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# Due date quotation in a dual-channel supply chain

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ABSTRACT

This paper studies reliable due date quotation for online customer orders in a two-echelon dual-channel supply chain when there is a threshold on quoted due dates. In this problem, there exist two delivery options for the e-tail customers with different cost and availability intervals, i.e., directly shipping by the manufacturer or through the retail store. The manufacturer has the capacity constraint for processing online orders. We adopt an online optimization perspective and propose algorithms to determine due date quotation coordinated with scheduling for e-tail customer with the objective of maximizing the total profit of completed orders considering linear due-date-sensitive revenue function and delivery costs. The approach of “competitive analysis” is performed to evaluate the proposed algorithms. We provide parametric bounds on the competitive ratio of any arbitrary online strategy, and investigate the competitive ratio of a specific online algorithm for single-type e-tail channel orders. Computational experiments illustrate the effectiveness of the proposed algorithms and analysis.

## 1. Introduction

In this paper, we study the problem of reliable and immediate due date quotation for online customer orders in a two-echelon dual-channel supply chain to maximize total profits. For the purposes of this investigation, we have one manufacturer and one retailer as the traditional channel, and online customers as the e-tail channel. Online customers place orders with the manufacturer; however, the products may be delivered to them directly by the manufacturer or through the retail store. In this problem, we try to maximize the due-date-sensitive profit function by quoting immediate and reliable due dates to online customers while considering capacity constraint and maximum acceptable lead time for online orders. The profit function decreases linearly as the quoted due dates increase and we consider single-type e-tail customers.

With the growth of e-business, many manufacturers using a traditional retail store distribution model are expanding into online (e-tail) channels to provide more convenient access to products for their customers. Firms following this dual-channel strategy are referred to as “click-and-mortar” companies, which is distinct from their traditional “brick-and-mortar” counterparts (Chand and Chhajed 1992). Although dual-channel supply chains may help companies increase their customers' awareness and shopping choices, this type of distribution model affects all business functions and operational decisions. Hill et al. (2002) introduced four main strategies for click-and-mortar companies. In the first strategy, firms separate retail and e-tail channels; each channel has its own warehouse, inventory control and pricing features. Some companies find it difficult to manage the same product in two different channels; therefore, in the second strategy, they outsource the e-tail channel—including all the order-fulfilment processes—to a third party. Drop-shipping is another strategy that some companies apply, in which the third party just picks, packs and delivers the orders to customers in the e-tail channel while all the distribution information is available. The final strategy is called the professional shopper strategy, where customers in the e-tail channel order online and then pick up the product(s) from the retail store. Offering both drop-shipping and professional shopper strategies for e-tail customers is a common business practice in many dual-channel supply chains. For example, STAPLES Canada provides two delivery options for online orders: “Ship to Address” and “Ship to Store.” “Ship to Store” lets the e-tail customer pick up the order at any STAPLES store the customer chooses; “Ship to Address” delivers the order to the customer directly. The advantages of having two delivery options include reducing shipping costs, increasing the capacity utilization of the online ordering system, and increasing delivery flexibility for e-tail customers.

Investigating dual-channel's impact on a company's performance is integrated with several related fields, including warehouse design, optimal inventory decisions, and pricing with several studies in each field (Chiang and Monahan 2005, Yue and Liu 2006, Hua et al. 2010, Fan et al.

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2008). Readers are referred to Niels et al. (2008), Yao et al. (2009), Qi et al. (2004) and Abdul-Jalbar et al. (2006) for more information. While growing in popularity, the addition of an e-tail channel is a complex matter but, at its base, most e-business failures are related to operational decisions, and one of the main reasons for early e-business failure is ineffective order fulfilment (Tarn et al. 2003); even a well-designed dual-channel supply chain is useless when it is not successful in delivering items as promised. Effective order fulfilment is closely related to accurate due date quotation, yet—according to Niels et al. (2008) and to the best of our knowledge—there is no study specifically addressing due date quotation in dual-channel supply chains. The existing literature focuses only on due date quotation in a traditional retail or standalone online channel. in this study, we examine both channels simultaneously, considering its effects on a more realistic modern retail model.

Accurate due date quotation is considered one of the main performance measures alongside cost and quality (Handfield et al. 1999, Stalk and Hout 1990), however scheduling orders to ensure that they meet a reasonably speedy due date is not an easy task, particularly in made-to-order environments when the demand trend is unknown (Kaminsky and Hochbaum 2004). The challenging part of these problems lies in the fact that Production capacity constraint makes it impossible to set ideal due dates, so there is a trade-off between sequencing jobs to meet the due dates and setting due dates so that sequencing is possible. In response to the highly competitive retail environment in recent years, the employment of a dual-channel supply chain is vastly increasing and one of the most important challenges for these facilities will be to quote and manage the most efficient due dates to get the competitive advantage in the market.

There are several studies in literature considering due date management when there exist only online customers, not applicable for dual-channel environments, which are briefly reviewed in the following. Scheduling coordinated with due date quotation for online orders, was first introduced by Keskinocak et al. (2001) on a single machine. They performed competitive analyses for a specific online algorithm to maximize due-date sensitive revenue. In their problem, it is assumed there is a threshold on the quoted due dates, and the order will be lost if it is not processed within a specific time interval. Kaminsky and Lee (2008) proposed an online heuristic model for the due date quotation problem, minimizing total quoted due dates, and investigated the conditions of asymptotical optimality of the suggested algorithm. Zheng et al. (2014) studied the same problem as Keskinocak et al. (2001) evaluating the competitive ratio of non-linear revenue functions in both discrete and continuous time points. Kapuscinski and Tayur (1997), Duenyas (1995) and Chand and Chhajed (1992) used analytical approaches in due date setting problems without any constraint on the time interval in which the due date should be quoted. There exist several studies on due date setting and sequencing problems, investigating the performance of online algorithms with methods rather than competitive analysis such as simulation, (Baker and Bertrand 1981, Bookbinder and Noor 1985, Weeks 1979, Ragatz and Mabert 1984). Hsu and Sha (2004) studied online scheduling and due date quotation problems applying an artificial neural network to minimize costs due to delays, and Chang et al. (2005) proposed a fuzzy modeling method embedded by a genetic algorithm for a due date assignment problem. Kaminsky and Kaya (2005) studied the problem of due date quotation and developed three online heuristics to minimize the total processing time; they then applied a probabilistic approach to investigate asymptotical optimality of the suggested heuristics. For a comprehensive review on papers related to due date management, readers are referred to Keskinocak and Tatur (2003) and Cheng and Gupta (1989).

### 1.1. Motivation

Having dual-channel supply chains with two delivery options for e-tail customers is a very common business practice, found in the apparel, courier and construction industries, for instance. So, our study of the due date setting problem in this business model is motivated by the real-world applicability of our solutions.

In the examples we examine, a customer is also a company, which is typical in the industrial supply chain. Consider a company that produces customized steel rolls for smaller mills (customers) worldwide which, in turn, manufacture various steel products. The roll producer has several production lines, however, in general, producing various types of steel rolls requires similar technology and therefore, processing times are almost deterministic, and similar for orders within each family of products. This company has two distinct types of customers, requiring two different delivery and inventory models. One set of customers' demands custom orders with different production requirements; thus, no inventory is kept. These customers can place their customized orders online (e-tail) requesting their order be shipped directly to them or to a retail store routinely supplied by the company. The second type of customer is more predictable, allowing the company to serve its retail channel in specific cycle times (e.g., every week or so). With more predictable demand (less demand variation), it is reasonable to apply inventory models with deterministic demand. In addition, in comparing online orders, we find the demand for the company's retail store is relatively high and stable; thus, the classic cycle inventory model can be implemented for delivering products to the retail stores.

In fact, the challenging issue in managing businesses of this type is not on the manufacturing side but in the coordination of manufacturing and customer service. When the (online) orders arrive, the Customer Service Representative (CSR) chooses which orders to accept and quotes a due date immediately. For more distant due dates, it is common practice to give price breaks to prevent losing customers who have other options. In the past, CSRs used to quote due dates without considering the shop floor status; this led to problems that could be resolved through the coordination of the customer service and manufacturing sides of the company. The motivation for producing our model was to support this coordination.

In this paper, we study the problem of reliable due date quotation in a two-echelon dual-channel supply chain to maximize total profit. In our model, there are two options for delivering items to the online customers: directly from the manufacturer or through the retail store. There is also a threshold on due dates (i.e., the online order will be lost if the quoted due date is after the latest acceptable time). The objective function includes due-date-sensitive revenue function and delivery cost. We consider single-type e-tail customers and capacity in our analyses, and use competitive analysis (Borodin and Yaniv 1998) to investigate the performance of the online heuristic algorithms.

This study will make three main contributions to research in this field: first, to the best of our knowledge, this is the first study of due date quotation in dual-channel supply chains (e-tail and retail) with two options for shipping online orders. The efficient completion of online orders will affect the total profitability and delivery flexibility. Efficient algorithms for due date quotation in dual-channel supply chains can increase a company's capacity for dealing with online orders and thus increase the manufacturer's/retailer's profit significantly. Second, we also considered production capacity constraint for the first time and were able to provide parametric bounds on the competitive ratio of any arbitrary online strategy. Third, we investigated the competitive ratio of a specific online algorithm for single-type e-tail channel orders where its profit was shown to be at most 2.24 of the optimal algorithm in the worst-case scenario.

The paper is organized as follows: Section 2 explains the problem in details and Section 2.1 provides the problem notations and mathematical model. Section 3 characterizes the profit function of both online and optimal offline algorithms. Section 4 presents a parametric upper bound and lower bound for the competitive ratio of any arbitrary online algorithm using concave fractional programing. Section 4.1 proposes a specific online algorithm and investigates its performance for single-type e-tail customers. A detailed computational experiment is provided in Section 5 and, finally, Section 6 concludes with a summary of the insights from the analysis and suggestions for future research.

## 2. Problem definition

In this paper, we study the problem of due date quotation for online customers in a two-echelon dual-channel supply chain while maximizing the profit function which contains due-date sensitive revenue and delivery costs of accepted online orders. We have one manufacturer and a retailer as the traditional channel and online customers as the e-tail channel who are served by the manufacturer. There exist two options for delivering products to the online customers: shipping directly from the manufacturer to online customers (available at any time), and delivering through the retail store (available at specific periods but imposes less cost to the system).

In our model we have the assumption of unknown demand for the e-tail channel (i.e., at any time we have no idea about the arrival of future online orders) and deterministic demand (with fixed profit) for the traditional channel. This assumption implies that there is an optimal cycle time $(T)$ for delivering items to the retail store, and thus the option of delivering items to online customers through the retail store is available every $T$-periods. There exists a maximum acceptable lead time $(L^{'})$ for e-tail customers, and if they are offered a due date after their desired lead time, they will not place the order; in fact, when there is no benefit in accepting the order, the manufacturer has the option of rejecting the online order by offering a due date after the desired lead time. The term *due date* in this paper refers to the time that the order is shipped to the customer (i.e., the time the order leaves the manufacturer); thus the quoted due date for each order may be different from the manufacturer's production completion time. Since the delivery option through the retail store is available at specific periods with less cost, the online order may be held by the manufacturer after its production is completed in order to use the most cost-effective delivery method.

We assume that the revenue will decrease linearly if the quoted due dates for e-tail customers increase (first used by Keskinocak et al. 2001; review on non-increasing revenue functions can be found in Keskinocak 1997). This linear change in revenue can be found in the courier industry: when the customer places emergency orders, the revenue decreases almost linearly based on shipment length/lead time. Another example can be found in online ordering for some materials in a tool and die room in an automotive company: the supervisor of the tool and die room orders materials or parts online for the emergent situation and the order price would almost linearly increase if the required lead time decreases. In order to illustrate the revenue in the objective function, let $r$ be the revenue that is lost for each unit of time if the online order is delayed before being delivered to the customer, and let $l$ be the time interval between the order's arrival time and its quoted due date, then the revenue will be $r(L^{'}-l)$. It is obvious that in this problem the maximum revenue one can obtain from each online order is $rL$, where $L=L^{'}-p$ and $p$ is the order production time. In this problem, we quote 100% reliable due dates to the online customers, i.e., there is no tardiness cost, and all orders should be delivered by the quoted due dates. We also consider capacity constraint of processing $N$ online orders at any time by the manufacturer. We consider a basic model, with single-type e-tail customers, i.e., all online orders have unit-length processing time, identical $L^{'}$, revenue and delivery cost parameters.

### 2.1. Mathematical model

In this sub-section, we introduce the batch definition in our problem, then the notations used in the rest of the paper are provided, followed by the developed mathematical model. Assuming that $\partial$ is the schedule of online orders generated by any algorithm, we can divide each schedule into batches, where each batch contains consecutively-scheduled orders. Let $s_i$ be the start time of the batch $B_i$, and let $s_i^{'}$ be the completion time of the last order in the batch. In batch $B_i$, the order which is processed at time $s_i$, has also arrived at $s_i$ and all the accepted orders arriving before $s_i$ are processed before, however they may leave the system after $s_i$. Batch definition in this paper is different from phase definition in Keskinocak et al. (2001), as the online orders scheduled in each batch may leave the system after the batch completion time. This is because in our problem the quoted due date for e-tail orders is the time that the orders leave the system, which may be different from their completion time.

If we assume that we have single-type e-tail customers, our problem of due date quotation for online customers will be reduced to determining how many online orders should be accepted, how many accepted orders should be processed, and how many processed orders should be shipped in each period. We use the following notations:

$i$ Time index, $i=1,2,\ldots,n$.
$T$ Optimal cycle time of shipments to the retail store.
$\pi$ Set of time indices that are multiples of $T$, $\{T,2T,3T,\ldots\}$.
$t_i$ Time interval between period $i$ and the next period of regular shipment to the retail store.
$r$ Penalty (or revenue that is lost) for each unit of time that the online order is delayed before being delivered to the customer.
$L$ Maximum acceptable lead time excluding the order processing time.
$c_1$ Delivery cost per online order shipped through the retail store.
$c_2$ Delivery cost per online order shipped directly from the manufacturer $(c_1<c_2)$.
$N$ Maximum number of online orders that can be processed at any time by the manufacturer.
$d_i$ Number of online orders (e-tail demand) that have arrived in period $i$.

$$\sigma(i)=\begin{cases}1 & \text{if } i\notin\pi\\ 0, & \text{otherwise}\end{cases}$$

Decision Variables:

$q_i$ Number of accepted online orders in period $i$.
$w_i$ Number of accepted online orders shifted from period $i$ to $i+1$ before being processed.
$u_i$ Number of online orders processed in period $i$ but not delivered to the customers.
$v_i$ Number of online orders processed in period $i$ and delivered to the customers.

For a schedule $\partial$ with $n$ periods, we can define the following mathematical model:$Max\ \sum_{i=1}^{n}C_i$, where $C_i=rLq_i-\ rw_i-\ u_i(rt_i+\ c_1)-v_i[\sigma(i)(c_2-c_1)+c_1]$,

$$s.t.\ u_i+v_i\le N\quad \forall i=1,2,\ldots,n$$
$$q_i+w_{i-1}-w_i=u_i+v_i\quad \forall i=1,2,\ldots,n,$$
$$q_i\le d_i\quad \forall i=1,2,\ldots,n$$

where the first term of the objective function is the maximum possible revenue one can obtain from any accepted online order. The expression $-rw_i-u_irt_i$ represents the revenue lost for the accepted online orders for each unit of time they spend in the manufacturer's system before being delivered. The terms $-u_i(c_1)-v_i\,[\sigma(i)\,(c_2-c_1)+c_1]$ are the delivery costs of online orders shipped directly by the manufacturer and through the retail store, respectively. The first set of constraints represents the capacity restriction. The second set of constraints represent the fact that at any time, the number of orders produced or delivered should be equal to the number of accepted orders or to the number of e-tail orders with a hold on delivery. The last constraints ensure that the number of accepted online orders at any time is less than the online arrivals (e-tail channel demand).

## 3. Preliminaries

In any online algorithm dealing with e-tail customers and unknown demand, making decisions regarding accepting or rejecting the order and quoting the due date must be done as soon as the order arrives, even though there is no information about the future orders. However, in offline algorithms, demand information of future orders are available in advance (demand arrival times or their amounts can be predicted or known). The performance of online algorithms is mainly evaluated by comparing the results of online and offline algorithms for specific instances.

In this Section, we study the mathematical model provided in Section 2.1 for both online and offline models. For a given batch with $n$ periods, let $Z_{(n)}$ denote the total profit obtained from an online algorithm and $Z^*_{(n)}$ the maximum profit one can obtain from the arrivals during the batch. First, we present some remarks and propositions to illustrate the features of the problem, and then in Lemma 1, we prove that for a given batch with $n$ periods, the lower bound of $Z_{(n)}$ is a linear combination of variables $q$ and $u$, where $q$ and $u$ are the column vectors of $n$ elements: $q = (q_1, \dots, q_n)$, $u = (u_1, \dots, u_n)$. Then, in Lemma 2, considering the offline model for a given batch, an upper bound for the optimal function of $Z^*_{(n)}$ is provided. These two Lemmas are then used for computing the bounds of the competitive ratio for any arbitrary online algorithm. Consider the following remarks and propositions for any online algorithm:

**Remark 1**. We know that the purpose of any online algorithm is to schedule orders as soon as possible to guarantee the available capacity for future arrivals, as they have no information about the future demand; therefore, in each period, if $w_i \geq 1$, it means that some orders were shifted to be processed in following periods, and, in this case, we should have used all available capacity in that period, i.e., in online strategies, if $w_i \geq 1$ then $u_i + v_i = N$.

**Remark 2**. In each period if $u_i > 0$, then $i \notin \pi$ and $\sigma(i) = 1$.

**Remark 3**. If $i \in \pi$, then $u_i = 0$ and $\sigma(i) = 0$.

**Remark 4**. $v_n = \sum_{i=1}^{n}(q_i) - (n-1)N - u_n$ where $n$ is the last period in a batch.

**Proposition 1**. *If n is the last period in a batch, then*

$$\sum_{i=1}^{n} w_i = \sum_{i=1}^{n}[(n-i)(q_i)] - N(n(n-1))/2$$

*Proof*: According to the batch definition, $w_i \geq 1$, for $i = 1, 2, \dots, n-1$ and $w_n = 0$. Therefore, from Remark 1, we have $u_i + v_i = N$ for $i = 1, 2, \dots, n-1$, and thus, $w_i = \sum_{j=1}^{i}(q_j - N)$ for $i = 1, 2, \dots, n-1$ and $w_n = 0$. Accordingly, $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n}\sum_{j=1}^{i}(q_j - N) = \sum_{i=1}^{n}[(n-i)(q_i)] - N(n(n-1))/2$.□

**Lemma 1**. *Considering any arbitrary online algorithm's schedule with single-type e-tail customers and $q \geq 0$, the lower bound of the profit function $Z_{(n)}$ for a batch with n periods can be written as $r''q + c'u + K$, where q, u, $r''$ and $c'$ are the column vectors of n elements and $r''_i = rL - r(n-i) - \sigma(n)(c_2 - c_1) - c_1$, $c''_i = -(r+h)t_i - c_1 + c_2$ and $K = rN(n(n-1))/2 + (1-\sigma(n))(N(n-1)(c_1 - c_2))$.*

*Proof*: *Assume that we have n periods in batch $B_l$. According to the batch definition, $w_n = 0$, and since the equality $u_i + v_i = N$ is true for $i = 1, 2, \dots, n-1$, we first consider $Z_{(n-1)}$ as the profit function of the first $n-1$ periods in $B_l$. Let Q and P be the sets $Q = \{i \mid u_i = 0, i \neq n\}$ and $P = \{i \mid u_i > 0, i \neq n\}$. Then*

$$\sum_{i\in Q} C_i = \sum_{i\in Q}\{rLq_i - rw_i - N[\sigma(i)(c_2 - c_1) + c_1]\}, \tag{1}$$

$$\sum_{i\in P} C_i = \sum_{i\in P}\{rLq_i - rw_i - u_i[rt_i + c_1] - (N - u_i)\ [\sigma(i)(c_2 - c_1) + c_1]\}. \tag{2}$$

*From Remark 2, we have* $\sum_{i\in P} C_i = \sum_{i\in P}\{rLq_i - rw_i - u_i[rt_i + c_1] - (N - u_i)\ (c_2)\}$. *Therefore,*

$$Z_{(n-1)} = \sum_{i\in P} C_i + \sum_{i\in Q} C_i = \sum_{i=1}^{n-1} rLq_i - \sum_{i=1}^{} rw_i n - 1 - \sum_{i\in Q} N\ [\sigma(i)(c_2 - c_1) + c_1] - \sum_{i\in P} u_i[rt_i + c_1] - \sum_{i\in P}(N - u_i)\ (c_2). \tag{3}$$

*For $i = n$, we have two Cases:*

*Case 1. $n \in \pi$: We have $\sigma(n) = 0$, and from Remark 3, $u_n = 0$., therefore, $C_n = rLq_n - v_n c_1$. From Remark 4, we have $C_n = rLq_n - [\sum_{i=1}^{n}(q_i) - (n-1)N][c_1]$, therefore,*

$$\begin{aligned} Z_{(n)} &= Z_{(n-1)} + C_n = Z_{(n-1)} + rLq_n - \left[\sum_{i=1}^{n}(q_i) - (n-1)N\right][c_1] \\ &= \sum_{i=1}^{n-1} rLq_i - \sum_{i=1}^{n-1} rw_i - \sum_{i\in Q} N\ [\sigma(i)(c_2 - c_1) + c_1] - \sum_{i\in P} u_i[rt_i + c_1] \\ &\quad - \sum_{i\in P}(N - u_i)\ (c_2) + rLq_n - \left[\sum_{i=1}^{n}(q_i) - (n-1)N\right][c_1]. \end{aligned} \tag{4}$$

*It is clear that we can write $\sum_{i\in P} u_i[rt_i + c_1]$ as $\sum_{i=1}^{n} u_i[rt_i + c_1]$, since if $i \in Q$ then $u_i = 0$ and in this Case $u_n = 0$. Also note that $\sum_{i=1}^{n-1} rw_i = \sum_{i=1}^{n} rw_i$ because $w_n = 0$. Thus, we can rewrite the Equation (4) as follows:*

$$\begin{aligned} Z_{(n)} &= \sum_{i=1}^{n} rLq_i - \sum_{i=1}^{n} rw_i - \sum_{i\in Q} N[\sigma(i)(c_2 - c_1) + c_1] - \sum_{i=1}^{n} u_i[rt_i + c_1] \\ &+ \sum_{i=1}^{n}(c_2 u_i) - \sum_{i\in P} Nc_2 - \left[\sum_{i=1}^{n}(q_i) - (n-1)N\right][c_1] = \sum_{i=1}^{n} rLq_i - \sum_{i=1}^{n} rq_i(n-i) + rN(n(n-1))/2 \\ &- \sum_{i\in Q} N[\sigma(i)(c_2 - c_1) + c_1] - \sum_{i=1}^{n} u_i[rt_i + c_1 - c_2] - \sum_{i\in P} Nc_2 - \left[\sum_{i=1}^{n}(q_i) - (n-1)N\right][c_1]. \end{aligned} \tag{5}$$

*Let $r'_i = rL - r(n-i)$, $c'_i = -rt_i - c_1 + c_2$, and $Q_1 = \{i \in Q \& i \in \pi\}$, $Q_2 = \{i \in Q \& i \notin \pi\}$ then the total profit function of the batch in Case 1 will be*

$$\begin{aligned} Z_{(n)} &= r'q + c'u - c_1 q - |P|Nc_2 - |Q_1|Nc_1 - |Q_2|Nc_2 + rN(n(n-1))/2 \\ &\quad + N(n-1)c_1, \end{aligned} \tag{6}$$

*where q, u, $r'$ and $c'$ are the column vectors of n elements: $q = (q_1, \dots, q_n)$, $u = (u_1, \dots, u_n)$, $r' = (r'_1, \dots, r'_n)$, $c' = (c'_1, \dots, c'_n)$.*

*Case 2. $n \notin \pi$: In this Case, $\sigma(n) = 1$ and $C_n = rLq_n - u_n(rt_n + c_1) - v_n c_2$. From Remark 4, we have $C_n = rLq_n - u_n(rt_n + c_1) - [\sum_{i=1}^{n}(q_i) - (n-1)N - u_n]c_2$. Therefore,*

$$\begin{aligned}Z_{(n)} &= Z_{(n-1)} + C_n = Z_{(n-1)} + rLq_n - u_n(rt_n + c_1)\\ &-\left[\sum_{i=1}^{n}(q_i) - (n-1)N) - u_n\right]c_2 = \sum_{i=1}^{n-1} rLq_i - \sum_{i=1}^{n-1} rw_i - \sum_{i\in Q} N[\sigma(i)(c_2 - c_1) + c_1] - \\ &\sum_{i\in P} u_i[rt_i + c_1] + \sum_{i\in P}(u_i - N)(c_2) + rLq_n - u_n(rt_n + c_1) - \left[\sum_{i=1}^{n}(q_i) - (n-1)N) - u_n\right]c_2.\end{aligned} \tag{7}$$

*Setting $r_i^{'}$,$c_i^{'}$, $Q_1$ and $Q_2$ as in Case 1, then the total profit function of the batch for Case 2 will be as follows:*

$$\begin{aligned}Z_{(n)} &= r^{'}q + c^{'}u - c_2 q - |P|Nc_2 - |Q|_1 Nc_1 - |Q_2|Nc_2 + rN(n(n-1))/2\\ &\quad + N(n-1)c_2.\end{aligned} \tag{8}$$

*Note that $(n-1) = |P| + |Q|_1 + |Q_2|$ and $c_1 < c_2$. In either Case 1 or 2, we can replace the expression $(-|P|Nc_2 - |Q_1|Nc_1 - |Q_2|Nc_2)$ by $(-|P|Nc_2 - |Q_1|Nc_2 - |Q_2|Nc_2)$ and determine the lower bound of $Z_{(n)}$ as follows:*

$$Z_{(n)} \geq \begin{cases} r^{'}q + c^{'}u - c_1 q + rN(n(n-1))/2 + N(n-1)(c_1 - c_2) & n \in \pi \\ r^{'}q + c^{'}u - c_2 q + rN(n(n-1))/2 & n \notin \pi. \end{cases} \tag{9}$$

*Therefore, for a given batch in an arbitrary online algorithm's schedule, we have $Z_{(n)} \geq r^{"}q + c^{'}u + K$, where $r_i^{"} = r_i^{'} - \sigma(n)(c_2 - c_1) - c_1$ and $K = rN(n(n-1))/2 + (1-\sigma(n))(N(n-1)(c_1 - c_2))$ $rN(n(n-1))/2 + (1-\sigma(n))(N(n-1)(c_1 - c_2))$ □*

**Lemma 2.** *The maximum profit one can obtain from the online arrivals during the given batch satisfies $Z^*_{(n)} \leq (rL - c_1)q^{'}$, where $Z^*_{(n)}$ is the profit of optimal offline algorithm, $q_i^{'}$ is the number of accepted online orders in period i by an optimal offline algorithm, and $q^{'}$ is the column vectors of n elements: $q^{'} = (q_1^{'}, \ldots, q_n^{'})$.*

*Proof: Assume that we have n periods in batch $B_l$. For the periods $\{i = 1, \ldots, n | u_i = 0\}$, if $i \in \pi$, delivery cost for each of the online orders in period i is $c_1$ and if $i \notin \pi$, delivery cost is $c_2$, thus considering $c_1$ as the delivery cost for all orders in these periods $\{i = 1, \ldots, n | u_i = 0\}$ does not reduce the profit function since $c_2 > c_1$. For periods $\{i = 1, \ldots, n | u_i > 0\}$, delivery cost is $c_1$, but we have also the cost $rt_i$ since $i \notin \pi$ and $t_i > 0$. Therefore, in this case, considering $c_1$ as the total delivery and holding cost for each of the online orders in these periods does not decrease the profit function as well. According to the batch definition, we know that $w_i \geq 1$ for $i = 1, 2, \ldots, n-1$ and $w_n = 0$, thus $rw \geq 0$. Therefore, based on the objective function in Section 2.1, it is clear that the maximum profit one can make from the online arrivals during a given batch has the following upper bound, $Z^*_{(n)} \leq (rL - c_1)q^{'}$.* □

## 4. Competitive analysis

In online optimization problems, online algorithm's performance is mainly evaluated via the competitive analysis, i.e., comparing an online algorithm's result with the offline model's optimal solution.

As explained in Section 1.1, the evaluation method used in this paper is called "competitive analysis" (Borodin and El-Yaniv, 1998). In online situations, the decision maker has no information about future online orders but needs to have an algorithm (strategy) to determine due dates as soon as the orders arrive. In the competitive analysis method, the decision maker will evaluate the performance of a selected strategy by comparing the outcome with the result of a situation where he/she already has the order's information in advance and could find the optimal plan (optimal offline algorithm). The result of that optimal plan is compared with the result of the online strategy by determining the competitive ratio: the gap between the result of the online strategy with the result of optimal offline algorithm).

For the problem in this paper, all the information about the online orders is available in advance to determine the optimal offline algorithms to obtain maximum possible profit. Given an instance $I$, let $Z_{(I)}$ denote the total profit obtained by using an online algorithm, and $Z^*_{(I)}$ denote the maximum profit obtained by an optimal offline algorithm. For maximization problems, the online algorithm is called $\rho-$ competitive if $Z^*_{(I)} \leq \rho Z_{(I)} + b$ where $\rho \geq 1$, and $b$ is a constant. We define the competitive ratio as $\rho = sup\rho\left(\frac{Z^*_{(I)}}{Z_{(I)}}\right)$ for $Z_{(I)} > 0$. Determining the bounds of the competitive ratio $(\rho)$ is the main challenge in online optimization problems.

According to the batch definition provided in Section 2.1, any schedule of orders generated by an online algorithm can be divided into batches. Therefore, if we investigate the competitive ratio of a given batch, we can generalize the results to determine the competitive performance of the online algorithm generating that batch. In this Section, we first investigate the competitive ratio of *any arbitrary* online algorithm and then for a specific online strategy; the parametric bounds of the competitive ratio are, thus, provided.

### 4.1. Competitive ratio of any arbitrary online algorithm

We review the concave fractional programming (Proposition 2) which is used in Lemma 3 to prove an upper bound for the competitive ratio of any arbitrary online algorithm. The lower bound for the competitive ratio is also provided in Lemma 4.

**Proposition 2.** *Concave Fractional Programming. If $x \in C$, $C \subset R^n$ is a convex set, f is a concave and non-negative function on C, and g is a positive and convex function on C, then the optimization problem $\max_{x\in C}\frac{f(x)}{g(x)}$ is equivalent to the following problem:*

$$\begin{aligned} &\min \lambda \\ s.t.\ & -\nabla f(x) + \lambda \nabla g(x) = 0 \\ & -f(x) + \lambda g(x) \geq 0 \\ & x \in C \\ & \lambda \geq 0 \end{aligned}.$$

*Proof: The proposition's proof and more general results on concave fractional programming can be found in Avriel and Diewert (1988).*

*In Lemma 3, we provide a parametric upper bound for the competitive ratio of any online algorithm using concave fractional programming.*

**Lemma 3.** *For an arbitrary online algorithm with single-type e-tail customers, if a finite competitive ratio exists, it satisfies $\rho \leq \frac{(rL-c_1)}{nr + r^{'}_{min} - c_2 - \frac{r}{2}}$, where $r^{'}_{min} = \min_i\{rL - r(n-i)\}$.*

*Proof: According to the competitive analysis description, the competitive ratio is defined as $\rho = sup\rho\left(\frac{Z^*_{(I)}}{Z_{(I)}}\right)$ for a given instance I. To find $\rho$, we can solve the optimization problem of $\max\frac{Z^*_{(I)}}{Z_{(I)}}$. According to Lemma 1, for each batch with n periods, $Z_{(n)} \geq r^{"}q + c^{'}u + K$, where $c^{'}$ is a column vector of n elements and $c_i^{'} = -rt_i - c_1 + c_2$. Note that $\forall i$, if $c_i^{'} \geq 0$, then $u_i \geq 0$, and if $c_i^{'} < 0$, the option of delivering items through the retail store is not cost-effective in any situation, so $u_i = 0$; therefore, $c^{'}u \geq 0$ and we have $Z_{(n)} \geq r^{"}q + K$. Also* *based on Lemma 2, for each batch, we have $Z^*_{(n)} \le (rL - c_1)q'$. Then it is obvious that the inequality of $\frac{Z^*_{(n)}}{Z_{(n)}} \le \frac{(rL-c_1)q'}{r^* q+K}$ is satisfied and thus $\rho \le max\left(\frac{(rL-c_1)q'}{r^* q+K}\right)$.*

*By the batch definition, we know that $\sum_{i=1}^n q \ge (n-1)N + 1_i$ and let $r'_{min} = \min_i\{r'_i\}$. From Lemma 1, we have*

$$Z_{(n)} \ge \begin{cases} (r'_{min} - c_1)((n-1)N+1) + rN(n(n-1))/2 + N(n-1)(c_1 - c_2) & n \in \pi \\ (r'_{min} - c_2)((n-1)N+1) + rN(n(n-1))/2 & n \notin \pi. \end{cases} \tag{10}$$

*As $N(n-1)(c_1 - c_2) - c_1((n-1)N+1) \ge -c_2((n-1)N+1)$; therefore, $Z_{(n)} \ge (r'_{min} - c_2)((n-1)N+1) + rN(n(n-1))/2$. By rearranging the inequality we have,*

$$\begin{aligned} Z_{(n)} &\ge (n-1)N\left(r'_{min} - c_2 + \frac{nr}{2}\right) + \left(r'_{min} - c_2\right) \\ &\ge \frac{Nrn^2}{2} + n\left(Nr'_{min} - Nc_2 - \frac{Nr}{2}\right) + Nc_2 - r'_{min}(N-1) - c_2. \end{aligned} \tag{11}$$

*Note that $\sum_{i=1}^n q'_i \ge \sum_{i=1}^n q_i$, and considering the batch definition, the maximum possible number of orders accepted from arrivals during the batch can be at most NL number of orders more than the accepted orders by any online algorithm, i.e., $\sum_{i=1}^n q'_i \le \sum_{i=1}^n q_i + NL$. Also we know that $\sum_{i=1}^n q_i \le nN$. Therefore, $\sum_{i=1}^n q'_i \le \sum_{i=1}^n q_i + NL \le Nn + NL$, and $Z^*_{(n)} \le (rL - c_1)q' \le (rL - c_1)(Nn + NL)$. Based on Proposition 2, optimization problem $\left(\rho \le max\left(\frac{(rL-c_1)q'}{r^* q+K}\right)\right)$, can be written as the following dual model if $Z^*_{(n)}$ and $Z_{(n)}$ are concave and convex functions, respectively.*

$$\begin{aligned} & min\lambda \\ s.t. & -\nabla Z^*_{(n)} + \lambda \nabla Z_{(n)} = 0 \\ & -Z^*_{(n)} + \lambda Z_{(n)} \ge 0 \\ & \lambda \ge 0 \end{aligned}$$

*Note that $\frac{d^2}{dn^2}(Z^*_{(n)}) = 0$ and $\frac{d^2}{dn^2}(Z_{(n)}) = Nr$ (the corresponding conditions are satisfied). Therefore, if a finite ratio $\rho$ exists, i.e., there would be a feasible solution for the above dual model, and we have $\rho \le \lambda = \frac{\nabla Z^*_{(n)}}{\nabla Z_{(n)}} = \frac{N(rL-c_1)}{Nnr + Nr'_{min} - Nc_2 - \frac{Nr}{2}} = \frac{(rL-c_1)}{nr + r'_{min} - c_2 - \frac{r}{2}}$.* □

*In Lemma 4, we provide a parametric lower bound for the competitive ratio of any arbitrary online algorithm.*

**Lemma 4**. *For any arbitrary online algorithm with single-type e-tail customers, the lower bound of the competitive ratio is $\rho \ge \frac{(L+1)/2 - L/k_2}{1 - 1/k_2} \ge 1.5 - \frac{1}{k_2}$ where $k_2 = \frac{rL}{c_2}$ and $k_2 \ge L \ge 2$.*

*Proof: To find the lower bound of the competitive ratio for any online algorithm, we adapt the rule that at any time, the adversary knows all the actions of the online algorithm and provides the worst possible arrivals of e-tail customer orders as an input to maximize the competitive ratio. Based on this rule, at any time, if the algorithm decides to accept even one of the online arrivals for processing at time $t$, we will have NL number of new online arrivals in each period afterwards until period $t$. At any time, if the algorithm decides to reserve the capacity for period $t$ by rejecting the available orders and using future arrivals for period $t$, there would be no more online arrivals afterwards. Note that if the algorithm decides to accept all the NL possible orders, it implies that $r \ge c_2$ (which is the worst case and is considered in this Lemma), otherwise, at any time only the number of orders that guarantees the profitability will be accepted.*

*Assume that in each T-period, the last $x$ orders have the option of delivery through the retail store, and we know that their delivery cost is $c'_i = (r)\left\lfloor\frac{i}{N}\right\rfloor + c_1$ for $i = 1, \dots, x$ where $\left\lfloor\frac{i}{N}\right\rfloor$ is the greatest integer which is less than $\frac{i}{N}$. This cost is replaced by $c_1$ for all $x$ orders in the online profit function, where $c_1 \le \min_i(c'_i)$ and is replaced by $c'$ for all $x$ orders in the optimal offline profit function, where $c' = \max_i(c'_i) = (r)\frac{x}{N} + c_1$.*

If at $t = 0$, the online algorithm decides to reserve the capacity for time $t \ge 1$ by rejecting the available e-tail orders and using new arrivals. Based on the adversary rule, there would be no more arrivals after $t = 0$, and, thus, the batch length is $n = 1$. In this case, the online profit is $Z_{(n)} \le NrL - \left\lfloor\frac{1}{T}\right\rfloor x(c_1) - \left(N - \left\lfloor\frac{1}{T}\right\rfloor x\right)c_2$. However, the maximum profit one can gain from the arrivals during the batch period (arrivals at $t = 0$) is $Z^*_{(n)} = \left(\frac{NL(L+1)}{2}\right)r - \left\lfloor\frac{L}{T}\right\rfloor xc' - \left(NL - \left\lfloor\frac{L}{T}\right\rfloor x\right)c_2$. At $t = 0$, the maximum possible orders that the offline algorithm can accept is $NL$. Therefore, the revenue gained from these accepted orders is $NL + N(L-1) + \dots + N(1) = \frac{NL(L+1)}{2}$. In this case, the length of consecutively-scheduled orders will be $L$. $\left\lfloor\frac{L}{T}\right\rfloor x$ determines the number of orders that have been delivered through the retail store in each $T$-period and $\left(NL - \left\lfloor\frac{L}{T}\right\rfloor x\right)$ are the rest of accepted orders shipped directly to the online customers.

If the batch ends at $t \ge 1$, it is clear that in this case, batch length is $n = t + 1$ and the maximum revenue one can gain from online algorithm is $NrL + Nt(r)(L-1)$, where $NrL$ is for the first period and $Nt(r)(L-1)$ denotes the maximum possible revenue for the next $t$ periods. Therefore, $Z_{(n)} = NrL - Nc_2 + Nt(r(L-1) - c_2) + \left\lfloor\frac{1+t}{T}\right\rfloor x(c_2 - c_1)$. For this case, the maximum number of orders that the offline algorithm can accept from the arrivals during the batch length is $N(t) + NL$. In the first $t$ periods that we have arrivals the revenue will be $Nrt$, and in the last period that we have any arrivals we will accept the maximum number which is $NL$, where its revenue will be $NL + N(L-1) + \dots + N(1) = \frac{NL(L+1)}{2}$. So, the partial schedule has $t + L$ periods and the maximum possible profit one can gain is $Z^*_{(n)} = \left(\frac{NL(L+1)}{2}\right)r - NLc_2 + Nt(Lr - c_2) + \left\lfloor\frac{L+t}{T}\right\rfloor x(c_2 - c')$.

It is clear that when $t$ and $N$ increase, the ratio $Z^*_{(n)}/Z_{(n)}$ increases, and when $T$ increases, the ratio will decrease. Therefore, the minimum ratio occurs when $t = 0$, $N = 1$ and $T$ equals to infinity. Thus, the lower bound of the competitive ratio for any online algorithm is $\frac{Z^*_{(n)}}{Z_{(n)}} \ge \frac{rL(L+1)/2 - Lc_2}{rL - c_2}$. Let $c_1 \times k_1 = rL$ and $c_2 \times k_2 = rL$, where $k_1 > k_2 > 1$, then $\frac{Z^*_{(n)}}{Z_{(n)}} \ge \frac{(L+1)/2 - L/k_2}{1 - 1/k_2}$. Note that we assumed that if the algorithm decides to accept orders at any time, all the possible $NL$ arrivals may be accepted; therefore, we should have $r \ge c_2$. Thus, $r \ge \frac{rL}{k_2}$, $k_2 \ge L$. In this situation, $\frac{Z^*_{(n)}}{Z_{(n)}}$ has the minimum amount at $L = 2$, and $\frac{Z^*_{(n)}}{Z_{(n)}} \ge \frac{(3) + 2 - 2/k_2}{1 - 1/k_2} \ge 1.5 - \frac{1}{k_2}$ where $k_2 \ge L \ge 2$. □

### *4.2. Due date quotation for customers (DQC) algorithm*

In this section, we introduce a specific online algorithm for single-type e-tail customers, called the Due Date Quotation (DQC), and we investigate its corresponding competitive ratio. Select $0 < \alpha < 1$. Among the orders that arrive at time $t$, the ones that yield at least $\alpha(rL - c_1)$ profit are accepted, and others will be rejected. The accepted orders will be scheduled at the earliest possible position. Note that $(rL - c_1)$ represents the maximum possible profit yield from any accepted order, which includes the maximum possible revenue $(rL)$ and the minimum delivery cost $(c_1)$ gained from delivery through the retail store without holding the order. We need to find the optimal figure so that the worst performance (competitive rate is smallest) is best. In fact, in this algorithm, an online order is accepted if a certain fraction of maximum profit is guaranteed and the rest of the arrived orders are rejected to keep the capacity for later orders that may yield more profit. The main idea of this algorithm is taken from that presented by Equation (6) for the problem of revenue maximization; however, there exist influential differences in details and assumptions.

**Lemma 5.** *The competitive ratio of online algorithm (DQC) is at most* $\frac{\left(1-\frac{1}{k_1}\right)}{\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{T}\left(\frac{1}{k_2}-\frac{1}{k_1}\right)}$, *where* $\alpha$ *satisfies the following Equation:*

$$\frac{\left(2\alpha^{''}-1+\frac{1}{L}+\frac{2}{Lk_1}-\frac{2\alpha^{''}}{k_1}-\left(\frac{1}{k_2}-\frac{1}{k_1}\right)^2\right)}{(1-\alpha^{''})^2+\frac{(1-3\alpha^{''})}{L}-\frac{2}{Lk_2}+\frac{2}{Lk_1}}=\frac{(rL-c_1)}{\alpha(rL-c_1)+\frac{1}{T}(c_2-c_1)},\text{ where }\alpha^{''}=\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{k_2}. \tag{12}$$

*Proof: The proof of this Lemma is provided in the* Appendix.

**Corollary 1.** *In* Lemma 5, *we assumed we have the option of holding the completed orders to be delivered through the retail store, which implies that* $c_2 > r + c_1$. *However, we may have two other cases:* $(c_2 > r\&c_2 < r + c_1)$ *and* $(c_2 < r, i.e., k_2 > L)$, *where holding the completed item is not profitable, and completed orders may be delivered through the retail store only if their completion time is set at* $\pi$. *For these two cases,* $Z^*_{(n)}$ *in* Lemma 5 *changes to* $Z^*_{(n)} \le \frac{NrL^2}{2}\left(2\alpha^{''}-1+\frac{1}{L}+\frac{2}{Lk_1}-\frac{2\alpha^{''}}{k_1}\right)+nN(rL-c_1)$, *and we have* $Z^*_{(n)} \le \frac{\left(1-\frac{1}{k_1}\right)}{\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{T}\left(\frac{1}{k_2}-\frac{1}{k_1}\right)}Z_{(n)}$ *where* $\alpha$ *is obtained from the equation* $\frac{\left(2\alpha^{''}-1+\frac{1}{L}+\frac{2}{Lk_1}-\frac{2\alpha^{''}}{k_1}\right)}{(1-\alpha^{''})^2+\frac{(1-3\alpha^{''})}{L}-\frac{2}{Lk_2}+\frac{2}{Lk_1}}=\frac{(rL-c_1)}{\alpha(rL-c_1)+\frac{1}{T}(c_2-c_1)}$.

**Corollary 2.** *Note that based on the results obtained from* Lemma 3, *the upper bound of the competitive ratio for any arbitrary online algorithm is* $\rho \le \frac{(rL-c_1)}{nr+r^{'}_{min}-c_2-\frac{r}{2}}$. *In DQC,* $r^{'}_{min}=\alpha^{''}rL$, *thus* $\rho \le \frac{(rL-c_1)}{nr+\alpha^{''}rL-c_2-\frac{r}{2}}=\frac{\left(1-\frac{1}{k_1}\right)}{\frac{n}{L}+\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{k_2}-\frac{1}{k_2}-\frac{1}{2L}}=\frac{\left(1-\frac{1}{k_1}\right)}{\left(\frac{n}{L}-\frac{1}{2L}\right)+\alpha\left(1-\frac{1}{k_1}\right)}$, *and it is clear that* $\rho \le \frac{1}{\alpha}$. *Therefore, results of both Lemmas 5 and 3 satisfy the inequality of* $Z^*_{(n)} \le \frac{1}{\alpha}Z_{(n)}$ *for the DQC online algorithm.*

## 5. Experimental results

In order to evaluate the competitive performance of the proposed algorithm, we define three different cases: 1) $c_2 > r + c_1$, 2) $c_2 > r\&c_2 < r + c_1$ and 3) $c_2 < r, i.e., k_2 > L$. We investigate the performance of the DQC algorithm by providing computational experiments on the upper bound of its competitive ratio (Lemma 5 and Corollary 1). Note that we may use different scenarios for each case, and, in each scenario, the maximum possible ratio is reported as the upper bound of the ratio considered for our performance evaluation. For all three cases, we noted that by increasing $k_1$ while other parameters are fixed, the ratio of the DQC algorithm increases, thus the maximum value for the ratio occurs with the delivery cost $c_1$ is minimized. In addition, by increasing $T$, the competitive ratio decreases ($\alpha$ increases) implying that the maximum ratio occurs when $T$ has the minimum amount. This result also satisfies the argument in Lemma 4, which denoted that the minimum ratio for any online algorithm occurs when $T$ goes to infinity. For each case, the competitive ratio is calculated for different amounts of $L$ and $k_2$, while $k_1$ and $T$ are set to be 10,000 and 2, respectively. (Note that $T = 1$ is not considered for this problem, as it eliminates the second option of direct delivery and $k_1 = 10000$ was examined and proved large enough for the analysis).

**Table 1**
Competitive ratio of *DQC* Algorithm

| $L$ | *Case*1 | *Case*2 | *Case* 3 |
|---|---|---|---|
| | $\rho_{mx}$ | $\rho_{mx}$ | $\rho_{mx}$ |
| 2 | 1.001 | 2.00128 | 1.990304 |
| 3 | 1.05722 | 1.955236 | 1.954316 |
| 4 | 1.066855 | 1.897095 | 1.896957 |
| 5 | 1.202256 | 1.857932 | 1.860584 |
| 6 | 1.307815 | 1.820385 | 1.818566 |
| 10 | 1.570692 | 1.743365 | 1.750954 |
| 50 | 2.060528 | 1.637027 | 1.64149 |
| 100 | 2.150173 | 1.625775 | 1.626146 |
| 500 | 2.228154 | 1.618276 | 1.613276 |
| 1000 | 2.238121 | 1.611491 | 1.6121 |
| 10000 | 2.247761 | 1.617716 | 1.617945 |

The results are provided in Table 1. For Case 1, we have $c_2 > r + c_1$ which means $\frac{1}{k_2} > \frac{1}{k_1} + \frac{1}{L}$, thus for a specific $L$, the parameter $k_2$ should be within $1 < k_2 < \frac{Lk_1}{L+k_1}$. In Cases 2 and 3, for a specific $L$, we have $k_2 > \frac{Lk_1}{L+k_1}$ and $k_2 > L$, respectively. In Case 1, the minimum amount of $k_2$ is set to be 1.01 and in Cases 2 and 3, the maximum amount of $k_2$ is set to be 10,000. For all the Cases, the maximum ratio ($\rho_{mx}$) is obtained from all possible amounts of $k_2$ for a specific $L$. As shown in Table 1, $L$ is changing from 2 to 10,000 and the maximum amount of the competitive ratios for Cases 1, 2 and 3 is at most 2.247761, 2.00128 and 1.990304 respectively. We can claim that the competitive ratio of the DQC algorithm is at most 2.247761, considering all different Cases. This result illustrates that following the DQC algorithm for accepting/rejecting the online arrivals is at most 2.24 worse when compared to the optimal situation if we had all the information in advance. Note that by increasing $L$, the competitive ratio in Cases 2 and 3 converges to 1.618, which is the number Keskinocak et al. (2001) reported as the competitive ratio of their problem (which is a special scenario of our problem in these two Cases). In their problem, the manufacturer is assumed to be a single machine and they consider only maximizing revenue in an e-tail channel, i.e., ($c_1 = c_2 = 0$).

In order to evaluate the performance of bounds provided in Lemmas 3 and 4 for Cases 2 and 3 of the data sets, the gap between the upper and lower bounds of the DQC's competitive ratio is presented in Figures 1 and 2. In these figures, $k_1$ and $T$ are set as mentioned above, and for each $L$ and its corresponding $k_2$ amounts, the maximum upper bound value and the minimum lower bound value is reported.

In Figure 1, it is shown that if Case 2 ($c_2 > r$) is the situation, the upper bound of the competitive ratio, in the worst case, is 2.003. In this case, also, both upper and lower bounds of the competitive ratio converge to 1.618. Figure 2 illustrates that if Case 3 ($c_2 < r, i.e., k_2 > L$) is the situation, the upper bound of the competitive ratio, in the worst case, is 1.999. In this case, also, both the upper and lower bounds of the competitive ratio converge to 1.618.

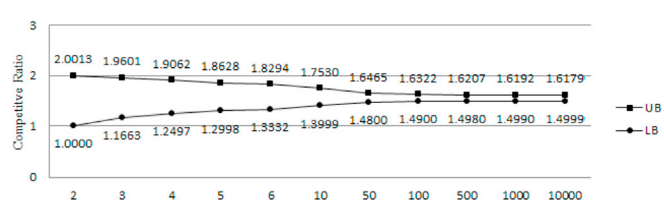


**Figure 1.** Upper and Lower bound of $\rho$ (Case 2).

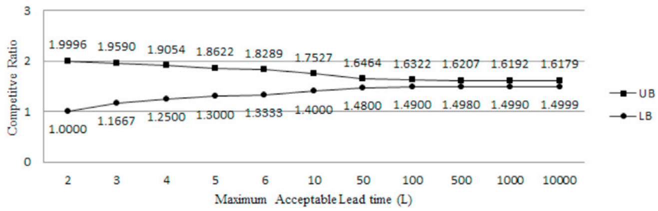


**Figure 2.** Upper and Lower bound of $\rho$ (Case 3).

The results of investigating all the different Cases illustrate that in the worst-case situation, the competitive ratio of the DQC algorithm is 2.24, meaning that the objective function (profit) of this algorithm in determining the due date and accepting/rejecting online orders, is 2.24 worse than the result of the optimal algorithm that knows the online orders in advance.

One of the main concerns for companies with an e-tail channel is determining the best strategy for accepting/rejecting orders and, more importantly, determining the gap between their strategy and the optimal situation for maximizing profit. In this study, we showed that following the DQC algorithm for acceptance/rejection of e-tail customers, (where an online order is accepted only if a certain fraction of maximum profit is guaranteed, while all others are rejected in order to keep the capacity for later orders that may yield more profit) the profit gained is (in the worst case) $\frac{1}{\alpha}$ less than the maximum possible profit by selecting any $\alpha$between 0 and 1. Also, by selecting the $\alpha$ as in Lemma 5, the profit gap between actual and optimal situations can be calculated using different cost parameters of the company, and is at most 2.24 of the optimal profit.

## 6. Conclusions

With the growth of e-business, many companies are adopting a dual-channel strategy, adding online (e-tail) channels to traditional retail channels to provide more convenient access to products. However, even a well-designed dual-channel supply chain is useless when it does not successfully deliver items as promised. One of the most important challenges for these facilities is to quote and manage the most efficient due dates to get the competitive advantage in the market. In this paper, we studied reliable due date quotation in two-echelon, dual-channel supply chains while there is an availability interval for online customers. We applied competitive analysis to this problem while maximizing the total profit. The profit function consists of linear due-date-sensitive revenue and delivery costs. In our analyses, we considered capacity constraint and two delivery options for e-tail customers with different costs and availability intervals. We provided parametric bounds on the competitive ratio of any arbitrary online strategy, and investigated the competitive ratio of a specific online algorithm for single-type e-tail channel orders. Computational experiments illustrate the effectiveness of the proposed analysis. Future research can consider different types of orders with different processing times and cost parameters. In order to analyze these types of problems, asymptotic probabilistic analysis of the model and heuristics can be helpful.

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## Appendix

Proof: For the batch $B_l$ with $n$ periods, let $Z_{(n)}$ be the profit obtained from the DQC algorithm and $Z^*_{(n)}$ the maximum possible profit one can gain from the arrivals during the batch. Note that all the orders accepted by algorithm DQC yield at least $\alpha(rL-c_1)$ profit. Assume $R$ is the revenue obtained from an order accepted by DQC, and in the worst case it is delivered directly to the customer, then $R-c_2 \geq \alpha(rL-c_1)$. Let $\alpha' = \frac{c_2-\alpha c_1}{rL}$, then $R \geq \alpha rL + \alpha' rL$, and any order accepted by algorithm DQC yields at least $\alpha'' rL$ revenue where $\alpha'' = \alpha + \alpha'$ and $0 < \alpha'' < 1$. Note that for determining the bounds of the competitive ratio, we must consider the worst-case situation. First assume that $n \geq \lfloor (1-\alpha'')L \rfloor + 1$, then the revenue we can get from the DQC algorithm is at least

$$\begin{aligned} Z_{(n)} &\geq rN\left(\frac{L(L+1)}{2} - \frac{\lfloor \alpha'' L \rfloor (\lfloor \alpha'' L \rfloor + 1)}{2}\right) + (n - \lfloor (1-\alpha'')L \rfloor - 1)\alpha'' rLN - nNc_2 + \left\lfloor \frac{n}{T} \right\rfloor N(c_2 - c_1) \\ &\geq rN\left(\frac{L(L+1)}{2} - \frac{\alpha'' L(\alpha'' L + 1)}{2}\right) + (n - (1-\alpha'')L - 1)\alpha'' rLN - nNc_2 + \left(\frac{n}{T} - 1\right)N(c_2 - c_1). \end{aligned} \quad (13)$$

In the worst-case situation, we assume all the orders scheduled in the first $\lfloor (1-\alpha'')L \rfloor + 1$ periods have arrived at $t = 1$; therefore, the revenue the DQC algorithm can gain from these orders will be $rN\left(\frac{L(L+1)}{2} - \frac{\lfloor \alpha'' L \rfloor (\lfloor \alpha'' L \rfloor + 1)}{2}\right) \geq rN\left(\frac{L(L+1)}{2} - \frac{\alpha'' L(\alpha'' L+1)}{2}\right)$, which is the first term in the right-hand side of Equation (13). $\alpha'' rL$ is the minimum revenue the DQC algorithm can get from the remaining periods $(n - \lfloor (1-\alpha'')L \rfloor - 1))$ in batch $B_l$, thus we have $(n - (1-\alpha'')L - 1))\alpha'' rLN$, as well. The expression $-nNc_2 + \left\lfloor \frac{n}{T} \right\rfloor N\left(c_2 - c_1\right)$ denotes the maximum delivery costs for all the orders scheduled in the batch, where we know that $\left\lfloor \frac{n}{T} \right\rfloor \geq \frac{n}{T} - 1$, and therefore, $\left\lfloor \frac{n}{T} \right\rfloor N(c_2 - c_1)$ is replaced by $\left(\frac{n}{T} - 1\right)N(c_2 - c_1)$ in Equation (13). Without loss of generality, let $c_1 \times k_1 = rL$ and $c_2 \times k_2 = rL$ where $k_1 > k_2 > 1$, then $-N(c_2 - c_1)$ in Equation (13) can be replaced by $\frac{NrL^2}{2}\left(\frac{-2}{Lk_2} + \frac{2}{Lk_1}\right)$. By rearranging Equation (13), we have

$$Z_{(n)} \geq \frac{NrL^2}{2}\left(\left(1-\alpha''\right)^2 + \frac{(1-3\alpha'')}{L} - \frac{2}{Lk_2} + \frac{2}{Lk_1}\right) + nNr\alpha'' L - nNc_2 + \frac{n}{T}N(c_2 - c_1). \quad (14)$$

Note that $\alpha'' rL = \alpha rL + c_2 - \alpha c_1$; therefore, the term $nNr\alpha'' L - nNc_2$ in Equation (14) is equal to $nN\alpha(rL - c_1)$, and we have

$$Z_{(n)} \geq \frac{NrL^2}{2}\left(\left(1-\alpha''\right)^2 + \frac{(1-3\alpha'')}{L} - \frac{2}{Lk_2} + \frac{2}{Lk_1}\right) + nN\alpha(rL - c_1) + \frac{n}{T}N(c_2 - c_1). \quad (15)$$

The maximum profit we can obtain from the arrivals during the batch $B_l$ (for the considered worst-case situation) is as follows

$$Z^*_{(n)} \le (n - \lfloor (1-\alpha^{"})L \rfloor) NrL + Nr\left(\frac{L(L-1)}{2}\right) - nNc_1 - \lfloor \alpha^{"} L \rfloor Nc_1 - rN\left\lceil \frac{n}{T} \right\rceil (1+2+\ldots+x). \tag{16}$$

Considering batch definition, capacity constraint $N$, and the worst-case situation mentioned above, there would be no arrivals during the last $\lfloor (1-\alpha^{"})L \rfloor$ periods of batch $B_l$. The first term in the right-hand side of Equation (16) shows that all the orders arriving in periods $1, 2, \ldots, n - \lfloor (1-\alpha^{"})L \rfloor$ can obtain the maximum amount of revenue which is $rL$, and the second term denotes that the maximum possible revenue one can get from the arrivals during the last $\lfloor (1-\alpha^{"})L \rfloor$ periods of the batch is $Nr\left(\frac{L(L-1)}{2}\right)$. The last period in the batch when we could have any arrivals is the period $(n - \lfloor (1-\alpha^{"})L \rfloor)^{th}$, and—to obtain maximum possible revenue—we assume the maximum number of orders we can accept ($NL$) has arrived in that period. Therefore, the maximum possible revenue for the arrivals during the last $\lfloor (1-\alpha^{"})L \rfloor$ periods is $Nr\left(L-1\right) + Nr\left(L-2\right) + \ldots + Nr\left(1\right) = Nr\left(\frac{L(L-1)}{2}\right)$. The terms $nNc_1$ and $\lfloor \alpha^{"} L \rfloor Nc_1$ are also the minimum delivery costs for all arrivals during the batch time. Some of the orders delivered through the retail store may have been held after their completion time, and it is obvious that their holding time would be $rN\left\lceil \frac{n}{T} \right\rceil (1+2+\ldots+x)$, where $x = \frac{c_2 - c_1}{r} = \left(\frac{1}{k_2} - \frac{1}{k_1}\right)L$.

Note that $-\lfloor \alpha^{"} L \rfloor Nc_1 \le (1-\alpha^{"} L)\Big) Nc_1 = \frac{NrL^2}{2}\left(\frac{2}{Lk_1} - \frac{2\alpha^{"}}{k_1}\right)$, and $-rN\left\lceil \frac{n}{T} \right\rceil \frac{x(x+1)}{2} \le -rN\frac{x^2}{2} = -\frac{NrL^2}{2}\left(\frac{1}{k_2} - \frac{1}{k_1}\right)^2$. By rearranging Equation (16), we have

$$Z^*_{(n)} \le \frac{NrL^2}{2}\left(2\alpha^{"} - 1 + \frac{1}{L} + \frac{2}{Lk_1} - \frac{2\alpha^{"}}{k_1} - \left(\frac{1}{k_2} - \frac{1}{k_1}\right)^2\right) + nN(rL - c_1). \tag{17}$$

Considering the right-hand side of Equations (15) and (17), If we set

$$\frac{\left(2\alpha^{"} - 1 + \frac{1}{L} + \frac{2}{Lk_1} - \frac{2\alpha^{"}}{k_1} - \left(\frac{1}{k_2} - \frac{1}{k_1}\right)^2\right)}{(1-\alpha^{"})^2 + \frac{(1-3\alpha^{"})}{L} - \frac{2}{Lk_2} + \frac{2}{Lk_1}} = \frac{(rL - c_1)}{\alpha(rL - c_1) + \frac{1}{T}(c_2 - c_1)} \tag{18}$$

We have $Z^*_{(n)} \le \left(\frac{(rL-c_1)}{\alpha\left(rL-c_1\right)+\frac{1}{T}\left(c_2-c_1\right)}\right) Z_{(n)}$. Note that $\frac{(rL-c_1)}{\alpha\left(rL-c_1\right)+\frac{1}{T}\left(c_2-c_1\right)} = \frac{\left(1-\frac{1}{k_1}\right)}{\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{T}\left(\frac{1}{k_2}-\frac{1}{k_1}\right)}$, then $Z^*_{(n)} \le \left(\frac{\left(1-\frac{1}{k_1}\right)}{\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{T}\left(\frac{1}{k_2}-\frac{1}{k_1}\right)}\right) Z_{(n)}$, where $\alpha$ is obtained from the quadratic Equation (19). By replacing $\alpha^{"} = \alpha + \frac{c_2 - \alpha c_1}{rL} = \alpha\left(1 - \frac{1}{k_1}\right) + \frac{1}{k_2}$ in Equation (18), we have

$$\begin{gathered}
\alpha^2(a) + \alpha(b) + (c) = 0, \\
a = \left(1 - \frac{1}{k_1}\right)^3 - 2\left(1 - \frac{1}{k_1}\right)^2 + \frac{2}{k_1}\left(1 - \frac{1}{k_1}\right)^2, \\
b = -2(1 - \frac{1}{k_1})(1 - \frac{1}{k_2})2 - \frac{3}{L}\left(1 - \frac{1}{k_1}\right)^2 - \frac{2}{k_1}\left(1 - \frac{1}{k_1}\right) + , \\
\left(1 - \frac{1}{k_1}\right) - \frac{1}{L}\left(1 - \frac{1}{k_1}\right) - \frac{2}{Lk_1}\left(1 - \frac{1}{k_1}\right) + \frac{2}{k_2 k_1}\left(1 - \frac{1}{k_1}\right) + , \\
\left(1 - \frac{1}{k_1}\right)\left(\frac{1}{k_2} - \frac{1}{k_1}\right)^2 - \frac{2}{T}\left(1 - \frac{1}{k_1}\right)\left(\frac{1}{k_2} - \frac{1}{k_1}\right) + \frac{2}{k_1 T}\left(1 - \frac{1}{k_1}\right)\left(\frac{1}{k_2} - \frac{1}{k_1}\right), \\
c = \left(1 - \frac{1}{k_1}\right)\left(1 - \frac{1}{k_2}\right)^2 + \frac{1}{L}\left(1 - \frac{1}{k_1}\right) - \frac{5}{Lk_2}\left(1 - \frac{1}{k_1}\right) + , \\
\frac{2}{Lk_1}\left(1 - \frac{1}{k_1}\right) - \frac{2}{Tk_1}\left(\frac{1}{k_2} - \frac{1}{k_1}\right) + \frac{1}{T}\left(\frac{1}{k_2} - \frac{1}{k_1}\right) - \frac{2}{TL}\left(\frac{1}{k_2} - \frac{1}{k_1}\right) - , \\
\frac{2}{LTk_1}\left(\frac{1}{k_2} - \frac{1}{k_1}\right) + \frac{2}{Tk_2 k_1}\left(\frac{1}{k_2} - \frac{1}{k_1}\right) + \frac{2}{T}\left(\frac{1}{k_2} - \frac{1}{k_1}\right)^3.
\end{gathered} \tag{19}$$

It is obvious that $\frac{1}{k_2} - \frac{1}{k_1} \ge 0$, then $\frac{\left(1-\frac{1}{k_1}\right)}{\alpha\left(1-\frac{1}{k_1}\right)+\frac{1}{T}\left(\frac{1}{k_2}-\frac{1}{k_1}\right)} \le \frac{1}{\alpha}$, and $Z^*_{(n)} \le \frac{1}{\alpha} Z_{(n)}$, where $\alpha$ is obtained from Equation (19).

Note that if $n < \lfloor (1-\alpha^{"})L \rfloor + 1$, then the online algorithm will accept all possible arrivals during the batch length and schedule them in non-decreasing order of their arrival times, because any arrival during the $\lfloor (1-\alpha^{"})L \rfloor + 1$ periods yield at least $\alpha^{"} rL$ revenue. It can be simply shown that the online algorithm in this case, gives an optimal solution, i.e., $\sum_{i=1}^{n} q_i^{'} = \sum_{i=1}^{n} q_i$ and $Z^*_{(n)} = Z_{(n)}$. □

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