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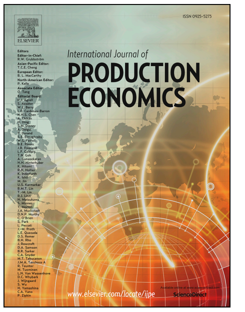

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Abstract: This paper examines cooperative advertising strategies in O2O supply chain made of a seller and an online platform agent. The seller sells a product to end market through both offline and online channels, the latter operated by the agent. We consider cooperative advertising strategies for this O2O supply chain and investigate three models, namely, Integration Model (I-Model), Unilateral co-op advertising model (U-Model), and Bilateral co-op advertising model (B-Model). We derive the optimal decisions of advertisement levels and participation rates between the supply chain members, and explore how they are linked to the interrelationship between the O2O channels and other system parameters. In addition, we provide comparison results among the three models and find that B-model can lead to significant benefits to the seller and the entire channel compared to U-model, especially under a high online profit share for the agent. However, the agent is undermined under B-model as he is obliged to share a portion of offline advertisement expenditure without corresponding compensations, which explains the difficulty in adopting bilateral advertising cooperation in real practice.

Keywords: Cooperative advertising; O2O supply chain; Bilateral participation; Game theory

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1. Introduction

Statistics show that 40 percent of worldwide internet users have bought products or goods online via desktop, mobile, tablet or other online devices. This amounts to more than 1 billion online buyers and is projected to continuously grow (Statistics.com, 2016). The transformative power of the internet continues to revolutionize industry, with new and better ways of doing business emerging on a daily basis. One such, O2O (Online to Offline), is making a splash in China and also flourishing globally as an innovative

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business mode. To simply put, O2O provides information, services, and booking discount to Internet users, who in return will be converted into the customers of the particular offline business partners. This business mode is particularly suitable to consumer goods and services, such as food and beverage, fitness, movies and beauty salon. A typical example is Meituan-Dianping, the largest group deals site in China, selling online vouchers of a wide range of services/entertainment products with the cooperation of providers conducting the services offline. In addition, many firms initiate the consolidation of their supply chains by developing online trading platforms and offline experience shops. For example, SAIC, an automobile manufacturing group in China, implements its O2O business model by integrating 4S shops with the website, www.chexiang.com, an online platform used to communicate with his consumers and facilitate e-payments. In western countries, O2O has not only been evolved by groupbuying websites like Groupon and LivingSocial but also traditional brick and mortar retailers like Nordstrom and Walmart.

Since O2O identifies customers in the online space, it incorporates online advertising techniques such as web banner and interstitial ads. More recently, many O2O companies including Meituan-Dianping and Ctrip have sought to merge their advertising messages into editorial content or valuable services through social media or mobile advertising, together with traditional promotions used in brick-and-mortar channel from the seller or brand owner. Hence, a vital issue is how to balance advertising efforts online and offline, as their influences can be intertwined in close relation to the profits of O2O supply chain members including the seller and the online platform agent. As has been shown by both practice and literature, vertical cooperative (co-op) advertising is a joint effort exerted by all members in a distribution channel to increase market demand and overall profits and it can be achieved by the upstream manufacturer sharing a portion of the downstream retailer’s advertising costs. In the current O2O environment, however, this problem is obviously more complicated since it relates to the offline/online channel relationship which is quite a delicate issue. For example, many platform firms in China such as Meituan.com and Ele.me (a platform that offers online food delivery service) ask the sellers to participate in their promotion and to bear a portion of the related expenditure. As such activity enhances the customers’ willingness-to-pay through Internet, it may cut the seller’s profit seized offline and the overall effects remain unclear. For example, it is reported that during the establishment of O2O systems Nordstrom and Walmart have attempted to combat the ill-effects of ``showrooming” wherein customers come to the store to just look at the products and go back home buying it online.

Motivated by above facts, this paper investigates cooperative advertising strategies in an O2O supply chain consisting of a seller and an online platform agent. The seller sells a product/service to end market through both offline channel (*e.g*, physical store) and online channel promoted by the agent. We develop three cooperative advertising models: Integration model (I-Model), Unilateral co-op advertising model (U-Model), and Bilateral co-op advertising model (B-Model). Then we derive the optimal decisions of advertisement levels and participation rates between the supply chain members, and explore how they are linked to the interrelationship between offline and online channels and the online channel profit shares between the seller and the agent. Furthermore, we compare the advertisement levels and channel member profits among three models, seeking to explore the following research questions: What are the relationships of the advertisement levels and demands among the three models? Can bilateral participation co-op advertising leads to an improved system performance compared to unilateral participation? If so, will the agent spontaneously join in such bilateral participation cooperation? How are these results related to the influence of each channel advertisement to the other channel?

The problem in this paper mainly relates to two streams of researches. One is on the supply chain co-op advertising using quantitative modelling approaches. The earliest literature is Berger (1972) that establishes a mathematical model for vertical co-op advertising problem on manufacturers' and retailers' price discounts and shows the proposed quantitative analysis can be applied in determining the optimal decisions appropriately. Other works include Huang and Li (2001), and Huang *et al.* (2002), and Xie and Ai (2006), which develop models to reflect different power structures and compare the corresponding ways of coordinating advertisement activities within a vertical supply chain. Zhang *et al.* (2013) extend the popular unilateral participation strategy in co-op advertising to bilateral participations and show that a properly designed bilateral participation has several advantages over unilateral participation. Yue *et al.* (2006) extend the model of Huang *et al.* (2002) by considering a price-sensitive demand and study the impact of price-discount conducted by the manufacturer. Some other papers also incorporate pricing decisions into co-op advertising strategy, including Xie and Neyret (2009), Xie and Wei (2009), SeyedEsfahani et al. (2011), Kunter (2012), *etc.* We refer the readers to Aust and Buscher (2014) for a comprehensive review for this research stream. Compared to the above literature, our paper examines co-op advertising strategies under an O2O supply chain environment which involves both online and offline channels under consideration of their interaction, which differs from the traditional vertical supply chain structure. We also investigate the performance of bilateral participation in this O2O supply chain, which is the first attempt among the related researches.

The other stream of research related to our paper is the omni-channel management which has received a lot of attention in recent years. Ofek et al. (2011) study the impact of product returns on a multichannel retailer and examine how pricing strategies and physical store assistance levels change as a result of the additional online outlet. Zhang et al. (2017) investigate a retailer's channel structure choice and pricing decisions in a supply chain under three possible alternatives: a pure offline channel, a pure online channel, and dual channels. Choi et al. (2017) explore online-offline fashion franchising supply chains without channel conflicts with the focal points on the choice of franchising contract and the ordering time. Xu et al. (2017) consider an inventory replenishment planning problem for retailers with online channels who are able to obtain advance demand information (ADI) in an environment of time-varying demands. Gao and Su (2017a) build a theoretical model to study the implementation of BOPS (Buy-Online-and-Pickup-in-Store) and its implications on channel coordination from a theoretical perspective. Gao and Su (2017b) further study the information mechanisms that retailers can use to deliver online and offline information to omni-channel consumers, who strategically choose whether to gather information online or offline and whether to buy products online or offline. Bell et al. (2017) explore the impact of physical showrooms on consumers' channel choice and find that it can be linked to a greater customer need for product information self-select into the physical channel, leading to reduced online returns and increased overall demand.

The paper is organized as follows. In the next section, we describe the problem and modelling assumptions. In section 3, we study three cooperative advertising models in the O2O supply chain, *i.e.*, I-Model, U-Model, and B-model, and derive the optimal decisions of the supply chain members. In section 4, we compare the three models and provide both analytical and numerical results. Finally, we conclude our findings and discuss further research in section 5.

2. Problem Description

Consider a seller (can be a product manufacturer or a service provider) that uses both offline and online channels to sell a product (or service) to end customers. The offline channel adopts direct sales, *e.g.*, via bricks and mortar stores, while the online channel promotes through an online platform agent, like www.meituan.com. The marginal profits for selling through offline and online channels are ρ_1 (the customers pay offline, *e.g.*, in brand store) and ρ_2 (the customers pay through the online platform), respectively. The seller hence takes all profit offline but leaves a fraction, $\theta \in (0,1)$, of the profit online to the agent, which is the major supply chain transaction pattern for O2O business. We assume these profits and sharing fraction are exogenous, since the main focus of this paper is the advertising efforts of the channel members and their co-op advertising policy. We assume $\rho_2/\rho_1 \in (0,2)$, which implies that the online channel profit cannot overweigh the offline counterpart too much.

Denote by A and a, respectively, the seller's and the agent's advertising levels, respectively. The consumer demand functions offline and online are denoted as q_1 and q_2, respectively, with the following forms:

$$\begin{cases} q_1 = \sqrt{\mathrm{A}} + k_1\sqrt{\mathrm{a}} \\ q_2 = \sqrt{a} + k_2\sqrt{A}. \end{cases} \tag{1}$$

This square root formulation of advertising response function depicts the diminishing returns to advertising expenditures and has been extensively adopted in marketing and OM literature, e.g. Fruchter and Kalish (1997), Zhao (2000) , and Xie and Wei (2009). In addition, k_1/k_2 represents the influence of the online/offline channel advertisement to the other channel. We assume that the seller's advertisement offline always boosts the online sales, so $k_2 \in (0,1)$. On the other hand, the agent advertisement may have positive or negative effect to the offline channel demand, so k_1 can be positive or negative. Therefore, we further assume $k_1 \in (\max[-\rho_2/\rho_1, -\rho_1/\rho_2, -1], 1)$, which implies that the cannibalism effect of online advertisement to offline market, if exists, cannot be too intense. This relationship resembles that between the national advertisement made by a brand manufacturer and the local promotion conducted by its retailer, in which the brand advertisement always benefits the retailer sales whereas the local promotion can be detrimental to the manufacturer (Jørgensen *et al.* 2003, Karray and Zaccour 2006).

We consider three types of cooperative advertising models. The first model is the Integration Model (I-Model), in which the seller and her agent cooperate in advertising in an integrated manner so that the whole supply chain profit can be maximized. The second model is Unilateral co-op advertising Model (U-Model), in which the seller not only makes its offline advertisement level A but also shares a fraction

t_1 for the agent's advertisement expenditure a. This is the usual pattern of cooperative advertisement in current O2O practice. The second model is Bilateral co-op advertising Model (B-Model), in which the seller shares a fraction t_1 for the agent's level, and versa vice, the agent also bears a fraction t_2 for the seller's advertisement. This bilateral co-op advertising pattern may occur when the seller's advertisement promotes for the agent and thus benefits its market prominently. Consistent with the existing literature (e.g. Zhang et al. 2013), we call t_1 and t_2 the seller's participation rate (for the online advertisement) and the agent's participation rate (for the offline advertisement), respectively. In the decentralized models, the seller is a Stackelberg leader who moves first (referred to as ``she'') and the agent is a Stackelberg follower who acts accordingly (referred to as ``he''). We assume that all the demand and cost information is common knowledge between two firms.

q_1: Market demand offline

q_2: Market demand online

A: Advertisement level offline, decision variable

a: Advertisement level online, decision variable

k_1: Influence factor of online advertisement to offline demand, $k_1 \in (\max[-\rho_2/\rho_1, -\rho_1/\rho_2], 1)$

k_2: Influence factor of offline advertisement to online demand, $k_2 \in (0,1)$

ρ_1: Marginal profit offline

ρ_2: Marginal profit online

θ: Proportion of online profit for the agent, $\theta \in (0,1)$

t_1: Seller's participation rate, $t_1 \in [0,1]$

t_2: Agent's participation rate, $t_2 \in [0,1]$

π_i^j: Profit of firm *i* under co-op advertising model *j*, $i = 1, 2, \varnothing$ denotes the seller, the agent, and the total channel, respectively, while $j = I, U, B$ denotes I-Model, U-Model, and B-model, respectively.

3. Model Analysis

3.1 I-Model

In this section, we focus on I-Model in which both the seller and the agent agree to make decisions that maximize the total channel profits. We have the channel profit depicted as:

$$\begin{aligned}\pi^I(A,a) &= \rho_1 q_1 + \rho_2 q_2 - A - a \\ &= \rho_1(\sqrt{A} + k_1\sqrt{a}) + \rho_2(\sqrt{a} + k_2\sqrt{A}) - A - a.\end{aligned} \tag{2}$$

Theorem 1. *In I-Model, the optimal advertisement levels offline and online are* $A^{I*} = \left(\frac{\rho_1 + k_2\rho_2}{2}\right)^2$ *and* $a^{I*} = \left(\frac{k_1\rho_1 + \rho_2}{2}\right)^2$*, respectively. The demands offline and online are* $q_1^{I*} = \frac{(k_1^2+1)\rho_1 + (k_1+k_2)\rho_2}{2}$ *and* $q_2^{I*} = \frac{(k_2^2+1)\rho_2 + (k_1+k_2)\rho_1}{2}$*, respectively. And the channel profit is* $\pi^{I*} = \frac{(\rho_1 + k_2\rho_2)^2 + (k_1\rho_1 + \rho_2)^2}{4} = A^{I*} + a^{I*}$.

Proof: In the formulation of the objective function (2), the two decision variables are not intertwined and thus can be solved independently. Taking derivatives of equation (2) with respect to *A* and *a*, respectively, we have

$$\begin{cases} \dfrac{\partial \pi}{\partial A} = \dfrac{\rho_1}{2\sqrt{A}} + \dfrac{k_2\rho_2}{2\sqrt{A}} - 1 \\ \dfrac{\partial \pi}{\partial a} = \dfrac{k_1\rho_1}{2\sqrt{a}} + \dfrac{\rho_2}{2\sqrt{a}} - 1 \end{cases} \tag{3}$$

and

$$\begin{cases} \dfrac{\partial^2 \pi}{\partial A^2} = -\dfrac{\rho_1}{4\sqrt{A^3}} - \dfrac{k_2\rho_2}{4\sqrt{A^3}} < 0 \\ \dfrac{\partial^2 \pi}{\partial a^2} = -\dfrac{k_1\rho_1}{4\sqrt{a^3}} - \dfrac{\rho_2}{4\sqrt{a^3}} < 0. \end{cases} \tag{4}$$

This shows that the objective function is convex with respect to *A* and *a*, respectively. We thus can obtain the optimal solutions by equalizing the first-order derivatives as zero, which yields the solutions in Theorem 1. Further note that $q_1^{I*} = \frac{(k_1^2+1)\rho_1 + (k_1+k_2)\rho_2}{2} > \frac{(k_1^2+1)\rho_1 + k_1\rho_2}{2} > 0$ because $\rho_2/\rho_1 \in (0,2)$. □

Theorem 1 provides closed-form solutions for the optimal advertisement levels and the resulting demands of the offline and online channels, as well as the entire channel profit. The following sensitivity results on the system parameters can also be obtained:

- The optimal advertisement levels are related to the inter-channel influence factors. That is, the optimal advertisement offline is increasing as its influence to online channel k_2 grows, and versa vice. In addition, the optimal offline advertisement is always increasing with the marginal profits of online and offline channels, ρ_1 and ρ_2, whereas the optimal online advertisement can be decreasing with the marginal profit of offline channel, ρ_1, when the corresponding influence factor is negative.
- The demands offline and online are linear combination of the channel marginal profits ρ_1 and ρ_2, and they are increasing with the influence factor of the offline advertisement to the online channel, k_2. The online channel demand is also increasing with the influence factor k_1, while the offline channel demand may not when this factor is negative.
- The channel profit coincides with the total advertisement expenditure offline and online, and hence is increasing with both marginal profits and the inter-channel influences.

3.2 U-Model

We now turn to U-Model, in which the seller not only invests her offline channel advertisement but also bears a fraction, t_1, of the online advertisement. The profit functions of the seller, the agent, and the entire channel are formulated by:

$$\pi_1^U(A,t_1) = \rho_1 q_1 + (1-\theta)\rho_2 q_2 - A - t_1 a, \tag{5}$$

$$\pi_2^U(a) = \theta\rho_2 q_2 - (1-t_1)a, \tag{6}$$

and $$\pi^U = \rho_1 q_1 + \rho_2 q_2 - A - a. \tag{7}$$

The sequence of events is first the seller proposes the offline advertisement level A and the participation rate t_1, and then the agent determines the online advertisement level a. Using a standard backward approach to solve this problem, we have the following result.

Theorem 2. *In U-Model, the optimal advertisement levels offline and online are* $A^{U*} = \left(\frac{\rho_1 + (1-\theta)k_2\rho_2}{2}\right)^2$ *and* $a^{U*} = \begin{cases} \left(\frac{(2-\theta)\rho_2 + 2k_1\rho_1}{4}\right)^2, & \theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3} \\ \left(\frac{\theta\rho_2}{2}\right)^2, & \text{else} \end{cases}$ *with the participation* *rate for the seller* $t_1^{U*} = \begin{cases} \dfrac{(2-3\theta)\rho_2 + 2k_1\rho_1}{(2-\theta)\rho_2 + 2k_1\rho_1} \in (0,1), & \theta < \dfrac{2\rho_1}{3\rho_2}k_1 + \dfrac{2}{3} \\ 0, & else \end{cases}$. *The demands offline and online are* $q_1^{U*} = \begin{cases} \dfrac{[(2-\theta)k_1 + 2(1\text{-}\theta)k_2]\rho_2 + 2(k_1^2+1)\rho_1}{4}, & \theta < \dfrac{2\rho_1}{3\rho_2}k_1 + \dfrac{2}{3} \\ \dfrac{\rho_1 + [\theta k_1 + (1\text{-}\theta)k_2]\rho_2}{2}, & else \end{cases}$ *and*

$q_2^{U*} = \begin{cases} \dfrac{2(k_1+k_2)\rho_1 + [2-\theta+2k_2^2(1\text{-}\theta)]\rho_2}{4}, & \theta < \dfrac{2\rho_1}{3\rho_2}k_1 + \dfrac{2}{3} \\ \dfrac{k_2\rho_1 + [\theta + k_2^2(1\text{-}\theta)]\rho_2}{2}, & else \end{cases}$, *respectively.*

Proof: Taking derivatives of equation (6) with respect to *a*, we have that π_2^U is a convex function of *a* and hence, equalizing the first order derivative to zero yields $\sqrt{a} = \dfrac{\theta\rho_2}{2(1-t_1)}$ under optimality. Substituting it into the equation (5), we have

$$\pi_1^U = [\rho_1 + (1-\theta)k_2\rho_2]\sqrt{A} - A + \frac{\theta\rho_2[k_1\rho_1 + (1-\theta)\rho_2]}{2(1-t_1)} - \frac{t_1\theta^2\rho_2^2}{4(1-t_1)^2}. \tag{8}$$

In this objective function, the two decision variables A and t_1 are not intertwined and thus can be solved independently. Taking derivatives yields

$$\begin{cases} \dfrac{\partial \pi_1^U}{\partial A} = \dfrac{\rho_1 + (1-\theta)k_2\rho_2}{2\sqrt{A}} - 1 \\ \dfrac{\partial \pi_1^U}{\partial t_1} = \dfrac{\theta\rho_2[2k_1\rho_1 + (2-3\theta)\rho_2]}{4(1-t_1)^2} - \dfrac{t_1(\theta\rho_2)^2}{2(1-t_1)^3}. \end{cases} \tag{9}$$

We have $\dfrac{\partial^2 \pi_1^U}{\partial A^2} = -\dfrac{\rho_1 + (1-\theta)k_2\rho_2}{4\sqrt{A^3}} < 0$ and consequently obtain $A^{U*} = \left(\dfrac{\rho_1 + (1-\theta)k_2\rho_2}{2}\right)^2$ by equalizing the first-order derivative to zero. In addition, we can see that $\dfrac{\partial \pi_1^U}{\partial t_1}$<0, so $t_1^{U*}=0$ when $\theta \geq \dfrac{2\rho_1}{3\rho_2}k_1 + \dfrac{2}{3}$; and $\dfrac{\partial^2 \pi_1^U}{\partial t_1^2} < 0$, so $t_1^{U*} = \dfrac{(2-3\theta)\rho_2 + 2k_1\rho_1}{(2-\theta)\rho_2 + 2k_1\rho_1} \in (0,1)$ when $\theta < \dfrac{2\rho_1}{3\rho_2}k_1 + \dfrac{2}{3}$. The formulations of optimal advertisement online and the demands can be obtained accordingly. Note that $k_2 > 0$, so $q_2^{U*} = \sqrt{a^{U*}} + k_2\sqrt{A^{U*}} > 0$ always holds. It is also easy to see $q_1^{U*} > 0$ holds when

$k_1 \geq 0$, we thus only need to show $q_1^{U*} > 0$ for $k_1 < 0$. This is true because when $\theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$ we have $q_1^{U*} = \frac{[(2-\theta)k_1 + 2(1\text{-}\theta)k_2]\rho_2 + 2(k_1^2+1)\rho_1}{4} > \frac{2k_1\rho_2 + 2(k_1^2+1)\rho_1}{4} > 0$, and when $\theta \geq \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$ we have $q_1^{U*} = \frac{\rho_1 + [\theta k_1 + (1\text{-}\theta)k_2]\rho_2}{2} > \frac{\rho_1 + k_1\rho_2}{4} > 0$ since $k_1 > -\rho_1/\rho_2$.□

Theorem 2 provides closed-form solutions for the optimal advertisement levels, the participation rate of the seller for online advertisement, and the demands of the two channels under U-model. The following results can also be obtained:

- The optimal participation rate depends on the proportion of the online profit for the agent θ. If this proportion is lower than a threshold related to k_1 and ρ_1/ρ_2, then the seller is willing to share a fraction of the online advertisement expenditure under consideration of the relatively high marginal profit of selling online, which leads to an online advertisement level related to the product of the influence factor of online advertisement and the offline profit, $k_1\rho_1$. In this case, it is interesting that this advertisement level is decreasing with the proportion θ. This is because a higher portion of profit allocation for the agent will increase his earned marginal profit which enhances the advertisement level, whereas it also decreases the advertisement expenditure share of the seller which discourages the advertisement level. It turns out the latter dominates the former, which implies the effectiveness of cost sharing in motivating a higher advertisement level. On the other hand, if the proportion of the online profit for the agent is higher than the threshold, then the online advertisement expenditure will be entirely borne by the agent and thus only relevant to his extracted profit, $\theta\rho_2$.
- The seller's offline advertisement level is increasing with the offline marginal profit ρ_1, the portion of earned online profit $(1-\theta)\rho_2$, and the influence factor of the offline advertisement k_2.
- The specific forms of market demand under optimality depend on whether the proportion of the online profit for the agent θ is lower than the threshold. In either case, the demands are linear combination of the marginal profits of offline and online channels, ρ_1 and ρ_2, and they are always positive which implies that both channels are utilized.

Corollary 1. *In U-Model, if* $k_1 > 0$ *and* $\theta < 2/3$, *we have* $t_1^{U*} = \dfrac{(2-3\theta)\rho_2 + 2k_1\rho_1}{(2-\theta)\rho_2 + 2k_1\rho_1} \in (0,1)$ and $a^{U*} = \left(\dfrac{(2-\theta)\rho_2 + 2k_1\rho_1}{4}\right)^2$ *; otherwise, if* $k_1 \le 0$ *and* $\theta \ge 2/3$ *, we have* t_1^{U*}=0 and $a^{U*} = \left(\dfrac{\theta\rho_2}{2}\right)^2$.

Proof: We have $\theta < \dfrac{2(k_1\rho_1 + \rho_2)}{3\rho_2}$ if $k_1 > 0$ and $\theta < 2/3$; otherwise, $\theta > \dfrac{2(k_1\rho_1 + \rho_2)}{3\rho_2}$ if $k_1 < 0$ and $\theta > 2/3$, which yields the result according to Theorem 2. □

Corollary 1 shows that if the online advertisement can also boost the offline sales and the agent's online profit share is not very large ($\theta < 2/3$), then the seller has incentive to share online advertisement expenditure to promote sales for both online and offline channels. Conversely, if the influence of the online advertisement to the offline is negative and the agent's profit share is dominant ($\theta \ge 2/3$), then the seller will not participate in the online advertisement cooperation. An intuitionistic diagram is shown by Figure 1, which provides the region of zero or positive unilateral participation rate for the seller mapping with two crucial parameters, k_1 and θ. It can also be seen the slope of dividing line is proportional to the ratio of the offline and online profits, ρ_1 / ρ_2.

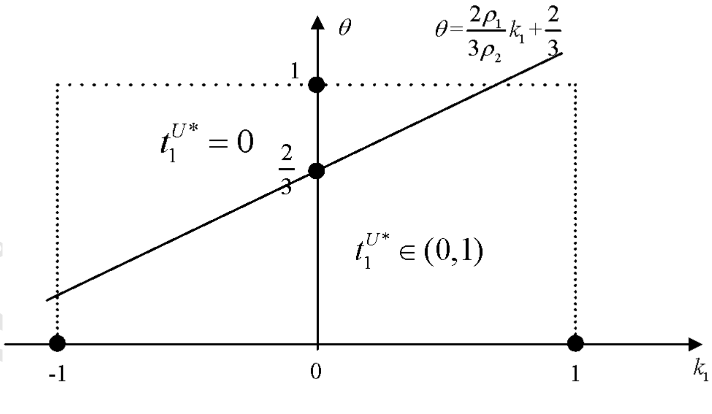


Figure 1. Region of unilateral participation rate mapping with k_1 and θ

3.3 B-Model

In the following we investigate B-Model, which extends the popular unilateral participation advertisement strategy to bilateral participations between the seller and the agent. Intuitively, if the offline advertisement does have effect on expanding online sales, the agent may have incentive to contribute to a portion of the corresponding expenditure. If so, the participation rate is denoted as t_2 parallel to that of the seller for the online advertisement t_1. Under B-Model, the profit functions of the seller, the agent, and the entire channel are formulated by:

$$\pi_1^B(t_1,t_2) = \rho_1 q_1 + (1-\theta)\rho_2 q_2 - (1-t_2)A - t_1 a, \tag{10}$$

$$\pi_2^B(A,a) = \theta\rho_2 q_2 - t_2 A - (1-t_1)a, \tag{11}$$

$$\pi^B = \rho_1 q_1 + \rho_2 q_2 - A - a. \tag{12}$$

This B-Model relates to four decision variables, including the advertisement levels offline and online, A and a, and the bilateral participation rates of the seller and agent, t_1 and t_2. According to Zhang *et al.* (2013) considering a single vertical channel, there are rules on the decision procedure that the supply chain members in bilateral co-op advertising game should follow to avoid trivial game results, *i.e.*, the game follower should not decide either member's participation rate, and any game player should make one of its own decisions: its own advertisement level or participation rate, but not both. These rules carry over to our O2O dual channel model, as any violation will lead to a trivial game. Since the seller is regarded as the Stackelberg leader, we end up with the assumption that the seller first proposes the bilateral participation rates t_1 and t_2, and then the agent accordingly determines the advertisement levels A and a.

Theorem 3. *In B-Model, the optimal advertisement levels offline and online are* $A^{B*} = \left(\frac{(2-\theta)k_2\rho_2 + 2\rho_1}{4}\right)^2$ *and* $a^{B*} = \begin{cases} \left(\frac{(2-\theta)\rho_2 + 2k_1\rho_1}{4}\right)^2, & \theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3} \\ \left(\frac{\theta\rho_2}{2}\right)^2, & else \end{cases}$*, respectively, with bilateral participation rates* $t_1^{B*} = \begin{cases} \frac{(2-3\theta)\rho_2 + 2k_1\rho_1}{(2-\theta)\rho_2 + 2k_1\rho_1} \in (0,1), & \theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3} \\ 0, & else \end{cases}$ and $t_2^{B*} = \frac{2\theta k_2\rho_2}{(2-\theta)k_2\rho_2 + 2\rho_1} \in (0,1)$. *The demands offline and online are*

$$q_1^{B*} = \begin{cases} \frac{(2-\theta)(k_1+k_2)\rho_2 + 2(1+k_1^2)\rho_1}{4}, & \theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3} \\ \frac{(2\theta k_1 + (2-\theta)k_2)\rho_2 + 2\rho_1}{4}, & else \end{cases}$$ *and*

$$q_2^{B*}=\begin{cases}\dfrac{(2-\theta)(1+k_2^2)\rho_2+2(k_1+k_2)\rho_1}{4}, & \theta<\dfrac{2\rho_1}{3\rho_2}k_1+\dfrac{2}{3}\\ \dfrac{(2\theta+(2-\theta)k_2^2)\rho_2+2k_2\rho_1}{4}, & else\end{cases}$$ *, respectively.*

Proof: Taking derivatives of equation (11) with respect to *A* and *a*, respectively, we have

$$\begin{cases}\dfrac{\partial\pi_2^B}{\partial A}=\dfrac{k_2\theta\rho_2}{2\sqrt{A}}-t_2=0\\ \dfrac{\partial\pi_2^B}{\partial a}=\dfrac{\theta\rho_2}{2\sqrt{a}}-(1-t_1)=0\end{cases}, \tag{13}$$

which yields

$$\begin{cases}\sqrt{A}=\dfrac{k_2\theta\rho_2}{2t_2}\\ \sqrt{a}=\dfrac{\theta\rho_2}{2(1-t_1)}\end{cases}. \tag{14}$$

Substituting it into equation (10), we can see that the two decision variables t_1 and t_2 are not intertwined and thus can be solved independently. Hence, we have

$$\begin{cases}\dfrac{\partial\pi_1^B}{\partial t_1}=\dfrac{\theta\rho_2[(2k_1\rho_1+(2-3\theta)\rho_2)-(2k_1\rho_1+(2-\theta)\rho_2)t_1]}{4(1-t_1)^3}\\ \dfrac{\partial\pi_1^B}{\partial t_2}=\dfrac{k_2\theta\rho_2[2\theta k_2\rho_2-(2\rho_1+(2-\theta)k_2\rho_2)t_2]}{4t_2^{\,3}},\end{cases} \tag{15}$$

which follows

$$t_1^{B*}=\begin{cases}\dfrac{(2-3\theta)\rho_2+2k_1\rho_1}{(2-\theta)\rho_2+2k_1\rho_1}, & \theta<\dfrac{2\rho_1}{3\rho_2}k_1+\dfrac{2}{3}\\ 0, & else.\end{cases} \tag{16}$$

On the other hand, we have

$$t_2^{B*}=\begin{cases}\dfrac{2\theta k_2\rho_2}{(2-\theta)k_2\rho_2+2\rho_1}, & \theta<\dfrac{2\rho_1}{3\rho_2}\dfrac{1}{k_2}+\dfrac{2}{3}\\ 1, & else\end{cases} \tag{17}$$

Since $\dfrac{\rho_1}{\rho_2}>\dfrac{1}{2}$ and $k_2\le 1$, it can be seen that $\theta\le 1<\dfrac{2\rho_1}{3\rho_2}\dfrac{1}{k_2}+\dfrac{2}{3}$ always holds so $t_2^{B*}=\dfrac{2\theta k_2\rho_2}{(2-\theta)k_2\rho_2+2\rho_1}\in(0,1)$. Substituting them into equation (14) yields the optimal solutions of A

and a, and q_1 and q_2 depicted in Theorem 3. Also note that $A^{B*} > A^{U*}$ so $q_1^{B*} = A^{B*} + k_1 a^{B*} > A^{U*} + k_1 a^{B*} = A^{U*} + k_1 a^{U*} = q_1^{U*} > 0$. □

Theorem 3 provides closed-form solutions for the optimal advertisement levels and the bilateral participation rates between the seller and the agent under B-model. The demands are also characterized and the following results can be obtained:

- The optimal participate rate for the agent is positive and it is increasing with the proportion of the online profit for the agent θ, the product of the online profit ρ_2 and the influence factor k_2, while decreasing with the offline profit ρ_1. In other words, the agent will be obliged to a higher share for the offline advertisement expenditure when her online earning is larger and this online market is more sensitive to the offline advertisement. It is also noticeable that this participation rate is irrelevant to the influence factor of the online advertisement to the offline demand. Moreover, the offline advertisement level is increasing with the online and offline profits ρ_1 and ρ_2, the influence factor k_2, while decreasing with the proportion of the online profit for the agent θ.
- The optimal participation rate for the seller and the corresponding online advertisement level are the same as those in U-model, which relate closely to the proportion of the online profit for the agent θ. The online advertisement expenditure can be undertaken solely by the agent when this proportion is lower than a threshold related k_1 and ρ_1 / ρ_2, which yields a lower advertisement level.
- The specific forms of market demand also depend on whether the agent's share of the online profit θ is lower than the threshold. In either case, the demands are linear combination of the marginal profits of offline and online channels, ρ_1 and ρ_2, and they are always positive.

4. Model Comparison

This section compares the optimal decision variables and the profits of supply chain members among the three models. The closed-form solutions of decisions and demands are listed in Table 1, with the comparison results shown in Theorem 4.

Table 1. The analytical solutions of decisions and demands among three models

	I-Model	U-Model		B-Model	
		$\theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$	$\theta \geq \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$	$\theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$	$\theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$
A	$\left(\frac{\rho_1 + k_2\rho_2}{2}\right)^2 \perp$	$\left(\frac{\rho_1 + (1-\theta)k_2\rho_2}{2}\right)^2$	$\left(\frac{\rho_1 + (1-\theta)k_2\rho_2}{2}\right)^2$	$\left(\frac{(2-\theta)k_2\rho_2 + 2\rho_1}{4}\right)^2$	$\left(\frac{(2-\theta)k_2\rho_2 + 2\rho_1}{4}\right)^2$
a	$\left(\frac{k_1\rho_1 + \rho_2}{2}\right)^2$	$\left(\frac{(2-\theta)\rho_2 + 2k_1\rho_1}{4}\right)^2$	$\left(\frac{\theta\rho_2}{2}\right)^2$	$\left(\frac{(2-\theta)\rho_2 + 2k_1\rho_1}{4}\right)^2$	$\left(\frac{\theta\rho_2}{2}\right)^2$
t_1	-	$\frac{(2-3\theta)\rho_2 + 2k_1\rho_1}{(2-\theta)\rho_2 + 2k_1\rho_1}$	0	$\frac{(2-3\theta)\rho_2 + 2k_1\rho_1}{(2-\theta)\rho_2 + 2k_1\rho_1}$	0
t_2	-	-	-	$\frac{2\theta k_2\rho_2}{(2-\theta)k_2\rho_2 + 2\rho_1}$	$\frac{2\theta k_2\rho_2}{(2-\theta)k_2\rho_2 + 2\rho_1}$
q_1	$\frac{(k_1^2+1)\rho_1 + (k_1+k_2)\rho_2}{2}$	$\frac{[(2-\theta)k_1 + 2(1\text{-}\theta)k_2]\rho_2}{4} + \frac{(k_1^2+1)\rho_1}{2}$	$\frac{\rho_1 + [\theta k_1 + (1\text{-}\theta)k_2]\rho_2}{2}$	$\frac{(2-\theta)(k_1+k_2)\rho_2}{4} + \frac{(k_1^2+1)\rho_1}{2}$	$\frac{(2\theta k_1 + (2-\theta)k_2)\rho_2 + 2\rho_1}{4}$
q_2	$\frac{(k_2^2+1)\rho_2 + (k_1+k_2)\rho_1}{2}$	$\frac{[2-\theta+2k_2^2(1\text{-}\theta)]\rho_2}{4} + \frac{(k_1+k_2)\rho_1}{2}$	$\frac{k_2\rho_1 + [\theta + k_2^2(1\text{-}\theta)]\rho_2}{2}$	$\frac{(2-\theta)(1+k_2^2)\rho_2}{4} + \frac{(k_1+k_2)\rho_1}{2}$	$\frac{(2\theta + (2-\theta)k_2^2)\rho_2 + 2k_2\rho_1}{4}$

Theorem 4. *We have (i)* $t_2^{B*} \in (0,1)$, $t_1^{B*} = t_1^{U*}$*; (ii)* $A^{I*} > A^{B*} > A^{U*}$ *and* $a^{I*} > a^{B*} = a^{U*}$*; (iii)* $q_2^{I*} > q_2^{B*} > q_2^{U*}$, $q_1^{B*} > q_1^{U*}$ *and* $q_1^{I*} + q_2^{I*} > q_1^{B*} + q_2^{B*} > q_1^{U*} + q_2^{U*}$, *and there exist thresholds* $\tilde{\tilde{k}} \leq \tilde{k} < 0$ *such that* $q_1^{I*} > q_1^{B*}$ *when* $k_1 > \tilde{k}$*; and* $q_1^{I*} > q_1^{U*}$ *when* $k_1 > \tilde{\tilde{k}}$.

Proof. The comparison results of (i) and (ii) can be easily obtained from comparing the analytical solutions in Table 1. We also have $q_2^{I*} > q_2^{B*} > q_2^{U*}$, $q_1^{B*} > q_1^{U*}$ and $q_1^{I*} + q_2^{I*} > q_1^{B*} + q_2^{B*} > q_1^{U*} + q_2^{U*}$ from $A^{I*} > A^{B*} > A^{U*}$ and $a^{I*} > a^{B*} > a^{U*}$. Moreover, it can be proven that $q_1^{I*} < q_1^{B*}$ if and only if $k_1 < -k_2$ and $q_1^{I*} < q_1^{U*}$ if and only if $k_1 < -2k_2$, when $\theta < \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$. On the other hand, when $\theta \geq \frac{2\rho_1}{3\rho_2}k_1 + \frac{2}{3}$, letting $\tilde{\tilde{k}} = \frac{-(1-\theta)\rho_2 + \sqrt{(1-\theta)^2\rho_2^{\,2} - 2\theta k_2\rho_1\rho_2}}{2\rho_1}$ and $\tilde{k} = \frac{-(1-\theta)\rho_2 + \sqrt{(1-\theta)^2\rho_2^{\,2} - 4\theta k_2\rho_1\rho_2}}{2\rho_1}$, we have $q_1^{I*} > q_1^{B*}$ if $k_1 > \tilde{k}$ and $q_1^{I*} > q_1^{U*}$ if $k_1 > \tilde{\tilde{k}}$

under the conditions $(1-\theta)^2 > 2\theta k_2\rho_1/\rho_2$ and $(1-\theta)^2 > 4\theta k_2\rho_1/\rho_2$, respectively ($\tilde{k}$ or $\tilde{\tilde{k}}$ should be the lower bound of k_1 if the respective condition is not satisfied). □

Theorem 4 shows that I-model yields the highest online and offline advertisement levels among three models, which demonstrates the superiority of channel integration. In comparison, B-model and U-model generate the same participation rate and advertisement level of the online channel for the seller. Hence, the main difference between these two models is that B-model always leads the agent to participate in the cooperative advertisement for the offline channel, which results in a higher offline advertisement level than U-channel. Moreover, both the online channel demand and total demand of the two channels are the highest in I-Model and the lowest in U-Model, respectively, which are the direct results corresponding to the participation rates and advertisement levels. When the influence factor of online advertisement to the offline demand is positive or the cannibalism effect is not large, the offline demand in I-Model is also higher than those in B-Model or U-Model. Hence, from the perspective of both input and output, bilateral participation in O2O advertisement cooperation can lead to a result that is closer to systematic optimality than unilateral participation from the perspective of the whole supply chain channel. The price of this system performance improvement, however, is paid by the agent since he will share a portion of the offline channel advertisement for the seller.

To see the insights more clearly, in the following we focus on the profits of the O2O supply chain members by numerical experiments, which is difficult to analyze through closed-form solutions. The base parameter setting is $\rho_1 = 5$, ρ_2=8, θ=$\{0.3, 0.7\}, k_1 \in \{-0.5, 0.5\}, k_2 = 0.5$. We use the following symbols to denote the profit gaps among different cooperative advertisement models: $\Delta_1^{BtoU} = \dfrac{(\pi_1^{B*} - \pi_1^{U*})}{\pi_1^{U*}} \times 100\%$, $\Delta_2^{UtoB} = \dfrac{(\pi_2^{U*} - \pi_2^{B*})}{\pi_2^{B*}} \times 100\%$, $\Delta^{ItoB} = \dfrac{(\pi^{I*} - \pi^{B*})}{\pi^{I*}} \times 100\%$, and $\Delta^{ItoU} = \dfrac{(\pi^{I*} - \pi^{U*})}{\pi^{I*}} \times 100\%$, in which π_i^j is the profit of firm *i* under co-op advertising model *j*, where $i = 1, 2, \varnothing$ denotes the seller, the agent, and the total channel, respectively, while $j = I, U, B$ denotes I-Model, U-Model, and B-model, respectively. Hence, Δ_1^{BtoU} is the percentage of the seller's profit improvement for B-Model compared to U-Model, while Δ^{ItoB} and Δ^{ItoU} are those of the channel profit improvement for I-Model compared to B-Model and U-Model, respectively. Noting that the agent's profit under U-model is higher than that under B-model, we use Δ_2^{UtoB} to denote the profit loss for the agent (which is a positive number) under B-Model compared to U-Model.

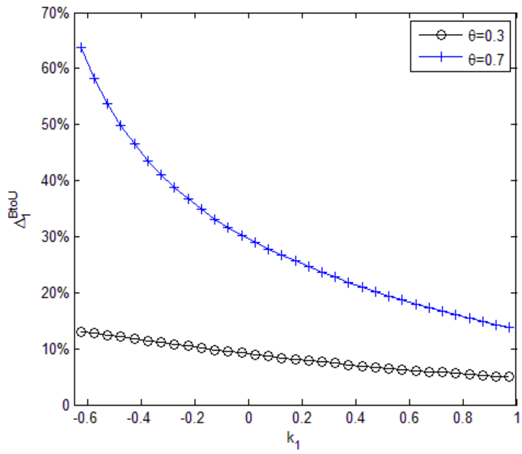


Figure 2. The value of Δ_1^{BtoU} as k_1 varies

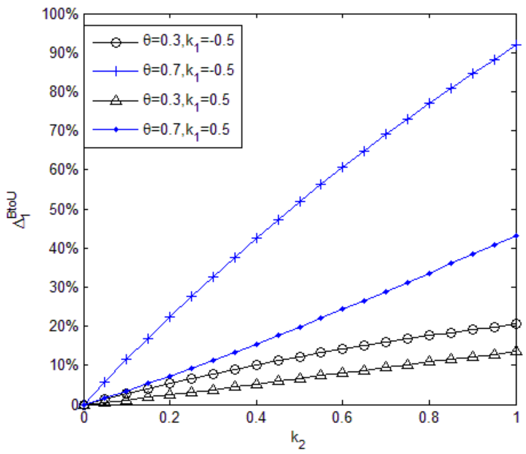


Figure 3. The value of Δ_1^{BtoU} as k_2 varies

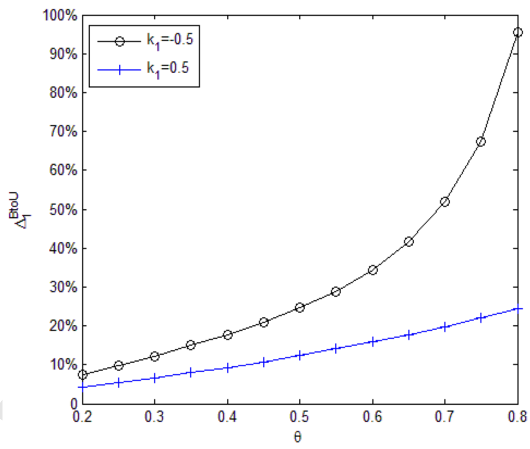


Figure 4. The value of Δ_1^{BtoU} as θ varies

We can see that the seller's profit in B-Model is always higher than that in U-Model, which indicates the benefit of bilateral participation in cooperative advertising for the seller. Figure 2 shows that the gap between B-Model and U-Model are decreasing with the influence factor of the online advertisement to the offline channel, which implies that the seller tends more to adopt bilateral participation if the online channel has a higher degree of cannibalistic effect to the offline channel. On the other hand, the benefit of bilateral participation is more prominent for the seller when the offline advertisement is more effective in boosting the online market demand, as indicated by Figure 3. Moreover, Figure 4 shows that the seller's profit improvement is increasing with the proportion of online profit extracted by the agent. In other words, the seller is more prone to the bilateral rather than unilateral advertisement cooperation as a compensation for the smaller share of online profit.

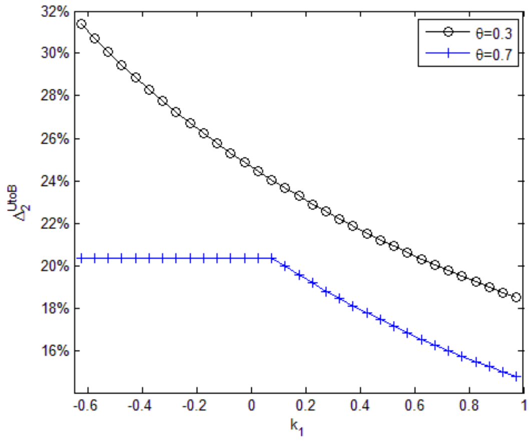


Figure 5. The value of Δ_2^{UtoB} as k_1 varies

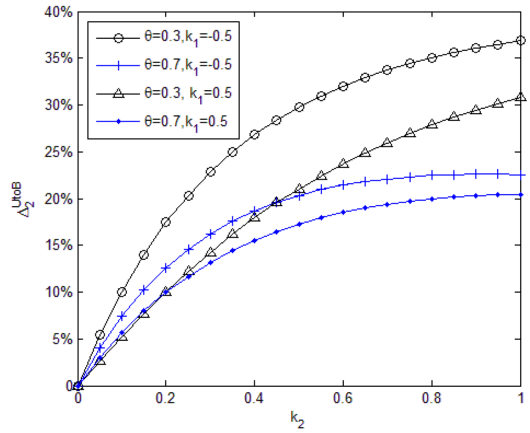


Figure 6. The value of Δ_2^{UtoB} as k_2 varies

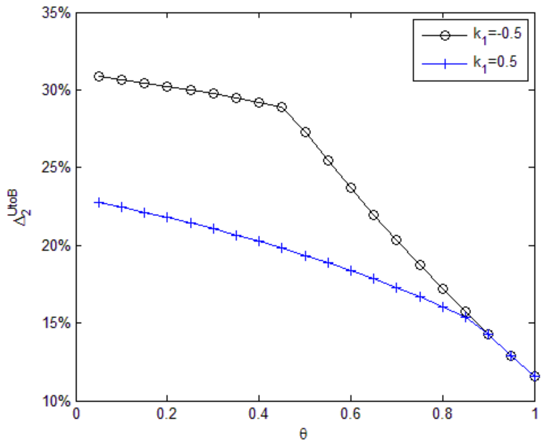
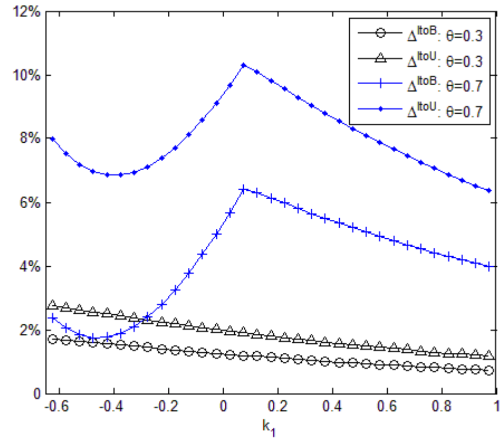


Figure 7. The value of Δ_2^{UtoB} as θ varies

Figures 5-7 indicate that the agent's profit in U-Model is always higher than that in B-Model, which suggests the reluctance of cooperation in the offline advertising for the agent. Figure 5 shows that the gap between the two models is decreasing (when the proportion of online profit for the agent is small) or first irrelevant then decreasing (when the proportion of online profit for the agent is large) with the influence factor of the online advertisement to the offline channel, and Figure 6 shows that this gap is increasing with the influence factor of the offline advertisement to the online channel. Furthermore, we can see from Figure 7 that the agent will be more reluctant in participating the bilateral cooperative advertisement as he is allocated with a smaller portion of the online profit.



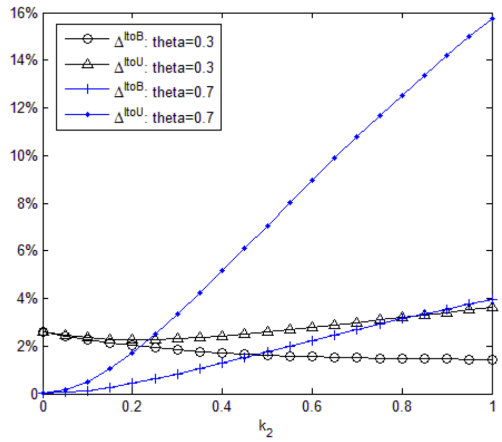
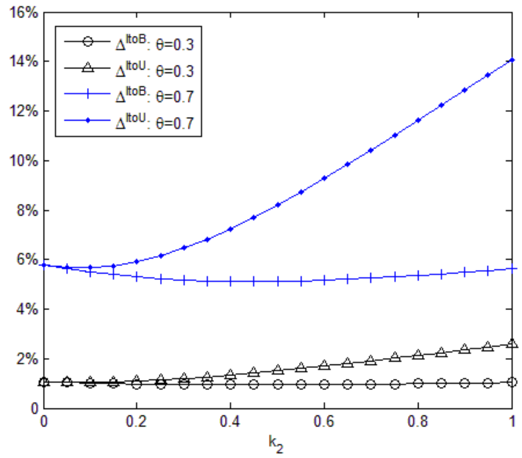


Figure 8. Comparison of channel profits as k_1 varies   Figure 9. Comparison of channel profit as k_2 varies when k_1=-0.5



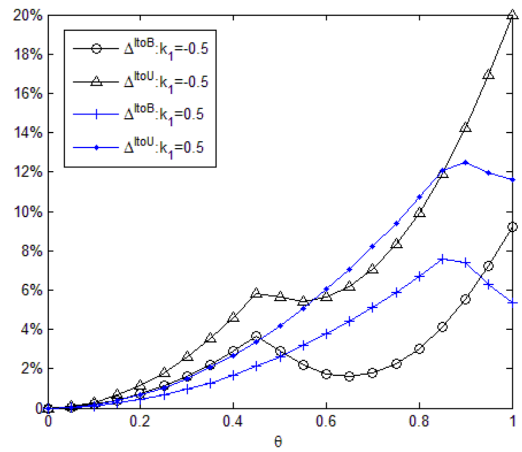


Figure 10. Comparison of channel profit as k_2 varies when k_1=0.5   Figure 11. Comparison of channel profits as θ varies

Figures 8-11 illustrate that the entire channel's profit in I-Model is higher than that in B-Model, which is in turn higher than that in U-Model. This result, combined with Theorem 4, shows again the advantage of bilateral participation over lateral participation from the perspective of channel overall performance. It is also shown that the profit improvement through channel integration can be increasing, decreasing, or zigzag with system parameters such as the inter-channel influence factors and the profit allocation of the online channel. Generally speaking, the gap between I-model and B-model is marginal when the agent's share of online channel profit is small, which implies that bilateral advertisement participation mechanism in O2O supply chain system can achieve a quite satisfactory system improvement when the seller is the dominant party for online sales. In contrast, Figures 9-11 show that the gap between I-Model and U-Model is particularly observable when the impact factor of the offline advertisement to online channel and the agent's proportion in the online channel profit are large. This result indicates the deficiency of unilateral cooperative advertisement as a performance improvement mechanism for the O2O dual-channel supply chain.

5. Conclusion

Cooperative advertising is a cost allocation mechanism widely adopted among supply chain members in a vertical supply chain. However, the prevalence of O2O business mode requires new research on the effect of the corresponding advertising strategies under consideration of the inter-channel relationship. This paper fills this gap by developing three cooperative advertising models including Integration co-op advertising (I-Model), Unilateral co-op advertising (U-Model), and Bilateral co-op advertising (B-Model), in an O2O supply chain consisting of a seller and an online agent. We derive the equilibrium solutions of the three models with the sensitivity analysis to the crucial system parameters, and provide comparison results on the effects of U-Model and B-Model in contrast with I-Model. We find that from the perspective of the entire supply chain performance, B-Model can lead to a result that is close to systematic optimality especially with a low online profit proportion for the agent. In contrast, U-Model provides the lowest channel profit among three models, which indicates unilateral co-op advertising is hardly effective as a channel coordination mechanism. However, it is shown that the rise of the seller's profit in B-Model is at the expense of the agent since the agent can earn more in U-Model without the participation in offline channel advertisement cooperation. This provides an explanation on why the bilateral participation is more of a theoretical co-op advertising scheme while its real use in industry is scarce and may need amending in specific terms (Zhang *et al.* 2013).

This paper can be regarded as a starting point for studying the supply chain advertising and marketing strategy in O2O era. Future researches include adding more factors such as pricing and service decisions into the model. A challenge in this regard is to obtain analytical results as the problems will become more complicated to resolve. In addition, we can further consider the issues including market demand randomness, information asymmetry between supply chain members, and multi-step dynamics. Finally, combining the problem with behavior factors such as strategic response of consumers and fairness concerns between supply chain members is also a promising future research direction, *e.g.*, following the research line of Yang *et al*. (2013).

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