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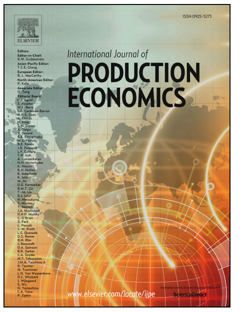

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Shaofu Du, Li Wang, Li Hu*

Shaofu Du, Ph.D., Professor
School of Management, University of Science and Technology of China, Hefei 230026, P.R. China
Telephone Number: +86-551-6360-1733
Email: sdu@ustc.edu.cn

Li Wang, Ph.D. Candidate,
School of Management, University of Science and Technology of China, Hefei 230026, P.R. China
Email: wendylee@mail.ustc.edu.cn

Li Hu, Ph.D. (Corresponding author)
School of Management, University of Science and Technology of China, Hefei 230026, P.R. China
Telephone Number: +86-159-5518-1830
Email: foxhu@mail.ustc.edu.cn

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Abstract

With the popularity of omnichannel retailing, consumers are sophisticated enough to make strategic channel decisions, considering the possible disappointment for an unexpected outcome induced by various uncertainties from online and offline channels. Value uncertainty online can trigger low-value disappointment, and availability uncertainty offline may cause stock-out disappointment. This study characterizes the effect of consumers' anticipated disappointment aversion behavior on the optimal pricing decisions of retailers with or without inventory constraint in the omnichannel environment. When a retailer operates in dual-channel and faces offline inventory constraint, the concern for consumers' homogeneous disappointment aversion changes the threshold above which the retailer implement different channel pricing strategies. Then, we show how the negative impact of disappointment on profit and market can be mitigated by physical showroom mechanism, which is increasingly adopted by omnichannel management. Introduction of a cost-effective physical showroom expands the market by relaxing the restriction of stock-out disappointment and improves profit as long as consumers' low-value disappointment is high enough. Interestingly, the low-value disappointment benefits the omnichannel in expanding the market coverage by alleviating the constraint exerted by stock-out disappointment. Further discussions regarding the disappointment behavior are derived from the extension of consumers' heterogeneity in disappointment-aversion.

Keywords: Omnichannel management; disappointment aversion; consumer strategic option; behavioral pricing

1. Introduction

With the increasing digitalization in marketing and retailing, the retail landscape continues to change with the integration of new phenomena, such as mobile shopping and social media, in online and offline channels. A new concept called online-to-offline is increasingly becoming popular. As consumers gradually take advantage of the seamless and unified shopping experience offered by the emerging business models, new opportunities and challenges have emerged for retailers (Bell et al., 2014; Harsha et al., 2016).

Based on existing online and offline channel management, an increasing number of companies are transforming into omnichannel operations, a new retail mode that emphasizes the interplay between channels and consumers. For instance, Wal-Mart, the worlds largest retailer and a model brick-and-mortar store, has explored its business online with new strategies. Wal-Mart offers a new service through which customers can order goods online and pay for them at a nearby store with cash. New features of the Wal-Mart App, such as savings catcher (a new price-check function) and Geo-fence, offer a convenient and interactive shopping experience. The solely e-commerce store of the New York-based eyeglasses brand Warby Parker has supplemented its operation with the establishment of stores for product display and brand promotion. Even dual-channel retailers have explored the supply and demand interactions from crossing channels and integrating price and inventory-sharing decisions to promote omnichannel efficiency and performance. Therefore, to thrive in this new environment, retailers of all types should take a broader perspective on channels and reexamine their strategies for delivering information and products to consumers, who move among channels in their search and buying process (Bell et al., 2014; Verhoef et al., 2015).

In an omnichannel retail environment, where the product is accessible from both online and offline channels, consumers may either purchase a product online directly or only browse a product online and choose visit the brick-and-mortar store to touch and feel the product before making their decisions. The practice of multi-channel shopping has taught consumers to be “omnichannel” in their outlook and behavior. Consumers are sophisticated enough to consider all the advantages and disadvantages of each channel, especially when they are confronted with the uncertainty of product value and availability in online and offline channels, respectively. They are aware that the actual outcome may not coincide with their previous expectation after choosing each channel. The consumers who choose online purchasing must sacrifice the benefit of physical inspection of the product, which increases the likelihood of dissatisfaction with the returns (Mukhopadhyay and Setaputra, 2007). According to ChinaIRN (China Industry Research Net) report, an approximately 30% average refund rate in e-commence channels is common, and this index even reaches 40% in garment products. However, in an online channel, consumers who are uninformed about value may find the purchased product unsuitable and return them. While consumers expect to take advantage of online shopping's delivery convenience in choosing an online channel, they eventually must spend extra time and cost to transport the product back to the online retailer, arousing a sense of disappointment about the poor outcome. In an offline channel, consumers visiting the store may find the product unavailable and end up with switching to an online channel. According to Bäckström and Johansson (2006), stockout situations (e.g.,missing commodities, sold-out special offer products) constitute critical incidents that contribute to negative shopping experiences. Consumers react substantially and negatively to stock-out, reporting low satisfaction with the decision process and showing a high likelihood of switching choices for subsequent shopping trips (Fitzsimons, 2000). Thus consumers commonly feel disappointed with the occurrence of stock-out. As consumers make a decision in anticipation of a possible negative outcome and the associated pessimistic emotion from each channel, value uncertainty may discourage purchase and availability uncertainty may discourage patronage (Gao and Su, 2016b). Hence, it is believed that the consumer' purchase decision can be affected by the emotion caused by the uncertainties incurred online and offline.

Disappointment has important implications in the study of decision making under uncertainty, as verified by Gill and Prowse (2012) through experimental research. They find that the anticipation of disappointment affects decision making. According to disappointment theory, disappointment is a psychological reaction to an outcome that does not match expectation. Conversely, if the outcome exceeds expectation, then a sense of elation arises. Consumers usually face uncertainties (about valuation and product availability) when making purchase decisions. They may compare the actual outcome with their expectations after their uncertainties are resolved. Elation and disappointment emerge concurrently to measure gain and loss relative to expectation, respectively. A good outcome, which is superior to previous expectation, triggers elation, whereas a poor result causes disappointment. Generally, the effect of disappointment on utility is stronger than that of elation. Thus, the compounding effect of disappointment and elation is captured as disappointment aversion behavior. The assumption that consumers are regarded as averse to disappointment is widely used in, for example, Kőszegi and Rabin (2007) and Liu and Shum (2013). Therefore, combined with the above-mentioned omnichannel context, in which emotionally rational consumers who do not learn their value and buy from the online channel directly anticipate low-value disappointment, while those who take the risk of a stock-out to visit the store predict stock-out disappointment, we seek to understand the following: What is the effect of disappointment-aversion on consumers' utility and purchase behavior? What if consumers show diverse levels of disappointment-aversion?

As consumers incur distinct types of disappointment-aversion behavior caused by the low-value and stock-out uncertainty incurred in online and offline channels, leading to heterogeneous channel preferences and values, omnichannel retailers may apply differential prices to allow consumers to self-select their preferred channel price combination. Considerate successful multichannel players such as Walmart, Tesco, and AT & T differentiate their prices across online and offline channels to increase profitability (Vogel and Paul, 2015). Various types of price differentiation have been widely used by retailers, such as coupons, special discounts, and bundling. Although there are empirical studies that suggest uniform prices should be charged to preserve channel consistency and avoid consumer irritation, channel-based price differentiation has been acknowledged as an opportunity to increase profits through theoretical research (Xie et al., 2017a; Zhang, 2009), and empirical studies that indicate that up to 60 percent of multi-channel retailers engage in channel-based price differentiation and that this trend is increasing (Wolk and Ebling, 2010). Given that consumer disappointment is a potential concern for the retailer, flexible pricing can become an indispensable tool for omnichannel retailers. This raises the second question addressed in this research, which is as follows: What is the effect of consumers' disappointment-aversion on profit, and how should the omnichannel retailer obtain optimal pricing decisions to respond to disappointment?

We also note that omnichannel retailers are aware of the negative effect of uncertainty and associated disappointment on consumer behavior, and they have begun to leverage this effect in their operational strategies. The physical showroom, which is designed to achieve channel integration, is precisely able to mitigate the disappointment caused by uncertainty. Typically, suppose there is a physical showroom in a store that allows consumers to inspect the entire product line even when the product is out-of-stock. The idea of a physical showroom has been adopted by omnichannel companies, especially for products that require service or have many touch-and-feel components, or both (Bell et al., 2014). For example, Apple INC. offers consumers hands-on experiences with Apple products in their showrooms and learning centers. Even if consumers are informed of the stock-out of newly launched Iphone, they can still visit resellers to enjoy the try-before-you-buy service to obtain a comprehensive sense of the performances of the desired smartphone.Once they find it to be their liking, they can make a purchase directly by placing an order at the Apple' website. It is reported that, since 2013, Canon has followed Apple's strategy, providing physical showroom of Canon products, including sculptural exhibits for testing point-and-shoots, DSLRs, lenses and accessories using interesting visual subjects. Many fashion e-tailers such as Bonobos and Warby Parker have also established product showrooms purely for display purposes in third-party locations. Using data on display showroom introductions by WarbyParker.com, Bell et al. (2015) shows that the introduction of a physical showroom takes an active role in reducing the return rate in the online channel and increasing the demand overall. This motivates the last question addressed in this research, which is as follows: Given consumers' disappointment-aversion emotion, does the physical showroom always benefit the retailer, and when should the omnichannel retailer implement the physical showroom mechanism?

Overall, The purpose of our paper is to study how the omnichannel retailer makes strategic pricing decisions to achieve optimal profit considering consumers' disappointment-aversion behavior incurred online and offline.

Specifically, our study relies on formal disappointment theory (Bell, 1985) to characterize the following:

- Consumers channel choice decision under anticipated disappointment in the omnichannel environment (§3);
- The effect of consumer disappointment aversion behavior on the pricing policy and profit of an omnichannel retailer with or without inventory constraint when consumers have the homogeneous (§4) or heterogeneous (§6) disappointment-aversion
- The role of a physical showroom in mitigating consumer disappointment and enhancing channel performance(§5)

We review the related literature in §2 and summarize the main findings in §7.

Our paper, as the first one to incorporate the disappointment-aversion emotion in an omnichannel environment, presents an in-depth discussion about the impact of the behavior on retailers' pricing decisions and profits. We discover that online and offline pricing decisions are fundamentally different from the benchmark when disappointment behavior is ignored. Specifically, when consumers share the same level of disappointment, optimal price and profit are negatively affected by consumers' low-value disappointment, and the stock-out disappointment will restrict the structure of retailers optimal channel polices. While when consumers are heterogeneous in their disappointment-aversion, which is drawn from a two-dimensional uniform distribution, the optimal pricing decision should be adjusted strategically to the maximum range of the population's disappointment-aversion level. In addition, we investigate the effect of implementing the physical showroom mechanism on the retailer from the perspectives of market coverage and profit promotion, respectively. When consumers are homogeneous in disappointment-aversion level, we find that the physical showroom can expand market coverage by mitigating the disappointment aversion emotion and what is counterintuitive is that consumers' low-value disappointment, which is criticized for the decline in channel profit, plays a positive role in increasing the demand under the physical showroom mechanism. Further, only when the cost of undertaking the mechanism is low enough and consumers are averse enough to the low-value disappointment, can the physical showroom help to dramatically boost the profit. Simply put, based on the concern about the behavior of strategic but disappointment-averse consumers who decide where to purchase, our paper provides pricing policy guidance to omnichannel retailers in the homogeneous and heterogeneous markets, respectively, and evaluate the emerging physical showroom mechanism from the perspective of disappointment behavior.

2. Literature Review

This study focuses on the management of online and offline channels. In the past decade, the advent of the online channel and new additional digital channels have significantly changed traditional retail business models, that is, consumer behavior and retail mix strategy (Verhoef et al., 2015; Sorescu et al., 2011). With this trend, many manufacturers or suppliers have introduced a direct selling channel to compete with their original retail partners (Gao and Su, 2016a,b). In this situation, different channels are operated by different companies, creating the basic business setting of a series of multi-channel studies. Chiang et al. (2003) pioneered a price-setting game between a manufacturer that used direct marketing and its independent retailer. Surprisingly, they find that direct marketing can be used for strategic channel control purposes even if no sale occurs in the direct channel. Many studies have focused on competition in multiple channels. Some interesting and specific pricing mechanisms, such as personalized pricing (Liu and Zhang, 2006), and price matching strategy (Cattani et al., 2006; Wang et al., 2016) , have also been widely discussed. The effect of product return on a multi-channel retailer is theoretically analyzed by (Ofek et al., 2011), who intensively discuss pricing strategies and physical store assistance levels. Other recent studies have explored related issues, such as the consumer channel migration (Chintagunta et al., 2012) and segmentation (Ansari et al., 2008; Konuş et al., 2008) and product selection and service strategy (Brynjolfsson et al., 2009; Chen et al., 2008). Aside from the view of competition, multi-channel coordination in cooperative advertising and service is another stream of this research area (Chen et al., 2016; Xie et al., 2017b).

In contrast to the research mentioned above, this study focuses on an emerging environment, omnichannel retailing, in which a single retailer operates both online and store channels in an integrated manner. According to Verhoef et al. (2015), omnichannel management is defined as *"the synergetic management of the numerous available channels and customer touchpoints, in such a way that the customer experience across channels and the performance over channels is optimized."* Arguably, channel integartion, which separates online and offline channels with no overlap, is one of key characteristics that distinguish omnichannel management from multichannel management. On one hand, due to the increased digitalization in marketing and retailing (Leeflang et al., 2014) with the dawn of the mobile channel, tablets, and social media, the distinctions between online and offline will vanish, propelling the model from a multi-channel to an omnichannel one (Verhoef et al., 2015; Bell et al., 2014; Rigby, 2011; Brynjolfsson et al., 2013). On the other hand, consumers pursue and enjoy a seamless shopping experience, making strategic purchase decisions by crossing channels. New opportunities and challenges call for innovations in retail business models (Sorescu et al., 2011), which have recently been studied and captured in omnichannel management. Firstly, the emerging fulfillment option, called the buy-online-and-pickup-in-store (BOPS) mechanism in omnichannel retail, has been explored from theoretical and empirical perspectives (Gao and Su, 2016a; Gallino and Moreno, 2014), These studies have found that it may not be profitable to implement BOPS on products that sell well in stores. Gallino and Moreno (2014) examine another similar option, called ship-to-store, and they analyze the effect of sharing reliable inventory availability information. Information mechanisms have also become an important issue in omnichannel retailing. Gao and Su (2016b) introduce three information mechanisms: physical showrooms, virtual showrooms, and availability information. They find that the optimal information structure may involve choosing only one of the three mechanisms when customers are homogeneous. Bell et al. (2015) mainly focused on the effect of the physical showroom mechanism on channel performance and return rate from an experimental perspective. In the present study, we depict the features of an omnichannel and analyze the effect of value and availibility uncertainty. The physical showroom mechanism has been discussed and evaluated in detail. In contrast to the existing omnichannel studies, we consider consumer behavior, that is, disappointment aversion caused by uncertainty, in modifying the expected utility of channel choice.

We join the growing stream of literatures that incorporate consumers' behavioral regularities into revenue management. Many studies have paid attention to forward-looking customer behavior, instead of viewing consumers as myopic. These studies mostly discuss firms' two-stage markdown pricing strategies in which consumers strategically choose when to purchase (Su and Zhang, 2009; Su, 2010; Cachon and Swinney, 2011). The issue of where strategic consumers buy products in the omnichannel scenario has also been explored (Gao and Su, 2016a,b). Similar to this paper, we investigate the consumer strategic option between online and offline channels after going through each channel utility. Also of concern here is consumers' disappointment aversion behavior, which is well established in behavioral decision theories. Bell (1985) was the first paper to define disappointment aversion behavior and examine its effect on utility development and model foundation. This theory has been enriched by decision theory and behavior economics (Loomes and Sugden, 1987; Gul, 1991; Kőszegi and Rabin, 2007). Herrmann et al. (2007) empirically demonstrate the influence of perceived price fairness on satisfaction or disappointment judgments. Gill and Prowse (2012) experimentally validate disappointment aversion behavior and its effect on decisions under uncertainty. Tavana et al. (2016) consider consumers' perception of the potential interactions between product characteristics and they use the corresponding certainty equivalents as their reference values when acquiring information to explain the disappointment expressed by loyal customers. Liu and Shum (2013) study the impact of disappointment aversion in consumer strategic purchasing behavior as well as firm pricing and capacity rationing decisions, with a focus on who executes the mark-up or markdown policies in advance pricing scenarios. Other behavioral factors are also widely considered in retail management, such as risk aversion (Xu et al., 2014; Gan et al., 2005; Choi, 2016; Masatlioglu and Raymond, 2016) and consumer regret (Nasiry and Popescu, 2012; Özer and Zheng, 2015). Among them, Choi (2016) incorporate retailers' risk-averse behavior into the optimization of the make-to-order quick-response fashion supply chain system, and Masatlioglu and Raymond (2016) examine the reference-dependent risk preference, focusing on choice-acclimating personal equilibria and discussing different psychological intuitions, while Nasiry and Popescu (2012)characterize the effect of anticipated regret in an advance selling context. Additionally, loss aversion, which is closely related to the concept of disappointment aversion, mainly refers to an economic agent's behavior in response to institutional phenomena (such as the pressure of performance evaluation on a short-term basis) whereby agent usually has a fixed reference point, while disappointment aversion is mainly based on the idea that reference points are determined endogenously (Fielding and Stracca, 2007). Thus we adopt disappointment aversion to depict consumers' attitude toward uncertainty. Investigating the effect of consumers' behavioral factors can lead to prescription for optimal polices that can work better in practice. The present paper advances the omnichannel management research in this this direction.

3. Consumer Channel Choice Behavior Under Disappointment

We consider a profit-seeking retailer that sells a product to a group of consumers through two channels, online and store, at static prices p_o and p_s , respectively. In the store channel, a capacitated inventory can exist under local and space constraints, and we assume the in-store stock to be limited to B units. Consumers who arrive sequentially to the market have the probability $f \in [0, 1]$ of gaining access to the product in the store. We assume that the online channel is supplied and delivered exogenously and always has enough capacity to satisfy the demand. Given the omnichannel firms policy, consumers with unit demand and uncertain value v decide whether to purchase online directly or to visit the store. Without a loss of generality, we normalize the fixed market size to 1. A fraction of θ population has a higher value H for the product, defined as high types, and the remaining consumers keep a low value L , defined as low types. Consumers are ex-ante homogeneous and do not know their actual type ex-ante until they receive the product after purchase online or check the product in store to realize its value. Consumers who choose to purchase online cannot directly touch and feel the actual product and may find the product to be unfit ex-post and return the product for a refund r , incurring low-value disappointment. For consumers intending to go to the store with a fill rate f , they are uninformed of the real-time inventory availability and may find the product unavailable ex-post, thus incurring stock-out disappointment with a probability of $1-f$. Those who actually encounter stock-out may instead switch to the online channel, leaving the online channel as a second option. Therefore, under omnichannel retailing, a key trade-off for consumers with value uncertainty is to buy the product directly online with the uncertainty of low-value versus to examining the product in-store with the risk of a stock-out. In the following, we introduce disappointment theory, incorporate the low-value (online) and stock-out (offline) disappointment aversion behavior into the consumer utility model, and examine consumers purchase decisions.

3.1. Disappointment Theory

According to psychological disappointment theory (Bell, 1985), disappointment (or elation) is a psychological reaction caused by comparing the actual outcome of a lottery to one's prior expectation when making decision under uncertainty. Bell's study is the first to integrate the concept of disappointment into utility theory in a prescriptive model. The model assumes that the total utility perceived by a consumer who faces uncertainty is the combination of economic surplus and psychological satisfaction, which is expressed as follows:

$$Utility = economic\ surplus + psychological\ satisfaction.$$

The following is a simple example to illustrate Bell's model. We consider consumers engaged in a lottery, in which they are likely to gain a good payoff x or a bad payoff y , which is lower than x , and $1-p$ (or p) is the respective probability that the bad (or good) outcome and disappointment (or elation) will occur. The expected economic surplus is $\mu = px + (1-p)\,y$. The additional psychological part indicates the compounding effect of disappointment (or elation) on utility:

$$Psychological\ satisfaction = p * Elation + (1-p) * Disappointment,$$

where disappointment (or elation) is proportional to the difference between realized payoff and expected economic surplus if the outcome turns out to be bad (or good).

In particular, when the actual outcome is preferred by consumers, they will incur a sense of elation given by:

$$Elation = e(x - \mu) = e(1 - p)(x - y).$$

Conversely, when the actual outcome is non-preferred, a sense of disappointment arises from the bad outcome:

$$Disappointment = d(\mu - y) = dp(x - y),$$

where $e > 0$ (or $d > 0$) is the degree to which a unit of elation or disappointment affects the consumer's economic utility, respectively. Therefore, the total expected utility is

$$px + (1 - p)\, y - p\,(1 - p)\,(d - e)\,(x - y)\,.$$

We use $\kappa = d - e$ to represent the difference in the effects of elation and disappointment. We assume that psychological elation is always dominated by negative disappointment in the same amount of economic surplus, i.e., κ is always positive. We define κ as the disappointment-aversion level, which is a common concept widely shared among Gill and Prowse (2012); Kőszegi and Rabin (2007); Liu and Shum (2013).

3.2. Low-value Disappointment When Purchasing Online

When purchasing online, consumers with an ex-ante unknown value become elated or disappointed depending on whether they hold a value that is higher or lower than the product's price. When $L < p_o \leq H$, if consumers like the product after receiving the product bought online, then they will realize a high value H and accept the product with a payoff $H - p_o$. Otherwise, they confirm a low value L and return the product with refund r. It is not uncommon that the return policies and the percent of refund applied in e-businesses vary across industries and markets. Some e-tailers adopt a 100% money-back return policy, while others return only a specific part of the paid price. For example, Zappos.com and Shoebacca.com offer a one-year return policy with 100% refund and free shipping on any size order, while Best Buy, Amazon and eBay offer partial return and charge for restocking, handling and shipping.(Pei et al., 2014) Furthermore, even under a full-refund policy, consumers should still pay the shipping fee to send the product back, resulting in additional loss for return. Here we assume implicitly that the partial refund is executed by the retailer,i.e., $r < p$, to emphasize the effect of the value uncertainty and the associated disappointment without conflict with industry observation. (There is no doubt that all of our analyses and conclusions are generally applied to the case where the retailer offers a full-refund online,i.e., $r = p$.) Therefore, the expected economic surplus of the consumer is $\theta(H - p_o) + (1 - \theta)(r - p_o)$, denoted by E_o. Considering the effect of behavior motives, high-type consumers will feel elation e_l if they realize a valuation higher than the price paid and complete their order online. In this equation, e_l measures the marginal value of high-value elation in comparison with the expected economic surplus. Conversely, consumers will return a product for a refund r if their valuation turns out to be lower than the refund, i.e., $L < r < p_o \leq H$. Consumers will suffer disappointment $d_l(r - p_o - E_o)$,where d_l measures the marginal value of low-value disappointment compared with the expected economic surplus. Therefore, the total expected utility is as follows:

$$E_o + \theta e_l (H - p_o - E_o) + (1 - \theta) d_l (r - p_o - E_o)$$

Substituting E_o in the above-mentioned expression, we can derive the expected utility of purchasing online for consumers when $L < p_o \leq H$ as follows:

$$\theta(H - p_o) + (1 - \theta)(r - p_o) - \theta(1 - \theta)\kappa_l(H - r)$$

where $\kappa_l = d_l - e_l$ is the compounding effect of psychological disappointment and elation for value uncertainty. We define $\kappa_l > 0$ as the low-value disappointment aversion level and consumers share the same κ_l .

When $p_o \leq L$, because all consumers certainly hold a value no smaller than L , they will purchase online and obtain the product without any disappointment with the payoff: $\theta H + (1 - \theta) L - p_o$. Therefore, when consumers value is drawn from a two-point distribution, specifically, $v = (H, \theta; L, 1 - \theta)$,the expected utility of purchasing online can be derived as follows:

$$U_o = \begin{cases} \theta(H - p_o) + (1 - \theta)(r - p_o) - \theta(1 - \theta)\kappa_l(H - r) & if \quad L < r < p_o \leq H; \\ \theta H + (1 - \theta) L - p_o & if \quad p_o \leq L. \end{cases} \tag{1}$$

3.3. Stock-out Disappointment When Purchasing Offline

When purchasing offline, consumers with rational expectation about the store fill rate f will feel disappointed if they find the product to be unavailable in the store. If the product is in stock, then consumers will examine the product and purchase it as long as they realize that the value is higher than the price listed in the store. If consumers do not obtain access to the product, then they will not buy anything from the offline channel. Therefore, consumers receive the expected economic surplus $fE[v-p_s]^+$, denoted by E_s. Specifically,

$$E_s = \begin{cases} f\theta\left(H-p_s\right) & if \quad L < p_s \leq H \\ f\left(\theta H + (1-\theta)L - p_s\right) & if \quad p_s \leq L. \end{cases}$$

With regard to consumers' psychological emotion, consumers feel lucky when they encounter a suitable product. Their elation is expressed as $e_s\left(v-p_s-E_s\right)$, where e_s is the in-stock elation parameter representing the degree to which a unit of positive emotion affects consumers utility. Consumers who do not buy the product in the store with zero surplus reduces the payoff of E_s amounts. A disappointment of $d_s\left(0-E_s\right)$ arises from the stock-out. In the equation, d_s is the stock-out disappointment parameter representing the degree to which a unit of negative emotion affects consumers utility. Nonetheless, under the omnichannel scenario, consumers faced with a stock-out cannot resolve the value uncertainty in the store; however, they can still switch to the online channel and buy the product with the expected surplus U_o. Thus, the total expected utility is

$$E_s + fe_s\left(E[v-p_s]^+ - E_s\right) + (1-f)\,d_s\left(0-E_s\right) + (1-f)\,U_o$$

where $1-f$ is the probability of encountering stock-out. When the consumer value is drawn from a two-point distribution, i.e.,$v=(H,\theta;L,1-\theta)$, the expected utility of visiting the store can be derived by substituting E_s into the above expression as follows:

$$U_s = \begin{cases} f\theta\left(H-p_s\right)\left[1-(1-f)\,\kappa_s\right] + (1-f)\,U_o & if \quad L < p_s \leq H \\ f\left(\theta H + (1-\theta)L - p_s\right)\left[1-(1-f)\,\kappa_s\right] + (1-f)\,U_o & if \quad p_s \leq L \end{cases} \tag{2}$$

Similarly, we define $\kappa_s = d_s - e_s > 0$ as the stock-out disappointment aversion level, and κ_s is identical for every consumer.

Confronted with the omnichannel environment, consumers decide on whether to examine the product in the store and where to buy by comparing the expected utility from either channel: online directly or visiting the store and switching to online if $U_s \geq U_o$. Specifically, consumers are willing to go to the store to check the product if and only if $\Delta U = U_s - U_o$ is not negative. In addition, when consumers actually encounter stock-out, $U_o \geq 0$ is the necessary condition for the online switching behavior.

In our analysis, we apply the concept of rational-expectation equilibrium, which has been used in recent studies on retail operations concerning consumer behavior. In particular, consumers simultaneously make channel choices based on the correct anticipation about the store channel's fill rate and other consumers' purchase behavior, while the omnichannel retailer optimizes the pricing decision with a belief consistent with consumers' purchasing behavior.

4. The effect of Disappointment Aversion Behavior on Retailers Decisions and Profit

In this section, we investigate the effect of anticipated disappointment aversion emotion on the omnichannel retailers pricing and inventory decisions when consumers are ex-ante homogeneous. We assume for simplicity that the retailer salvages products that are returned at $s \geq 0$ because of the online refund policy. We ignore marginal production cost to magnify the effects of disappointment. Therefore, the omnichannel retailers marginal revenue from the online and store channels can be written as follows:

$$R(p_1, p_2) = \begin{cases} p_o - s - (r - s) F(r) & \text{online} \\ p_s - s & \text{offline} \end{cases} \tag{3}$$

where $F(r)$ is the probability of return in which consumer value turns out to be lower than the refund r. Under our settings in which consumer value is drawn from a two-point distribution, i.e., $v = (H, \theta; L, 1 - \theta)$, low-value consumers will return the product for a refund r with a probability of $1 - \theta$ if and only if $L < r < p_o \leq H$.

4.1. Uncapacitated Retailers

In the absence of the capacity constraint,consumers choosing the offline channel will never face the stock-out problem, i.e.,$B = f = 1$. Given the retailer's pricing policy (p_o, p_s), ex-ante homogeneous consumers will either visit the store to examine the product and buy it if they like it or purchase the product online directly and return it if they dislike it. The firm will sell in only one channel without considering the stock-out switching behavior. According to the above-mentioned analysis, if $\Delta U = U_s - U_o \geq 0$,then all the consumers choose the offline channel; otherwise, the online channel is chosen. Thus, the retailer optimizes the online and offline prices separately considering online channel low-value disappointment.

Lemma 1. *(optimal pricing strategy offline) In the store channel scenario, the optimal strategy is charging price at $p_s^* = H$ when $\theta^s < \theta \leq 1$ and price at $p_s^* = L$ otherwise, where $\theta^s = \frac{L-s}{H-s}$.*

Accordingly, the maximum profit obtained from the offline channel for the retailer without inventory constraint is

$$\pi_s^* = \begin{cases} (H - s)\,\theta & if\ \theta^s < \theta \leq 1 \\ L - s & otherwise \end{cases} \tag{4}$$

Lemma 2. *(optimal pricing strategy online) In the store channel scenario under the premise $0 < \kappa_l < \frac{1}{1-\theta}$, the optimal strategy is charging price at $p_o^* = r + (H - r)\,\theta\,[1 - (1 - \theta)\,\kappa_l]$, when $\theta^o < \theta < 1$ and price at $p_o^* = L$ otherwise, where*

$$\theta^o = \frac{-H + s}{2\,(H - r)\,\kappa_l} + \frac{1}{2}\left(1 + \sqrt{1 + \frac{(H - s)^2 - 2\,(H - r)\,(H - 2L + s)\,\kappa_l}{(H - r)^2 \kappa_l^2}}\right)$$

Accordingly,the retailer's maximum profit from the online channel is

$$\pi_o^*\,(p_o^*) = \begin{cases} \theta\,(H - s - (H - r)\,(1 - \theta)\,\kappa_l) & if\ \theta^o < \theta < 1 \\ L - s & otherwise \end{cases} \tag{5}$$

The following discussion is under the premise of $0 < \kappa_l < \frac{1}{1-\theta}$, which ensures that $p_o^* = r + (H - r)\,\theta\,[1 - (1 - \theta)\,\kappa_l]$ is larger than r. Otherwise, the refund will be illogical.

The result of lemma 2 shows that the optimal online price and profit are always decreasing in κ_l, thus indicating that the retailer will never benefit from low-value disappointment. In addition, the online threshold θ^o increases with the low-value disappointment aversion level κ_l , i.e., $\frac{\partial \theta^o}{\partial \kappa_l} > 0$. This result illustrates that when consumers' disappointment is more prominent, the retailer charges a low price online more frequently to reduce return. By comparing the pricing strategy between online and offline channels, we observe that $\theta^o > \theta^s$, which reveals that the anticipated low-value disappointment makes consumers more reluctant to purchase online, which is represented as the increase in the threshold for charging a higher price. Furthermore, considering the low-value disappointment aversion behavior, the profit earned from the online channel is always no larger than that from the offline channel. We show that the online channel is always dominated by the offline channel without inventory restriction.

Proposition 1. *(optimal pricing strategy for uncapacitated retailer) In the absence of an inventory constraint, a price-setting retailer sells only in the store channel at* $p_s^* = H$, *when* $\theta^s < \theta \leq 1$, *where* $\theta^s = \frac{L-s}{H-s}$. *Otherwise,the retailer sells through the online or offline channel at a clearance price of* L *without making any difference.*

It can be easily inferred that the profit earned from the online channel is identical to the offline store when ignoring consumer disappointment aversion. However, the results drawn in Proposition 1 are more in line with the reality. Actually, quite a few omnichannel retailers are used to selling products online for a discount, even retaining the online shops as outlets. In particular, the product of great value uncertainty that consumers need to "touch and feel" is obviously inappropriate to sell purely online. Too much low-value disappointment may cause a negative effect on consumer satisfaction and brand image, and the online returns can also get the retailer into trouble.

4.2. The Inventory Constraint Retailer

In this section, we extend our analysis to a more realistic case in which the store channel faces an inventory constraint with a booking limit B $(0 \leq B < 1)$. Limited supply and out-of-stock are common phenomena in brick-and-mortar stores because of supply chain uncertainty and limited item presentation. This rather common temporary unavailability of items rates high on shoppers' irritation lists and decreases the level of consumer satisfaction (Fitzsimons, 2000), thus becoming a source of stock-out disappointment. For an omnichannel seller, the online channel provides consumers who encounter a stock-out in the store channel with the alternative to obtain the product as a backup option. We focus on investigating the manner in which the exogenous limiting sales in the store channel interferes with disappointment aversion behavior in affecting consumers' stock-out switching behavior and the retailer's optimal dual-channel pricing policy. We assume that all consumers are present in front of the store and attempt to examine the product in stock, i.e., $\Delta U \geq 0$. Otherwise, the firm does not need to allocate the stock in store, i.e., $B = 0$. In addition, this case degenerates to the online only pricing policy, which is discussed in lemma 2 and turns out to be never optimal.For simplicity of expression, we denote the pricing policy to be that the online price is charged higher than the offline price as the mark-down policy and the opposite case as the mark-up policy throughout the paper.

Proposition 2. *(Optimal dual-channel pricing strategy for an omnichannel retailer with stock constraint) When there is an offline capacity limit B,and consumers share the same level of disappointment,a price-setting retailer's optimal price p_o^* and p_s^* are given as follows:*

(i) When $0 < B < \theta < 1$,i.e. the stock level is low,if $\Delta_1 > 0$, $p_s^* = H$ and $p_o^* = \varpi_o$,otherwise $\Delta_1 < 0$ and $0 < \kappa_s < \frac{1}{1-B}$, $p_s^* = L$ and $p_o^* = \varpi_o$,

(ii) When $0 < \theta < B < 1$,i.e. the stock level is high, if $\Delta_2 > 0$,$p_s^* = H$ (store only),otherwise $\Delta_2 < 0$ and $0 < \kappa_s < \frac{1}{1-B}$,$p_s^* = L$ and $p_o^* = \varpi_o$,

$$\textit{where} \qquad \varpi_o = r + (H - r)\,\theta\,(1 + (-1 + \theta)\,\kappa_l)$$

$$\Delta_1 = B\left(-L + s + H\theta - s\theta + (H - r)(-1 + \theta)^2 \kappa_l\right)$$

$$\Delta_2 = B\,(-L + s + H\theta - s\theta) + (1 - B)\,(H - r)\,(1 - \theta)\,\theta\kappa_l.$$

we note that only when consumers' stock-out disappointment-aversion is low enough will the omnichannel retailer be able to implement the mark-up policy. The mark-up policy generates more demand at a cheaper price in the store but causes a high rate of stock-out. Thus, consumers with a high stock-out disappointment would rather purchase online directly at a premium price to ensure availability.

Now we investigate the condition in proposition 2. We concentrate on the case of low level stock, where Δ_1 can be reduced as$\frac{L-s}{H-s} < \theta < 1$ or $1 - \frac{H-L}{r-s} < \theta < \frac{L-s}{H-s}$, and $\underline{\kappa_l} < \kappa_l < \frac{1}{1-\theta}$, with $\underline{\kappa_l} = \frac{L-s(1-\theta)-H\theta}{(H-r)(1-\theta)^2}$. A benchmark case is described below to explain the effect of consumers disappointment-aversion on the retailer's pricing decision.

Benchmark:

When ignoring consumer disappointment-aversion, we can derive the omnichannel retailers optimal decision and profit with an inventory constraint of B: If $\frac{L-s}{H-s} < \theta < 1$,the retailer charge ϖ_o^B online, H offline with profit of $\pi_-^B = (H-s)\,\theta$, otherwise the retailer charge ϖ_o^B online, L offline with profit of $\pi_+^B = B\,(L-s) + (1-B)\,(H-s)\,\theta$, where $\varpi_o = \theta H + (1-\theta)\,r$.

In the benchmark case without consumers' disappointment, the threshold that determine the retailer's mark-up or mark-down policy is $\frac{L-s}{H-s}$. It can be easily derived through a comparison such that when consumers incur disappointment-aversion, except for popular products $\left(\frac{L-s}{H-s} < \theta < 1\right)$, as the percentage of high type consumers satisfies: $1 - \frac{H-L}{r-s} < \theta < \frac{L-s}{H-s}$, and the low-value disappointment-aversion level κ_l is higher than $\underline{\kappa_l}$, the mark-down policy still dominates the mark-up policy. Compared to the benchmark, considering consumers' disappointment-aversion emotion will enlarge the scope of application of mark-down policy. In other words, when consumers' low-value disappointment-aversion level is high enough, the emotion will urge consumers to pay a premium price in-store. Thus, although the product is not as popular, the omnichannel can still charge a higher offline price to mitigate the impact of low-value disappointment-aversion online.

Similarly, for the case of high levels of stock, consumers low-value disappointment also increases opportunities for the retailer to only sell offline.

Corollary 1. *The more consumers are averse to the low-value disappointment, the more retailers will be likely to implement a mark-down policy. In other words, when consumers show strong low-value disappointment aversion, the omnichannel retailer charges a high price in the store channel to filter low-value consumers instead of applying a mark-up policy.*

It can be derived that when offline stock is confined to B, the threshold that determines the mark-up and mark-down policy is θ_l^T for the low stock level and θ_h^T for the high stock level, where

$$\theta_h^T = \frac{B(-H+s)}{2(-1+B)(H-r)\kappa_l} + \frac{1}{2}\sqrt{B^2(H-s)^2 + (-1+B)(H-r)\kappa_l(-2B(H-2L+s)+(-1+B)(H-r)\kappa_l)}$$

$$\theta_l^T = \frac{-H+s}{2(H-r)\kappa_l} + \frac{1}{2}\left(2 + \sqrt{\frac{(H-s)^2 - 4(H-L)(H-r)\kappa_l}{(H-r)^2\kappa_l^2}}\right)$$

$$\varpi_o = r + (H-r)\theta[1-(1-\theta)\kappa_l]$$

This finding can be deduced from $\frac{\partial\theta_l^T}{\partial\kappa_l} < 0$ and $\frac{\partial\theta_h^T}{\partial\kappa_l} < 0$. A mark-up policy is intended to encourage the unmet consumer in the store to switch to online. However, if consumers show strong disappointment aversion about the value uncertainty online, then they may abandon the substitution opportunity and consequently cancel the purchases or shift to competitors. However, the mark-down policy can mitigate the negative effect by making more consumers realize their value. Instead, the mark-up policy is suitable for the case in which consumers low-value disappointment is weak.

Corollary 2. *Optimal price and profit decrease with κ_l.*

The optimal profits earned from the mark-down and mark-up strategy, respectively, are

$$\pi_-^* = (H-s)\theta + (H-r)(-1+\theta)(-B+\theta)\kappa_l$$

$$\pi_+^* = B(L-s) + (1-B)\theta(H-s+(H-r)(-1+\theta)\kappa_l)$$

For $\frac{\partial\pi_-^*}{\partial\kappa_l} < 0$, $\frac{\partial\pi_+^*}{\partial\kappa_l} < 0$, and $\frac{\partial\varpi_o}{\partial\kappa_l} < 0$, we illustrate that the omnichannel retailer will never benefit from consumers' low-value disappointment.

Thus, the omnichannel retailer gains more profit from exploring the store channel to mitigate the negative effect of consumers' low-value disappointment caused by value uncertainty. Given the low-value uncertainty and the associated disappointment, the traditional pure online channel is never optimal. However,many "virtual fitting rooms," based on the "virtual reality" technology, have been emerging from the traditional internet, which provide online consumers with accurate fit information and size recommendations to screen out a subset of consumers who do not like the product in advance of any buying decision. In essence, the virtual showroom enhances the percentage of high-type consumers online and reduces online uncertainty. Once the product value uncertainty and associated low-value disappointment are reduced online, the online channel may dominate the offline as long as the virtual showroom is sufficiently informative and cost-effective. There is already evidence that "virtual fitting rooms" do help to reduce online returns and improve consumer satisfaction (Bell et al., 2014).

5. The Mechanism to Mitigate Disappointment: The Physical Showroom

In this section, we propose the physical showroom mechanism which is designed to achieve channel integration by mitigating the overlapping disappointment aversion caused by low-value uncertainty after facing stock-out in the store. Thereafter, we investigate the effect of the introduction of a physical showroom on consumer behavior and retailer strategy. We focus on the retailer's pricing decision when the stock inventory is exogenously confined to B. Consumers' expected utility of choosing to purchase online remains unchanged. However, for the store channel choice, consumers can still realize a product's value even during stock-out, and they can buy the product directly online when they find the product's value to be higher than the price online without suffering return. Therefore, the expected utility of visiting the product in the store with a physical showroom for consumers can be derived as follows:

$$U_s^p = \begin{cases} f\theta\left(H - p_s\right)\left[1 - \left(1 - f\right)\kappa_s\right] + \left(1 - f\right)E[v - p_o]^+ & if \quad L < p_s \leq H \\ f\left(\theta H + \left(1 - \theta\right)L - p_s\right)\left[1 - \left(1 - f\right)\kappa_s\right] + \left(1 - f\right)E[v - p_o]^+ & if \quad p_s \leq L. \end{cases} \tag{6}$$

Further, additional cost would be incurred to offer the physical showroom mechanism, which results from the reserved space for the samples, the specialized service required for managing the physical showroom and the associated recruiting cost, and the leftover handling cost of the samples at the end of selling season. We assume that the omnichannel retailer will pay the marginal cost of c to introduce each consumer who encounters the stock-out in the store to enjoy the physical showroom service to verify the product's value.To ensure the positive profit margin, we assume $0 < c < \theta (p_o - s)$ as a precondition in this model.

Proposition 3. *(optimal dual-channel pricing strategy for an omnichannel retailer with a physical showroom) When there is an offline capacity limit B,and consumers share the same level of disappointment,a price-setting retailer's optimal price p_o^* and p_s^* are given as follows:*

(i) When $0 < B < \theta < 1$,i.e. the stock level is low,if $\Delta_1^P > 0$,$p_s^* = p_o^* = H$, otherwise $\Delta_1^P < 0$ and $0 < \kappa_s < G(\kappa_l)$, $p_s^* = L$ and $p_o^* = H$;

(ii) When $0 < \theta < B < 1$,i.e. the stock level is high,if $\Delta_2^P > 0$,$p_s^* = H$ (store only),otherwise $\Delta_2^P < 0$ and $0 < \kappa_s < G(\kappa_l)$, $p_s^* = L$ and $p_o^* = H$;

where

$$G(\kappa_l) = \frac{r + BL\theta - r\theta + H(-1 + \theta - B\theta) + (H - r)(-1 + \theta)\theta\kappa_l}{(-1 + B)B(H - L)\theta}$$

$$\Delta_1^P = -L + s + c\left(-1 + \frac{1}{\theta}\right) + H\theta - s\theta$$

$$\Delta_2^P = c - Bc + B(-L + s + H\theta - s\theta)$$

Corollary 3. *With a physical showroom, the restriction of consumers' stock-out disappointment-aversion level imposed on the implementation of mark-up policy is relaxed, which acts to expand the whole market. Furthermore, the low-value disappointment aversion plays a positive role in the fight to capture a greater market share.*

In particular, as presented in the basic model, it is only when consumers with a low enough stock-out disappointment aversion emotion (i.e.,$0 < \kappa_s \leq \frac{1}{1-B}$) that ensures the expected utility in store to be positive, the store channel with the mark-up policy should be implemented. However, the physical showroom prompts more consumers to the market as long as their disappointment aversion is not greater than $G(\kappa_l)$,which expands the market share by $\frac{(H-r)(1-\theta)(1+\theta\kappa_l)}{(1-B)B(H-L)\theta}$. With a physical showroom, it may be rational for consumers to visit the store even if they correctly expect little chance of obtaining the product offline.

In contrast to the negative impact of the low-value disappointment aversion on retailer's profit, the emotion plays a positive role in the fight to capture a greater market share when implementing mark-up policy. This is because the market expansion magnitude increases in κ_l. Interestingly, in this case, consumers low-value disappointment aversion behavior has a positive effect on mitigating the negative restriction triggered by the stock-out disappointment, which is counterintuitive.

Hence, when the omnichannel has a limited supply in the store channel and consumers share the same level of disappointment aversion, the positive role of physical showroom in market expansion can be seen. Then, we turn to study the effect of physical showroom on profit improvement in such a homogeneous market. We compare the retailer's optimal profits with or without physical showroom case by case, given by π^* and π^{P*}, respectively, which are derived as follows:

(i) In the scenario of $0 < B < \theta < 1$, i.e. the stock level is low:

(a) When $\frac{L-s}{H-s} \leq \theta < 1$ or $\theta < \frac{L-s}{H-s}$, $c > c^L$,and $\kappa_l > \kappa_l^L$, If $0 < c < \bar{c}$,$\pi_l^{P*} > \pi_l^*$;

(b) When $\theta < \frac{L-s}{H-s}$, $0 < c < c^L$,and $\kappa_l > \kappa_l^L$,$\pi_l^{P*} > \pi_l^*$;

(c) When $\theta < \frac{L-s}{H-s}$, $c^L < c < \theta(H-s)$,and $0 < \kappa_l < \kappa_l^L$,$\pi_l^{P*} < \pi_l^*$;

(d) When $\theta < \frac{L-s}{H-s}$, $0 < c < c^L$,and $0 < \kappa_l < \kappa_l^L$, If $0 < c < \bar{c}$,$\pi_l^{P*} > \pi_l^*$.

Where $c^L = \frac{\theta(\theta H - L - \theta s + s)}{\theta - 1}$, $\kappa_l^L = \frac{-\theta H + L + (\theta-1)s}{(\theta-1)^2(H-r)}$, and $\bar{c} = (1-\theta)\theta(H-r)\kappa_l$.

(ii) In the scenario of $0 < \theta < B < 1$, i.e. the stock level is high:

(a) When $\frac{L-s}{H-s} \leq \theta < 1$ or $\theta < \frac{L-s}{H-s}$, $c > c^H$, and $\kappa_l > \kappa_l^H$, $\pi_h^{P*} = \pi_h^*$;

(b) When $\theta < \frac{L-s}{H-s}$ and $0 < c < c^H$,$\kappa_l > \kappa_l^H$,$\pi_l^{P*} > \pi_l^*$;

(c) When $\theta < \frac{L-s}{H-s}$ and $c^H < c < \theta(H-s)$,$0 < \kappa_l < \kappa_l^H$,$\pi_l^{P*} < \pi_l^*$;

(d) When $\theta < \frac{L-s}{H-s}$ and $0 < c < c^H$,$0 < \kappa_l < \kappa_l^H$, If $0 < c < \bar{c}$,$\pi_l^{P*} > \pi_l^*$.

Where $c^H = \frac{B(\theta H - L - \theta s + s)}{B-1}$, $\kappa_l^H = \frac{B(-\theta H + L + (\theta-1)s)}{(B-1)(\theta-1)\theta(H-r)}$, and $\bar{c} = (1-\theta)\theta(H-r)\kappa_l$.

Corollary 4. *When the omnichannel has a limited supply in the store channel and consumers share the same level of disappointment aversion, whether the omnichannel retailer may gain profit promotion with the physical showroom mechanism depends on the market environment, which corresponds to the cases such that* $\pi^{P*} \geq \pi^*$ *are satisfied in the above analyses.*

It can be recalled that when consumers share the same level of low-value disappointment κ_l, the disappointment emotion decreases the retailers' revenue by $(H-r)(1-\theta)\theta\kappa_l$, whereas the physical showroom may compromise the negative effect by offering try-on service to help consumers learn the products' value and eliminate the low-value uncertainty. However, it can be derived that the physical showroom is not always profitable. Specifically, when the product is popular $\left(\frac{L-s}{H-s} \leq \theta < 1\right)$, and the cost of implementing a physical showroom is sufficiently low, specifically not large than $(H-r)(1-\theta)\theta\kappa_l$, the physical showroom will work to offset the loss of profit caused by the disappointment and increase the retailers revenue. While once the product is only highly valued by a small population $\left(\theta < \frac{L-s}{H-s}\right)$, the application of physical showroom should be confined to a market where the consumers' low-value disappointment-aversion is large and the cost for the mechanism is low, as the physical showroom presents the consumers' second chance to realize their value and eliminate the low-value disappointment incurred in switching online. The retailer tends to be better off investing in the physical showroom mechanism if the low-value disappointment is shown to be more intensively.

6. Disappointment Aversion Heterogeneity

In this section, we incorporate consumers' heterogeneity in their disappointment-aversion level. It is easily inferred that some consumers show strong aversion to the shortage of their favorite product in-store, while others do not care as much about this occurrence and naturally switch to the online channel. However, some consumers are frustrated to find the product purchased online to defy expectation; however, the rest of consumers will not bother to obtain a refund as patient experimenters. Thus, we assume that the low-value and stock-out disappointment aversion level κ_l and κ_s differ among the consumers in an omnichannel. Specifically, consumers are uniformly distributed across the following rectangular $R = \{(X, Y) | \kappa_s \in [0, X], \kappa_l \in [0, Y]\}$.We consider only the scenario in which the omnichannel retailer charges a price exceeding L in the online and offline channel, i.e.,$L < r \leq p_o \leq H$ and $L \leq p_s \leq H$. Given consumers' heterogeneity in disappointment aversion, the consumers whose consumption value is drawn from a two-point distribution have three options as follows: go to the store, buy online and leave the market directly. Note that customers who find the store to be out-of-stock will buy online only if doing so is preferred over leaving the market. The corresponding utilities are as follows: $U_o = \theta(H - p_o) + (1-\theta)(r - p_o) - \theta(1-\theta)\kappa_l(H - r)$,$U_s = f\theta(H - p_s)(1 - (1 - f)\kappa_s) + (1 - f)U_o^+$,and $U_l = 0$.

Specifically, there are "pure online" cconsumers (who buy online directly), "store $\rightarrow$ online" consumers (who visit the store but switch online when the store is out of stock), "pure store" consumers (who visit the store exclusively), as well as consumers who simply leave the market. The fractions of these four types of customer are denoted as λ_s, λ_{so}, λ_o and λ_l, respectively, which are defined as follows:

(a) Store only λ_s: $\Delta U = U_s - U_o > 0$,$U_o < 0$,and $U_s > 0$;

(b) Store to online if stockout λ_{so}: $\Delta U = U_s - U_o > 0$,$U_o > 0$ and $U_s > 0$;

(c) Store only λ_o: $\Delta U = U_s - U_o < 0$,$U_o < 0$;

(d) Simply leave the market λ_l: $U_o < 0$ and $U_s < 0$.

On the supply side, the retailer makes the pricing decisions in the dual channel to achieve the profit maximization when the offline inventory capacity is confined to B. We concentrate the case of $0 < f < 1$,i.e.,$f = \frac{B}{\theta(\lambda_s + \lambda_{so})}$, and the retailer faces the following profit function:

$$\begin{aligned}\Pi(p_o, p_s) = {} & (p_o - s - (r - s)(1 - \theta))\lambda_o + B(p_s - s) + \\ & (p_o - s - (r - s)(1 - \theta))\tfrac{\lambda_{so}}{\lambda_s + \lambda_{so}}\left(\lambda_s + \lambda_{so} - \tfrac{B}{\theta}\right)\end{aligned} \quad (7)$$

For simplicity of expression, we continue to use the concept of mark-up and mark-down policy to describe price differentiation between online and offline. We further compute the fraction of consumer' types by comparing the prices that are charged in dual channels and discuss the mark-up and mark-down policies separately. Then, the specific consumers' segments are substituted into the profit function and the optimal pricing decisions are resolved through algebra. The goal of this section is to study the impact of disappointment-aversion on the omnichannel retailer with offline inventory constraint in such a heterogeneous market.

Proposition 4. *(optimal pricing policy for omnichannel retailer in heterogeneous market) When there is an offline capacity limit and the fill rate f in store is always less than 1, considering the consumers heterogeneity in their disappointment aversion level, the retailer may have four options of the optimal online and offline prices p_o^* and p_s^* given as follows:*

(i)

$$p_s^* = H, p_o^* = \frac{1}{2}\left(2r + H\theta - 2r\theta + s\theta\right) \tag{8}$$

(ii)

$$\begin{cases} p_s^* = L \\ p_o^* = \frac{1}{3f}\left(\begin{array}{l} 3fr + (-L + 3fL - 3fr + s + 2(-1+f)(-H+L)X)\theta - \\ \hline \theta\sqrt{\left(\begin{array}{l} f^2(H-L)\left(6s(1+X) - L(3+2X)^2 + H(3+6X+4X^2)\right) \\ +(L - s - 2HX + 2LX)^2 - 2fZ \end{array} \right)} \end{array} \right) \end{cases} \tag{9}$$

where $Z = H^2X(3+4X) - Ls(3+5X) + L^2(3+4X(2+X)) + 3rsY(1-\theta)$
$-HL(3 + X(11+8X) - 3Y(1-\theta)) - 3LrY(1-\theta) + Hs(3+5X-3Y(1-\theta))$

(iii) when $L < p_s < H$ and $\max\{r, \theta p_s + (1-\theta)r\} < p_o < \theta H + (1-\theta)r$,

If $0 < X < \frac{3f}{2-2f}$ and $Y > \frac{2(2(f-1)X+3f)^2(-\theta H+(\theta-1)r+p_o^*)}{f^2(\theta-1)\theta(H-r)}$ or $X \geq \frac{3f}{2-2f}$ where p_o^* and p_s^* are the optimal prices charged by retailer, which is determined by the *simultaneous equations:*

$$\begin{cases} p_o^* = \frac{1}{3f}\left[\begin{array}{c} 3fr + \theta\left(-3fr - 2\left(-1+f\right)HX + \left(3f + 2\left(-1+f\right)X\right)p_s^*\right) - \\ \sqrt{\theta^2\left(H - p_s^*\right)\left(\begin{array}{c} 3f^2\left(H+2s\right) + 6\left(-1+f\right)f\left(H+s\right)X \\ +4(-1+f)^2HX^2 - \left(3f+2\left(-1+f\right)X\right)^2 p_s^* \end{array}\right)} \end{array}\right] \\ p_s^* = H - \frac{(r - p_o^* + H\theta - r\theta)^3}{\sqrt{2}\sqrt{(H-r)Y(1-\theta)\theta^3(r-p_o^*+H\theta-r\theta)^3}} \end{cases} \tag{10}$$

(iv) When $p_o < p_s < H$ *and* $r < p_o < \theta p_s + (1-\theta)\,r$*, If* $X > \frac{9f}{8(1-f)}$*,*

$$\begin{cases} p_o^* = \frac{8rX(-1+\theta) - 4(H+s)X\theta + f(4s(1+X)\theta + H(5+4X+6Y(-1+\theta))\theta - r(-1+\theta)(9+8X+6Y\theta))}{9f+8(-1+f)X} \\ p_s^* = \frac{-2X(H+3s-4HY(-1+\theta)+4rY(-1+\theta)) + f(6s(1+X)+8rXY(-1+\theta)+H(3+X(2+8Y-8Y\theta)))}{9f+8(-1+f)X} \end{cases} \tag{11}$$

In general, considering consumers' heterogeneity in the low-value and stock-out disappointment aversion level, the optimal pricing decisions are chosen from the above options through comparing the optimal profits obtained by Eq. $(8 \sim 11)$, respectively. The threshold conditions are implicitly characterized by the combination of the market circumstances, the specific store fill rate, and the consumers' average disappointment level.

We note that the retailer is likely to sell only through the online channel (case i) in such a heterogeneous market, which is different from the previous homogeneous case in which selling purely online is never optimal. Although the omnichannel retailer declares the offline price of H, considering low fill-rate in store, consumers would rather flood the online shopping channel than visit the store. In this case, the markdown policy degenerates into the online-only policy, with the optimal profit unaffected by the population's average level of stock-out disappointment X. Thus as long as the maxmimum range of the population's low-value disappontment Y is mild, the policy of selling through online purely and regarding offline channel as a information signal, is inclined to dominate. However, as X and Y satisfy the specific conditions, the offline price in such a heterogeneous market is no longer simply either H or L in case (iii&iv). Instead, a delicate value between H and L may be charged offline as well as the online channel to make the retailer achieve profit optimality. On one hand, as X becomes sufficiently large, the offline price should be cut down to mitigate the negative effect of reduced offline demand caused by stock-out disappointment. On the other hand, as Y becomes relatively high, while X remains at a low level, a price less than H should still be charged offline to encourage consumers stock-out switching to alleviate the impact of unmet consumers' leaving due to low-value disappointment. Actually, the low-value and stock-out disappointment-aversion, which are uniformly distributed in online and offline channel respectively, will have a counteraction effect on consumers' strategic channel choice. Thus the retailer can take advantage of market segmentation with effective a differential pricing decision and extract more consumers' surplus by encouraging channel switching.

Corollary 5. *In above case (iii&iv) of such a heterogeneous market, the optimal prices charged online p_o^* and offline p_s^* all decrease with the maximum range of the population's stock-out disappointment X, while they all increase with the maximum range of population's low-value disappointment Y. Additionally, the omnichannel retailer's profit is decreases with X as well as Y.*

We substitute the optimal prices into the market segments and omnichannel retailer's profit function to investigate the effect of the maximum range of the population's disappointment X and Y. Interestingly, we find that the more separately consumers' low-value disappointment aversion level is distributed, the higher the price should be charged across the channels, however, the case is opposite for stock-out disappointment. First, it can be easily derived that once the maximum value of population's stock-out disappointment tends to be higher, the number of consumers purchasing offline will be decrease,i.e.$\frac{\partial \lambda_s}{\partial X} < 0$. The retailer reduces the offline price to attract more consumers to the store to learn the product's value. At the same time, the online price should also be brought down to encourage consumers who encounter the stock-out to switch to an online channel, instead of leaving the market directly. We then find that when the scope of the population's low-value disappointment is enlarged, the market segment in "store only" will be extended, while the consumer segment in "store → online" will be curtailed, however, their sum will continue to expand, i.e., $\frac{\partial \lambda_s}{\partial Y} > 0$, $\frac{\partial \lambda_{so}}{\partial Y} < 0$, and $\frac{\partial(\lambda_s+\lambda_{so})}{\partial Y} > 0$. An increasing number of consumers will be willing to visit the store to realize their value, presenting the retailer an opportunity to raise offline price, whereas the decreasing number of "stock-out switching" motivates the retailer to keep a higher online revenue margin.

Corollary 6. *In such a heterogeneous market,the physical showroom may abandon all the online direct marketing and attract all the consumers to visit the store by charging H both online and offline to earn $\Pi_P^0 = -c + cf - s\theta + (\theta - f\theta)\,H + f\theta H$ regardless of population's disappointment aversion level.*

With the physical showroom, the option for offline will be more appealing to consumers because they can still inspect the product and switch to online with informed value even encountering the stock-out. The retailer could take the advantage to increase the offline price to the maximum revenue margin H, and optimize the online price to segment the consumers elaborately. Further, the price online could be raised to H and all the consumers are convinced to offline with higher utility regardless of their possible disappointment aversion. Thus, the retailer would at least earn Π_P^0 with physical showroom, which provides a steady lower bound of the profit.

Observation. *Given consumers' heterogeneity in disappointment,*

(i) A higher X, as well as Y , leads to lower profit for omnichannel retailer.

(ii) A low-budget physical showroom will always improve retailer's profit.

(iii) As the maximum value of the population's low-value disappointment level Y enlarges, introducing the physical showroom will become more profitable for omnichannel retailer.

As we can see from Fig.B.1,as the maximum range of population's low-value and stock-out disappointment aversion expand, the profit earned by omnichannel retailer without physical showroom monotonically decreases. It means that the retailer is inclined to gain more profit when the disappointment-aversion level is distributed in a more concentrated manner, which is consistent with the homogeneous case.

The observation (ii&iii) are illustrated in Fig.B.2 (in Appendix B), which shows the impact of varying Y on the profit of omnichannel retailer with or without physical showroom respectively. As we can see, given the fixed fill rate, the curves of omnichannel retailer's profit in both cases decrease with maximum value of disappointment-aversion. Specifically, the profit of retailer without physical showroom declines significantly, while for the retailer with physical showroom, the detriment gradually levels off once the low-value disappointment value turns slightly larger (more than 1 in the example shown).This diminishing effect is absent under physical showroom since the benefit of inspecting all the product line is sufficient to essentially perform the strategy analyzed in corollary 6, that the retailer induces all the consumers visiting stores to abandon online-direct orders and introduces those facing stock-out to switch to online by charging the price of H in online and offline. Thus as Y becomes larger, the gap between the optimal profits in two cases seems to be gradually widening, indicating that the option to implement physical showroom would be more valuable with higher average level of population's low-value disappointment.

7. Conclusions and discussion

In this paper, we develop a model in which ex-ante homogeneous consumers, who are uncertain about their value and store availability, anticipate low-value and stock-out disappointment when making strategic channel choices between online and offline channels under the omnichannel environment. We characterize consumers' disappointment aversion behavior and discuss the optimal pricing policy of the omnichannel retailer with or without offline inventory constraint,then extend the analysis to the heterogeneous market where consumers' disappointment-aversion levels are uniformly distributed. We also study the optimal decision of the retailer under the disappointment-mitigating mechanism of a physical showroom, which is increasingly widespread in omnichannel management. Our main findings are summarized as follows:

(1) Uncapacitated retailers should only sell products through the offline channel in consideration of the negative effect of consumers low-value disappointment-

aversion behavior triggered by value uncertainty in the online channel. Our model also explains the full-refund policy generally adopted by pure e-commerce as well as multi-channel retailers to reduce both the economic and reputation lost obtained by the disappointment, even though it seems to cause a negative margin.

(2) When the omnichannel retailer faces offline inventory constraint, the decision to implement the mark-up or mark-down policy depends on the proportion of high-type consumers, which is further determined by consumers' low-value disappointment-aversion level. Moreover, when consumers reveal disappointment aversion to value uncertainty more strongly, a higher price is more likely to be charged offline. The retailer's profit never benefits from consumers low-value disappointment, and only when consumer's stock-out disappointment is sufficiently low can the retailer use the mark-up policy that attracts consumers to the store as a clearance price and encourages the unmet consumer to switch to the online channel. Considering consumers' heterogeneity in disappointment-aversion level, we find that the retailer should adjust the strategic mark-down or mark-up pricing policy according to the distribution of the population's average disappointment-aversion level. And once the population's low-value disappointment-aversion level is sufficiently low, selling purely online may be the option.

(3) The introduction of a physical showroom into the omnichannel may expand the market share by ensuring consumers to realize their value even encountering stock-out. This leads consumers who expect a slight chance of finding the product to be available offline and originally drop the purchase to conversely visit the store. It is counterintuitive that consumers low-value disappointment benefits the expansion of the market with a physical showroom by mitigating the negative effect posed by the stock-out disappointment, which seems to be another surprise brought by the physical showroom. When consumers have the same level of disappointment, the retailer can improve profit with a cost-effective physical showroom as long as the consumers' low-value disappointment-aversion

is sufficiently large, and the similar conclusion is drawn in such a heterogeneous market.

Our study focuses on the optimal pricing decision of omnichannel retailers with regard to consumers' disappointment aversion behavior and there are some limitations for future research. First, we assume that the online channel is operated exogenously and that the store channel is confined to an exogenous booking limit. Future research could extend to the endogenous capacity scenario and consider the pricing and capacity rationing joint decision. The two-point distribution of consumers' valuation could also be worth extending in a more general way. In addition, our study concentrates on the consumer stock-out switching and disappointment behavior in an omnichannel environment. Nevertheless, another important phenomenon, the "browse-and-switch behavior," has also recently gained attention. This behavior, also called "showrooming" means that after browsing the item at the store, consumers have the option to switch to an e-retailer to purchase the item at a cheaper price (Balakrishnan et al., 2014). In our study, when the retailer implements a physical showroom mechanism, the optimal pricing strategies (uniform or replacement) turn out to be immune from consumers' showrooming behavior. Future research could consider combining consumers' disappointment aversion emotion with the "showrooming" impulse to investigate omnichannel retailers' pricing or rationing decisions.

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Appendix A. Proofs

Appendix A.1. Proof of Lemma 1

For simplicity, we ignore marginal costs, which actually magnify the effects of disappointment emotion.

Then the profit from the store channel is

$$\pi_s(p_s) = \begin{cases} (p_s - s)\,\theta & L < p_s \le H \\ p_s - s & p_s \le L \end{cases} \tag{A.1}$$

When $L < p_s \le H, \frac{\partial \pi_s(p_s)}{p_s} > 0$,then $p_s^* = H, \pi_s^*(p_s = H) = (H - s)\,\theta$.

Likewise, $p_s^* = L$ when $p_s \le L$, and $\pi_s^*(p_s = L) = L - s$.

Consider the both cases, $p_s^* = H$,if and only if $\pi_s^*(p_s = H) \ge \pi_s^*(p_s = L)$,i.e. $\frac{L-s}{H-s} \le \theta < 1$.

Appendix A.2. Proof of Lemma 2

The consumer purchase the product from the online channel if $U_s \ge 0$. When $L < p_o \le H$,

$$U_o = \theta(H - p_o) + (1-\theta)(r - p_o) - \theta(1-\theta)\kappa_l(H - r) \ge 0.$$

Through rearranging, we derive that

$$p_o \le r + H\theta - r\theta + (H - r)(-1 + \theta)\,\theta\kappa_l$$

When $p_o \le L$, $U_o = \theta H + (1-\theta)L - p_o \ge 0$. Through rearranging, we derive that $p_o \le \theta H + (1-\theta)L$ Hence, we derive the maximum price inducing the online purchase:

$$p_o^{\max} = \begin{cases} r + H\theta - r\theta + (H - r)(-1 + \theta)\,\theta\kappa_l & L < p_o \le H \\ L & p_s \le L \end{cases} \tag{A.2}$$

Then the profit from the online channel is

$$\pi_o(p_o) = \begin{cases} r + H\theta - r\theta + (H - r)(-1 + \theta)\,\theta\kappa_l & L < p_o \le H \\ p_o - s & p_s \le L \end{cases} \tag{A.3}$$

According to $\frac{\partial \pi_o}{\partial p_o} > 0$, The optimal price of online channel is:

$$p_o^* = p_o^{\max} = \begin{cases} r + H\theta - r\theta + (H-r)(-1+\theta)\theta\kappa_l & L < p_o \le H \\ L & p_s \le L \end{cases} \tag{A.4}$$

Then the optimal profit online is:

$$\pi_o^*(p_o) = \begin{cases} \theta(H - s + (H-r)(-1+\theta)\kappa_l) & L < p_o^* \le H \\ L - s & p_o^* \le L \end{cases} \tag{A.5}$$

In this case, $\pi_o^*(p_o^* = \varpi_o) \ge \pi_o^*(p_o^* = L)$,if and only if

$$\theta(H - s + (H-r)(-1+\theta)\kappa_l) \ge L - s$$

i.e.$\theta^o < \theta < 1$, where

$$\varpi_o = r + H\theta - r\theta + (H-r)(-1+\theta)\theta\kappa_l$$

$$\theta^o = \frac{-H+s}{2(H-r)\kappa_l} + \frac{1}{2}\left(1 + \sqrt{1 + \frac{(H-s)^2 - 2(H-r)(H-2L+s)\kappa_l}{(H-r)^2\kappa_l^2}}\right)$$

Appendix A.3. Proof of proposition 1

According to the Lemma 1 and Lemma 2, the optimal profit earned form the offline and online respectively is:

$$\pi_s^*(p_s) = \begin{cases} \theta(H-s) & \theta_s \le \theta < 1 \\ L - s & otherwise \end{cases} \tag{A.6}$$

$$\pi_o^*(p_o) = \begin{cases} \theta(H - s + (H-r)(-1+\theta)\kappa_l) & \theta_o \le \theta < 1 \\ L - s & otherwise \end{cases} \tag{A.7}$$

Then the optimal profit is $\pi^{\max} = \max\{\pi_s^*(p_s^*), \pi_o^*(p_o^*)\}$. Comparing the both cases: Because $\theta_s < \theta_o$,and $\theta(H - s + (H-r)(-1+\theta)\kappa_l) < (H-s)\theta$, If $\theta_s \le \theta < 1$, $\pi^{\max} = \pi_s^*(p_s^* = H) = \theta(H-s)$; otherwise, $\pi^{\max} = \pi_s^*(p_s^* = L) = \pi_o^*(p_o^* = L) = L - s$.

Appendix A.4. Proof of proposition 2

The optimal price in the store channel p_s^* is either H or L.

Considering the booking limit B,

Case 1: For $p_s^* = H$, the demand in store channel is θ .

$U_s = (1-f)U_o$, $\Delta U = U_s - U_o = -fU_o \ge 0$,

(1) When $0 < B < \theta < 1$the fill rate offline is $f = \frac{B}{\theta}$. If $L < p_o \leq H$, $\Delta U \geq 0$ and $U_o \geq 0$ simplifies to

$$f\left(-r - H\theta + r\theta + p_o + (-H + r)(-1 + \theta)\theta\kappa_l\right) \geq 0$$

Solving these inequalities for p_o gives

$$\begin{aligned}
&p_o^* = p_o^{\max} = r + H\theta - r\theta + (H - r)(-1 + \theta)\theta\kappa_l. \\
&\pi_r^* = (p_s - s)B + \left(1 - \tfrac{B}{\theta}\right)(p_o - s - (r - s)(1 - \theta)) \\
&= (H - s)\theta + (H - r)(-1 + \theta)(-B + \theta)\kappa_l
\end{aligned} \quad \text{(A.8)}$$

If $p_o \leq L$, from $\Delta U \geq 0$, we derive $p_o = L + (H - L)\theta > L$, which is contradict with$p_o \leq L$, exceeding the initial domain beyond the discussion.

(2) When $0 < \theta < B < 1$the fill rate offline is $f = 1$. In this case, the retailer will sell only offline with the profit $\pi_s^* = (H - s)\theta$.

Case 2: For $p_s^* = L$, the demand in store channel is 1, the fill rate offline is $f = B$.

$$U_s = f(H - L)\theta(1 + (-1 + f)\kappa_s) + (1 - f)U_o$$

$$\Delta U = U_s - U_o = f(H - L)\theta(1 + (-1 + f)\kappa_s) - fU_o \geq 0$$

If $L < p_o \leq H$, $\Delta U \geq 0$ simplifies to

$$B\left(-r - H\theta + r\theta + p_o + (-H + r)(-1 + \theta)\theta\kappa_l\right) \geq 0.$$

Solving these inequalities for p_o gives

$$p_o \geq r + L\theta - r\theta + (H - r)(-1 + \theta)\theta\kappa_l + (-1 + B)(-H + L)\theta\kappa_s.$$

While $U_o \geq 0$ needs $p_o \leq r + H\theta - r\theta + (H - r)(-1 + \theta)\theta\kappa_l$. Through Solving these inequalities for p_o gives: If and only if $0 < \kappa_s < \frac{1}{1-B}$,

$$\begin{aligned}
&p_o^* = r + H\theta - r\theta + (H - r)(-1 + \theta)\theta\kappa_l. \\
&\pi_f^* = (p_s - s)B + (1 - B)(p_o - s - (r - s)(1 - \theta)) \\
&= B(L - s) + (1 - B)\theta(H - s + (H - r)(-1 + \theta)\kappa_l)
\end{aligned} \quad \text{(A.9)}$$

If $p_o \leq L$, from $\Delta U \geq 0$, we derive $p_o \geq L - (-1 + B)(H - L)\theta\kappa_s > L$, which is contradict with$p_o \leq L$, exceeding the initial domain beyond the discussion. Finally, through comparing the profit denoted by Eq.(A.8)and Eq.(A.9), we derive the threshold above which the retailer implement different channel pricing strategies.

Appendix A.5. Proof of proposition 4

The optimal price in the store channel p_s^* is either H or L.

Considering the booking limit B with a physical showroom,

Case 1:mark-down policy

For $p_s^* = H$, the demand in store channel is θ ,$U_s = (1 - f)[v - p_o]^+$,and $\Delta U = (1 - f)[v - p_o]^+ - U_o \geq 0$.

(1) When $0 < B < \theta < 1$, the fill rate offline is $f = \frac{B}{\theta}$. If $L < p_o \leq H$, $\Delta U \geq 0$ and $U_o \geq 0$ simplifies to

$$-BH + r(-1 + \theta) + (1 + B - \theta)p_o - (H - r)(-1 + \theta)\theta\kappa_l \geq 0$$

Solving these inequalities for p_o gives

$$p_o \geq \frac{BH + r - r\theta + (H - r)(-1 + \theta)\theta\kappa_l}{1 + B - \theta}.$$

We notice that with physical showroom, the expected utility from online channel after switching $[v - p_o]^+$ is always positive. Consumers who encounter stock-out would switch to online channel without hesitation. Because

$$\frac{BH + r - r\theta + (H - r)(-1 + \theta)\theta\kappa_l}{1 + B - \theta} < H,$$

The optimal price online is $p_o^* = p_o^{\max} = H$. In this case, the online and offline share the same high price.Considering the cost incurred in practicing physical showroom, the retailer's optimal profit is derived as follow:

$$\pi_-^P = \left(1 - \frac{B}{\theta}\right)(\theta(H - s) - c) + B(H - s) = c\left(\frac{B}{\theta} - 1\right) + \theta(H - s)$$

If $p_o \leq L$, from $\Delta U \geq 0$, we derive $p_o = L + (H - L)\theta > L$, which is contradict with$p_o \leq L$, exceeding the initial domain beyond the discussion.

(2) When $0 < \theta < B < 1$, the fill rate offline is $f = 1$. In this case, the retailer will sell only offline with the profit $\pi_-^P = (H - s)\theta$.

Case 2:mark-up policy

For $p_s^* = L$, the demand in store channel is 1, the fill rate offline is $f = B$. The offline utility $U_s = f(H-L)\theta(1+(-1+f)\kappa_s)+(1-f)[v-p_o]^+$,and $\Delta U = U_s - U_o = f(H-L)\theta(1+(-1+f)\kappa_s)+(1-f)[v-p_o]^+ - U_o \geq 0$.

If $L < p_o \leq H$, $\Delta U \geq 0$ simplifies to

$$-BL+(1+(-1+B)\theta)p_o+(-1+\theta)(-BL+r+(-H+r)\theta\kappa_l)+(-1+B)B(H-L)\theta\kappa_s \geq 0.$$

Solving these inequalities for p_o gives

$$p_o \geq \frac{r+BL\theta-r\theta+(H-r)(-1+\theta)\theta\kappa_l+(-1+B)B(-H+L)\theta\kappa_s}{1+(-1+B)\theta}$$

While $p_o \leq H$, Through Solving these inequalities for p_o gives: If and only if

$$0 < \kappa_s < \frac{r+BL\theta-r\theta+H(-1+\theta-B\theta)+(H-r)(-1+\theta)\theta\kappa_l}{(-1+B)B(H-L)\theta}$$

the optimal price $p_o^* = H$.Considering the cost incurred in practicing physical showroom, the retailer's optimal profit is derived as follow:

$$\pi_+^P = (1-B)(\theta(H-s)-c)+B(L-s).$$

If $p_o \leq L$, from $\Delta U \geq 0$, we derive $p_o \geq L-(-1+B)(H-L)\theta\kappa_s > L$ which is contradict with $p_o \leq L$ exceeding the initial domain beyond the discussion.

Finally, through comparing the profit case by case, we derive the threshold above which the retailer implement different channel pricing strategies.

Appendix A.6. Proof of corollary 4

We compare the profit functions with or without physical showroom under the pricing policy of mark-up or mark-down respectively.

Comparison under lower stock level

When $0 < B < \theta < 1$, the stock level is low,the optimal profit function earned from omnichannel with or without physical showroom is derives as follows:

$$\pi_l^* = \begin{cases} \theta(H-s)-(\theta-1)(B-\theta)(H-r)\kappa_l & if \Delta_1 > 0 \\ (B-1)\theta(-(\theta-1)(H-r)\kappa_l-H+s)+B(L-s) & otherwise \end{cases} \tag{A.10}$$

$$\pi_l^{P*} = \begin{cases} \left(1-\frac{B}{\theta}\right)(\theta(H-s)-c)+B(H-s) & if\Delta_1^P>0 \\ (1-B)(\theta(H-s)-c)+B(L-s) & otherwise \end{cases} \tag{A.11}$$

Where

$$\Delta_1 = B\left(-L+s+H\theta-s\theta+(H-r)(-1+\theta)^2\kappa_l\right)$$
$$\Delta_1^P = B\left(-L+s+c\left(-1+\frac{1}{\theta}\right)+H\theta-s\theta\right)$$

(i) When $\Delta_1>0$ and $\Delta_1^P>0$, $\Delta\pi_l^P=\pi_l^*-\pi_l^{P*}=\left(1-\frac{B}{\theta}\right)(c-(1-\theta)\theta(H-r)\kappa_l)$. we find that if and only if $0<c<(1-\theta)\theta(H-r)\kappa_l$,$\pi_l^{P*}>\pi_l^*$.

(ii) When $\Delta_1>0$ and $\Delta_1^P<0$, $\Delta\pi_l^P=-Bc-(\theta-1)(B-\theta)(H-r)\kappa_l+B(\theta H-L-\theta s+s)+c<0$ we know $\pi_l^{P*}>\pi_l^*$ in this case.

(iii) When $\Delta_1<0$ and $\Delta_1^P>0$, $\Delta\pi_l^P=-\frac{Bc}{\theta}-(B-1)(\theta-1)\theta(H-r)\kappa_l+B(-\theta H+L+(\theta-1)s)+c>0$ we know $\pi_l^{P*}<\pi_l^*$ in this case.

(iv) When $\Delta_1<0$ and $\Delta_1^P<0$, $\Delta\pi_l^P=(1-B)(c-(1-\theta)\theta(H-r)\kappa_l)$ we find that if and only if $0<c<(1-\theta)\theta(H-r)\kappa_l$,$\pi_l^{P*}>\pi_l^*$.

Comparison under higher stock level

When $0<\theta<B<1$, the stock level is high,the optimal profit function earned from omnichannel with or without physical showroom is derives as follows:

$$\pi_h^* = \begin{cases} \theta(H-s) & if\Delta_2>0 \\ (B-1)\theta(-(\theta-1)(H-r)\kappa_l-H+s)+B(L-s) & otherwise \end{cases} \tag{A.12}$$

$$\pi_h^{P*} = \begin{cases} \theta(H-s) & if\Delta_2^P>0 \\ (1-B)(\theta(H-s)-c)+B(L-s) & otherwise \end{cases} \tag{A.13}$$

Where $\Delta_2 = B(-L+s+H\theta-s\theta)+(-1+B)(H-r)(-1+\theta)\theta\kappa_l$
$\Delta_2^P = c-Bc+B(-L+s+H\theta-s\theta)$

(i) When $\Delta_2>0$ and $\Delta_2^P>0$, we know $\pi_h^{P*}=\pi_h^*$ in this case.

(ii) When $\Delta_2 > 0$ and $\Delta_2^P < 0$, $\Delta\pi_h^P = -Bc + B(\theta H - L - \theta s + s) + c < 0$ we know $\pi_h^{P*} > \pi_h^*$ in this case.

(iii) When $\Delta_2 < 0$ and $\Delta_2^P > 0$, $\Delta\pi_h^P = (B-1)(\theta-1)\theta(H-r)\kappa_l + B(\theta H - L - \theta s + s) > 0$ we know $\pi_h^{P*} < \pi_h^*$ in this case.

(iv) When $\Delta_2 < 0$ and $\Delta_2^P < 0$, $\Delta\pi_h^P = (1-B)\left(c - (1-\theta)\theta(H-r)\kappa_l\right)$ we find that if and only if $0 < c < (1-\theta)\theta(H-r)\kappa_l$,$\pi_h^{P*} > \pi_h^*$.

Appendix A.7. Proof of proposition 4

Case 1:mark-up policy

We only concentrate on the case that the fill rate is less than 1 in store channel, i.e.$0 < f = \frac{B}{\theta(\lambda_s+\lambda_{so})} < 1$.

When $p_s^* = L$,through comparing consumers' utility derived from online and offline, we can calculate the sizes of these three customer segments as follows:

$$\begin{aligned} \lambda_s &= \tfrac{(\theta(-H)+(\theta-1)(\theta Y(r-H)+r)+p_o)(\theta((2f-1)H-2fL)+(\theta-1)(\theta Y(r-H)+r)+p_o)}{2(f-1)f(\theta-1)\theta^2 XY(H-L)(H-r)} \\ \lambda_o &= \tfrac{(-\theta H+p_o+(\theta-1)r)(\theta(2(f-1)X(H-L)+H-2L+r)+p_o-r)}{2(f-1)(\theta-1)\theta^2 XY(H-L)(H-r)} \\ \lambda_{so} &= -\tfrac{(-\theta H+p_o+(\theta-1)r)(\theta(H-2L)+p_o+(\theta-1)r)}{2(f-1)(\theta-1)\theta^2 XY(H-L)(H-r)} \end{aligned} \tag{A.14}$$

Then substituting consumers' segmentsλ_s,λ_o and λ_{so} into retailer's profit function and taking the derivative of it with respect to p_o, we compute the first-condition of profit function and solve the critical point given as Eq.(9).Then we sustititute the critical point into the second-order derivative of the profit function to ensure the solution's optimality.

$$\frac{\partial \Pi_1^2(p_o)}{p_o^2} = -\frac{\theta\sqrt{\begin{pmatrix} f^2(H-L)\left(6s(1+X) - L(3+2X)^2 + H\left(3+6X+4X^2\right)\right) \\ +(L-s-2HX+2LX)^2 - 2fZ \end{pmatrix}}}{(-1+f)(H-L)(H-r)XY(-1+\theta)\theta^2)} \tag{A.15}$$

It can be easily seen that the second-order derivative of the profit function at the critical point is always negative.Thus the optimal online price p_o^* is determined by Eq.(9) when $p_s^* = L$.

When $L < p_s < H$ and $p_s < p_o < H$,through comparing consumers' utility derived from online and offline with the mark-up policy implemented, we can calculate the sizes of these three customer segments as follows:

(i) When $0 < \theta \leq \frac{-r+p_o}{H-r}$,$\lambda_s = \frac{1}{(1-f)X}$ and $\lambda_{so} = \lambda_o = 0$; (store only)

(ii) When $\frac{-r+p_o}{H-r} < \theta < 1$,

$$\begin{aligned}
\lambda_s &= \tfrac{Y(H-r)(1-\theta)\theta+p_o-r(1-\theta)-H\theta}{(1-f)XY(H-r)(1-\theta)\theta} \\
\lambda_o &= \tfrac{(r(-1+\theta)-H\theta+p_o)(-r+(H+r+2(-1+f)HX)\theta+p_o+2(-1+X-fX)\theta p_s)}{2(-1+f)(H-r)XY(-1+\theta)\theta^2(H-p_s)} \\
\lambda_{so} &= \tfrac{(r+H\theta-r\theta-p_o)(-r+H\theta+r\theta+p_o-2\theta p_s)}{2(-1+f)(H-r)XY(-1+\theta)\theta^2(H-p_s)}
\end{aligned} \tag{A.16}$$

Then substituting consumers' segmentsλ_s,λ_o and λ_{so} into retailer's profit function and taking the derivative of it with respect to p_o and p_s, we compute the first-order condition of profit function as follows:

$$\frac{\partial \Pi_1(p_o,p_s)}{\partial p_o} = \frac{\begin{pmatrix} -3f(r-p_o)^2 + 2\left(3f\left(-p_s+r\right) - 2\left(-1+f\right)\left(p_s-H\right)X\right)\left(r-p_o\right)\theta \\ +f\left(H^2-3r^2+2Hs-2p_s\left(2H-3r+s\right)\right) \\ -2\left(-1+f\right)\left(p_s-H\right)\left(H-2r+s\right)X)\theta^2 \end{pmatrix}}{2\left(-1+f\right)\left(p_s-H\right)\left(H-r\right)XY\left(-1+\theta\right)\theta^2} \tag{A.17}$$

$$\frac{\partial \Pi_1(p_o,p_s)}{\partial p_s} = \frac{f\begin{pmatrix} -(r-p_o)^3 + 3\left(-H+r\right)\left(r-p_o\right)^2\theta - 3(H-r)^2\left(r-p_o\right)\theta^2 + \\ \left(-H+r\right)\left((H-r)^2 - 2(p_s-H)^2Y\right)\theta^3 + 2(p_s-H)^2\left(-H+r\right)Y\theta^4 \end{pmatrix}}{2\left(-1+f\right)\left(p_s-H\right)\left(H-r\right)XY\left(-1+\theta\right)\theta^2} \tag{A.18}$$

Thus as is shown in Proposition 4, the optimal price can be solved by combining and reducing the Eq.(A.17)and Eq. (A.18).

Taking the second-order derivatives of the profit function as follows, which are denoted by A,B and C.

$$A = \frac{\partial \Pi_1^2(p_o,p_s)}{\partial {p_s}^2} = \frac{f(r-p_o+H\theta-r\theta)^3}{(1-f)\left(p_s-H\right)^3\left(H-r\right)XY\left(1-\theta\right)\theta^2} < 0$$

$$B = \frac{\partial \Pi_1^2(p_o,p_s)}{\partial p_s p_o} = \frac{3f(p_o+r\left(-1+\theta\right)-H\theta)^2}{2\left(-1+f\right)\left(p_s-H\right)^2\left(H-r\right)XY\left(-1+\theta\right)\theta^2}$$

$$C = \frac{\partial \Pi_1^2 (p_o, p_s)}{\partial {p_o}^2} = \frac{6f(r - p_o) - 2(3f(-p_s + r) - 2(-1 + f)(p_s - H)X)\theta}{2(-1 + f)(p_s - H)(H - r)XY(-1 + \theta)\theta^2}$$

We turn to verify the determinant of Hessian matrix negative definite to discuss the existence condition of profit function's extremum. To prove the critical point determined by the first-order conditions achieves local maximization, the conditions of $A < 0$ and $AC - B^2 > 0$ should be satisfied, where $A < 0$ is satisfied naturally, and $AC - B^2$ can be computed as follow:

$$AC - B^2 = \frac{f(p_o + r(-1 + \theta) - H\theta)^3 \begin{pmatrix} 3f(-r + p_o) + 3f(-4p_s + 3H + r)\theta \\ -8(-1 + f)(p_s - H)X\theta \end{pmatrix}}{4(-1 + f)^2 (p_s - H)^4 (H - r)^2 X^2 Y^2 (-1 + \theta)^2 \theta^4} \tag{A.19}$$

To ensure $AC - B^2$ positive , We acquire

$$R(p_s^*, p_o^*) = 3f(-r + p_o^*) + 3f(-4p_s^* + 3H + r)\theta - 8(-1 + f)(p_s^* - H)X\theta < 0$$

Substituting $p_s^* = H - \frac{(r - p_o^* + H\theta - r\theta)^3}{\sqrt{2}\sqrt{-(H-r)Y(-1+\theta)\theta^3 (r - p_o^* + H\theta - r\theta)^3}}$ into $R(p_s^*, p_o^*)$,We obtain $R(p_o^*)$ as follow:

$$\begin{aligned} R(p_o^*) = 3f(-r + Z) \frac{4\sqrt{2}(-1+f)X\theta(p_o^* + r(-1+\theta) - H\theta)^3}{\sqrt{(H-r)Y(1-\theta)\theta^3(r - p_o^* + H\theta - r\theta)^3}} + \\ 3f\theta\left(3H + r - 4\left(H + \frac{(p_o^* + r(-1+\theta) - H\theta)^3}{\sqrt{2}\sqrt{(H-r)Y(1-\theta)\theta^3(r - p_o^* + H\theta - r\theta)^3}}\right)\right) \end{aligned} \tag{A.20}$$

Furthermore, we find that if and only if $X \geq \frac{3f}{2-2f}$ or $0 < X \leq \frac{3f}{2-2f}$ and $Y > \frac{2(3f - 2(1-f)X)^2 (r(1-\theta) + H\theta - p_o^*)}{f^2 (H-r)(1-\theta)\theta}$, $R''(p_o^*) < 0$ and when $r < p_o^* < r + (H - \theta)r$, $R'(p_o^*) < 0$. Besides,When $p_o^* \to r + \theta(H - r)$,we compute the limit as $R(p_o^*) \to 0$. Hence, $\forall p \in [r, r + \theta(H - r)]$,$R(p_o^*) = AC - B^2 > 0$.

Case 2:mark-down policy

We only concentrate on the case that the fill rate is less than 1 in store channel, i.e.$0 < f = \frac{B}{\theta(\lambda_s + \lambda_{so})} < 1$.

When $p_s^* = H$,considering consumers' heterogeneity in disappointment-aversion,the offline utility $((1 - f)U_o^+)$ is always less than the online utility.Thus there are only λ_o fraction of consumers purchase online, and the rest of them leave the market directly, where $\lambda_o = \frac{r(-1+\theta) - H\theta + p_o}{(H-r)Y(-1+\theta)\theta}$. The profit function of retailer is $\frac{(-\theta H + p_o + (\theta-1)r)(p_o + (\theta-1)r - \theta s)}{(\theta-1)\theta Y(H-r)}$.

We take the first and second-order of the profit function respectively, obtaining $\frac{-\theta(H+s)+2p_o+2(\theta-1)r}{(\theta-1)\theta Y(H-r)}$ and $\frac{2}{(\theta-1)\theta Y(H-r)}$. Because the second-order derivetive is always negative,we derive that the optimal online price is achieved at the critical point given by $\frac{1}{2}(\theta H - 2\theta r + 2r + \theta s)$.

When $r < p_o < p_s < H$,through comparing consumers' utility derived from online and offline with the mark-down policy implemented, we can calculate the sizes of these three customer segments as follows:

(i) When $0 < \theta \leq \frac{-r+p_o}{H-r}$, $\lambda_s = \frac{1}{(1-f)X}$ and $\lambda_{so} = \lambda_o = 0$; (store only)

(ii) When $\frac{-r+p_o}{H-r} < \theta < \frac{-r+p_o}{p_s-r}$,

$$\begin{aligned} \lambda_s &= \frac{Y(H-r)(1-\theta)\theta+p_o-r(1-\theta)-H\theta}{(1-f)XY(H-r)(1-\theta)\theta} \\ \lambda_o &= \frac{(r(-1+\theta)-H\theta+p_o)(-r+(H+r+2(-1+f)HX)\theta+p_o+2(-1+X-fX)\theta p_s)}{2(-1+f)(H-r)XY(-1+\theta)\theta^2(H-p_s)} \\ \lambda_{so} &= \frac{(r+H\theta-r\theta-p_o)(-r+H\theta+r\theta+p_o-2\theta p_s)}{2(-1+f)(H-r)XY(-1+\theta)\theta^2(H-p_s)} \end{aligned} \tag{A.21}$$

(iii) When $\frac{-r+p_o}{p_s-r} < \theta < 1$

$$\begin{aligned} \lambda_s &= \frac{Y(H-r)(1-\theta)\theta+p_o-r(1-\theta)-H\theta}{(1-f)XY(H-r)(1-\theta)\theta} \\ \lambda_o &= \frac{-H\theta+2(-1+f)X(r(-1+\theta)-H\theta)+2(-1+f)Xp_o+\theta p_s}{2(-1+f)(H-r)XY(-1+\theta)\theta} \\ \lambda_{so} &= \frac{H-p_s}{2(-1+f)(H-r)XY(-1+\theta)} \end{aligned} \tag{A.22}$$

It can be easily derived that the case (i) and (ii) in the mark-down policy are identical with the previous mark-up policy. We can focus on the case (iii) in the mark-down setting. Likewise, we substitute consumers' segmentsλ_s,λ_o and λ_{so} into retailer's profit function and taking the derivative of it with respect to p_o and p_s, we compute the first-condition of profit function as follows:

$$\begin{cases} \frac{\partial \Pi_2(p_o,p_s)}{\partial p_s} = \frac{f(-H\theta+(-1+\theta)(3r+2(-H+r)Y\theta)+3p_o-2\theta p_s)}{2(-1+f)(H-r)XY(-1+\theta)} = 0 \\ \frac{\partial \Pi_2(p_o,p_s)}{\partial p_o} = \frac{4(-1+f)rX(-1+\theta)-(f(H+2s)+2(-1+f)(H+s)X)\theta+4(-1+f)Xp_o+3f\theta p_s}{2(-1+f)(H-r)XY(-1+\theta)\theta} = 0 \end{cases} \tag{A.23}$$

The optimal prices can be resolved by computing the Eq.(A.21) as follow:

$$\begin{cases} p_o = \frac{f(\theta H(4X+6(\theta-1)Y+5)-(\theta-1)r(8X+6\theta Y+9)+4\theta s(X+1))-4\theta X(H+s)+8(\theta-1)rX}{8(f-1)X+9f} \\ p_s = \frac{f(H(X(-8\theta Y+8Y+2)+3)+8(\theta-1)rXY+6s(X+1))-2X(-4(\theta-1)HY+H+4(\theta-1)rY+3s)}{8(f-1)X+9f} \end{cases} \tag{A.24}$$

Taking the second-order derivatives of the profit function , we obtain $A = \frac{\partial \Pi_2^2(p_o,p_s)}{\partial {p_s}^2} = -\frac{f\theta}{(-1+f)(H-r)XY(-1+\theta)}$,$B = \frac{\partial \Pi_1^2(p_o,p_s)}{\partial p_s p_o} = \frac{3f}{2(-1+f)(H-r)XY(-1+\theta)}$, and $C = \frac{\partial \Pi_1^2(p_o,p_s)}{\partial {p_o}^2} = \frac{2}{(H-r)Y(-1+\theta)\theta}$.

Then $AC - B^2$ can be computed as

$$AC - B^2 = -\frac{f\left(9f + 8\left(-1+f\right)X\right)}{4{(-1+f)}^2{(H-r)}^2X^2Y^2{(-1+\theta)}^2}.$$

It can be easily seen that $A < 0$ is satisfied naturally, and $AC - B^2 > 0$ can be guaranteed if and only if $X > \frac{9f}{8(1-f)}$.

Appendix A.8. Proof of Corollory6

The profit function of omnichannel retailer with physical showroom can be denoted as:

$$\begin{aligned} &\Pi\left(p_o,p_s\right) = \left[p_o - s - \left(r-s\right)\left(1-\theta\right)\right]\tau_o\left(p_o,p_s\right) \\ &+ f\theta\left(p_s - s\right)\tau_{so}\left(p_o,p_s\right) + \left(1-f\right)\left[\theta\left(p_o - s\right) - c\right]\tau_{so}\left(p_o,p_s\right) \end{aligned} \tag{A.25}$$

where $\tau_{so}\left(p_o,p_s\right) = 1 - \tau_o\left(p_o,p_s\right)$.

Take the derivative of $\Pi\left(p_o,p_s\right)$ with respect to p_s and p_o,we have:

$$\begin{aligned} &\tfrac{\partial\Pi(p_o,p_s)}{\partial p_s} = f\theta\left[1-\tau_o\left(p_o,p_s\right)\right] + H\left(p_s\right)\tfrac{\partial\tau_o(p_o,p_s)}{\partial p_s} \\ &\tfrac{\partial\Pi(p_o,p_s)}{\partial p_o} = \theta\left(1-f\right) + \left(1-\left(1-f\right)\theta\right)\tau_o\left(p_o,p_s\right) + H\left(p_s\right)\tfrac{\partial\tau_o(p_o,p_s)}{\partial p_o} \\ &where \quad H\left(p_s\right) = c - cf + r\left(-1+\theta\right) + \left(1+\left(-1+f\right)\theta\right)p_o - f\theta p_s > 0 \end{aligned} \tag{A.26}$$

Case 1:mark-down policy

Considering the optimal price for online and offline, if ${\tau_o}'\left(p_s\right) > 0$,it can be easily inferred that $\frac{\partial\Pi(p_o,p_s)}{\partial p_s} > 0$ will be satisfied for all p_s.Then the optimal offline price should definitely be H. When $p_s^* = H$,and $L < r < p_o < H$,we derive the offline utility with physical showroom is $U_s^P = f\theta\left(H-p_s\right)\left(1-\left(1-f\right)\kappa_s\right) + \left(1-f\right)\theta\left(H-p_o\right) = \left(1-f\right)\theta\left(H-p_o\right)$. Through some algebra, we find the consumer segments are:

(a) If $\frac{r-p_o}{-fH+(f-1)p_o+r} < \theta < 1$,

$$\begin{aligned} \tau_o &= \tfrac{-f\theta H + ((f-1)\theta+1)p_o + (\theta-1)r}{(\theta-1)\theta Y(H-r)} \\ \tau_{\rm so} &= \tfrac{Y - \frac{-f\theta H + ((f-1)\theta+1)p_o + (\theta-1)r}{(\theta-1)\theta(H-r)}}{Y} \end{aligned} \tag{A.27}$$

(b) else $\tau_o = 0$ and$\tau_{\mathrm{so}} = 1$

It can be easily derived that the retailer's profit function under the condition of $0 < \theta < \frac{r-p_o}{-fH+r+(-1+f)p_o}$, which is

$$\Pi_P^* = -c + cf - s\theta + (\theta - f\theta)\,H + f\theta H \tag{A.28}$$

Then we substitutes the consumers' segments into the retailer's profit function under the condition of $\frac{r-p_o}{-fH+(f-1)p_o+r} < \theta < 1$:

$$\Pi_P = (p_o - s - (r-s)(1-\theta))\,\tau_o + (p_s - s)\,B + (\theta\,(p_o - s) - c)\,(1-f)\,\tau_{so} \tag{A.29}$$

Take the first derivative of the profit function,the critical point is:

$$p_o^P = \frac{\begin{pmatrix} 2fH\theta + (-1+f)\,H\,(2f + Y\,(-1+\theta))\,\theta^2 + c\,(-1+f)\,(1 + (-1+f)\,\theta) \\ -r\,(-1+\theta)\,(2 + (-1+f)\,\theta\,(2 + Y\theta)) \end{pmatrix}}{2(1 + (-1+f)\,\theta)^2} \tag{A.30}$$

and the second derivative is proved to be always negative:$\frac{\partial^2 \Pi_P}{\partial p_o^2} = \frac{2(1+(-1+f)\theta)^2}{(H-r)Y(-1+\theta)\theta} < 0$. Given the optimal online price $p_o^* \in \left[r, \frac{r+fH\theta-r\theta}{1-\theta+f\theta}\right]$, we derive the optimal profit earned by the retailer with physical showroom in such a heterogeneous market is as follow:

(a) If $0 < Y < \frac{c(-1+\theta-f\theta)}{(H-r)(-1+\theta)\theta^2}$, and $K\,(c,Y) > 0$, $\Pi_P^* = \Pi_P\left(p_o^P\right)$,i.e.,

$$\Pi_P^* = \frac{1}{4}\left(-\frac{c^2(f-1)^2}{(\theta-1)\theta Y(H-r)} + \frac{2c(f-1)((f-1)\theta+2)}{(f-1)\theta+1} - \frac{\theta\big(-4(f-1)\theta(fH-fr+2r-2s)+(f-1)^2\theta^2(-HY+r(Y-4)+4s)+4(f(r-H)-r+s)+(f-1)^2\theta^3Y(H-r)\big)}{((f-1)\theta+1)^2} \right) \tag{A.31}$$

(b) If $Y > \frac{c(-f\theta+\theta-1)}{(\theta-1)\theta^2(H-r)}$,

$$\Pi_P^* = \Pi_P\left(\frac{r + fH\theta - r\theta}{1 - \theta + f\theta}\right) = \frac{\theta\,(fH + r - fr - s - (1-f)\,(r-s)\,\theta)}{1 - (1-f)\,\theta} - c\,(1-f) \tag{A.32}$$

(c) If $K\,(c,Y) < 0$,

$$\Pi_P^* = \Pi_P\,(r) = \frac{\begin{pmatrix} c\,(-1+f)\,(f + Y\,(-1+\theta)) + \\ (f^2\,(H-r) + f\,(H-r)\,Y\,(-1+\theta) + (r-s)\,Y\,(-1+\theta))\,\theta \end{pmatrix}}{Y\,(-1+\theta)} \tag{A.33}$$

$$\begin{aligned} &where \quad K(c,Y) = c(-1+f)(1+(-1+f)\theta) \\ &+ (H-r)\theta(2f + (-1+f)(2f-Y)\theta + (-1+f)Y\theta^2) \end{aligned}$$

When $0 < r < p_o < p_o < H$, given the consumer's utility on online and offline:

$$U_o = \theta(H - p_o) + (1-\theta)(r - p_o) - \theta(1-\theta)\kappa_l(H - r)$$

$$U_s^P = f\theta(H - p_s)(1 - (1-f)\kappa_s) + (1-f)\theta(H - p_o),$$

we derive the online-only consumer segment as follow:

(a) when $\frac{-r+p_o}{H-r} < \theta \leq \frac{r-p_o}{r+(-1+f)p_o-fp_s}$,

$$\tau_o = \frac{(r(1-\theta) + H\theta - p_o)(-r + H\theta + r\theta + (1+2(-1+f)\theta)p_o - 2f\theta p_s)}{2(1-f)f(H-r)XY(1-\theta)\theta^2(H-p_s)} \tag{A.34}$$

(b) when $\frac{r-p_o}{r+(-1+f)p_o-fp_s} < \theta < 1$,

$$\tau_o = \frac{\begin{pmatrix} \theta(-H-(-1+f)p_o + fp_s)J(p_s) \\ +2(-1+f)fX(H-p_s)(r(-1+\theta) + (1+(-1+f)\theta)p_o - f\theta p_s) \end{pmatrix}}{2(1-f)fX(H-p_s)(H-r)Y(1-\theta)\theta} \tag{A.35}$$

where $\quad J(p_s) = H + 2(-1+f)fHX + (-1+f)p_o + f(-1+2X-2fX)p_s$

Case 2:mark-up policy

If $\frac{-r+p_o}{H-r} < \theta < 1$,

$$\tau_o = \frac{(r(1-\theta) + H\theta - p_o)\begin{pmatrix} r - (H + r + 2(-1+f)fHX)\theta \\ -(1+2(-1+f)\theta)p_o - 2f(-1+X-fX)\theta p_s \end{pmatrix}}{2(-1+f)f(H-r)XY(-1+\theta)\theta^2(H-p_s)} \tag{A.36}$$

we substitute consumers segments τ_o into Eq.(A.25)to derive the retailer's profit function. Thus, the corollary 6 can be easily derived by comparing the above optimal profit function with those derived in Proposition 4(A numerical example is shown in Appendix B).

Appendix B. Example

The sample used to generate these figures is $c = 0.5, s = 3, L = 6, r = 8, H = 10.5, \theta = 0.3, f = 0.7$.

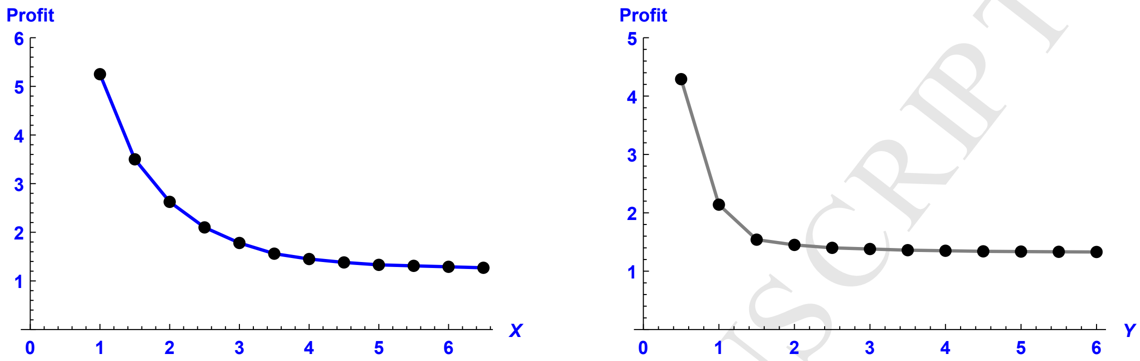

Figure B.1: Note:The optimal profit of omnichannel retailer without physical showroom regarding maximum range of populations's stock-out disappointment respectively. We set $Y = 2$ to investigate the effect of varying X (left)and set $X = 4$ to investigate the effect of varying Y (right).

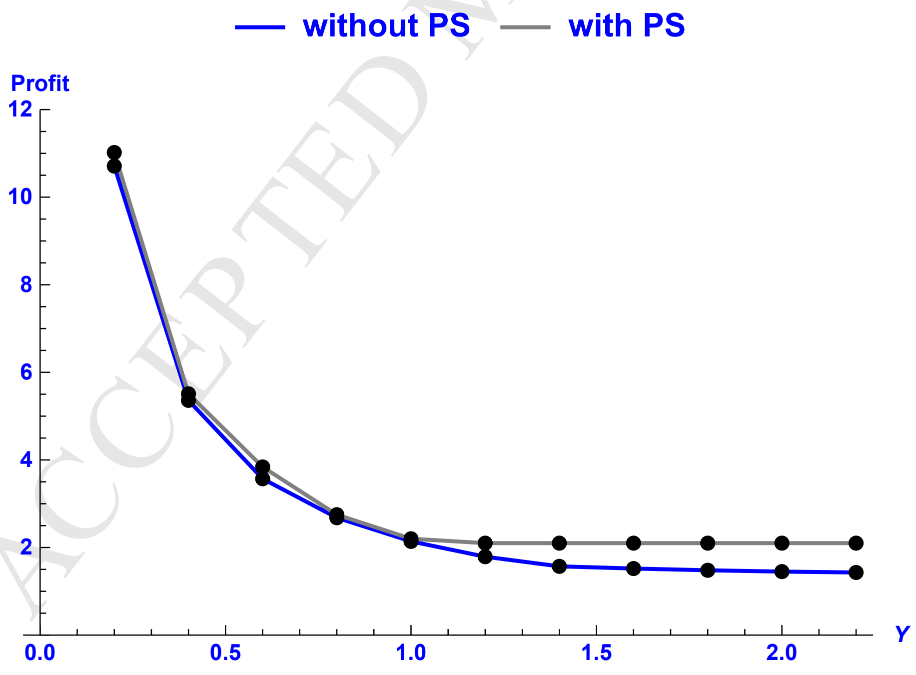

Figure B.2: Notes:The two curves display the optimal profits of omnichannel retailer with or without physical showroom regarding maximum range of populations's low-value disappointment respectively.(The "PS" in legend denote "Physical Showroom")

Highlight

- Disappointment aversion behavior is incorporated into omnichannel management.
- Low-value DA reduces profit while benefits omni-retailer with physical showroom.
- Online selling purely may be an option for retailer in heterogeneous DA market.