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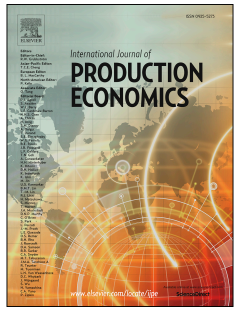

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Online-Offline Fashion Franchising Supply Chains without Channel Conflicts: Choices on Postponement and Contracts

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***Abstract* –** Online-offline operations are known to induce channel conflicts if the same products are offered by them. Under many franchising arrangements in the fashion industry, to avoid channel conflicts and cannibalization between the franchisee and the brand owner, the brand owner will first supply the product for the franchisee to sell offline in the first period. After that, the brand owner will sell the product online directly in the second period. We explore this online-offline model with the focal points on the choice of franchising contract and the ordering time. By modelling the choices under four different scenarios, we derive the analytical conditions in which one scenario is preferred to another scenario with respect to contract type and ordering time option. We examine the problem from the perspectives of the brand owner, the franchisee and the supply chain. We identify the situations in which the optimal choices of the brand owner and the supply chain are the same, as well as the conditions when Pareto improvement is achievable.

Keywords: Online-to-offline operations, O2O, franchise, supply chain management, wholesale pricing contract, profit sharing royalty, choice of contracts, information updating.

1. Introduction

Fashion retailing has stepped into the omni-channel retailing era. Online and offline operations of many fashion brands and international retailers are both well-developed. For example, Sears, one of the largest American fashion retailer, has invested heavily in information systems so that its online and offline operations are both enhanced and integrated together (Laudon and Laudon 2014). Uniqlo, the world's leading fashion brand originated from Japan, also employs the O2O strategy as the sharp tool for its rapid expansion in the China market. The core of Uniqlo's O2O strategy lies in leading the offline customer traffic to retail shops by the online services including new arrival promotion with the mobile social networking software (e.g., WeChat), APP coupons and big data analysis on consumer buying behaviours. Besides, JD[1], the leading e-commerce platform in China, has established the strategic cooperation with two famous sportswear brands, "Lining" and "Xstep", to integrate the online and offline channels and provide better experience to consumers with an improved logistics solution and a more effective inventory circulation.

A lot of fashion brands, in both fashion apparel and footwear (Khazaei Pool et al. 2016), are implementing franchise operations. Franchising is a licensing arrangement with which the franchisee can operate the brand as well as the retail format in a specific market. Fashion brands, like Bossini, are known to be expanding by relying heavily on franchising. Under the franchising arrangement, the fashion brand (the franchisor) will make a profit by some means, such as charging the franchisee a royalty or making a profit margin by the wholesale price. Some fashion brands also prefer to sell in a market offline via the franchisee, and online by its website directly.

In the China market, almost all fashion brands have established their e-commerce platforms such as TMall and JD. Meters/bonwe, one of the biggest casual wear brands in China, launches its online shopping platform www.banggo.com, which is also regarded as an interactive tool with the offline

[1] In this paper, we have included several real brands and companies, especially those in the China market, in the discussion. The details of these cases are mainly based on our discussions with the people in the industry as well as our own observations. Even though we believe that these case details should be true, some personal biases may inevitably be present. Readers should understand this point and interpret these cases with care.

shops. Moreover, many fashion brands work proactively to adopt the O2O approach in their operations. Bestseller Group, operating several popular fashion brands like Only, Vero Moda, Jack & Jones and Selected in China, develops their WeChat public account as the service platform for individual consumers. Customers can obtain personal and customized advices for product and matching recommendations, which may induce them to purchase in offline shops or online platforms. In order to avoid channel conflicts and cannibalization between the franchisee and the franchisor (i.e. the fashion brand), some additional arrangements need to be made. For instance, one rather commonly seen measure is: During the same selling season, the fashion brand and the franchisee will not sell the same product at the same time. For example, A.Yilian, one of the biggest young women's wear brands in China, develops special collections only for its online channels to avoid channel conflicts. Trendiano, a fashion men's wear brand, maintains different inventory portfolios for online and offline channels, respectively. Vip.com, the e-commerce platform operating in China, is specialized in post-selling season inventory sales. The platform is the strategic partner of many fashion brands for selling exclusive products that will not be sold through any other channels of the cooperating brands.

If we focus on one particular product item (e.g., a thick warm-keeping jacket), suppose that the fashion brand first supplies it to the franchisee and lets it sell offline in a market like Hong Kong during the winter (December). After the selling season in Hong Kong has ended, the fashion brand can sell its product online. Markets like Australia will have the winter in June and the fashion brand can sell to this market. In addition, some fashion brands apply O2O solutions to evade the problem of channel conflicts. For instance, PinkMary, the famous women's wear brand in Taiwan, shares the online orders with the franchisees. Once the online order is confirmed by the customer, the system will automatically release the details in the sharing platform. All the franchising shops in the region have the opportunity to grab the order and deliver the product to the customer. GXG, a fast-developing men's wear fashion brand in China, establishes an O2O interactive mechanism, especially for the "Singles' Day Shopping Festival". Customers can go to any shop to try on the products before the Singles' Day. If the customer finds the products satisfactory, he or she can pay the deposit first and then pay the balance on Singles' Day to enjoy the discount. The purchase will be delivered to the customer by express services. This kind of offline and online operations can avoid having the channel conflicts between the franchisor and the franchisee. Plus, potentially, the franchisor can also consider postponing its ordering decision and employing the demand information from the franchisee’s offline sales, to improve its demand forecast for the online market. However, under order postponement with a shortened lead time, the product cost is more expensive. So, a classic tradeoff between ordering cost and the forecast accuracy exists (Choi et al. 2003). As a remark, the situation considered in this paper is tricky and more challenging than Choi et al. (2003), because it involves two different sales channels and hence two different demands.

Moreover, for franchising contracts, the simplest format includes the pure wholesale pricing contract in which the fashion brand offers the franchise right to the franchisee, and makes a profit margin from the wholesale price. Another popular contract is the profit sharing contract. Under the profit sharing contract, we consider the case in which the fashion brand supplies the product at cost and then shares the profit of the franchisee. Based on prior studies, the profit sharing contract may be a more versatile contract compared to the wholesale pricing contract because it can dampen the double marginalization effect and improve profits of the supply chain members.

In this paper, motivated by the fact that offline-online operations are emerging as a critical part of fashion business, and avoiding channel conflicts is a critical issue, we explore the situation when channel conflicts are avoided as the product is sold online and offline in different seasons. Further motivated by the importance of information updating and the popularity of franchising arrangement in fashion retailing, we explore the optimal ordering time and the best franchising contract to choose for the fashion brand. Impacts on the supply chain are also explored. To the best of our knowledge, this paper is the first one which explores the online-offline mode of operations in this specific setting with franchising arrangement and information updating. The insights are important, especially to the fashion franchising operations, and the findings contribute to the literature.

The rest of this paper is structured as shown below. In Section 2, we review the related literature in three parts. In Section 3, we present the basic model which includes the supply chain structure and the information updating model. In Section 4, we introduce the two ordering cases (which relate to the optimal ordering time point) and the two franchise contracts. The resulting four scenarios are also defined in Section 4. In Section 5, we conduct the scenario analysis with the goal of highlighting when to choose which scenario and the respective optimal decision. In Section 6, we explore from the supply chain perspective how different scenarios affect the supply chain's performance. In Section 7, we conclude and discuss future research directions. To enhance presentation, we put all major technical proofs in Appendix (A1).

2. Literature Review

This paper relates to franchising, online-offline dual channel operations, and the use of information under postponement. We concisely review some related studies as follows.

Franchising is a very well established scheme in fashion business. It has been shown that it occupies one third of the retail sales in the US (Huang 1997). Under a typical franchising arrangement, the franchisor will issue a contract to the franchisee who is granted the right to operate the retail business under the franchisor's brand and sell its products. The specific franchise contract takes different forms which include the wholesale pricing contract (Zhao et al. 2014), the fixed lump sum contract, the performance based royalty contract, etc. In the literature, Huang (1997) is one of the very important papers which explore coordination challenges with the compensation plans as specified in the contracts. The author explores both the fixed lump sum fee and the performance based royalty contracts. They also reveal that channel coordination can be achieved by using a bargaining model. Shane et al. (2006) empirically explore how the new franchisor's partnering strategies affect the size of franchising system. One important result that is obtained by Shane et al. (2006) is that the franchisors which grow bigger tend to partner well and even finance their franchisees. In recent years, some papers are devoted to examining franchising in operations management. For instance, Babich and Tang (2015) study a franchise system which includes the franchisor, the entrepreneur and the bank. The authors formulate the problem as a sequential-move game. They analytically show that the franchise contracts favour the performance based royalty contract over the fixed lump sum fee contract. Based on the related literature in franchising operations, this paper also considers the franchising contracts. To be specific, we investigate two kinds of related contracts, namely the simplest wholesale pricing contract, and a performance based royalty contract called the profit sharing contract.

Online-offline operations are widely seen nowadays and have been known as an emerging trend in the fashion industry. In the literature, many studies explore how the dual channel strategies can be implemented in a supply chain context. For example, recently, Yan and Pei (2015) study the strategic value of cooperative advertisement in a dual channel system with competition. Taleizadeh et al. (2016) explore the impacts of marketing effort decisions on a dual channel closed loop supply chain. However, channel conflicts exist between the online and offline channels which would lead to serious problems which include harmful channel competition, losing profit margins and even the cannibalization problem. In operations management, Tsay and Agrawal (2004) pioneer an important study on the channel conflict and channel coordination issues when the manufacturer adds a direct sales channel online. The authors propose that a change of the supply contract might help dampen the channel conflicts. Luo et al. (2016) explore the free-riding effect in a dual-channel supply chain. In the presence of e-commerce, the authors analytically study the supply chain coordination challenge. Even though there are reports showing that the existence of dual channels can be beneficial to the supplier and the original retailer (e.g., Soysal and Krishnamurthi 2016), it is commonly known that franchisees usually do not prefer to have competition with the franchisor in the same market. This calls for including terms and measures in the franchise contracts to avoid channel conflicts. In this paper, we consider the situation under which the franchisor and the franchisee will adopt an operations mode where no channel conflicts exist.

In supply chain management, the use of information is a big topic which receives a lot of attention over the past several decades (Scarf 1959; Murray 1966; Azoury 1985; Bourland et al. 1996; Yue and Liu 2006; Mishra et al. 2009; Shaltayev and Sox 2010). In many cases, by postponing the final inventory decision, operational improvement can be made by using market information (Saghiri and Barnes 2016; Edirisinghe and Atkins 2017) which also helps to reduce risk (Asian and Nie 2014; Paul et al. 2017). Among the different related fields of studies, the use of market information to improve demand forecast via "information updating" is a very important and popular area (Gurnani and Tang (1999); Vlachos and Tagaras (2001); Choi et al.(2003, 2006); see the review by Choi and Sethi (2010) for more information). For example, based on the fashion industry's practices on accurate response and quick response, Hammond (1990), Fisher and Raman (1996), Iyer and Bergen (1997), Eppen and Iyer (1997a), Eppen and Iyer (1997b), Kim (2003), Tang et al. (2004), Choi (2007), and Cachon and Swinney (2011) all study the use of market information, usually with the concept of postponing the ordering decision time point, to improve inventory planning in fashion operations. They derive the optimal inventory policies under the respective setting and generate insights by examining how the use of information improves the supply chain performance and/or the measures to coordinate the channel. In recent few years, several papers have explored the use of market information in a quick response environment. For example, Lin and Parlarturk (2012) investigate the role played by quick response in a competitive market environment. Yang et al. (2015) study the quick response policy in the presence of strategic forward looking consumers. Choi (2016) investigates the impacts of inventory service targets on quick response fashion supply chains. Chen et al. (2016) reveal how the inventory subsidizing contract can be used to coordinate a just-in-time quick response supply chain with multiple shipments. Following the above stream of literature, this paper also studies the use of market information in improving demand forecast. Different from all of the above studies on quick response and information updating, we consider the franchising arrangement and the online-offline operations.

3. Basic Model

3.1. Supply Chain Structure

We consider a simple fashion supply chain with a fashion brand (B) which supplies to and grants a franchise right to the franchisee (F). The fashion brand operates an online-offline system in which its products are sold online and offline. However, to avoid channel conflicts, the fashion brand sells the same product to the consumers either online by itself or offline via the franchisee. Thus, the two channels do not sell the same product at the same time. This is a rather usual industrial practice under franchising arrangement.

Under this channel conflict avoidance strategy, we consider the case when the franchisee will get a fashionable product from the fashion brand at a time point called "Stage 0". Ordering at Stage 0 means the franchisee will definitely be able to get the ordered quantity when the season starts. The product's unit selling price in the market is p. By the end of the selling season, any leftover will be salvaged at a price v. Demand is uncertain and we will discuss its distribution later.

For the franchising business, the fashion brand supplies the product to the franchisee and makes a profit. There are two different, mutually exclusive franchise contracts being considered in this paper, namely the wholesale pricing contract and the profit sharing contract. For the wholesale pricing contract, the fashion brand offers a constant wholesale price which is higher than the product cost for each item supplied to the franchisee. Thus, the wholesale pricing contract is a simple one which guarantees that the fashion brand can make a certain profit margin (Shen et al. 2016). For the profit sharing contract (Wei and Choi 2010), the fashion brand supplies the product to the franchisee at cost (i.e. the wholesale price is the same as the product cost for the fashion brand). In order to make a profit, the fashion brand charges the franchisee a share of its profit generated by the product. In this paper, we employ the notation as shown in Table 3.1.

Table 3.1. Notation

Category	**Notation**	**Meaning**
Distribution	$f_N(X,Y)$	The normal distribution with mean X and

		variance Y.
	$\phi(\cdot)$	The standard normal density function
	$\Phi(\cdot)$	The standard normal cumulative distribution function
	$\Phi^{-1}(\cdot)$	The inverse function of $\Phi(\cdot)$
	$\Psi(x)$	The right linear loss function of the standard normal: $\Psi(x)=\int_x^\infty (z-x)d\Psi(z)$
Supply Chain & Its Members	B	The fashion brand
	F	The franchisee
	SC	The supply chain
Contracts	WP	Wholesale pricing
	PS	Profit sharing
Time Point	0	Stage 0
	1	Stage 1 (closer to the selling season)
Ordering Case	OC1	Ordering Case 1
	OC2	Ordering Case 2
Quantity	q_B	Order quantity by the fashion brand
	q_F	Order quantity by the franchisee

3.2. Demand Distributions and Information Updating

Owing to the lead time requirement, the franchisee has to order at Stage 0. However, since the fashion brand will sell the same product online after the selling season in the franchisee has finished, the fashion brand can consider the ordering time point. To be specific, the fashion brand can order at Stage 0, the same as what the franchisee does. At Stage 0, we model the demand distribution for the franchisee's market (which is offline) as follows:

$$x_F \sim f_N(\theta,\delta),$$

where x_F represents the random seasonal demand of the product in the offline market faced by the franchisee, $f_N(\theta,\delta)$ is the distribution of x_F with mean θ and variance δ.

Following Iyer and Bergen (1997), Choi et al. (2006) and Choi (2007), we further model θ (the mean of x_F) as a random variable and it follows a normal distribution with mean μ_0 and variance d_0,

$$\theta \sim f_N(\mu_0,d_0). \tag{3.1}$$

Thus, at Stage 0, the marginal distribution of x_F is given to be:

$$x_F \sim f_N(\mu_0, \sigma_0^2)\,, \tag{3.2}$$

where $\sigma_0 = \sqrt{d_0 + \delta}$. (3.3)

For the fashion brand, since it will sell the same product online after the franchisee's offline season is over, it will improve its forecast if it decides to order later and observe the market information from the sales of the franchisee. Following the linear demand relationship as proposed by Choi (2007), we consider the case when the demand faced by the fashion brand in the online market at Stage 0 ($x_{B,0}$) to be related to x_F as follows:

$$x_{B,0} \sim f_N(a\theta + b, k\delta)\,. \tag{3.4}$$

As θ is a random variable and its distribution is shown above in (3.1), we can derive the marginal demand distribution for $x_{B,0}$ as follows (see Choi 2007):

$$x_{B,0} \sim f_N(m_0, s_0^2)\,, \tag{3.5}$$

where $m_0 = a\mu_0 + b$, and $s_0 = \sqrt{a^2 d_0 + k\delta}$. (3.6)

For the fashion brand, if it decides to postpone its ordering decision to a time point Stage 1, market demand information from the franchisee's market will be observed. We denote the observation as $\tilde{x}_F$. Based on the Bayesian conjugate pair theory with the normal process with unknown mean and known variance (Pratt et al. 1995; Choi 2007), we have the demand distribution for the fashion brand's online sales at Stage 1 as follows:

$$x_{B,1} \sim f_N(m_1, s_1^2)\,, \tag{3.7}$$

where $m_1 = a\left\{\left(\dfrac{d_0}{d_0 + \delta}\right)\tilde{x}_F + \left(\dfrac{\delta}{d_0 + \delta}\right)\mu_0\right\} + b$, $d_1 = \delta\, d_0 /(d_0 + \delta)$, and $s_1 = \sqrt{a^2 d_1 + k\delta}$. (3.8)

Notice that in (3.8), m_1 represents the posterior demand mean which is a function of the prior mean μ_0 and the market observation $\tilde{x}_F$; s_1 is the posterior demand standard deviation. As a remark, the above Bayesian model has been well-established in the literature, and we just follow and use it in this study. For the theoretical background, please refer to Pratt et al. (1995).

4. Scenarios

4.1. Two Ordering Cases

Under our model setting, the franchisee F always orders at Stage 0 with the prior demand information whereas the fashion brand B may order earlier at Stage 0 or later at Stage 1. Thus, there are two ordering cases.

Case 1 refers to the case when the fashion brand B follows the franchisee F to order at Stage 0. Owing to the benefit of economy of scale, B can accumulate quantity (from F and itself) when the order is placed. Under this arrangement, the unit ordering cost for the product at Stage 0 is denoted by $\hat{c}_0$. Obviously, the benefit of adopting Case 1 ordering is to enjoy a lower unit ordering cost even though at the time of ordering, both the fashion brand and the franchisee do not make use of market information.

Case 2 is the case when the fashion brand postpones the ordering time and places the order at Stage 1 after observing the demand information in the market. Under Case 2, the unit ordering cost for the franchisee at Stage 0 is c_0 and the unit ordering cost for the fashion brand at Stage 1 is c_1. Following the usual situation in real world, the unit ordering cost is a decreasing function of lead time, and hence we have: $c_0 < c_1$. As $\hat{c}_0$ is smaller than c_0 owing to economy of scale, we have: $\hat{c}_0 < c_0 < c_1$. Table 4.1 summarizes the two ordering cases.

Table 4.1. The Two Ordering Cases

Cases	Ordering Time Point	Product Cost	Demand Uncertainty
Ordering Case 1 (OC1)	Fashion Brand: Stage 0	$\hat{c}_0$	$s_0 = \sqrt{a^2 d_0 + k\delta}$
	Franchisee: Stage 0	$\hat{c}_0$	$\sigma_0 = \sqrt{d_0 + \delta}$
Ordering Case 2 (OC2)	Fashion Brand: Stage 1	c_1	$s_1 = \sqrt{a^2 d_1 + k\delta}$
	Franchisee: Stage 0	c_0	$\sigma_0 = \sqrt{d_0 + \delta}$

4.2. Two Contract Options

In this paper, we consider two franchise contracts, which are commonly seen under a franchise arrangement in the fashion supply chain. The first one is the wholesale pricing (WP) contract. The WP contract is the simplest franchise contract in which the franchisee F pays a unit wholesale price to the fashion brand B for each unit of supply. We denote this unit wholesale price by w. Undoubtedly, the fashion brand B makes a profit margin by having w larger than the product ordering cost from the manufacturer.

The second franchise contract is the profit sharing (PS) contract (Wei and Choi 2010). Under the PS contract, we consider the situation in which the fashion brand B supplies the product to the franchisee F at cost (i.e., B does not make a profit from the wholesale price). In order for the fashion brand to make a reasonable profit, the fashion brand will receive a certain percentage of the franchisee's profit, and this percentage, denoted by λ, is well-written in the PS contract. Table 4.2 summarizes the details of the two franchise contracts.

Table 4.2. The Two Franchise Contracts

Contracts	**Contract Parameters and Characteristics**
Wholesale pricing (WP)	The unit wholesale price w, which is larger than the product cost. As this wholesale price is fixed, the fashion brand is guaranteed to make a profit.
Profit sharing (PS)	The product is supplied at cost and the fashion brand receives a proportion λ of the franchisee's profit. As the profit of the franchisee is random, the amount that the fashion brand receives is also random.

4.3. Four Scenarios

In the presence of the two ordering cases and the two franchise contracts, we have four probable scenarios, namely Scenario α, Scenario β, Scenario γ, and Scenario ξ, as shown in Table 4.3.

	Contracts	
	WP	**PS**

Ordering Cases	OC1	Scenario α	Scenario β
	OC2	Scenario γ	Scenario ξ

Table 4.3. A Table Showing the Four-Scenario Matrix Considered in this Paper.

In this paper, we aim to analytically explore and develop insights on when to choose which scenario among the four different scenarios. Notice that we do not aim to optimize the supply chain by adjusting contract parameters under each scenario. Instead, we take the contract for each scenario as given and explore the situation under which one scenario is preferred to the other scenario. The perspectives from the fashion brand, the franchisee as well as the whole supply chain will be examined.

5. Scenario Analysis: When to Choose Which Scenario?

5.1. Scenarios

(A) SCENARIO α

For the scenario under Ordering Case 1 with the WP contract, noting that the problem can be formulated as the standard newsvendor model setting. We can easily find that the expected profits for the franchisee and the fashion brand are given as follows:

$$\Pi_F^{\alpha}(q_F) = (p-v)\left[\mu_0 - \sigma_0 \Psi\left(\frac{q_F - \mu_0}{\sigma_0}\right)\right] - (w-v)q_F, \tag{5.1a}$$

$$\Pi_B^{\alpha}(q_B) = (p-v)\left[m_0 - s_0 \Psi\left(\frac{q_B - m_0}{s_0}\right)\right] - (\hat{c}_0 - v)q_B + (w-\hat{c}_0)q_F. \tag{5.1b}$$

Following the newsvendor model, it is straightforward to find that (5.1a) and (5.1b) are concave functions of q_F and q_B, respectively. Thus, the respective expected profit maximizing quantities for the fashion brand and the franchisee are found by solving the first order condition as follows:

$$q_{F*}^{\alpha} = \mu_0 + \sigma_0 \Phi^{-1}\left(\frac{p-w}{p-v}\right), \tag{5.2a}$$

$$q_{B*}^{\alpha} = m_0 + s_0 \Phi^{-1}\left(\frac{p - \hat{c}_0}{p - v}\right). \tag{5.2b}$$

As a remark, the terms $(p-w)/(p-v)$ in (5.2a) represents the inventory service level achieved by the ordering quantity q_{F*}^{α}. As the inventory service level in the real world should not be too low, we consider in this paper the situation when the inventory service level is above 50%. In other words, we assume that $(p-w)/(p-v) > 0.5$.

Putting (5.2a) and (5.2b) respectively into (5.1a) and (5.1b), it is easy to find that the optimal expected profit for the fashion brand and the franchisee under Scenario α are shown below:

$$\Pi_{B*}^{\alpha} = (p - \hat{c}_0)m_0 - s_0\left\{(\hat{c}_0 - v)\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right) + (p-v)\Psi\left[\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right)\right]\right\}$$
$$+ (w - \hat{c}_0)\left[\mu_0 + \sigma_0 \Phi^{-1}\left(\frac{p-w}{p-v}\right)\right], \tag{5.3}$$

$$\Pi_{F*}^{\alpha} = (p-w)\mu_0 - \sigma_0\left\{(w-v)\Phi^{-1}\left(\frac{p-w}{p-v}\right) + (p-v)\Psi\left[\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right]\right\}. \tag{5.4}$$

Notice that the expressions of (5.3) and (5.4) take the similar form as the ones derived in the literature (see, e.g., Iyer and Bergen (1997)). Define (5.5) and we have Lemma 5.1.

$$\Delta(x) = \left\{(x-v)\Phi^{-1}\left(\frac{p-x}{p-v}\right) + (p-v)\Psi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]\right\}. \tag{5.5}$$

Lemma 5.1. $\Delta(x) = (p-v)\phi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]$.

Proof of Lemma 5.1: All proofs are placed in the appendix.

Lemma 5.1 shows a compact form of an important term which will be used in many analytical derivations in subsequent sections. For the structural properties of $\Delta(x)$: First, it is positive. Second, since $\phi(z)$ is a decreasing function for any positive argument *z*, $\Delta(x)$ is an increasing function of *x* in the range when $\Phi^{-1}\left(\frac{p-x}{p-v}\right)$ is positive.

Using the result from Lemma 5.1, and the above analytical expressions, we have Corollary 5.1.

Corollary 5.1. *Under Scenario* α *, the optimal expected profits for the fashion brand and the franchisee are given as follows*:

$$\Pi_{B*}^{\alpha} = (p-\hat{c}_0)m_0 - s_0\Delta(\hat{c}_0) + (w-\hat{c}_0)\left[\mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right], \tag{5.6}$$

$$\Pi_{F*}^{\alpha} = (p-w)\mu_0 - \sigma_0\Delta(w). \tag{5.7}$$

From Corollary 5.1, we can see that the optimal expected profits for the fashion brand and the franchisee under Scenario α can be expressed in terms of $\Delta(\cdot)$, which is a function defined in (5.5). Notice that under Scenario α, the fashion brand enjoys a benefit of having the lowest unit product cost $\hat{c}_0$ and hence $\hat{c}_0$ appears in (5.6).

(B) SCENARIO γ

Under the scenario with Ordering Case 2 and the WP contract, we have Scenario γ. Similar to Scenario α, we can express the respective expected profits for the franchisee and the fashion brand in the following:

$$\Pi_{F}^{\gamma}(q_F) = (p-v)\left[\mu_0 - \sigma_0\Psi\left(\frac{q_F-\mu_0}{\sigma_0}\right)\right] - (w-v)q_F,$$

$$\Pi_{B}^{\gamma}(q_B) = (p-v)\left[m_1 - s_1\Psi\left(\frac{q_B-m_1}{s_1}\right)\right] - (c_1-v)q_B + (w-c_0)q_F.$$

The corresponding expected profit maximizing quantities for the fashion brand and the franchisee under Scenario γ can be found to be the following:

$$q_{B*}^{\gamma} \mid m_1 = m_1 + s_1\Phi^{-1}\left(\frac{p-c_1}{p-v}\right), \tag{5.8}$$

$$q_{F*}^{\gamma} = \mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right). \tag{5.9}$$

With (5.8) and (5.9), we can find the optimal expected profit for the fashion brand and the franchisee under Scenario γ to be the following:

$$\Pi_{B*}^{\gamma} \mid m_1 = (p-c_1)m_1 - s_1\Delta(c_1) + (w-c_0)q_{F*}^{\gamma}, \tag{5.10}$$

$$\Pi_{F*}^{\gamma} = (p-w)\mu_0 - \sigma_0\Delta(w). \tag{5.11}$$

Un-conditioning (5.10) yields:

$$\Pi_{B*}^{\gamma} = (p-c_1)m_0 - s_1\Delta(c_1) + (w-c_0)\left[\mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right]. \tag{5.12}$$

We summarize the findings in Corollary 5.2.

Corollary 5.2. *Under Scenario* γ*, the optimal expected profits for the fashion brand B and the franchisee F are*:

$$\Pi_{B*}^{\gamma} = (p-c_1)m_0 - s_1\Delta(c_1) + (w-c_0)\left[\mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right],$$

$$\Pi_{F*}^{\gamma} = \Pi_{F*}^{\alpha}.$$

From Corollary 5.2, we can see that the optimal expected profit for the franchisee under Scenario γ is the same as the one under Scenario α as its ordering is confirmed at Stage 0 and it pays the fashion brand a unit wholesale price *w* (which implies that the fashion brand makes a profit margin of $(w-c_0)$ for each unit ordered by the franchisee). However, for the fashion brand, it postpones the ordering decision so that it can observe market information and improve its forecast. However, in this case, the fashion brand has to pay a unit product cost c_1 which is also the highest one.

(C) SCENARIO β

Scenario β refers to the case with Ordering Case 1 and the PS contract. We can express the expected profits for the franchisee and the fashion brand in the following:

$$\Pi_F^{\beta}(q_F) = (1-\lambda)\left\{(p-v)\left[\mu_0 - \sigma_0\Psi\left(\frac{q_F-\mu_0}{\sigma_0}\right)\right] - (w-v)q_F\right\},$$

$$\Pi_B^{\beta}(q_B) = (p-v)\left[m_0 - s_0\Psi\left(\frac{q_B-m_0}{s_0}\right)\right] - (\hat{c}_0 - v)q_B +$$

$$\lambda\left\{(p-v)\left[\mu_0 - \sigma_0\Psi\left(\frac{q_F-\mu_0}{\sigma_0}\right)\right] - (w-v)q_F\right\}.$$

The expected profit maximizing quantities for the franchisee and the fashion brand can be derived to be the following:

$$q_{F*}^{\beta} = \mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right), \tag{5.13}$$

$$q_{B*}^{\beta} = m_0 + s_0\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right). \tag{5.14}$$

With (5.13) and (5.14), we can easily derive the optimal expected profits for the fashion brand and the franchisee under Scenario β below:

$$\Pi_{B*}^{\beta} = (p-\hat{c}_0)m_0 - s_0\Delta(\hat{c}_0) + \lambda\overline{\Pi}_{F*}^{\beta}, \tag{5.15}$$

$$\overline{\Pi}_{F*}^{\beta} = (p-\hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0) \;. \tag{5.16}$$

$$\Pi_{F*}^{\beta} = (1-\lambda)\overline{\Pi}_{F*}^{\beta} \;. \tag{5.17}$$

We have Corollary 5.3.

Corollary 5.3. *Under Scenario* β*, the optimal expected profits for the franchisee F and the fashion brand B are given as follows*:

$$\Pi_{F*}^{\beta} = (1-\lambda)\{(p-\hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0)\},$$

$$\Pi_{B*}^{\beta} = (p-\hat{c}_0)m_0 - s_0\Delta(\hat{c}_0) + \lambda[(p-\hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0)].$$

Similar to Corollary 5.1, we can see that under Scenario β, the fashion brand commits the ordering quantities (for itself and the franchisee) at Stage 0. Thus, the unit product cost is the lowest one (i.e. $\hat{c}_0$). As the fashion brand's supply business to the franchisee is based on the profit sharing contract under Scenario β, it is interesting to note that the franchisee's expected profit also depends on $\hat{c}_0$, which means it enjoys the lowest product cost in getting the supply.

(D) SCENARIO ξ

The last scenario refers to the case with Ordering Case 2 and the PS contract and we call it Scenario ξ. Similar to other scenarios, the expected profits for the franchisee and the fashion brand are listed in the following:

$$\Pi_F^{\xi}(q_F) = (1-\lambda)\left\{(p-v)\left[\mu_0 - \sigma_0\Psi\left(\frac{q_F-\mu_0}{\sigma_0}\right)\right] - (c_0-v)q_F\right\},$$

$$\Pi_B^{\xi}(q_B) = (p-v)\left[m_1 - s_1\Psi\left(\frac{q_B-m_1}{s_1}\right)\right] - (c_1-v)q_B +$$

$$\lambda\left\{(p-v)\left[\mu_0 - \sigma_0\Psi\left(\frac{q_F-\mu_0}{\sigma_0}\right)\right] - (c_0-v)q_F\right\}.$$

The respective expected profit maximizing quantities for the fashion brand and the franchisee under Scenario ξ can be found to be the following:

$$q_{B^*}^{\xi} \mid m_1 = m_1 + s_1\Phi^{-1}\left(\frac{p-c_1}{p-v}\right), \tag{5.18}$$

$$q_{F^*}^{\xi} = \mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-c_0}{p-v}\right). \tag{5.19}$$

Define:

$$\overline{\Pi}_{F^*}^{\xi} = (p-c_0)\mu_0 - \sigma_0\Delta(c_0). \tag{5.20}$$

With (5.18), (5.19) and (5.20), we can find the optimal expected profit for the franchisee and the fashion brand under Scenario ξ to be the following:

$$\Pi_{F^*}^{\xi} = (1-\lambda)\overline{\Pi}_{F^*}^{\xi},$$

$$\Pi_{B^*}^{\xi} \mid m_1 = (p-c_1)m_1 - s_1\Delta(c_1) + \lambda\overline{\Pi}_{F^*}^{\xi}. \tag{5.21}$$

Un-conditioning (5.21) yields:

$$\Pi^{\xi}_{B*} = (p-c_1)m_0 - s_1\Delta(c_1) + \lambda\overline{\Pi}^{\xi}_{F*}. \tag{5.22}$$

We summarize the findings in Corollary 5.4.

Corollary 5.4. *Under Scenario* ξ*, the optimal expected profits for the fashion brand B and the franchisee F are*:

$$\Pi^{\xi}_{B*} = (p-c_1)m_0 - s_1\Delta(c_1) + \lambda[(p-c_0)\mu_0 - \sigma_0\Delta(c_0)],$$

$$\Pi^{\xi}_{F*} = (1-\lambda)[(p-c_0)\mu_0 - \sigma_0\Delta(c_0)].$$

In Corollary 5.4, observe that even though the franchisee places the order at Stage 0, as the fashion brand places its own order at Stage 1, the unit product cost for the franchisee is only c_0, but not the lowest one (i.e., $\hat{c}_0$). This means compared to Scenario β, the franchisee enjoys a smaller "product cost advantage" under Scenario ξ.

5.2. Scenario Analysis – Fashion Brand's Perspective

(A) OPTIMAL ORDERING CASE

We have two ordering cases under two contracts. To compare choices on the ordering case, we first explore Scenario α versus Scenario γ, which are both using the WP contract. By directly comparing the expected profits for the fashion brand under these two scenarios, we have Proposition 5.1.

Proposition 5.1. *In the presence of the WP contract, in deciding the optimal ordering case, the fashion brand will prefer Scenario* α *(OC1) to Scenario* γ *(OC2) if and only if*

$(c_1-\hat{c}_0)m_0 + (c_0-\hat{c}_0)\left\{\mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right\} \geq s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$*; otherwise, the fashion brand will prefer Scenario* γ *(OC2) to Scenario* α *(OC1).*

Proposition 5.1 is intuitive and the results are based on the tradeoff between the product cost advantage and the demand uncertainty reduction advantage. To be specific, when the product cost savings (i.e., $(c_1 - \hat{c}_0)$, $(c_0 - \hat{c}_0)$) are sufficiently big compared to the demand uncertainty reduction (as reflected by $s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$), Scenario α is preferred to Scenario γ which means Ordering Case 1 (at Stage 0) is more beneficial. If the product cost savings are relatively small with respect to the demand uncertainty reduction, Scenario γ is the more preferred option.

For the cases with the PS contract, to reveal the optimal choice on the ordering case, we explore Scenario β versus Scenario ξ. By checking the corresponding expected profits for the fashion brand under these two scenarios, we get Proposition 5.2.

Proposition 5.2. *In the presence of the PS contract, in deciding the optimal ordering case, the fashion brand will prefer Scenario* β *(OC1) to Scenario* ξ *(OC2) if and only if* $(c_1 - \hat{c}_0)m_0 + \lambda(c_0 - \hat{c}_0)\mu_0 \geq (s_0 + \lambda\sigma_0)\Delta(\hat{c}_0) - (s_1 - \lambda\sigma_0)\Delta(c_1)$ *; otherwise, the fashion brand will prefer Scenario* ξ *(OC2) to Scenario* β *(OC1).*

Similar to the findings in Proposition 5.1 (for the case with the WP contract), Proposition 5.2 shows the tradeoff between the product cost advantage and the demand uncertainty reduction. As a remark, the profit sharing rate λ also plays a critical role as shown in the analytical condition in Proposition 5.2.

(B) OPTIMAL CONTRACT

Next, we consider the optimal choice on contracts. We have two contracts and two ordering cases. To compare choices on the franchise contract, we compare Scenario α (with the WP contract) versus Scenario β (with the PS contract), which are both under Ordering Case 1. To enhance presentation, we define the following: $J(x) = \dfrac{(w-x)Q(w)}{(p-x)\mu_0 - \sigma_0\Delta(x)}$. By directly comparing the expected profits for the fashion brand under these two respective scenarios, we have Proposition 5.3.

As a remark, when we compare between the WP and PS contracts, as the fashion brand does not face any uncertainty (and hence has no risk) under the WP contract, whenever the expected profit it earns under the WP contract is equal to or larger than the expected profit it earns under the PS contract, the WP contract will be the more preferred choice.

Proposition 5.3. *Under Ordering Case 1, in deciding the optimal contract, the fashion brand will prefer Scenario* α *(WP contract) to Scenario* β *(PS contract) if and only if* $\lambda \le J(\hat{c}_0)$ *;otherwise, the fashion brand will prefer Scenario* β *(PS contract) to Scenario* α *(WP contract).*

Proposition 5.3 shows that if the profit sharing rate λ is relatively small compared to the unit wholesale price, the WP contract is preferred. If the profit sharing rate is sufficiently big, the PS contract will be the fashion brand's optimal choice.

Under Ordering Case 2, to compare choices on the franchise contract, we compare Scenario γ (with the WP contract) versus Scenario ξ (with the PS contract), which are both under Ordering Case 1. By comparing the respective expected profits for the fashion brand under Scenario γ and Scenario ξ, we have Proposition 5.4.

Proposition 5.4. *Under Ordering Case 2, in deciding the optimal contract, the fashion brand will prefer Scenario* γ *(WP contract) to Scenario* ξ *(PS contract) if and only if* $\lambda \le J(c_0)$ *; otherwise, the fashion brand will prefer Scenario* ξ *(PS contract) to Scenario* γ *(WP contract).*

Proposition 5.4 indicates when it is optimal to choose the PS contract or the WP contract under Ordering Case 2. The result is consistent with our intuition and the findings in Proposition 5.3 where depending on the value of profit sharing rate λ, an optimal choice can be made.

For the sake of notational simplicity, we define:

$$\Omega(w) = \Phi^{-1}\left(\frac{p-w}{p-v}\right), \text{ and} \tag{5.23}$$

$$Q(w) = \mu_0 + \sigma_0 \Omega(w) . \tag{5.24}$$

Table 5.1 summarizes the necessary and sufficient conditions, as revealed by Propositions 5.1 to 5.4, for the fashion brand to prefer one scenario to another one.

Table 5.1. Scenario Preferences and Optimal Choices: From the Perspective of the Fashion Brand

Choices	Optimal Choices	Scenario Preferences	Necessary and Sufficient Conditions
Ordering Cases	OC1	$\alpha \succ \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) \geq s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$
	OC2	$\alpha \prec \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) < s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$
	OC1	$\beta \succ \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + \lambda(c_0 - \hat{c}_0)\mu_0 \geq (s_0 + \lambda\sigma_0)\Delta(\hat{c}_0) - (s_1 - \lambda\sigma_0)\Delta(c_1)$
	OC2	$\beta \prec \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + \lambda(c_0 - \hat{c}_0)\mu_0 < (s_0 + \lambda\sigma_0)\Delta(\hat{c}_0) - (s_1 - \lambda\sigma_0)\Delta(c_1)$
Contracts	WP	$\alpha \succ \beta$ (under OC1)	$\lambda \leq J(\hat{c}_0)$
	PS	$\alpha \prec \beta$ (under OC1)	$\lambda > J(\hat{c}_0)$
	WP	$\gamma \succ \xi$ (under OC2)	$\lambda \leq J(c_0)$
	PS	$\gamma \prec \xi$ (under OC2)	$\lambda > J(c_0)$

5.3. Scenario Analysis – Franchisee's Perspective

(A) OPTIMAL ORDERING CASE

In Section 5.2, we have examined the scenarios from the fashion brand's perspective. We now proceed to examine the scenarios from the franchisee's perspective. Adopting the similar approach, to compare choices on the ordering case, we first investigate Scenario α versus Scenario γ, which are both using the WP contract. Comparing the franchisee's expected profits under these two scenarios, we have Proposition 5.5.

Proposition 5.5. *In the presence of the WP contract, for the optimal ordering case, the franchisee is indifferent between Scenario α (OC1) and Scenario γ (OC2).*

Proposition 5.5 is a direct result from the fact that the expected profits of the franchisee under Scenario α (OC1) and Scenario γ (OC2) are the same.

Under the PS contract, we explore Scenario β versus Scenario ξ to reveal the optimal ordering case from the franchisee's perspective. By checking the respective franchisee's expected profits under these two scenarios, we yield Proposition 5.6.

Proposition 5.6. *In the presence of the PS contract, in deciding the optimal ordering case, the franchisee will always prefer Scenario β (OC1) to Scenario ξ (OC2).*

Proposition 5.6 is a strong and clear finding. For any given PS contract with a fixed profit sharing rate, because the wholesale price under OC1 is lower than OC2, the franchisee's expected profit under OC1 is larger than the OC2 counterpart. Thus, OC1 is always preferred for any given PS contracts.

(B) OPTIMAL CONTRACT

After considering the optimal ordering case for the franchisee, we now examine the optimal choice on contracts. To compare choices on the franchise contract, we compare Scenario α (WP contract) versus Scenario β (PS contract), which are both under OC1. For a notational purpose, we define the following:

$T(\hat{c}_0) = [(w+\hat{c}_0)\mu_0 + \sigma_0(\Delta(w) - \Delta(\hat{c}_0))]/[(p-\hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0)]$.

By directly comparing the franchisee's expected profits under these two scenarios, we have Proposition 5.7.

Proposition 5.7. *Under OC1, in deciding the optimal contract, the franchisee will prefer Scenario α (WP contract) to Scenario β (PS contract) if and only if $\lambda > T(\hat{c}_0)$; otherwise, the franchisee will prefer Scenario β (PS contract) to Scenario α (WP contract).*

Proposition 5.7 shows that if λ, the profit sharing rate for the fashion brand, is sufficiently big, the WP contract is preferred from the perspective of the franchisee. If the fashion brand's profit sharing rate is sufficiently small, the PS contract will be the franchisee's optimal choice. This finding is intuitive and reasonable.

Under OC2, to find the optimal franchise contract, we compare Scenario γ (WP contract) versus Scenario ξ (PS contract). By comparing the respective expected profits for the franchisee under Scenario γ and Scenario ξ, we have Proposition 5.8.

Proposition 5.8. *Under OC 2, in deciding the optimal contract, the franchisee will prefer Scenario γ (WP contract) to Scenario ξ (PS contract) if and only if $\lambda > T(c_0)$; otherwise, the franchisee will prefer Scenario ξ (PS contract) to Scenario γ (WP contract).*

Proposition 5.8 is similar to Proposition 5.7, and the interpretation is similar. In short, if the profit sharing rate (for the fashion brand) is sufficiently small, the PS contract is preferred by the franchisee; otherwise, the WP contract is preferred.

Table 5.2 summarizes the findings on scenario preferences and optimal choices from Propositions 5.5 to 5.8. It is interesting to observe that for the optimal decision on "ordering case", the franchisee faces a very simple decision making problem: (i) Under the WP contract, both OC1 and OC2 are equally good and hence the franchisee can choose either one of them without any trouble. (ii) Under the PS contract, OC1 is always preferred to OC2, which is a straightforward decision.

Table 5.2. Scenario Preferences and Optimal Choices: From the Perspective of the Franchisee

Choices	**Optimal Choices**	**Scenario Preferences**	**Necessary and Sufficient Conditions**
Ordering Cases	OC1	$\alpha \succ \gamma$ (under WP)	Never happens
	OC2	$\alpha \prec \gamma$ (under WP)	Never happens
	OC1 and OC2 are equally good	$\alpha = \gamma$ (under WP)	Always
	OC1	$\beta \succ \xi$ (under PS)	Always
	OC2	$\beta \prec \xi$ (under PS)	Never happens
Contracts	WP	$\alpha \succ \beta$ (under OC1)	$\lambda > T(\hat{c}_0)$
	PS	$\alpha \prec \beta$ (under OC1)	$\lambda < T(\hat{c}_0)$
	WP	$\gamma \succ \xi$	$\lambda > T(c_0)$

		(under OC2)	
	PS	$\gamma \prec \xi$ (under OC2)	$\lambda < T(c_0)$

6. Supply Chain Systems Analysis

6.1. Supply Chain Best Scenarios

In Section 5, we have explored the four scenarios and derived the conditions under which one scenario is preferred to another one under the choice either on the ordering case or the franchise contract.

However, the perspective of decision making is only from the fashion brand's or the franchisee's perspective, but not considering the whole supply chain system. In this section, we explore how the fashion brand's choices on the scenarios would affect the supply chain's performance.

By definition, under each scenario, the supply chain's expected profit is equal to the sum of the expected profits of the fashion brand and the franchisee. We thus have:

$$\Pi_{SC}^{i} = \Pi_{B*}^{i} + \Pi_{F*}^{i}, \text{ where } i = \{\alpha, \beta, \gamma, \xi\}. \tag{6.1}$$

Following the definition as given by (6.1), Table 6.1 shows the analytical expressions of Π_{SC}^{i} for all scenarios.

Table 6.1. Supply Chain Expected Profits under Different Scenarios

Scenarios	Supply Chain Expected Profits
α	$\Pi_{SC}^{\alpha} = (p-\hat{c}_0)m_0 - s_0\Delta(\hat{c}_0) + \sigma_0\left[(w-\hat{c}_0)\Omega(w) - \Delta(w)\right] + (p-w)\mu_0$
β	$\Pi_{SC}^{\beta} = (p-\hat{c}_0)(m_0+\mu_0) - (s_0+\sigma_0)\Delta(\hat{c}_0)$
γ	$\Pi_{SC}^{\gamma} = (p-c_1)m_0 - s_1\Delta(c_1) + \sigma_0\left[(w-c_0)\Omega(w) - \Delta(w)\right] + (p-c_0)\mu_0$
ξ	$\Pi_{SC}^{\xi} = (p-c_1)m_0 + (p-c_0)\mu_0 - s_1\Delta(c_1) - \sigma_0\Delta(c_0)$

From the supply chain's perspective, Table 6.2 shows the necessary and sufficient condition for the supply chain to prefer one scenario to another one. Combining Table 5.1 and Table 6.2, we have the necessary and sufficient condition in which the optimal choice is the best for both the fashion brand and the supply chain system. The results are summarized in Table 6.3 and we have Proposition 6.1.

Table 6.2. Scenario Preferences and Optimal Choices: From the Perspective of the Supply Chain

Choices	Optimal Choices	Scenario Preferences	Necessary and Sufficient Conditions
Ordering Cases	OC1	$\alpha \succ \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (w - c_0)\mu_0 + s_1\Delta(c_1) - s_0\Delta(\hat{c}_0) + \sigma_0(c_0 - \hat{c}_0)\Omega(w) \geq 0$
	OC2	$\alpha \prec \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (w - c_0)\mu_0 + s_1\Delta(c_1) - s_0\Delta(\hat{c}_0) + \sigma_0(c_0 - \hat{c}_0)\Omega(w) < 0$
	OC1	$\beta \succ \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + (p - \hat{c}_0)\mu_0 + s_1\Delta(c_1) - s_0\Delta(\hat{c}_0) - \sigma_0(w - \hat{c}_0)\Omega(w) + \sigma_0[\Delta(w) - \Delta(\hat{c}_0)] \geq 0$
	OC2	$\beta \prec \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + (p - \hat{c}_0)\mu_0 + s_1\Delta(c_1) - s_0\Delta(\hat{c}_0) - \sigma_0(w - \hat{c}_0)\Omega(w) + \sigma_0[\Delta(w) - \Delta(\hat{c}_0)] < 0$
Contracts	WP	$\alpha \succ \beta$ (under OC1)	$(w - \hat{c}_0)\left[\sigma_0\Omega(w) - \mu_0\right] - \sigma_0[\Delta(w) - \Delta(\hat{c}_0)] \geq 0$
	PS	$\alpha \prec \beta$ (under OC1)	$(w - \hat{c}_0)\left[\sigma_0\Omega(w) - \mu_0\right] - \sigma_0[\Delta(w) - \Delta(\hat{c}_0)] < 0$
	WP	$\gamma \succ \xi$ (under OC2)	$(w - c_0)\ \Omega(w) - [\Delta(w) - \Delta(c_0)] \geq 0$
	PS	$\gamma \prec \xi$ (under OC2)	$(w - c_0)\ \Omega(w) - [\Delta(w) - \Delta(c_0)] < 0$

Table 6.3. Scenario Preferences and Optimal Choices which are the Best for both the Supply Chain and the Fashion Brand

Choices	Optimal Choices	Scenario Preferences	Necessary and Sufficient Conditions
Ordering Cases	OC1	$\alpha \succ \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) \geq s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$ (same as the one for the fashion brand).
	OC2	$\alpha \prec \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) < s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$ (same as the one for the fashion brand).
	OC1	$\beta \succ \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + \lambda(c_0 - \hat{c}_0)\mu_0 \geq (s_0 + \lambda\sigma_0)\Delta(\hat{c}_0) - (s_1 - \lambda\sigma_0)\Delta(c_1)$ (same as the one for the fashion brand).
	OC2	$\beta \prec \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + (p - \hat{c}_0)\mu_0 + s_1\Delta(c_1) - s_0\Delta(\hat{c}_0) - \sigma_0(w - \hat{c}_0)\Omega(w) + \sigma_0[\Delta(w) - \Delta(\hat{c}_0)] < 0$ and $(c_1 - \hat{c}_0)m_0 + \lambda(c_0 - \hat{c}_0)\mu_0 < (s_0 + \lambda\sigma_0)\Delta(\hat{c}_0) - (s_1 - \lambda\sigma_0)\Delta(c_1)$
Contracts	WP	$\alpha \succ \beta$ (under OC1)	$\lambda \leq \frac{(w - \hat{c}_0)Q(w)}{(p - \hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0)}$ and $(w - \hat{c}_0)\left[\sigma_0\Omega(w) - \mu_0\right] - \sigma_0[\Delta(w) - \Delta(\hat{c}_0)] \geq 0$

	PS	$\alpha \prec \beta$ (under OC1)	$\lambda > \frac{(w-\hat{c}_0)Q(w)}{(p-\hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0)}$ and $(w-\hat{c}_0)\left[\sigma_0\Omega(w)-\mu_0\right]-\sigma_0[\Delta(w)-\Delta(\hat{c}_0)]<0$
	WP	$\gamma \succ \xi$ (under OC2)	$\lambda \le \frac{(w-c_0)Q(w)}{(p-c_0)\mu_0 - \sigma_0\Delta(c_0)}$ and $(w-c_0)\ \Omega(w)-[\Delta(w)-\Delta(c_0)]\ge 0$
	PS	$\gamma \prec \xi$ (under OC2)	$\lambda > \frac{(w-c_0)Q(w)}{(p-c_0)\mu_0 - \sigma_0\Delta(c_0)}$ and $(w-c_0)\ \Omega(w)-[\Delta(w)-\Delta(c_0)]<0$

Proposition 6.1. *In deciding the optimal ordering case under the WP contract: Comparing between Scenario α and Scenario γ, if it is optimal for the fashion brand to choose Scenario l, for $l \in (\alpha,\gamma)$, it will also be the optimal scenario for the supply chain. Under the PS contract, if it is optimal for the fashion brand to choose OC1, it will also be optimal for the supply chain.*

Proposition 6.1 shows that for the ordering case optimization problem in the presence of the WP contract, the fashion brand's optimal ordering case decision is consistent with the supply chain's optimal decision. This finding hence shows the beauty behind the WP contract: It is not only a simple contract easy to implement, in our model, we also achieve the consistency between the optimal ordering case decisions of the fashion brand and the supply chain. Under the PS contract, the "consistency" situation between the optimal ordering case choices of the fashion brand and the supply chain occurs only for the case when it is optimal for the fashion brand to choose OC1 but not OC2. This shows that a natural difference usually exists between the fashion brand's optimal choice and the supply chain's under the PS contract. For the other cases, the respective necessary and sufficient conditions need to be examined. Table 6.3 shows the respective analytical conditions.

6.2. Pareto Improving Scenarios

In Section 6.1, we look at the necessary and sufficient conditions for the optimal choices of the supply chain, and find the common conditions with which the optimal choices of the fashion brand and the supply chain are the same. However, an optimal choice for the fashion brand and the supply chain need not always benefit the franchisee. In this section, we examine the Pareto improving conditions for the choices (and scenario preferences). Here, we say that a choice is Pareto improving when after taking it, both the fashion brand and the franchisee are either both strictly benefited (i.e. win-win), or at least one of them is strictly benefited and the other is not worse off in expected profit.

To establish this result, we make use of the findings from Tables 5.1 and 5.2. The result is summarized in Table 6.4 and we have Proposition 6.2.

Table 6.4. Scenario Preferences and Choices which are Pareto Improving

Choices	**Optimal Choices**	**Scenario Preferences**	**Necessary and Sufficient Conditions**
Ordering Cases	OC1	$\alpha \succ \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) \geq s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$ (same as the one for the fashion brand).
	OC2	$\alpha \prec \gamma$ (under WP)	$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) < s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$ (same as the one for the fashion brand).
	OC1	$\beta \succ \xi$ (under PS)	$(c_1 - \hat{c}_0)m_0 + \lambda(c_0 - \hat{c}_0)\mu_0 \geq (s_0 + \lambda\sigma_0)\Delta(\hat{c}_0) - (s_1 - \lambda\sigma_0)\Delta(c_1)$ (same as the one for the fashion brand).
	OC2	$\beta \prec \xi$ (under PS)	Never happens
Contracts	WP	$\alpha \succ \beta$ (under OC1)	$T(\hat{c}_0) < \lambda \leq J(\hat{c}_0)$
	PS	$\alpha \prec \beta$ (under OC1)	$J(\hat{c}_0) < \lambda < T(\hat{c}_0)$
	WP	$\gamma \succ \xi$ (under OC2)	$T(c_0) < \lambda \leq J(c_0)$
	PS	$\gamma \prec \xi$ (under OC2)	$J(c_0) < \lambda < T(c_0)$

Proposition 6.2. (a) *In deciding the optimal ordering case under the WP contract: Comparing between Scenario* α *and Scenario* γ, *if it is optimal for the fashion brand to choose Scenario* l, *for* $l \in (\alpha, \gamma)$, *it will be a Pareto improving scenario.* (b) *In deciding the optimal ordering case under the PS contract: Only OC1 can be a Pareto improving scenario and OC2 is never Pareto improving.* (c) *In deciding the optimal contract, the Pareto improving condition depends on the profit sharing rate.*

Pareto improvement can be achieved only when the profit sharing rate is neither not too high nor too small.

Proposition 6.2 shows that for the ordering case selection problem in the presence of the WP contract, the fashion brand's optimal choice is also a Pareto improving choice. From Proposition 6.1, this is also an optimal choice for the decentralized supply chain. So, both propositions illustrate the nice feature of the WP contract in the optimal ordering case decision. However, for the case with the PS contract, this Pareto improving situation only appears for OC1, but not for OC2 because the franchisee always suffers under OC2. For the contract selection problem, when the profit sharing rate (for the fashion brand) is bounded in the range as specified in the respective case in Table 6.3, Pareto improvement can be achieved. This result is very intuitive because a very big profit sharing rate benefits the fashion brand but hurts the franchisee, whereas a very small profit sharing rate hurts the fashion brand but benefits the franchisee. So, Pareto improvement appears only when the profit sharing rate is neither too big nor too small.

7. Conclusion and Future Research

Motivated by the importance of online-offline operations and the problems associated with channel conflicts with dual channels under franchising arrangement, we have examined in this paper a fashion franchising supply chain in which no channel conflicts exist and the franchisor may make use of the franchisee's demand information to improve its own inventory planning. To be specific, we have studied the case in which the fashion brand (i.e. the franchisor) first supplies the product for the franchisee to sell offline in the first period. After that, the fashion brand will sell the same product online in the second period. The fashion brand can choose to order the product for its own online channel at the same time as the franchisee so that the unit ordering cost is lower (from economy of scale). Alternatively, the fashion brand can choose to postpone its ordering decision to a later time point so that it can improve its demand forecast and reduce demand uncertainty. A tradeoff hence exists between ordering cost and forecast accuracy.

For the optimization problems, we have focused on exploring the choice of franchising contract and the ordering time. We have modelled the choices under four different scenarios, and derived the analytical closed-form conditions in which one scenario is preferred to another scenario with respect to contract type and ordering time option.

In the explorations, we have examined the scenario choices from the perspectives of the fashion brand owner, the franchisee and the supply chain. We have identified the situations in which the optimal choices of the fashion brand and the supply chain are consistent. In particular, from Proposition 6.1 and Proposition 6.2, we have shown that under the WP contract, the optimal ordering case decisions among the fashion brand, the franchisee and the supply chain are consistent. However, this situation is not always true under the PS contract. This result is a bit counter-intuitive because it shows that the simple WP contract is in fact capable of achieving Pareto improvement for the optimal ordering case decisions.

We have further uncovered the conditions and cases in which Pareto improvement appears. All the conditions are derived in closed form and hence provide theoretically solid and practical guidance to decision makers. Notice that various insights have been derived and reported in the proposed corollaries and propositions. The managerial implications and tradeoffs have also been elaborated in the respective sections.

For future research, one may consider other channel conflicts avoiding measures. For instance, one may consider the case when the fashion brand offers multiple products and each channel is responsible for different related but not the same products. One may also consider other channel integration measures, such as ordering online and picking up in store operations and the corresponding incentive alignment schemes. This paper considers only one franchisee while in general, a supply chain usually includes several franchisees. In such a situation, issues such as inventory allocation (Somarin et al. 2016, 2017a and 2017b) and contracting mechanisms need deeper explorations. In this paper, we have not considered the existence of external competitors who might observe the supply chain's actions and make other moves. This is an important issue and we relegate it to future research. Last but not least, in this paper, we focus on examining the decentralized supply chain operations from the fashion brand's perspective, and hence we have not considered the important supply chain coordination challenge (Xu et al. 2010; Wu 2013; Asian and Nie 2014; Pfeiffer 2016). Future research can hence be conducted to investigate the channel coordination issue.[2]

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Appendix (A1): Technical Proofs

Proof of Lemma 5.1:

From (5.5), we have:

$$\Delta(x) = \left\{ (x-v)\Phi^{-1}\left(\frac{p-x}{p-v}\right) + (p-v)\Psi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]\right\}. \tag{A1}$$

Noting that $\Psi(y) = \phi(y) - y(1-\Phi(y))$, we can hence rewrite $\Psi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]$ as follows;

$$\Psi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right] = \phi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right] - \left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]\left[1-\Phi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]\right]$$

$$= \phi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right] - \left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]\left(\frac{x-v}{p-v}\right). \tag{A2}$$

Putting (A2) into (A1) and after simplification, we have: $\Delta(x) = (p-v)\phi\left[\Phi^{-1}\left(\frac{p-x}{p-v}\right)\right]$. (Q.E.D.)

Proof of Corollary 5.1: The result directly follows the analytical expressions. First, from (5.3) and (5.4), we have:

$$\Pi_{B^*}^{\alpha} = (p-\hat{c}_0)m_0 - s_0\left\{(\hat{c}_0 - v)\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right) + (p-v)\Psi\left[\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right)\right]\right\}$$

$$+\ (w-\hat{c}_0)\left[\mu_0+\sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right],$$

$$\Pi_{F*}^{\alpha} = (p-w)\mu_0-\sigma_0\left\{(w-v)\Phi^{-1}\left(\frac{p-w}{p-v}\right)+(p-v)\Psi\left[\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right]\right\}\ .$$

Using the result of Lemma 5.1 and employing $\Delta(\cdot)$, we have: $\Pi_{F*}^{\alpha} = (p-w)\mu_0-\sigma_0\Delta(w)$ and

$$\Pi_{B*}^{\alpha} = (p-\hat{c}_0)m_0-s_0\Delta(\hat{c}_0)+\ (w-\hat{c}_0)\left[\mu_0+\sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right]. \qquad \text{(Q.E.D.)}$$

Proof of Corollary 5.2: First, it is straightforward to note that $\Pi_{F*}^{\gamma} = \Pi_{F*}^{\alpha}$. Second, from (5.12), we have: $\Pi_{B*}^{\gamma} = (p-c_1)m_0-s_1\Delta(c_1)\ +\ (w-c_0)q_{F*}^{\gamma}$. From (5.9), we have $q_{F*}^{\gamma} = \mu_0+\sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)$. Putting it into Π_{B*}^{γ} completes the proof. (Q.E.D.)

Proof of Corollary 5.3. Under Scenario β, we have: $\Pi_{B*}^{\beta} = (p-\hat{c}_0)m_0-s_0\Delta(\hat{c}_0)+\lambda\overline{\Pi}_{F*}^{\beta}$, $\overline{\Pi}_{F*}^{\beta} = (p-\hat{c}_0)\mu_0-\sigma_0\Delta(\hat{c}_0)$, and $\Pi_{F*}^{\beta} = (1-\lambda)\overline{\Pi}_{F*}^{\beta}$. Simplifying the expressions yields the result.

(Q.E.D.)

Proof of Corollary 5.4. Similar to Corollary 5.3. (Q.E.D.)

Proof of Proposition 5.1:

The finding is based on a direct comparison between the expected profits earned by the fashion brand under Scenario α and Scenario γ. From Corollary 5.1, we have:

$\Pi_{B*}^{\alpha} = (p-\hat{c}_0)m_0-s_0\Delta(\hat{c}_0)+\ (w-\hat{c}_0)\left[\mu_0+\sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right]$. From Corollary 5.2, we have:

$\Pi^{\gamma}_{B*} = (p-c_1)m_0 - s_1\Delta(c_1) + (w-c_0)\left[\mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right]$. It is straightforward to see that:

$\Pi^{\alpha}_{B*} > \Pi^{\gamma}_{B*}$ if and only if $(c_1-\hat{c}_0)m_0 + (c_0-\hat{c}_0)\left\{\mu_0 + \sigma_0\Phi^{-1}\left(\frac{p-w}{p-v}\right)\right\} \geq s_0\Delta(\hat{c}_0) - s_1\Delta(c_1)$;

otherwise, $\Pi^{\alpha}_{B*} \leq \Pi^{\gamma}_{B*}$. (Q.E.D.)

Proofs of Propositions 5.2 – 5.4: Using the results from Corollaries 5.2-5.4 and following the similar approach in the proof of Proposition 5.1, we can derive them. (Q.E.D.)

Proof of Proposition 5.5: Notice that under the WP contract, the expected profits of the franchisee under Scenario α (OC1) and Scenario γ (OC2) are the same. Thus, in this case, the franchisee is indifferent between Scenario α (OC1) and Scenario γ (OC2). (Q.E.D.)

Proof of Proposition 5.6: From Corollary 5.3 and Corollary 5.4, we have:

$\Pi^{\beta}_{F*} = (1-\lambda)\{(p-\hat{c}_0)\mu_0 - \sigma_0\Delta(\hat{c}_0)\}$ and $\Pi^{\xi}_{F*} = (1-\lambda)[(p-c_0)\mu_0 - \sigma_0\Delta(c_0)]$.

First of all, we have: $\hat{c}_0 < c_0$. Then, notice that in this paper, as we consider the situation when the inventory service level $(p-w)/(p-v) > 0.5$ (Section 5.1), we have: $\left(\frac{p-\hat{c}_0}{p-v}\right) > \left(\frac{p-c_0}{p-v}\right) > 0.5$. As a consequence, the following is true:

$$\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right) > \Phi^{-1}\left(\frac{p-c_0}{p-v}\right) > 0$$

$$\Rightarrow \phi\left[\Phi^{-1}\left(\frac{p-\hat{c}_0}{p-v}\right)\right] < \phi\left[\Phi^{-1}\left(\frac{p-c_0}{p-v}\right)\right]$$

$$\Rightarrow \Delta(\hat{c}_0) < \Delta(c_0).$$

Combining $\hat{c}_0 < c_0$ and $\Delta(\hat{c}_0) < \Delta(c_0)$, we have: $\Pi^{\beta}_{F*} > \Pi^{\xi}_{F*}$, which means that in the presence of the PS contract, in deciding the optimal ordering case, the franchisee will always prefer Scenario β (OC1) to Scenario ξ (OC2). (Q.E.D.)

Proofs of Proposition 5.7 and Proposition 5.8: We can derive them by directly checking and comparing the franchisee's expected profit functions under the respective cases. (Q.E.D.)

Proof of Proposition 6.1:

From Table 5.1, we have: From the fashion brand's perspective, $\alpha \succ \gamma$ if and only if

$$(c_1 - \hat{c}_0)m_0 + (c_0 - \hat{c}_0)Q(w) \geq s_0\Delta(\hat{c}_0) - s_1\Delta(c_1). \tag{A3}$$

From Table 6.2, we have: From the supply chain's perspective, $\alpha \succ \gamma$ if and only if

$$(c_1 - \hat{c}_0)m_0 + (w - c_0)\mu_0 + s_1\Delta(c_1) - s_0\Delta(\hat{c}_0) + \sigma_0(c_0 - \hat{c}_0)\Omega(w) \geq 0. \tag{A4}$$

Since (A3) $\Rightarrow$ (A4), we have Proposition 6.1 which states that comparing between Scenario α and Scenario γ, when it is optimal for the fashion brand to choose Scenario l, for $l \in (\alpha, \gamma)$, it is also the optimal scenario for the supply chain. Under the PS contract, since for the franchisee, it always prefers Scenario β to Scenario ξ, we thus know that if it is optimal for the fashion brand to prefer Scenario β to Scenario ξ (i.e. $\beta \succ \xi$), the supply chain will also prefer Scenario β to Scenario ξ because the supply chain's expected profit is the sum of expected profits of the fashion brand and the franchisee.

(Q.E.D.)

Proof of Proposition 6.2: Directly from Table 6.4 and following the same approach as in the proof of Proposition 6.1, we can derive Proposition 6.2. (Q.E.D.)

Highlights:

1. Explore online-offline franchising without channel conflicts.
2. Consider tradeoffs between postponement and ordering cost advantage.
3. Compare different franchising contracts.
4. Study different scenarios and find the solution.
5. Derive conditions for Pareto improvement.