha_f+(8-10k^2+3k^4)^2\alpha_n^2\right)}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_6 < A < +\infty$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$

Table A2. The equilibrium profit of the OEM when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < A_5$	$\frac{\alpha_n^2}{12}$	0	0	0
$A_5 < A < A_3$	$\frac{\alpha_n^2}{12}$	0	0	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{32(2-k^2)(3-5k^2+2k^4)}$
$A_3 < A < A_1$	$\frac{\alpha_n^2}{12}$	0	$\frac{\left((4-k^2)\alpha_n-3k\alpha_f\right)^2}{48(4-5k^2+k^4)}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{32(2-k^2)(3-5k^2+2k^4)}$
$A_1 < A < A_4$	$\frac{\alpha_n^2}{12}$	$\frac{(2-k^2)(\alpha_n-k\alpha_f)^2}{8(3-4k^2+k^4)}$	$\frac{\left((4-k^2)\alpha_n-3k\alpha_f\right)^2}{48(4-5k^2+k^4)}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{32(2-k^2)(3-5k^2+2k^4)}$
$A_4 < A < A_2$	$\frac{\alpha_n^2}{12}$	$\frac{(2-k^2)(\alpha_n-k\alpha_f)^2}{8(3-4k^2+k^4)}$	$\frac{\alpha_n^2}{12}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{32(2-k^2)(3-5k^2+2k^4)}$
$A_2 < A < A_6$	$\frac{\alpha_n^2}{12}$	$\frac{\alpha_n^2}{12}$	$\frac{\alpha_n^2}{12}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{32(2-k^2)(3-5k^2+2k^4)}$
$A_6 < A < +\infty$	$\frac{\alpha_n^2}{12}$	$\frac{\alpha_n^2}{12}$	$\frac{\alpha_n^2}{12}$	$\frac{\alpha_n^2}{12}$

Table A3. The equilibrium profit of the retailer when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < A_5$	$\frac{\alpha_n^2}{36}$	0	0	0
$A_5 < A < A_3$	$\frac{\alpha_n^2}{36}$	0	0	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{64(1-k^2)(3-2k^2)^2}$
$A_3 < A < A_1$	$\frac{\alpha_n^2}{36}$	0	$\frac{\left(\begin{array}{c}9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2+k^4)\alpha_n\alpha_f\\+(4-k^2)^2(16-7k^2)\alpha_n^2\end{array}\right)}{576(4-k^2)^2(1-k^2)}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{64(1-k^2)(3-2k^2)^2}$
$A_1 < A < A_4$	$\frac{\alpha_n^2}{36}$	$\frac{(2-k^2)(\alpha_n-k\alpha_f)^2}{8(3-k^2)^2(1-k^2)}$	$\frac{\left(\begin{array}{c}9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2+k^4)\alpha_n\alpha_f\\+(4-k^2)^2(16-7k^2)\alpha_n^2\end{array}\right)}{576(4-k^2)^2(1-k^2)}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{64(1-k^2)(3-2k^2)^2}$
$A_4 < A < A_2$	$\frac{\alpha_n^2}{36}$	$\frac{(2-k^2)(\alpha_n-k\alpha_f)^2}{8(3-k^2)^2(1-k^2)}$	$\frac{\alpha_n^2}{36}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{64(1-k^2)(3-2k^2)^2}$
$A_2 < A < A_6$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{(4-3k^2)\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{64(1-k^2)(3-2k^2)^2}$
$A_6 < A < +\infty$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$	$\frac{\alpha_n^2}{36}$

Proof of Proposition 5. Using the equilibrium profits showed in Tables A1-A3, we can show the dominant strategy of each player of the supply chain. ■

Lemma A2. Given $0 < A < +\infty$, the equilibrium supply-chain profit and the consumer surplus are summarized in Tables A5 and A6.

Table A4. The equilibrium profit of the whole supply-chain system when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < A_5$	$\frac{5\alpha_n^2}{36}$	$\frac{\alpha_f^2}{4}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{8}$
$A_5 < A < A_3$	$\frac{5\alpha_n^2}{36}$	$\frac{\alpha_f^2}{4}$	$\frac{\alpha_f^2}{8}$	$\frac{\begin{pmatrix}(288-592k^2+400k^4-89k^6)\alpha_f^2-2k(2-k^2)(4-3k^2)(20-13k^2)\alpha_f\alpha_n\\+(2-k^2)^2(80-112k^2+39k^4)\alpha_n^2\end{pmatrix}}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_3 < A < A_1$	$\frac{5\alpha_n^2}{36}$	$\frac{\alpha_f^2}{4}$	$\frac{\begin{pmatrix}9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2-k^4)\alpha_f\alpha_n\\+(16-7k^2)(4-k^2)^2\alpha_n^2\end{pmatrix}}{1152(1-k^2)(3-2k^2)^2}$	$\frac{\begin{pmatrix}(288-592k^2+400k^4-89k^6)\alpha_f^2-2k(2-k^2)(4-3k^2)(20-13k^2)\alpha_f\alpha_n\\+(2-k^2)^2(80-112k^2+39k^4)\alpha_n^2\end{pmatrix}}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_1 < A < A_4$	$\frac{5\alpha_n^2}{36}$	$\frac{\begin{pmatrix}(36-40k^2+12k^4-k^6)\alpha_f^2-2k(20-16k^2+3k^4)\alpha_f\alpha_n\\+(20-16k^2+3k^4)\alpha_n^2\end{pmatrix}}{16(1-k^2)(3-k^2)^2}$	$\frac{\begin{pmatrix}9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2-k^4)\alpha_f\alpha_n\\+(16-7k^2)(4-k^2)^2\alpha_n^2\end{pmatrix}}{1152(1-k^2)(3-2k^2)^2}$	$\frac{\begin{pmatrix}(288-592k^2+400k^4-89k^6)\alpha_f^2-2k(2-k^2)(4-3k^2)(20-13k^2)\alpha_f\alpha_n\\+(2-k^2)^2(80-112k^2+39k^4)\alpha_n^2\end{pmatrix}}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_4 < A < A_2$	$\frac{5\alpha_n^2}{36}$	$\frac{\begin{pmatrix}(36-40k^2+12k^4-k^6)\alpha_f^2-2k(20-16k^2+3k^4)\alpha_f\alpha_n\\+(20-16k^2+3k^4)\alpha_n^2\end{pmatrix}}{16(1-k^2)(3-k^2)^2}$	$\frac{5\alpha_n^2}{36}$	$\frac{\begin{pmatrix}(288-592k^2+400k^4-89k^6)\alpha_f^2-2k(2-k^2)(4-3k^2)(20-13k^2)\alpha_f\alpha_n\\+(2-k^2)^2(80-112k^2+39k^4)\alpha_n^2\end{pmatrix}}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_2 < A < A_6$	$\frac{5\alpha_n^2}{36}$	$\frac{5\alpha_n^2}{36}$	$\frac{5\alpha_n^2}{36}$	$\frac{\begin{pmatrix}(288-592k^2+400k^4-89k^6)\alpha_f^2-2k(2-k^2)(4-3k^2)(20-13k^2)\alpha_f\alpha_n\\+(2-k^2)^2(80-112k^2+39k^4)\alpha_n^2\end{pmatrix}}{128(3-2k^2)^2(2-3k^2+k^4)}$
$A_6 < A < +\infty$	$\frac{5\alpha_n^2}{36}$	$\frac{5\alpha_n^2}{36}$	$\frac{5\alpha_n^2}{36}$	$\frac{5\alpha_n^2}{36}$

Table A5. The equilibrium consumer surplus when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < A_5$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{32}$	$\frac{\alpha_f^2}{32}$
$A_5 < A < A_3$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{32}$	$\frac{\left(\begin{array}{c}(576-1472k^2+1344k^4-516k^6+69k^8)\alpha_f^2-2k(4-3k^2)(2-k^2)(8-16k^2+7k^4)\alpha_n\alpha_f\\+(2-k^2)^2(4-3k^2)^3\alpha_n^2\end{array}\right)}{512(1-k^2)(6-7k^2+2k^4)^2}$
$A_3 < A < A_1$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_f^2}{8}$	$\frac{\left(\begin{array}{c}9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2+k^4)\alpha_n\alpha_f\\+(4-k^2)^2(16-7k^2)\alpha_n^2\end{array}\right)}{576(4-k^2)^2(1-k^2)}$	$\frac{\left(\begin{array}{c}(576-1472k^2+1344k^4-516k^6+69k^8)\alpha_f^2-2k(4-3k^2)(2-k^2)(8-16k^2+7k^4)\alpha_n\alpha_f\\+(2-k^2)^2(4-3k^2)^3\alpha_n^2\end{array}\right)}{512(1-k^2)(6-7k^2+2k^4)^2}$
$A_1 < A < A_4$	$\frac{\alpha_n^2}{72}$	$\frac{\left(\begin{array}{c}(36-56k^2+24k^4-3k^6)\alpha_f^2+2k(2-k^2)^2\alpha_n\alpha_f\\-(2-k^2)^2\alpha_n^2\end{array}\right)}{32(1-k^2)(3-k^2)^2}$	$\frac{\left(\begin{array}{c}9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2+k^4)\alpha_n\alpha_f\\+(4-k^2)^2(16-7k^2)\alpha_n^2\end{array}\right)}{576(4-k^2)^2(1-k^2)}$	$\frac{\left(\begin{array}{c}(576-1472k^2+1344k^4-516k^6+69k^8)\alpha_f^2-2k(4-3k^2)(2-k^2)(8-16k^2+7k^4)\alpha_n\alpha_f\\+(2-k^2)^2(4-3k^2)^3\alpha_n^2\end{array}\right)}{512(1-k^2)(6-7k^2+2k^4)^2}$
$A_4 < A < A_2$	$\frac{\alpha_n^2}{72}$	$\frac{\left(\begin{array}{c}(36-56k^2+24k^4-3k^6)\alpha_f^2+2k(2-k^2)^2\alpha_n\alpha_f\\-(2-k^2)^2\alpha_n^2\end{array}\right)}{32(1-k^2)(3-k^2)^2}$	$\frac{\alpha_n^2}{72}$	$\frac{\left(\begin{array}{c}(576-1472k^2+1344k^4-516k^6+69k^8)\alpha_f^2-2k(4-3k^2)(2-k^2)(8-16k^2+7k^4)\alpha_n\alpha_f\\+(2-k^2)^2(4-3k^2)^3\alpha_n^2\end{array}\right)}{512(1-k^2)(6-7k^2+2k^4)^2}$
$A_2 < A < A_6$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_n^2}{72}$	$\frac{\left(\begin{array}{c}(576-1472k^2+1344k^4-516k^6+69k^8)\alpha_f^2-2k(4-3k^2)(2-k^2)(8-16k^2+7k^4)\alpha_n\alpha_f\\+(2-k^2)^2(4-3k^2)^3\alpha_n^2\end{array}\right)}{512(1-k^2)(6-7k^2+2k^4)^2}$
$A_6 < A < +\infty$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_n^2}{72}$	$\frac{\alpha_n^2}{72}$

Proof of Proposition 6. Using the equilibrium profits showed in Tables A4 and A5, we can show the dominant strategies for the supply-chain system and the consumer. ■

Proof of Corollary 1. Follows directly from the results of Propositions 5 and 6. ■

Proof of Proposition 7. We here only provide the proof of Part (i). The other two parts can be proved by using the similar approach. Subtracting $\hat{\pi}_C^{NE}$ from $\hat{\pi}_C^{ED}$ yields $\hat{\pi}_C^{ED} - \hat{\pi}_C^{NE} = \frac{\bar{L}_1(A,k)\alpha_f^2}{16(1-k^2)}$, where $\bar{L}_1(A,k) = (4-3k^2) - 2Ak + k^2A^2$. Note $0 \le k < 1$. Thus, $\bar{L}_1(A,k) \ge 1 - 2Ak + k^2A^2 = (1-kA)^2 \ge 0$, which implies that $\hat{\pi}_C^{ED} \ge \hat{\pi}_C^{NE}$. For the relation of $\hat{\pi}_{IO}^{NE}$ and $\hat{\pi}_{IO}^{ED}$, we have $\hat{\pi}_{IO}^{ED} - \hat{\pi}_{IO}^{NE} = \frac{\bar{L}_2(A,k)k\alpha_f^2}{8(1-k^2)}$, where $\bar{L}_2(A,k) = k - 2A + kA^2$. Solving the equation $k - 2A + kA^2 = 0$ yields two roots: $\bar{T}_1 = \frac{1-\sqrt{1-k^2}}{k}$ and $\bar{T}_2 = \frac{1+\sqrt{1-k^2}}{k}$. Note that $\frac{\partial^2 \bar{L}_2(A,k)}{\partial A^2} = k \ge 0$ and $k \le \bar{T}_1 < \bar{T}_2 \le \frac{2-k^2}{k}$ when $0 \le k < 1$. Therefore, we know that $\bar{L}_2(A,k) \le 0$ for $0 \le k < 1$, which implies that $\hat{\pi}_{IO}^{ED} \le \hat{\pi}_{IO}^{NE}$. Employing the similar approach, we can also derive the relations $\hat{\pi}_C^{ED} + \hat{\pi}_{IO}^{ED} > \hat{\pi}_C^{NE} + \hat{\pi}_{IO}^{NE}$ and $\hat{U}^{ED} > \hat{U}^{NE}$. Hence, Part (i) follows. ■

Lemma A3. Given $0 < A < +\infty$, the equilibrium profits and the consumer surplus are summarized in Tables A6-A9.

Table A6. The equilibrium profit of the CM when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < \frac{k}{2-k^2}$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_f^2}{4}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{8}$
$\frac{k}{2-k^2} < A < k$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_f^2}{4}$	$\frac{\alpha_f^2}{8}$	$\frac{(8-7k^2)\alpha_f^2 - 2k(2-k^2)\alpha_f\alpha_n + (2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$k < A < \frac{1}{k}$	$\frac{\alpha_n^2}{16}$	$\frac{(4-3k^2)\alpha_f^2 - 2k\alpha_f\alpha_n + \alpha_n^2}{16(1-k^2)}$	$\frac{2\alpha_f^2 - 4k\alpha_f\alpha_n + (1+k^2)\alpha_n^2}{16(1-k^2)}$	$\frac{(8-7k^2)\alpha_f^2 - 2k(2-k^2)\alpha_f\alpha_n + (2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$\frac{1}{k} < A < \frac{2-k^2}{k}$	$\frac{\alpha_n^2}{16}$	$\frac{(4-3k^2)\alpha_f^2 - 2k\alpha_f\alpha_n + \alpha_n^2}{16(1-k^2)}$	$\frac{\alpha_n^2}{16}$	$\frac{(8-7k^2)\alpha_f^2 - 2k(2-k^2)\alpha_f\alpha_n + (2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$\frac{2-k^2}{k} < A < \frac{4-3k^2}{k(2-k^2)}$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_n^2}{16}$	$\frac{(8-7k^2)\alpha_f^2 - 2k(2-k^2)\alpha_f\alpha_n + (2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$A > \frac{4-3k^2}{k(2-k^2)}$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_n^2}{16}$	$\frac{\alpha_n^2}{16}$

Table A7. The equilibrium profit of the integrated OEM when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < \frac{k}{2-k^2}$	$\frac{\alpha_n^2}{8}$	0	$\frac{\alpha_f^2}{16}$	0
$\frac{k}{2-k^2} < A < k$	$\frac{\alpha_n^2}{8}$	0	$\frac{\alpha_f^2}{16}$	$\frac{\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{16\left(2-3k^2+k^4\right)}$
$k < A < \frac{1}{k}$	$\frac{\alpha_n^2}{8}$	$\frac{\left(\alpha_n-k\alpha_f\right)^2}{8\left(1-k^2\right)}$	$\frac{\left(\alpha_n-k\alpha_f\right)^2}{8\left(1-k^2\right)}$	$\frac{\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{16\left(2-3k^2+k^4\right)}$
$\frac{1}{k} < A < \frac{2-k^2}{k}$	$\frac{\alpha_n^2}{8}$	$\frac{\left(\alpha_n-k\alpha_f\right)^2}{8\left(1-k^2\right)}$	$\frac{\alpha_n^2}{8}$	$\frac{\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{16\left(2-3k^2+k^4\right)}$
$\frac{2-k^2}{k} < A < \frac{4-3k^2}{k(2-k^2)}$	$\frac{\alpha_n^2}{8}$	$\frac{\alpha_n^2}{8}$	$\frac{\alpha_n^2}{8}$	$\frac{\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{16\left(2-3k^2+k^4\right)}$
$A > \frac{4-3k^2}{k(2-k^2)}$	$\frac{\alpha_n^2}{8}$	$\frac{\alpha_n^2}{8}$	$\frac{\alpha_n^2}{8}$	$\frac{\alpha_n^2}{8}$

Table A8. The equilibrium profit of the whole supply-chain system when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < \frac{k}{2-k^2}$	$\frac{3\alpha_n^2}{16}$	$\frac{\alpha_f^2}{4}$	$\frac{3\alpha_f^2}{16}$	$\frac{\alpha_f^2}{8}$
$\frac{k}{2-k^2} < A < k$	$\frac{3\alpha_n^2}{16}$	$\frac{\alpha_f^2}{4}$	$\frac{3\alpha_f^2}{16}$	$\frac{(8-5k^2)\alpha_f^2-6k(2-k^2)\alpha_f\alpha_n+3(2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$k < A < \frac{1}{k}$	$\frac{3\alpha_n^2}{16}$	$\frac{(4-k^2)\alpha_f^2-6k\alpha_f\alpha_n+3\alpha_n^2}{16\left(1-k^2\right)}$	$\frac{3\alpha_f^2-3k\alpha_f\alpha_n+3\alpha_n^2}{16\left(1-k^2\right)}$	$\frac{(8-5k^2)\alpha_f^2-6k(2-k^2)\alpha_f\alpha_n+3(2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$\frac{1}{k} < A < \frac{2-k^2}{k}$	$\frac{3\alpha_n^2}{16}$	$\frac{(4-k^2)\alpha_f^2-6k\alpha_f\alpha_n+3\alpha_n^2}{16\left(1-k^2\right)}$	$\frac{3\alpha_n^2}{16}$	$\frac{(8-5k^2)\alpha_f^2-6k(2-k^2)\alpha_f\alpha_n+3(2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$\frac{2-k^2}{k} < A < \frac{4-3k^2}{k(2-k^2)}$	$\frac{3\alpha_n^2}{16}$	$\frac{3\alpha_n^2}{16}$	$\frac{3\alpha_n^2}{16}$	$\frac{(8-5k^2)\alpha_f^2-6k(2-k^2)\alpha_f\alpha_n+3(2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)}$
$A > \frac{4-3k^2}{k(2-k^2)}$	$\frac{3\alpha_n^2}{16}$	$\frac{3\alpha_n^2}{16}$	$\frac{3\alpha_n^2}{16}$	$\frac{3\alpha_n^2}{16}$

Table A9. The equilibrium consumer surplus when $0 < A < +\infty$

Item	Scenario NE	Scenario ED	Scenario EC	Scenario EE
$0 < A < \frac{k}{2-k^2}$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{32}$	$\frac{\alpha_f^2}{32}$
$\frac{k}{2-k^2} < A < k$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_f^2}{8}$	$\frac{\alpha_f^2}{32}$	$\frac{(16-20k^2+5k^4)\alpha_f^2-2k^3(2-k^2)\alpha_f\alpha_n+(2-k^2)^2(4-3k^2)\alpha_n^2}{128(2-k^2)^2(1-k^2)}$
$k < A < \frac{1}{k}$	$\frac{\alpha_n^2}{32}$	$\frac{(4-3k^2)\alpha_f^2-2k\alpha_f\alpha_n+\alpha_n^2}{32(1-k^2)}$	$\frac{\alpha_f^2-2k\alpha_f\alpha_n+\alpha_n^2}{32(1-k^2)}$	$\frac{(16-20k^2+5k^4)\alpha_f^2-2k^3(2-k^2)\alpha_f\alpha_n+(2-k^2)^2(4-3k^2)\alpha_n^2}{128(2-k^2)^2(1-k^2)}$
$\frac{1}{k} < A < \frac{2-k^2}{k}$	$\frac{\alpha_n^2}{32}$	$\frac{(4-3k^2)\alpha_f^2-2k\alpha_f\alpha_n+\alpha_n^2}{32(1-k^2)}$	$\frac{\alpha_n^2}{32}$	$\frac{(16-20k^2+5k^4)\alpha_f^2-2k^3(2-k^2)\alpha_f\alpha_n+(2-k^2)^2(4-3k^2)\alpha_n^2}{128(2-k^2)^2(1-k^2)}$
$\frac{2-k^2}{k} < A < \frac{4-3k^2}{k(2-k^2)}$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_n^2}{32}$	$\frac{(16-20k^2+5k^4)\alpha_f^2-2k^3(2-k^2)\alpha_f\alpha_n+(2-k^2)^2(4-3k^2)\alpha_n^2}{128(2-k^2)^2(1-k^2)}$
$A > \frac{4-3k^2}{k(2-k^2)}$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_n^2}{32}$	$\frac{\alpha_n^2}{32}$

Proof of Proposition 8. Using the equilibrium outcomes showed in Lemma A3, we can show the dominant strategies for the supply-chain players, the entire supply-chain system and the consumer. ■

Appendix B. Equilibrium outcomes of the decentralized supply chain

Scenario NE: Note that $p_n^{NE} = w_n^{NE} + m_n^{NE} + r_n^{NE}$. Substituting Eq. (8) into the expressions of sales price, demand quantity, profits, and consumer utility, we have

$$p_n^{NE} = \frac{5\alpha_n}{6},\ Q_n^{NE} = \frac{\alpha_n}{6};\ \pi_C^{NE} = \frac{\alpha_n^2}{36},\ \pi_O^{NE} = \frac{\alpha_n^2}{12},\ \pi_R^{NE} = \frac{\alpha_n^2}{36};\ U^{NE} = \frac{\alpha_n^2}{72}.$$

Scenario ED: Note that $p_n^{ED} = w_n^{ED} + m_n^{ED} + r_n^{ED}$. Thus, we derive the equilibrium sales prices, demand quantities, profits, and consumer utility as

$$p_n^{ED} = \begin{cases} \text{N/A} & \text{if } A \in (0, A_1] \\ \frac{(10-3k^2)\alpha_n - k(4-k^2)\alpha_f}{4(3-k^2)} & \text{if } A \in (A_1, A_2) \\ p_n^{NE} & \text{if } A \in [A_2, +\infty) \end{cases},$$

$$Q_n^{ED} = \begin{cases} 0 & \text{if } A \in (0, A_1] \\ \frac{(2-k^2)(\alpha_n - k\alpha_f)}{4(3-4k^2+k^4)} & \text{if } A \in (A_1, A_2) \\ Q_n^{NE} & \text{if } A \in [A_2, +\infty) \end{cases},$$

$$Q_f^{ED}=\begin{cases}\frac{\alpha_f}{2} & \text{if } A\in(0,A_1] \\ \frac{(6-6k^2+k^4)\alpha_f-k(2-k^2)\alpha_n}{4(3-4k^2+k^4)} & \text{if } A\in(A_1,A_2) \\ 0 & \text{if } A\in[A_2,+\infty)\end{cases};$$

$$\pi_C^{ED}=\begin{cases}\frac{\alpha_f^2}{4} & \text{if } A\in(0,A_1] \\ \frac{(36-56k^2+24k^4-3k^6)\alpha_f^2-2k(2-k^2)^2\alpha_n\alpha_f+(2-k^2)^2\alpha_n^2}{16(1-k^2)(3-k^2)^2} & \text{if } A\in(A_1,A_2) \\ \pi_C^{NE} & \text{if } A\in[A_2,+\infty)\end{cases},$$

$$\pi_O^{ED}=\begin{cases}0 & \text{if } A\in(0,A_1] \\ \frac{(2-k^2)(\alpha_n-k\alpha_f)^2}{8(3-4k^2+k^4)} & \text{if } A\in(A_1,A_2) \\ \pi_O^{NE} & \text{if } A\in[A_2,+\infty)\end{cases},\quad \pi_R^{ED}=\begin{cases}0 & \text{if } A\in(0,A_1] \\ \frac{(2-k^2)(\alpha_n-k\alpha_f)^2}{8(3-k^2)^2(1-k^2)} & \text{if } A\in(A_1,A_2) \\ \pi_R^{NE} & \text{if } A\in[A_2,+\infty)\end{cases};$$

$$U^{ED}=\begin{cases}\frac{\alpha_f^2}{8} & \text{if } A\in(0,A_1] \\ \frac{(36-56k^2+24k^4-3k^6)\alpha_f^2-2k(2-k^2)^2\alpha_n\alpha_f+(2-k^2)^2\alpha_n^2}{32(1-k^2)(3-k^2)^2} & \text{if } A\in(A_1,A_2) \\ U^{NE} & \text{if } A\in[A_2,+\infty)\end{cases}.$$

Scenario EC: Note that $p_n^{EC}=w_n^{EC}+m_n^{EC}+r_n^{EC}$. Thus, we derive the equilibrium sales prices, demand quantities, profits, and consumer utility as

$$p_n^{EC}=\begin{cases}\text{N/A} & \text{if } A\in(0,A_3] \\ \frac{5\alpha_n}{6}-\frac{k\alpha_f}{2(4-k^2)} & \text{if } A\in(A_3,A_4) \\ p_n^{NE} & \text{if } A\in[A_4,+\infty)\end{cases};$$

$$Q_n^{EC}=\begin{cases}0 & \text{if } A\in(0,A_3] \\ \frac{(4-k^2)\alpha_n-3k\alpha_f}{24(1-k^2)} & \text{if } A\in(A_3,A_4) \\ Q_n^{NE} & \text{if } A\in[A_4,+\infty)\end{cases},\quad Q_f^{EC}=\begin{cases}\frac{\alpha_f}{4} & \text{if } A\in(0,A_3] \\ \frac{(8-5k^2)\alpha_f-k(4-k^2)\alpha_n}{8(4-5k^2+k^4)} & \text{if } A\in(A_3,A_4) \\ 0 & \text{if } A\in[A_4,+\infty)\end{cases};$$

$$\pi_C^{EC}=\begin{cases}\frac{\alpha_f^2}{8} & \text{if } A\in(0,A_3] \\ \frac{9(64-72k^2+17k^4)\alpha_f^2-6k(64-44k^2+7k^4)\alpha_n\alpha_f+(4-k^2)^2(8+k^2)\alpha_n^2}{288(4-k^2)^2(1-k^2)} & \text{if } A\in(A_3,A_4) \\ \pi_C^{NE} & \text{if } A\in[A_4,+\infty)\end{cases},$$

$$\pi_O^{EC}=\begin{cases}0 & \text{if } A\in(0,A_3] \\ \frac{((4-k^2)\alpha_n-3k\alpha_f)^2}{48(4-5k^2+k^4)} & \text{if } A\in(A_3,A_4) \\ \pi_C^{NE} & \text{if } A\in[A_4,+\infty)\end{cases},$$

$$\pi_R^{EC}=\begin{cases}\frac{\alpha_f^2}{16} & \text{if } A\in(0,A_3] \\ \frac{9(64-64k^2+9k^4)\alpha_f^2-6k(32-4k^2+k^4)\alpha_n\alpha_f+(4-k^2)^2(16-7k^2)\alpha_n^2}{1152(4-k^2)^2(1-k^2)} & \text{if } A\in(A_3,A_4) \\ \pi_C^{NE} & \text{if } A\in[A_4,+\infty)\end{cases};$$

$$U^{EC} = \begin{cases} \frac{\alpha_f^2}{32} & \text{if } A \in (0, A_3] \\ \frac{9\left(64-64k^2+9k^4\right)\alpha_f^2-6k\left(32-4k^2+k^4\right)\alpha_n\alpha_f+\left(4-k^2\right)^2\left(16-7k^2\right)\alpha_n^2}{576(4-k^2)^2(1-k^2)} & \text{if } A \in (A_3, A_4) \\ U^{NE} & \text{if } A \in [A_4, +\infty) \end{cases}.$$

Scenario EE: Note that $p_n^{EE} = w_n^{EE} + m_n^{EE} + r_n^{EE}$. Thus, we derive the equilibrium sales prices, demand quantities, profits, and consumer utility as

$$p_n^{EE} = \begin{cases} \text{N/A} & \text{if } A \in (0, A_5] \\ \frac{\left(40-46k^2+13k^4\right)\alpha_n-k\left(8-5k^2\right)\alpha_f}{8(6-7k^2+2k^4)} & \text{if } A \in (A_5, A_6) \\ p_n^{EC} & \text{if } A \in [A_6, +\infty) \end{cases};$$

$$Q_n^{EE} = \begin{cases} 0 & \text{if } A \in (0, A_5] \\ \frac{\left(4-3k^2\right)\left(\left(2-k^2\right)\alpha_n-k\alpha_f\right)}{16(3-5k^2+2k^4)} & \text{if } A \in (A_5, A_6) \\ Q_n^{EC} & \text{if } A \in [A_6, +\infty) \end{cases},$$

$$Q_f^{EE} = \begin{cases} \frac{\alpha_f}{4} & \text{if } A \in (0, A_5] \\ \frac{\left(24-36k^2+13k^4\right)\alpha_f-k\left(8-10k^2+3k^4\right)\alpha_n}{16(6-13k^2+9k^4-2k^6)} & \text{if } A \in (A_5, A_6) \\ 0 & \text{if } A \in [A_6, +\infty) \end{cases};$$

$$\pi_C^{EE} = \begin{cases} \frac{\alpha_f^2}{8} & \text{if } A \in (0, A_5] \\ \frac{\left(288-656k^2+488k^4-119k^6\right)\alpha_f^2-2k\left(4-3k^2\right)^2\left(2-k^2\right)\alpha_n\alpha_f+\left(8-10k^2+3k^4\right)^2\alpha_n^2}{128(3-2k^2)^2(2-3k^2+k^4)} & \text{if } A \in (A_5, A_6) \\ \pi_C^{NE} & \text{if } A \in [A_6, +\infty) \end{cases},$$

$$\pi_O^{EE} = \begin{cases} 0 & \text{if } A \in (0, A_5] \\ \frac{\left(4-3k^2\right)\left(\left(2-k^2\right)\alpha_n-k\alpha_f\right)^2}{32(2-k^2)(3-5k^2+2k^4)} & \text{if } A \in (A_5, A_6) \\ \pi_O^{NE} & \text{if } A \in [A_6, +\infty) \end{cases},$$

$$\pi_R^{EE} = \begin{cases} 0 & \text{if } A \in (0, A_5] \\ \frac{\left(4-3k^2\right)\left(\left(2-k^2\right)\alpha_n-k\alpha_f\right)^2}{64(1-k^2)(3-2k^2)^2} & \text{if } A \in (A_5, A_6) \\ \pi_R^{NE} & \text{if } A \in [A_6, +\infty) \end{cases},$$

$$\pi_{RE}^{EE} = \begin{cases} \frac{\alpha_f^2}{16} & \text{if } A \in (0, A_5] \\ \frac{\left(\left(24-36k^2+13k^4\right)\alpha_f-k\left(8-10k^2-3k^4\right)\alpha_f\right)^2}{256(1-k^2)^2(6-7k^2+2k^4)^2} & \text{if } A \in (A_5, A_6) \\ 0 & \text{if } A \in [A_6, +\infty) \end{cases};$$

$$U^{EE} = \begin{cases} \frac{\alpha_f^2}{32} & \text{if } A \in (0, A_5] \\ \frac{(576-1472k^2+1344k^4-516k^6+69k^8)\alpha_f^2-2k(4-3k^2)(2-k^2)(8-16k^2+7k^4)\alpha_n\alpha_f+(2-k^2)^2(4-3k^2)^3\alpha_n^2}{512(1-k^2)(6-7k^2+2k^4)^2} & \text{if } A \in (A_5, A_6) \\ U^{NE} & \text{if } A \in [A_6, +\infty) \end{cases}.$$

Appendix C. Equilibrium outcomes of the supply chain with the integrated downstream firms

Note: In the following descriptions, we use the notations $\widehat{\ }$ to denote the equilibrium outcomes of the case of the integrated downstream firms. Moreover, the subscript IO denote the integrated OEM.

Scenario NE: Denote $\widehat{w}_n$ and $\widehat{m}_n$ as the CM's wholesale price and the integrated OEM's markup of the NB product, respectively. Thus, we have the equilibrium solutions: $\widehat{w}_n^{NE} = \frac{\alpha_n}{4}$ and $\widehat{m}_n^{NE} = \frac{\alpha_n}{2}$. We further derive all other equilibrium outcomes: $\hat{p}_n^{NE} = \frac{3\alpha_n}{4}$, $\hat{q}_n^{NE} = \frac{\alpha_n}{4}$, $\hat{\pi}_C^{NE} = \frac{\alpha_n^2}{16}$, $\hat{\pi}_{IO}^{NE} = \frac{\alpha_n^2}{8}$, $\widehat{U}^{NE} = \frac{\alpha_n^2}{32}$.

Scenario ED: The equilibrium outcomes are summarized as follows:

$$\widehat{m}_n^{ED} = \begin{cases} \text{N/A} & \text{if } A \in (0,k] \\ \frac{\alpha_n - k\alpha_f}{2} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \widehat{m}_n^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases}, \quad \widehat{w}_n^{ED} = \begin{cases} \text{N/A} & \text{if } A \in (0,k] \\ \frac{\alpha_n + k\alpha_f}{4} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \widehat{w}_n^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases},$$

$$\hat{p}_n^{ED} = \begin{cases} \text{N/A} & \text{if } A \in (0,k] \\ \frac{3\alpha_n - k\alpha_f}{4} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \hat{p}_n^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases}, \quad \hat{p}_f^{ED} = \begin{cases} \frac{\alpha_f}{2} & \text{if } A \in \left(0, \frac{2-k^2}{k}\right) \\ \text{N/A} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases},$$

$$\widehat{Q}_n^{ED} = \begin{cases} 0 & \text{if } A \in (0,k] \\ \frac{\alpha_n - k\alpha_f}{4(1-k^2)} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \widehat{Q}_n^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases}, \quad \widehat{Q}_f^{ED} = \begin{cases} \frac{\alpha_f}{2} & \text{if } A \in (0,k] \\ \frac{(2-k^2)\alpha_f - k\alpha_n}{4(1-k^2)} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ 0 & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases},$$

$$\hat{\pi}_{IO}^{ED} = \begin{cases} 0 & \text{if } A \in (0,k] \\ \frac{(\alpha_n - k\alpha_f)^2}{8(1-k^2)} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \hat{\pi}_{IO}^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases}, \quad \hat{\pi}_C^{ED} = \begin{cases} \frac{\alpha_f^2}{4} & \text{if } A \in (0,k] \\ \frac{(4-3k^2)\alpha_f^2 - 2k\alpha_f\alpha_n + \alpha_n^2}{16(1-k^2)} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \hat{\pi}_C^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases},$$

$$\widehat{U}^{ED} = \begin{cases} \frac{\alpha_f^2}{8} & \text{if } A \in (0,k] \\ \frac{(4-3k^2)\alpha_f^2 - 2k\alpha_f\alpha_n + \alpha_n^2}{32(1-k^2)} & \text{if } A \in \left(k, \frac{2-k^2}{k}\right) \\ \widehat{U}^{NE} & \text{if } A \in \left[\frac{2-k^2}{k}, +\infty\right) \end{cases}.$$

Scenario EC: The equilibrium outcomes are summarized as follows:

$$\hat{m}_n^{EC} = \begin{cases} \text{N/A} & \text{if } A \in (0,k] \\ \frac{\alpha_n}{2} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{m}_n^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases}, \quad \hat{w}_n^{EC} = \begin{cases} \text{N/A} & \text{if } A \in (0,k] \\ \frac{\alpha_n}{4} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{w}_n^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases},$$

$$\hat{p}_n^{EC} = \begin{cases} \text{N/A} & \text{if } A \in (0,k] \\ \frac{3\alpha_n}{4} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{p}_n^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases}, \quad \hat{w}_f^{EC} = \begin{cases} \frac{\alpha_f}{2} & \text{if } A \in (0,k] \\ \frac{2\alpha_f - k\alpha_n}{4} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \text{N/A} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases},$$

$$\hat{p}_f^{EC} = \begin{cases} \frac{3\alpha_f}{4} & \text{if } A \in \left(0, \frac{1}{k}\right) \\ \text{N/A} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases},$$

$$\hat{Q}_n^{EC} = \begin{cases} 0 & \text{if } A \in (0,k] \\ \frac{\alpha_n - k\alpha_f}{4\left(1-k^2\right)} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{Q}_n^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases}, \quad \hat{Q}_f^{EC} = \begin{cases} \frac{\alpha_f}{2} & \text{if } A \in (0,k] \\ \frac{\alpha_f - k\alpha_n}{4\left(1-k^2\right)} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ 0 & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases},$$

$$\hat{\pi}_{IO}^{EC} = \begin{cases} \frac{\alpha_f^2}{16} & \text{if } A \in (0,k] \\ \frac{\alpha_f^2 - 2k\alpha_f\alpha_n + \left(2-k^2\right)\alpha_n^2}{16\left(1-k^2\right)} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{\pi}_{IO}^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases}, \quad \hat{\pi}_C^{EC} = \begin{cases} \frac{\alpha_f^2}{8} & \text{if } A \in (0,k] \\ \frac{2\alpha_f^2 - 4k\alpha_f\alpha_n + \left(1+k^2\right)\alpha_n^2}{16\left(1-k^2\right)} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{\pi}_C^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases},$$

$$\hat{U}^{EC} = \begin{cases} \frac{\alpha_f^2}{32} & \text{if } A \in (0,k] \\ \frac{\alpha_f^2 - 2k\alpha_f\alpha_n + \alpha_n^2}{32\left(1-k^2\right)} & \text{if } A \in \left(k, \frac{1}{k}\right) \\ \hat{U}^{NE} & \text{if } A \in \left[\frac{1}{k}, +\infty\right) \end{cases}.$$

Scenario EE: The equilibrium outcomes are summarized as follows:

$$\hat{m}_n^{EE} = \begin{cases} \text{N/A} & \text{if } A \in \left(0, \frac{k}{2-k^2}\right] \\ \frac{1}{2}\left(\alpha_n - \frac{k\alpha_f}{2-k^2}\right) & \text{if } A \in \left(\frac{k}{2-k^2}, \frac{4-3k^2}{k\left(2-k^2\right)}\right) \\ \hat{m}_n^{NE} & \text{if } A \in \left[\frac{4-3k^2}{k\left(2-k^2\right)}, +\infty\right) \end{cases},$$

$$\hat{w}_n^{EE} = \begin{cases} \text{N/A} & \text{if } A \in \left(0, \frac{k}{2-k^2}\right] \\ \frac{1}{4}\left(\alpha_n + \frac{k\alpha_f}{2-k^2}\right) & \text{if } A \in \left(\frac{k}{2-k^2}, \frac{4-3k^2}{k\left(2-k^2\right)}\right) \\ \hat{w}_n^{NE} & \text{if } A \in \left[\frac{4-3k^2}{k\left(2-k^2\right)}, +\infty\right) \end{cases},$$

$$\hat{p}_n^{EE}=\begin{cases}\text{N/A} & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{1}{4}\left(3\alpha_n-\frac{k\alpha_f}{2-k^2}\right) & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right),\\ \hat{p}_n^{NE} & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\hat{w}_f^{EE}=\begin{cases}\frac{\alpha_f}{2} & \text{if } A\in\left(0,\frac{4-3k^2}{k(2-k^2)}\right)\\ 0 & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases},\quad \hat{p}_f^{EE}=\begin{cases}\frac{3\alpha_f}{4} & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{1}{8}\left(\frac{(12-7k^2)\alpha_f}{2-k^2}-k\alpha_n\right) & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right)\\ \text{N/A} & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\hat{Q}_n^{EE}=\begin{cases}0 & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{(2-k^2)\alpha_n-k\alpha_f}{8(1-k^2)} & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right),\\ \hat{Q}_n^{NE} & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\hat{Q}_f^{EE}=\begin{cases}\frac{\alpha_f}{4} & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{(4-3k^2)\alpha_f-k(2-k^2)\alpha_n}{8(2-3k^2+k^4)} & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right),\\ 0 & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\hat{\pi}_{IO}^{EE}=\begin{cases}0 & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{\left((2-k^2)\alpha_n-k\alpha_f\right)^2}{16(2-3k^2+k^4)} & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right),\\ \hat{\pi}_{IO}^{NE} & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\hat{\pi}_C^{EE}=\begin{cases}\frac{\alpha_f^2}{8} & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{(8-7k^2)\alpha_f^2-2k(2-k^2)\alpha_f\alpha_n+(2-k^2)^2\alpha_n^2}{32(2-3k^2+k^4)} & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right),\\ \hat{\pi}_C^{NE} & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\hat{\pi}_{RE}^{EE}=\begin{cases}\frac{\alpha_f^2}{16} & \text{if } A\in(0,\frac{k}{2-k^2}]\\ \frac{\left((4-3k^2)\alpha_f-k(2-k^2)\alpha_n\right)^2}{64(1-k^2)(2-k^2)^2} & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k(2-k^2)}\right),\\ \hat{\pi}_{IO}^{NE} & \text{if } A\in\left[\frac{4-3k^2}{k(2-k^2)},+\infty\right)\end{cases}$$

$$\widehat{U}^{EE}=\begin{cases}\frac{\alpha_f^2}{32} & \text{if } A\in\left(0,\frac{k}{2-k^2}\right] \\ \frac{\left(16-20k^2+5k^4\right)\alpha_f^2-2k^3(2-k^2)\alpha_f\alpha_n+\left(2-k^2\right)^2(4-3k^2)\alpha_n^2}{128\left(2-k^2\right)^2\left(1-k^2\right)} & \text{if } A\in\left(\frac{k}{2-k^2},\frac{4-3k^2}{k\left(2-k^2\right)}\right) \\ \widehat{U}^{NE} & \text{if } A\in\left[\frac{4-3k^2}{k\left(2-k^2\right)},+\infty\right)\end{cases}.$$